CS5760: Cryptanalysis of DES and DES-like Iterated Cryptosystems

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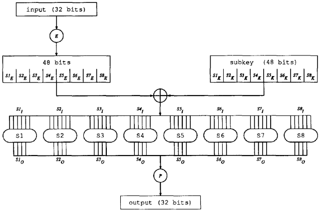
   - *Invariant* in key mixing with subkey S_K to get $S_I = S_E \oplus S_K$.
   - *Linear* in permutation P on S_O after S boxes.
   - *Invariant* in XOR operation connecting rounds.



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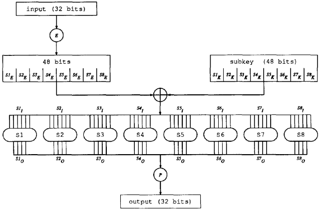
4. S boxes are *nonlinear*. Probability analysis performed between input and output XOR.



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4. i^{th} S box contributes probability p_i for $Si'_I \to Si'_O$.
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   - Over n rounds, $P = \prod_{i=1}^{n} P_i$.

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Desirable for cryptanalysis: high P with large n.

Characteristic

Definition 1 (Characteristic)

An *n-round chracteristic* is a tuple $\Omega = (\Omega_P, \Omega_\Lambda, \Omega_T)$ where $\Omega_P = (L', R')$ and $\Omega_T = (l', r')$ are m bit numbers, $\Omega_\Lambda = (\Lambda_1, \dots, \Lambda_n)$, $\Lambda_i = (\lambda_I^i, \lambda_O^i)$ and $\lambda_I^i, \lambda_O^i, L', R', l', r'$ are $\frac{m}{2}$ bit numbers and m is the block size of the cryptosystem satisfying

$$\lambda_I^1 = R' \tag{1}$$

$$\lambda_I^2 = L' \oplus \lambda_O^1 \tag{2}$$

$$\lambda_I^n = r' \tag{3}$$

$$\lambda_I^{n-1} = l' \oplus \lambda_O^n \tag{4}$$

$$\forall\ 1 < i < n,\ \lambda_O^i = \lambda_I^{i-1} \oplus \lambda_I^{i+1} \tag{5}$$

Characteristic

Definition 2 (Right Pair)

A *right pair with respect to an n-round characteristic* $\Omega = (\Omega_P, \Omega_\Lambda, \Omega_T)$ *and an independent key* K is a pair for which $P' = \Omega_P$ and for each round i of the first n rounds of the encryption of the pair using K the input XOR of the i^{th} round equals λ_I^i and the output XOR of the F function equals λ_O^i. Pairs that do not satisfy these conditions are called *wrong pairs*.

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Definition 3 (Probability of a Round of a Characteristic)

Round i of an n-round characteristic Ω has probability p_i^Ω if $\lambda_I^i \to \lambda_O^i$ with probability p_i^Ω by the F function.

Probability of a Characteristic

Definition 4 (Probability of a Characteristic)

An n-round characteristic Ω has probability p^{Ω} given by

$$p^{\Omega} = \prod_{i=1}^{n} p_i^{\Omega} \tag{6}$$

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Theorem 5 (Probability of a Characteristic and Right Pairs)

The formally defined probability of a characteristic $\Omega = (\Omega_P, \Omega_\Lambda, \Omega_T)$ is the probability that any fixed plaintext pair satisfying $P' = \Omega_P$ is a right pair when random independent keys are used.

