

Circle Assignment

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Abstract—This document contains the solution to Question 12 of Exercise 5 in Chapter 10 of the class 9 NCERT textbook.

1) Prove that a cyclic parallelogram is a rectangle.

Solution: Consider the points \mathbf{P}_i , $1 \leq i \leq 4$ on the unit circle. Thus, for $1 \leq i \leq 4$,

$$\|\mathbf{P}_i\|^2 = 1 \quad (1)$$

Since $P_1P_2P_3P_4$ is a parallelogram, its diagonals bisect each other. Thus,

$$\frac{\mathbf{P}_1 + \mathbf{P}_3}{2} = \frac{\mathbf{P}_2 + \mathbf{P}_4}{2} \quad (2)$$

$$\Rightarrow \mathbf{P}_1 + \mathbf{P}_3 = \mathbf{P}_2 + \mathbf{P}_4 \quad (3)$$

$$\Rightarrow \|\mathbf{P}_1 + \mathbf{P}_3\|^2 = \|\mathbf{P}_2 + \mathbf{P}_4\|^2 \quad (4)$$

Using (1), (4) becomes

$$\mathbf{P}_1^\top \mathbf{P}_3 = \mathbf{P}_2^\top \mathbf{P}_4 \quad (5)$$

Hence, using (1) and (5)

$$\|\mathbf{P}_1 - \mathbf{P}_3\|^2 = \|\mathbf{P}_1\|^2 - 2\mathbf{P}_1^\top \mathbf{P}_3 + \|\mathbf{P}_3\|^2 \quad (6)$$

$$= \|\mathbf{P}_2\|^2 - 2\mathbf{P}_2^\top \mathbf{P}_4 + \|\mathbf{P}_4\|^2 \quad (7)$$

$$= \|\mathbf{P}_2 - \mathbf{P}_4\|^2 \quad (8)$$

From (8), we see that $P_1P_3 = P_2P_4$, or the diagonals of the parallelogram are equal. Thus, $P_1P_2P_3P_4$ is in fact a rectangle.

The situation is demonstrated in Fig. 1, plotted by the Python code `codes/circle.py`.

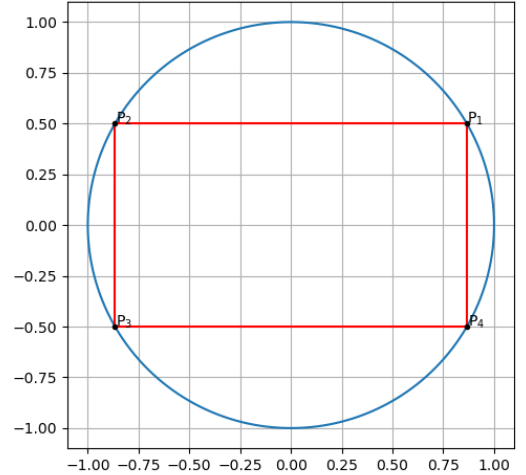


Fig. 1: $P_1P_2P_3P_4$ is a rectangle.