Assignment 11

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Outline

Problem Statement

(Papoulis/Pillai Example 3-4, Generalised Version) Given n experiments $S_1, S_2, \ldots, S_n$ we define experiment S as

$$S = S_1 \times \ldots \times S_n \tag{1}$$

where events in S are of the form $A_1 \times \ldots \times A_n$ where $A_i \subset S_i$. If the experiments are independent and $\Pr_i(A_i)$ is the probability of the event A_i in experiment S_i, then

$$\Pr(A_1 \times \ldots \times A_n) = \prod_{i=1}^{i=n} \Pr_i(A_i) \tag{2}$$

Solution

Note that since all the experiments are independent,

$$\Pr_i(A_i) = \Pr(S_1 \times \ldots \times A_i \times \ldots \times S_n) \tag{3}$$

$$\Pr_i(A_i)\Pr_j(A_j) = \Pr(A_i A_j) \tag{4}$$

and also, for n-component Boolean Algebra,

$$(A_1 \ldots A_n) \times (B_1 \ldots B_n) = (A_1 \times \ldots \times A_n)(B_1 \times \ldots \times B_n) \tag{5}$$

Therefore, since all the $S_i = 1$,

$$\prod_{i=1}^{i=n} \Pr_i(A_i) = \prod_{i=1}^{i=n} \Pr(S_1 \times \ldots \times A_i \times \ldots \times S_n) \tag{6}$$

$$= \Pr((A_1 \times \ldots \times S_n) \ldots (S_1 \times \ldots \times A_n)) \tag{7}$$

$$= \Pr((A_1 S_1) \times (A_2 S_2) \times \ldots \times (A_n S_n)) \tag{8}$$

$$= \Pr(A_1 \times \ldots \times A_n) \tag{9}$$

as desired.