Computer Aided Geometric Design/Geometric Modelling

Chapter 1: Introduction to Geometric Primitives

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History of Curves and Surfaces

- ***A History of Curves and Surfaces in CAGD*** : *Please read on this article by Gerald Farin.* `http://www.farinhansford.com/gerald/papers/history.pdf`

Old Vs New drafting process

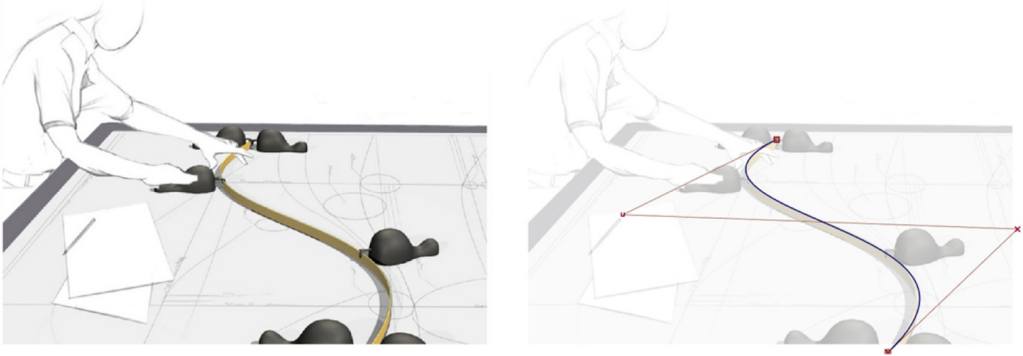


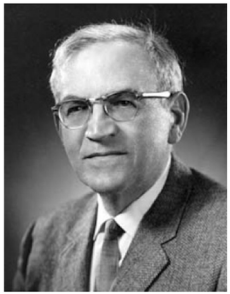
Figure: Ducks with elastic band Versus Bezier curves

Bezier Curves

- born 1930 in Besançon, France.
- In 1959 Citroën hired Paul de Faget de Casteljau, formulate a curve for design who just Fresh from his PhD studies.

- Pierre Étienne Bézier (Sept.1, 1910 – Nov. 25, 1999)
- As an engineer at Renault, he became a leader in the transformation of design and manufacturing, through mathematics and computing tools, into computer-aided design and three-dimensional modeling.

B-Splines



- Isaac Jacob Schoenberg (April 21, 1903 – Feb. 21, 1990) was a Romanian-American mathematician

- Carl-Wilhelm Reinhold de Boor (born 3 Dec. 1937) is a German-American mathematician found a better way to evaluate B-Splines.
- Presently a professor emeritus at the University of Wisconsin–Madison
- ISI Highly Cited Author in Mathematics

Timeline: Curve and surface design

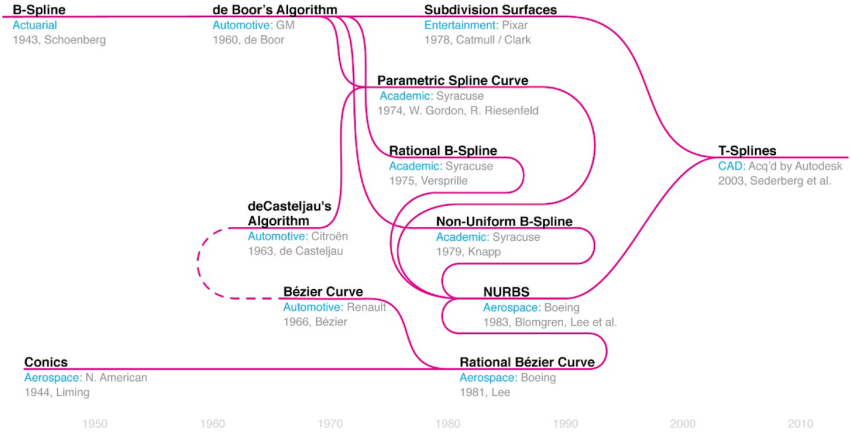


Figure: Progrsssion of curve and surface design

Euclidean Geometry

- **Euclidean geometry** is a mathematical system attributed to Euclid, which he described in his textbook on geometry: the Elements.
- Euclidean geometry is the study of **geometrical shapes**. Geometry is derived from the Greek words 'geo' which means earth and 'metrein' which means 'to measure'.
- Euclidean geometry is an **axiomatic system**, in which all theorems are derived from **five** postulates (axioms).
- **Postulates:** the assumptions that are obvious universal truths, but they are not proved.

