

Line Assignment

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Abstract—This document contains the solution to Question 16 of Exercise 2 in Chapter 11 of the class 12 NCERT textbook.

- 1) Find the shortest distance between the lines whose vector equations are

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \quad (1)$$

and

$$\mathbf{r} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad (2)$$

Solution: The shortest distance between the two lines would be along the vector normal to both direction vectors. The given direction vectors are

$$\mathbf{A} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad (3)$$

Hence, their cross product is

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = \begin{pmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \\ \mathbf{A}_{12} & \mathbf{B}_{12} \end{pmatrix} \quad (4)$$

$$= \begin{pmatrix} \begin{vmatrix} -3 & 3 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ -3 & 3 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -9 \\ 3 \\ 9 \end{pmatrix} \quad (5)$$

where

$$\mathbf{P}_{ij} \triangleq \begin{pmatrix} p_i \\ p_j \end{pmatrix} \quad (6)$$

From (1) and (2), the points on the first and second line respectively are

$$\mathbf{P} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \quad (7)$$

Hence, the vector joining \mathbf{P} and \mathbf{Q} is given by

$$\mathbf{D} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \quad (8)$$

Therefore, the magnitude of the vector component of \mathbf{D} along \mathbf{C} is the required shortest distance, and is given by

$$d = \frac{\|\mathbf{C}^T \mathbf{D}\|}{\|\mathbf{C}\|} \quad (9)$$

$$= \frac{9}{\sqrt{(-9)^2 + 3^2 + 9^2}} \quad (10)$$

$$= \frac{3}{\sqrt{13}} \text{ units} \quad (11)$$

The situation is depicted in Fig. 1, which is generated using the Python code codes/plot_3d.py.

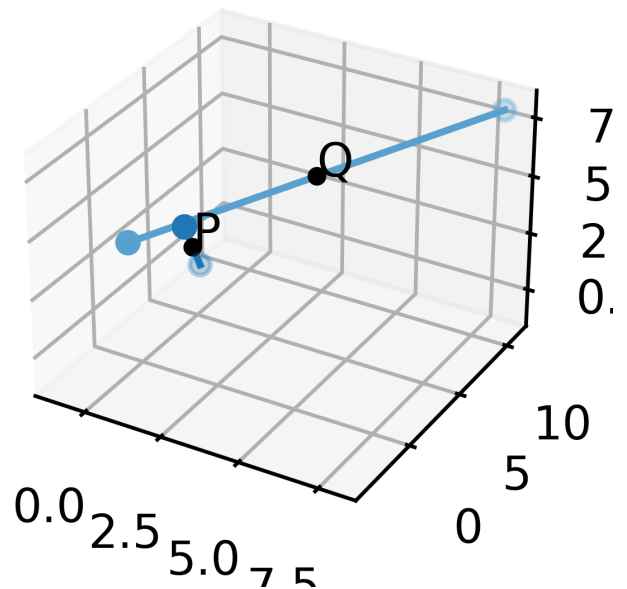


Fig. 1: Finding the shortest distance between two skew lines.