Causal Inference in Three-Wave Panel Designs

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Section 1

The Fundamental Problem of Causal Inference

Causality Requires a Contrast of Exposures

$$Y_{\text{you}}(1) - Y_{\text{you}}(0)$$

But Individuals Experience Only **One** Exposure

$$Y_i|A_i = 1 \implies Y_i(0)|A_i = 1 \text{ is counterfactual}$$

Average Treatment Effect in Randomised Controlled Experiments Work From Assumptions

$$\text{Average Treatment Effect} = \left[\left(\underbrace{\mathbb{E}[Y(1)|A=1]}_{\text{observed}} + \underbrace{\mathbb{E}[Y(1)|A=0]}_{\text{unobserved}} \right) - \left(\underbrace{\mathbb{E}[Y(0)|A=0]}_{\text{observed}} + \underbrace{\mathbb{E}[Y(0)|A=1]}_{\text{unobserved}} \right) \right]$$

Section 2

The Three Fundemental Assumptions of Causal Inference

Causal Consistency

$$Y_i^{observed}|A_i = \begin{cases} Y_i(a^*) & \text{if } A_i = a^* \\ Y_i(a) & \text{if } A_i = a \end{cases}$$

Conditional Exchangeabilty

$$Y(a) \coprod A|L \quad \text{or equivalently} \quad A \coprod Y(a)|L$$

Positivity

$$0 < Pr(A = a|L = l) < 1, \quad \forall a, l \text{ with } Pr(L = l) > 0$$

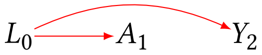
Section 3

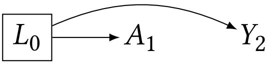
Confounding

Common Cause

$$
\begin{tikzcd}
L_0 \arrow[r] \arrow[rr, bend left] & A_1 & Y_2
\end{tikzcd}
$$

Common Cause: Longitudinal Solution





Reverse Causation Solution

$$Y_1 \longrightarrow A_2$$

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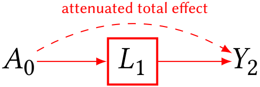
$$Y_1 \longrightarrow A_2 \qquad\qquad\qquad A_1 \qquad Y_2$$

Mediator Bias



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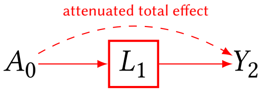
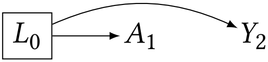
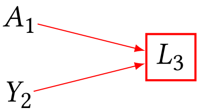




Collider Bias



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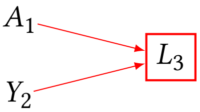
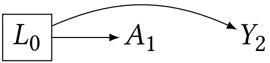
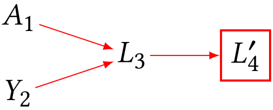


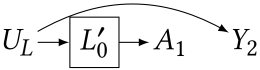


Collider Bias Proxy

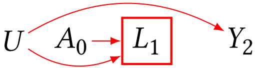
$$
\begin{tikzcd}
A_1 \arrow[dr] & & \\
& L_3 \arrow[r] & \boxed{L'_4} \\
Y_2 \arrow[ur] & &
\end{tikzcd}
$$

Collider Bias Proxy Solution

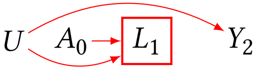


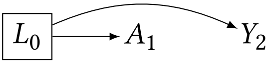


Post Exposure Collider Bias



Post Exposure Collider Bias Solution





Unmeasured Common Cause

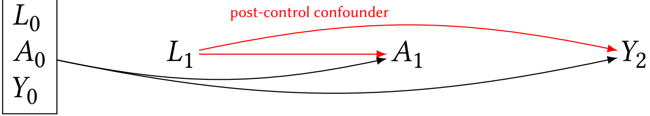
$$
\begin{tikzcd}
U \arrow[r] \arrow[rr, bend left] & A_1 & Y_2
\end{tikzcd}
$$

Unmeasured Common Cause Solution

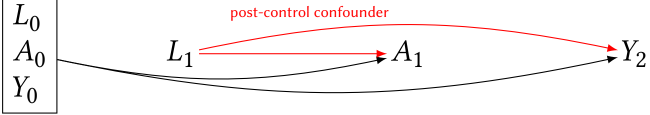
$$\begin{tikzcd} U \arrow[r] \arrow[rr, bend left] & A_1 & Y_2 \end{tikzcd} \qquad \begin{tikzcd} \boxed{L_0} \arrow[r] \arrow[rr, bend left] & A_1 & Y_2 \end{tikzcd}$$

