$$\begin{aligned}
\sum_{k=1}^{2n} \frac{(-1)^{k+1}}{k} &= \sum_{\substack{k=1 \\ \text{impair}}}^{2n} \frac{1}{k} + \sum_{\substack{k=1 \\ \text{pair}}}^{2n} \frac{-1}{k} \\
&= \sum_{k=1}^{2n} \frac{1}{k} - 2 \sum_{\substack{k=1 \\ \text{pair}}}^{2n} \frac{1}{k} \\
&= \sum_{k=1}^{2n} \frac{1}{k} - 2 \sum_{p=1}^{n} \frac{1}{2p} \\
&= \sum_{k=1}^{2n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k}
\end{aligned}$$