

Circle Assignment

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Abstract—This document contains the solution to Question 6 of Exercise 4 in Chapter 10 of the class 9 NCERT textbook.

- 1) A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution: Let the position vectors of the boys be

$$\mathbf{A} = \begin{pmatrix} r \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} r \cos \beta \\ r \sin \beta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} r \cos \gamma \\ r \sin \gamma \end{pmatrix} \quad (1)$$

We have,

$$\|\mathbf{A} - \mathbf{S}\|^2 = \|\mathbf{A} - \mathbf{D}\|^2 \quad (2)$$

$$\Rightarrow \mathbf{A}^\top \mathbf{S} = \mathbf{A}^\top \mathbf{D} \quad (3)$$

$$\Rightarrow \cos \beta = \cos \gamma \quad (4)$$

$$\Rightarrow \sin \beta = -\sin \gamma \quad (5)$$

Since $\beta \neq \gamma$. We can also write using (4) and (5),

$$\|\mathbf{A} - \mathbf{S}\|^2 = \|\mathbf{S} - \mathbf{D}\|^2 \quad (6)$$

$$\Rightarrow \mathbf{A}^\top \mathbf{S} = \mathbf{D}^\top \mathbf{S} \quad (7)$$

$$\Rightarrow \cos \beta = \cos(\beta - \gamma) \quad (8)$$

$$\Rightarrow \cos \beta = \cos^2 \beta - \sin^2 \beta \quad (9)$$

$$\Rightarrow 2 \cos^2 \beta - \cos \beta - 1 = 0 \quad (10)$$

$$\Rightarrow (2 \cos \beta + 1)(\cos \beta - 1) = 0 \quad (11)$$

From (11), if $\cos \beta = 1$, then $\sin \beta = \sin \gamma = 0$, a contradiction. Therefore, since $\beta, \gamma \leq 2\pi$,

$$\cos \beta = \cos \gamma = -\frac{1}{2} \quad (12)$$

$$\Rightarrow \beta, \gamma \in \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\} \quad (13)$$

Therefore, the length of the thread is

$$\|\mathbf{S} - \mathbf{D}\| = 2r \sin \beta = r\sqrt{3} \quad (14)$$

Here, $r = 20$ m. Thus, the length is $20\sqrt{3}$ m.

The situation is demonstrated in Fig. 1, plotted by the Python code `codes/equilateral.py`. Here, the values used for construction are shown in Table (I)

Parameter	Value
r	20
β	$\frac{2\pi}{3}$
γ	$\frac{4\pi}{3}$

TABLE I: Parameters used in the construction of Fig. 1.

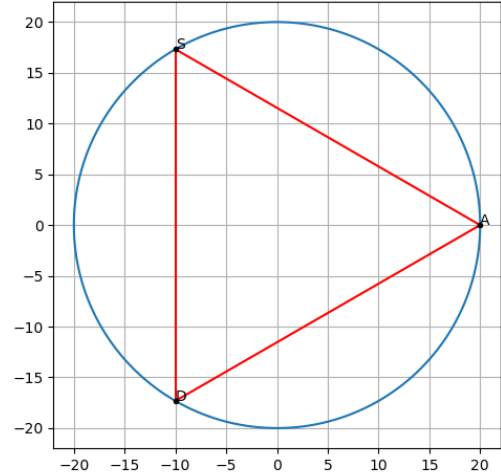


Fig. 1: ASD is an equilateral triangle of side $20\sqrt{3}$ m.