Example of a Characteristic

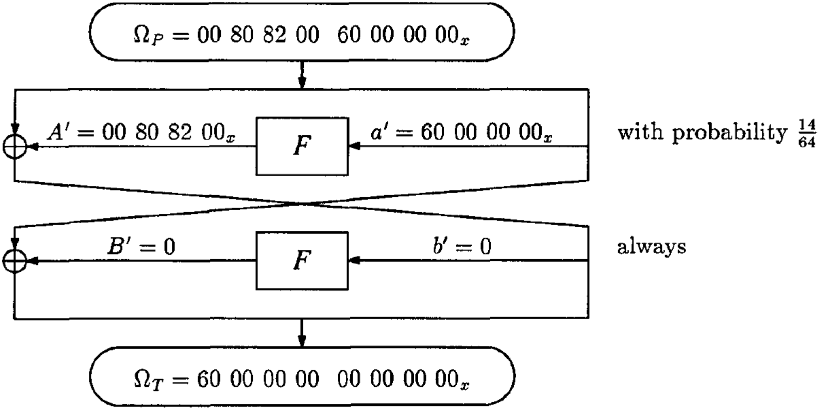

Figure 2: Example of a two-round characteristic with probability $\frac{14}{64}$.

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Definition 6 (Signal-to-Noise Ratio)

The ratio between the number of right pairs and the average count of incorrect subkeys in a counting scheme is called the *signal to noise ratio of the counting scheme* and is denoted by S/N.

Computing the SNR

Consider the variables shown in Table 1.

Variable	Definition
p	Probability of the characteristic
m	Number of created pairs
α	Average count per analyzed pair
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Then,

$$S/N = \frac{m \cdot p}{\frac{m \cdot \beta \cdot \alpha}{2^k}} = \frac{2^k \cdot p}{\alpha \cdot \beta} \tag{7}$$

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Definition 7 (Quartet and Octet)

A *quartet* is a structure of four ciphertexts that simultaneously contains two ciphertext pairs of one characteristic and two ciphertext pairs of a second characteristic. An *octet* is a structure of eight ciphertexts that simultaneously contains four ciphertext pairs of each of three characteristics.

3. As an example, $(P, P \oplus \Omega_P^1, P \oplus \Omega_P^2, P \oplus \Omega_P^1 \oplus \Omega_P^2)$ is a quartet.

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4. Quartets save $\frac{1}{2}$ of the data and octets save $\frac{2}{3}$ of the data.

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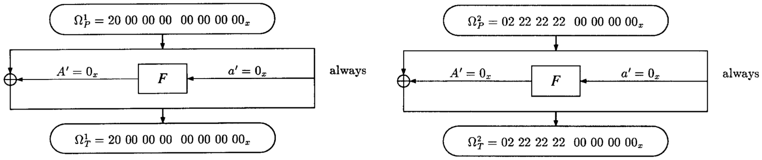

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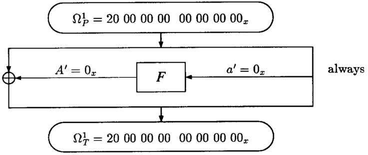
3. Example of a *3R-attack*. There are *three* extra rounds after the characteristic is applied.



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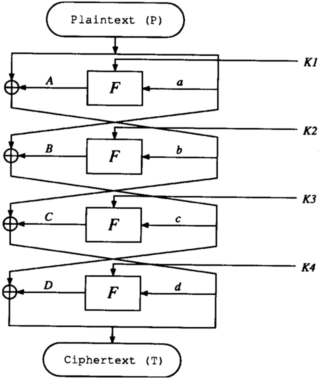

Figure 4: DES reduced to four rounds.

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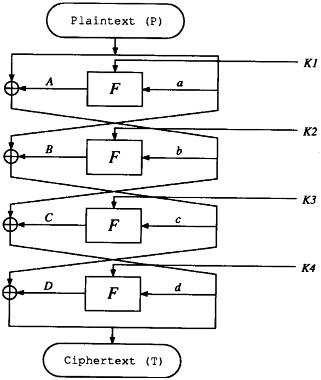
1. Using Ω^1, we have

$$c' = D' \oplus l' = a' \oplus B' \implies D' = B' \oplus l' \quad (8)$$



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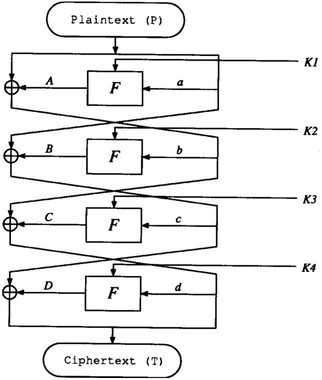

