CS5760: Topics in Cryptanalysis **19 January 2025**

Lecture 2: Cryptanalysis of Reduced-Round DES

Instructors: Maria Francis and M. V. Panduranga Rao *Scribe: Gautam Singh*

2.1 Cryptanalysis of DES Reduced to 4 Rounds

The notation for this reduced DES cryptosystem is shown in Figure 2.1.

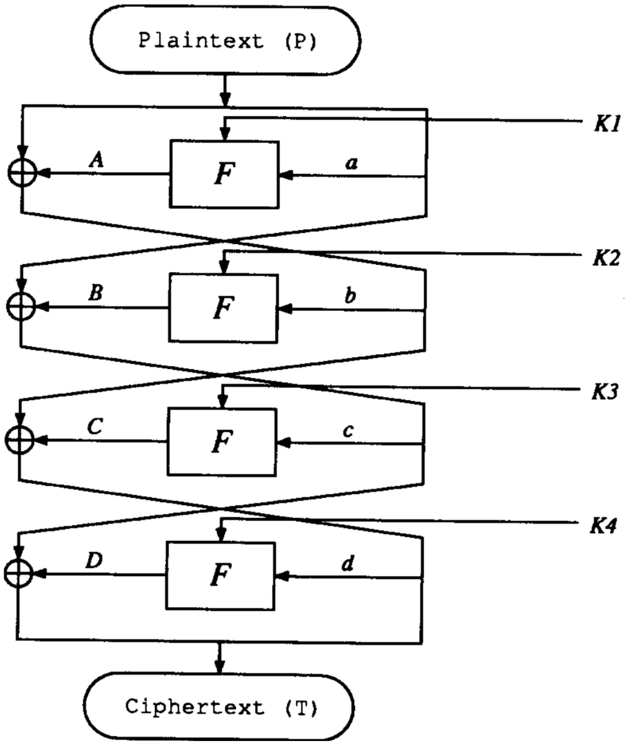


Figure 2.1: DES reduced to four rounds.

To find the master key, we make use of the characteristic shown in Figure 2.2. Using this characteristic, we have $a' = 0_x \implies A' = 0_x$. Thus, $b =$ `20 00 00 00` necessarily, and the single bit difference only diffuses from here on.

Since $a' = 0$, we write

$$c' = D' \oplus l' = a' \oplus B' \tag{2.1}$$

$$\implies D' = l' \oplus B', \tag{2.2}$$

where $T' = (l', r')$ is the ciphertext XOR. Further, we have $d' = r'$, so d' is completely known. Observe that $S'_{Eb} = 0$ for S2, ..., S8. Thus, $S'_{Ob} = 0$ always for 28 bits. Hence, S'_{Od} is known for S2, ..., S8. We find the 6-bit subkey blocks corresponding S_{Kd} using bruteforce to verify (2.3).

$$S(S_E \oplus S_K) \oplus S(S_E^* \oplus S_K) = S_O'. \tag{2.3}$$

Since Ω_P^1 has probability 1 and (d', D') is a right pair, we will find the right value of S_{Kd} with probability 1. Thus, we have found 42 bits of the subkey $K4$. If the DES key-scheduling algorithm is followed, these correspond to 42 key bits of the master key K. Finding the other 14 bits can be done by exhaustively searching the 2^{14} possibilities and verifying that the plaintexts are correctly encrypted. This leads to an attack with 2^{14} encryptions, which runs efficiently.

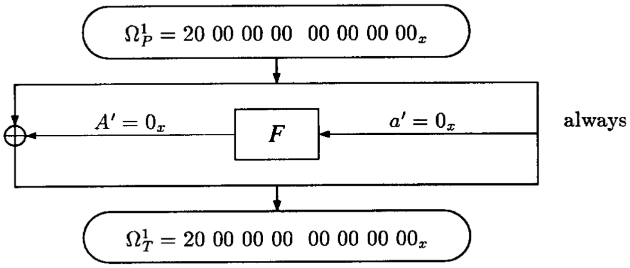

$\Omega_P^1 = $ 20 00 00 00 00 00 00 00_x
$A' = 0_x$
F
$a' = 0_x$
always
$\Omega_T^1 = $ 20 00 00 00 00 00 00 00_x

Figure 2.2: Characteristic used for cryptanalysis of DES reduced to four rounds.

2.1.1 DES With Independent Subkeys

Differential cryptanalysis can also work if the subkeys $K1, \ldots, K4$ are generated independently and do not depend on a key-scheduling algorithm. As before, we can find 42 bits of $K4$. To find the remaining 6 bits, we use Ω_P^2 shown in Figure 2.3. With this characteristic, we have $S1'_{Eb} = 0$, thus using a similar argument we can find $S1'_{Od}$ and apply the counting approach to get $S1_{Kd}$, which will completely find $K4$.

