LaTeX Assignment

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***Abstract*—This document contains the solution to Exercises 10.4 of the Class 9 NCERT Mathematics textbook.**

Problem 1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Solution: Let the centres of the two circles be $\begin{pmatrix}0\\0\end{pmatrix}$ and $\begin{pmatrix}4\\0\end{pmatrix}$. The situation is shown in Fig. (1).

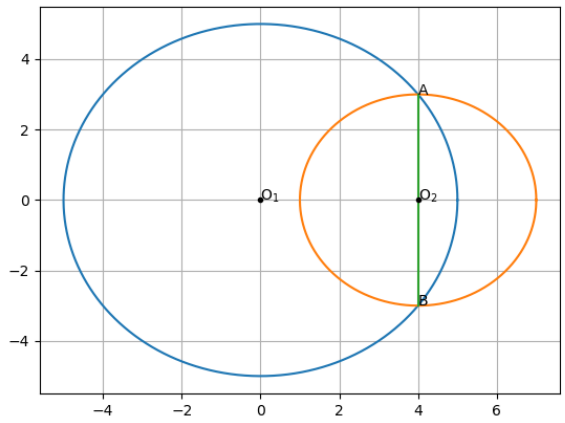

Fig. 1: The common chord in this situation is AB.

The equations of the circles O_1 and O_2 are

$$\|\mathbf{x}\|^2 - 25 = 0 \tag{1}$$

$$\|\mathbf{x}\|^2 - 2\begin{pmatrix}4\\0\end{pmatrix}^T.\mathbf{x} + 7 = 0 \tag{2}$$

$$\tag{3}$$

Substituting (1) in (2), we get

$$\begin{pmatrix}4\\0\end{pmatrix}^T.\mathbf{x} = 16 \tag{4}$$

$$\tag{5}$$

Using (1), we see that the intersection points of the two circles are

$$\mathbf{x_1} = \begin{pmatrix}4\\3\end{pmatrix} \qquad \mathbf{x_2} = \begin{pmatrix}4\\-3\end{pmatrix} \tag{6}$$

Thus, the length of the common chord is $\|\mathbf{x_1} - \mathbf{x_2}\| = 6$.

Problem 2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution: We know that equal chords of a circle subtend equal angles at the centre of the circle (see Fig. (2)). The area of the minor and major segments are (here $[.]$ denotes the area of region enclosed by . and $[O]$ denotes the area of the circle centered at O)

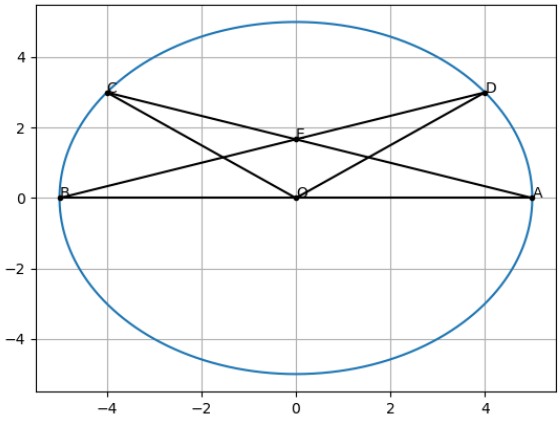

Fig. 2: Two equal chords of a circle create equal segments.

$$[AECDA] = [OCDAO] - [OAECO] \tag{7}$$

$$= [OBCDO] - [OBEDO] \tag{8}$$

$$= [BEDCB] \tag{9}$$

since sectors subtending equal angles at the centre of the circle are of equal area and equal chords of a circle are equidistant from the centre of the circle.

Using (9),

$$[AECBA] = [O] - [AECDA] \quad (10)$$
$$= [O] - [BEDCB] \quad (11)$$
$$= [BEDAB] \quad (12)$$

and the conclusion follows.

Problem 3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution: See Fig. (3). In triangles OP_1E and OP_2E,

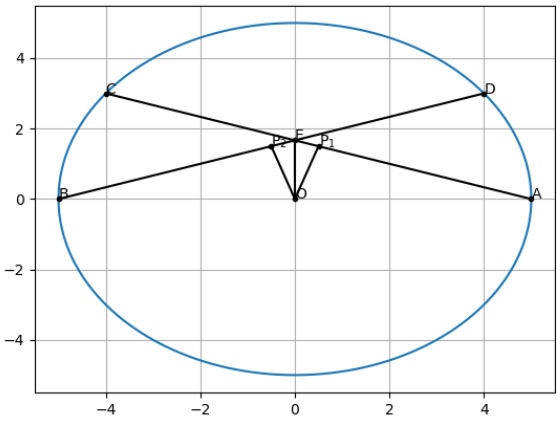

Fig. 3: Equal intersecting chords of a circle make equal angles with OE.

$$\angle OP_1E = \angle OP_2E \quad \text{[Right angles]} \quad (13)$$
$$OE = OE \quad \text{[Common]} \quad (14)$$
$$OP_1 = OP_2 \quad [\because AC = BD] \quad (15)$$

Thus, $\Delta OP_1E \cong \Delta OP_2E$ by RHS criterion. Equating corresponding parts, $\angle AEO = \angle P_1EO = \angle P_2EO = \angle BEO$ as desired.

Problem 4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. (4)).

