The Retracing Boomerang Attack

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2. Brings the attack complexity down to $2^{16.5}$ encryptions.
3. Uncovers a hidden relationship between boomerang attacks and two other cryptanalysis techniques: yoyo game and mixture differentials.

The Yoyo Game[1]

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[1] Sondre Rønjom, Navid Ghaedi Bardeh, and Tor Helleseth. *Yoyo Tricks with AES*. 2017. URL: https://eprint.iacr.org/2017/980 (visited on 04/14/2025). Pre-published.

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3. All pairs of intermediate values (X_{2l+1}, X_{2l+2}) satisfy some property (such as zero difference in some part).
4. Probabilities are low with large l. Still, the yoyo technique has been used to attack AES reduced to 5 rounds.

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Mixture

Definition 1 (Mixture)

Suppose $P_i \triangleq (\rho_1^i, \rho_2^i, \ldots, \rho_t^i)$. Given a plaintext pair (P_1, P_2), we say (P_3, P_4) is a *mixture counterpart* of (P_1, P_2) if for each $1 \leq j \leq t$, the quartet $(\rho_j^1, \rho_j^2, \rho_j^3, \rho_j^4)$ consists of two pairs of equal values or of four equal values. The quartet (P_1, P_2, P_3, P_4) is called a *mixture*.

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1. If (P_1, P_2, P_3, P_4) is a mixture, then XOR of the intermediate values (X_1, X_2, X_3, X_4) is zero.
2. $X_1 \oplus X_3 = \gamma \implies X_2 \oplus X_4 = \gamma$. Hence, for $\gamma \xrightarrow{q} \delta$ in E_1, $C_1 \oplus C_3 = C_2 \oplus C_4 = \delta$ with probability q^2.

The SimpleSWAP Algorithm[2]

Algorithm 1 is a primitive used to generate mixture counterparts.

Algorithm 1 Swaps the first word where texts are different and returns one word.

function $\textsc{SimpleSWAP}(x^0, x^1)$ $\triangleright$ $x^0 \neq x^1$
    $x'^0, x'^1 \leftarrow x^0, x^1$
    for i from 0 to 3 **do**
        if $x_i^0 \neq x_i^1$ **then**
            $x_i'^0, x_i'^1 \leftarrow x_i^1, x_i^0$
            return x'^0, x'^1

[2] Rønjom, Bardeh, and Helleseth, *Yoyo Tricks with AES*.

The Retracing Boomerang Framework

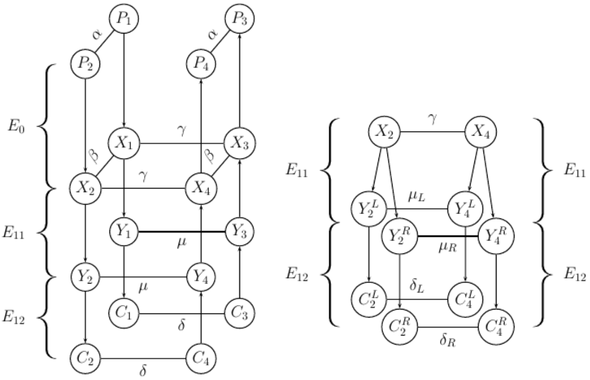

Figure 1: The retracing boomerang attack.

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2. Although the additional split looks restrictive, it applies for a wide class of block ciphers such as SASAS constructions.
3. It is assumed that E_{12} can be split into two parts of size b and $n - b$ bits, call these functions E_{12}^L and E_{12}^R, with characteristic probabilities q_2^L and q_2^R respectively.

The Shifting Retracing Boomerang Attack

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3. *If one of these pairs satisfies $\delta_L \xrightarrow{q_2^L} \mu_L$, the other pair will too!*. Increases the probability of the boomerang distinguisher by $(q_2^L)^{-1}$.

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4. Higher signal to noise ratio (SNR) and lower data complexity due to filtering.

The Shifting Retracing Boomerang Attack

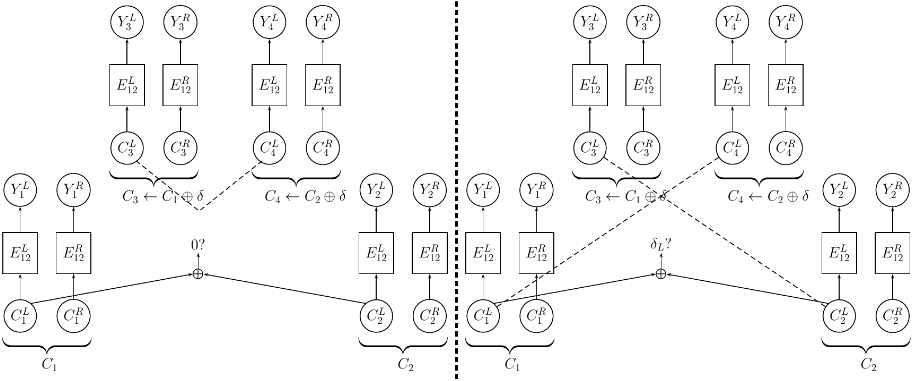

Figure 2: A shifted quartet (dashed lines indicate equality).

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$$C_3 = (C_3^L, C_3^R) = (C_1^L \oplus (C_1^L \oplus C_2^L), C_1^R) = (C_2^L, C_1^R), \tag{1}$$

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5. Similar to the core step used in the yoyo attack on AES.