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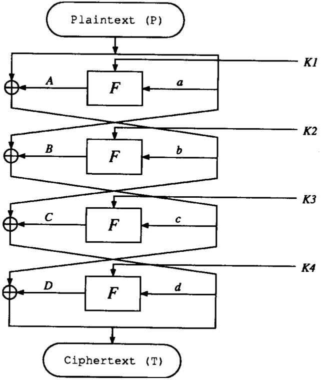

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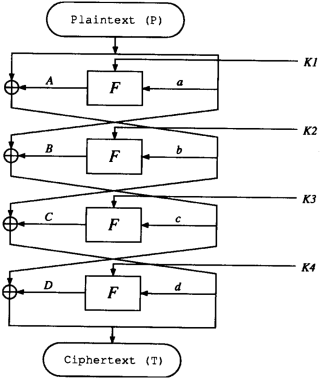
    - 28 bits of B' are zero and hence we can find *28 bits of D'*.



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   - In the second round S2, . . . , S8 receive zero XOR input.
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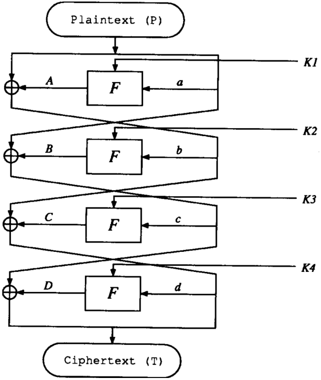
   - We already know $d' = r'$. So, we employ a counting approach to get $K4$.



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DES Reduced to Four Rounds

1. To get Si_{Kd} for $2 \leq i \leq 8$, we verify (9).

$$S(S_E \oplus S_K) \oplus S(S_E^* \oplus S_K) = S_O' \tag{9}$$

2. Only *one* plaintext pair is needed since characteristic probability is 1.
3. We recover $7 \times 6 = 42$ key bits of $K4$, which correspond to 42 bits of the master key.
4. Exhaustively search the other 14 key bits to get the entire master key.
5. We have used the key schedule to our advantage here? *What if all the keys were independent?*

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3. Since $a' = A' = 0_x$, all keys are equally likely. Other characteristics Ω^3 and Ω^4 are chosen such that
   - $S'_{Ea} \neq 0_x$ for all S boxes for both characteristics.
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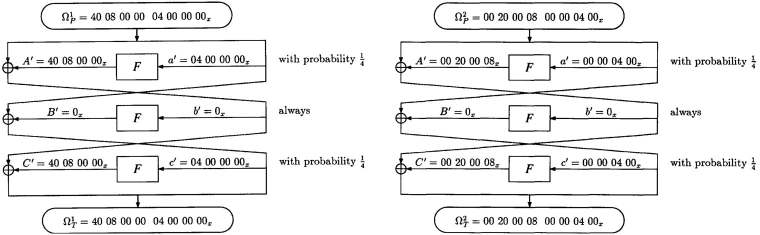
   - $S'_{Ea} \neq 0_x$ for all S boxes for both characteristics.
   - For every S box, the S'_{Ea} values differ between the characteristics.
   - Similar counting methods used to get $K1$ and $K2$.
4. 16 chosen plaintexts are needed for this attack.
   - 8 pairs of Ω^1 and Ω^2 each.
   - 4 pairs of Ω^3 and Ω^4 each.

To reduce the data needed, two octets are used.

