

Causal Inference in Three-Wave Panel Designs

Section 1

The Fundamental Problem of Causal Inference

Causality Requires a Contrast of Exposures

$$Y_{\text{you}}(1) - Y_{\text{you}}(0)$$

But Individuals Experience Only **One** Exposure

$Y_i|A_i = 1 \implies Y_i(0)|A_i = 1$ is counterfactual

Average Treatment Effect in Randomised Controlled Experiments Work From Assumptions

$$\text{Average Treatment Effect} = \left[\begin{array}{c} \left(\underbrace{\mathbb{E}[Y(1)|A=1]}_{\text{observed}} + \underbrace{\mathbb{E}[Y(1)|A=0]}_{\text{unobserved}} \right) \\ - \left(\underbrace{\mathbb{E}[Y(0)|A=0]}_{\text{observed}} + \underbrace{\mathbb{E}[Y(0)|A=1]}_{\text{unobserved}} \right) \end{array} \right]$$

Section 2

The Three Fundamental Assumptions of Causal Inference

Causal Consistency

$$Y_i^{observed}|A_i = \begin{cases} Y_i(a^*) & \text{if } A_i = a^* \\ Y_i(a) & \text{if } A_i = a \end{cases}$$

Conditional Exchangeability

$$Y(a) \coprod A|L \quad \text{or equivalently} \quad A \coprod Y(a)|L$$

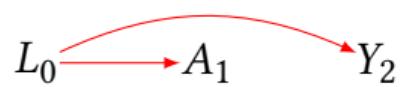
Positivity

$$0 < Pr(A = a | L = l) < 1, \quad \forall a, l \text{ with } Pr(L = l) > 0$$

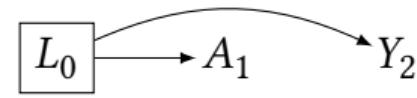
Section 3

Confounding

Common Cause



Common Cause: Longitudinal Solution



Reverse Causation Solution

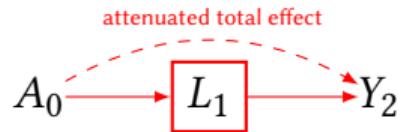
$$Y_1 \longrightarrow A_2$$

Reverse Causation Solution

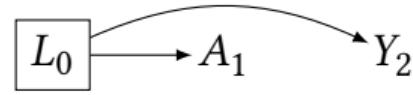
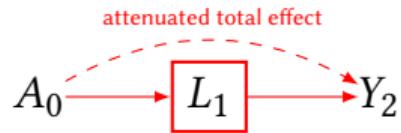
$$Y_1 \longrightarrow A_2$$

$$A_1 \qquad Y_2$$

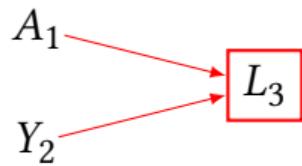
Mediator Bias



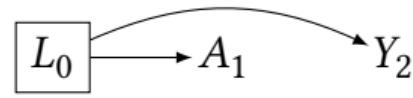
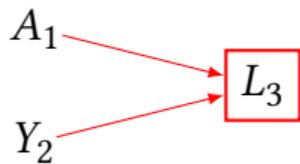
Mediator Bias Solution



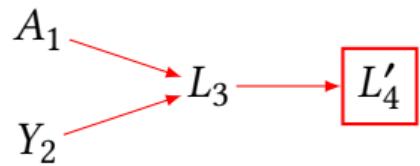
Collider Bias



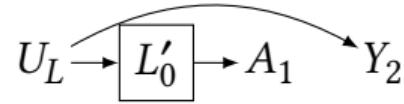
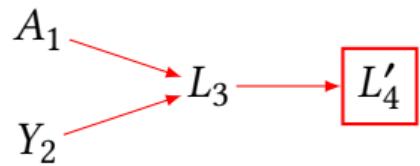
Collider Bias Solution



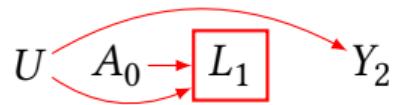
Collider Bias Proxy



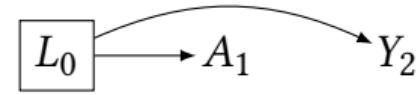
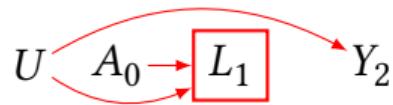
Collider Bias Proxy Solution



