速算方法介绍

1. 平方差法
2. 和十速算法
3. 凑十速算法
   (a) 逢五凑十法
   (b) 大数凑十法
   (c) 双向凑十法
4. 因数分解法
5. 二项式法

平方差法

平方差公式：$a^2 - b^2 = (a+b)(a-b)$

或：$(a+b)(a-b) = a^2 - b^2$

或：$(a-b)(a+b) = a^2 - b^2$

或：$a^2 = (a-b)(a+b) + b^2$

例题：

1. $28 \times 32 = (30-2)(30+2) = 30^2 - 2^2 = 900 - 4 = 896$
2. $39 \times 41 = (40-1)(40+1) = 40^2 - 1^2 = 1600 - 1 = 1599$
3. $45 \times 55 = (50-5)(50+5) = 50^2 - 5^2 = 2500 - 25 = 2475$
4. $62 \times 78 = (70-8)(70+8) = 70^2 - 8^2 = 4900 - 64 = 4836$
5. $63 \times 87 = (75-12)(75+12) = 75^2 - 12^2 = 5625 - 144 = 5481$
6. $45 \times 45 = 45^2 - 5^2 + 5^2 = (45+5)(45-5) + 25 = 50 \times 40 + 25 = 2000 + 25 = 2025$

总结利用平方差发的数字特点，每人出 2 道类似的题目

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总结利用平方差发的数字特点，每人出 2 道类似的题目

平均数为整十或整五，可以方便使用平方差法。

习题

1. $15 \times 25 =$

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2. $25 \times 35 = (30-5)(30+5) = 30^2 - 5^2 = 900 - 25 = 875$
3. $24 \times 36 = (30-6)(30+6) = 30^2 - 6^2 = 900 - 36 = 864$
4. $35 \times 45 =$

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5. $15 \times 35 = (25-10)(25+10) = 25^2 - 10^2 = 625 - 100 = 525$
6. $24 \times 46 =$

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6. $24 \times 46 = (35-11)(35+11) = 35^2 - 11^2 = 1225 - 121 = 1104$
7. $75 \times 75 =$

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7. $75 \times 75 = 70 \times 80 + 5 \times 5 = 5600 + 25 = 5625$
8. $85 \times 85 =$

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7. $75 \times 75 = 70 \times 80 + 5 \times 5 = 5600 + 25 = 5625$
8. $85 \times 85 = 80 \times 90 + 5 \times 5 = 7200 + 25 = 7225$
9. $95 \times 95 =$

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8. $85 \times 85 = 80 \times 90 + 5 \times 5 = 7200 + 25 = 7225$
9. $95 \times 95 = 90 \times 100 + 5 \times 5 = 9000 + 25 = 9025$

思考：还有没有其它速算方法可以计算上述代数式?

和十速算法

计算：$\overline{ab} \times \overline{ad}$

当个位数之和等于 10，即：$b + d = 10$ 时，可以使用《和十速算法》，即：

设：$e = a + 1$，$a(a + 1) = \overline{AB}$，$bd = \overline{CD}$ 则：

$$\overline{ab} \times \overline{ad} = (10a + b)(10a + d) = 100a^2 + 10a(b + d) + bd = 100a(a + 1) + bd = \overline{ABCD}$$

例题：

1. $21 \times 29 = 100 \times 2 \times 3 + 1 \times 9 = 600 + 9 = 609$
2. $32 \times 38 = 100 \times 3 \times 4 + 2 \times 8 = 1200 + 16 = 1216$
3. $43 \times 47 = 100 \times 4 \times 5 + 3 \times 7 = 2000 + 21 = 2021$
4. $45 \times 45 = 100 \times 4 \times 5 + 5 \times 5 = 2000 + 25 = 2025$
5. $74 \times 76 = 100 \times 7 \times 8 + 4 \times 6 = 5600 + 24 = 5624$

思考：和十速算法速利用了乘法的什么规律？数字有什么特点？

每人出 2 道类似的题目

和十速算法

1. 21×29

和十速算法

1. 21×29

   $\because 2 \times 3 = 6, \quad 1 \times 9 = 9, \quad \therefore 21 \times 29 = \overline{06} \quad \overline{09} = 609$

2. 32×38

和十速算法

1. 21×29

   $\because 2 \times 3 = 6, \quad 1 \times 9 = 9, \quad \therefore 21 \times 29 = \overline{06} \quad \overline{09} = 609$

2. 32×38

   $\because 3 \times 4 = 12, \quad 2 \times 8 = 16, \quad \therefore 32 \times 38 = \overline{12} \quad \overline{16} = 1216$

3. $43 \times 47 =$

和十速算法

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   $\because 3 \times 4 = 12, \quad 2 \times 8 = 16, \quad \therefore 32 \times 38 = \overline{12} \quad \overline{16} = 1216$

3. $43 \times 47 =$

   $\because 4 \times 5 = 20, \quad 3 \times 7 = 21, \quad \therefore 43 \times 47 = \overline{20} \quad \overline{21} = 2021$

