Circle Assignment

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***Abstract*—This document contains the solution to Question 12 of Exercise 5 in Chapter 10 of the class 9 NCERT textbook.**

1) Prove that a cyclic paralellogram is a rectangle.
Solution: Consider the points $\mathbf{P_i}$, $1 \le i \le 4$ in anticlockwise order on the unit circle. Thus, for $1 \le i \le 4$,

$$\mathbf{P_i} = \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \end{pmatrix} \tag{1}$$

where $\theta_i \in (-\pi, \pi]$. Since $P_1P_2P_3P_4$ is a parallelogram, its diagonals bisect each other. Thus, using (1)

$$\frac{\mathbf{P_1} + \mathbf{P_3}}{2} = \frac{\mathbf{P_2} + \mathbf{P_4}}{2} \tag{2}$$

$$\implies \mathbf{P_1} + \mathbf{P_3} = \mathbf{P_2} + \mathbf{P_4} \tag{3}$$

$$\implies \begin{pmatrix} \cos\theta_1 + \cos\theta_3 \\ \sin\theta_1 + \sin\theta_3 \end{pmatrix} = \begin{pmatrix} \cos\theta_2 + \cos\theta_4 \\ \sin\theta_2 + \sin\theta_4 \end{pmatrix} \tag{4}$$

Using (4), we have

$$(\cos\theta_1 + \cos\theta_3)^2 + (\sin\theta_1 + \sin\theta_3)^2$$
$$= (\cos\theta_2 + \cos\theta_4)^2 + (\sin\theta_2 + \sin\theta_4)^2 \tag{5}$$

$$\implies \cos(\theta_1 - \theta_3) = \cos(\theta_2 - \theta_4) \tag{6}$$

$$\implies (\theta_1 - \theta_3) = (\theta_2 - \theta_4) + 2n\pi \tag{7}$$

Hence, using (1) and (7),

$$\|\mathbf{P_1} - \mathbf{P_3}\|^2 = \|\mathbf{P_1}\|^2 - 2\mathbf{P_1}^\top\mathbf{P_3} + \|\mathbf{P_3}\|^2 \tag{8}$$

$$= \|\mathbf{P_2}\|^2 - 2\mathbf{P_2}^\top\mathbf{P_4} + \|\mathbf{P_4}\|^2 \tag{9}$$

$$= \|\mathbf{P_2} - \mathbf{P_4}\|^2 \tag{10}$$

From (10), we see that $P_1P_3 = P_2P_4$, or the diagonals of the parallelogram are equal. Thus, $P_1P_2P_3P_4$ is in fact a rectangle.
The situation is demonstrated in Fig. 1, plotted by the Python code `codes/circle.py`.

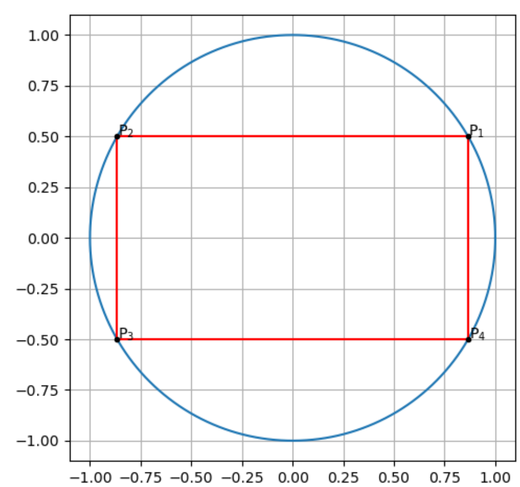

Fig. 1: $P_1P_2P_3P_4$ is a rectangle.