CS5760: Topics in Cryptanalysis **2025**

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3.1 Cryptanalysis of DES Reduced to 8 Rounds

DES reduced to 8 rounds uses a 5-round characteristic with probability approximately $\frac{1}{10486}$ as shown in Figure 3.1.

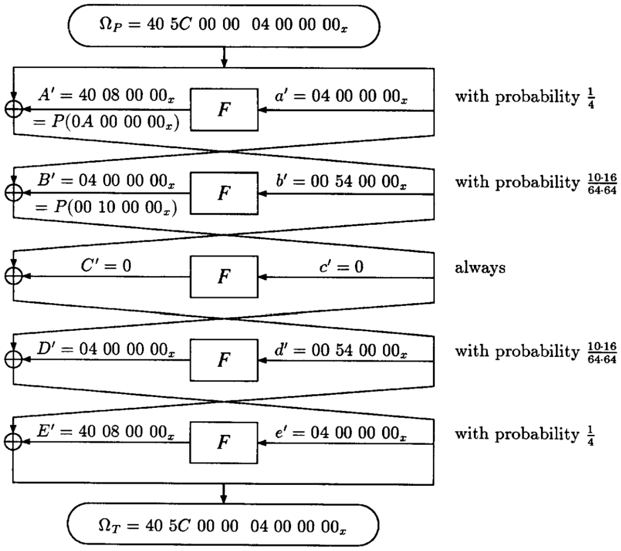


Figure 3.1: 5 round characteristic used to cryptanalyze DES reduced to 8 rounds.

From the characteristic, it is evident that

$$f' = d' \oplus E' = b' \oplus A' = L' = \texttt{40 5C 00 00}. \tag{3.1}$$

Thus, for a right pair, five S boxes S2, S5, ..., S8 have zero input XORs in the sixth round. Using

$$H' = l' \oplus g' = l' \oplus e' \oplus F' \tag{3.2}$$

and the fact that $h' = r'$, we can count on $5 \cdot 6 = 30$ key bits of $K8$. The signal to noise ratio is $S/N = \frac{2^{30}}{4^5 \cdot 10486} \approx 100$. However, due to the large memory requirement of 2^{30} locations, we count on fewer key bits. Further, due to the small probability of the characteristic, we require many plaintexts, which makes the clique method slow. Notice that each S box discards 20 % of wrong pairs. Thus, counting on 24 key bits has $S/N = \frac{2^{24}}{4^4 \cdot 0.8 \cdot 10486} \approx 7.8$ and counting on 18 key bits has $S/N = \frac{2^{18}}{4^3 \cdot 0.8^2 \cdot 10486} \approx 0.6$.

3.1.1 Modifying the Characteristic

By reducing the number of key bits to count, we can also choose which key bits are to be counted in order to improve the signal to noise ratio. Notice that

$$e' = \texttt{04 00 00 00} \rightarrow E' = P(\texttt{0W 00 00 00}) = \texttt{X0 0Y Z0 00} \tag{3.3}$$

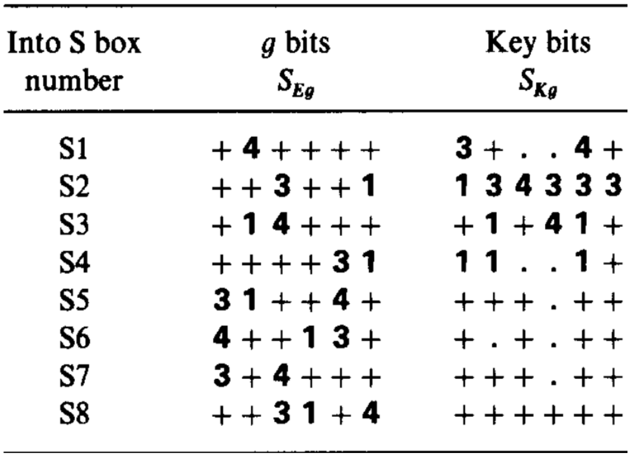
where $W \in \{0,1,2,3,8,9,A,B\}$, $X,Z \in \{0,4\}$, $Y \in \{0,8\}$. Hence, we have $f' = d' \oplus E' = \texttt{X0 5V Z0 00}$ where $V = Y \oplus 4$. If $Z = 0$, then necessarily $E' = \texttt{40 08 00 00}$ and this happens with probability $\frac{16}{64}$. All other combinations involving $Z = 4$ occur with probability $\frac{20}{64}$.

Although we cannot count on $S5_{Kh}$, one can check $S5'_{Eh} \rightarrow S5'_{Oh}$ which is satisfied by approximately 80 % of the pairs. Thus, the modified probability of $e' \rightarrow E'$ is $\frac{16}{64} + 0.8\frac{20}{64} = \frac{1}{2}$. This doubles the probability of the characteristic Ω_P to $\frac{1}{5243}$ and consequently doubles the S/N for counting on 24 bits and 18 bits of $K8$ to 15.6 and 1.2 respectively.

Counting on 24 subkey bits, we only require about five right pairs due to the high S/N. This gives us approximately 25000 plaintext pairs. For 18 subkey bits, we need about 150000 pairs. The average count per key is $\frac{150000 \times 4^3 \times 0.8^2}{2^18} = 24$ and the right key is counted an additional $\frac{150000}{5243} = 29$ times, giving a total count of $24 + 29 = 53$ for the right key.

However, this finds us 18 subkey bits, say entering S6, S7 and S8. To find the other 12 subkey bits entering S2 and S5 in the eighth round, we filter the pairs that correspond to the subkey values in S6, S7 and S8. The expected number of pairs is then 53. Counting on the remaining 12 bits using these right pairs leads to a higher S/N which can filter more pairs.

Now, using the known 30 subkey bits of $K8$, we can find the dependence of the bits of g and S_{Kg} on $K8$. This dependence is shown in Figure 3.2.



Into S box number	g bits S_{Eg}	Key bits S_{Kg}
S1	+ 4 + + + +	3 + . . 4 +
S2	+ + 3 + + 1	1 3 4 3 3 3
S3	+ 1 4 + + +	+ 1 + 4 1 +
S4	+ + + + 3 1	1 1 . . 1 +
S5	3 1 + + 4 +	+ + + . + +
S6	4 + + 1 3 +	+ . + . + +
S7	3 + 4 + + +	+ + + . + +
S8	+ + 3 1 + 4	+ + + + + +

Figure 3.2: Dependence of bits at the seventh round on those of the eighth round. '+' indicates dependence on known key bits, '.' indicates dependence on other key bits and a number indicates dependence on unknown key bits that enter the corresponding S box in the eighth round.