Probability Assignment

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***Abstract*—This document contains the solution to Question 15 of Exercise 1 in Chapter 13 of the class 12 NCERT textbook.**

1) Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.
Solution: Define random variables X and Y to be the outcomes of the first and second events repectively as shown in Tables 1 and 2.

$X = 1$	Die shows 1.
$X = 2$	Die shows 2.
$X = 3$	Die shows 3.
$X = 4$	Die shows 4.
$X = 5$	Die shows 5.
$X = 6$	Die shows 6.

TABLE 1: Definition of X.

$Y = 1$	Coin shows heads.
$Y = 2$	Coin shows tails.
$Y = 3$	Die shows 1.
$Y = 4$	Die shows 2.
$Y = 5$	Die shows 3.
$Y = 6$	Die shows 4.
$Y = 7$	Die shows 5.
$Y = 8$	Die shows 6.

TABLE 2: Definition of Y.

We are required to find $\Pr(Y = 1|X = 3)$. Since both the coin and die are fair, the joint pmf of X and Y is as shown in Table 3.
Clearly, $\Pr(X = 3,\ Y = 1) = 0$, thus

$$\Pr(Y = 1|X = 3) = \frac{\Pr(X = 3,\ Y = 1)}{\Pr(X = 3)} = 0 \tag{1}$$

Y \ X	1	2	3	4	5	6
1	$\frac{1}{12}$	$\frac{1}{12}$	0	$\frac{1}{12}$	$\frac{1}{12}$	0
2	$\frac{1}{12}$	$\frac{1}{12}$	0	$\frac{1}{12}$	$\frac{1}{12}$	0
3	0	0	$\frac{1}{36}$	0	0	$\frac{1}{36}$
4	0	0	$\frac{1}{36}$	0	0	$\frac{1}{36}$
5	0	0	$\frac{1}{36}$	0	0	$\frac{1}{36}$
6	0	0	$\frac{1}{36}$	0	0	$\frac{1}{36}$
7	0	0	$\frac{1}{36}$	0	0	$\frac{1}{36}$
8	0	0	$\frac{1}{36}$	0	0	$\frac{1}{36}$

TABLE 3: Joint pmf of X and Y.