

速算方法介绍

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平方差公式的证明

平方差公式是： $a^2 - b^2 = (a + b)(a - b)$ ，如何证明呢？

思考：如果从左向右证明一下找不到思路，那么可以先从右边向左边验证：
证明：

$$\begin{aligned}(a + b)(a - b) &= a(a - b) + b(a - b) \\ &= a^2 - ab + ba - b^2 \\ &= a^2 - ab + ab - b^2 \\ &= a^2 - b^2\end{aligned}$$

有了上述的验证过程，从下往上写，就找到构造公式，并予以证明。
证明：

$$\begin{aligned}a^2 - b^2 &= a^2 - ab + ab - b^2 \\ &= a^2 - ab + ba - b^2 \\ &= a(a - b) + b(a - b) \\ &= (a + b)(a - b)\end{aligned}$$

平方差法

平方差公式： $a^2 - b^2 = (a + b)(a - b)$

或： $(a + b)(a - b) = a^2 - b^2$

或： $(a - b)(a + b) = a^2 - b^2$

或反向平方差： $a^2 = (a - b)(a + b) + b^2$

例题：

$$1. 28 \times 32 = (30 - 2)(30 + 2) = 30^2 - 2^2 = 900 - 4 = 896$$

$$2. 39 \times 41 = (40 - 1)(40 + 1) = 40^2 - 1^2 = 1600 - 1 = 1599$$

$$3. 45 \times 55 = (50 - 5)(50 + 5) = 50^2 - 5^2 = 2500 - 25 = 2475$$

$$4. 62 \times 78 = (70 - 8)(70 + 8) = 70^2 - 8^2 = 4900 - 64 = 4836$$

$$5. 63 \times 87 = (75 - 12)(75 + 12) = 75^2 - 12^2 = 5625 - 144 = 5481$$

$$6. 45 \times 45 = 45^2 - 5^2 + 5^2 = (45 + 5)(45 - 5) + 25 = 50 \times 40 + 25 = 2000 + 25 = 2025$$

总结利用平方差法的数字特点，每人出 2 道类似的题目

习题

$$1. 15 \times 25 = (20 - 5)(20 + 5) = 20^2 - 5^2 = 400 - 25 = 375$$

$$2. 25 \times 35 = (30 - 5)(30 + 5) = 30^2 - 5^2 = 900 - 25 = 875$$

$$3. 24 \times 36 = (30 - 6)(30 + 6) = 30^2 - 6^2 = 900 - 36 = 864$$

$$4. 35 \times 45 = (40 - 5)(40 + 5) = 40^2 - 5^2 = 1600 - 25 = 1575$$

$$5. 15 \times 35 = (25 - 10)(25 + 10) = 25^2 - 10^2 = 625 - 100 = 525$$

$$6. 24 \times 46 = (35 - 11)(35 + 11) = 35^2 - 11^2 = 1225 - 121 = 1104$$

$$7. 75 \times 75 = 70 \times 80 + 5 \times 5 = 5600 + 25 = 5625$$

$$8. 85 \times 85 = 80 \times 90 + 5 \times 5 = 7200 + 25 = 7225$$

$$9. 95 \times 95 = 90 \times 100 + 5 \times 5 = 9000 + 25 = 9025$$

思考：还有没有其它速算方法可以计算上述代数式？

和十速算法

计算： $\overline{ab} \times \overline{ad}$

当个位数之和等于 10，即： $b + d = 10$ 时，可以使用《和十速算法》，即：

设： $e = a + 1$ ， $a(a + 1) = \overline{AB}$ ， $bd = \overline{CD}$ 则：

$$\overline{ab} \times \overline{ad} = (10a + b)(10a + d) = 100a^2 + 10a(b + d) + bd = 100a(a + 1) + bd = \overline{ABCD}$$

