Samples to Digital Symbols:
Symbol Clock Recovery and Improved Symbol Synchronization Blocks

Andy Walls
12 Sept 2017

Outline

SILVERBLOCK
SYSTEMS

- Advertisement
  - About SilverBlock
- Theoretical
  - Problem Statement
  - Symbol Synchronization Overview
  - PLL Symbol Synchronizer
  - Clock Tracking PLL Model
  - Timing Error Detector
  - Interpolating Resampler

Outline (continued)

SILVERBLOCK SYSTEMS

- Practical
  - GNURadio Sync Blocks
  - Existing Blocks' Deficiencies
  - New Symbol Sync Blocks Overview
  - New Symbol Sync Blocks Architecture
  - Adding a New Timing Error Detector
  - Adding a New Resampler
  - Using a Different Slicer
  - Existing Blocks to New Blocks
  - Usage Hints and Gotchas
  - Experimental Tuning Example
  - TED S-Curve Simulation

SilverBlock Systems

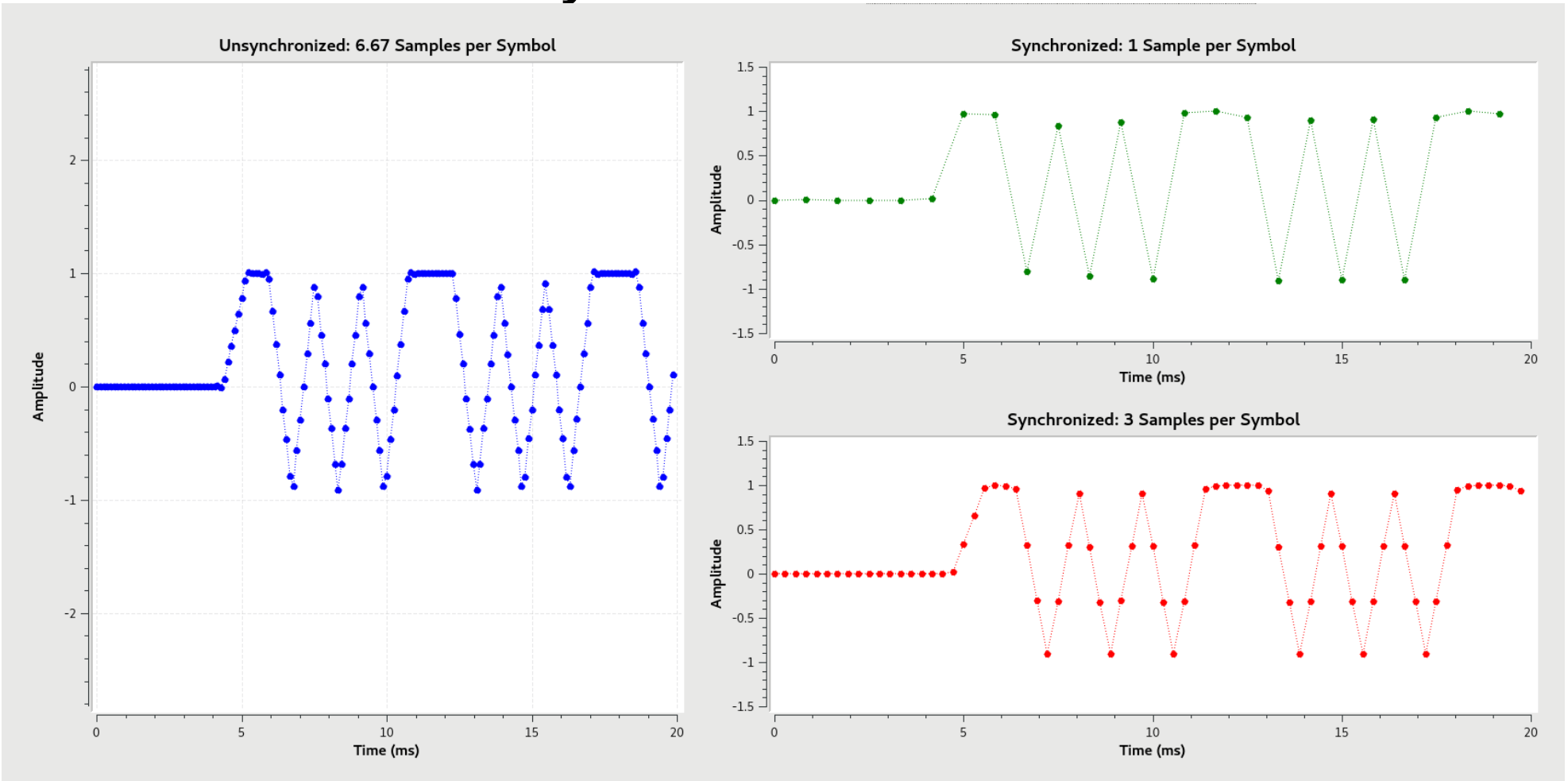
- Small company founded in 2009
  - Leonardtown, MD (0.3 miles from the waterfront)

    https://www.visitstmarysmd.com/see-do/towns-communities/leonardtown-area/
  - Rochester, NY
- What we build
  - Distributed Mission Computing and Display Software
  - RADAR Processing Algorithms and GPU Implementations
  - Experimental Communications Systems
  - Experimental Airborne Sensors and Systems
- Looking to hire 1 or 2 experienced software engineers
  - C++ required. US Citizenship required.
  - contact@silverblocksystems.net

Problem Statement

SILVERBLOCK SYSTEMS

- Output a sample stream synchronized with the center of data symbols



Symbol Synch Overview

SILVERBLOCK SYSTEMS

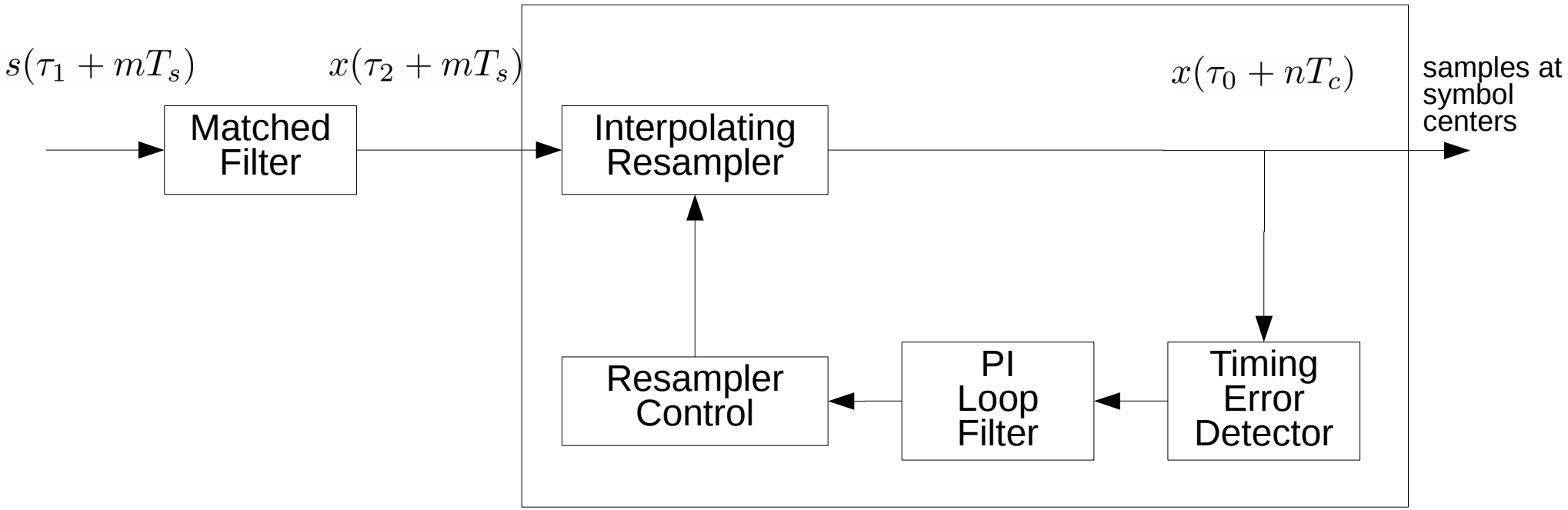
- Two broad categories of algorithms
  - Feedfoward (Open Loop)
    - Block oriented
    - Operate on samples for a number of symbols at a time
    - Non-tracking, but can be computationally complex
    - Burst mode communications, or initial acquisition of synchronization
  - Feedback (Closed Loop)
    - Stream oriented
    - Operate on immediate incoming sample or symbol
    - Tracking, and not computationally complex for any 1 input
    - Continual stream of symbols, or tracking after initial acquisition

Symbol Synch Overview

SILVERBLOCK SYSTEMS

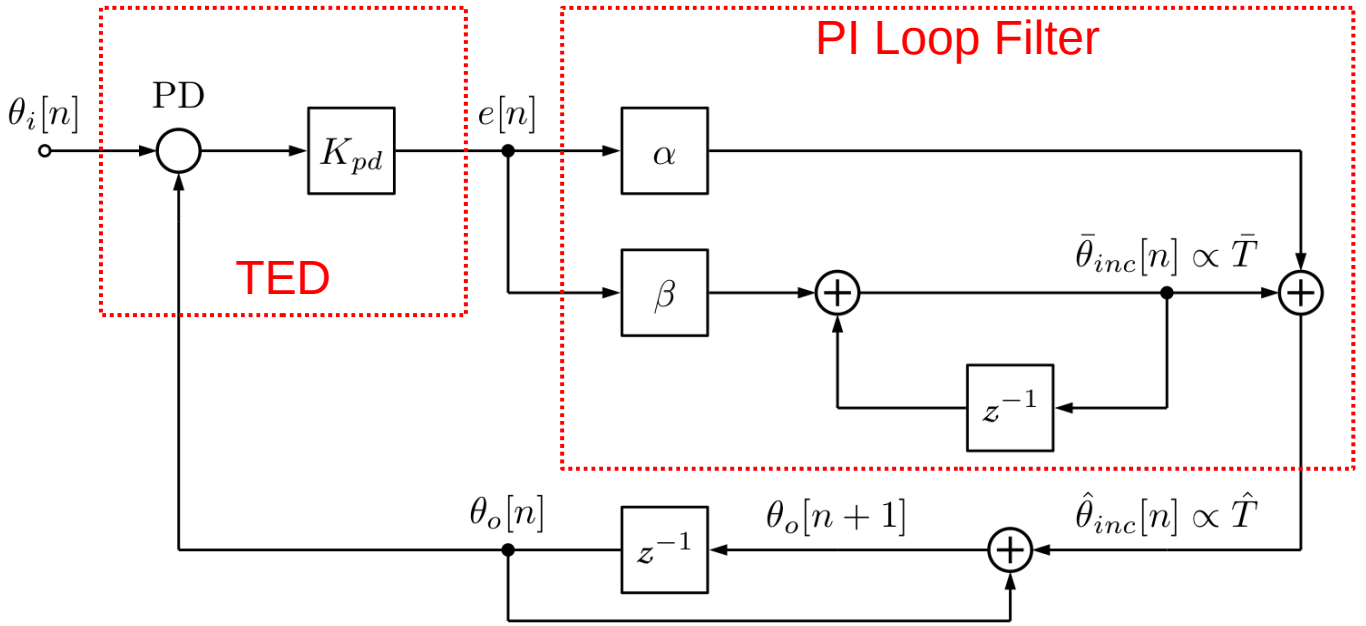
- Algorithm and Implementation Requirements
  - Resampling with interpolation
    - Imposes signal bandwidth requirements
  - Symbol clock timing estimation or timing error estimation
    - Imposes signal conditioning requirements
  - Offline modeling, simulation, and analysis by the designer !
- Correlation-based Feedforward Algorithm and Implementation
  - GNURadio has ~1 implementation
- PLL-based Feedback Algorithm and Implementation
  - GNURadio now has 4 implementations

