

Conic Assignment

Gautam Singh

Abstract—This document contains the solution to Question 12 of Exercise 2 in Chapter 11 of the class 11 NCERT textbook.

- 1) Find the equation of the parabola with vertex

$$\mathbf{V} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1)$$

and passing through the point

$$\mathbf{P} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \quad (2)$$

and symmetric to the y-axis.

Solution: Let the equation of the conic with focus \mathbf{F} , directrix $\mathbf{n}^\top \mathbf{x} = c$ and eccentricity e be

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (3)$$

where

$$\mathbf{V} \triangleq \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^\top \quad (4)$$

$$\mathbf{u} \triangleq c e^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \quad (5)$$

$$f \triangleq \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \quad (6)$$

Since the conic is a parabola symmetric to the y-axis, we have

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad e = 1 \quad (7)$$

From (4),

$$\mathbf{V} = \mathbf{I} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (8)$$

$$\mathbf{u} = c \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \mathbf{F} \quad (9)$$

$$f = \|\mathbf{F}\|^2 - c^2 \quad (10)$$

Putting $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ in (3) gives $f = 0$, thus

$$\|\mathbf{F}\|^2 = c^2 \quad (11)$$

Using the symmetry of the conic, we see that $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ lie on the conic. Substituting both

points, we get

$$\mathbf{F}^\top \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \frac{25}{2} - 2c \quad (12)$$

$$\mathbf{F}^\top \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \frac{25}{2} - 2c \quad (13)$$

Adding and subtracting (12) and (13),

$$\mathbf{F}^\top \begin{pmatrix} 0 \\ 4 \end{pmatrix} = 25 - 4c \quad (14)$$

$$\mathbf{F}^\top \begin{pmatrix} 10 \\ 0 \end{pmatrix} = 0 \quad (15)$$

$$\Rightarrow \mathbf{F} = \begin{pmatrix} 0 \\ \frac{25}{4} - c \end{pmatrix} \quad (16)$$

Using (11),

$$\frac{25}{4} - c \pm c = 0 \quad (17)$$

$$\Rightarrow c = \frac{25}{8} \quad (18)$$

Thus,

$$\mathbf{F} = \begin{pmatrix} 0 \\ \frac{25}{8} \end{pmatrix} \quad (19)$$

The conic is plotted in Fig. 1 using the Python code `codes/conic.py`.

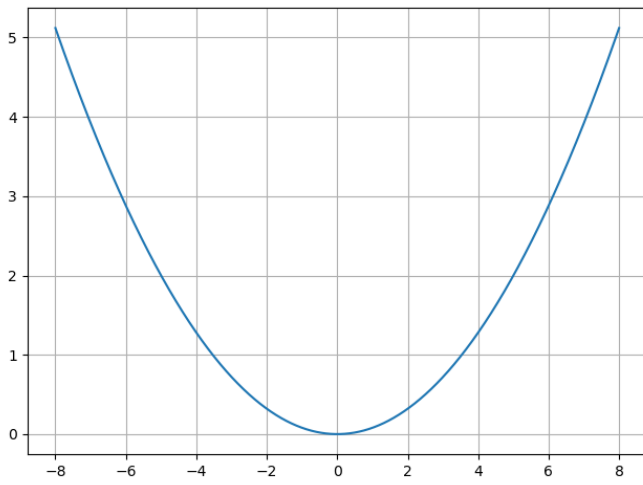


Fig. 1: Locus of the required parabola.