Post Exposure Collider Bias



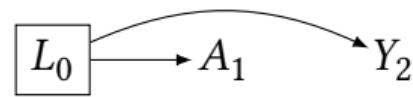
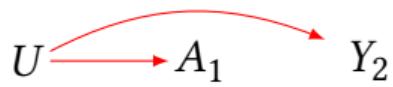
Post Exposure Collider Bias Solution



Unmeasured Common Cause

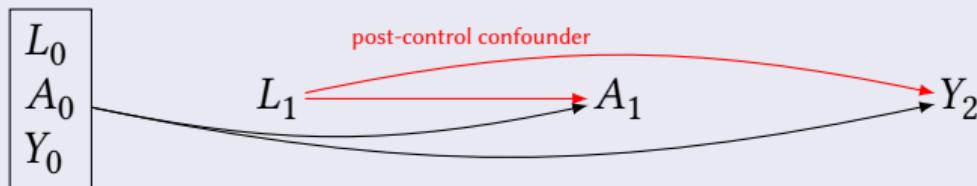


Unmeasured Common Cause Solution

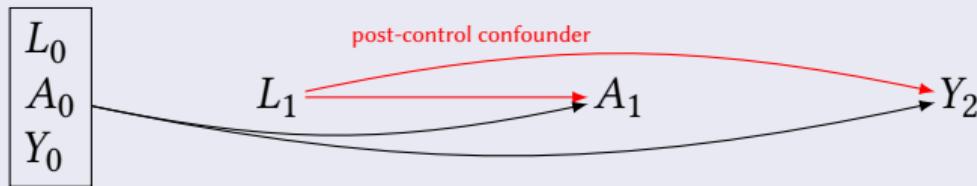


Longitudinal Data Bring Their Own Problems

Timing of Confounder

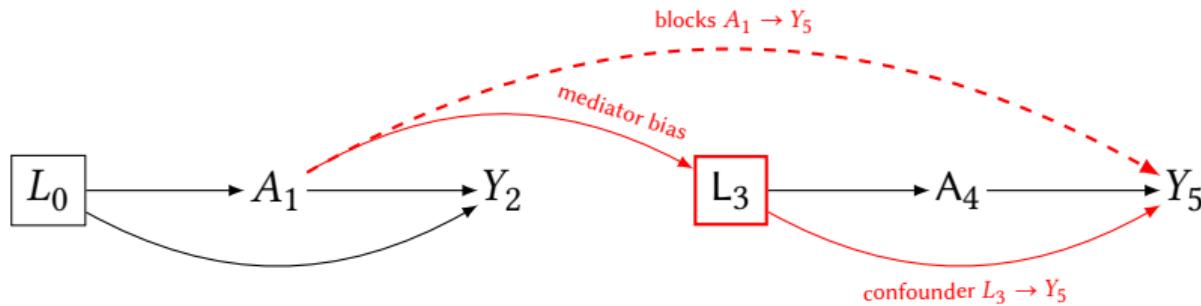


Timing of Mediator



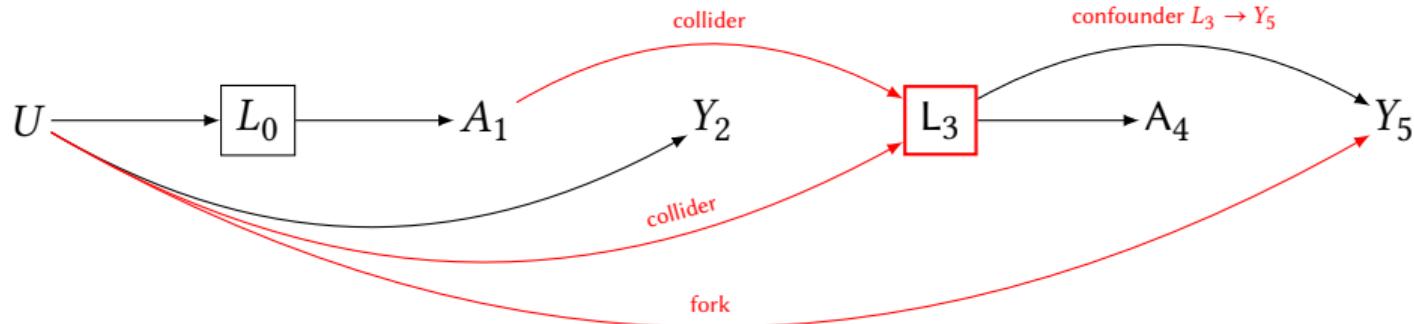
Treatment Confounder Bias

Exposure 1 affects confounder of Exposure 2: biasing true causality



