The Retracing Boomerang Attack

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3. Uncovers a hidden relationship between boomerang attacks and two other cryptanalysis techniques: yoyo game and mixture differentials.

The Boomerang Attack

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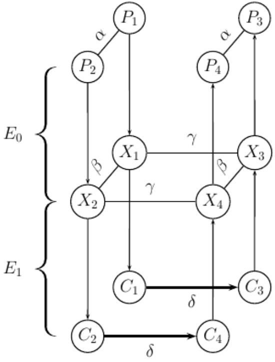

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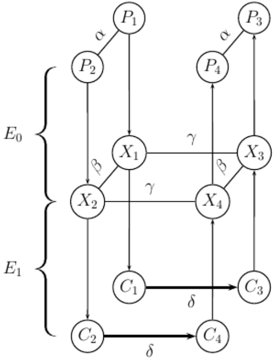
2. We can build a distinguisher that can distinguish E from a truly random permutation in $\mathcal{O}((pq)^{-2})$ plaintext pairs.



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The Boomerang Distinguisher

Algorithm 1 The Boomerang Attack Distinguisher

Initialize a counter $ctr \leftarrow 0$.
Generate $(pq)^{-2}$ plaintext pairs (P_1, P_2) such that $P_1 \oplus P_2 = \alpha$.
for all pairs (P_1, P_2) **do**
    Ask for the encryption of (P_1, P_2) to (C_1, C_2).
    Compute $C_3 = C_1 \oplus \delta$ and $C_4 = C_2 \oplus \delta$. ▷ *δ-shift*
    Ask for the decryption of (C_3, C_4) to (P_3, P_4).
    if $P_3 \oplus P_4 = \alpha$ **then**
        Increment ctr
if $ctr > 0$ **then**
    return This is the cipher E
else
    return This is a random permutation

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3. Denoting this part of the intermediate state by X_j,

$$(X_1)_j \oplus (X_2)_j = (X_1)_j \oplus (X_3)_j = (X_2)_j \oplus (X_4)_j = \Delta \qquad (1)$$

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5. Distinguisher probability increases by a factor of $(q')^{-1}$, where q' is the probability of the differential characteristic in f_j.

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4. Probabilities are low with large l. Still, the yoyo technique has been used to attack AES reduced to 5 rounds.

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Definition 1 (Mixture)

Suppose $P_i \triangleq (\rho_1^i, \rho_2^i, \ldots, \rho_t^i)$. Given a plaintext pair (P_1, P_2), we say (P_3, P_4) is a *mixture counterpart* of (P_1, P_2) if for each $1 \leq j \leq t$, the quartet $(\rho_j^1, \rho_j^2, \rho_j^3, \rho_j^4)$ consists of two pairs of equal values or of four equal values. The quartet (P_1, P_2, P_3, P_4) is called a *mixture*.

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3. Mixture differentials have been applied to AES reduced up to 6 rounds.

The Retracing Boomerang Framework

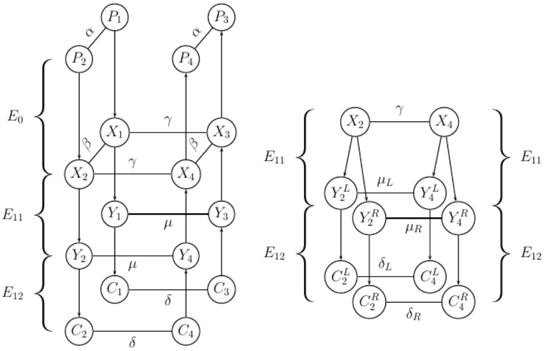

Figure 2: The retracing boomerang attack.

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2. Both attacks use the setup shown in Figure 2.
3. Although the additional split looks restrictive, it applies for a wide class of block ciphers such as SASAS constructions.
4. Further, we assume that E_{12} can be split into two parts of size b and $n - b$ bits, call these functions E_{12}^L and E_{12}^R, with characteristic probabilities q_2^L and q_2^R respectively.