DES Reduced to Six Rounds

1. Two three-round characteristics used, each with probability $\frac{1}{16}$.



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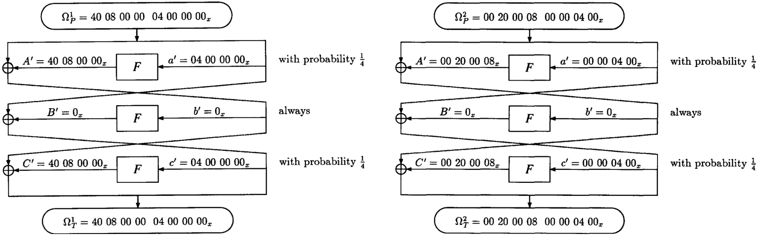
2. We have,

$$e' = c' \oplus D' = F' \oplus l' \implies F' = c' \oplus D' \oplus l' \tag{10}$$



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1. In the fourth round,
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4. Due to higher S/N, fewer plaintext pairs are analyzed. *This is exploited to get a faster counting algorithm*.

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5. Apply this method for both Ω^1 and Ω^2, ensuring that the suggested keys at S2, S5 and S6 match. Otherwise, use more data.

Completing the Cryptanalysis

1. 42 key bits have been found, can exhaustively search remaining 14 bits.

Into S box number	e bits S_{Ee}	Key bits S_{Ke}
S1	+ + + + + +	3 + . . + +
S2	+ + 3 + + +	+ 3 + 3 3 3
S3	+ + + + + +	+ + + + + +
S4	+ + + + 3 +	+ + . . + +
S5	3 + + + + +	+ + + . + +
S6	+ + + + 3 +	+ . + . + +
S7	3 + + + + +	+ + + . + +
S8	+ + 3 + + +	+ + + + + +

Figure 6: Dependence of $K5$ on $K6$. '3' indicates dependence on $S3_{Kf}$, '.' indicates bits unused in K6 and '+' indicates dependence on known key bits of K6.

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   - Exhaustively search remaining 8 bits.

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S5	3 + + + + + +	+ + + . + +
S6	+ + + + + 3 +	+ . + . + +
S7	3 + + + + + +	+ + + . + +
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Figure 6: Dependence of $K5$ on $K6$. '3' indicates dependence on $S3_{Kf}$, '.' indicates bits unused in K6 and '+' indicates dependence on known key bits of K6.

Completing the Cryptanalysis

1. 42 key bits have been found, can exhaustively search remaining 14 bits.
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   - Leaves $\frac{1}{16}$ of the pairs, boosts S/N.

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Data Requirements

1. The first phase has

$$S/N = \frac{2^{30} \cdot \frac{1}{16}}{4^5} = 2^{16}. \tag{11}$$

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3. We can reduce the data required by using quartets. In total, about 240 ciphertexts are needed.

DES Reduced to Eight Rounds

1. We use a 5-round characteristic with probability $\approx \frac{1}{10486}$.
2. From Figure 7, a right pair has $f' = d' \oplus E' = 40\ 5C\ 00\ 00_x$.
   - In the sixth round, S2, S5, ..., S8 have zero input XORs.
3. We have,

$$g' = e' \oplus F' = H' \oplus I' \tag{13}$$

$$\implies H' = e' \oplus F' \oplus I'. \tag{14}$$

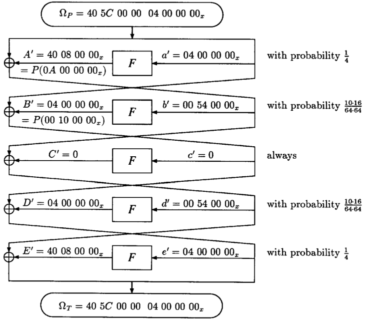

Figure 7: 5 round characteristic to cryptanalyze DES reduced to 8 rounds.

Improving the Signal to Noise Ratio

1. Signal to noise ratio for
   - $k = 30$ is $S/N = \frac{2^{30}}{4^5 \cdot 10486} \approx 100$. Requires 2^{30} counters.
   - $k = 24$ is $S/N = \frac{2^{24}}{4^4 \cdot 0.8 \cdot 10486} \approx 7.8$. Requires 2^{24} counters.
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4. Thus, $f' = d' \oplus E' = X0\ 5V\ Z0\ 00_x$ where $V = Y \oplus 4$.
   - $Z = 0 \implies E' = 40\ 08\ 00\ 00_x$. This happens with probability $\frac{16}{64}$.
   - All other possiblities having $Z = 4$ happen with probability $\frac{20}{64}$.