Euclidean Geometry

- the postulates are:
  - : To draw a straight line from any point to any point.
  - : To produce (extend) a finite straight line continuously in a straight line.
  - : To describe a circle with any centre and distance (radius).
  - : That all right angles are equal to one another.
  - : The parallel postulate: That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

Euclidean Space

- Examples of Euclidean Space:
  - : Euclidean 1-space R^1: The set of all real numbers, i.e., the **real line**. For example, $-1.2, 1, \frac{2}{3}$, are all elements of R^1.
  - : Euclidean 2-space R^2: The collection of ordered pairs of real numbers, (x_1, x_2) is denoted R^2, also known as **plane**. For example, $(-1.2, 1)$ and $(\frac{2}{3}, \frac{2}{3}) \in R^2$)
  - : Euclidean 3-space R^3: The collection of all ordered triplets, (x_1, x_2, x_3), of real numbers is denoted R^3. Euclidean 3-space is also called **space**. For example, $(-1, 2, 3) \in R^3$.

Non-Euclidean: Hyperbolic and Elliptic Geometry

- **non-Euclidean geometry** arises by either relaxing the metric requirement, or replacing the parallel postulate with an alternative. If we replace parallel postulate:
  - In Euclidean geometry, the lines remain at a constant distance from each other (meaning that a line drawn perpendicular to one line at any point will intersect the other line and the length of the line segment joining the points of intersection remains constant) and are known as **parallels**.
  - In hyperbolic geometry, they "curve away" from each other, increasing in distance as one moves further from the points of intersection with the common perpendicular; these lines are often called **ultraparallels**.
  - In elliptic geometry, the lines "curve toward" each other and **intersect**.

Non-Euclidean: Hyperbolic and Elliptic Geometry

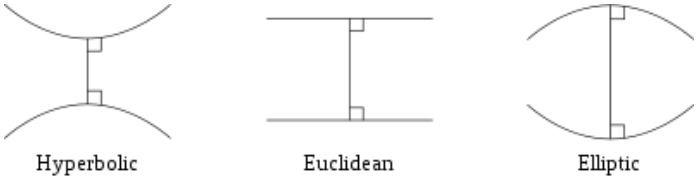


Figure: Behavior of lines with a common perpendicular in each of the three types of geometry

Comparison of Euclidean and Non-Euclidean Geometry

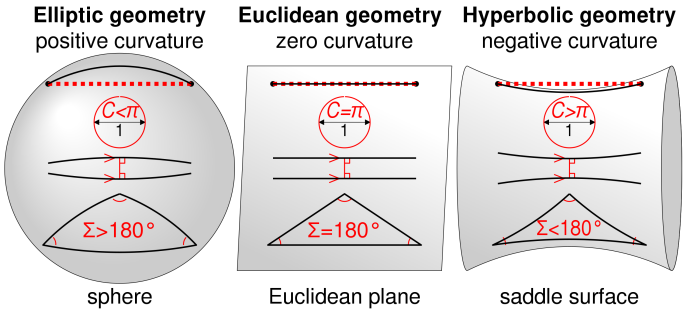


Figure: Behavior of lines with a common perpendicular in each of the three types of geometry

Terminology: Point as a location in Euclidean Space

The most basic geometry entity is a point represents a location.

- Notation: A point in R^n is denoted by the ordered pair $(x_1, x_2, ..., x_n)$; however, depending on the context, this notation can also be used to represent a vector.
- A bold letter: $p = (x_1, x_2)$represents a point.
- For example $p = (-3, 3)$ is a point in R^2 or a vector in R^2 depending on the context.
- A point can also be considered a row-matrix: $[x_1, x_2...x_n]$. A point can also be written as a column matrix.

Terminology: A vector a translation of points

Vectors are used in many disciplines such as physics and engineering.

- Vectors are directed line segments that have both a magnitude and a direction.
- The tail of the vector is the initial point, and the head is the terminal point.
- A letter with an arrow over v top represents a vector.
- The difference between two points can be represented as a vector.
- The translation from initial point to terminal point can also be represented as a vector.

Summary Chapter 1

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Thanks

Doubts and Suggestions?

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References

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Gerald Farin

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Appendices

Online links: `https://www.wolframcloud.com/`

`https://www.wolframalpha.com/`