Optimization Assignment

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***Abstract*—This document contains the solution to Question 4 of Exercise 2 in Chapter 10 of the class 11 NCERT textbook.**

1) Find the coordinates of the foot of perpendicular from the point

$$\mathbf{P} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{1}$$

to the line

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = 16 \tag{2}$$

Solution: We rewrite the problem as

$$\min_{\mathbf{x}} h(\mathbf{x}) \triangleq \|\mathbf{x} - \mathbf{P}\|^2 \tag{3}$$

$$\text{s.t. } g(\mathbf{x}) \triangleq \mathbf{n}^\top \mathbf{x} - c = 0 \tag{4}$$

Define

$$C(\mathbf{x}, \lambda) = h(\mathbf{x}) - \lambda g(\mathbf{x}) \tag{5}$$

and note that

$$\nabla h(\mathbf{x}) = 2(\mathbf{x} - \mathbf{P}) \tag{6}$$

$$\nabla g(\mathbf{x}) = \mathbf{n} \tag{7}$$

We are required to find $\lambda \in \mathbb{R}$ such that

$$\nabla C(\mathbf{x}, \lambda) = 0 \tag{8}$$

$$\implies 2(\mathbf{x} - \mathbf{P}) - \lambda \mathbf{n} = 0 \tag{9}$$

However, $\mathbf{x}$ lies on the line (2). Thus, from (9),

$$\mathbf{n}^\top \left(\frac{\lambda}{2} \mathbf{n} + \mathbf{P} \right) - c = 0 \tag{10}$$

$$\implies \frac{25\lambda}{2} - 15 - 16 = 0 \tag{11}$$

$$\implies \lambda = \frac{62}{25} \tag{12}$$

Substituting (12) in (9), the optimal point is given by

$$\mathbf{Q} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \frac{31}{25} \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{13}$$

$$= \frac{1}{25} \begin{pmatrix} 68 \\ -49 \end{pmatrix} \tag{14}$$

To find $\mathbf{Q}$ graphically, we use constrained gradient descent, with learning rate $\alpha = 0.01$. The results are shown in Fig. 1, plotted using the Python code `codes/gd_lagrange.py`.

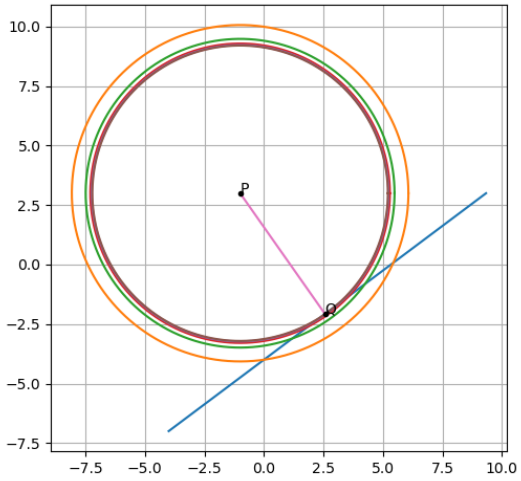

Fig. 1: Constrained gradient descent to find optimal $\mathbf{Q}$.