

Line Assignment

Gautam Singh

Abstract—This document contains the solution to Question 16 of Exercise 3 in Chapter 10 of the class 12 NCERT textbook.

1) Show that the points $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$, and

$\mathbf{C} = \begin{pmatrix} 3 \\ 10 \\ -1 \end{pmatrix}$ are collinear.

Solution: Points \mathbf{A} , \mathbf{B} and \mathbf{C} are on a line if

$$\text{rank}(\mathbf{A} \ \mathbf{B} \ \mathbf{C}) < 3 \quad (1)$$

Substituting, we must find the rank of

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 7 & 3 & -1 \end{bmatrix} \quad (2)$$

Using row reduction methods to bring \mathbf{M} into row-reduced echelon form,

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 7 & 3 & -1 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 7 & 3 & -1 \end{bmatrix} \quad (3)$$

$$\xrightarrow{R_3 \rightarrow R_3 - 7R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & -11 & -22 \end{bmatrix} \quad (4)$$

$$\xrightarrow{R_1 \rightarrow R_1 - R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 4 \\ 0 & -11 & -22 \end{bmatrix} \quad (5)$$

$$\xrightarrow{R_3 \rightarrow R_3 + \frac{11}{2}R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

Clearly, the rank of \mathbf{M} is 2, and hence the given points are collinear. The rank of \mathbf{M} is verified in the Python code `codes/rank.py`.

Fig. 1 generated by the Python code `codes/line_3d.py` verifies that the three points are indeed collinear as claimed.

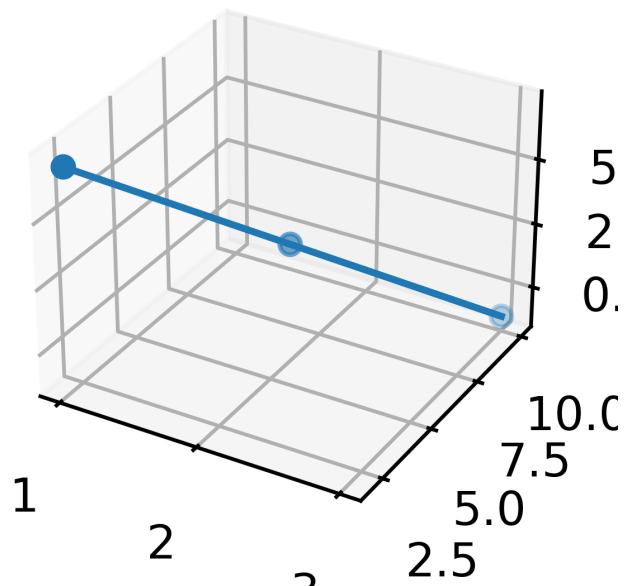


Fig. 1: Points \mathbf{A} , \mathbf{B} and \mathbf{C} are collinear.