Circle Assignment

Gautam Singh

Abstract—This document contains the solution to Question 6 of Exercise 4 in Chapter 10 of the class 9 NCERT textbook.

1) A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

   Solution: Let the position vectors of the boys be

   $$\mathbf{A} = \begin{pmatrix} r \\ 0 \end{pmatrix}, \ \mathbf{S} = \begin{pmatrix} r\cos\beta \\ r\sin\beta \end{pmatrix}, \ \mathbf{D} = \begin{pmatrix} r\cos\gamma \\ r\sin\gamma \end{pmatrix} \quad (1)$$

   where

   $$\beta, \gamma \in [0, 2\pi), \ \beta, \gamma \neq 0 \quad (2)$$

   We have,

   $$\|\mathbf{A} - \mathbf{S}\|^2 = \|\mathbf{A} - \mathbf{D}\|^2 \quad (3)$$
   $$\implies \mathbf{A}^\top \mathbf{S} = \mathbf{A}^\top \mathbf{D} \quad (4)$$
   $$\implies \cos\beta = \cos\gamma \quad (5)$$
   $$\implies \sin\frac{\beta+\gamma}{2} \sin\frac{\beta-\gamma}{2} = 0 \quad (6)$$

   Since $\beta \neq \gamma$ and from (2), $\frac{\beta-\gamma}{2} \in (-\pi, \pi)$, we get that

   $$\frac{\beta+\gamma}{2} = n\pi \quad (7)$$
   $$\implies \beta + \gamma = 2n\pi \quad (8)$$

   However, (2) gives $\beta+\gamma \in (0, 4\pi)$, which forces $n = 1$ in (8), and thus

   $$\beta + \gamma = 2\pi \quad (9)$$

   Therefore, using (9)

   $$\|\mathbf{A} - \mathbf{S}\|^2 = \|\mathbf{S} - \mathbf{D}\|^2 \quad (10)$$
   $$\implies \mathbf{A}^\top \mathbf{S} = \mathbf{D}^\top \mathbf{S} \quad (11)$$
   $$\implies \cos\beta = \cos(\beta - \gamma) \quad (12)$$
   $$\implies \sin\frac{3\beta - 2\pi}{2} \sin\frac{\gamma}{2} = 0 \quad (13)$$

From (13) and (2),

$$3\beta - 2\pi = 2m\pi \quad (14)$$
$$\implies 3\beta = 2k\pi \quad (15)$$
$$\implies \beta = \frac{2k\pi}{3} \quad (16)$$

where $k \in \mathbb{Z}$. From (2), $k \in \{1, 2\}$. Thus,

$$\beta, \gamma \in \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\} \quad (17)$$

Therefore, the length of the thread from (17) is

$$\|\mathbf{S} - \mathbf{D}\| = \left\| r \begin{pmatrix} \cos\beta - \cos\gamma \\ \sin\beta - \sin\gamma \end{pmatrix} \right\| \quad (18)$$
$$= r\sqrt{3} \quad (19)$$

Here, $r = 20$ m. Thus, the length is $20\sqrt{3}$ m. The situation is demonstrated in Fig. 1, plotted by the Python code codes/equilateral.py. Here, the values used for construction are shown in Table (I).

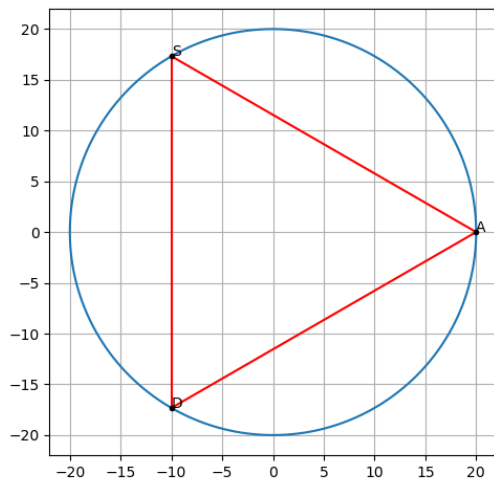| Parameter	Value
r	20
β	$\frac{2\pi}{3}$
γ	$\frac{4\pi}{3}$

TABLE I: Parameters used in the construction of Fig. 1.

Fig. 1: ASD is an equilateral triangle of side $20\sqrt{3}$ m.