Numerical Linear Algebra: Quiz 2

Due on Aug 19, 2014

Lecture time: 6:00 pm

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Problem 1

(i)

The payoff matrix is:

$$M = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 32 & 38 & 42 & 44 \\ 0 & 2 & 6 & 8 \\ 8 & 2 & 0 & 0 \end{pmatrix} \tag{1}$$

It is easy to verify that $det(M) = 16 \neq 0$, which implies $rank(M) = 4$, therefore the four assets are non-redundant.

(ii)

Let s_τ be the price vector at time τ of the replicable derivative security, then

$$s_\tau = (0, 4, 6, 6) \tag{2}$$

Let Θ be the positions vector of the replicating portfolio, then

$$s_\tau = \Theta^t M_\tau \tag{3}$$

We derive the positions vector as

$$\Theta^t = s_\tau M_\tau^{-1} = (24, -0.5, 0.5, -1)^t \tag{4}$$

(iii)

The price vector of the securities at time t_0 is

$$S_{t_0} = (1, 40, 8, 5)^t \tag{5}$$

To decide whether this market model is arbitrage-free, we must solve the linear system $M_\tau Q = S_{t_0}$ and check whether all the entries of the vector Q are positive, as stated

$$Q = M_\tau^{-1} S_{t_0} = (1, -1.5, 0.5, 1)^t \tag{6}$$

since not all the entries of Q are positive, we conclude that the one period market model is not arbitrage-free.

(iv)

A possible arbitrage can be a short position of three months call on the stock with strike 36 at value \$8. It is easy to verify the payoff is always at least 0 for all states, since even if the underlying asset rises to the highest price \$44, the P&L of this option is still 0.
Formally, we can also apply the conclusion of state price Q to continue constructing an arbitrage portfolio, let Θ be the positions vector of the arbitrage portfolio, let the payoff at τ as V_τ then

$$V_\tau = (0, 1, 0, 0)^t \tag{7}$$

since the second entry of Q is negative. Then according to $\Theta^t M_\tau = V_\tau^t$

$$\Theta = (-36, 1, -1, 0.5)^t \tag{8}$$

which is the argitrage portfolio generating \$1.5 at time τ, therefore this is a type II arbitrage.