Assignment 7 (NCERT Class 12)

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Abstract—This document contains the solution to Question 5 of Exercise 13.1 in Chapter 13 (Probability) of the NCERT Class 12 Mathematics Textbook.

Exercise 13.1, Q5. If $\Pr(A) = \frac{6}{11}$, $\Pr(B) = \frac{5}{11}$ and $\Pr(A \cup B) = \frac{7}{11}$, find

(i) $\Pr(A \cap B)$
(ii) $\Pr(A|B)$
(iii) $\Pr(B|A)$

Solution: Define the random variables X and Y as follows:

$$X = \begin{cases} 0, & \text{event A occurs} \\ 1, & \text{otherwise} \end{cases} \tag{1}$$

$$Y = \begin{cases} 0, & \text{event B occurs} \\ 1, & \text{otherwise} \end{cases} \tag{2}$$

We know that, given events A and B,

$$\Pr(AB) = \Pr(A) + \Pr(B) - \Pr(A + B) \tag{3}$$

and also,

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{4}$$

Using (2) and (3), we get

$$\Pr(X = 0, Y = 0) = \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{4}{11} \tag{5}$$

From (4) and (5),

$$\Pr(X = 0|Y = 0) = \frac{\Pr(X = 0, Y = 0)}{\Pr(Y = 0)} = \frac{4}{5} \tag{6}$$

and

$$\Pr(X = 0|Y = 0) = \frac{\Pr(X = 0, Y = 0)}{\Pr(X = 0)} = \frac{2}{3} \tag{7}$$

The C code `./codes/7_1.c` verifies the solution, within limits of float precision.

Note: Derivation of (3) using Boolean Algebra:

$$LHS = A \vee B = \neg(\neg A \wedge \neg B) \tag{8}$$
$$= 1 - (1 - A)(1 - B) \tag{9}$$
$$= A + B - AB = RHS \tag{10}$$

The following identities were used:

$$\text{(De-Morgan's Law)} \ \neg(A \vee B) = \neg A \wedge \neg B \tag{11}$$
$$\neg A = 1 - A \tag{12}$$
$$A \wedge B = AB \tag{13}$$