Circle Assignment

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***Abstract*—This document contains the solution to Question 12 of Exercise 5 in Chapter 10 of the class 9 NCERT textbook.**

1) Prove that a cyclic paralellogram is a rectangle.
Solution: Consider the points $\mathbf{P_i}$, $1 \le i \le 4$ on the unit circle. Thus, for $1 \le i \le 4$,

$$\mathbf{P_i} \triangleq \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \end{pmatrix} \tag{1}$$

where all the θ_i's are distinct. Choose the axes in such a way that P_1P_4 and P_2P_3 are parallel to the x-axis. Suppose P_1P_4 lies on the line

$$\mathbf{n}^\top \mathbf{x} = c \tag{2}$$

where

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3}$$

Since $\mathbf{P_1}$ and $\mathbf{P_4}$ also lie on the unit circle, substituting $\mathbf{P_1}$ and $\mathbf{P_4}$ into (2) gives

$$c = \sin\theta_1 = \sin\theta_4 \tag{4}$$

Similarly for $\mathbf{P_2}$ and $\mathbf{P_3}$,

$$\sin\theta_2 = \sin\theta_3 \tag{5}$$

From (4),

$$\cos^2\theta_1 = 1 - \sin^2\theta_1 \tag{6}$$
$$= 1 - \sin^2\theta_4 \tag{7}$$
$$= \cos^2\theta_4 \tag{8}$$

Thus $\cos\theta_1 = \pm\cos\theta_4$. But since $\theta_1 \neq \theta_4$, we must have

$$\cos\theta_1 = -\cos\theta_4 \tag{9}$$

Similarly, we have

$$\cos\theta_2 = -\cos\theta_3 \tag{10}$$

Since $P_1P_2P_3P_4$ form a parallelogram, $P_1P_2 \parallel P_3P_4$. Equating direction vectors and using equations (4), (5), (9) and (10),

$$\begin{pmatrix} \cos\theta_1 - \cos\theta_2 \\ \sin\theta_1 - \sin\theta_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_4 - \cos\theta_3 \\ \sin\theta_4 - \sin\theta_3 \end{pmatrix} \tag{11}$$
$$\implies \cos\theta_1 - \cos\theta_2 = \cos\theta_4 - \cos\theta_3 \tag{12}$$
$$\implies 2\cos\theta_1 = 2\cos\theta_2 \tag{13}$$
$$\implies \cos\theta_1 = \cos\theta_2 \tag{14}$$

Using (14), the direction vector of P_1P_2 is

$$\mathbf{P_1} - \mathbf{P_2} = \begin{pmatrix} \cos\theta_1 - \cos\theta_2 \\ \sin\theta_1 - \sin\theta_2 \end{pmatrix} \tag{15}$$
$$= \begin{pmatrix} 0 \\ \sin\theta_1 - \sin\theta_2 \end{pmatrix} \tag{16}$$

Equation (16) clearly shows that P_1P_2 is parallel to the y-axis. Thus, $P_1P_2 \perp P_1P_4$ and therefore, $P_1P_2P_3P_4$ is a rectangle, as required. The situation is demonstrated in Fig. 1, plotted by the Python code `codes/circle.py`.

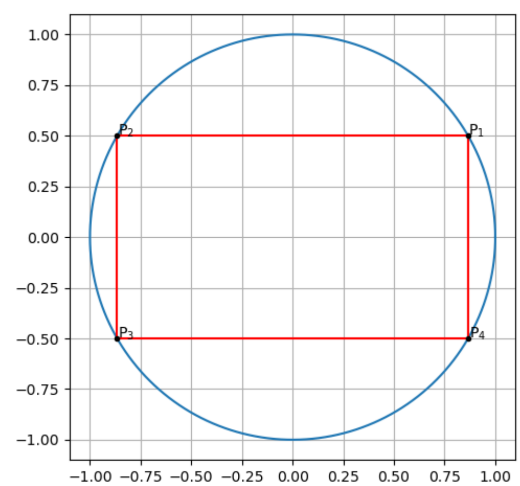

Fig. 1: $P_1P_2P_3P_4$ is a rectangle.