

Line Assignment

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Abstract—This document contains the solution to Question 24 of Exercise 4 in Chapter 10 of the class 11 NCERT textbook.

- 1) A person standing at the junction (crossing) of two straight paths represented by the equations

$$(2 \ -3)\mathbf{x} = -4 \quad (1)$$

and

$$(3 \ 4)\mathbf{x} = 5 \quad (2)$$

wants to reach the path whose equation is

$$(6 \ -7)\mathbf{x} = -8 \quad (3)$$

Find equation of the path that he should follow.

Solution: We first find the coordinates of the intersection of (1) and (2). Using the augmented matrix and row reduction methods,

$$\left(\begin{array}{cc|c} 2 & -3 & -4 \\ 3 & 4 & 5 \end{array} \right) \xrightarrow{R_2 \rightarrow 2R_2 - 3R_1} \left(\begin{array}{cc|c} 2 & -3 & -4 \\ 0 & 17 & 22 \end{array} \right) \quad (4)$$

$$\xrightarrow{R_1 \rightarrow 17R_1 + 3R_2} \left(\begin{array}{cc|c} 17 & 0 & -1 \\ 0 & 17 & 22 \end{array} \right) \quad (5)$$

$$\xrightarrow{\begin{array}{l} R_1 \rightarrow \frac{R_1}{17} \\ R_2 \rightarrow \frac{R_2}{17} \end{array}} \left(\begin{array}{cc|c} 1 & 0 & -\frac{1}{17} \\ 0 & 1 & \frac{22}{17} \end{array} \right) \quad (6)$$

the intersection of the lines is

$$\mathbf{a} = \frac{1}{17} \begin{pmatrix} -1 \\ 22 \end{pmatrix} \quad (7)$$

Clearly, the man should follow the path perpendicular to (3) from \mathbf{a} to reach it in the shortest time. The normal vector of (3) is

$$\mathbf{m} = \begin{pmatrix} 6 \\ -7 \end{pmatrix} \quad (8)$$

which is consequently the direction vector of the required line. Therefore, the required normal vector is given by

$$\mathbf{n} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \quad (9)$$

and hence, the equation of the line is

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{a} \quad (10)$$

$$\Rightarrow (7 \ 6)\mathbf{x} = \frac{1}{17}(7 \ 6) \begin{pmatrix} -1 \\ 22 \end{pmatrix} = \frac{125}{17} \quad (11)$$

The Python code `codes/crossing.py` verifies the solution and depicts the situation in Fig. 1.

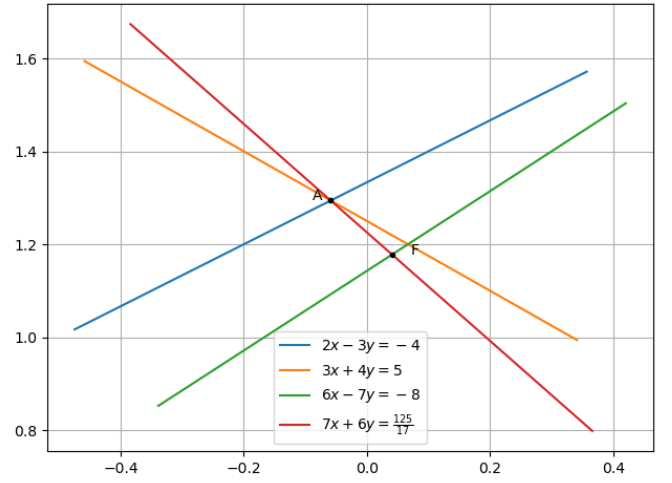


Fig. 1: AF is the required line.