MTH 9821 Numerical Methods for Finance

Fall 2014

Homework 1
Assigned: August 27; Due: September 11

This homework is to be done as a group. Each team will hand in one homework solution, and each member of the team should write at least one problem. On the cover page of the homework, please indicate the members of the team and who wrote each problem.

(1) Let L_1 and L_2 be nonsingular lower triangular matrices and let U_1 and U_2 be nonsingular upper triangular matrices. If $L_1U_1 = L_2U_2$, show that there exists a nonsingular diagonal matrix D such that

$$L_1 = L_2D \quad \text{and} \quad U_1 = D^{-1}U_2.$$

(2) Find the LU decomposition without pivoting of the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ -1 & -1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 \end{pmatrix}.$$

Note: This is the 5×5 version of the classic example of a matrix whose LU decomposition is unstable.

(3) Let

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -2 & 1 & 3 \\ 4 & 0 & -1 \end{pmatrix}.$$

(i) Show that the 2×2 leading principal minor of A is 0, i.e., show that

$$\det \begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix} = 0.$$

(ii) Attempt to do the LU decomposition without pivoting of the matrix A, and show that the division by $U(2,2)$ cannot be performed when trying to compute the second row of L.

(iii) Show that the matrix A is nonsingular, and compute the LU decomposition with row pivoting of A.

(4) Write the pseudocode for the forward substitution corresponding to a lower triangular banded matrix of band m, i.e., for solving $Lx = b$ where b is an $n \times 1$ vector and L is an $n \times n$ lower triangular matrix such that

$$L(j,k) = 0, \ \forall \ 1 \leq j,k \leq n \quad \text{with} \quad j - k > m.$$

What is the corresponding operation count?

(5) Write the pseudocode for the backward substitution corresponding to an upper triangular banded matrix of band m, i.e., for solving $Ux = b$ where b is an $n \times 1$ vector and

U is an $n \times n$ upper triangular matrix such that

$$U(j,k) \;=\; 0,\; \forall\; 1 \leq j,k \leq n \quad \text{with} \quad k-j > m.$$

What is the corresponding operation count?

(6) Write the pseudocode for the LU decomposition without pivoting for banded matrices of band m. What is the corresponding operation count?

Use (without proving) the fact that the L and U factors from the LU decomposition without pivoting of a banded matrix of band m are a banded lower triangular matrix of band m and a banded upper triangular matrix of band m, respectively.

What is the corresponding operation count?

(7) Write C++ codes for backward and forward substitutions, called forward_subst and backward_subst.

(8) Write C++ codes called lu_no_pivoting and lu_row_pivoting to compute the LU decomposition without pivoting of a matrix and the LU decomposition with row pivoting of a matrix.

TESTING THE CODES: To test whether your codes run properly or not, your teaching assistant will post an Excel data file on the class forum on Saturday, September 13, at 12noon. The file will contain data to solve the following problems using the C++ codes you designed:

(1) multiply a matrix by a vector;
(2) solve a linear system using forward substitution;
(3) solve a linear system using backward substitution;
(4) find the LU decomposition without pivoting of a full matrix;
(5) find the LU decomposition with row pivoting of a full matrix;
(6) find the LU decomposition with row pivoting of a banded matrix.

You should email your answer to the Teaching Assistant within one hour of downloading the file. Unless you clear it with me ahead of time, the TA should receive an email from you before 10pm on Sunday.