PLL Symbol Synchronizer



- Timing Error Detector: analogous to PLL phase detector
  - *Estimates* the symbol clock timing error, a hidden quantity
- Resampler Control: analogous to PLL phase accumulator/NCO
- T_s input sample clock period
- T_c symbol clock period
- τ_0 optimal symbol sampling offset

Clock Tracking PLL Model



- Timing Error Detector (TED) modeled by phase detector (PD) and K_{pd} gain
  - Outputs symbol clock timing error estimate, e[n], once per symbol
- Proportional-Integral (PI) Loop Filter
  - Proportional arm gain, α
  - Integral arm gain, β
  - Integral arm output: estimate of average period of symbol clock, T_{avg}
  - Filter output: estimate of instantaneous period of symbol clock, T_{inst}
- Resampler & Control maps to phase accumulator and phase signals

Clock Tracking PLL Model

SILVERBLOCK SYSTEMS

- Loop Phase Transfer Function (PI gains)

$$H(z) = \frac{\Theta_o(z)}{\Theta_i(z)} = K_{pd}(\alpha+\beta)z^{-1} \cdot \frac{1 - \frac{\alpha}{\alpha+\beta}z^{-1}}{1 - 2\left(1 - K_{pd}\frac{\alpha+\beta}{2}\right)z^{-1} + (1 - K_{pd}\alpha)z^{-2}}$$

- Zeros, Poles, and Critical Damping (PI gains)

$$z_1 = \frac{\alpha}{\alpha+\beta}$$

$$z_2 \to \infty$$

$$p_{1,2} = \left(1 - K_{pd}\frac{\alpha+\beta}{2}\right) \pm \sqrt{\left(1 - K_{pd}\frac{\alpha+\beta}{2}\right)^2 - (1 - K_{pd}\alpha)}$$

$$\alpha = -\beta \pm \frac{2}{|K_{pd}|}\sqrt{K_{pd}\beta} \quad \text{for critical damping}$$

Clock Tracking PLL Model

SILVERBLOCK SYSTEMS

- Mapping Poles of a 2nd Order Analog Control System to z-plane

$$s_{1,2} = -\zeta\omega_n \pm j\omega_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

ζ : damping factor (1.0 is critically damped)

ω_n : natural frequency (radius from origin, approx 1-sided BW for underdamped system)

ω_d : damped frequency of oscillation (ringing of underdamped impulse response)

$$z = e^{sT}$$

$$p_{1,2} = e^{-\zeta\omega_n T \pm j\omega_d T}$$

- Loop Phase Transfer Function (2nd Order Control System)

$$H(z) = \begin{cases} \dfrac{[2-2\cos(\omega_d T)e^{-\zeta\omega_n T}]z - 2\sinh(\zeta\omega_n T)e^{-\zeta\omega_n T}}{z^2 - 2\cos(\omega_d T)e^{-\zeta\omega_n T}z + e^{-2\zeta\omega_n T}} & \text{for} \quad \zeta < 1 \quad \text{with} \quad \omega_d T = \omega_n T\sqrt{1-\zeta^2} \\ \dfrac{[2-2(1)e^{-\zeta\omega_n T}]z - 2\sinh(\zeta\omega_n T)e^{-\zeta\omega_n T}}{z^2 - 2(1)e^{-\zeta\omega_n T}z + e^{-2\zeta\omega_n T}} & \text{for} \quad \zeta = 1 \quad \text{with} \quad \omega_d T = 0 \\ \dfrac{[2-2\cosh(\omega_d T)e^{-\zeta\omega_n T}]z - 2\sinh(\zeta\omega_n T)e^{-\zeta\omega_n T}}{z^2 - 2\cosh(\omega_d T)e^{-\zeta\omega_n T}z + e^{-2\zeta\omega_n T}} & \text{for} \quad \zeta > 1 \quad \text{with} \quad \omega_d T = \omega_n T\sqrt{\zeta^2-1} \end{cases}$$

Clock Tracking PLL Model

SILVERBLOCK SYSTEMS

- PI Gains from 2nd Order Digital Control Loop parameters:

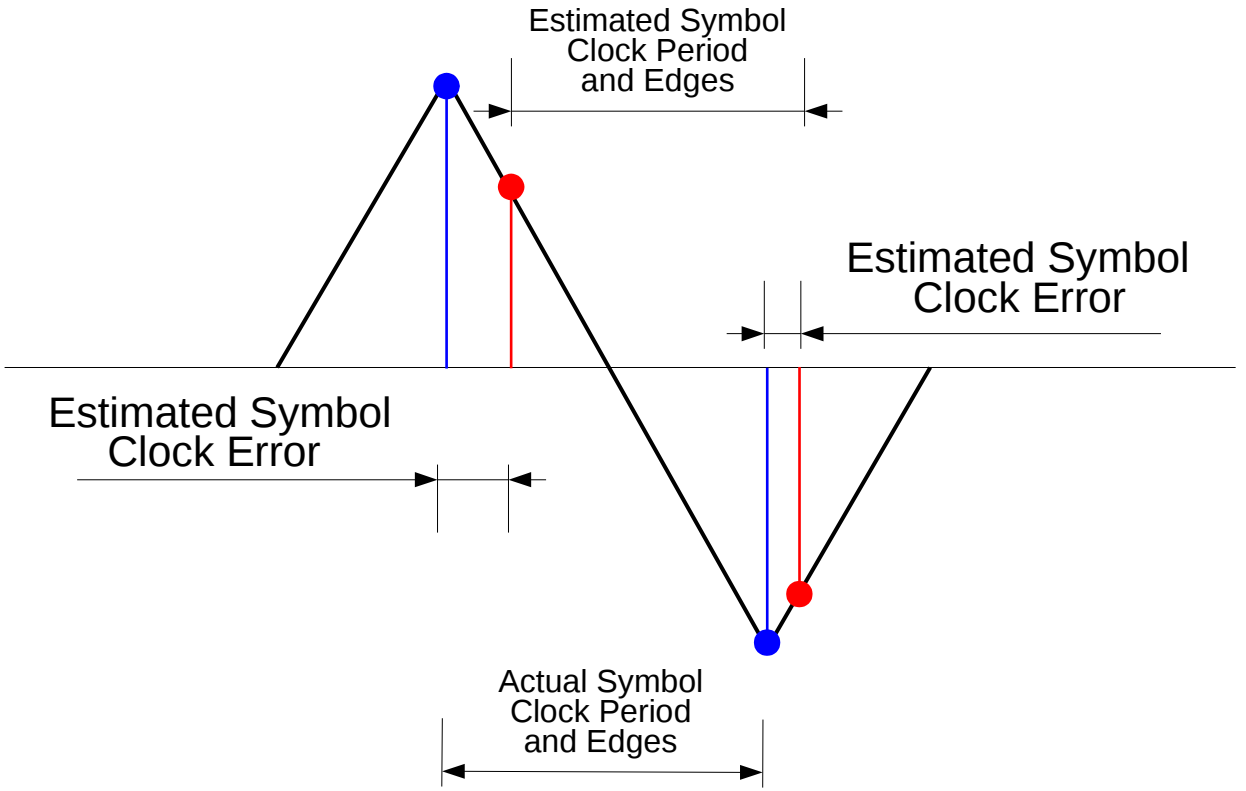
$$\alpha = \frac{2}{K_{pd}} e^{-\zeta\omega_n T} \sinh(\zeta\omega_n T)$$

$$\beta = \begin{cases} \frac{2}{K_{pd}} \left(1 - e^{-\zeta\omega_n T}[\sinh(\zeta\omega_n T) + \cos(\omega_d T)]\right) & \text{for} \quad \zeta < 1 \quad (under\ damped) \\ \frac{2}{K_{pd}} \left(1 - e^{-\zeta\omega_n T}[\sinh(\zeta\omega_n T) + 1]\right) & \text{for} \quad \zeta = 1 \quad (critically\ damped) \\ \frac{2}{K_{pd}} \left(1 - e^{-\zeta\omega_n T}[\sinh(\zeta\omega_n T) + \cosh(\omega_d T)]\right) & \text{for} \quad \zeta > 1 \quad (over\ damped) \end{cases}$$

Timing Error Detector

SILVERBLOCK SYSTEMS

- A TED emits an error value *proportional* to the time difference between
  - Optimal current symbol sampling time (blue)
  - Actual current symbol sampling time (red)



Timing Error Detector

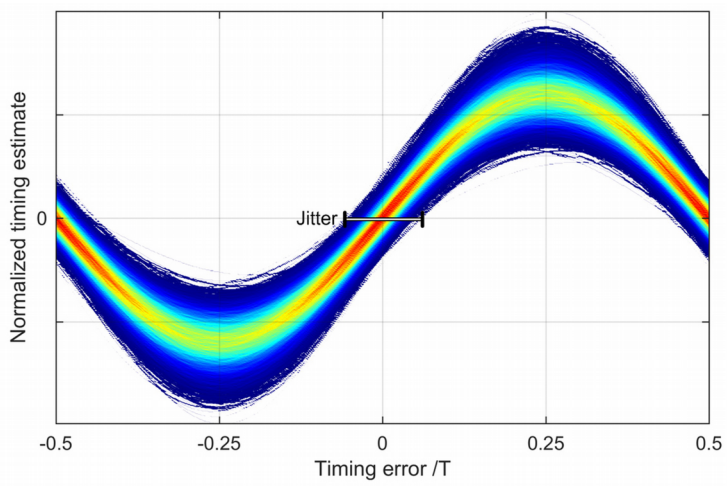
SILVERBLOCK SYSTEMS

- A TED is just an expression for the error value, e[n]
  - Formally derived - *Pay attention to the assumptions!*
  - Can include factors such as
    - Current symbol estimate
    - Nearby samples or symbol estimates
    - Current symbol decision (Decision Directed TEDs)
    - Nearby symbol decisions (Decision Directed TEDs)
    - Estimate of signal slope at symbol sampling time
    - Estimate of E_s/N_0
  - Usually a simple expression in the end
- Examples
  - Mueller and Müller $e[n] = \mathrm{slice}(x[n-1])x[n] - \mathrm{slice}(x[n])x[n-1]$
  - Small signal ML approximation $e[n] = x[n]x'[n]$

