

Assignment 9

Gautam Singh (CS21BTECH11018)

May 23, 2022

Outline

1 Problem

2 Solution

Problem Statement

(NCERT Class 12, Exercise 13.5.3) There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Solution

Random Variables

- ① X_i : Bernoulli random variables with parameter $p, 1 \leq i \leq N$
- ② Y : Binomial random variable given by $Y = \sum_{i=1}^N X_i$

Moment Generating Function of X_i and Y

$$M_Z(X_i) = \sum_{k=-\infty}^{k=\infty} z^{-k} P_X(k) \quad (1)$$

$$= P_X(0) + z^{-1} P_X(1) = (1 - p) + pz^{-1} \quad (2)$$

$$(3)$$

Moment Generating Function of Y

$$M_Y(Z) = E(Z^{-Y}) = E(Z^{-\sum_{i=1}^N X_i}) \quad (4)$$

$$= \prod_{i=1}^N E(Z^{-X_i}) \quad (5)$$

$$= [(1-p) + pz^{-1}]^N \quad (6)$$

$$= \sum_{k=0}^N z^{-k} \binom{N}{k} (1-p)^{N-k} p^k \quad (7)$$

PMF of Y

$$\Pr(Y = k) = \begin{cases} \binom{N}{k} (1-p)^{N-k} p^k, & 0 \leq k \leq N \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

CDF of Y

$$F_Y(k) = \sum_{i=-\infty}^{i=k} \Pr(Y = i) = \begin{cases} 0, & k < 0 \\ \sum_{K=0}^{K=k} \binom{N}{K} (1-p)^{N-K} p^K, & 0 \leq k < N \\ 1, & k \geq N \end{cases} \quad (9)$$

Problem parameters

Given:

- ① $p = 0.05$
- ② $N = 10$

To find: $F_Y(1)$

Graphs for PMF and CDF

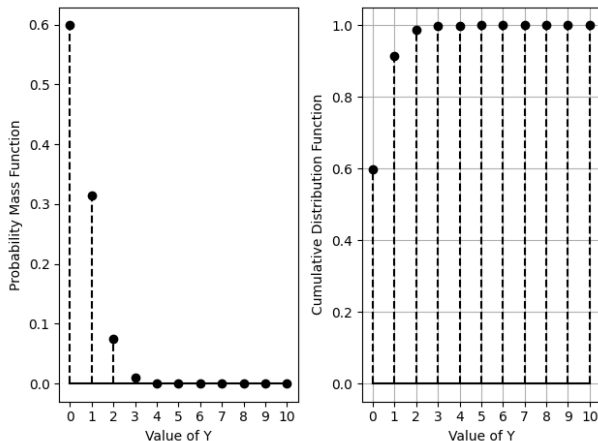


Figure: Graphs of PMF and CDF of Binomial Random Variable Y

Solution

$$F_Y(1) = \sum_{i=0}^{i=1} \binom{10}{i} (1 - 0.05)^{10-i} (0.05)^i \quad (10)$$

$$= (0.95)^{10} + 10(0.95)^9(0.05) = 0.914 \quad (11)$$