

Probability Assignment

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Abstract—This document contains the solution to Question 5 of Exercise 6 in Chapter 13 of the class 12 NCERT textbook.

- 1) An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

- all will bear 'X' mark.
- not more than 2 will bear 'Y' mark.
- at least one ball will bear 'Y' mark.
- the number of balls with 'X' mark and 'Y' mark will be equal.

Solution: Let the random variable X denote one single draw with p being the probability that a ball marked 'X' is drawn. Then,

$$X \sim \text{Ber}(p) \quad (1)$$

Suppose $X_i, 1 \leq i \leq n$ represent each of the n draws. Define Y as

$$Y = \sum_{i=1}^n X_i \quad (2)$$

Then, since the X_i are iid, the pmf of Y is given by

$$Y \sim \text{Bin}(n, p) \quad (3)$$

In this case,

$$p = \frac{2}{5}, \quad n = 6 \quad (4)$$

- a) We require $\Pr(Y = 6)$. Thus,

$$\Pr(Y = 6) = \binom{n}{6} p^6 (1 - p)^0 \quad (5)$$

$$= 0.004096 \quad (6)$$

- b) We require $\Pr(Y \geq 4)$. Thus,

$$\Pr(Y \geq 4) = \sum_{k=4}^6 \Pr(Y = k) \quad (7)$$

$$= \sum_{k=4}^6 \binom{n}{k} p^k (1 - p)^{n-k} \quad (8)$$

$$= 0.1792 \quad (9)$$

- c) We require $\Pr(Y \leq 5)$. Since $n = 6$ in (3),

$$\Pr(Y \leq 5) = 1 - \Pr(Y = 6) = 0.995904 \quad (10)$$

- d) We require $\Pr(Y = 3)$. Thus,

$$\Pr(Y = 3) = \binom{n}{3} p^3 (1 - p)^{n-3} \quad (11)$$

$$= 0.27648 \quad (12)$$