Timing Error Detector

SILVERBLOCK SYSTEMS

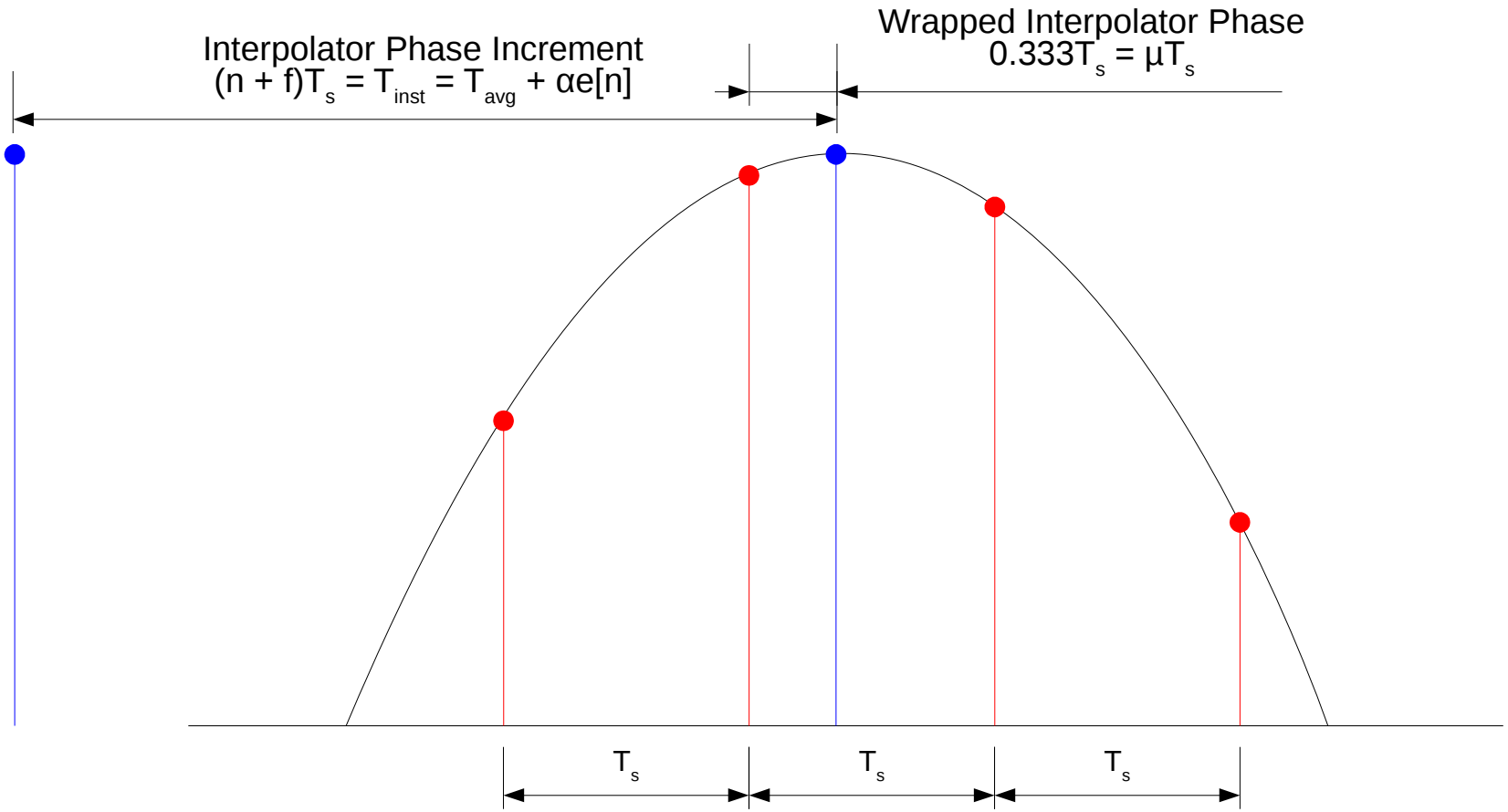
- A TED is characterized by its S-curve: $S(\tau_\epsilon) = E\{e[n]|\tau_\epsilon\}$
  - y-axis: expected value of timing error output, given the normalized symbol clock timing offset
  - τ-axis: normalized symbol clock timing offset
  - Slope at timing offset τ = 0 is TED gain, K_{ted} (= K_{pd})
    - Units of K_{ted} might not match the units required by the loop – scaling needed
- S-Curve shape and central slope at τ = 0 depend on
  - TED's error estimator expression
  - Input signal amplitude
  - Pulse shaping filter / pulse shape
  - Input E_s/N_0
  - Other factors
- Simulation required to find TED gain
  - Octave, MatLab, R, Python, or whatever



Interpolating Resampler

SILVERBLOCK SYSTEMS

- Symbol synchronization process reduces sample rate
- Input samples (red) not at optimal symbol sampling (blue)
- Need to resample input between samples



Interpolating Resampler

SILVERBLOCK SYSTEMS

- Interpolation implemented with FIR filters
  - Fractional Delay (FD)
    - MMSE interpolator polyphase filterbank
    - Matched filter/interpolator polyphase filterbank
  - Polynomial Interpolation using Farrow structure
    - Lagrange
    - B-spline

- Practical interpolation filters impose bandwidth constraint
  - Input signal bandwidth must be some fraction of F_s
  - Bounds the error in the interpolated output samples

Interpolating Resampler

SILVERBLOCK SYSTEMS

- Ideal FD interpolation filter frequency responses
  - Interpolator $H(\omega) = 1 \cdot e^{j\omega\mu} \quad \text{for} \quad -\pi \leq \omega \leq \pi$
  - Differentiating interpolator $H(\omega) = j\omega \cdot e^{j\omega\mu} \quad \text{for} \quad -\pi \leq \omega \leq \pi$
  - μ is intersample interpolation fraction in [0.0, 1.0]

- Ideal FD interpolation filter impulse responses
  - Interpolator $h[n] = \text{sinc}\,(n + \mu)$
  - Differentiating interpolator

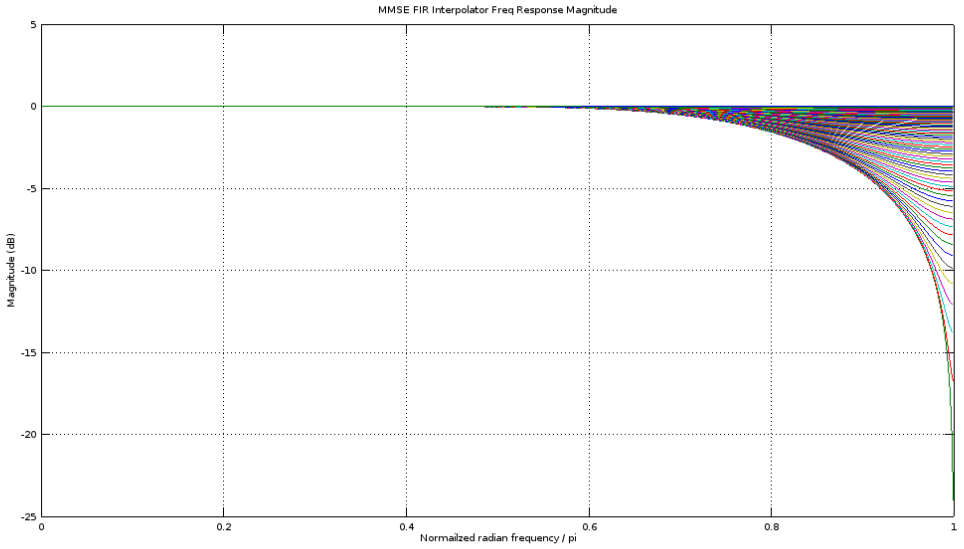
$$h[n] = \frac{1}{n+\mu}\left[\cos\left(\pi[n+\mu]\right) - \text{sinc}(n+\mu)\right]$$

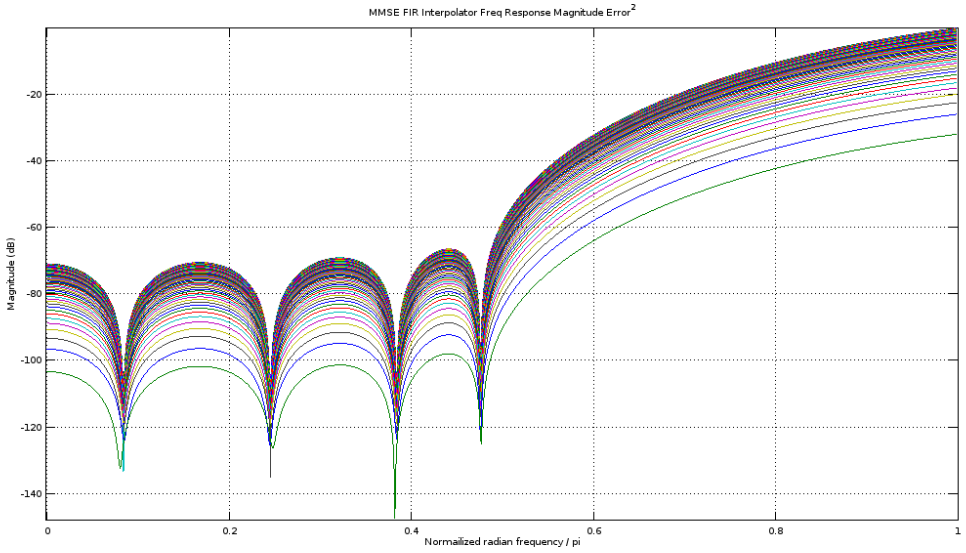
  - For a particular μ in [0.0, 1.0]
  - These ideal responses from IDTFT are infinite length

Interpolating Resampler

SILVERBLOCK SYSTEMS

- GR Minimum Mean Squared Error interpolators
  - Truncated, MMSE version of ideal filters
  - Bank of 129, 8 tap FIR filters
  - For μ in {0/128, 1/128, 2/128, …, 127/128, 128/128}
  - Each filter has a MMSE H(ω) *only* in $[-F_s/4, F_s/4]$
- One sided freq response and error^2 of the filters