Longitudinal Data Bring Their Own Problems

Timing of Confounder

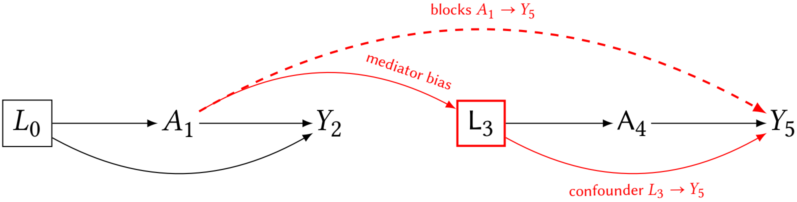


Timing of Mediator



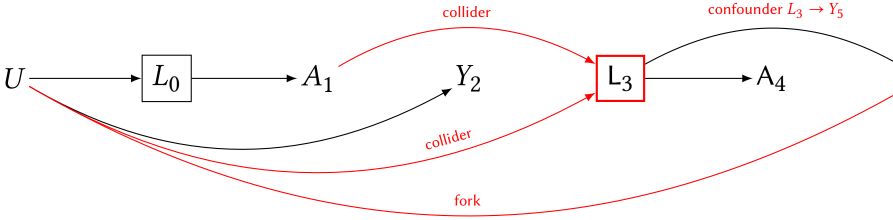
Treatment Confounder Bias

Exposure 1 affects confounder of Exposure 2: biasing true causality



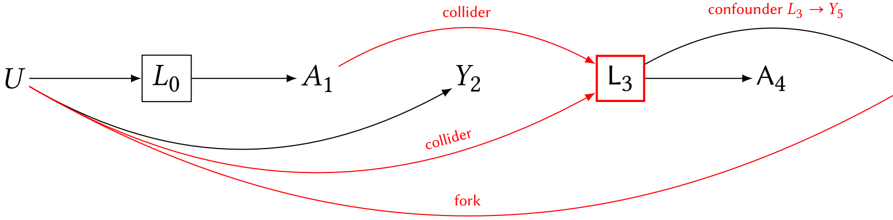
Treatment Confounder Feedback

Exposure 1 affects confounder of Exposure 2 when true causality absent



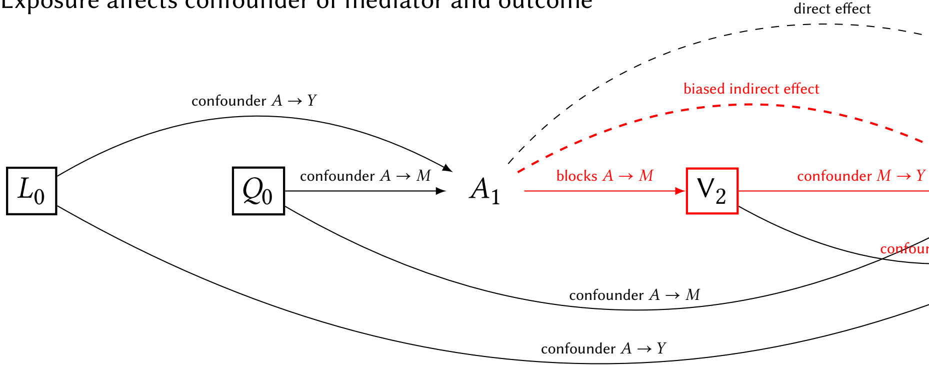
Treatment Confounder Feedback Variation

Exposure 1 affects confounder of Exposure 2 when true causality absent



Mediation

Exposure affects confounder of mediator and outcome



Fork Chain

$$\begin{tikzcd} A \arrow[r] \arrow[rr, bend left] & B & C \end{tikzcd}$$

$$\begin{tikzcd} A \arrow[r] & B \arrow[r] & C \end{tikzcd}$$

The New Zealand Attitudes and Values Study

- Planned 20-year longitudinal study, currently in its 14^{th} year.