例题：

1. $21 \times 29 = 100 \times 2 \times 3 + 1 \times 9 = 600 + 9 = 609$

2. $32 \times 38 = 100 \times 3 \times 4 + 2 \times 8 = 1200 + 16 = 1216$

3. $43 \times 47 = 100 \times 4 \times 5 + 3 \times 7 = 2000 + 21 = 2021$

4. $45 \times 45 = 100 \times 4 \times 5 + 5 \times 5 = 2000 + 25 = 2025$

5. $74 \times 76 = 100 \times 7 \times 8 + 4 \times 6 = 5600 + 24 = 5624$

思考：和十速算法速利用了乘法的什么规律？数字有什么特点？

每人出 2 道类似的题目

和十速算法

1. 21×29

$$\because 2 \times 3 = 6, \quad 1 \times 9 = 9, \quad \therefore 21 \times 29 = \overline{06} \quad \overline{09} = 609$$

2. 32×38

$$\because 3 \times 4 = 12, \quad 2 \times 8 = 16, \quad \therefore 32 \times 38 = \overline{12} \quad \overline{16} = 1216$$

3. $43 \times 47 =$

$$\because 4 \times 5 = 20, \quad 3 \times 7 = 21, \quad \therefore 43 \times 47 = \overline{20} \quad \overline{21} = 2021$$

4. 45×45

$$\because 4 \times 5 = 20, \quad 5 \times 5 = 25, \quad \therefore 45 \times 45 = \overline{20} \quad \overline{25} = 2025$$

5. 74×76

逢五凑十法

计算： $\overline{ab} \times \overline{cd}$

当 $b = 5$ ， d 为偶数时，通常可以使用《逢五凑十法》，即：

对 \overline{cd} 先除以 2，再乘以 2，即： $\overline{cd} = \overline{ef} \times 2$ ，则：

$$\overline{ab} \times \overline{cd} = \overline{ab} \times \overline{cd} \div 2 \times 2 = (\overline{ab} \times 2) \times \overline{ef}$$

例题：

1. $15 \times 18 = 15 \times 2 \times 9 = 30 \times 9 = 270$

2. $25 \times 34 = 25 \times 2 \times 17 = 50 \times 17 = 850$

3. $35 \times 16 = 35 \times 2 \times 8 = 70 \times 8 = 560$

4. $75 \times 14 = 75 \times 2 \times 7 = 150 \times 7 = 1050$

思考：逢五凑十法本质就是乘五等于乘十除二，利用了乘法的什么规律？

总结逢五凑十法的数字特点，每人出 2 道类似的题目

习题

计算下列各式的值：

$$1. 25 \times 48 = 25 \times 4 \times 12 = 100 \times 12 = 1200$$

$$2. 35 \times 28 = 35 \times 2 \times 14 = 70 \times 14 = 980$$

$$3. 45 \times 16 = 45 \times 2 \times 8 = 90 \times 8 = 720$$

$$4. 55 \times 18 = 55 \times 2 \times 9 = 110 \times 9 = 990$$

$$5. 25 \times 36 = 25 \times 4 \times 9 = 100 \times 9 = 900$$

$$6. 23 \times 44 = (25 - 2) \times 44 = 25 \times 44 - 2 \times 44 = 1100 - 88 = 1012$$

$$7. 22 \times 36 = 22 \times (35 + 1) = 22 \times 35 + 22 \times 1 = 770 + 22 = 792$$

$$8. 22 \times 37 = 22 \times (35 + 2) = 22 \times 35 + 22 \times 2 = 770 + 44 = 814$$

$$9. 24 \times 28 = (25 - 1) \times 28 = 25 \times 4 \times 7 - 28 = 700 - 28 = 672$$

大数凑十法

计算： $\overline{ab} \times \overline{cd}$

当尾数（或个位数） $b \geq 6$ 时，通常可以使用《大数凑十法》，即：

设： $e = a + 1, f = 10 - b$ ，则：

$$\overline{ab} \times \overline{cd} = \overline{e0} \times \overline{cd} - \overline{f} \times \overline{cd}$$

