

Straight Lines Assignment

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Abstract—This document contains the solution to Question 13 of Exercise 1 in Chapter 10 of the class 11 NCERT textbook.

- 1) If three points $\mathbf{A} = \begin{pmatrix} h \\ 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 0 \\ k \end{pmatrix}$ lie on a line, show that

$$\frac{a}{h} + \frac{b}{k} = 1 \quad (1)$$

Solution: The collinearity matrix formed by the three points is singular. Using row reduction,

$$\begin{pmatrix} h & a & 0 \\ 0 & b & k \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow[R_2 \rightarrow R_2/b]{R_1 \rightarrow R_1/h} \begin{pmatrix} 1 & \frac{a}{h} & 0 \\ 0 & 1 & \frac{k}{b} \\ 1 & 1 & 1 \end{pmatrix} \quad (2)$$

$$\xrightarrow{R_3 \rightarrow R_3 - R_1} \begin{pmatrix} 1 & \frac{a}{h} & 0 \\ 0 & 1 & \frac{k}{b} \\ 0 & 1 - \frac{a}{h} & 1 \end{pmatrix} \quad (3)$$

$$\xrightarrow{R_3 \rightarrow R_3 - (1 - \frac{a}{h})R_2} \begin{pmatrix} 1 & \frac{a}{h} & 0 \\ 0 & 1 & \frac{k}{b} \\ 0 & 0 & 1 - \frac{k}{b} \left(1 - \frac{a}{h}\right) \end{pmatrix} \quad (4)$$

Since the matrix is singular,

$$1 - \frac{k}{b} \left(1 - \frac{a}{h}\right) = 0 \quad (5)$$

$$\implies \frac{a}{h} + \frac{b}{k} = 1 \quad (6)$$

as desired. A sample illustration is shown in Fig. 1. The figure is generated using the Python code `codes/intercept.py`. The input parameters used are shown in Table 1.

Parameter	Value
a	2
b	3
h	5
k	5

TABLE 1: Input parameters used in the construction of Fig. 1.

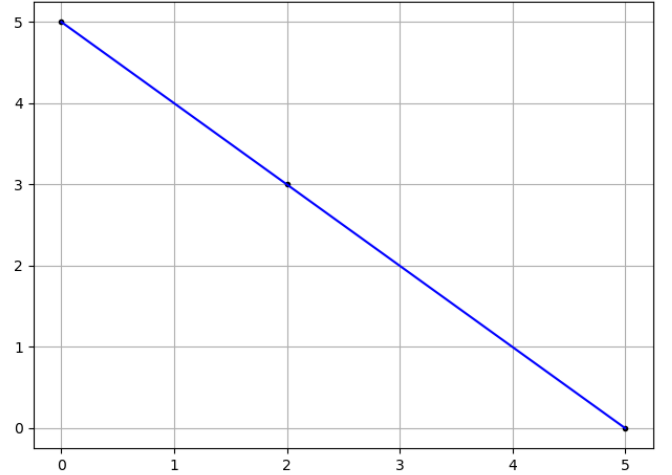


Fig. 1: Plot for $a = 2$, $b = 3$, $h = k = 5$.