4. 45×45

和十速算法

1. 21×29

   $\because 2 \times 3 = 6, \quad 1 \times 9 = 9, \quad \therefore 21 \times 29 = \overline{06} \quad \overline{09} = 609$

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   $\because 3 \times 4 = 12, \quad 2 \times 8 = 16, \quad \therefore 32 \times 38 = \overline{12} \quad \overline{16} = 1216$

3. $43 \times 47 =$

   $\because 4 \times 5 = 20, \quad 3 \times 7 = 21, \quad \therefore 43 \times 47 = \overline{20} \quad \overline{21} = 2021$

4. 45×45

   $\because 4 \times 5 = 20, \quad 5 \times 5 = 25, \quad \therefore 45 \times 45 = \overline{20} \quad \overline{25} = 2025$

5. 74×76

和十速算法

1. 21×29

   $\because 2 \times 3 = 6, \quad 1 \times 9 = 9, \quad \therefore 21 \times 29 = \overline{06} \quad \overline{09} = 609$

2. 32×38

   $\because 3 \times 4 = 12, \quad 2 \times 8 = 16, \quad \therefore 32 \times 38 = \overline{12} \quad \overline{16} = 1216$

3. $43 \times 47 =$

   $\because 4 \times 5 = 20, \quad 3 \times 7 = 21, \quad \therefore 43 \times 47 = \overline{20} \quad \overline{21} = 2021$

4. 45×45

   $\because 4 \times 5 = 20, \quad 5 \times 5 = 25, \quad \therefore 45 \times 45 = \overline{20} \quad \overline{25} = 2025$

5. 74×76

   $\because 7 \times 8 = 56, \quad 4 \times 6 = 24, \quad \therefore 74 \times 76 = \overline{56} \quad \overline{24} = 5624$

逢五凑十法

计算：$\overline{ab} \times \overline{cd}$

当 $b = 5$，d 为偶数时，通常可以使用《逢五凑十法》，即：

对 $\overline{cd}$ 先除以 2，再乘以 2，即：$\overline{cd} = \overline{ef} \times 2$，则：

$\overline{ab} \times \overline{cd} = \overline{ab} \times 2 \times \overline{ef}$ 例题：

1. $15 \times 18 = 15 \times 2 \times 9 = 30 \times 9 = 270$
2. $25 \times 34 = 25 \times 2 \times 17 = 50 \times 17 = 850$
3. $35 \times 16 = 35 \times 2 \times 8 = 70 \times 8 = 560$
4. $75 \times 14 = 75 \times 2 \times 7 = 150 \times 7 = 1050$

思考：逢五凑十法本质就是乘五除二，利用了乘法的什么规律?

总结逢五凑十法的数字特点，每人出 2 道类似的题目

习题

计算下列各式的值：

1. 23×44

习题

计算下列各式的值：

1. $23 \times 44 = (25 - 2) \times 44 = 25 \times 44 - 2 \times 44 = 1100 - 88 = 1012$
2. 22×36

习题

计算下列各式的值：

1. $23 \times 44 = (25 - 2) \times 44 = 25 \times 44 - 2 \times 44 = 1100 - 88 = 1012$
2. $22 \times 36 = 22 \times (35 + 1) = 22 \times 35 + 22 \times 1 = 770 + 22 = 792$
3. 22×37

习题

计算下列各式的值：

1. $23 \times 44 = (25 - 2) \times 44 = 25 \times 44 - 2 \times 44 = 1100 - 88 = 1012$
2. $22 \times 36 = 22 \times (35 + 1) = 22 \times 35 + 22 \times 1 = 770 + 22 = 792$
3. $22 \times 37 = 22 \times (35 + 2) = 22 \times 35 + 22 \times 2 = 770 + 44 = 814$
4. 24×28

习题

计算下列各式的值：

1. $23 \times 44 = (25 - 2) \times 44 = 25 \times 44 - 2 \times 44 = 1100 - 88 = 1012$
2. $22 \times 36 = 22 \times (35 + 1) = 22 \times 35 + 22 \times 1 = 770 + 22 = 792$
3. $22 \times 37 = 22 \times (35 + 2) = 22 \times 35 + 22 \times 2 = 770 + 44 = 814$
4. $24 \times 28 = (25 - 1) \times 28 = 25 \times 4 \times 7 - 28 = 700 - 28 = 672$

大数凑十法

计算：$\overline{ab} \times \overline{cd}$

当尾数（或个位数）$b \geq 6$ 时，通常可以使用《大数凑十法》，即：

设：$e = a + 1, f = 10 - b$，则：

$\overline{ab} \times \overline{cd} = \overline{e0} \times \overline{cd} - \overline{f} \times \overline{cd}$ 例题：

1. $32 \times 39 = 32 \times (40 - 1) = 32 \times 40 - 32 = 1280 - 32 = 1248$
2. $52 \times 39 = 52 \times (40 - 1) = 52 \times 40 - 52 = 2080 - 52 = 2028$
3. $48 \times 43 = (50 - 2) \times 43 = 50 \times 43 - 2 \times 43 = 2150 - 86 = 2064$
4. $37 \times 37 = (40 - 3)(40 - 3) = 40^2 - 2 \times 40 \times 3 + 3^2 = 1600 - 240 + 9 = 1369$