例题：

$$1. 22 \times 37 = 22 \times (40 - 3) = 22 \times 40 - 22 \times 3 = 880 - 66 = 814$$

$$2. 22 \times 37 = 22 \times (35 + 2) = 22 \times 35 + 22 \times 2 = 770 + 44 = 814$$

$$3. 32 \times 39 = 32 \times (40 - 1) = 32 \times 40 - 32 = 1280 - 32 = 1248$$

$$4. 52 \times 39 = 52 \times (40 - 1) = 52 \times 40 - 52 = 2080 - 52 = 2028$$

$$5. 48 \times 43 = (50 - 2) \times 43 = 50 \times 43 - 2 \times 43 = 2150 - 86 = 2064$$

$$6. 37 \times 37 = (40 - 3)(40 - 3) = 40^2 - 2 \times 40 \times 3 + 3^2 = 1600 - 240 + 9 = 1369$$

思考：大数凑十法速利用了乘法的什么规律？数字有什么特点？

每人出 2 道类似的题目

习题

计算下列各式的值：

$$1. 15 \times 49 = 15 \times (50 - 1) = 15 \times 50 - 15 = 750 - 15 = 735$$

$$2. 23 \times 29 = 23 \times (30 - 1) = 23 \times 30 - 23 = 690 - 23 = 667$$

$$3. 24 \times 28 = 24 \times (30 - 2) = 24 \times 30 - 24 \times 2 = 720 - 48 = 672$$

$$4. 32 \times 57 = 32 \times (60 - 3) = 32 \times 60 - 32 \times 3 = 1920 - 96 = 1824$$

双向凑十法

计算： $\overline{a9} \times \overline{c9}$

当两个尾数都等于 9 时，通常可以使用《双向凑十法》，即：

设： $e = a + 1, f = d + 1$ ，则：

$$\overline{a9} \times \overline{c9} = (\overline{e0} - 1) \times (\overline{f0} - 1) = \overline{e0} \times \overline{f0} - \overline{e0} - \overline{f0} + 1$$

例题：

$$1. 29 \times 39 = (30 - 1) \times (40 - 1) = 30 \times 40 - 30 - 40 + 1 = 1200 - 70 + 1 = 1131$$

$$2. 19 \times 59 = (20 - 1) \times (60 - 1) = 20 \times 60 - 20 - 60 + 1 = 1200 - 80 + 1 = 1121$$

$$3. 29 \times 69 = (30 - 1) \times (70 - 1) = 30 \times 70 - 30 - 70 + 1 = 2100 - 100 + 1 = 2001$$

思考：双向凑十法速利用了乘法的什么规律？

每人出 2 道类似的题目

因数分解法

例题：

$$1. 15 \times 28 = 15 \times 4 \times 7 = 60 \times 7 = 420$$

$$2. 25 \times 36 = 25 \times 4 \times 9 = 100 \times 9 = 900$$

$$3. 33 \times 12 = 3 \times 11 \times 12 = 3 \times 121 = 363$$

$$4. 74 \times 27 = 2 \times 37 \times 27 = 2 \times 999 = 1998$$

$$5. 91 \times 22 = 91 \times 11 \times 2 = 1001 \times 2 = 2002$$

总结利用因数分解法的数字特点，每人出 2 道类似的题目

习题

1. $125 \times 72 = 125 \times 8 \times 9 = 1000 \times 9 = 9000$
2. $14 \times 28 = 7 \times 2 \times 7 \times 4 = 49 \times 8 = 400 - 8 = 392$

思考：逢五凑十法是不是因数分解法的特例？

二项式速算法

$$(a + b)(a + c) = a^2 + a(b + c) + bc$$

$$1. 26 \times 26 = (25 + 1)(25 + 1) = 25^2 + 2 \times 25 + 1^2 = 625 + 50 + 1 = 676$$

$$2. 27 \times 27 = (25 + 2)(25 + 2) = 25^2 + 2 \times 25 \times 2 + 2^2 = 625 + 100 + 4 = 729$$

$$3. 36 \times 36 = (35 + 1)(35 + 1) = 35^2 + 2 \times 35 + 1^2 = 1225 + 70 + 1 = 1296$$

$$4. 37 \times 37 = (35 + 2)(35 + 2) = 35^2 + 2 \times 35 \times 2 + 2^2 = 1225 + 140 + 4 = 1369$$

$$5. 37 \times 38 = (40 - 3)(40 - 2) = 40^2 - 40 \times (3 + 2) + 3 \times 2 = 1600 - 200 + 6 = 1406$$

$$6. 27 \times 38 = (30 - 3)(40 - 2) = 30 \times 40 - 2 \times 30 - 3 \times 40 + 3^2 = 1200 - 180 + 6 = 1026$$

$$7. 27 \times 48 = (30 - 3)(50 - 2) = 30 \times 50 - 30 \times 2 - 3 \times 50 + 3 \times 2 = 1500 - 60 - 150 + 6 = 1296$$