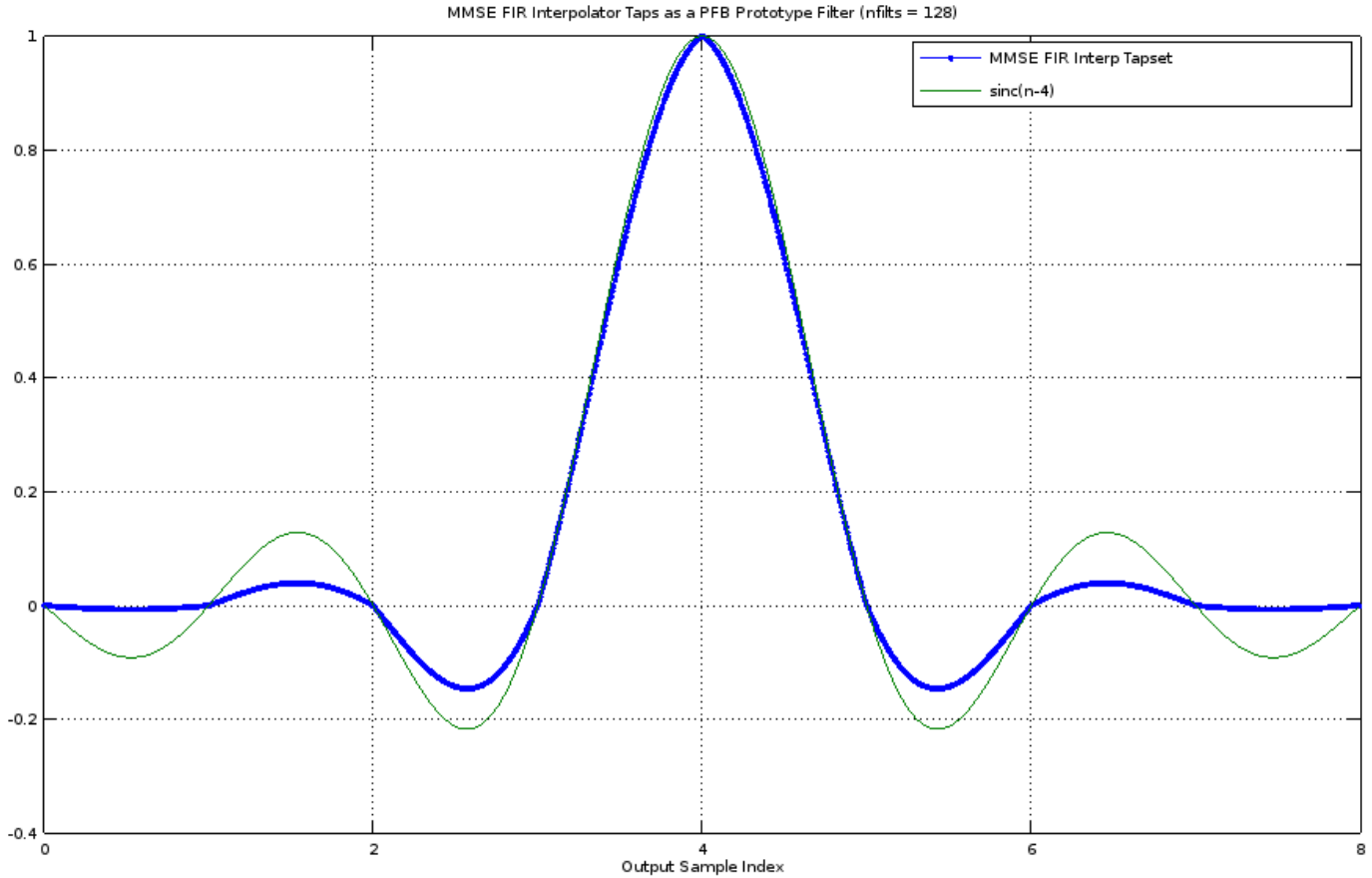




Interpolating Resampler

SILVERBLOCK SYSTEMS

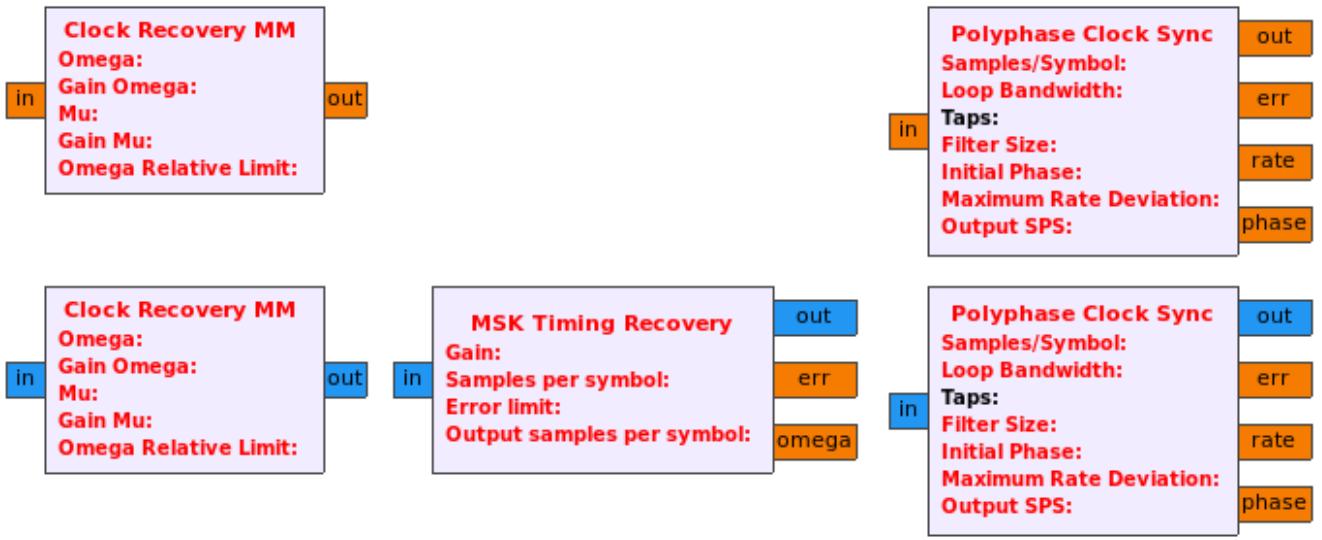
- GR's MMSE interpolator is a polyphase filter bank
- Equivalent PFB prototype filter vs truncated ideal



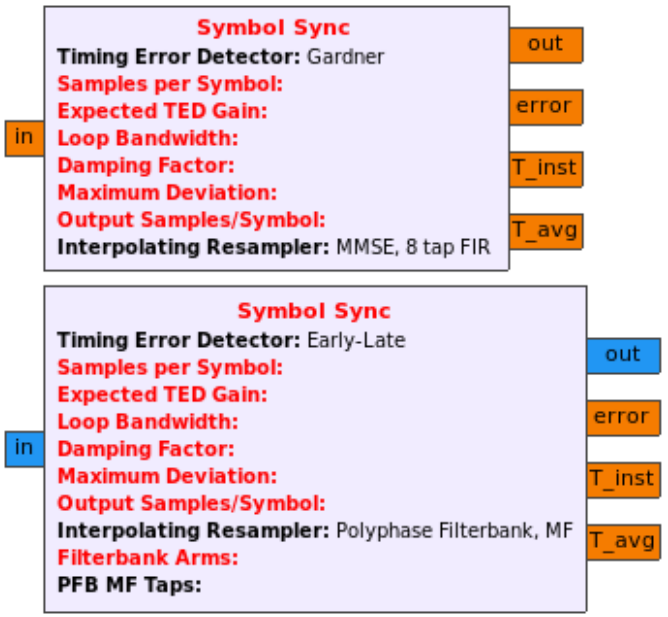
GNURadio Sync Blocks

SILVERBLOCK SYSTEMS

Existing



New



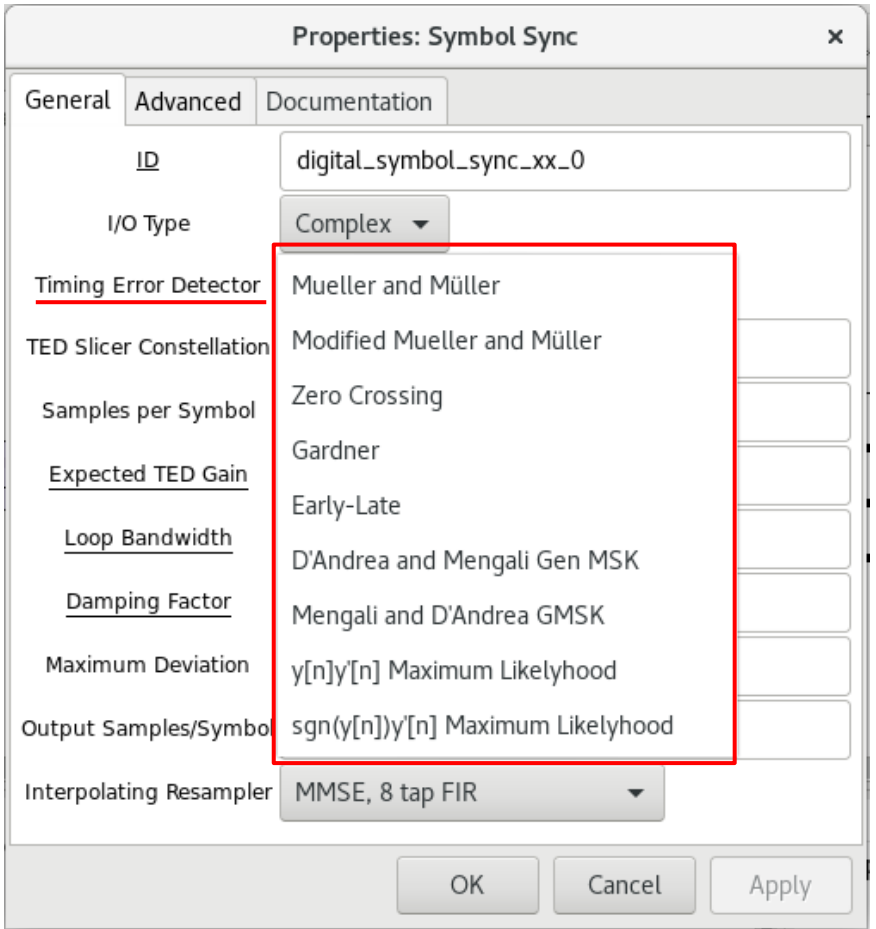
Existing Blocks' Deficiencies

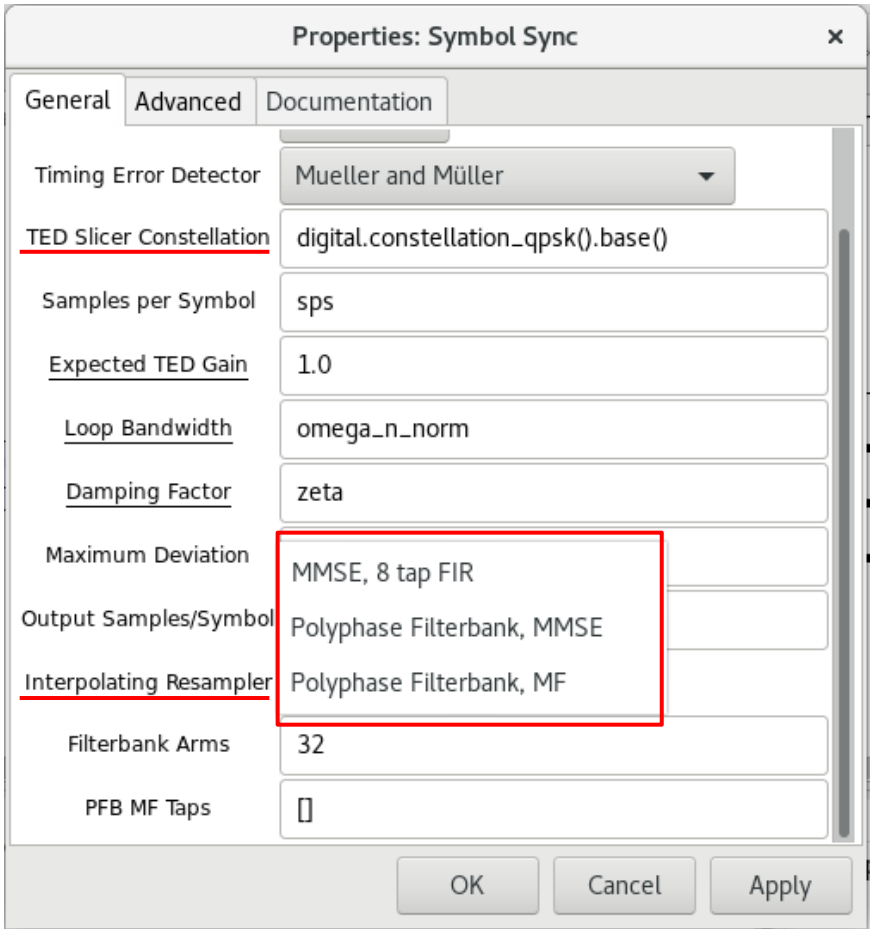
- Incorrect tag propagation
- Conflates symbol clock phase and interpolator phase
  - Self noise & Unable to stay locked on a clock pattern
- Incorrect decision slicer constellation
- Drops some input, when > 8 samples/symbol
- No reset on receipt of time_est tag
- No way to change TED, slicer, or resampler
  - Whole new blocks needed - bringing new bugs
- Initializes to very overdamped loop filter
- PI filter gain computations ignored TED gain
- Restricted to 1 or 2 samples/symbol on output

