

Quadratic Programming Assignment

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Abstract—This document contains the solution to Question 21 of Exercise 6 in Chapter 6 of the class 12 NCERT textbook using quadratic programming.

1) The line

$$y = mx + 1 \quad (1)$$

is a tangent to the curve

$$y^2 = 4x \quad (2)$$

if the value of m is

- a) 1
- b) 2
- c) 3
- d) $\frac{1}{2}$

Solution: Rewriting (1) and (2) in standard forms, we get,

$$\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix}, \quad c = -1 \quad (3)$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \quad f = 0 \quad (4)$$

Hence, the equivalent optimization problem is

$$\min_{\mathbf{x}} \frac{(\mathbf{n}^\top \mathbf{x} - c)^2}{\|\mathbf{n}\|^2} \quad (5)$$

$$\text{s.t. } \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (6)$$

In quadratic form, (5) becomes

$$\min_{\mathbf{x}} \mathbf{x}^\top \left(\frac{\mathbf{n}\mathbf{n}^\top}{\|\mathbf{n}\|^2} \right) \mathbf{x} + 2 \left(-\frac{c\mathbf{n}^\top}{\|\mathbf{n}\|^2} \right) \mathbf{x} + \frac{c^2}{\|\mathbf{n}\|^2} \quad (7)$$

We solve this problem using quadratic programming. The Python code `codes/qp.py` finds the value of m that has the least cost by plotting Fig. 1. We see that the optimal slope is $m = 1$. Hence, **a)** is the correct answer.

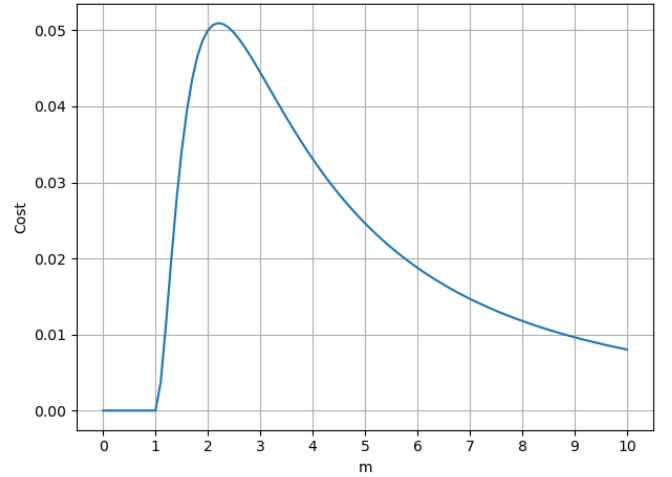


Fig. 1: Cost as a function of m .