



Curves and Surfaces for CAD systems



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Computational Mechanics and Numerical Methods

**Curves and Surfaces
for CAD Systems**



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...via FB live / webex platform

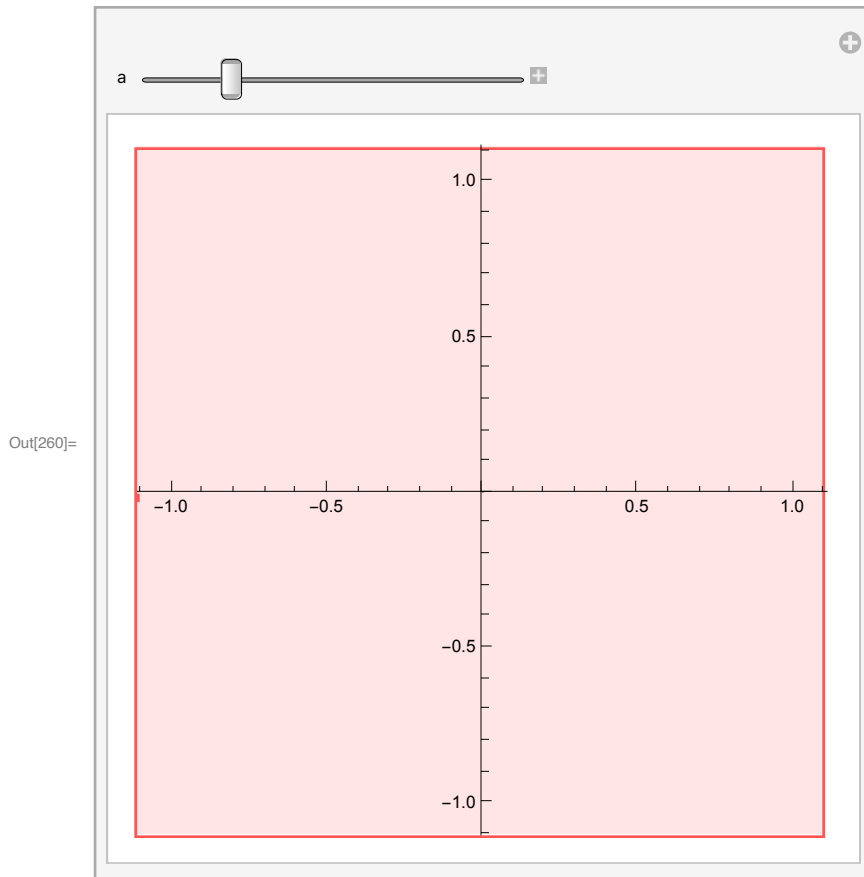
Curve for Design

1) Parametric Curve and its properties

```

In[252]:= ClearAll["Global`*"]
x1 =  $\theta$ ;
y1 = Sin[ $\theta$ ];
MyCurve[ $\theta$ _] = {x1, y1};
UnitTangent[ $\theta$ _] :=  $\left\{ \frac{1}{\sqrt{1 + \cos[\theta]^2}}, \frac{\cos[\theta]}{\sqrt{1 + \cos[\theta]^2}} \right\}$ 
UnitNormal[ $\theta$ _] :=  $\left\{ -\frac{\cos[\theta]}{\sqrt{1 + \cos[\theta]^2}}, \frac{1}{\sqrt{1 + \cos[\theta]^2}} \right\}$ 
UnitTangentDeriv[ $\theta$ _] :=
 $\left\{ \sqrt{1 + \cos[\theta]^2} \cot[\theta] \sqrt{\frac{\sin[\theta]^2}{(1 + \cos[\theta]^2)^2}}, -\sqrt{1 + \cos[\theta]^2} \csc[\theta] \sqrt{\frac{\sin[\theta]^2}{(1 + \cos[\theta]^2)^2}} \right\};$ 
CurveElp[ $\theta$ _] :=  $-\frac{\sin[\theta]}{(1 + \cos[\theta]^2)^{3/2}}$ 
Quiet[Manipulate[
  Show[
    {ParametricPlot[MyCurve[ $\theta$ ], { $\theta$ ,  $-\pi$ ,  $\pi$ }, PlotRange  $\rightarrow$  All],
    Graphics[{Red, Arrow[{MyCurve[a], MyCurve[a] + UnitTangent[a]}]}],
    Graphics[{Black, Arrow[{MyCurve[a], MyCurve[a] + UnitNormal[a]}]}],
    Graphics[{Green, Arrow[{MyCurve[a], MyCurve[a] + UnitTangentDeriv[a]}]}],
    Graphics[Circle[MyCurve[a] +
      (UnitTangentDeriv[a]) * Abs[1 / CurveElp[a]], Abs[1 / CurveElp[a]]],
    Graphics[Point[MyCurve[a] + (UnitTangentDeriv[a]) * Abs[1 / CurveElp[a]]],
    Graphics[Point[MyCurve[a]]]], {{a, -1.8},  $-\pi + 0.1$ ,  $\pi - 0.1$ ]}]

