Assignment 10

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Outline

Problem Statement

(Papoulis/Pillai Exercise 2-26) Show that a set S with n elements has

$$\frac{n(n-1)\dots(n-k+1)}{1.2\dots k} = \frac{n!}{k!(n-k)!} \tag{1}$$

subsets of k elements.

Solution

We begin by choosing any k elements of a set consisting of n elements. The first element can be chosen in n ways, the second in $n-1$ ways, and so on. The k^{th} element can be chosen in $n-k+1$ ways. However, the order of elements in a set does not matter. This gives a total of

$$\frac{n(n-1)\dots(n-k+1)}{1.2\dots k} \tag{2}$$

$$= \frac{n(n-1)\dots 1}{(1.2\dots k)(1.2\dots(n-k))} \tag{3}$$

$$= \frac{n!}{k!(n-k)!} = \binom{n}{k} \tag{4}$$

subsets containing k elements. Here, $0 \leq k \leq n$. The Python code `codes/10_1.py` verifies the identity.