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3. For $k = 24$, we need 25000 pairs. For $k = 18$, we need 150000 pairs, where
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   - *Hint: What is the probability that a wrong pair survives both counting stages?*

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3. The weighting function reduces number of analyzed pairs to 7500, leading to improvements in runtime.

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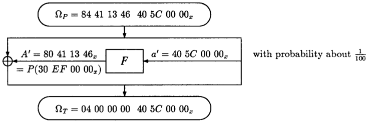
2. Main idea:
   - Find relation between input bits for a high probability entry in the pairs XOR distribution table.
   - Find information about the key bits at those positions (this could be found earlier).
   - Choose plaintexts accordingly to boost characteristic probability and signal to noise ratio.

Extension to Nine Rounds

1. Characteristic shown in Figure 7 extended with extra round shown in Figure 9.



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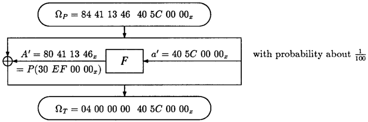

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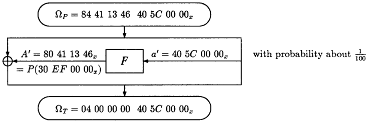
3. This attack requires a lot of data and memory, hence it is unrealistic.



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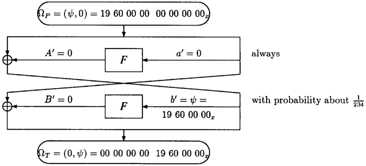

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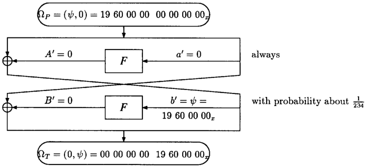

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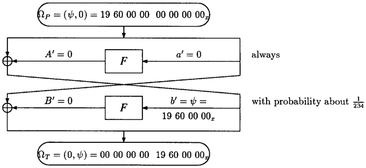

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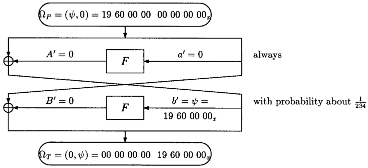

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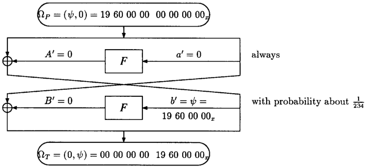
4. 15-round extension has probability 2^{-56}. *Just the iterative characteristic is not enough*!



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2. Not advisable for larger rounds due to small S/N.
3. More powerful compared to 0R/1R/2R-attacks due to smaller characteristic length.
   - For fixed number of iterations in a cryptosystem, 3R-attacks are the most useful.

2R-Attacks

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2. Wrong pairs discarded if input XORs of S boxes in the previous round may not cause expected output XORs.
3. Example: DES reduced to 9 rounds.
   - 7-round iterative characteristic has probability $\approx 2^{-24}$.
   - Five S boxes in the eighth round have $S'_{Eh} = S'_{Ih} = 0$, thus $S'_{Oh} = 0$. This happens for wrong pairs with probability $\frac{1}{16}$. For other S boxes, this probability is 0.8.
   - Counting on 48 bits of $K9$ has $S/N = \frac{2^{48} \cdot 2^{-24}}{4^8 \cdot 0.8^3 \cdot (\frac{1}{16})^5} \approx 2^{29}$.
   - Counting on 18 bits of $K9$ has $S/N = \frac{2^{18} \cdot 2^{-24}}{4^3 \cdot 0.8^5 \cdot 0.8^3 \cdot (\frac{1}{16})^5} \approx 2^{11}$.
   - Total of 2^{26} pairs needed. Filtering on last two rounds leaves $0.8^3 \cdot (\frac{1}{16})^5 \cdot 0.8^8 \approx 2^{-24}$ of wrong pairs. *The clique method can be used since there are few pairs.*

