Circle Assignment

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***Abstract*—This document contains the solution to Question 12 of Exercise 5 in Chapter 10 of the class 9 NCERT textbook.**

1) Prove that a cyclic paralellogram is a rectangle.
   Solution: Consider the points $\mathbf{P_i}, \ 1 \le i \le 4$ on the unit circle, where

$$\mathbf{P_i} \triangleq \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \end{pmatrix} \tag{1}$$

Now, since $P_1P_2P_3P_4$ form a parallelogram,

$$\left\|\mathbf{P_1} - \mathbf{P_2}\right\| = \left\|\mathbf{P_3} - \mathbf{P_4}\right\| \tag{2}$$

$$\implies \mathbf{P_1}^\top \mathbf{P_2} = \mathbf{P_3}^\top \mathbf{P_4} \tag{3}$$

and

$$\left\|\mathbf{P_1} - \mathbf{P_4}\right\| = \left\|\mathbf{P_2} - \mathbf{P_3}\right\| \tag{4}$$

$$\implies \mathbf{P_1}^\top \mathbf{P_4} = \mathbf{P_2}^\top \mathbf{P_3} \tag{5}$$

Also, it is given that the parallelogram is cyclic. So,

$$\left(\mathbf{P_4} - \mathbf{P_1}\right)^\top \left(\mathbf{P_1} - \mathbf{P_2}\right) + \left(\mathbf{P_4} - \mathbf{P_3}\right)^\top \left(\mathbf{P_3} - \mathbf{P_2}\right) = 0 \tag{6}$$

Using (3) and (5) in (6), and noting that $\mathbf{P_i}^\top \mathbf{P_i} = 1, \ 1 \le i \le 4$,

$$\mathbf{P_4}^\top \mathbf{P_1} - \mathbf{P_4}^\top \mathbf{P_2} + \mathbf{P_1}^\top \mathbf{P_2} - 1 = 0 \tag{7}$$

$$\mathbf{P_4}^\top \mathbf{P_1} - \mathbf{P_4}^\top \mathbf{P_2} + \mathbf{P_1}^\top \mathbf{P_2} - \mathbf{P_1}^\top \mathbf{P_1} = 0 \tag{8}$$

$$\left(\mathbf{P_4} - \mathbf{P_1}\right)^\top \left(\mathbf{P_1} - \mathbf{P_2}\right) = 0 \tag{9}$$

Hence $P_1P_2 \perp P_1P_4$, and thus, the parallelogram is indeed a rectangle. The situation is demonstrated in Fig. 1, plotted by the Python code `codes/circle.py`.

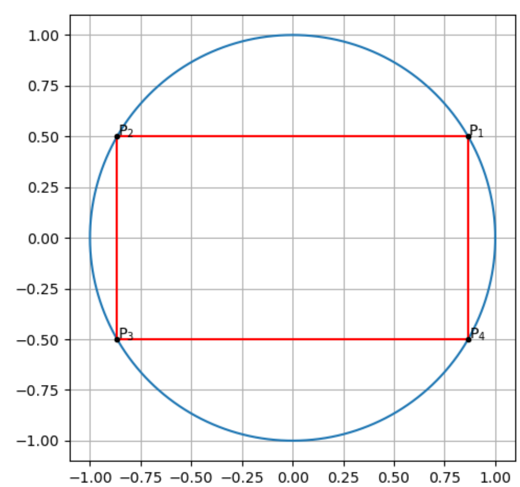

Fig. 1: $P_1P_2P_3P_4$ is a rectangle.