Zone Encryption with Anonymous Authentication for V2V Communication

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Introduction

Preliminaries

- Zone Encryption
- 4 Dynamic Group Signatures with Attributes

Conclusion

Summary

V2X Related Terminology

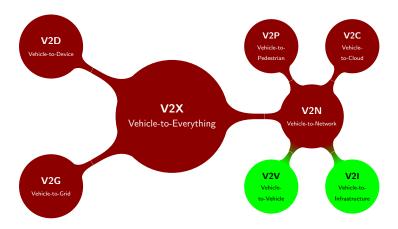


Figure 1: A breakdown of V2X.

Message Types in V2X

- Cooperative Awareness Messages (CAMs)¹ and Basic Safety Messages (BSMs)².
 - Exchanged between vehicles to create awareness and support cooperative performance of vehicles in the road network.
 - Includes status information such as time, position, speed, active systems, vehicle dimensions, etc.

¹European Telecommunications Standards Institute. "Intelligent Transport Systems (ITS); Vehicular Communications; Basic Set of Applications; Part 2: Specification of Cooperative Awareness Basic Service". In: ETSI EN 302 637-2 V1.4.1 (2019). URL: https://www.etsi.org/deliver/etsi en/302600 302699/30263702/01.04.01 60/en 30263702v010401p.pdf.

²J2735_202309: V2X Communications Message Set Dictionary - SAE International. URL: https://www.sae.org/standards/content/j2735_202309/ (visited on 04/15/2024).□ → ← □ → ← □

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 - Exchanged between vehicles to create awareness and support cooperative performance of vehicles in the road network.
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- Other types of messages
 - Signal Phase and Timing (SPaT)
 - Roadside Infrastructure Information (MAP)

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- Encryption impractical, since CAMs must be decrypted by nearby vehicles in a highly dynamic environment.
 - But CAMs have to be encrypted because of the data they carry!
- 3 Instead, focus on privacy-preserving authentication.
 - Ensuring a message is issued by a "genuine" vehicle.
 - "Genuine" vehicles must be untraceable.

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- Proposed systems
 - Stronger privacy and security guarantees.
 - Do not meet the stringent bandwidth constraint of 300 bytes per CAM, thus impractical.

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- Address problems of authenticity and confidentiality in combination for the first time.
- Meet (bandwidth) requirements.
- Negligible storage and bandwidth overheads.
- Efficient encryption scheme (symmetric-key crypto).
- Better security guarantees (privacy, authenticity, confidentiality).



Preliminaries

- Pairing-based Cryptography
- Hardness Assumptions
 - Symmetric Discrete Logarithm (SDL) assumption
 - Modified q-Strong Diffie-Hellman (q-MSDH-1) assumption
- Deterministic Authenticated Encryption (DAE)
- PS Signatures
- Opposition of the property of the property

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Overall Flow of Zone Encryption

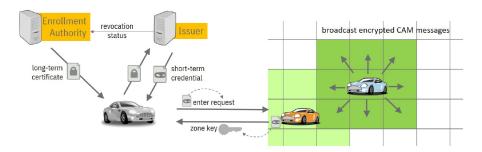


Figure 2: Illustration of Zone Encryption with its Anonymous-Authentication Approach.

Notation

Notation	Meaning
Z	Set of zones covering the road network
\mathcal{P}	Payload/message space
Epoch	Set of epochs
T	Set of timestamps
$K_{z,t}$	Zone key for zone z at time t
L _K	List of zone keys known to a vehicle, stored as $(z, t, K_{z,t})$
\mathcal{E}	Enrollment authority
\mathcal{I}	Issuer
$\mathcal{V} \in \left\{0,1\right\}^*$	Vehicle identity
$\mathit{cert}_\mathcal{V}$	Long-term certificate of ${\mathcal V}$
$\mathit{cred}_{\mathcal{V}}$	Short-term credential of ${\mathcal V}$

Zones, Epochs, Zone Keys

A zone z is a continuous geographical area covering part of a road network (shown as squares alongside).

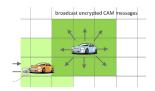


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 - An epoch is denoted by [e, e+1). Each time instance t satisfies $e \le t < e+1$ for a unique e. This is denoted as e(t).
 - Vehicles need $K_{z,t}$ for secure communication when they are in zone z at time t.