```



1) Extra: Curve Synthesis

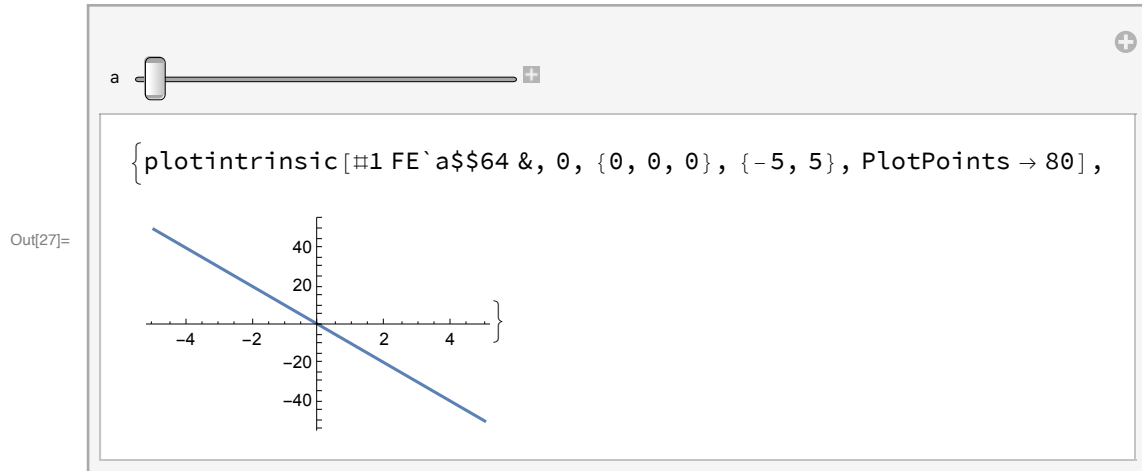
```

In[261]:= intrinsic[fun_, a_ : 0, {c_ : 0, d_ : 0, theta0_ : 0}, optsnd___, {smin_, smax_}][t_] :=
  Flatten[Module[{x, y, theta},
    {x[t], y[t]} /. NDSolve[{x'[ss] == Cos[theta[ss]], y'[ss] == Sin[theta[ss]],
      theta'[ss] == fun[ss], x[a] == c, y[a] == d, theta[a] == theta0},
    {x, y, theta}, {ss, smin, smax}, optsnd]]]
plotintrinsic[fun_, a_ : 0, {c_ : 0, d_ : 0, theta0_ : 0}, optsnd___,
  {smin_, smax_}, optspp___] := ParametricPlot[Module[{x, y, theta},
  {x[t], y[t]} /. NDSolve[{x'[ss] == Cos[theta[ss]], y'[ss] == Sin[theta[ss]],
    theta'[ss] == fun[ss], x[a] == c, y[a] == d, theta[a] == theta0},
  {x, y, theta}, {ss, smin, smax}, optsnd, MaxSteps -> 1000]] // Evaluate,
  {t, smin, smax}, PlotRange -> All, AspectRatio -> Automatic,
  PlotStyle -> Directive[Black, Thick], optspp];