New Symbol Sync Blocks

SILVERBLOCK SYSTEMS

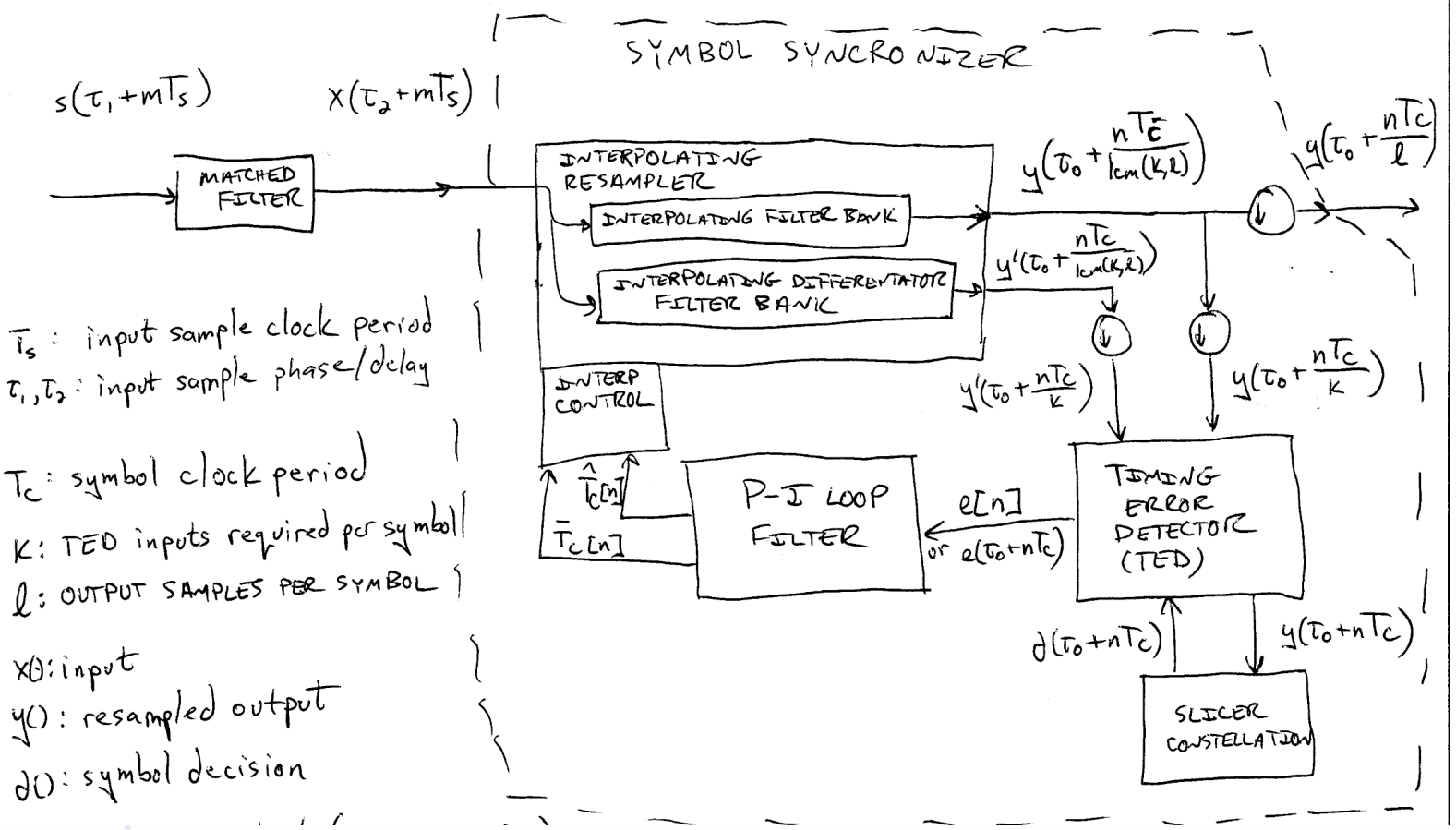
- Fixed, replacement superset of existing blocks
  - Except can't change MF taps on the fly (yet)
- Selectable TED, slicer, and resampler





New Symbol Sync Blocks

SILVERBLOCK SYSTEMS



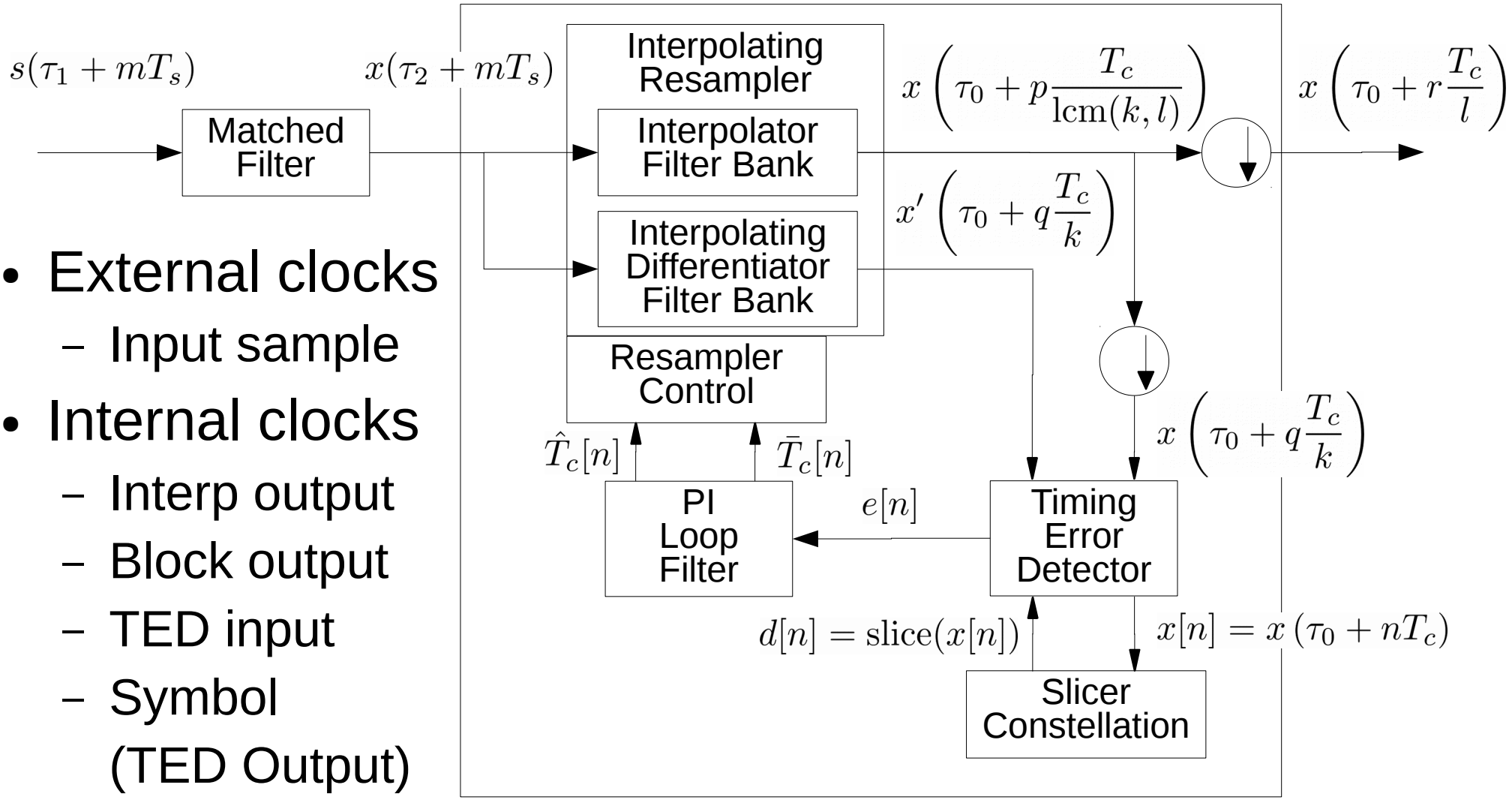
jmcorgan commented on Apr 23

@awalls-cx18 👎 for not using an actual napkin for that diagram.

New Symbol Sync Blocks

SILVERBLOCK SYSTEMS

- External clocks
  - Input sample
- Internal clocks
  - Interp output
  - Block output
  - TED input
  - Symbol (TED Output)



Adding a New TED

SILVERBLOCK SYSTEMS

- Modify the following files
  - gr-digital/include/.../timing_error_detector_type.h
  - gr-digital/grc/digital_symbol_sync_xx.xml
  - gr-digital/lib/timing_error_detector.*
- Your new derived TED class only needs
  - A simple constructor
  - Two methods to compute the error output term
    - Complex input
    - Float input
- Leave the symbol sync blocks' code alone
  - tag handling, slicer, resampler, & loop filter – all done!

Adding a New Resampler

SILVERBLOCK SYSTEMS

- Modify the following files
  - gr-digital/include/.../interpolating_resampler_type.h
  - gr-digital/grc/digital_symbol_sync_xx.xml
  - gr-digital/lib/interpolating_resampler.*
- Your two (1 float, 1 complex) new derived resampler classes each need
  - A constructor
  - A simple ntaps() method
  - An interpolate() method
  - A differentiate() method (interpolating differentiator)
- Leave the symbol sync blocks' code alone
  - tag handling, slicer, TED, & loop filter – all done!