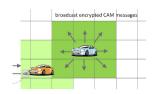


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- Vechicles can communicate securely with other vehicles in surrounding zones also.

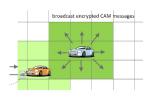


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Entities and Credentials

- **1** An enrollment authority \mathcal{E} issues long-term certificates to vehicle $\mathcal{V} \in \{0,1\}^*$.
 - **1** Long-term certificate $cert_{\mathcal{V}}$ obtained.
 - ② Can be used to check revocation status.

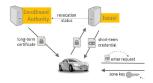


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 - \circ cred_V is valid only for the epoch e in which it was issued.



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Credential Issuance

- $\P \ \, \langle \mathsf{Enroll.V} \left(\mathsf{pk}_{\mathcal{E}}, \mathcal{V} \right) \leftrightharpoons \mathsf{Enroll.E} \left(\mathsf{sk}_{\mathcal{E}}, \mathsf{st}_{\mathcal{E}}, \mathcal{V} \right) \rangle \to \langle \mathsf{cert}_{\mathcal{V}}, \mathsf{st}_{\mathcal{E}}' \rangle$
- ② $\langle \text{Authorize.V}(\textit{cert}_{\mathcal{V}}, e, \textit{pk}_{\mathcal{I}}) \leftrightharpoons \text{Authorize.I}(\textit{sk}_{\mathcal{I}}, \textit{st}_{\mathcal{I}}, \mathcal{V}, e, \textit{pk}_{\mathcal{E}}) \rangle \rightarrow \langle \textit{cred}_{\mathcal{V}}, \textit{st}_{\mathcal{I}}' \rangle$



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 - Vehicle uses certificate to obtain credentials.
 - Issuer checks certificate using public key of enrollment authority.

Gautam Singh (IITH) Zone Encryption April 27, 2024 12 / 22

Entering and Exiting Zones

- **1** ⟨Enter.V ($cred_{\mathcal{V}}$, $L_{\mathcal{K}}$, $pk_{\mathcal{I}}$, z, t, requester) \leftrightharpoons Enter.W ($cred_{\mathcal{W}_i}$, $L_{\mathcal{K}_i}$, $pk_{\mathcal{I}}$, z, t, $responder_i$) $_{i \ge 0}$ ⟩ \rightarrow ⟨ $L_{\mathcal{K}}$, \bot ⟩
 - Why $i \ge 0$?

Entering and Exiting Zones

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 - Why $i \ge 0$?
- 2 Exit $(L_K, z, t) \rightarrow L'_K$

Sending and Receiving Payloads

- **1** Send $(L_K, P, Y \subseteq Z, t)$ → γ / \bot
- 2 Receive $(L_K, \gamma) \rightarrow P/\perp$
- 3 It's all symmteric key cryptography!



Identity Escrow

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 - To revoke certificates of misbehaving vehicles.
 - To provide concrete court evidence.
- Assuming identity escrow is rare, Open need not be efficient in terms of time/storage complexity.

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- **1** Payload-Hiding security against Chosen-Ciphertext Attacks (PH-CCA): No efficient adversary can infer about the underlying payload without knowing $K_{z,t}$.

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 - Why not do DAE on payloads?
 - Remember the CAM length constraint!

Summary of ZE

Parameter	Zone Encryption	C-ITS Proposal
Encrypted CAM	Yes	No
Anonymity	Yes	No
Pseudonyms per Week	Unlimited	100 (EU) / 20 (US)
CAM Authentication	DAE	ECDSA
Overhead per CAM	224 Bytes	160 Bytes
+ per entered Zones	284 (Request) / 300 (Response) Bytes	N/A

Table 1: Comparison of zone encryption to current C-ITS proposals at a 128-bit security level.

• **Group Signatures**³: A scheme where a user can sign a message anonymously on behalf of the group.

³ David Chaum and Eugène van Heyst. "Group Signatures". In: *Advances in Cryptology — EUROCRYPT '91*. Ed. by Donald W. Davies. Berlin, Heidelberg: Springer, 1991, pp. 257–265. ISBN: 978-3-540-46416-7. DOI: 10.1007/3-540-46416-6. 22.

