Assignment 9 (NCERT Class 12)

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Abstract—**This document contains the solution to Question 3 of Exercise 13.5 in Chapter 13 (Probability) of the NCERT Class 12 Mathematics Textbook.**

Exercise 13.5, Q3. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Solution: Let $X_i, 1 \le i \le N$ represent N Bernoulli random variables with parameter p. Then,

$$\Pr(X_i = k) = \begin{cases} 1-p, & k=0 \\ p, & k=1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Let Y be a random variable given by

$$Y = \sum_{i=1}^{i=N} X_i \tag{2}$$

Using Equation 1 the moment generating function of X_i is given by

$$M_Z(X_i) = \sum_{k=-\infty}^{k=\infty} z^{-k} P_X(k) \tag{3}$$
$$= P_X(0) + z^{-1}P_X(1) = (1-p) + pz^{-1} \tag{4}$$

Since all the X_i are independent and identically distributed, the moment generating function of Y is

$$M_Y(Z) = E(Z^{-Y}) = E(Z^{-\sum_{i=1}^{i=N} X_i}) \tag{5}$$
$$= \prod_{i=1}^{i=N} E(Z^{-X_i}) \tag{6}$$
$$= [(1-p) + pz^{-1}]^N \tag{7}$$
$$= \sum_{k=0}^{k=N} z^{-k}(\binom{N}{k}(1-p)^{N-k}p^k) \tag{8}$$

The PMF of the Binomial random variable Y is

$$\Pr(Y = k) = \begin{cases} \binom{N}{k}(1-p)^{N-k}p^k, & 0 \le k \le N \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

Therefore, the CDF of Y is given by

$$\mathrm{F}_Y(k) = \sum_{i=-\infty}^{i=k} \Pr(Y = i)$$
$$= \begin{cases} 0, & k < 0 \\ \sum_{K=0}^{K=k}\binom{N}{K}(1-p)^{N-K}p^K, & 0 \le k < N \\ 1, & k \ge N \end{cases} \tag{10}$$

For this problem we have $N = 10$ and $p = \frac{5}{100} = 0.05$. As the number of items is large, the value of p does not change, and we can apply the binomial distribution. We are required to find $\mathrm{F}_Y(1)$. Hence,

$$\mathrm{F}_Y(1) = \sum_{i=0}^{i=1} \binom{10}{i}(1-0.05)^{10-i}(0.05)^i \tag{11}$$
$$= (0.95)^{10} + 10(0.95)^9(0.05) = 0.914 \tag{12}$$

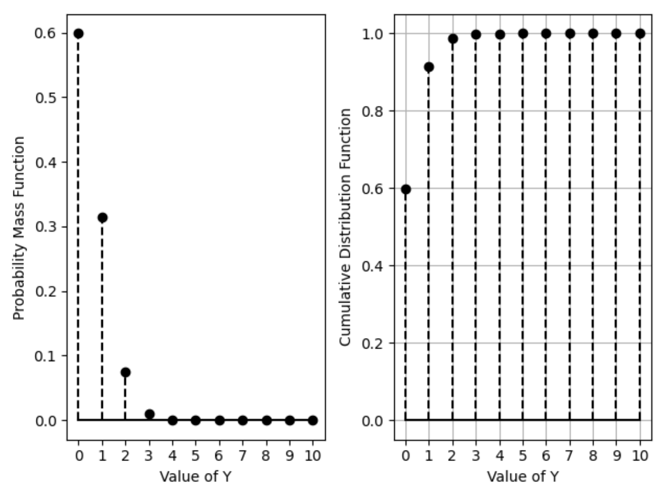

Fig. 1: PMF and CDF for the given situation. Code: `codes/9_1.py`

The answer is verified in `codes/9_2.c` (to 3 d.p.).