Using a Different Slicer

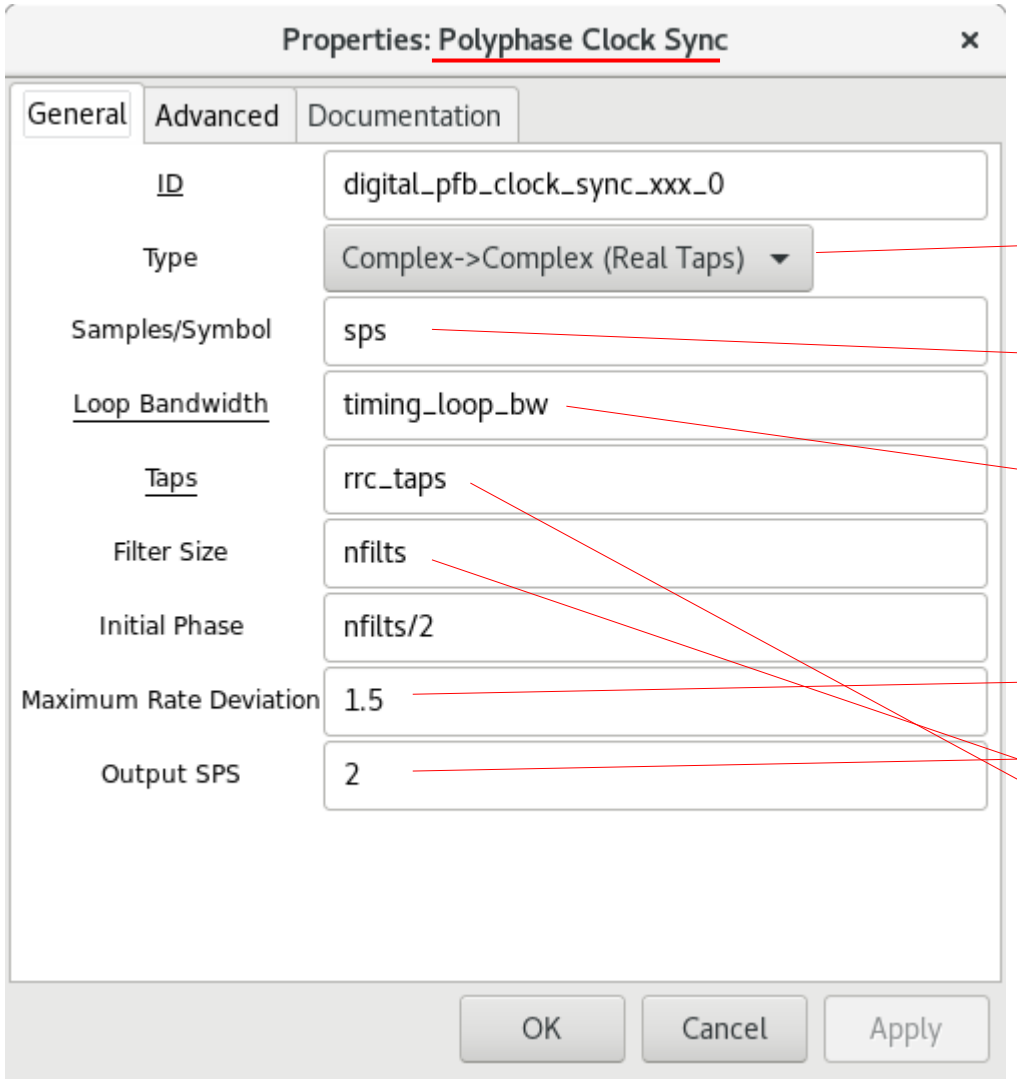
SILVERBLOCK SYSTEMS

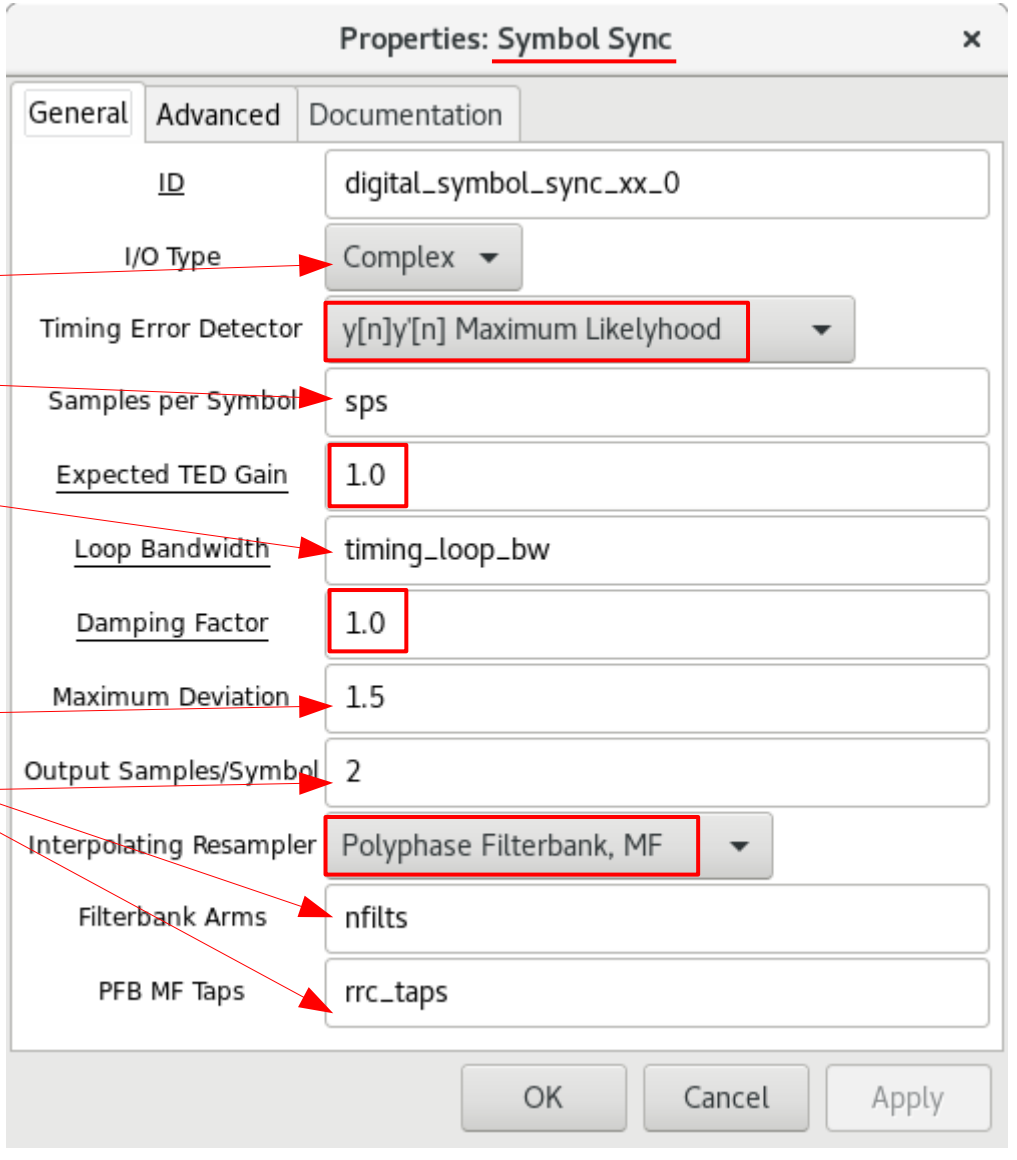
- Instantiate a custom Constellation Object
  - Slicer only needed for decision directed TEDs though
    - M&M, Modified M&M, Zero Crossing
- Pass in the constellation object as the TED slicer
- Leave the symbol sync blocks' code alone
  - tag handling, resampler, TED, & loop filter – all done!

Existing Block to New Block

SILVERBLOCK SYSTEMS

- Polyphase Clock Sync

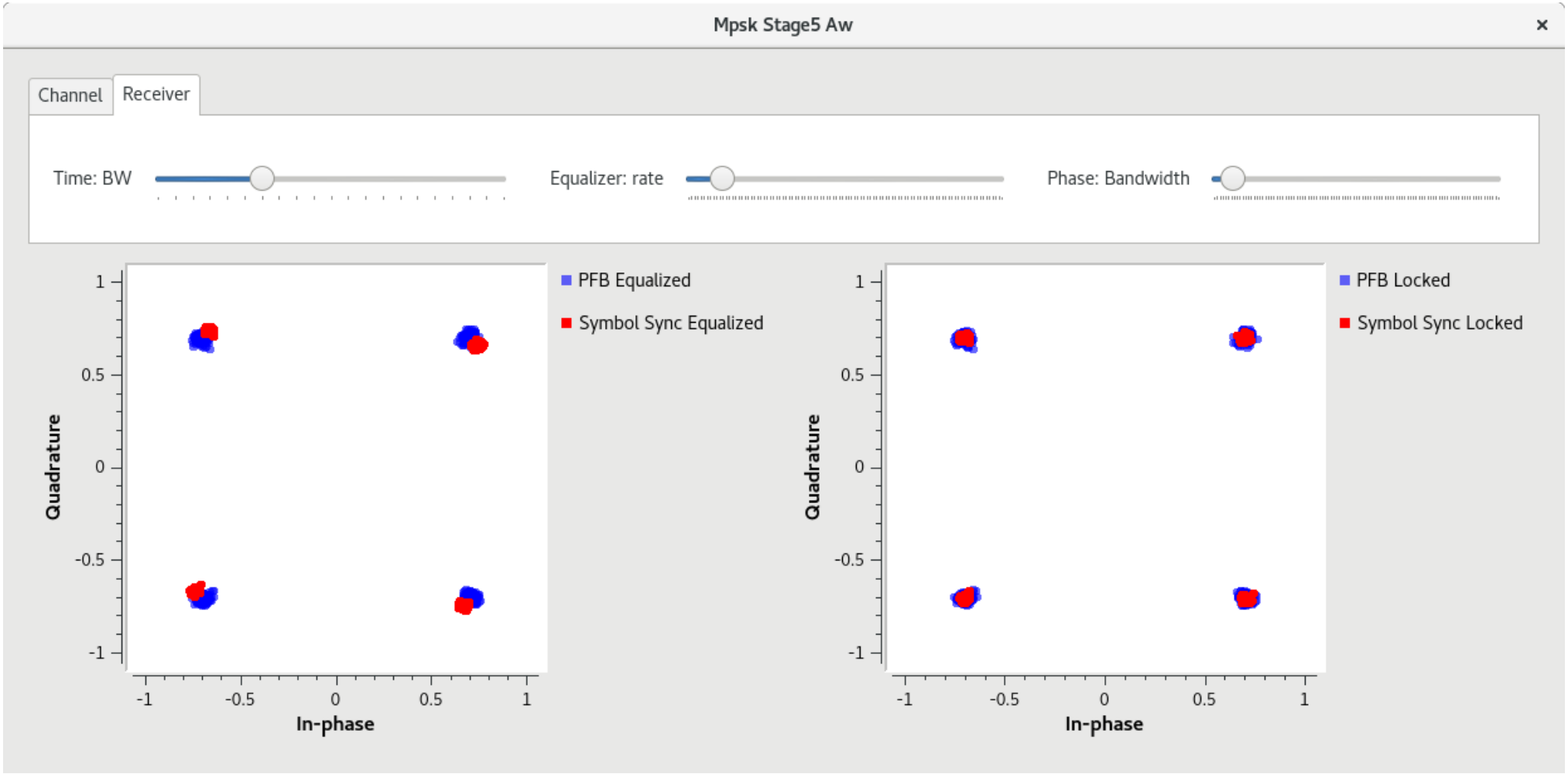




Existing Block to New Block

SILVERBLOCK SYSTEMS

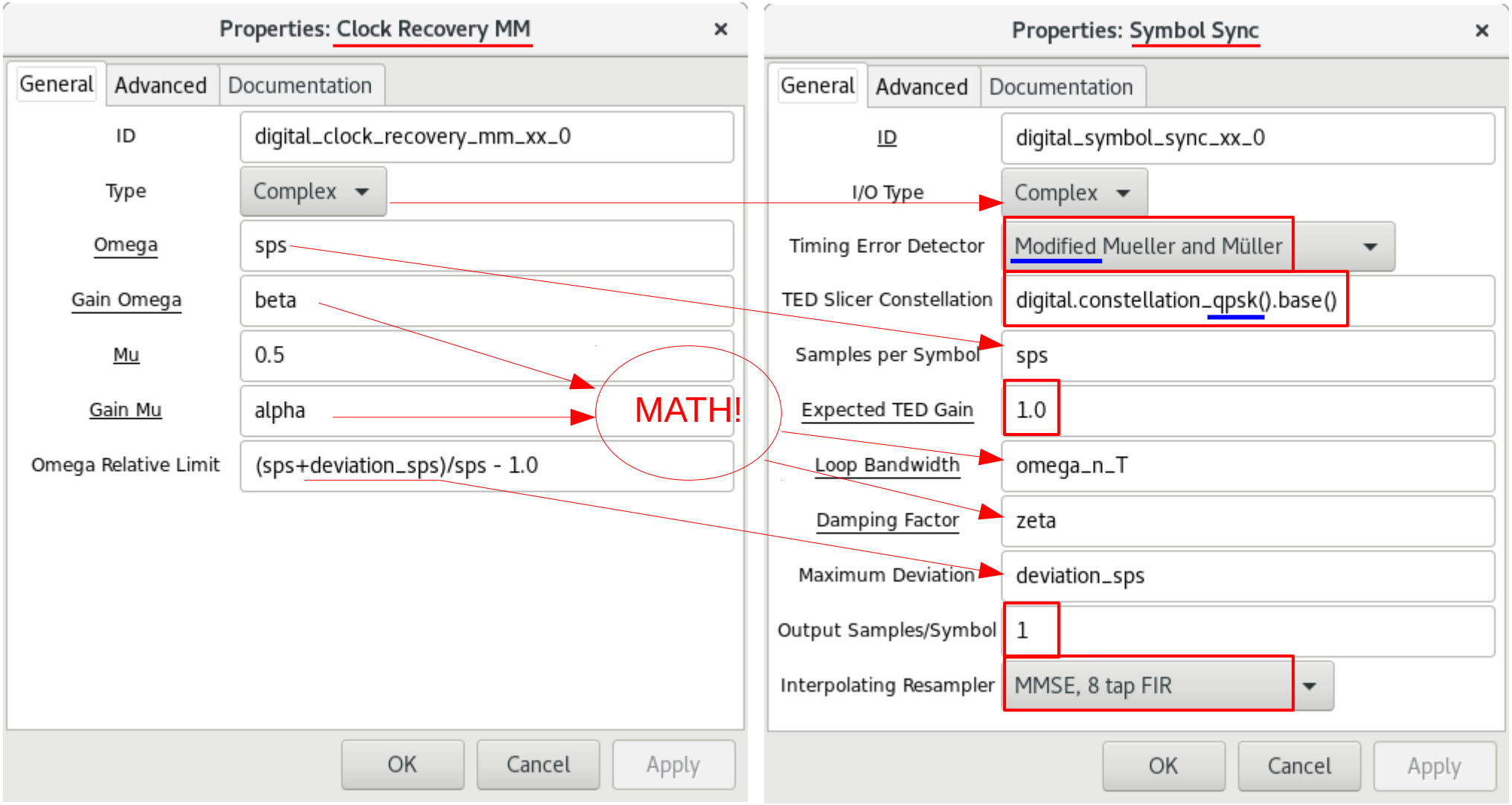
- Polyphase Clock Sync comparison (MPSK tutorial)