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   - 9-round iterative characteristic has probability $\approx 2^{-32}$.
   - Right pairs have $r' = \psi$ and 20 btits in l' going out of S4, . . . , S8 are zero.
   - Wrong pairs pass these checks with probability 2^{-52}. Thus, counting on 18 key bits has $S/N = \frac{2^{18} \cdot 2^{-32}}{4^3 \cdot 2^{-52}} = 2^3 2$. 2^{34} pairs are needed.

Complexity of Differential Cryptanalysis Attacks So Far

No. of rounds	No. pairs needed	No. pairs used	No. bits found	Characteristics		S/N		Comments
4	2^3	2^3	42	1	1	16	[6]	
6	2^7	2^7	30	3	1/16	2^{16}	*	
8	2^{15}	2^{13}	30	5	1/10,486	15.6	[24]	
8	2^{17}	2^{13}	30	5	1/10,486	1.2	[18]	
8	2^{20}	2^{19}	30	5	1/55,000	1.5	[24]	The iterative characteristic
9	2^{25}	2^{24}	30	6	1/1,000,000	1.0	[30]	Extension to six rounds
9	2^{26}	8	48	7	2^{-24}	2^{29}	*	
10	2^{34}	4	18	9	2^{-32}	2^{32}	*	
11	2^{35}	2^{11}	48	9	2^{-32}	2^{21}	*	
12	2^{42}	4	18	11	2^{-40}	2^{24}	*	
13	2^{43}	2^{19}	48	11	2^{-40}	4	[30]	
14	2^{50}	4	18	13	2^{-48}	2^{16}	*	
15	2^{51}	2^{27}	48	13	2^{-48}	2.5	[42]	Needs a huge memory. With less memory needs 2^{57} pairs
16	2^{57}	2^5	18	15	2^{-56}	2^8	*	Slower than exhaustive search

Figure 11: Summary of time and space complexity of differential cryptanalysis on DES.

Main Idea of the New Attack

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4. This attack has two phases: *data collection* and *data analysis*.

Data Collection Phase

1. Want to generate plaintexts that are fed to 15-round attack after first round.

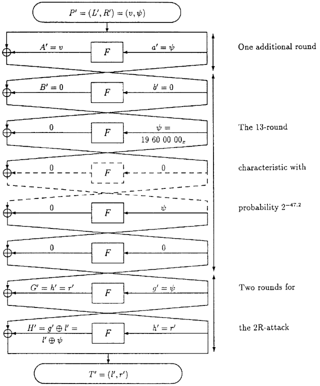

Figure 12: Modified 2R-attack on DES.

Data Collection Phase

1. Want to generate plaintexts that are fed to 15-round attack after first round.
2. Let $v_0, \ldots, v_{4095}$ be the 2^{12} 32-bit constants consisting of all possible values at the 12 output positions of S1, S2 and S3 after the first round, and zero elsewhere.

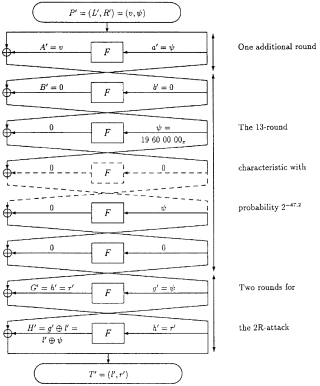

Figure 12: Modified 2R-attack on DES.

Data Collection Phase

1. For arbitrary 64-bit P, define

$$P_i \triangleq P \oplus (v_i, 0) \tag{16}$$
$$\bar{P}_i \triangleq (P \oplus (v_i, 0)) \oplus (0, \psi) \tag{17}$$
$$T_i \triangleq DES(P_i, K) \tag{18}$$
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3. *We still don't know the best k for which $\psi \to v_k$.*
   - Exhaustive search over the 2^{24} pairs is too slow.
   - Exploit the cross-product structure to speed up the search.

