Assignment 8

Gautam Singh (CS21BTECH11018)

May 17, 2022

Outline

1. Problem
2. Solution

Problem Statement

(NCERT Class 12, Exercise 13.5.3) There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Solution

Random Variables

1. X_i: Bernoulli random variables with parameter $p, 1 \leq i \leq N$
2. Y: Binomial random variable given by $Y = \sum_{i=1}^{i=N} X_i$

Moment Generating Function of X_i and Y

$$M_Z(X_i) = \sum_{k=-\infty}^{k=\infty} z^{-k} P_X(k) \tag{1}$$

$$= P_X(0) + z^{-1} P_X(1) = (1-p) + pz^{-1} \tag{2}$$

$$\tag{3}$$

Moment Generating Function of Y

$$M_Y(Z) = E(Z^{-Y}) = E(Z^{-\sum_{i=1}^{i=N} X_i}) \tag{4}$$

$$= \prod_{i=1}^{i=N} E(Z^{-X_i}) \tag{5}$$

$$= [(1-p) + pz^{-1}]^N \tag{6}$$

$$= \sum_{k=0}^{k=N} z^{-k}(\binom{N}{k}(1-p)^{N-k}p^k) \tag{7}$$

PMF of Y

$$\Pr(Y = k) = \begin{cases} \binom{N}{k}(1-p)^{N-k}p^k, & 0 \leq k \leq N \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

CDF of Y

$$F_Y(k) = \sum_{i=-\infty}^{i=k} \Pr(Y = i) = \begin{cases} 0, & k < 0 \\ \sum_{K=0}^{K=k} \binom{N}{K}(1-p)^{N-K}p^K, & 0 \leq k < N \\ 1, & k \geq N \end{cases} \tag{9}$$

Problem parameters

Given:

1. $p = 0.05$
2. $N = 10$

To find: $F_Y(1)$

Solution

$$F_Y(1) = \sum_{i=0}^{i=1} \binom{10}{i} (1-0.05)^{10-i}(0.05)^i \tag{10}$$

$$= (0.95)^{10} + 10(0.95)^9(0.05) = 0.914 \tag{11}$$