Vector Assignment

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***Abstract*—This document contains the solution to Question 17 of Exercise 2 in Chapter 10 of the class 12 NCERT textbook.**

1) Show that the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ with position vectors $\mathbf{A} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$ form the vertices of a right angled triangle.
Solution: We write the direction vectors of the three sides as

$$\mathbf{c} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \tag{1}$$

$$\mathbf{a} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} -1 \\ -2 \\ -6 \end{pmatrix} \tag{2}$$

$$\mathbf{b} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} \tag{3}$$

Taking the inner product of each pair of vectors,

$$\langle \mathbf{c}, \mathbf{a} \rangle = \mathbf{c}^\top \mathbf{a} = -35 \tag{4}$$

$$\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^\top \mathbf{b} = 6 \tag{5}$$

$$\langle \mathbf{b}, \mathbf{c} \rangle = \mathbf{b}^\top \mathbf{c} = 0 \tag{6}$$

From (6), $\langle \mathbf{b}, \mathbf{c} \rangle = 0$, which implies that $\mathbf{b} \perp \mathbf{c}$. Hence, ΔABC is right angled at A. The Python code `codes/perp.py` verifies the answer.