# Quadratic Programming Assignment

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***Abstract*—This document contains the solution to Question 21 of Exercise 6 in Chapter 6 of the class 12 NCERT textbook using quadratic programming.**

1) The line

$$y = mx + 1 \tag{1}$$

is a tangent to the curve

$$y^2 = 4x \tag{2}$$

if the value of $m$ is

a) 1
b) 2
c) 3
d) $\frac{1}{2}$

**Solution:** Rewriting (1) and (2) in standard forms, we get,

$$\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix}, \ c = -1 \tag{3}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, \ f = 0 \tag{4}$$

Hence, the equivalent optimization problem is

$$\min_{\mathbf{x}} \frac{(\mathbf{n}^\top \mathbf{x} - c)^2}{\|\mathbf{n}\|^2} \tag{5}$$

$$\text{s.t. } \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{6}$$

Using

$$\mathbf{y} \triangleq \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} \tag{7}$$

In quadratic form, (5) becomes

$$\min_{\mathbf{x}} \mathbf{x}^\top \left( \frac{\mathbf{n}\mathbf{n}^\top}{\|\mathbf{n}\|^2} \right) \mathbf{x} + 2\left( -\frac{c\mathbf{n}^\top}{\|\mathbf{n}\|^2} \right) \mathbf{x} + \frac{c^2}{\|\mathbf{n}\|^2} \tag{8}$$

We solve this problem using quadratic programming. The Python code `codes/qp.py` finds the value of $m$ that has the least cost by plotting Fig. 1. We see that the optimal slope is $m = 1$. Hence, **a)** is the correct answer.

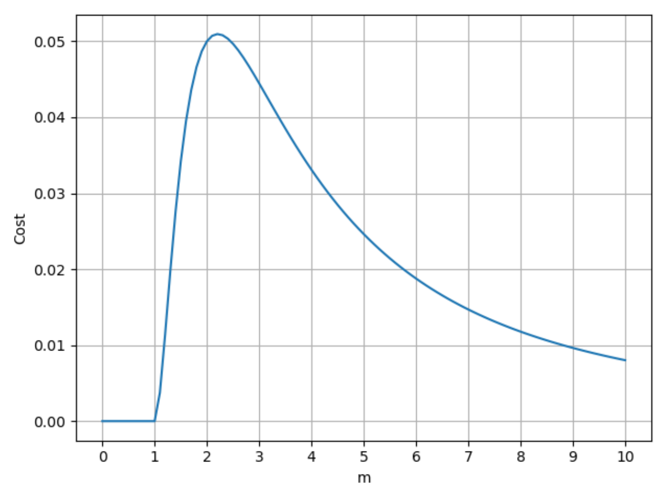


Fig. 1: Cost as a function of $m$.