Assignment 11

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Outline

Problem Statement

(Papoulis/Pillai Exercise 3-4) A coin with $\Pr(h) = p = 1 - q$ is tossed n times. Show that the probability that the number of heads is even equals $0.5[1 + (q - p)^n]$.

Solution

PMF of Y

$$\Pr(Y = k) = \begin{cases} \binom{n}{k} p^{(n-k)} q^k, & 0 \le k \le n \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$\implies \Pr(Y \equiv 0 \pmod 2) = \binom{n}{0} p^n + \binom{n}{2} p^{(n-2)} q^2 + \dots \tag{2}$$

We have,

$$\Pr(Y \equiv 0 \pmod 2) = \binom{n}{0} p^n + \binom{n}{2} p^{(n-2)} q^2 + \dots \tag{3}$$

$$= \frac{1}{2}(2(\binom{n}{0} p^n + \binom{n}{2} p^{(n-2)} q^2 + \dots)) \tag{4}$$

$$= 0.5\Bigg[\left(\binom{n}{0}p^n + \binom{n}{1}p^{(n-1)}q + \dots\right) + \left(\binom{n}{0}p^n - \binom{n}{1}p^{(n-1)}q + \dots\right)\Bigg] \tag{5}$$

$$= 0.5\left[\sum_{k=0}^{k=n}\binom{n}{k}q^{(n-k)}p^k + \sum_{k=0}^{k=n}\binom{n}{k}(-1)^k q^{(n-k)}p^k\right] \tag{6}$$

$$= 0.5[(q+p)^n + (q-p)^n] \tag{7}$$

$$= 0.5[1 + (q-p)^n] \tag{8}$$

as deisred. This is verified in `codes/11_1.py`.