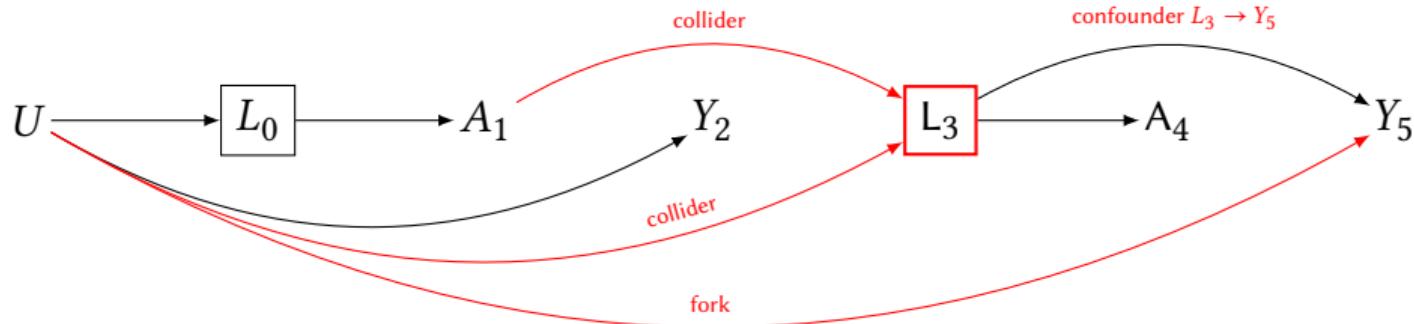
Treatment Confounder Feedback

Exposure 1 affects confounder of Exposure 2 when true causality absent



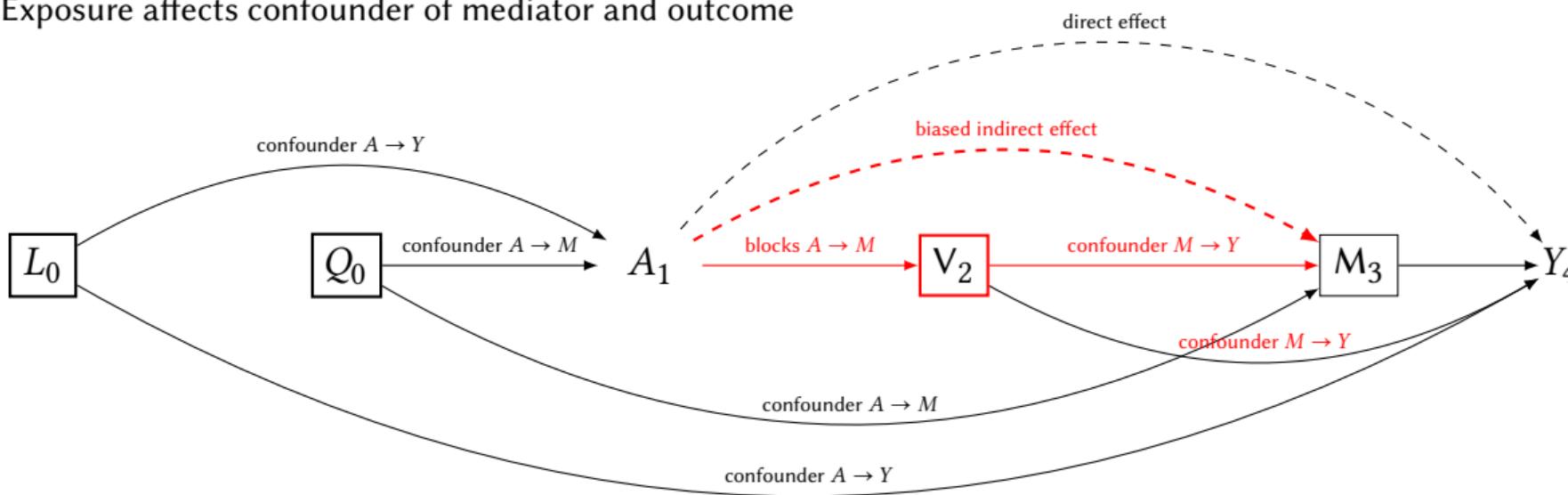
Treatment Confounder Feedback Variation

Exposure 1 affects confounder of Exposure 2 when true causality absent

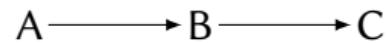
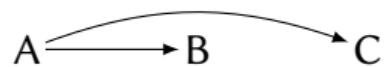


Mediation

Exposure affects confounder of mediator and outcome



Fork Chain



The New Zealand Attitudes and Values Study

- Planned 20-year longitudinal study, currently in its 14th year.

