Line Assignment

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Abstract—This document contains the solution to Question 24 of Exercise 4 in Chapter 10 of the class 11 NCERT textbook.

1) A person standing at the junction (crossing) of two straight paths represented by the equations

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = -4 \tag{1}$$

and

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} = 5 \tag{2}$$

wants to reach the path whose equation is

$$\begin{pmatrix} 6 & -7 \end{pmatrix} \mathbf{x} = -8 \tag{3}$$

Find equation of the path that he should follow.
Solution: We first find the coordinates of the intersection of (1) and (2). Call their intersection point $\mathbf{a}$. Then,

$$\begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix} \mathbf{a} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} \tag{4}$$

$$\implies \mathbf{a} = \begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}^{-1} \begin{pmatrix} -4 \\ 5 \end{pmatrix} \tag{5}$$

$$= \frac{1}{17} \begin{pmatrix} 4 & 3 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} -4 \\ 5 \end{pmatrix} \tag{6}$$

$$= \frac{1}{17} \begin{pmatrix} -1 \\ 22 \end{pmatrix} \tag{7}$$

Clearly, the man should follow the path perpendicular to (3) from $\mathbf{a}$ to reach it in the shortest time. The normal vector of (3) is $\mathbf{m} = \begin{pmatrix} 6 \\ -7 \end{pmatrix}$, which is consequently the direction vector of the required line. Therefore, the required normal vector is given by $\mathbf{n} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$, and hence, the equation of the line is

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{a} \tag{8}$$

$$\implies \begin{pmatrix} 7 & 6 \end{pmatrix} \mathbf{x} = \frac{1}{17} \begin{pmatrix} 7 & 6 \end{pmatrix} \begin{pmatrix} -1 \\ 22 \end{pmatrix} = \frac{125}{17} \tag{9}$$
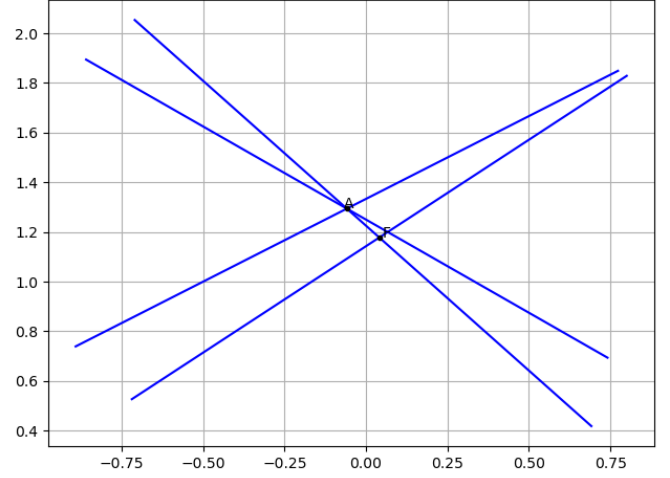
The Python code `codes/crossing.py` veri-fies the solution and depicts the situation in Fig. 1.

Fig. 1: AF is the required line.