Conic Assignment

Gautam Singh

***Abstract*—This document contains the solution to Question 6 of Exercise 5 in Chapter 11 of the class 11 NCERT textbook.**

1) Find the area of the triangle formed by the lines joining the vertex of the parabola

$$x^2 = 12y \tag{1}$$

to the ends of its latus rectum.
Solution: Rewriting (1) in matrix form,

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} 0 & -6 \end{pmatrix} \mathbf{x} = 0 \tag{2}$$

Since the parabola is clearly symmetric about the y-axis, we see that the directrix is parallel to the x-axis, thus

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3}$$

Using the standard definition of the conic and equating $\mathbf{u}$ and f,

$$\begin{pmatrix} 0 \\ -6 \end{pmatrix} = c\begin{pmatrix} 0 \\ 1 \end{pmatrix} - \mathbf{F} \tag{4}$$

$$0 = \|\mathbf{F}\|^2 - c^2 \tag{5}$$

From (4), we have

$$\mathbf{F} = \begin{pmatrix} 0 \\ c+6 \end{pmatrix} \tag{6}$$

Using (6) in (5),

$$(c+6)^2 = c^2 \tag{7}$$

$$\implies c = -3 \tag{8}$$

Thus,

$$\mathbf{F} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{9}$$

The latus rectum of the parabola is the chord passing through the focus parallel to the directrix. Its equation is given by

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 3 \end{pmatrix} = 3 \tag{10}$$

Adding (2) to 12 times (10),

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = 36 \tag{11}$$

$$\implies \left(\mathbf{x}^\top \begin{pmatrix} 1 \\ 0 \end{pmatrix} 0\right) \mathbf{x} = 36 \tag{12}$$

$$\implies \mathbf{x}^\top \begin{pmatrix} 1 \\ 0 \end{pmatrix} \left(\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}\right) = 36 \tag{13}$$

$$\implies \left(\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}\right)^2 = 36 \tag{14}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = \pm 6 \tag{15}$$

Combining (10) and (15), the ends of the latus rectum are

$$\mathbf{x} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} \pm 6 \\ 3 \end{pmatrix} \tag{16}$$

Since the vertex of the parabola is at $\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, we see that the area of the required triangle is

$$A = \frac{1}{2} \begin{vmatrix} 6 & 3 \\ -6 & 3 \end{vmatrix} = 18 \text{ sq. units} \tag{17}$$

The situation is illustrated in Fig. 1, plotted using the Python code `codes/parabola.py`.

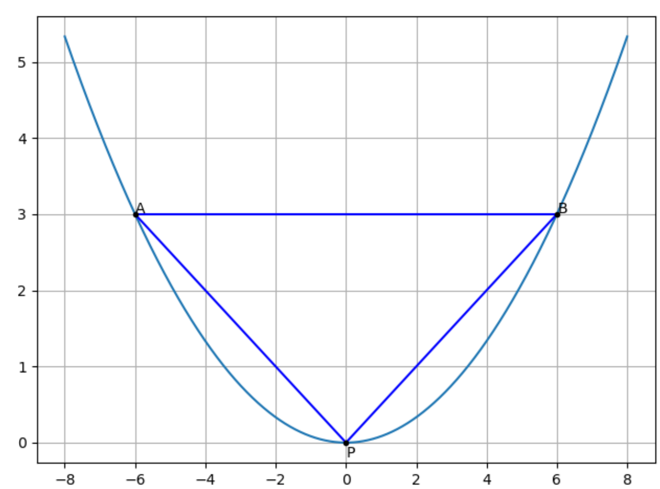

Fig. 1: PAB is the triangle whose area is to be found.