RegEx

Format:

1. $a \in \Sigma$, "a" is a RegEx over Σ, where L(a) = {a}
2. $b \in \Sigma$, "b" is a RegEx over Σ, where L(b) = {b}
3. $c \in \Sigma$, "c" is a RegEx over Σ, where L(c) = {c}
4. $\lambda \in \Sigma$, "λ" is a RegEx over Σ, where L(λ) = {λ}
5. "Σ" is a RegEx, L(Σ) = Σ = {a, b, c} (of length 1, so $\Sigma \circ \Sigma$ has length 2)
6. "Σ" is a RegEx (line 8), so "$(\Sigma)^\star$" is a a RegEx over Σ, L($(\Sigma)^\star$) = $\Sigma^\star$, set of all strings
7. "b" and "a" are RegEx (lines 2, 1), so "(b ∪ a)" is a RegEx over Σ, L(b ∪ a) = L(b) ∪ L(a)
8. "(b ∪ a)" is a RegEx (line 4), so "$((b \cup a))^\star$" is a RegEx over Σ, L($((b \cup a))^\star$) = $\{b,c\}^\star$, set of all strings that do not include an "a"
9. "a" and "c" are RegEx (lines 1, 3), so "(a ∘ c)" is a RegEx over Σ, L(a ∘ c) = L(a) ∘ L(b), set of size one
10. "$((b \cup a))^\star$" and "(a ∘ c)" are RegEx (lines 5, 4), so "$(((b \cup a))^\star \circ (a \circ c))$" is a RegEx over Σ, L($((b \cup a))^\star$) ∘ L(a ∘ c), set of strings where any of "b" or "a" come before the last "a", followed by a "c"
11. "c" and "λ" are RegEx (line 3, 4), so "$(c \cup \lambda)$" is a RegEx over Σ, L($c \cup \lambda$) = L(c) ∪ L(λ), set of strings that include at most 1 (or no) "c's"

DFA

Sanity Check 1: If it has a finite number of subsets

Sanity Check 2: Every string belongs to exactly one subset

Sanity Check 3:

Sanity Check 4

DFA NFA Equivalence

Format:

1. 1st state introduced: initally, $\hat{Q} = \emptyset$, $\hat{R} = \{\hat{q_0}\}$, so $\hat{q_0}$ is $Cl\lambda(q_0) = \{q_0\}$.
   Our current DFA: (Drawing of DFA w/ start state)
   We pick $\hat{q_0}$ since it's the only state in $\hat{R}$ so far:
   (a) Transitions out for $\omega = a$ for $\hat{q_0}$
   $$\begin{aligned}\bigcup_{r\in S_0}\bigcup_{s\in\delta(r,a)} Cl\lambda(s) &= \bigcup_{r\in\{q_0\}}\bigcup_{s\in\delta(r,a)} Cl\lambda(s)\\ &= \bigcup_{s\in\delta(q_0,a)} Cl\lambda(s)\\ &= \bigcup_{s\in\{q_0\}} Cl\lambda(s)\\ &= Cl\lambda(q_0)\\ &= \{q_0\} = S_0\end{aligned}$$
   Same as S_0, corresponding to state $\hat{q_0}$. Thus, $\hat{\delta}(\hat{q_0}, a) = \hat{q_0}$
   (b) Transitions out for $\omega = b$ for $\hat{q_0}$ (Same outcome as $\omega = a$)
   (c) Transitions out for $\omega = c$ for $\hat{q_0}$
   $$\begin{aligned}\bigcup_{r\in S_0}\bigcup_{s\in\delta(r,c)} Cl\lambda(s) &= \bigcup_{r\in\{q_0\}}\bigcup_{s\in\delta(r,c)} Cl\lambda(s)\\ &= \bigcup_{s\in\delta(q_0,c)} Cl\lambda(s)\\ &= \bigcup_{s\in\{q_0,q_1\}} Cl\lambda(s)\\ &= Cl\lambda(q_0)\cup Cl\lambda(q_1)\\ &= \{q_0\}\cup\{q_1\} = \{q_0, q_1\}\end{aligned}$$
   This set does not correspond to any states in $\hat{Q}\cup\hat{R}$, so it will be a new set $S_1 = \{q_0, q_1\}$ corresponding to the state $\hat{q_1}$.

   Now, $\hat{Q} = \{\hat{q_0}\}, \hat{R} = \{\hat{q_1}\}$. Our DFA: (draw DFA w/ state $\hat{q_0} \to \hat{q_1}$)
2. Since only $\hat{R} = \{\hat{q_1}\}$, set state to $\hat{q_1}$
   (a) Transitions out for $\omega = a$ for $\hat{q_1} \to$ same process, set $S_2 = \{q_0, q_2\}$
   (b) Transitions out for $\omega = b$ for $\hat{q_1} \to$ corresponds to state $\hat{q_0}$ (b transition back to $\hat{q_0}$)
   (c) Transitions out for $\omega = c$ for $\hat{q_1} \to$ corresponds to state $\hat{q_1}$ (c transition remain in $\hat{q_1}$)

   Now, $\hat{Q} = \{\hat{q_0}, \hat{q_1}\}, \hat{R} = \{\hat{q_2}\}$. Our DFA: (draw DFA w/ state $\hat{q_2}$)
3. Repeat until no more new states found

- Since $\hat{R} = \emptyset$, we have to find set $\hat{F}$ of accepting states:
  - $S_0 = \{q_0\}$ corresponds to $\hat{q_0}$, and $S_0 \cap F = \{q_0\} \cap \{q_2\} = \emptyset$
  - $S_1 = \{q_0, q_1\}$ corresponds to $\hat{q_1}$, and $S_0 \cap F = \{q_0, q_1\} \cap \{q_2\} = \emptyset$
  - $S_2 = \{q_0, q_2\}$ corresponds to $\hat{q_2}$, and $S_0 \cap F = \{q_0, q_2\} \cap \{q_2\} = \hat{\{q_2\}} \neq \emptyset, \hat{q_2} \in \hat{F}$
  - $S_3 = \{q_0, q_1, q_2\}$ corresponds to $\hat{q_3}$, and $S_0 \cap F = \{q_0, q_1, q_2\} \cap \{q_2\} = \hat{\{q_2\}} \neq \emptyset, \hat{q_3} \in \hat{F}$

  Thus, $\hat{F} = \{\hat{q_2}, \hat{q_3}\}$. Final DFA: