Circle Assignment

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Abstract—**This document contains the solution to Question 6 of Exercise 4 in Chapter 10 of the class 9 NCERT textbook.**

1) A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

   Solution: Let the position vectors of the boys be

$$\mathbf{A} = \begin{pmatrix} r \\ 0 \end{pmatrix}, \ \mathbf{S} = \begin{pmatrix} r\cos\beta \\ r\sin\beta \end{pmatrix}, \ \mathbf{D} = \begin{pmatrix} r\cos\gamma \\ r\sin\gamma \end{pmatrix} \tag{1}$$

   We have,

$$\|\mathbf{A}-\mathbf{S}\|^2 = \|\mathbf{A}-\mathbf{D}\|^2 \tag{2}$$
$$\implies \mathbf{A}^\top\mathbf{S} = \mathbf{A}^\top\mathbf{D} \tag{3}$$
$$\implies \cos\beta = \cos\gamma \tag{4}$$
$$\implies \sin\beta = -\sin\gamma \tag{5}$$

   Since $\beta \neq \gamma$. We can also write using (4) and (5),

$$\|\mathbf{A}-\mathbf{S}\|^2 = \|\mathbf{S}-\mathbf{D}\|^2 \tag{6}$$
$$\implies \mathbf{A}^\top\mathbf{S} = \mathbf{D}^\top\mathbf{S} \tag{7}$$
$$\implies \cos\beta = \cos(\beta-\gamma) \tag{8}$$
$$\implies \cos\beta = \cos^2\beta - \sin^2\beta \tag{9}$$
$$\implies 2\cos^2\beta - \cos\beta - 1 = 0 \tag{10}$$
$$\implies (2\cos\beta+1)(\cos\beta-1) = 0 \tag{11}$$

   From (11), if $\cos\beta = 1$, then $\sin\beta = \sin\gamma = 0$, a contradiction. Therefore, since $\beta, \gamma \leq 2\pi$,

$$\cos\beta = \cos\gamma = -\frac{1}{2} \tag{12}$$
$$\implies \beta, \gamma \in \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\} \tag{13}$$

   Therefore, the length of the thread is

$$\|\mathbf{S}-\mathbf{D}\| = 2r\sin\beta = r\sqrt{3} \tag{14}$$

   Here, $r = 20$ m. Thus, the length is $20\sqrt{3}$ m.

The situation is demonstrated in Fig. 1, plotted by the Python code `codes/equilateral.py`.

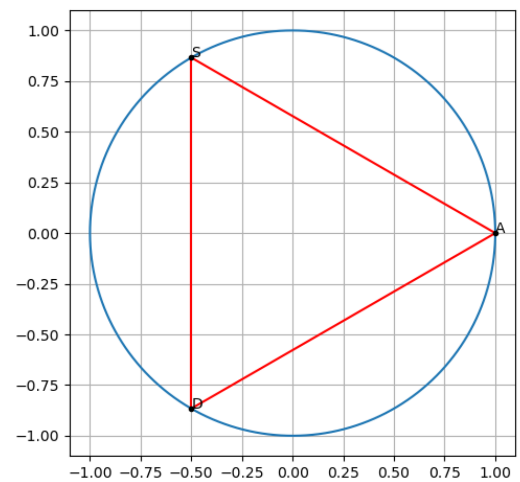

Fig. 1: ASD is an equilateral triangle of side $20\sqrt{3}$ m.