Existing Block to New Block

SILVERBLOCK SYSTEMS

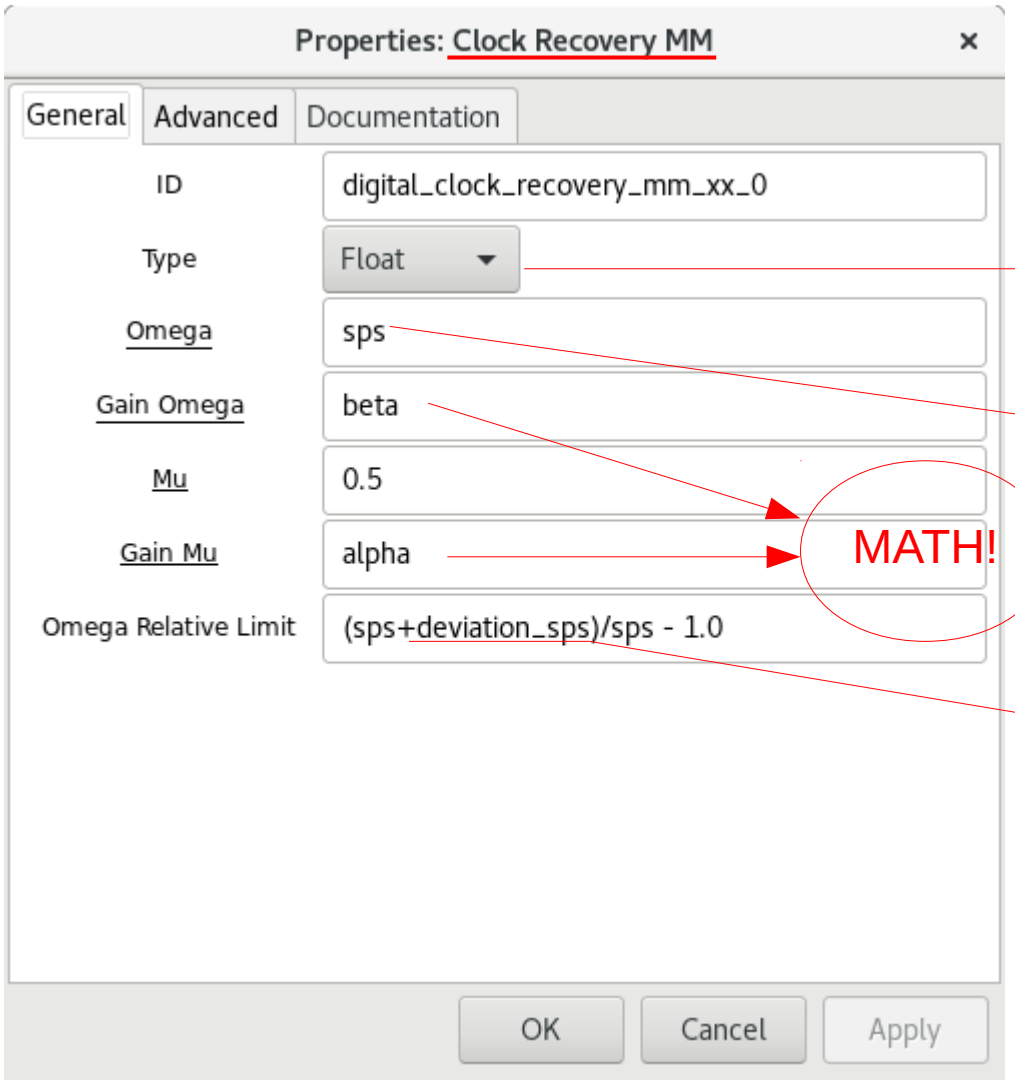
- Clock Recovery MM, Complex I/O



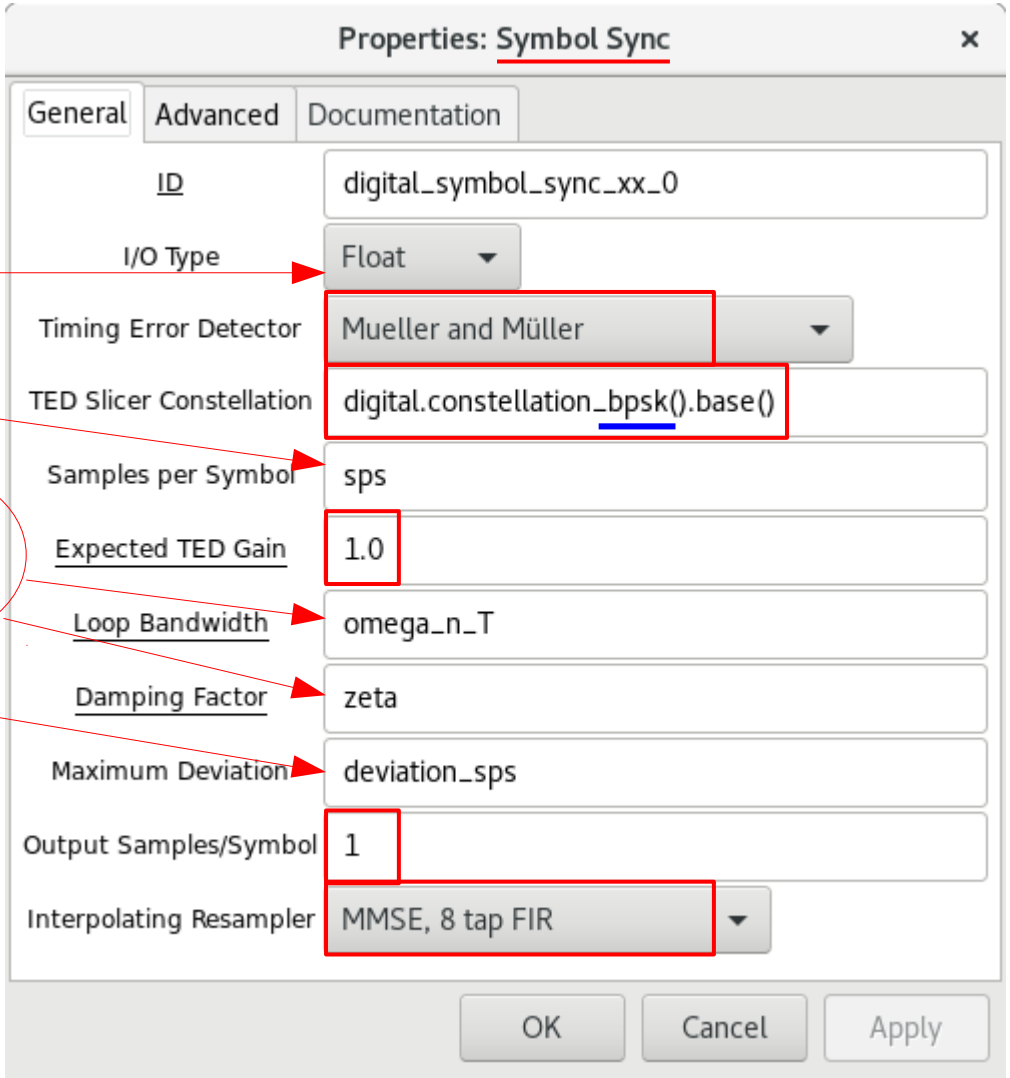
Existing Block to New Block

SILVERBLOCK SYSTEMS

- Clock Recovery MM, Float I/O



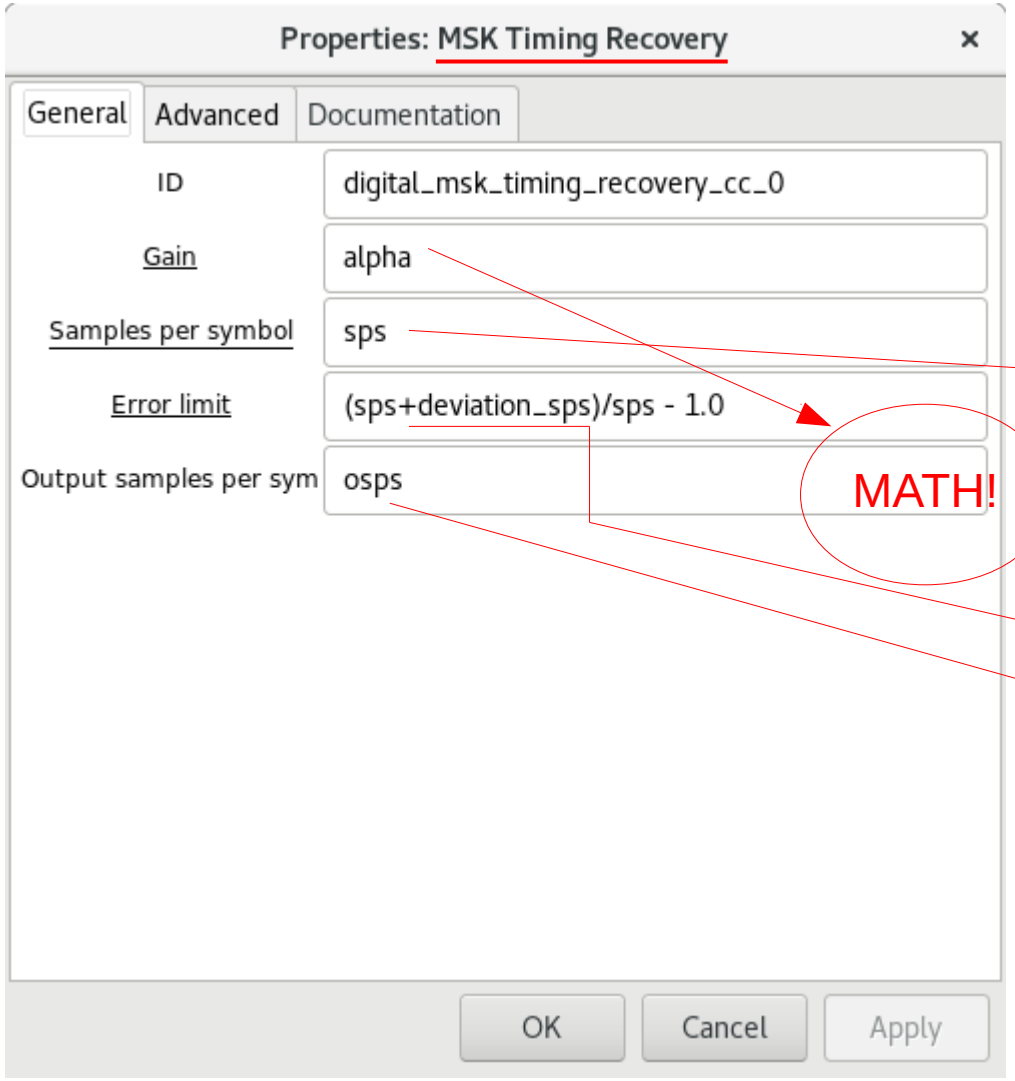
MATH!



