Quadratic Programming Assignment

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***Abstract*—This document contains the solution to Question 27 of Exercise 5 in Chapter 6 of the class 12 NCERT textbook.**

1) The point on the curve

$$x^2 = 2y \tag{1}$$

which is nearest to the point $\mathbf{P} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ is

a) $\begin{pmatrix} 2\sqrt{2} \\ 4 \end{pmatrix}$
b) $\begin{pmatrix} 2\sqrt{2} \\ 0 \end{pmatrix}$
c) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
d) $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

Solution: We need to find

$$\min_{\mathbf{x}} g(\mathbf{x}) = \|\mathbf{x} - \mathbf{P}\|^2 \tag{2}$$

$$\text{s.t. } h(\mathbf{x}) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} = 0 \tag{3}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{4}$$

We find the required minima using constrained gradient descent in Fig. 1, plotted using the Python code `codes/grad_pits.py`. The constrained gradient descent update equation is

$$\mathbf{x_{n+1}} = \mathbf{x_n} - \alpha \nabla h(\mathbf{x_n}) \tag{5}$$

$$\mathbf{x_{n+1}} = \mathbf{x_n} - 2\alpha \operatorname{sgn}\left((\mathbf{x_n} - \mathbf{P})^\top (\mathbf{V}\mathbf{x_n} + \mathbf{u})\right) (\mathbf{V}\mathbf{x_n} + \mathbf{u}) \tag{6}$$

where (6) follows from (3) and the fact that the signum function is used to choose the direction in which gradient descent is followed.

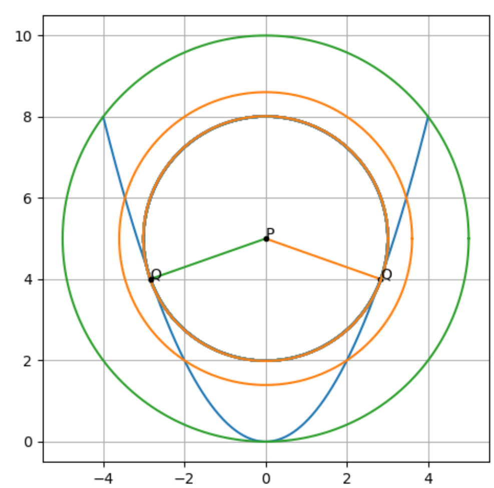

Fig. 1: Gradient descent for a nonconvex optimization problem.