Semidefinite Programming Assignment

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Abstract—This document contains the solution to Question 23 of Exercise 3 in Chapter 6 of the class 12 NCERT textbook.

1) Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

   Solution: We use semidefinite programming. The given curves in matrix form are

   $$\mathbf{x}^\top \mathbf{V_i} \mathbf{x} + 2\mathbf{u_i}^\top \mathbf{x} + f_i = 0 \qquad (1)$$

   where

   $$\mathbf{V_1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ \mathbf{u_1} = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}, \ f_1 = 0 \qquad (2)$$

   $$\mathbf{V_2} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}, \ \mathbf{u_2} = \mathbf{0}, \ f_2 = -k \qquad (3)$$