PT-100 Lab Assignment

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Abstract—**This document contains a lab report on the modeling of the voltage-temperature characteristics of the PT-100 RTD (Resistance Temperature Detector) using least squares method.**

1 TRAINING DATA

The training data gathered by the PT-100 to train the Arduino is shown in Table 1.

Temperature (°C)	Voltage (V)
66	1.85
27	1.76
2	1.66
23	1.72
56	1.82
34	1.76
33	1.75
31	1.74

TABLE 1: Training data.

The C++ source `codes/data.cpp` was used along with *platformio* to drive the Arduino.

2 MODEL

For the PT-100, we use the Callendar-Van Dusen equation

$$V(T) = V(0)\left(1 + AT + BT^2\right) \tag{1}$$
$$\implies c = \mathbf{n}^\top \mathbf{x} \tag{2}$$

where

$$c = V(T), \quad \mathbf{n} = V(0)\begin{pmatrix} 1 \\ A \\ B \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 1 \\ T \\ T^2 \end{pmatrix} \tag{3}$$

For multiple points, (2) becomes

$$\mathbf{X}^\top \mathbf{n} = \mathbf{C} \tag{4}$$

where

$$\mathbf{X} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ T_1 & T_2 & \dots & T_n \\ T_1^2 & T_2^2 & \dots & T_n^2 \end{pmatrix} \tag{5}$$

$$\mathbf{C} = \begin{pmatrix} V(T_1) \\ V(T_2) \\ \vdots \\ V(T_n) \end{pmatrix} \tag{6}$$

and $\mathbf{n}$ is the unknown.

3 SOLUTION

We approximate $\mathbf{n}$ by using the least sqaures method.

3.1 Pseuso-Inverse Method

Using the pseudo-inverse method, the solution to (4) is

$$\mathbf{n} = \left(\mathbf{X}\mathbf{X}^\top\right)^{-1}\mathbf{X}\mathbf{C} \tag{7}$$

The Python code `codes/lsq.py` solves for $\mathbf{n}$.
The calculated value of $\mathbf{n}$ is

$$\mathbf{n} = \begin{pmatrix} 1.6547 \\ 3.199 \times 10^{-3} \\ -3.9599 \times 10^{-6} \end{pmatrix} \tag{8}$$

The approximation is shown in Fig. 1.
Notice in (8) that the optimal value of C in (2) is negative, and hence the governing equation is strictly concave. Hence, we cannot use gradient descent methods to solve this problem.

4 VALIDATION

The validation dataset is shown in Table 2. The results of the validation are shown in Fig. 2.
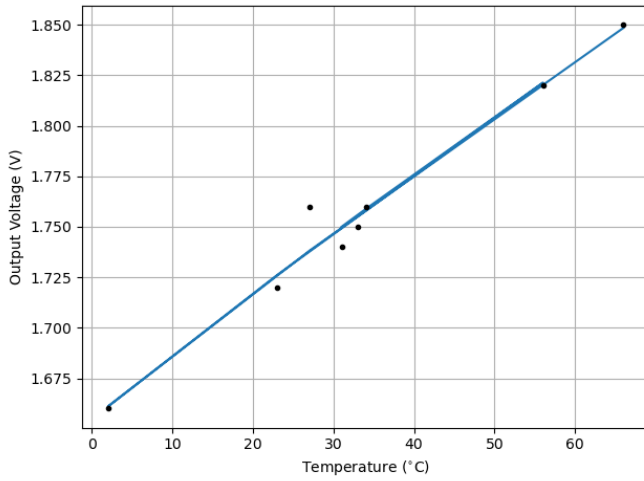
Fig. 1: Training the model.

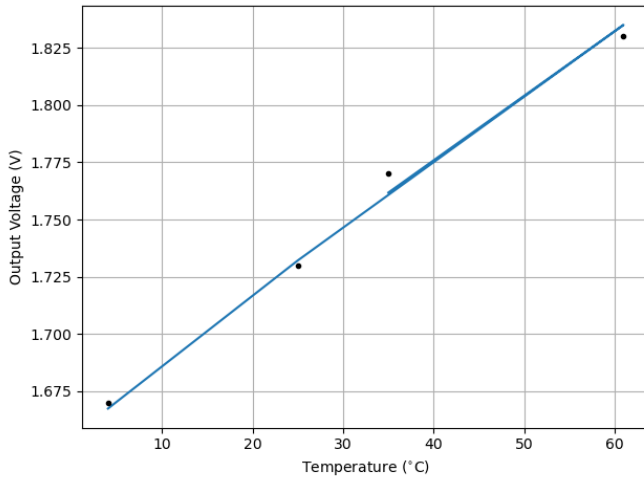| Temperature (°C)	Voltage (V)
4	1.67
25	1.73
61	1.83
35	1.77

TABLE 2: Validation data.

Fig. 2: Validating the model.

5 Conclusion

This lab experiment demonstrates how machine learning methods can be used to model the behaviour of an unknown device, and find the right parameters that fit the model. It also shows how to use Python libraries and frameworks to collect data and perform optimization.