EE6367: Topics in Data Storage and Communications **2023**

Lecture 18: 06 November 2023

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18.1 DME With Known Distribution on Inputs

Consider the DME problem where users U_i have samples X_i which are iid from a distribution with unknown mean. Suppose that each user sends one bit of information Y_i. We require to minimize the maximum MSE, defined as

$$\text{MaxMSE} \triangleq \max_{\mu \in \mathbb{M}} \left|\hat{\mu} - \mu\right|^2 . \tag{18.1}$$

18.1.1 Location Family

Let f_X denote a pdf with zero mean. Then, the location family corresponding to f_X is given by.

$$\mathcal{L}\left(f_X\right) \triangleq \left\{f_{X,\mu}\left(x\right) = f_X\left(x-\mu\right)\right\},\ \mu \in \mathbb{M}. \tag{18.2}$$

Thus, $cL\left(f_X\right)$ is a collection of pdfs where μ is the *only* unknown variable.

18.1.2 DME From MSE

In this situation, we have

$$\begin{aligned}
\text{MSE}\left(\mu\right) &= \frac{1}{m^2}\mathbb{E}\left[\sum_{i=1}^{m}\left(\hat{X}_i - \mu\right)\right]^2 && (18.3)\\
&= \frac{1}{m^2}\mathbb{E}\left[\sum_{i=1}^{m}\left(\hat{X}_i - \mu\right)^2 + \sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}\left(\hat{X}_i - \mu\right)\left(\hat{X}_j - \mu\right)\right] && (18.4)\\
&= \frac{1}{m}\mathbb{E}\left[\left(\hat{X}_1 - \mu\right)^2\right] + \frac{m-1}{m}\left(\mathbb{E}\left[\hat{X}_1 - \mu\right]\right)^2 . && (18.5)
\end{aligned}$$

Consider an encoder

$$Y_i = \begin{cases} 1 & X_i \leqslant \theta \\ 0 & X_i > \theta \end{cases} . \tag{18.6}$$

Thus, if F_X is invertible,

$$\frac{1}{m}\sum_{i=1}^{m} Y_i \xrightarrow{\mathrm{P}} F_X\left(\theta - \mu\right) \tag{18.7}$$

$$\implies \hat{\mu} = \theta - F_X^{-1}\left(\frac{1}{m}\sum_{i=1}^{m} Y_i\right) \tag{18.8}$$