Line Assignment

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Abstract—**This document contains the solution to Question 24 of Exercise 4 in Chapter 10 of the class 11 NCERT textbook.**

1) A person standing at the junction (crossing) of two straight paths represented by the equations

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = -4 \tag{1}$$

and

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} = 5 \tag{2}$$

wants to reach the path whose equation is

$$\begin{pmatrix} 6 & -7 \end{pmatrix} \mathbf{x} = -8 \tag{3}$$

Find equation of the path that he should follow.
Solution: We first find the coordinates of the intersection of (1) and (2). Using the augmented matrix and row reduction methods,

$$\begin{pmatrix} 2 & -3 & \bigm| & -4 \\ 3 & 4 & \bigm| & 5 \end{pmatrix} \xleftarrow{R_2 \to 2R_2 - 3R_1} \begin{pmatrix} 2 & -3 & \bigm| & -4 \\ 0 & 17 & \bigm| & 22 \end{pmatrix} \tag{4}$$

$$\xleftarrow{R_1 \to 17R_1 + 3R_2} \begin{pmatrix} 17 & 0 & \bigm| & -1 \\ 0 & 17 & \bigm| & 22 \end{pmatrix} \tag{5}$$

$$\xleftarrow[R_2 \to \frac{R_2}{17}]{R_1 \to \frac{R_1}{17}} \begin{pmatrix} 1 & 0 & \bigm| & -\frac{1}{17} \\ 0 & 1 & \bigm| & \frac{22}{17} \end{pmatrix} \tag{6}$$

the intersection of the lines is $\mathbf{a} = \frac{1}{17}\begin{pmatrix} -1 \\ 22 \end{pmatrix}$.

Clearly, the man should follow the path perpendicular to (3) from $\mathbf{a}$ to reach it in the shortest time. The normal vector of (3) is $\mathbf{m} = \begin{pmatrix} 6 \\ -7 \end{pmatrix}$, which is consequently the direction vector of the required line. Therefore, the required normal vector is given by $\mathbf{n} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$, and hence, the equation of the line is

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{a} \tag{7}$$

$$\implies \begin{pmatrix} 7 & 6 \end{pmatrix} \mathbf{x} = \frac{1}{17}\begin{pmatrix} 7 & 6 \end{pmatrix}\begin{pmatrix} -1 \\ 22 \end{pmatrix} = \frac{125}{17} \tag{8}$$
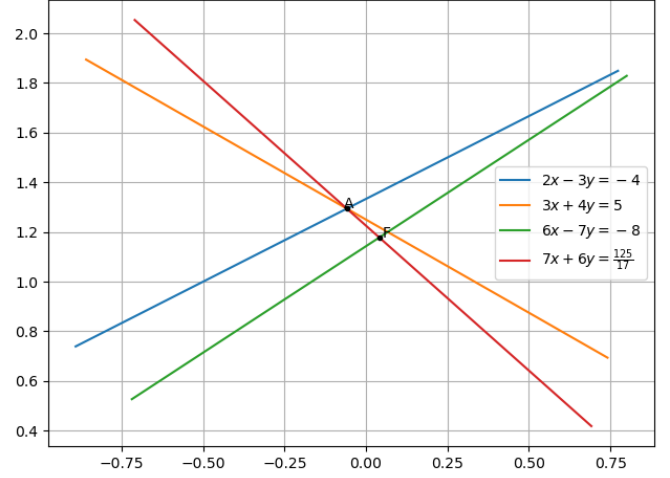
The Python code `codes/crossing.py` veri-

fies the solution and depicts the situation in Fig. 1.

Fig. 1: AF is the required line.