Line Assignment

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***Abstract*—This document contains the solution to Question 16 of Exercise 3 in Chapter 10 of the class 12 NCERT textbook.**

1) Show that the points $\mathbf{A} = \begin{pmatrix}1\\2\\7\end{pmatrix}$, $\mathbf{B} = \begin{pmatrix}2\\6\\3\end{pmatrix}$, and $\mathbf{C} = \begin{pmatrix}3\\10\\-1\end{pmatrix}$ are collinear.

Solution: Points $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are on a line if

$$\text{rank}\begin{pmatrix}\mathbf{A} & \mathbf{B} & \mathbf{C}\end{pmatrix} < 3 \tag{1}$$

Substituting, we must find the rank of

$$\mathbf{M} = \begin{bmatrix}1 & 2 & 3\\2 & 6 & 10\\7 & 3 & -1\end{bmatrix} \tag{2}$$

Using row reduction methods to bring $\mathbf{M}$ into row-reduced echelon form,

$$\begin{bmatrix}1 & 2 & 3\\2 & 6 & 10\\7 & 3 & -1\end{bmatrix} \xleftrightarrow{R_2 \to R_2 - 2R_1} \begin{bmatrix}1 & 2 & 3\\0 & 2 & 4\\7 & 3 & -1\end{bmatrix} \tag{3}$$

$$\xleftrightarrow{R_3 \to R_3 - 7R_1} \begin{bmatrix}1 & 2 & 3\\0 & 2 & 4\\0 & -11 & -22\end{bmatrix} \tag{4}$$

$$\xleftrightarrow{R_1 \to R_1 - R_2} \begin{bmatrix}1 & 0 & -1\\0 & 2 & 4\\0 & -11 & -22\end{bmatrix} \tag{5}$$

$$\xleftrightarrow{R_3 \to R_3 + \frac{11}{2}R_2} \begin{bmatrix}1 & 0 & -1\\0 & 2 & 4\\0 & 0 & 0\end{bmatrix} \tag{6}$$

Clearly, the rank of $\mathbf{M}$ is 2, and hence the given points are collinear. The rank of $\mathbf{M}$ is verified in the Python code `codes/rank.py`.