The New Zealand Attitudes and Values Study

- Planned 20-year longitudinal study, currently in its 14th year.
- Sample frame drawn randomly from NZ Electoral Roll.

The New Zealand Attitudes and Values Study

- Planned 20-year longitudinal study, currently in its 14th year.
- Sample frame drawn randomly from NZ Electoral Roll.
- Postal questionnaire (coverage; retention ~ 80%)

The New Zealand Attitudes and Values Study

- Planned 20-year longitudinal study, currently in its 14th year.
- Sample frame drawn randomly from NZ Electoral Roll.
- Postal questionnaire (coverage; retention ~ 80%)
- Large multidisciplinary research team (40 +)

The New Zealand Attitudes and Values Study

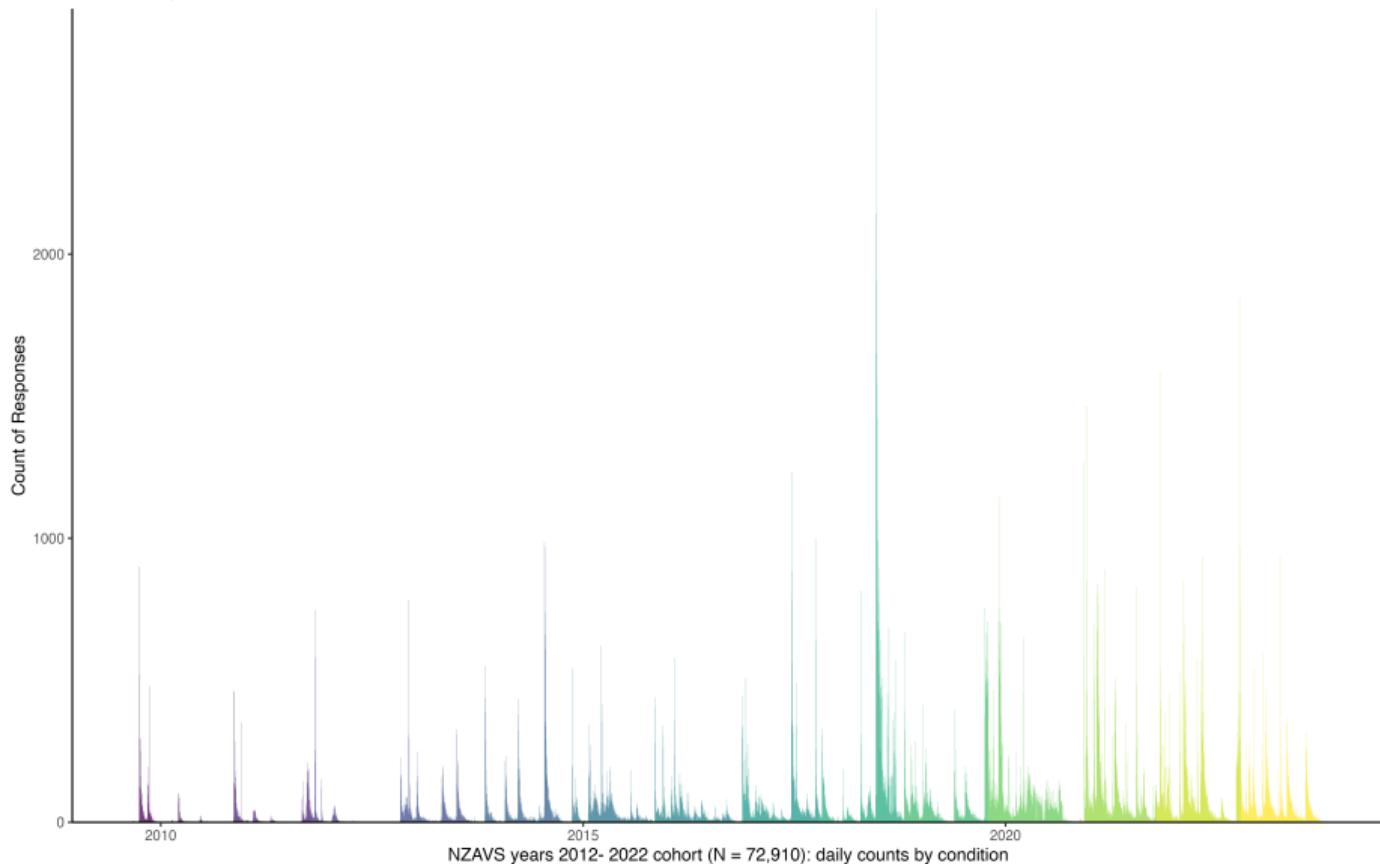
- Planned 20-year longitudinal study, currently in its 14th year.
- Sample frame drawn randomly from NZ Electoral Roll.
- Postal questionnaire (coverage; retention ~ 80%)
- Large multidisciplinary research team (40 +)
- Focus on personality, social attitudes, values, religion, employment, prejudice ...