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5. *Input to data analsyis phase contains mix of right and wrong pairs.*

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   - 28 bits of left key register are used as inputs to S1, S2 and S3 of first and fifteenth rounds, and S1, . . . , S4 of sixteenth round.

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5. Multiplying them together, each pair suggests 0.84 values for these 52 key bits. In total, each structure suggests $1.19 \cdot 0.84 \cdot 2^4 = 16$ values.

Data Analysis Phase

1. Verify each key by peeling up two rounds and checking against output of 13-round characteristic. Costs $16 \cdot \frac{2}{16} \cdot 2 = 4$ equivalent DES operations.

		K16									
		Left Key Register					Right Key Register				
		S1	S2	S3	S4	X	S5	S6	S7	S8	X
K1	S1		2	1	1	2					
	S2	2		1	2	1					
	S3	2			3	1					
	S4	2	3	1							
	X		1	3							
	S5							1	2	2	1
	S6						3		2	1	
	S7							2		2	2
	S8						2	3			1
	X						1		2	1	

Figure 13: Common bits between $K1$ and $K16$.

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K1	S1		2	1	1	2					
	S2	2		1	2	1					
	S3	2			3	1					
	S4	2	3	1							
	X		1	3							
	S5							1	2	2	1
	S6						3		2	1	
	S7							2		2	2
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	S3	2			3	1					
	S4	2	3	1							
	X		1	3							
	S5							1	2	2	1
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3. Using common key bits, we can speed up the data analysis, as shown in Figure 13.

		K16									
		Left Key Register					Right Key Register				
		S1	S2	S3	S4	X	S5	S6	S7	S8	X
K1	S1		2	1	1	2					
	S2	2		1	2	1					
	S3	2			3	1					
	S4	2	3	1							
	X		1	3							
	S5							1	2	2	1
	S6						3		2	1	
	S7							2		2	2
	S8						2	3			1
	X						1		2	1	

Figure 13: Common bits between $K1$ and $K16$.

General Form of the Attack

Theorem 8

Given a characteristic with probability p *and signal-to-noise ratio* S/N *for an iterated cryptosystem with* k *key bits, we can apply an attack which encrypts* $\frac{2}{p}$ *chosen plaintexts in the data collection phase and whose complexity is* $\frac{2^k}{S/N}$ *encryptions during the data analysis phase.*

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Given a characteristic with probability p and signal-to-noise ratio S/N for an iterated cryptosystem with k key bits, we can apply an attack which encrypts $\frac{2}{p}$ chosen plaintexts in the data collection phase and whose complexity is $\frac{2^k}{S/N}$ encryptions during the data analysis phase.

Appropriately chosen metastructures can reduce the number of plaintexts to $\frac{1}{p}$. Further, the effective time complexity can be reduced by a factor of $f \leq 1$ if a wrong key can be discarded by carrying out a fraction f of the rounds.

Results

Rounds	Chosen Plaintexts	Analyzed Plaintexts	Complexity of Analysis	Best Previous Time	Best Previous Space
8	2^{14}	4	2^{9}	2^{16}	2^{24}
9	2^{24}	2	2^{32}	2^{26}	2^{30}
10	2^{24}	2^{14}	2^{15}	2^{35}	–
11	2^{31}	2	2^{32}	2^{36}	–
12	2^{31}	2^{21}	2^{21}	2^{43}	–
13	2^{39}	2	2^{32}	2^{44}	2^{30}
14	2^{39}	2^{29}	2^{29}	2^{51}	–
15	2^{47}	2^{7}	2^{37}	2^{52}	2^{42}
16	2^{47}	2^{36}	2^{37}	2^{58}	–

Figure 14: Results of memoryless DES attack.