Apuntes de clase Miércoles 13/03/2019

Módulo

Definición del valor absoluto de un numero real

$$| -7 |= 7$$

$$| 3 |= 3$$

$$| a |= \begin{cases} a & si\, a > 0 \\ -a & si\, a < 0 \end{cases}$$

Propiedades del valor absoluto

Propiedad 1

$$| a |= k \quad con\, k > 0 \iff a = \pm k \tag{1}$$

Ej. 1

$$| a |= 2 \quad ecuación$$

$$\begin{array}{rcl} Si\, a \geq 0 & \vee & Si\, a < 0 \\ | a |= 2 & & | a |= 2 \\ a = 2 & & -a = 2 \\ & & a = -2 \end{array}$$

Ej. 2

$$
\begin{array}{rcl}
Si\, 2x+3 \geq 0 & \vee & Si\, 2x+3 < 0 \\
x \geq \dfrac{-3}{2} & & x < \dfrac{-3}{2} \\
\left[\dfrac{-3}{2}; +\infty\right) & & \left(-\infty; \dfrac{-3}{2}\right] \\
\mid 2x+3 \mid = 7 & & \mid 2x+3 \mid = 7 \\
2x+3 = 7 & & -(2x+3) = 7 \\
2x = 7-3 & & -2x-3 = 7 \\
x = \dfrac{4}{2} & & -2x = 10 \\
\boxed{x = 2} & & x = \dfrac{-10}{2} \\
\in \left[\dfrac{-3}{2}; +\infty\right) & & \boxed{x = -5} \\
 & & \in \left(+\infty; \dfrac{-3}{2}\right]
\end{array}
$$

$$\boxed{S = S_1 \cup S_2 = \{2; -5\}}$$

Ej. 3

$$\mid x \mid = k \quad con \;\; k > 0$$

$$
\begin{array}{rcl}
si\, x \geq 0 & \vee & si\, x < 0 \\
\mid x \mid = k & & \mid x \mid = k \\
x = k & & -x = k \\
 & & x = -k
\end{array}
$$

$$S = S_1 \cup S_2 = \{+k; -k\}$$

$$\boxed{x = \pm k}$$

Usando la propiedad

$$\mid 3x-2 \mid = 4$$

$$\boxed{3x-2 = \pm 4}$$

$$
\begin{array}{ll}
3x-2 = 4 & 3x-2 = -4 \\
3x = 4+2 & 3x = -4+2 \\
x = \dfrac{6}{3} & 3x = -2 \\
x = 2 & x = \dfrac{-2}{3}
\end{array}
$$

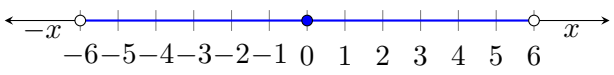
Propiedad 2

$$| x |< k \quad con\, k > 0 \Rightarrow -k < x < k \tag{2}$$

Ej. 1

$$| x |< 6$$

$Si\, x \geq 0$	$Si\, x < 0$
$\| x \|< 6$	$\| x \|< 6$
$x < 6$	$-x < 6$
$\boxed{S_1 = [0; 6)}$	$x > -6$
	$\boxed{S_2 = (-6; 0)}$



$$-6 < x < 6$$

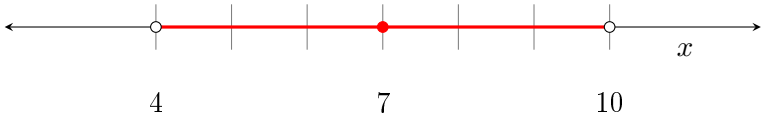
Entorno

Se llama Entorno de centro a y radio r y se escribe $E(a; r)$ al conjunto de números reales que verifican:

$$E(a; r) = \{x \in \mathbb{R} / \mid x - a \mid < r\} \quad con\, a = centro; r = radio$$

Ej.

$$E(7, 3)$$



$$E(7; 3) = (4; 10)$$
$$E(7; 3) = \{x \in \mathbb{R} / \mid x - 7 \mid < 3\}$$
$$| x - 7 |< 3$$
$$\boxed{-3 < x - 7 < 3}$$
$$-3 + 7 < x < 3 + 7$$
$$\boxed{4 < x < 10}$$
$$(4; 10)$$

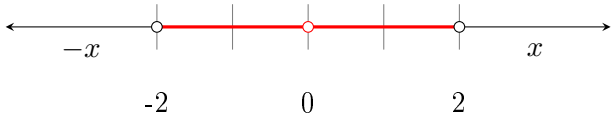
Entorno Reducido

Se llama Entorno Reducido de centro a y radio r y se escribe $E^*(a;r)$ al conjunto de números reales que verifican:

$$E^*(a;r) = \{x \in \mathbb{R} / 0 <\mid x - a \mid < r\}$$

Ej.

$$E^*(0;2) = \{x \in \mathbb{R} / 0 <\mid x - 0 \mid < 2\}$$
$$(-2;0) \cup (0;2)$$



Concepto de Límite

$$f(x) = x + 3$$

$$\lim_{x \to 4} f(x) \Rightarrow \lim_{x \to 4} (x + 3)$$

$x\, tiende\, a,\ x\, se\, acerca\, a$

x	$f(x)$	
3.99	6.99	$L_1 = \lim_{x \to 4^-} (x + 3) = 7$
3.999	6.999	
3.9999	6.9999	
4	7	
4.0001	7.0001	$L_2 = \lim_{x \to 4^+} (x + 3) = 7$
4.001	7.001	
4.01	7.01	

Existen límites laterales y son iguales.

$$L_1 = L_2 \Rightarrow \exists \lim_{x \to 4} (x + 3) = 7$$