```

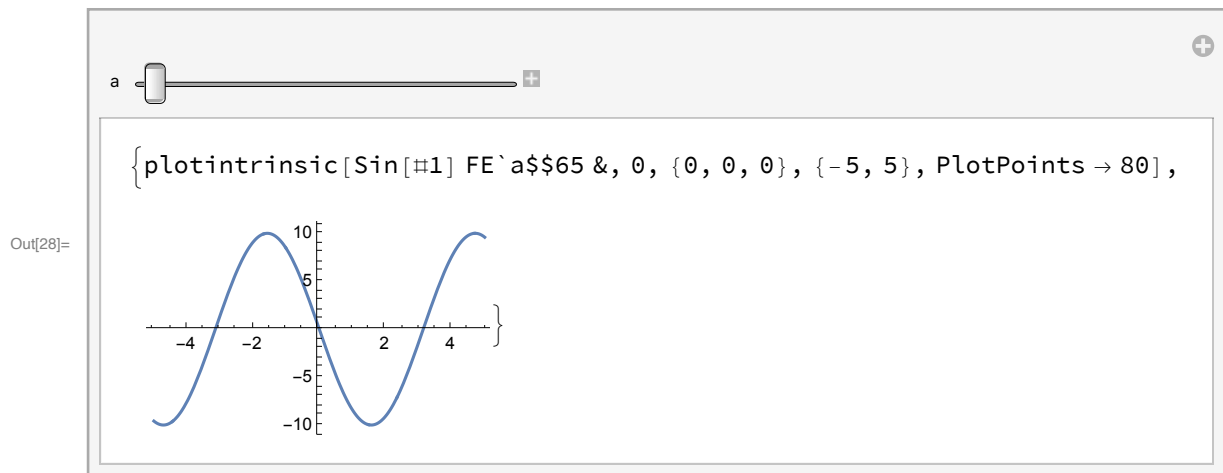
Curvature for case $[s] = a s$

In[27]:= `Quiet[Manipulate[{plotintrinsic[# * a &, 0, {0, 0, 0}, {-5, 5}, PlotPoints → 80],
Plot[s * a, {s, -5, 5}, PlotRange → All]}, {a, -10, 10}]]`



Curvature for case $[s] = a \sin[s]$

In[28]:= `Quiet[Manipulate[{plotintrinsic[Sin[#] * a &, 0, {0, 0, 0}, {-5, 5}, PlotPoints → 80],
Plot[Sin[s] * a, {s, -5, 5}, PlotRange → All]}, {a, -10, 10}]]`



Bezier Curves

1) Linear Bezier

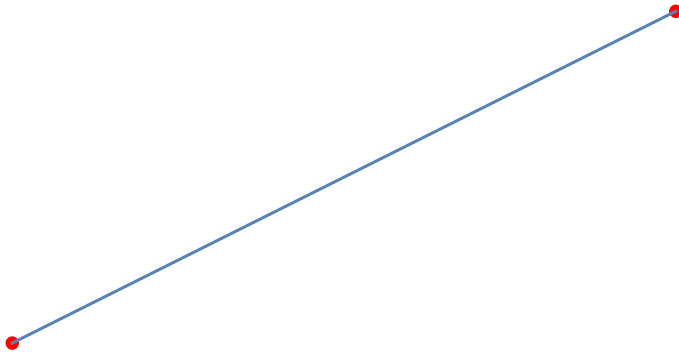
Define the linear Bezier equation.

```
In[29]:= B1[t_, P0_, P1_] := (1 - t) P0 + t * P1
```

Draw a linear Bezier line connecting (0,0) and (2,1).
Show the curve and it's control points.

```
In[30]:= P1 = {p10, p11} = {{0, 0}, {2, 1}};  
f11 = ParametricPlot[B1[t, p10, p11], {t, 0, 1}];  
f12 = Graphics[{Red, PointSize[Large], Point[P1]}];  
Show[f12, f11]
```

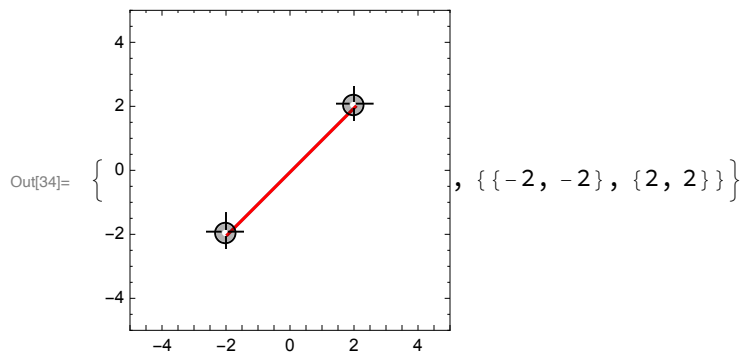
Out[33]=



```

In[34]:= DynamicModule[
  {P0 = {-2, -2}, P1 = {2, 2}},
  LocatorPane[
    Dynamic[{P0, P1}],
    Dynamic[{
      Show[{
        ParametricPlot[(1 - t) P0 + t * P1, {t, 0, 1},
          PlotRange -> {{-5, 5}, {-5, 5}}, Axes -> False, Frame -> True],
        ListPlot[{P0, P1}, Joined -> True, PlotStyle -> Red]
      }],
      Dynamic[{P0, P1}]
    ]
  ]
]

```



2) Quadratic Bezier

Define the quadratic Bezier equation.