The New Zealand Attitudes and Values Study

- Planned 20-year longitudinal study, currently in its 14th year.
- Sample frame drawn randomly from NZ Electoral Roll.
- Postal questionnaire (coverage; retention ~ 80%)
- Large multidisciplinary research team (40 +)
- Focus on personality, social attitudes, values, religion, employment, prejudice ...
- Current sample contains > 72,290,000 unique people, and ~ 38,000 in the longitudinal study

New Zealand Attitudes and Values Study (panel)

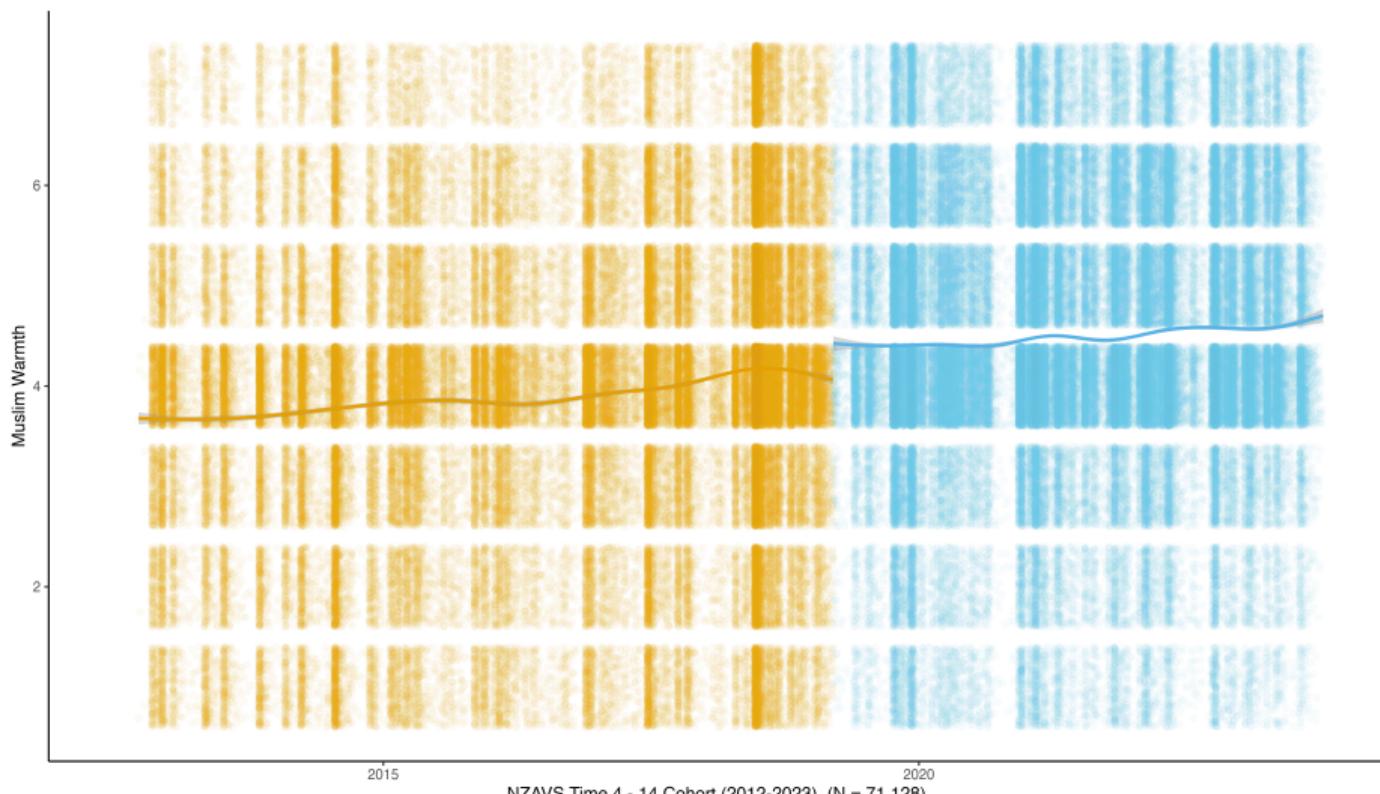
N = 72,910; years 2012-2022



Discontinuity at attacks (GAM)

Boost to Warmth increase in the years following the attacks: FULL SAMPLE

attack_condition 0 1



Key

Symbol

X	Any variable.
A	The treatment or, equivalently, the exposure.
Y	The outcome.
$Y(a)$	The potential outcome when $A = a$.
L	Measured confounder(s): typically comprises a set of variables.
U	Unmeasured confounder.
Z	Effect-modifier (or ‘moderator’) of A on Y .
M	Mediator of A on Y .
\bar{X}	Sequential variables, e.g. $\bar{A} = \{A_1, A_2, A_3\}; \bar{L} = \{L_0, L_1, L_2\}$.
\mathcal{R}	Denotes randomisation into treatment event.

Thanks

Section 4

Extra Slides

Section 5

Interaction

Interaction: potential outcomes

$$\left(\underbrace{\mathbb{E}[Y(1, 1)]}_{\text{joint exposure}} - \underbrace{\mathbb{E}[Y(0, 0)]}_{\text{neither exposed}} \right) - \left[\left(\underbrace{\mathbb{E}[Y(1, 0)]}_{\text{only A exposed}} - \underbrace{\mathbb{E}[Y(0, 0)]}_{\text{neither exposed}} \right) + \left(\underbrace{\mathbb{E}[Y(0, 1)]}_{\text{only B exposed}} - \underbrace{\mathbb{E}[Y(0, 0)]}_{\text{neither exposed}} \right) \right] =$$