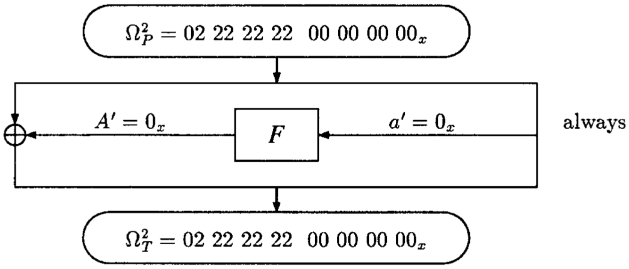

$\Omega_P^2 = $ 02 22 22 22 00 00 00 00_x
$A' = 0_x$
F
$a' = 0_x$
always
$\Omega_T^2 = $ 02 22 22 22 00 00 00 00_x

Figure 2.3: Second characteristic used to find K3 and K4 completely.

Finding $K3$ using Ω_P^2 is straightforward. At this point, we can decrypt the fourth round to completely find c' and $C' = b' \oplus D'$. A similar counting argument can be used to find $K3$ completely.

To find $K1$ and $K2$ we will need to choose different characteristics, since both characteristics have $a' = 0 \rightarrow A' = 0$ and thus all keys are equally likely for $K1$. Similarly, some S boxes in the second round have zero XOR inputs and make all keys equally likely. To overcome these, we choose characteristics Ω_P^3 and Ω_P^4 arbitrarily such that

1. $S'_{Ea} \neq 0$ for all S boxes for both characteristics.
2. For every S box the S'_{Ea} values differ between the characteristics.

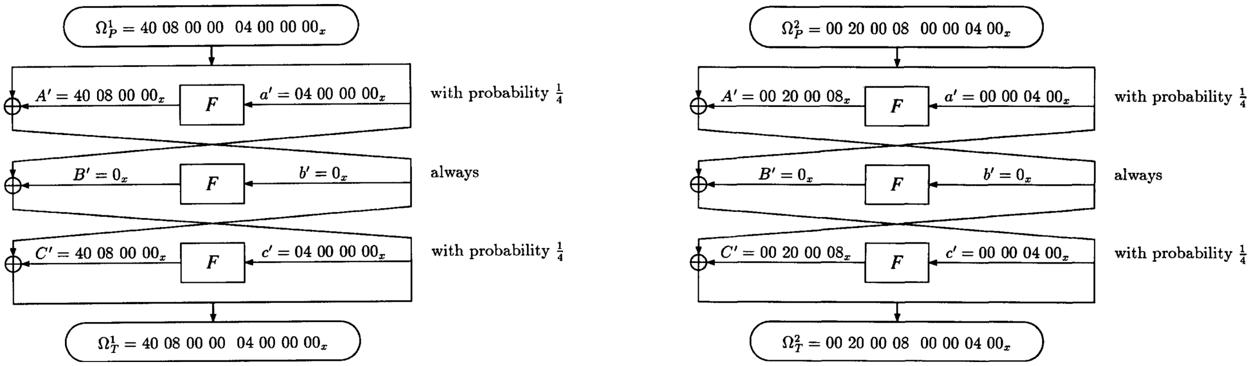
Knowing the value of b' after decryption of the third round, we can find $B' = c' \oplus a' = c' \oplus R'$. A similar counting argument will find the complete $K2$. Similarly, $A' = L' \oplus b'$ and thus the complete $K1$ can also be found. One can verify the keys have been found by encrypting plaintexts with these values and checking the outputs. This completes the cryptanalysis of DES reduced to 4 rounds. It also shows that differential cryptanalysis can work even if the round subkeys in DES are independently chosen.

For the cryptanalysis, a total of 16 encryptions are needed to find the keys with high probability: 8 pairs each of Ω^1 and Ω^2 and 4 pairs each of Ω^3 and Ω^4.

2.2 Cryptanalysis of DES Reduced to 6 Rounds

2.2.1 Characteristics for Recovering K6

For the cryptanalysis of DES reduced to 6 rounds, we make use of two characteristics, both with probability $\frac{1}{16}$ as shown in Figure 2.4.



$\Omega_P^1 = 40\ 08\ 00\ 00\ \ 04\ 00\ 00\ 00_x$
$A' = 40\ 08\ 00\ 00_x$
F
$a' = 04\ 00\ 00\ 00_x$
with probability $\frac{1}{4}$
$B' = 0_x$
F
$b' = 0_x$
always
$C' = 40\ 08\ 00\ 00_x$
F
$c' = 04\ 00\ 00\ 00_x$
with probability $\frac{1}{4}$
$\Omega_T^1 = 40\ 08\ 00\ 00\ \ 04\ 00\ 00\ 00_x$
$\Omega_P^2 = 00\ 20\ 00\ 08\ \ 00\ 00\ 04\ 00_x$
$A' = 00\ 20\ 00\ 08_x$
F
$a' = 00\ 00\ 04\ 00_x$
with probability $\frac{1}{4}$
$B' = 0_x$
F
$b' = 0_x$
always
$C' = 00\ 20\ 00\ 08_x$
F
$c' = 00\ 00\ 04\ 00_x$
with probability $\frac{1}{4}$
$\Omega_T^2 = 00\ 20\ 00\ 08\ \ 00\ 00\ 04\ 00_x$

Figure 2.4: Characteristics used for cryptanalysis of DES reduced to 6 rounds.

