

Quadratic Programming Assignment

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Abstract—This document contains the solution to Question 27 of Exercise 5 in Chapter 6 of the class 12 NCERT textbook.

1) The point on the curve

$$x^2 = 2y \quad (1)$$

which is nearest to the point $\mathbf{P} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ is

- a) $\begin{pmatrix} 2\sqrt{2} \\ 4 \end{pmatrix}$
- b) $\begin{pmatrix} 2\sqrt{2} \\ 0 \end{pmatrix}$
- c) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
- d) $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

Solution: We need to find

$$\min_{\mathbf{x}} g(\mathbf{x}) = \|\mathbf{x} - \mathbf{P}\|^2 \quad (2)$$

$$\text{s.t. } h(\mathbf{x}) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} = 0 \quad (3)$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad (4)$$

We find the required minima using constrained gradient descent in Fig. 1, plotted using the Python code `codes/grad_pits.py`. The constrained gradient descent update equation is

$$\mathbf{x}_{n+1} = \mathbf{x}_n - 2\alpha \frac{\text{sgn}((\mathbf{x}_n - \mathbf{P})^\top (\mathbf{V}\mathbf{x}_n + \mathbf{u}))}{(\mathbf{V}\mathbf{x}_n + \mathbf{u})} \quad (5)$$

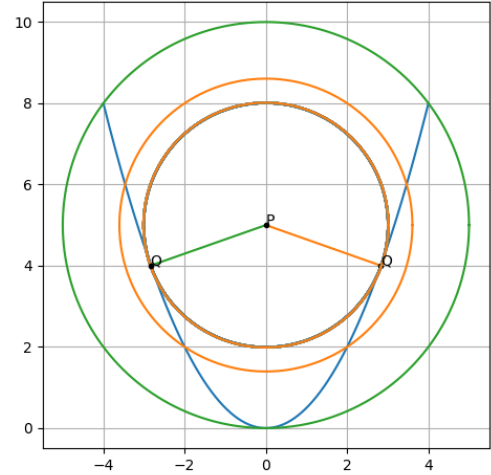


Fig. 1: Gradient descent for a nonconvex optimization problem.