

Linear Programming Assignment

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Abstract—This document contains the solution to Question 10 of Exercise 1 in Chapter 12 of the class 12 NCERT textbook.

1) Find

$$\max_{x,y} Z = x + y \quad (1)$$

$$\text{s.t. } x - y \leq -1 \quad (2)$$

$$-x + y \leq 0 \quad (3)$$

$$x, y \geq 0 \quad (4)$$

Solution: We use the simplex method. Introducing slack variables, we rewrite (1), (2) and (3).

$$x - y + s_1 = -1 \quad (5)$$

$$-x + y + s_2 = 0 \quad (6)$$

$$-x - y + Z = 0 \quad (7)$$

$$x, y, s_1, s_2, Z \geq 0 \quad (8)$$

The augmented matrix becomes

$$\mathbf{S} = \begin{pmatrix} 1 & -1 & 1 & 0 & 0 & -1 \\ -1 & \textcircled{1} & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (9)$$

Here, choosing the pivot column of \mathbf{S} to be the second column, the ratios by row numbers are

$$r_1 = \frac{-1}{1} = -1 \quad (10)$$

$$r_2 = \frac{0}{1} = 0 \quad (11)$$

$$r_3 = \frac{0}{-1} = 0 \quad (12)$$

From (10) to (12), we see that row 2 is suitable to be assigned a pivot row. Hence, the pivot

element is circled in (9). By row reduction,

$$\mathbf{S} \xrightarrow{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & -1 \\ -1 & 1 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (13)$$

$$\xrightarrow{R_3 \leftarrow R_3 + R_2} \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & -1 \\ -1 & 1 & 0 & 1 & 0 & 0 \\ -2 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (14)$$

Again, the pivot column in (14) is the first column. However, the first column has all negative elements, and therefore, there is no solution to the optimization problem. This is justified by Fig. 1, plotted using the Python code `codes/lp.py`. This is also verified using

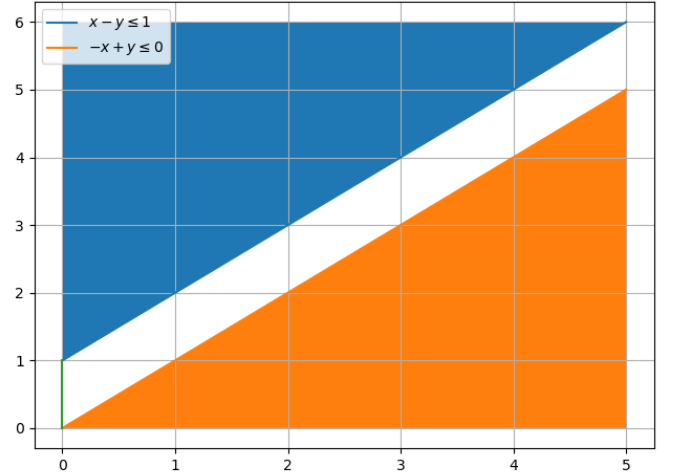


Fig. 1: The given optimization problem has no solution.

`cvxpy` in the Python code `codes/lp_cvx.py`.