Circle Assignment

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***Abstract*—This document contains the solution to Question 6 of Exercise 4 in Chapter 10 of the class 9 NCERT textbook.**

1) A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution: Let the position vectors of the boys be

$$\mathbf{A} = \begin{pmatrix} r \\ 0 \end{pmatrix}, \ \mathbf{S} = \begin{pmatrix} r\cos\beta \\ r\sin\beta \end{pmatrix}, \ \mathbf{D} = \begin{pmatrix} r\cos\gamma \\ r\sin\gamma \end{pmatrix} \tag{1}$$

where

$$\beta, \gamma \in (0, 2\pi) \tag{2}$$

We have,

$$\|\mathbf{A}-\mathbf{S}\|^2 = \|\mathbf{A}-\mathbf{D}\|^2 \tag{3}$$
$$\implies \mathbf{A}^\top\mathbf{S} = \mathbf{A}^\top\mathbf{D} \tag{4}$$
$$\implies \cos\beta = \cos\gamma \tag{5}$$
$$\implies \beta = 2n\pi \pm \gamma \tag{6}$$

where $n \in \mathbb{Z}$. From (2), we force $n = 1$ and consider the negative sign to get

$$\beta + \gamma = 2\pi \tag{7}$$

Therefore, using (7)

$$\|\mathbf{A}-\mathbf{S}\|^2 = \|\mathbf{S}-\mathbf{D}\|^2 \tag{8}$$
$$\implies \mathbf{A}^\top\mathbf{S} = \mathbf{D}^\top\mathbf{S} \tag{9}$$
$$\implies \cos\beta = \cos(\beta-\gamma) \tag{10}$$
$$\implies 2\beta - 2\pi = 2m\pi \pm \beta \tag{11}$$
$$\implies 2\beta \pm \beta = 2k\pi \tag{12}$$

where $k \in \mathbb{Z}$. From (2), we can only consider the plus sign and $k \in \{1, 2\}$ to get

$$\beta, \gamma \in \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\} \tag{13}$$

Therefore, the length of the thread from (13) is

$$\|\mathbf{S}-\mathbf{D}\| = \left\| r\begin{pmatrix} \cos\beta - \cos\gamma \\ \sin\beta - \sin\gamma \end{pmatrix} \right\| \tag{14}$$
$$= r\sqrt{3} \tag{15}$$

Here, $r = 20$ m. Thus, the length is $20\sqrt{3}$ m. The situation is demonstrated in Fig. 1, plotted by the Python code `codes/equilateral.py`. The values used for construction are shown in Table 1.

Parameter	Value
r	20
β	$\frac{2\pi}{3}$
γ	$\frac{4\pi}{3}$

TABLE 1: Parameters used in the construction of Fig. 1.

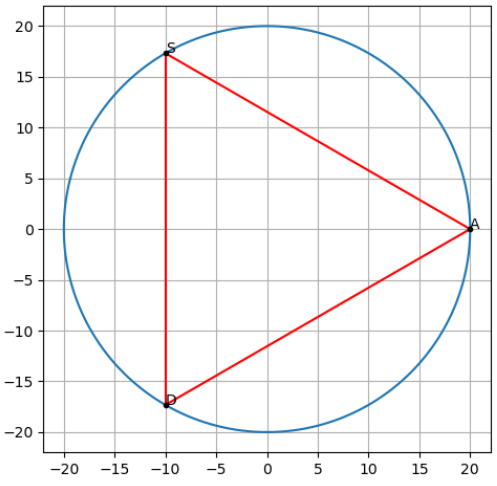

Fig. 1: ASD is an equilateral triangle of side $20\sqrt{3}$ m.