Conic Assignment

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***Abstract*—This document contains the solution to Question 12 of Exercise 2 in Chapter 11 of the class 11 NCERT textbook.**

1) Find the equation of the parabola with vertex

$$\mathbf{V} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

and passing through the point

$$\mathbf{P} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \tag{2}$$

and is symmetric to the y-axis.

Solution: Let the equation of the conic with focus $\mathbf{F}$, directrix $\mathbf{n}^\top \mathbf{x} = c$ and eccentricity e be

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{3}$$

where

$$\mathbf{V} \triangleq \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n}\mathbf{n}^\top \tag{4}$$

$$\mathbf{u} \triangleq ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \tag{5}$$

$$f \triangleq \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \tag{6}$$

Since the conic is a parabola symmetric to the y-axis, we have

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad e = 1 \tag{7}$$

From (4),

$$\mathbf{V} = \mathbf{I} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{8}$$

$$\mathbf{u} = c \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \mathbf{F} \tag{9}$$

$$f = \|\mathbf{F}\|^2 - c^2 \tag{10}$$

Putting $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ in (3) gives $f = 0$, thus

$$\|\mathbf{F}\|^2 = c^2 \tag{11}$$

Using the symmetry of the conic, we see that $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ lie on the conic. Substituing both points, we get

$$\mathbf{F}^\top \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \frac{25}{2} - 2c \tag{12}$$

$$\mathbf{F}^\top \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \frac{25}{2} - 2c \tag{13}$$

Adding and subtracting (12) and (13),

$$\mathbf{F}^\top \begin{pmatrix} 0 \\ 4 \end{pmatrix} = 25 - 4c \tag{14}$$

$$\mathbf{F}^\top \begin{pmatrix} 10 \\ 0 \end{pmatrix} = 0 \tag{15}$$

$$\implies \mathbf{F} = \begin{pmatrix} 0 \\ \frac{25}{4} - c \end{pmatrix} \tag{16}$$

Using (11),

$$\frac{25}{4} - c \pm c = 0 \tag{17}$$

$$\implies c = \frac{25}{8} \tag{18}$$

Thus,

$$\mathbf{F} = \begin{pmatrix} 0 \\ \frac{25}{8} \end{pmatrix} \tag{19}$$

The conic is plotted in Fig. 1. The plot id generated using the Python code `codes/conic.py`.

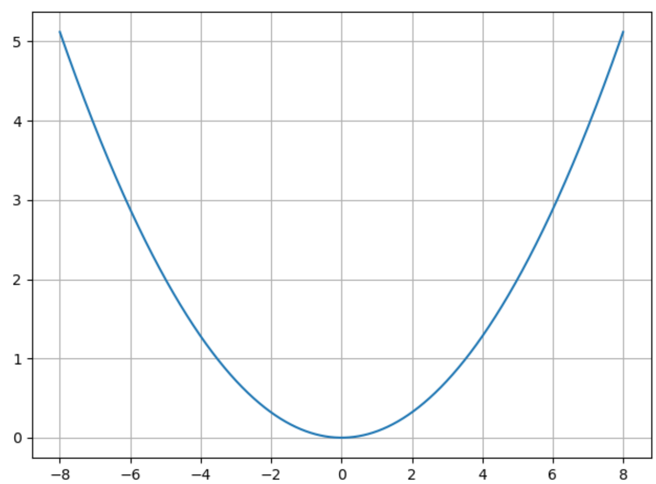

Fig. 1: Locus of the required parabola.