EE6367: Topics in Data Storage and Communications **2023**

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21.1 General Single-Bit Quantization Schemes

Any one-bit quantization scheme takes the form

$$Y_i = \mathbb{1}_{\{X_i \in \mathcal{A}_i\}}. \tag{21.1}$$

For a fully distributed scheme, the $\mathcal{A}_i$ are fixed and independent of the samples X_i. However, in an interactive scheme the $\mathcal{A}_i$ can depend on the X_i.

21.1.1 A Potential One-Bit Scheme

The mean of a random variable X can be written in another form as

$$\begin{aligned}
\mathbb{E}[X] &= \int_{-\infty}^{\infty} x f_X(x)\, dx && (21.2)\\
&= \int_0^{\infty} \Pr(X > t)\, dt - \int_{-\infty}^{0} \Pr(X < t)\, dt && (21.3)\\
&= \int_0^{\infty} \mathbb{E}\left[\mathbb{1}_{\{X>t\}}\right] dt - \int_{-\infty}^{0} \mathbb{E}\left[\mathbb{1}_{\{X<t\}}\right] dt && (21.4)\\
&\approx \sum_{j=0}^{\alpha} \mathbb{E}\left[\mathbb{1}_{\{X>j\Delta\}}\right] \Delta - \sum_{j=-\beta}^{0} \mathbb{E}\left[\mathbb{1}_{\{X<j\Delta\}}\right] \Delta && (21.5)
\end{aligned}$$

Thus, we can create a potential scheme in which the j^{th} user transmits $\mathbb{1}_{\{X_j<j\Delta\}}$.

21.2 Threshold Based Schemes

In threshold-based schemes, we have $\mathcal{A}_i = (-\infty, \theta_i]$, and $Y_i = \mathbb{1}_{(X_i \leqslant \theta_i)}$. We state the following claim.

Theorem 21.1. *For any threshold-based scheme and symmetric log-concave f_X, we have*

$$\sqrt{m}\left(\hat{\mu} - \mu\right) \xrightarrow{\mathrm{d}} \frac{1}{\kappa(\mu)} \tag{21.6}$$

as $m \to \infty$. Further,

$$\lim_{m\to\infty} \mathrm{MSE} \to \frac{1}{\kappa(\mu)} \tag{21.7}$$

$$\kappa(\mu) \triangleq \int_{-\infty}^{\infty} \frac{f_X^2(t-\mu)}{F_X(t-\mu)\,F_X(\mu-t)} \lambda(t)\,dt \tag{21.8}$$

$$\Lambda_m(\tau) \triangleq \frac{1}{m}\sum_{i=1}^{m} \mathbb{1}_{\theta_i \leqslant \tau} \tag{21.9}$$

$$\lambda(\tau) \triangleq \frac{d}{d\tau}\lim_{m\to\infty} \Lambda_m(\tau) = \frac{d\Lambda(\tau)}{d\tau}. \tag{21.10}$$

Note that $\Lambda_m(z)$ is the approximation of a CDF. Further, if θ_i is uniform over $[-\beta,\alpha]$, then

$$\kappa(\mu) = \int_{-\beta}^{\alpha} \frac{f_X^2(t-\mu)}{F_X(t-\mu)\,F_X(\mu-t)} \frac{1}{\alpha+\beta} dt. \tag{21.11}$$