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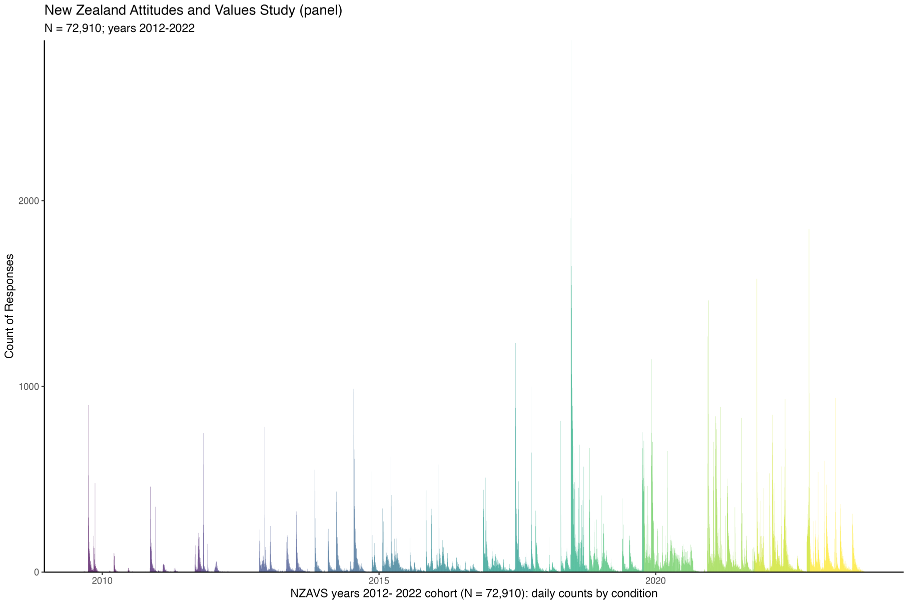
- Current sample contains > 72,290,000 unique people, and ~ 38,000 in the longitudinal study


New Zealand Attitudes and Values Study (panel)
N = 72,910; years 2012-2022
Count of Responses
2000
1000
0
2010
2015
2020
NZAVS years 2012- 2022 cohort (N = 72,910): daily counts by condition


Discontinuity at attacks (GAM)
Boost to Warmth increase in the years following the attacks: FULL SAMPLE
attack_condition 0 1
Muslim Warmth
2
4
6
2015
2020
NZAVS Time 4 - 14 Cohort (2012-2023), (N = 71,128)

Key

Symbol	
X	Any variable.
A	The treatment or, equivalently, the exposure.
Y	The outcome.
$Y(a)$	The potential outcome when $A = a$.
L	Measured confounder(s): typically comprises a set of variables.
U	Unmeasured confounder.
Z	Effect-modifier (or 'moderator') of A on Y.
M	Mediator of A on Y.
$\bar{X}$	Sequential variables, e.g. $\bar{A} = \{A_1, A_2, A_3\}$; $\bar{L} = \{L_0, L_1, L_2\}$.
$\mathscr{R}$	Denotes randomisation into treatment event.

Thanks

Section 4

Extra Slides

Section 5

Interaction

Interaction: potential outcomes

$$\left(\underbrace{\mathbb{E}[Y(1,1)]}_{\text{joint exposure}} - \underbrace{\mathbb{E}[Y(0,0)]}_{\text{neither exposed}} \right) - \Bigg[\left(\underbrace{\mathbb{E}[Y(1,0)]}_{\text{only A exposed}} - \underbrace{\mathbb{E}[Y(0,0)]}_{\text{neither exposed}} \right) + \left(\underbrace{\mathbb{E}[Y(0,1)]}_{\text{only B expose}} \right.$$

Interaction: simplifies to

$$\underbrace{\mathbb{E}[Y(1,1)]}_{\text{joint exposure}} - \underbrace{\mathbb{E}[Y(1,0)]}_{\text{only A exposed}} - \underbrace{\mathbb{E}[Y(0,1)]}_{\text{only B exposed}} + \underbrace{\mathbb{E}[Y(0,0)]}_{\text{neither exposed}} \neq 0$$

Section 6

Mediation

Total Effect

$$TE = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$

Total Effect Considering Mediator

$$TE = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$

$$\mathbb{E}[Y(1)] = \mathbb{E}[Y(1, M(1))]$$

Natural Direct Effect

Natural Direct Effect (NDE) is the effect of the treatment on the outcome while maintaining the mediator at the level it would have been if the treatment had *not* been applied:

$$NDE = \mathbb{E}[Y(1, M(0))] - \mathbb{E}[Y(0, M(0))]$$

Natural Indirect Effect

Natural Indirect Effect (NIE): is the effect of the exposure on the outcome that is mediated. To obtain these quantities we must compare the potential outcome Y under treatment, where the mediator assumes its natural level under treatment with the potential outcome when the mediator assumes its natural value under no treatment is given:

$$NIE = \mathbb{E}[Y(1, M(1))] - \mathbb{E}[Y(1, M(0))]$$

Decomposition

$$\text{Total Effect (TE)} = \underbrace{\left\{\mathbb{E}[Y(1, M(1))] - \mathbb{E}[Y(1, M(0))]\right\}}_{\text{Natural Indirect Effect (NIE)}} + \underbrace{\left\{\mathbb{E}[Y(1, M(0))] - \mathbb{E}[Y(}_{\text{Natural Direct Effect}}$$

Why Mediation is Difficult

$$

$$