CS5760: Cryptanalysis of DES and DES-like Iterated Cryptosystems

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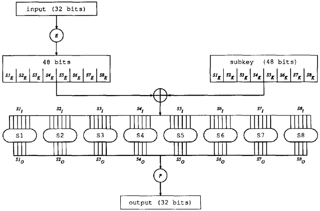
   - *Invariant* in key mixing with subkey S_K to get $S_I = S_E \oplus S_K$.
   - *Linear* in permutation P on S_O after S boxes.
   - *Invariant* in XOR operation connecting rounds.



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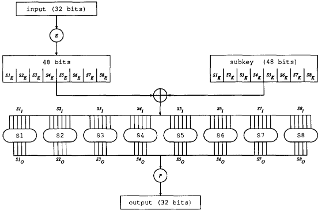
4. S boxes are *nonlinear*. Probability analysis performed between input and output XOR.



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3. This joint PMF can reduce the number of possible (sub)keys. Used to drive choice for the plaintext XOR.
   - $\approx 80\%$ entries are non-zero/possible for each S box (some have lesser percentages).
   - Given Si'_I and Si'_O, we can narrow down Si_K to a few possibilities.

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1. Suppose $Si_I' = Si_I \oplus Si_I^*$ is the input XOR to the i^{th} S box, and Si_O' is the output XOR $(1 \le i \le 8)$.
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   - $\approx 80\%$ entries are non-zero/possible for each S box (some have lesser percentages).
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4. i^{th} S box contributes probability p_i for $Si_I' \to Si_O'$.
   - For $X \to Y$ over a round, $P = \prod_i p_i$.
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Desirable for cryptanalysis: high P with large n.

Characteristic

Formalizes notion of high-probability plaintext XORs.

Definition 1 (Characteristic)

An *n-round chracteristic* is a tuple $\Omega = (\Omega_P, \Omega_\Lambda, \Omega_T)$ where $\Omega_P = (L', R')$ and $\Omega_T = (l', r')$ are m bit numbers, $\Omega_\Lambda = (\Lambda_1, \dots, \Lambda_n)$, $\Lambda_i = (\lambda_I^i, \lambda_O^i)$ and $\lambda_I^i, \lambda_O^i, L', R', l', r'$ are $\frac{m}{2}$ bit numbers and m is the block size of the cryptosystem satisfying

$$\lambda_I^1 = R' \tag{1}$$

$$\lambda_I^2 = L' \oplus \lambda_O^1 \tag{2}$$

$$\lambda_I^n = r' \tag{3}$$

$$\lambda_I^{n-1} = l' \oplus \lambda_O^n \tag{4}$$

$$\forall\ 1 < i < n,\ \lambda_O^i = \lambda_I^{i-1} \oplus \lambda_I^{i+1} \tag{5}$$

Characteristic

Definition 2 (Right Pair)

A *right pair with respect to an n-round characteristic* $\Omega = (\Omega_P, \Omega_\Lambda, \Omega_T)$ *and an independent key* K is a pair for which $P' = \Omega_P$ and for each round i of the first n rounds of the encryption of the pair using K the input XOR of the i^{th} round equals λ_I^i and the output XOR of the F function equals λ_O^i. Pairs that do not satisfy these conditions are called *wrong pairs*.

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Definition 3 (Probability of a Round of a Characteristic)

Round i of an n-round characteristic Ω has probability p_i^Ω if $\lambda_I^i \to \lambda_O^i$ with probability p_i^Ω by the F function.

Probability of a Characteristic

Definition 4 (Probability of a Characteristic)

An n-round characteristic Ω has probability p^{Ω} given by

$$p^{\Omega} = \prod_{i=1}^{n} p_i^{\Omega} \tag{6}$$

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Theorem 5 (Probability of a Characteristic and Right Pairs)

The formally defined probability of a characteristic $\Omega = (\Omega_P, \Omega_\Lambda, \Omega_T)$ is the probability that any fixed plaintext pair satisfying $P' = \Omega_P$ is a right pair when random independent keys are used.

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Proof Idea.

Keys *randomize* the inputs to the S boxes in each round. □

Example of a Characteristic

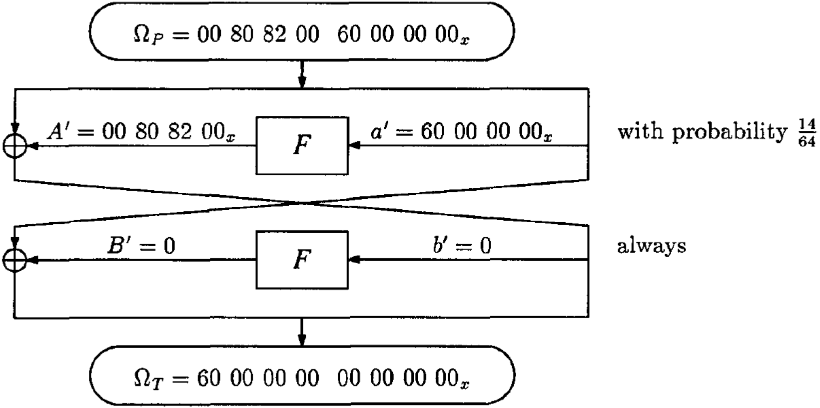

Figure 2: Example of a two-round characteristic with probability $\frac{14}{64}$.

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Definition 6 (Signal-to-Noise Ratio)

The ratio between the number of right pairs and the average count of incorrect subkeys in a counting scheme is called the *signal to noise ratio of the counting scheme* and is denoted by S/N.

Computing the SNR

Consider the variables shown in Table 1.