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5. Increases the probability of the boomerang distinguisher by $(q_2^L)^{-1}$.
6. Any possible characteristic of E_{12}^L has probability at least 2^{-b+1}, thus overall probability increases by a factor of at most 2^{b-1}. On the other hand, filtering only leaves 2^{-b+1} of the pairs, so *no apparent gain?*

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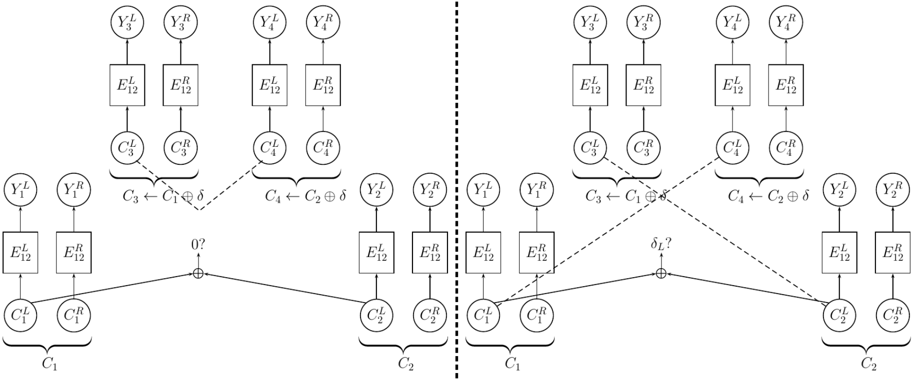

Figure 3: A shifted quartet (dashed lines indicate equality).

Advantages of Filtering

1. *Improving the signal to noise ratio.* Improving probability by a factor of $(q_2^L)^{-1}$ improves SNR which ensures a higher fraction of the filtered pairs on average satisfy $P_3 \oplus P_4 = \alpha$. Then, $\beta \xrightarrow{p} \alpha$ in the backward direction for (X_3, X_4) can be replaced by a truncated characteristic $\beta \xrightarrow{p'} \alpha'$ of higher probability.

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3. *Reducing the time complexity.* Filtering can also reduce time complexity if dominated by the analysis of (P_3, P_4).

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4. Similar to the core step used in the yoyo attack on AES.

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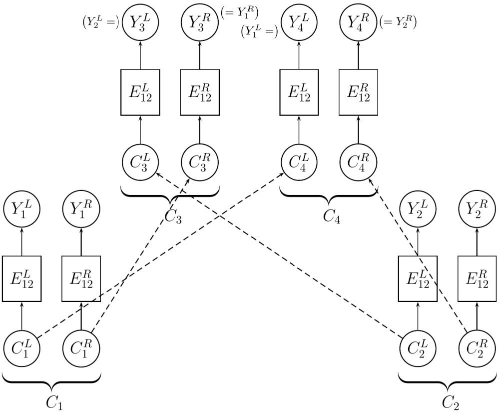

Figure 4: A mixture quartet of ciphertexts (dashed lines indicate equality).

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   - More 'friend pairs' can be constructed in the shifting variant.

Description of AES

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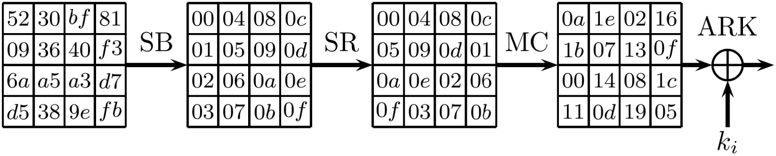

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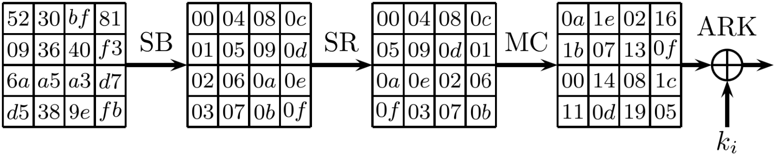

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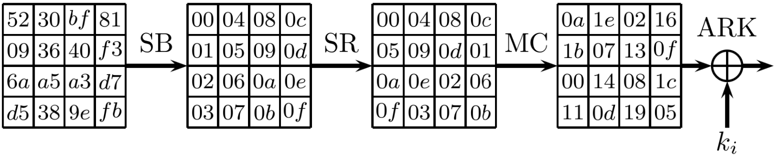

Figure 5: An AES round.

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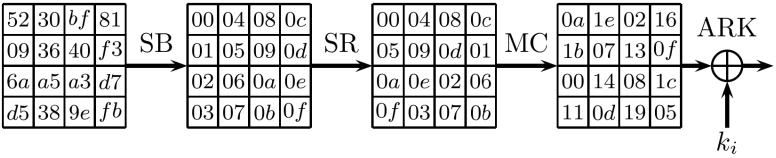

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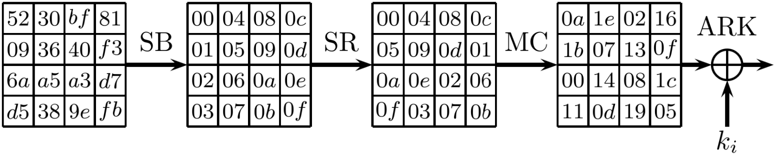

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6. Attack quartets of k_{-1}. Friend pairs of (Z_3, Z_4) used to get more information.