思考：大数凑十法速利用了乘法的什么规律？数字有什么特点？

每人出 2 道类似的题目

32×39 还可以用什么方法速算？

48×43 还可以用什么方法速算？

习题

计算下列各式的值：

1. 15×49

习题

计算下列各式的值：

1. $15 \times 49 = 15 \times (50 - 1) = 15 \times 50 - 15 = 750 - 15 = 735$
2. 23×29

习题

计算下列各式的值：

1. $15 \times 49 = 15 \times (50 - 1) = 15 \times 50 - 15 = 750 - 15 = 735$
2. $23 \times 29 = 23 \times (30 - 1) = 23 \times 30 - 23 = 690 - 23 = 667$
3. 24×28

习题

计算下列各式的值：

1. $15 \times 49 = 15 \times (50 - 1) = 15 \times 50 - 15 = 750 - 15 = 735$
2. $23 \times 29 = 23 \times (30 - 1) = 23 \times 30 - 23 = 690 - 23 = 667$
3. $24 \times 28 = 24 \times (30 - 2) = 24 \times 30 - 24 \times 2 = 720 - 48 = 672$
4. 32×57

习题

计算下列各式的值：

1. $15 \times 49 = 15 \times (50 - 1) = 15 \times 50 - 15 = 750 - 15 = 735$
2. $23 \times 29 = 23 \times (30 - 1) = 23 \times 30 - 23 = 690 - 23 = 667$
3. $24 \times 28 = 24 \times (30 - 2) = 24 \times 30 - 24 \times 2 = 720 - 48 = 672$
4. $32 \times 57 = 32 \times (60 - 3) = 32 \times 60 - 32 \times 3 = 1920 - 96 = 1824$

双向凑十法

计算：$\overline{a9} \times \overline{c9}$

当两个尾数都等于 9 时，通常可以使用《双向凑十法》，即：

设：$e = a + 1, f = d + 1$，则：

$\overline{a9} \times \overline{c9} = (\overline{e0} - 1) \times (\overline{f0} - 1) = \overline{e0} \times \overline{f0} - \overline{e0} - \overline{f0} + 1$ 例题：

1. $29 \times 39 = (30 - 1) \times (40 - 1) = 30 \times 40 - 30 - 40 + 1 = 1200 - 70 + 1 = 1131$
2. $19 \times 59 = (20 - 1) \times (60 - 1) = 20 \times 60 - 20 - 60 + 1 = 1200 - 80 + 1 = 1121$
3. $29 \times 69 = (30 - 1) \times (70 - 1) = 30 \times 70 - 30 - 70 + 1 = 2100 - 100 + 1 = 2001$

思考：双向凑十法速利用了乘法的什么规律?

每人出 2 道类似的题目

因数分解法

例题：

1. $15 \times 28 = 15 \times 4 \times 7 = 60 \times 7 = 420$
2. $25 \times 36 = 25 \times 4 \times 9 = 100 \times 9 = 900$
3. $33 \times 12 = 3 \times 11 \times 12 = 3 \times 121 = 363$
4. $74 \times 27 = 2 \times 37 \times 27 = 2 \times 999 = 1998$
5. $91 \times 22 = 91 \times 11 \times 2 = 1001 \times 2 = 2002$

总结利用因数分解法的数字特点，每人出 2 道类似的题目

习题

1. $125 \times 72 =$

习题

1. $125 \times 72 = 125 \times 8 \times 9 = 1000 \times 9 = 9000$
2. $14 \times 28 =$

习题

1. $125 \times 72 = 125 \times 8 \times 9 = 1000 \times 9 = 9000$
2. $14 \times 28 = 7 \times 2 \times 7 \times 4 = 49 \times 8 = 400 - 8 = 392$

思考：逢五凑十法是不是因数分解法的特例?

二项式速算法

$(a+b)(a+c)=a^2+a(b+c)+bc$

1. $26\times 26=(25+1)(25+1)=25^2+2\times 25+1^2=625+50+1=676$
2. $27\times 27=(25+2)(25+2)=25^2+2\times 25\times 2+2^2=625+100+4=729$
3. $36\times 36=(35+1)(35+1)=35^2+2\times 35+1^2=1225+70+1=1296$
4. $37\times 37=(35+2)(35+2)=35^2+2\times 35\times 2+2^2=1225+140+4=1369$
5. $37\times 38=(40-3)(40-2)=40^2-40\times(3+2)+3\times 2=1600-200+6=1406$
6. $27\times 38=(30-3)(40-2)=30\times 40-2\times 30-3\times 40+3^2=1200-180+6=1026$
7. $27\times 48=(30-3)(50-2)=30\times 50-30\times 2-3\times 50+3\times 2=1500-60-150+6=1296$

总结利用二项式速算法的数字特点，每人出 2 道类似的题目