Interaction: simplifies to

$$\underbrace{\mathbb{E}[Y(1, 1)]}_{\text{joint exposure}} - \underbrace{\mathbb{E}[Y(1, 0)]}_{\text{only A exposed}} - \underbrace{\mathbb{E}[Y(0, 1)]}_{\text{only B exposed}} + \underbrace{\mathbb{E}[Y(0, 0)]}_{\text{neither exposed}} \neq 0$$

Section 6

Mediation

Total Effect

$$TE = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$

Total Effect Considering Mediator

$$TE = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$

$$\mathbb{E}[Y(1)] = \mathbb{E}[Y(1, M(1))]$$

Natural Direct Effect

Natural Direct Effect (NDE) is the effect of the treatment on the outcome while maintaining the mediator at the level it would have been if the treatment had *not* been applied:

$$NDE = \mathbb{E}[Y(1, M(0))] - \mathbb{E}[Y(0, M(0))]$$

Natural Indirect Effect

Natural Indirect Effect (NIE): is the effect of the exposure on the outcome that is mediated. To obtain these quantities we must compare the potential outcome Y under treatment, where the mediator assumes its natural level under treatment with the potential outcome when the mediator assumes its natural value under no treatment is given:

$$NIE = \mathbb{E}[Y(1, M(1))] - \mathbb{E}[Y(1, M(0))]$$

Decomposition

$$\text{Total Effect (TE)} = \underbrace{\left\{ \mathbb{E}[Y(1, M(1))] - \mathbb{E}[Y(1, M(0))] \right\}}_{\text{Natural Indirect Effect (NIE)}} + \underbrace{\left\{ \mathbb{E}[Y(1, M(0))] - \mathbb{E}[Y(0, M(0))] \right\}}_{\text{Natural Direct Effect (NDE)}}$$

Why Mediation is Difficult

\$\$

\$\$