Algortihm of Yoyo Attack

Algorithm 2 Yoyo Attack on Five Round AES

Ask for the encryption of 2^6 pairs (P_1, P_2) of chosen plaintexts with non-zero difference only in bytes $0, 5, 10, 15$.
for all corresponding ciphertext pairs (C_1, C_2) **do**
    Let (C_3^j, C_4^j), $j = 1, 2, 3, 4$ be the mixture counterparts of the pair (C_1, C_2).
    Ask for the decryption of the ciphertext pairs and consider the pairs (Z_3^j, Z_4^j).
    for all $l \in \{0, 1, 2, 3\}$ **do**
        Assume all four pairs (Z_3^j, Z_4^j) and the pair (Z_1, Z_2) have zero difference in byte l.
        Use the assumption to extract bytes $0, 5, 10, 15$ of k_{-1}.
        if a contradiction is reached **then**
            Increment l
            if $l > 3$ **then** Discard the pair
        else
            Using $Z_3^j \oplus Z_4^j = 0$ in the entire l-th inverse shifted column, attack the three remaining columns of round 0 (sequentially) and decude the rest of k_{-1}.

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3. Denote the value of byte m before MC operation of round 0 by W_m, and WLOG let $l = 0$. Then,

$$Z_0 = 02_x \cdot W_0 \oplus 03_x \cdot W_1 \oplus 01_x \cdot W_2 \oplus 01_x \cdot W_3. \tag{4}$$

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4. Adversary guesses $k_{-1,\{0,5\}}$ by computing the following for $j = 1, 2, 3$ and storing the concatenated 24-bit value in a hash table.

$$02_x \cdot ((W_3^j)_0 \oplus (W_4^j)_0) \oplus 03_x \cdot ((W_3^j)_1 \oplus (W_4^j)_1) \tag{5}$$

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5. Similarly, the adversary does this for $k_{-1,\{10,15\}}$, computing

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7. This 24-bit filtering leaves 2^8 candidates for $k_{-1,\{0,5,10,15\}}$. They are checked using the conditions $(Z_3^4)_0 = (Z_4^4)_0$ and $(Z_1)_0 = (Z_2)_0$.

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9. The time complexity is now reduced to $2^6 \cdot 4 \cdot 2^{16} = 2^{24}$ operations, which is roughly equivalent to less than 2^{23} encryptions.

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   - Eliminating key bytes using friend pairs.

Specific Choice of Plaintexts

1. Choose plaintexts with non-zero difference *only in bytes 0 and 5.* Here, $(Z_1)_0 = (Z_2)_0$ leaves 2^8 candidates for $k_{-1,\{0,5\}}$, given by

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6. Obtain 2^8 candidates for $k_{-1,\{0,5\}}$ in about 2^8 operations per pair.

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7. Each pair requires 2^7 friend pairs to find one that satisfies (8) with high probability. Total data complexity is increased to about 2^{15}.

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3. For each plaintext pair, create 2^7 friend pairs (P_1^j, P_2^j) such that for each j, $P_1^j \oplus P_2^j = P_1 \oplus P_2$ and $(P_1^j)_{\{0,5,10,15\}} = (P_1)_{\{0,5,10,15\}}$.

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   3. Find a j for which (8) is satisfied. Perform an MITM attack on column 0 of round 0 using (P_3^j, P_4^j) to obtain 2^8 candidates for $k_{-1,\{0,5,15\}}$.

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   5. If contradiction, go to the next value of l. If contradiction for all l, discard this pair and go to the next pair.

Attack Algorithm

4. For each plaintext pair (P_1, P_2) and for each $l \in \{0, 1, 2, 3\}$, do the following. ($l = 0$ taken below)
   1. Use (7) to compute and store all 2^8 candidates for $k_{-1,\{0,5\}}$ in a table.
   2. Use the boomerang process to obtain pairs (P_3, P_4) and (P_3^j, P_4^j).
   3. Find a j for which (8) is satisfied. Perform an MITM attack on column 0 of round 0 using (P_3^j, P_4^j) to obtain 2^8 candidates for $k_{-1,\{0,5,15\}}$.
   4. Perform another MITM attack on column 0 of round 0 using two plaintext pairs $(P_3^{j'}, P_4^{j'})$. This gives a possible value for $k_{-1,\{0,5,10,15\}}$.
   5. If contradiction, go to the next value of l. If contradiction for all l, discard this pair and go to the next pair.
5. Using a pair (P_1, P_2) for which no contradiction occurred, perform MITM attacks on columns 1, 2 and 3 of round 0 using the fact that $Z_3 \oplus Z_4$ equals 0 in the l-th inverse shifted column to recover k_{-1}.