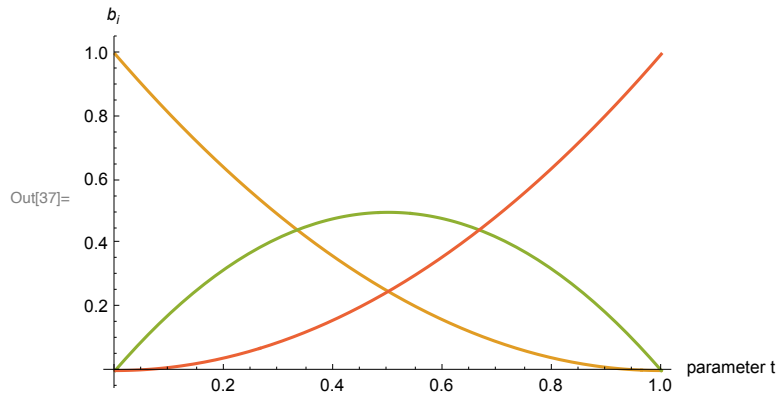
```

In[35]:= B2[t_, p0_, p1_, p2_] := p0 (1 - t)^2 + 2 t (1 - t) p1 + t^2 p2
basisFunction = {b0[t_] = (1 - t)^2, b1[t_] = 2 t (1 - t), b2[t_] = t^2};

```

Basis Function plot

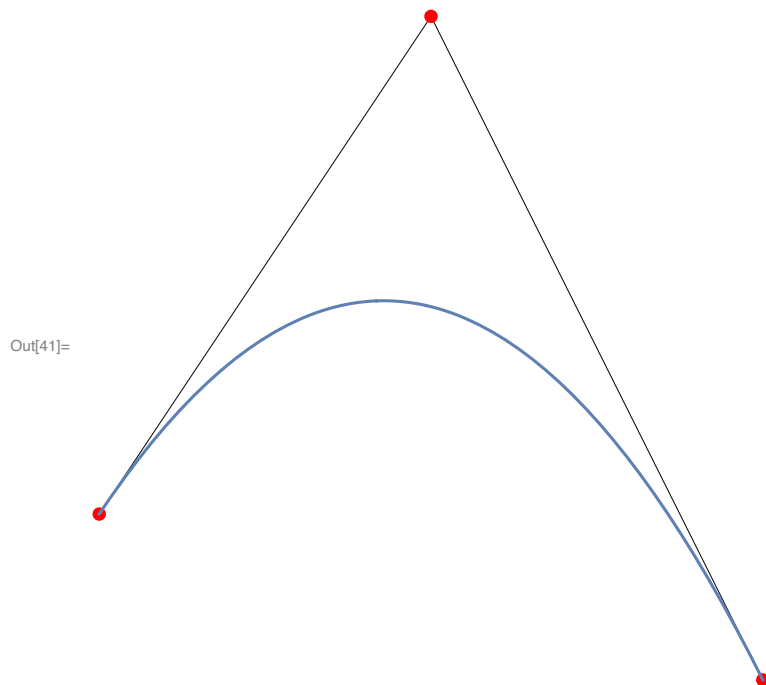
```
In[37]:= Plot[Evaluate[Table[basisFunction[n], {n, 0, 3}]],  
             {t, 0, 1}, PlotRange -> All, AxesLabel -> {"parameter t", "bi"}]
```



Draw a quadratic Bezier with control points (0,0), (2,3), and (4,-1).

Draw the control points and polygons, then show the curve, control points and polygons in a single plot.

```
In[38]:= P2 = {p20, p21, p22} = {{0, 0}, {2, 3}, {4, -1}};  
f21 = ParametricPlot[B2[t, p20, p21, p22], {t, 0, 1}];  
f22 = Graphics[{Line[P2], Red, PointSize[Large], Point[P2]}];  
Show[f22, f21]
```