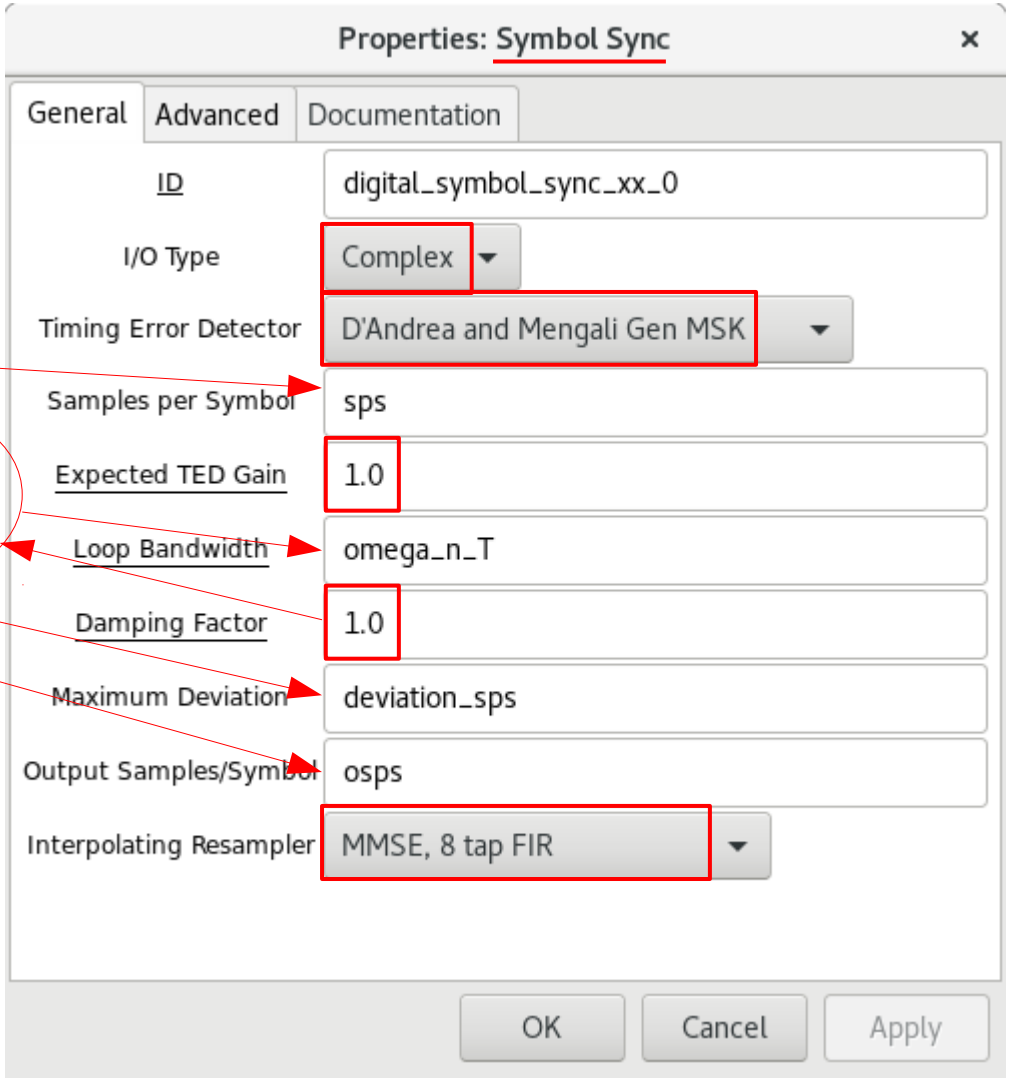
Existing Block to New Block

SILVERBLOCK SYSTEMS

- MSK Timing Recovery



MATH!



Usage Hints and Gotchas

- Easy stuff
  - Output samples/symbol can be in [1, 2, 3, 4, 5, 6, …]
    - Normally set to 1; or to 2, if upstream from an equalizer block
  - Maximum deviation is in units of samples/symbol
    - Smaller is better for acquiring lock at start of burst
    - Too small misses data when symbol clock is far from nominal
  - Tracking resets on a “time_est” or “clock_est” tag
    - time_est tag value is a PMT double
      - Sample offset estimate, in [-1.0, 1.0] samples, relative to tagged sample
    - clock_est tag value is a PMT 2-tuple of doubles
      - Sample offset estimate, in [-1.0, 1.0] samples, relative to tagged sample
      - Symbol clock period estimate, in samples/symbol
    - clock_est tag has priority over time_est tag

Usage Hints and Gotchas

SILVERBLOCK SYSTEMS

- Input signal conditioning and filtering
  - MSK signals and MSK TEDs don't use matched filters
    - But a narrow IF filter can be beneficial
  - Input signal should be at a consistent amplitude (e.g. +/- 1.0)
    - AGC
    - TEDs have specific assumptions about input amplitudes !!!
  - Input signal amplitude should match constellation
    - Only for decision directed TEDs: M&M, Modified M&M, Zero Crossing
    - GNURadio's Constellation Object silently scales your constellation !!!
  - Input signal should normally be NRZ (no DC offset)
  - Input signal should be peaked at symbol centers
    - Except for MSK signals and MSK TEDs
    - Normally accomplished with a matched filter
  - Sync block's "PFB, MF" resampler can do the matched filtering
    - Except for rectangular pulse filter and a TED that needs a derivative !!!

Usage Hints and Gotchs

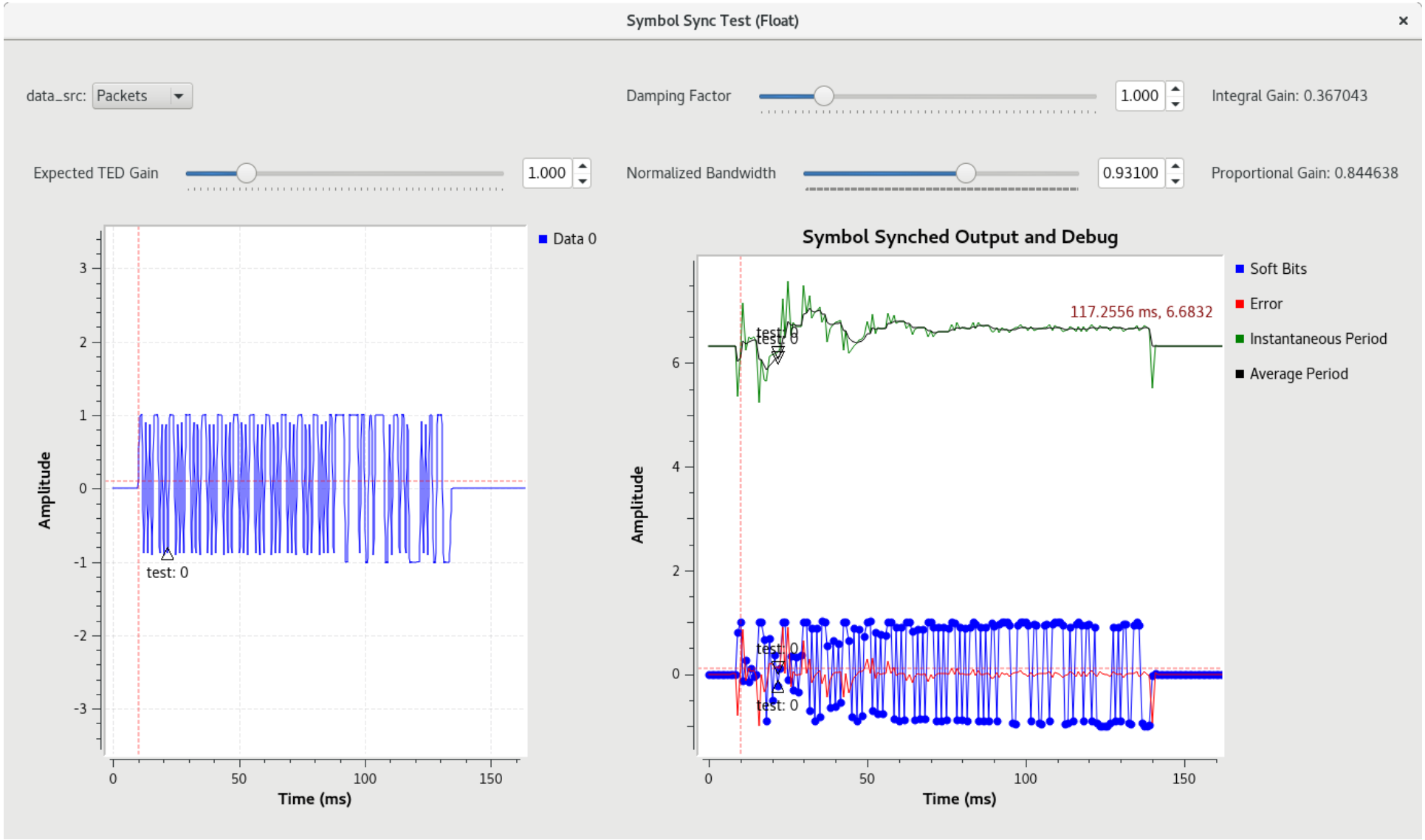
SILVERBLOCK SYSTEMS

- Loop parameters and tuning
  - Use simulation to determine TED gain, K_{ted}
    - Cannot know damping regime without it !!!
    - Ensure TED gain is scaled to the proper units for the loop !!!
  - Start with a critically damped, or over damped, loop
    - Damping factor, ζ, of 1.0, or greater than 1.0
    - An under damped loop *usually* isn't desirable for timing recovery
  - Use a Loop BW, $\omega_n T$, in [0.0, π(?)], usually closer to 0.0
  - Use simulation to determine optimal ζ & $\omega_n T$ for best BER vs. Es/No
  - If you just want to play around and can accept suboptimal results
    - Start with K_{ted} = 1.0, ζ = 1.0, $\omega_n T$ = a number close to 0.0
    - Use GUI sliders to control all 3 of those values
    - Send all 4 outputs of the block to a single Time Sink/Scope
    - Adjust $\omega_n T$ slider first, observing the primary, T_inst, and T_avg traces
    - Adjust ζ to 1.3 or 1.5 or 2.0 (or 0.707 or 0.5), and try adjusting $\omega_n T$ again
    - See gr-digital/examples/demod/symbol_sync_test_float.grc

Experimental Tuning Example

SILVERBLOCK SYSTEMS

- Intentionally terrible loop BW example



TED S-Curve Simulation

SILVERBLOCK SYSTEMS

- gr-digital/examples/demod/*_ted_gain.m
  - M&M TED gain: K_{ted} = 0.28271 sample^{-1}
  - Gardner TED gain: K_{ted} = 0.11810 sample^{-1}