Circle Assignment

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Abstract—**This document contains the solution to Question 12 of Exercise 5 in Chapter 10 of the class 9 NCERT textbook.**

1) Prove that a cyclic paralellogram is a rectangle.
   Solution: Consider the points $\mathbf{P_i}$, $1 \le i \le 4$ on the unit circle, where

   $$\mathbf{P_i} \triangleq \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \end{pmatrix} \quad (1)$$

   Now, since $P_1P_2P_3P_4$ form a parallelogram,

   $$\|\mathbf{P_1} - \mathbf{P_2}\| = \|\mathbf{P_3} - \mathbf{P_4}\| \quad (2)$$
   $$\implies \mathbf{P_1}^\top\mathbf{P_2} = \mathbf{P_3}^\top\mathbf{P_4} \quad (3)$$

   and

   $$\|\mathbf{P_1} - \mathbf{P_4}\| = \|\mathbf{P_2} - \mathbf{P_3}\| \quad (4)$$
   $$\implies \mathbf{P_1}^\top\mathbf{P_4} = \mathbf{P_2}^\top\mathbf{P_3} \quad (5)$$

   Also, it is given that the parallelogram is cyclic. So,

   $$(\mathbf{P_4} - \mathbf{P_1})^\top (\mathbf{P_1} - \mathbf{P_2}) + (\mathbf{P_4} - \mathbf{P_3})^\top (\mathbf{P_3} - \mathbf{P_2}) = 0 \quad (6)$$

   Using (3) and (5) in (6), and noting that $\mathbf{P_i}^\top\mathbf{P_i} = 1$, $1 \le i \le 4$,

   $$\mathbf{P_4}^\top\mathbf{P_1} - \mathbf{P_4}^\top\mathbf{P_2} + \mathbf{P_1}^\top\mathbf{P_2} - 1 = 0 \quad (7)$$
   $$\mathbf{P_4}^\top\mathbf{P_1} - \mathbf{P_4}^\top\mathbf{P_2} + \mathbf{P_1}^\top\mathbf{P_2} - \mathbf{P_1}^\top\mathbf{P_1} = 0 \quad (8)$$
   $$(\mathbf{P_4} - \mathbf{P_1})^\top (\mathbf{P_1} - \mathbf{P_2}) = 0 \quad (9)$$

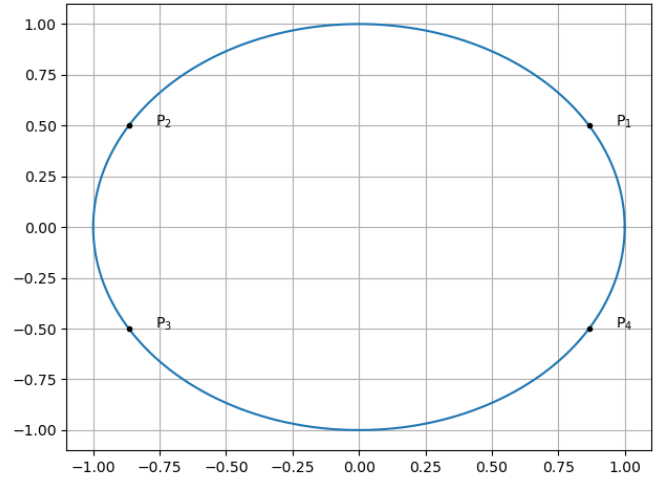   Hence $P_1P_2 \perp P_1P_4$, and thus, the parallelogram is indeed a rectangle. The situation is demonstrated in Fig. 1, plotted by the Python code `codes/circle.py`.

Fig. 1: $P_1P_2P_3P_4$ is a rectangle.