Solution: We know that the perpendicular dropped from the centre of a circle to a chord bisects the chord. Therefore, in Fig. (4),

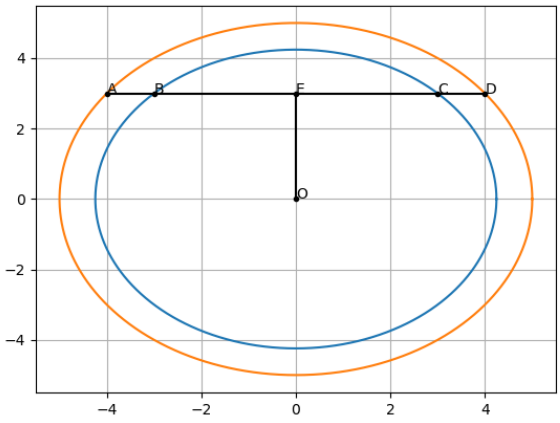

Fig. 4: ABCD intersects the two circles at A, B, C, D. E is the foot of the perpendicular on AD from O.

$$AE = ED \quad (16)$$
$$BE = EC \quad (17)$$
$$\implies AB = AE - BE = ED - EC = CD \quad (18)$$

as required.

Problem 5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Solution: Suppose that Reshma, Salma and Mandip are located at R, S and M as shown in Fig. (5), and have position vectors $\mathbf{r}, \mathbf{s} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and $\mathbf{m}$ respectively.

$\mathbf{r}$ and $\mathbf{m}$ satisfy

$$\|\mathbf{x}\|^2 = 25 \quad (19)$$
$$\|\mathbf{x} - \mathbf{s}\|^2 = 36 \quad (20)$$

Using (19) in (20),

$$\mathbf{s}^T\mathbf{x} = 7 \quad (21)$$

Using (21) along with (19) gives two solutions

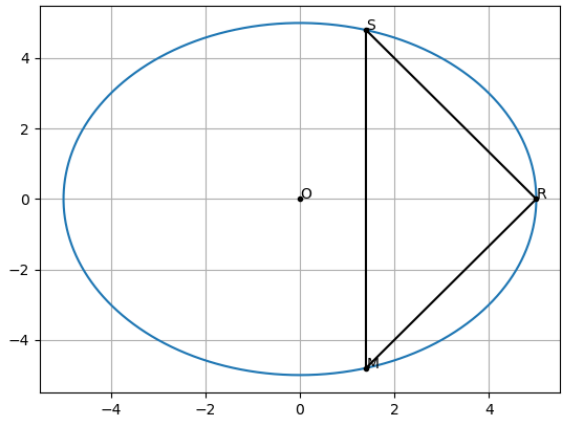

Fig. 5: $RS = SM = 6$

for $\mathbf{x}$,

$$\mathbf{r} = \frac{1}{5}\begin{pmatrix}7\\24\end{pmatrix} \qquad \mathbf{m} = \frac{1}{5}\begin{pmatrix}7\\-24\end{pmatrix} \tag{22}$$

Thus, the required distance is $\|\mathbf{r}-\mathbf{m}\| = 9.6$m.

Problem 6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution: The situation is depicted in Fig. (6).

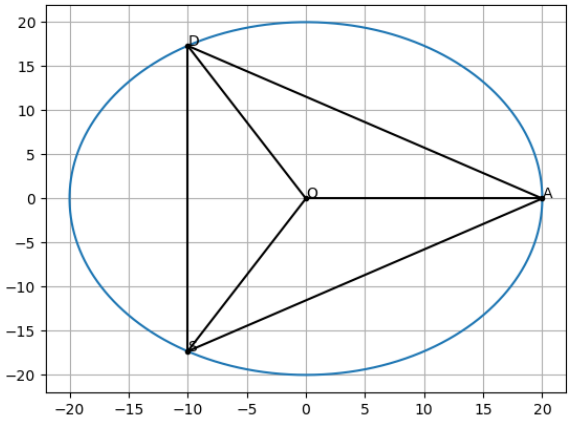

Fig. 6: Triangle ASD is equilateral.

Suppose the position vectors of Ankur, Syed and David are $\mathbf{a}, \mathbf{s}$ and $\mathbf{d}$ respectively. Then,

$$\|\mathbf{a}\| = \|\mathbf{s}\| = \|\mathbf{d}\| = 20 \tag{23}$$

$$\|\mathbf{a}-\mathbf{s}\| = \|\mathbf{s}-\mathbf{d}\| = \|\mathbf{d}-\mathbf{a}\| \tag{24}$$

$$\tag{25}$$

From (24), we get

$$\mathbf{a}^T\mathbf{d} = \mathbf{d}^T\mathbf{s} = \mathbf{s}^T\mathbf{a} \tag{26}$$

$$\implies \angle AOD = \angle DOS = \angle AOS = \frac{2\pi}{3} \tag{27}$$

Using (23),

$$\|\mathbf{a}-\mathbf{d}\| = \sqrt{\|\mathbf{a}\|^2 + \|\mathbf{d}\|^2 + 2\mathbf{a}^T\mathbf{d}} \tag{28}$$

$$= \sqrt{400 + 400 + 2\times 400 \times \frac{-1}{2}} \tag{29}$$

$$= 20\sqrt{3} \tag{30}$$

Hence, the required length of the string is $20\sqrt{3}$m.