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 - Other attributes of the user need not be revealed.
- DGS+A using PS⁵ scheme.
 - Can sign *k* message blocks at once.
 - No hash functions needed and signatures are randomizable.
 - Also doubles up as a ZKPoK of σ on m.

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Syntax of DGS+A

Note: An issuer \mathcal{I} is a trusted party that issues credentials to users and can find the user that generated a given signature for a given message.

Setup and Key Generation

- **1** Setup $(1^{\lambda}, k) \to pp$: Generate public parameters.
- ② KG.I(pp) \rightarrow (pk,(sk,st)): Generate key pair for \mathcal{I} .

Credential Issuance

 $\left\langle \text{Issue.I}\left(sk,st,id,A=\left(a_i\right)_{i=1}^k\right)\leftrightharpoons \text{Issue.U}\left(id,A,pk\right)\right\rangle \to cred$: Interactive protocol between a user $\mathcal U$ and issuer $\mathcal I$ to obtain credentials cred.

Syntax of DGS+A

Signing and Verification

- Auth (pk, cred, m) → tok: Generate an authentication token or signature on m.
- ② Vf $(pk, m, A, tok) \rightarrow b \in \{0, 1\}$: Verify whether tok has been properly generated for the given m and A.

Opening

Open $(sk, st, m, A, tok) \rightarrow id/\perp$: Check whether tok was generated properly and recover the identity id of the user that generated tok. **Note**: Time complexity of Open is $\mathcal{O}(|ID|)$.

- Security Properties: Correctness, Traceability, Anonymity.
- Application to ZE: 216 Byte token size at 128-bit security level.
- Extension to threshold opening.

Challenges and Future Improvements

- Key Agreement Strategy
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- Robustness / Implementation Details
 - Encrypting payloads under zone keys in a region.
 - Overlapping time periods for smooth transition.
 - Robust communication medium and retransmission mechanisms.
- 3 Do we really need to encrypt CAMs?
 - Google (Maps) may already be profiling us!
 - Focus on more sensitive messages and information sent less frequently.
 - Avoid complexities in implementation of ZE.

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- OBS+A
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Pointcheval-Sanders Signatures

Consider $\Gamma = (q, \mathbb{G}_0, \mathbb{G}_1, \mathbb{G}_T, e : \mathbb{G}_0 \times \mathbb{G}_1 \to \mathbb{G}_T) \leftarrow G(1^{\lambda})$, where G is a type-3 pairing group generator and λ is the *security parameter*, PS consists of the following algorithms.

Algorithm 1 PS.KG

Input: Pairing group Γ and number of message blocks k.

Output: Signing and verification key (vk, sk).

- 1: Generate $g_1 \in_R \mathbb{G}_1$, $x, y_1, \dots, y_{k+1} \in_R \mathbb{Z}_q$.
- 2: Compute $X \leftarrow g_1^X$ and $Y_j \leftarrow g_1^J$ for $j \in \{1, \dots, k+1\}$.
- 3: **return** $sk \leftarrow (x, y_1, \dots, y_{k+1})$ and $vk \leftarrow (X, Y_1, \dots, Y_{k+1})$.

Pointcheval-Sanders Signatures

Algorithm 2 PS.Sign

Input: Signing key sk and message $m = (m_1, \ldots, m_k)$.

Output: Signature σ on m.

- 1: Generate $h \in_R \mathbb{G}_1, m' \in_R \mathbb{Z}_q$.
- 2: **return** $\sigma \leftarrow \left(m', h, h^{x + \sum_{j=1}^{k} y_j m_j + y_{k+1} m'}\right)$.

Algorithm 3 PS.Vf

Input: Verification key vk, message $m = (m_1, ..., m_k)$, signature $\sigma = (m', \sigma_1, \sigma_2)$ on m.

Output: $b \in \{0, 1\}$.

1: **return**
$$b \leftarrow e\left(\sigma_1, X \prod_{j=1}^k Y_j^{m_j} Y_{k+1}^{m'}\right) \stackrel{?}{=} e\left(\sigma_2, g_1\right)$$