The Mixing Retracing Boomerang Attack

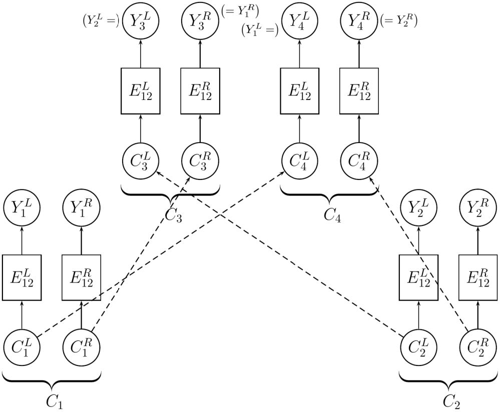

Figure 3: A mixture quartet of ciphertexts (dashed lines indicate equality).

Description of AES[3]

1. Byte ordering shown after *SB* in Figure 4 (column major).

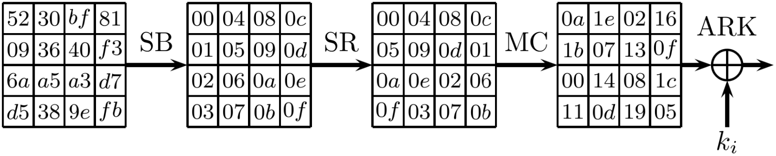

Figure 4: An AES round.

[3] National Institute of Standards and Technology. *Advanced Encryption Standard (AES)*. Federal Information Processing Standard (FIPS) 197. U.S. Department of Commerce, May 9, 2023. DOI: 10.6028/NIST.FIPS.197-upd1. URL: https://csrc.nist.gov/pubs/fips/197/final (visited on 04/14/2025).

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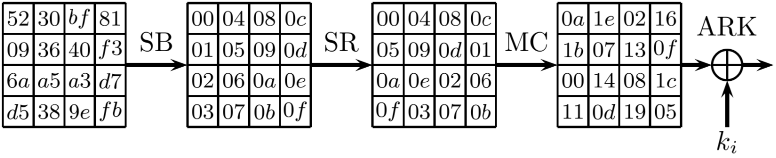

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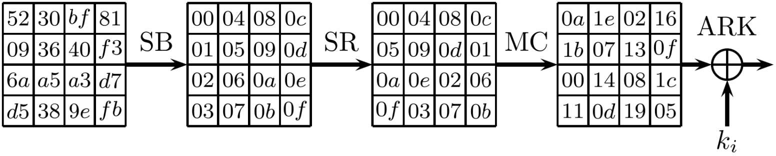
3. Round subkeys are $k_{-1}, k_0, \ldots$.



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3. Attack inverse shifted columns of k_{-1}. Friend pairs used to get more information.

Meet in the Middle Improvement on Yoyo Attack

1. Denote the value of byte m before MC operation of round 0 by W_m and the corresponding output by Z_m. Then,

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2. Adversary guesses $k_{-1,\{0,5\}}$ by computing the following for $j = 1, 2, 3$ and storing the concatenated 24-bit value in a hash table (each j represents a friend pair).

$$02_x \cdot ((W_3^j)_0 \oplus (W_4^j)_0) \oplus 03_x \cdot ((W_3^j)_1 \oplus (W_4^j)_1) \tag{4}$$

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    - Eliminating key bytes using friend pairs.

Specific Choice of Plaintexts

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6. Obtain 2^8 candidates for $k_{-1,\{0,5\}}$ in about 2^8 operations per pair.

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5. Similar MITM procedure followed with another friend pair to obtain the unique value of $k_{-1,\{0,5,10,15\}}$ by isolating $k_{-1,10}$.

Eliminating Key Bytes Using Friend Pairs

1. To reduce the number of candidates for $k_{-1,\{10,15\}}$, the boomerang process is used to return multiple friend pairs (P_3^j, P_4^j).
2. Choose one such pair for which
$$(P_3^j)_{10} \oplus (P_4^j)_{10} = 0 \quad \text{or} \quad (P_3^j)_{15} \oplus (P_4^j)_{15} = 0. \tag{6}$$
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6. Perform 2^8 operations for each pair (P_1, P_2) and for each value of l. Total time complexity of about 2^{16} operations.
7. Each pair requires 2^7 friend pairs to find one that satisfies (6) with high probability. Total data complexity is increased to about 2^{15}.

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3. For each plaintext pair, create 2^7 friend pairs (P_1^j, P_2^j) such that for each j, $P_1^j \oplus P_2^j = P_1 \oplus P_2$ and $(P_1^j)_{\{0,5,10,15\}} = (P_1)_{\{0,5,10,15\}}$.

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   5. If contradiction, go to the next value of l. If contradiction for all l, discard this pair and go to the next pair.
5. Using a pair (P_1, P_2) for which no contradiction occurred, perform MITM attacks on columns 1, 2 and 3 of round 0 using the fact that $Z_3 \oplus Z_4$ equals 0 in the l-th inverse shifted column to recover k_{-1}.

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2. Increasing the number of initial pairs and friend pairs per initial pair boosts success probability. With 64 pairs and 128 friend pairs per initial pair, the probability of success is $(1 - e^{-1})^2 \approx 0.4$
3. Another way to boost succees probability is to find other ways to cancel terms in (3). For instance, if there exist j, j' such that $\{(P_3^j)_{10}, (P_4^j)_{10}\} = \{(P_3^{j'})_{10}, (P_4^{j'})_{10}\}$, we can take the XOR of (3) to cancel the effect of $k_{-1,10}$, thus increasing the success probability even when there is no pair that satisfies (6).

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6. Memory complexity of the attack remains at 2^9, like yoyo attack.
7. Time complexity dominated by MITM attacks that take 2^{16} operations each. Taking one AES operation equivalent to 80 S-box lookups and adding it to the number of queries gives us a total of $2^{16.5}$ encryptions.