Numerical Linear Algebra: Homework 1

Due on Aug 14, 2014

Lecture time: 6:00 pm

Weiyi Chen

Problem 1

Find the correlation matrix of these random variables.

Answer

The correlation matrix is

$$\Omega_X = D_{\sigma_X}^{-1} \Sigma_X D_{\sigma_X}^{-1} = \begin{pmatrix} 1 & -0.35 & 0.15 \\ -0.35 & 1 & 0.05 \\ 0.15 & 0.05 & 1 \end{pmatrix} \tag{1}$$

where

$$D_{\sigma_X} = diag(\sqrt{\Sigma_X(i,i)})_{i=1:n} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2.5 \end{pmatrix} \tag{2}$$

Problem 2

The file indeces-jul26-aug9-2012.xlsx from fepress.org/nla-primer contains the July 26, 2012 August 9, 2012 end of day values of Dow Jones, Nasdaq, and S&P 500...

(i)

The daily percentage returns X is:

	Dow Jones	NASDAQ	S&P 500
0	0.014566342	0.022410784	0.019080602
1	-0.000202667	-0.004141186	-0.000483416
2	-0.004920825	-0.002145398	-0.004316754
3	-0.002502175	-0.0065691	-0.00289998
4	-0.00749453	-0.003575085	-0.007503708
5	0.016871809	0.019977524	0.019040293
6	0.001629484	0.007416018	0.002329276
7	0.003894794	0.008679191	0.005106761
8	0.000534605	-0.001528586	0.00062083
9	-0.00079313	0.00245413	0.00041363

(ii)

The covariance matrix is computed by

$$\hat{\Sigma}_X = \frac{1}{N-1} \overline{T}_X^t \overline{T}_X \tag{3}$$

where $\overline{T}_X^t(i,k) = X_k(t_i) - \hat{\mu}_k$. The result is:

	Dow Jones	NASDAQ	S&P 500
Dow Jones	6.1742e-05	7.4276e-05	7.1106e-05
NASDAQ	7.4276e-05	0.000103667	8.7988e-05
S&P 500	7.1106e-05	8.7988e-05	8.2637e-05

Python code attached:

```
import numpy as np
import pandas as pd
from numpy.linalg import inv

#1
df_price = pd.read_csv('indeces-jul26-aug9-2012.csv')
del df_price['Date']
df_rets = df_price.shift(-1) / df_price -1
df_rets = df_rets.drop(df_rets.index[-1])
df_rets.to_csv('output.csv', sep=',')

#2
df_norm = df_rets - df_rets.mean()
N = df_norm.shape[0]
mat_norm = np.matrix(df_norm)
mat_cov = np.round(1.0/(N-1) * mat_norm.transpose() * mat_norm,9)
pd.DataFrame(mat_cov,columns=df_rets.columns,index=df_rets.columns).to_csv('output.csv', sep=',')
```

(iii)

Similar to part(i), the daily log returns X is:

	Dow Jones	NASDAQ	S&P 500
0	0.014461272	0.022163352	0.01890085
1	-0.000202687	-0.004149784	-0.000483533
2	-0.004932973	-0.002147703	-0.004326099
3	-0.002505311	-0.006590771	-0.002904193
4	-0.007522755	-0.003581491	-0.007532003
5	0.016731061	0.019780592	0.018861295
6	0.001628158	0.007388655	0.002326568
7	0.003887229	0.008641743	0.005093766
8	0.000534462	-0.001529755	0.000620637
9	-0.000793445	0.002451124	0.000413544

(iv)

Similar to part(ii), the covariance matrix of the daily log returns is:

	Dow Jones	NASDAQ	S&P 500
Dow Jones	6.1075e-05	7.3212e-05	7.0216e-05
NASDAQ	7.3212e-05	0.000102023	8.6587e-05
S&P 500	7.0216e-05	8.6587e-05	8.1456e-05

Python code attached:

```
#3
df_price = pd.read_csv('indeces-jul26-aug9-2012.csv')
del df_price['Date']
df_rets = np.log(df_price.shift(-1) / df_price)
df_rets = df_rets.drop(df_rets.index[-1])
```

```
df_rets.to_csv('output.csv', sep=',')

#4
df_norm = df_rets - df_rets.mean()
N = df_norm.shape[0]
mat_norm = np.matrix(df_norm)
mat_cov = 1.0/(N-1) * mat_norm.transpose() * mat_norm
pd.DataFrame(mat_cov,columns=df_rets.columns,index=df_rets.columns).to_csv('output.csv', sep=',')
```

Problem 3

(i)

States of the market are:

- Asset at \$30 (state ω^1)
- Asset at \$35 (state ω^2)
- Asset at \$40 (state ω^3)
- Asset at \$42 (state ω^4)
- Asset at \$45 (state ω^5)
- Asset at \$50 (state ω^6)

The payoff matrix of this model is:

$$M_{1/2} = \begin{pmatrix} S_{1,1/2} \\ S_{2,1/2} \\ S_{3,1/2} \\ S_{4,1/2} \end{pmatrix} = \begin{pmatrix} 1.03 & 1.03 & 1.03 & 1.03 & 1.03 & 1.03 \\ 30 & 35 & 40 & 42 & 45 & 50 \\ 0 & 0 & 0 & 2 & 5 & 10 \\ 10 & 5 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4}$$

(ii)

It is easy to check that

$$rank(M_{1/2}) = 3 < S = 6 \tag{5}$$

where S is the number of states, then the market is incomplete.

(iii)

Since

$$S_{2,1/2} + S_{4,1/2} - S_{3,1/2} = \frac{40}{1.03} S_{1,1/2} \tag{6}$$

the payoff matrix is not linear independent, therefore the four securities are not non-redundant.