Optimization Assignment

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***Abstract*—This document contains the solution to Question 4 of Exercise 2 in Chapter 10 of the class 11 NCERT textbook.**

1) Find the coordinates of the foot of perpendicular from the point

$$\mathbf{P} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{1}$$

to the line

$$\begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{x} = 16 \tag{2}$$

Solution: Any point on (2) is clearly of the form

$$\mathbf{Q} = \mathbf{A} + \lambda \mathbf{m} \tag{3}$$

where $\lambda \in \mathbb{R}$ and

$$\mathbf{A} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}, \ \mathbf{m} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{4}$$

Thus,

$$\|\mathbf{Q} - \mathbf{P}\|^2 = \|\mathbf{A} - \mathbf{P} + \lambda \mathbf{m}\|^2 \tag{5}$$

$$= \left\| \begin{pmatrix} 4\lambda + 1 \\ 3\lambda - 7 \end{pmatrix} \right\|^2 \tag{6}$$

$$= 25\lambda^2 - 34\lambda + 50 = f(\lambda) \tag{7}$$

Since (7) is convex, we use the gradient descent function on λ to converge at the minimum of $f(\lambda)$.

$$\lambda_{n+1} = \lambda_n - \alpha f'(\lambda_n) \tag{8}$$

$$= (1 - 50\alpha)\lambda_n + 34\alpha \tag{9}$$

Taking the one-sided Z-transform on both sides of (9),

$$z\Lambda(z) = (1 - 50\alpha)\Lambda(z) + \frac{34\alpha}{1 - z^{-1}} \tag{10}$$

$$\Lambda(z) = \frac{34\alpha z^{-1}}{(1 - z^{-1})(1 - (1 - 50\alpha)z^{-1})} \tag{11}$$

$$= \frac{17}{25}\left(\frac{1}{1 - (1 - 50\alpha)z^{-1}} - \frac{1}{1 - z^{-1}}\right) \tag{12}$$

$$= \frac{17}{25}\sum_{k=0}^{\infty}\left(1 - (1 - 50\alpha)^k\right)z^{-k} \tag{13}$$

From (11), the ROC is

$$|z| > \max\{1, 1 - 50\alpha\} \tag{14}$$

$$\implies 0 < 1 - 50\alpha < 1 \tag{15}$$

$$\implies \alpha \in (0, 0.02) \tag{16}$$

Thus, if $\alpha > 0$, then from (13)

$$\lim_{n \to \infty} \lambda_n = \lim_{n \to \infty} \frac{17}{25}(1 - (1 - 50\alpha)^n) \tag{17}$$

$$= \frac{17}{25} \tag{18}$$

We select the following parameters to arrive at the optimal λ, where N is the number of iterations and ϵ is the convergence limit. The gradient descent is demonstrated in Fig. 1, plotted by the Python code `codes/grad_desc.py`. The relevant parameters are shown in Table 1.

Parameter	**Value**
λ_0	0
α	0.1
N	1000000
ϵ	10^{-6}

TABLE 1: Parameters for Gradient Descent

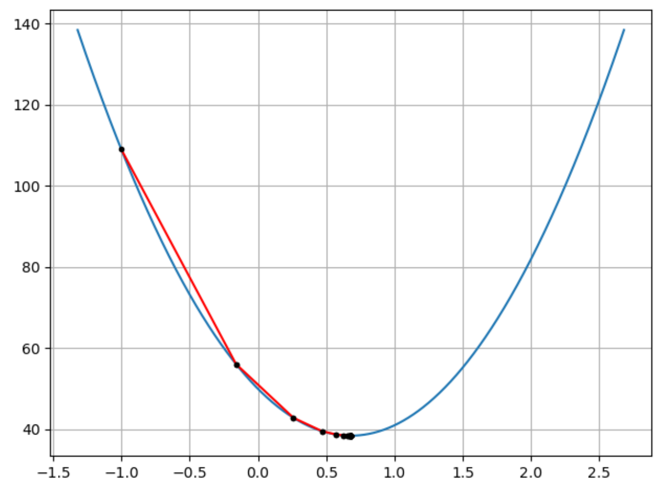

Fig. 1: Gradient descent to get the optimal λ.