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2. Increasing the number of initial pairs and friend pairs per initial pair boosts success probability. With 64 pairs and 128 friend pairs per initial pair, the probability of success is $(1-e^{-1})^2 \approx 0.4$
3. Another way to boost succees probability is to find other ways to cancel terms in (6). For instance, if there exist j, j' such that $\{(P_3^j)_{10}, (P_4^j)_{10}\} = \{(P_3^{j'})_{10}, (P_4^{j'})_{10}\}$, we can take the XOR of (6) to cancel the effect of $k_{-1,10}$, thus increasing the success probability even when there is no pair that satisfies (8).

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6. Memory complexity of the attack remains at 2^9, like yoyo attack.
7. Time complexity dominated by MITM attacks that take 2^{16} operations each. Taking one AES operation equivalent to 80 S-box lookups and adding it to the number of queries gives us a total of $2^{16.5}$ encryptions.

Attack on Five Round AES with a Secret S-box

1. Retracing boomerang attack recovers the secret key without fully recovering the secret S-box (the S-box is recovered upto an affine transformation in $GF(2^8)$).

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2. Idea is to exploit the fact that with probability 2^{-6}, the pair (Z_3, Z_4) has zero difference in an inverse shifted column.
3. This does not depend on the specific structure of MC and SB operations, hence it can be applied to key-dependent variants as well.

Setting up a System of Linear Equations

1. Assume WLOG the retracing boomerang produces zero difference in byte 0 of state Z, or $(Z_3)_0 \oplus (Z_4)_0 = 0$. (4) can be rewritten as

$$\begin{aligned} 0 &= (Z_3)_0 \oplus (Z_4)_0 \qquad (9) \\ &= 02_x \cdot ((W_3)_0 \oplus (W_4)_0) \oplus 03_x \cdot ((W_3)_1 \oplus (W_4)_1) \\ &\quad \oplus 01_x \cdot ((W_3)_2 \oplus (W_4)_2) \oplus 01_x \cdot ((W_3)_3 \oplus (W_4)_3). \qquad (10) \end{aligned}$$

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2. Note that $(W_3)_j = SB(P_3 \oplus k_{-1,j'})$ for $j = 0, 1, 2, 3$ where $j' = SR^{-1}(j)$.
3. Define $4 \cdot 256 = 1024$ variables $x_{m,j} = SB(m \oplus k_{-1,j'})$ for $m \in \mathbb{F}_q$ and $j = 0, 1, 2, 3$. Each plaintext pair P_1, P_2 satisfying (10) provides a linear equation in $x_{m,j}$.

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6. Since differences are used (10), we can recover the S-box with an invertible linear transformation over $GF(2^8)$. That is, we can only obtain functions S_0, S_1, S_2, S_3 such that

$$S_j(x) = L_0(SB(x \oplus k_{-1,j'})), \tag{11}$$

for some unknown linear transformation L_0. Similar linear transformations L_t will be obtained for column t.

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5. In about 2^{12} operations, k_{-1} is determined upto the value of $k_{-1,0}$. These 2^8 possibilities can be exhaustively searched.

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5. Memory complexity dominated by the memory required for solving the equations, which is about 2^{17} 128-bit blocks.

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5. Time complexity of this attack is $\mathcal{O}(2^{n/2}(pq)^{-1})$ using hash tables.

The Retracing Rectangle Attack

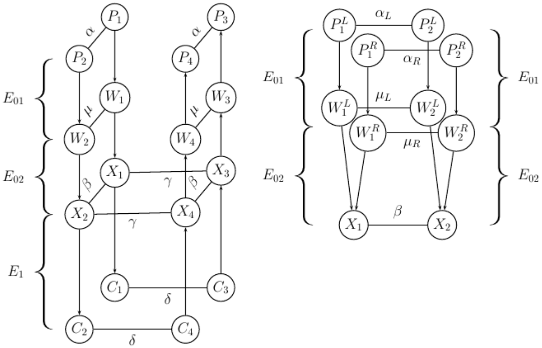

Figure 6: The retracing rectangle attack.

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8. Instead of checking all $\binom{|\mathcal{S}|}{2}$ pairs, hash table can check all quartets in $\mathcal{O}(|\mathcal{S}|)$ time.

Mixing Variant and Relation to Mixture Differentials

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2. As this choice forces $\{P_1^R, P_2^R\} = \{P_3^R, P_4^R\}$, the probability of the rectangle distinguisher is increased by a factor of $(p_1^L p_1^R)^{-1}$.