For the characteristic Ω_P^1, the S boxes S2, S5, ..., S8 have zero input XORs and for Ω_P^2, the S boxes S1, S2, S4, S5 and S6 have zero input XORs in the fourth round. We write

$$e' = F' \oplus l' = c' \oplus D' \tag{2.4}$$

$$\implies F' = c' \oplus D' \oplus l' \tag{2.5}$$

2.2.2 Finding K6 Efficiently

From both characteristics, we can find 42 bits of $K6$. However, using a direct 2^{30} counting approach for each characteristic is memory intensive. The authors proposed a graph-based algorithm to work around this. Consider a graph $G = (V, E)$ where

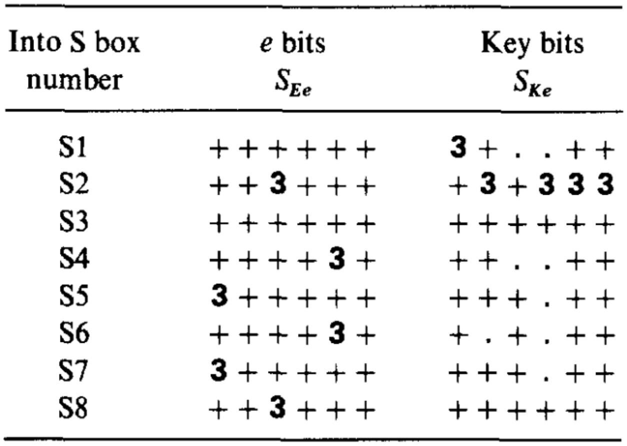
1. Each $v \in V$ represents a plaintext pair.
2. The edge (u, v) is labelled using a 64-bit mask for each S box, with each bit representing one of the possible 64 6-bit keys that enter that particular S box. A set bit implies that the corresponding key is suggested by the pair. Thus, each edge has five 64-bit masks associated with it.

3. We define a *clique* in G to be a set of vertices where the bitwise "and" of all edge labels for each of the five masks is nonzero.

Thus, the task is to find the largest such clique, which corresponds to the most likely values of S_{Kf} for the corresponding S boxes with zero XOR inputs. This can be done with a recursive clique finding algorithm. This works for smaller graphs with few edges as the complexity of this algorithm is exponential. Applying this algorithm for both characteristics, if the S_{Kf} values match for the common S boxes S2, S5 and S6, we have found 42 bits of $K6$ with high probability.

2.2.3 Speeding up the Cryptanalysis

Though brute forcing the other 14 bits of the master key is already fast enough, we can do better by finding the other 6 key bits entering S3 in the sixth round. First, we filter out pairs satisfying (2.5), leaving us with $\frac{1}{16}$ possibly right pairs. Then, for each possible value of $S3_{Kf}$, we decrypt the sixth round to get e and e^*, using which we verify the equality $E' = d' \oplus f'$. Specifically, we verify that Si'_{Oe} for $i \in \{2, 3, 8\}$ match the computed value of E'. Only one of the 64 possibilities will survive with high probability since the chances of another possibility also satisfying the given inequality is 2^{-20}. This works because of the dependence of S_{Ke} on the key bits in S_{kf} as shown in Figure 2.5.



Into S box number	*e* bits S_{Ee}	Key bits S_{Ke}
S1	+ + + + + +	**3** + . . + +
S2	+ + **3** + + +	+ **3** + **3** **3** **3**
S3	+ + + + + +	+ + + + + +
S4	+ + + + **3** +	+ + . . + +
S5	**3** + + + + +	+ + + . + +
S6	+ + + + **3** +	+ . + . + +
S7	**3** + + + + +	+ + + . + +
S8	+ + **3** + + +	+ + + + + +

Figure 2.5: Dependence of $K5$ on bits of $K6$. The '3' indicates dependence on $S3_{Kf}$, the '.' indicates bits unused in $K6$ and the '+' indicates dependence on known key bits of $K6$.

After finding $K6$ entirely, one can now brute force the other 8 missing key bits of the master key, leading to a faster cryptanalysis of DES reduced to six rounds.

2.2.4 Amount of Data Required

The signal to noise ratio for finding the 30 subkey bits using Ω_P^1 and Ω_P^2 is

$$S/N = \frac{2^{30} \cdot \frac{1}{16}}{4^5} = 2^{14}. \tag{2.6}$$

The S/N is high and about 7-8 pairs are needed for each characteristic. Since each characteristic has probability $\frac{1}{16}$, we require about 120 pairs of plaintexts.

For finding the remaining 6 key bits of $K6$, the signal to noise ratio is

$$S/N = \frac{2^6 \cdot 1}{4} = 16. \tag{2.7}$$

Though it is much lesser, we can still find 7-8 right pairs and find a clique. In total, we need 240 plaintexts for the cryptanalysis.