总结利用二项式速算法的数字特点，每人出 2 道类似的题目

速算习题

1. $19 \times 24 = (20 - 1) \times 24 = 480 - 24 = 456$
2. $16 \times 39 = 16 \times (40 - 1) = 640 - 16 = 624$
3. $17 \times 47 = 17 \times (50 - 3) = 850 - 51 = 799$
4. $24 \times 25 = 6 \times 4 \times 25 = 6 \times 100 = 600$
5. $27 \times 23 = 20 \times 30 + 7 \times 3 = 600 + 21 = 621$
6. $26 \times 29 = 26 \times (30 - 1) = 780 - 26 = 754$
7. $37 \times 43 = (40 - 3)(40 + 3) = 1600 - 9 = 1591$
8. $36 \times 44 = (40 - 4)(40 + 4) = 1600 - 16 = 1584$
9. $42 \times 25 = 21 \times 2 \times 25 = 21 \times 50 = 1050$

速算习题

1. $36 \times 24 = (30 + 6) \times (30 - 6) = 30^2 - 6^2 = 900 - 36 = 864$
2. $36 \times 24 = 36 \times (25 - 1) = 9 \times 4 \times 25 - 36 = 900 - 36 = 864$
3. $26 \times 48 = 26 \times (50 - 2) = 26 \times 50 - 52 = 1300 - 52 = 1248$
4. $26 \times 22 = (25 + 1) \times 22 = 25 \times 22 + 22 = 550 + 22 = 572$
5. $43 \times 27 = (23 + 20) \times 27 = 23 \times 27 + 20 \times 27 = 621 + 540 = 1161$
6. $33 \times 28 = 33 \times (30 - 2) = 33 \times 30 - 33 \times 2 = 990 - 66 = 924$
7. $48 \times 28 = (50 - 2) \times 28 = 50 \times 28 - 2 \times 28 = 1400 - 56 = 1344$
8. $44 \times 14 = (45 - 1) \times 14 = 45 \times 2 \times 7 - 14 = 90 \times 7 - 14 = 630 - 14 = 616$
9. $48 \times 45 = 48 \times (50 - 5) = 2400 - 240 = 2160$

速算习题

1. $38 \times 19 = 38 \times (20 - 1) = 38 \times 20 - 38 = 760 - 38 = 722$
2. $46 \times 27 = 46 \times (25 + 2) = 46 \times 25 + 46 \times 2 = 1150 + 92 = 1242$
3. $44 \times 23 = 44 \times (25 - 2) = 44 \times 25 - 44 \times 2 = 1100 - 88 = 1012$
4. $44 \times 39 = 44 \times (40 - 1) = 44 \times 40 - 44 = 1760 - 44 = 1716$
5. $36 \times 48 = 36 \times (50 - 2) = 36 \times 50 - 72 = 1800 - 72 = 1728$
6. $47 \times 48 = (50 - 3) \times 48 = 50 \times 48 - 3 \times 48 = 2400 - 144 = 2256$
7. $47 \times 48 = (50 - 3)(50 - 2) = 2500 - 50(3 + 2) + 6 = 2500 - 250 + 6 = 2256$
8. $47 \times 48 = (42 + 5) \times 48 = 42 \times 48 + 5 \times 48 = 2016 + 240 = 2256$
9. $37 \times 48 = 37 \times (50 - 2) = 37 \times 50 - 74 = 1850 - 74 = 1776$
10. $37 \times 48 = (40 - 3)(50 - 2) = 2000 - 80 - 150 + 6 = 2000 - 200 - 30 + 6 = 1800 - 30 + 6 = 1776$