Variable	Definition
p	Probability of the characteristic
m	Number of created pairs
α	Average count per analyzed pair
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Table 1: Table of variables to compute the SNR.

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Then,

$$S/N = \frac{m \cdot p}{\frac{m \cdot \beta \cdot \alpha}{2^k}} = \frac{2^k \cdot p}{\alpha \cdot \beta} \tag{7}$$

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Definition 7 (Quartet and Octet)

A *quartet* is a structure of four ciphertexts that simultaneously contains two ciphertext pairs of one characteristic and two ciphertext pairs of a second characteristic. An *octet* is a structure of eight ciphertexts that simultaneously contains four ciphertext pairs of each of three characteristics.

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4. Quartets save $\frac{1}{2}$ of the data and octets save $\frac{2}{3}$ of the data.

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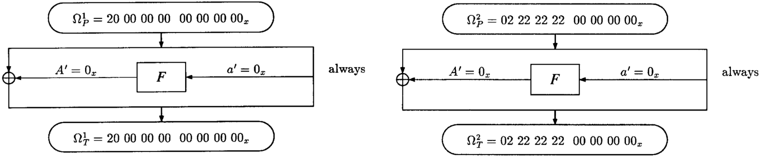

Figure 3: Characteristics used for cryptanalysis of DES reduced to four rounds.

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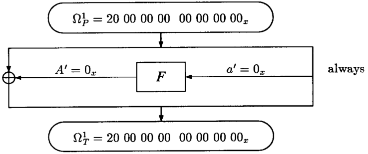
3. Example of a *3R-attack*. There are *three* extra rounds after the characteristic is applied.



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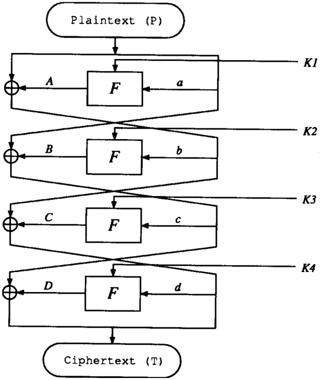

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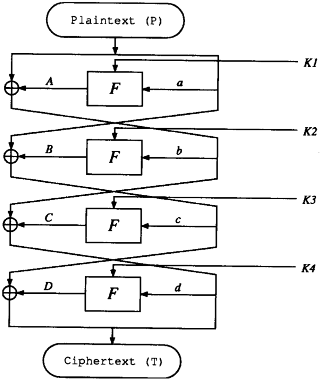
$$c' = D' \oplus l' = a' \oplus B' \implies D' = B' \oplus l' \quad (8)$$



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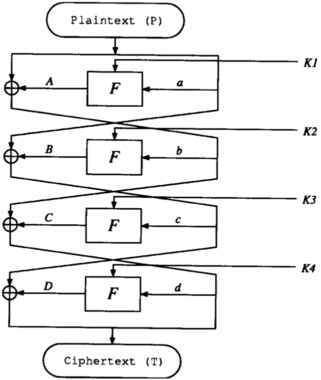

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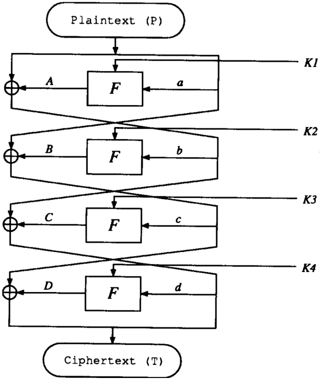

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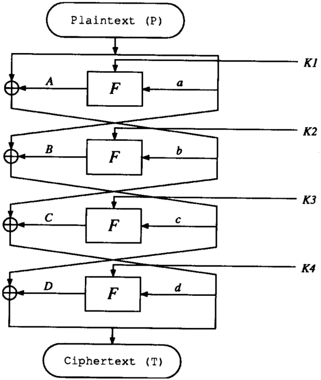
   - 28 bits of B' are zero and hence we can find *28 bits of D'*.



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   - In the second round S2, ..., S8 receive zero XOR input.
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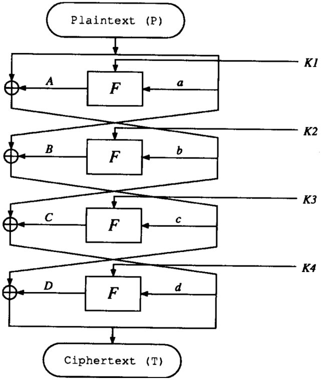
   - We already know $d' = r'$. So, we employ a counting approach to get $K4$.



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1. To get Si_{Kd} for $2 \leq i \leq 8$, we verify (9).

$$S(S_{Ed} \oplus S_{Kd}) \oplus S(S^*_{Ed} \oplus S_{Kd}) = S'_{Od} \tag{9}$$

2. Only *one* plaintext pair is needed since characteristic probability is 1.
3. We recover $7 \times 6 = 42$ key bits of $K4$, which correspond to 42 bits of the master key.
4. Exhaustively search the other 14 key bits to get the entire master key.
5. We have used the key schedule to our advantage here? *What if all the keys were independent?*

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3. Since $a' = A' = 0_x$, all keys are equally likely. Other characteristics Ω^3 and Ω^4 are chosen such that
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    - $S'_{Ea} \neq 0_x$ for all S boxes for both characteristics.
    - For every S box, the S'_{Ea} values differ between the characteristics.
    - Similar counting methods used to get $K1$ and $K2$.
4. 16 chosen plaintexts are needed for this attack.
    - 8 pairs of Ω^1 and Ω^2 each.
    - 4 pairs of Ω^3 and Ω^4 each.

To reduce the data needed, two octets are used.

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