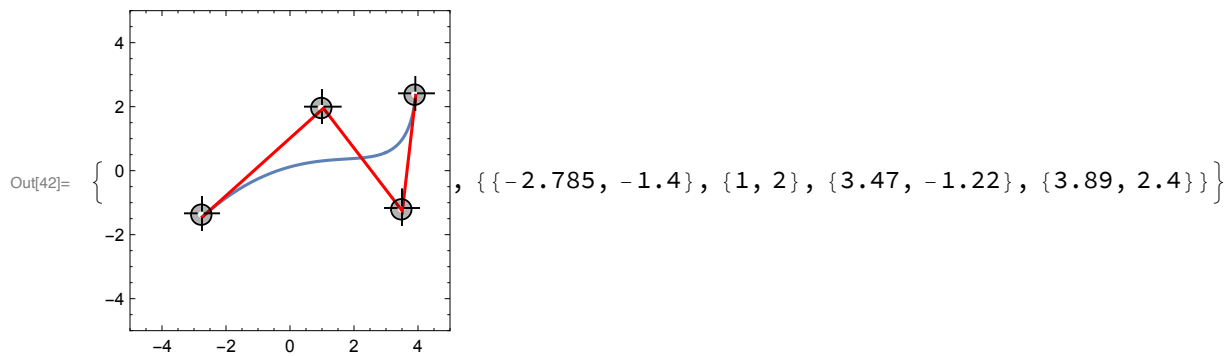


3) Interactive Bezier Cubic

```

In[42]:= DynamicModule[
  {P0 = {0, 0}, P1 = {1, 2}, P2 = {3, 0}, P3 = {5, 2}},
  LocatorPane[
    Dynamic[{P0, P1, P2, P3}],
    Dynamic[{
      Show[
        ParametricPlot[(1 - t)^3 P0 + 3 t (1 - t)^2 * P1 + 3 t^2 (1 - t) * P2 + t^3 P3, {t, 0, 1},
          PlotRange -> {{-5, 5}, {-5, 5}}, Axes -> False, Frame -> True],
        ListPlot[{P0, P1, P2, P3}, Joined -> True, PlotStyle -> Red]
      ],
      Dynamic[{P0, P1, P2, P3}]
    ]
  ]
]

```



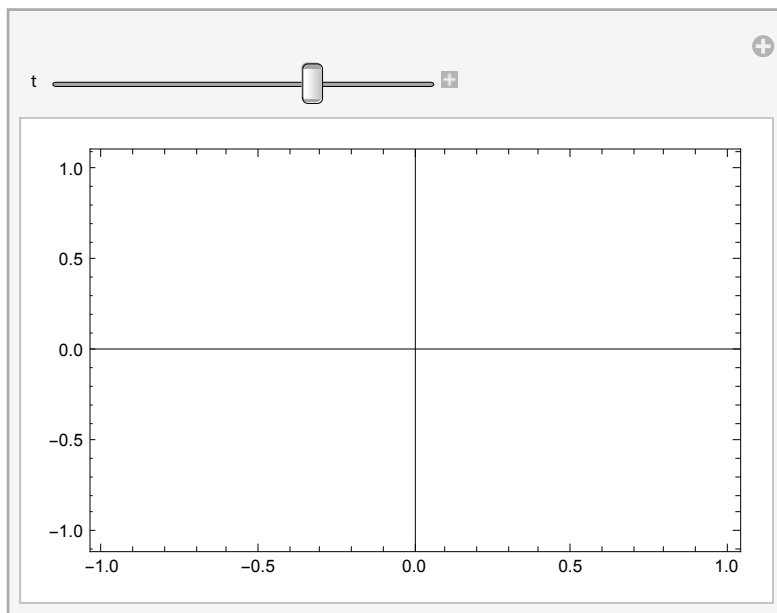
4) de Casteljau Algo

```

In[263]:= ClearAll["Global`*"]
deCasteljau[t_, P0_, P1_] := (1 - t) P0 + t P1
{p0, p1, p2, p3} = {{0, 0}, {1, 2}, {3, 0}, {5, 2}};
Manipulate[Show[{ListPlot[{p0, p1, p2, p3}, Joined → True, PlotStyle → Red],
  ListPlot[{p10 = (1 - t) p0 + t p1,
    p11 = (1 - t) p1 + t p2, p12 = (1 - t) p2 + t p3}, PlotStyle → Black],
  ListPlot[{p10, p11, p12}, Joined → True, PlotStyle → Blue],
  ListPlot[{p20 = (1 - t) p10 + t p11, p21 = (1 - t) p11 + t p12}, PlotStyle → Black],
  ListPlot[{p20, p21}, Joined → True, PlotStyle → Green],
  ListPlot[{p30 = (1 - t) p20 + t p21}, PlotStyle → Red],
  ParametricPlot[{(1 - t)3 p0 + 3 t (1 - t)2 * p1 + 3 t2 (1 - t) * p2 + t3 p3}, {t, 0, 1}]
}, Frame → True, PlotRange → All], {t, 0, 1}]

```

Out[266]=



5) Rational Bezier

```

In[267]:= ClearAll["Global`*"]
Bez[t_, w0_, w1_, w2_, w3_] :=
  
$$\frac{w0 * (1 - t)^3 P0 + w1 * 3 t (1 - t)^2 P1 + w2 * 3 t^2 (1 - t) P2 + w3 * t^3 P3}{w0 * (1 - t)^3 + w1 * 3 t (1 - t)^2 + w2 * 3 t^2 (1 - t) + w3 * t^3}$$

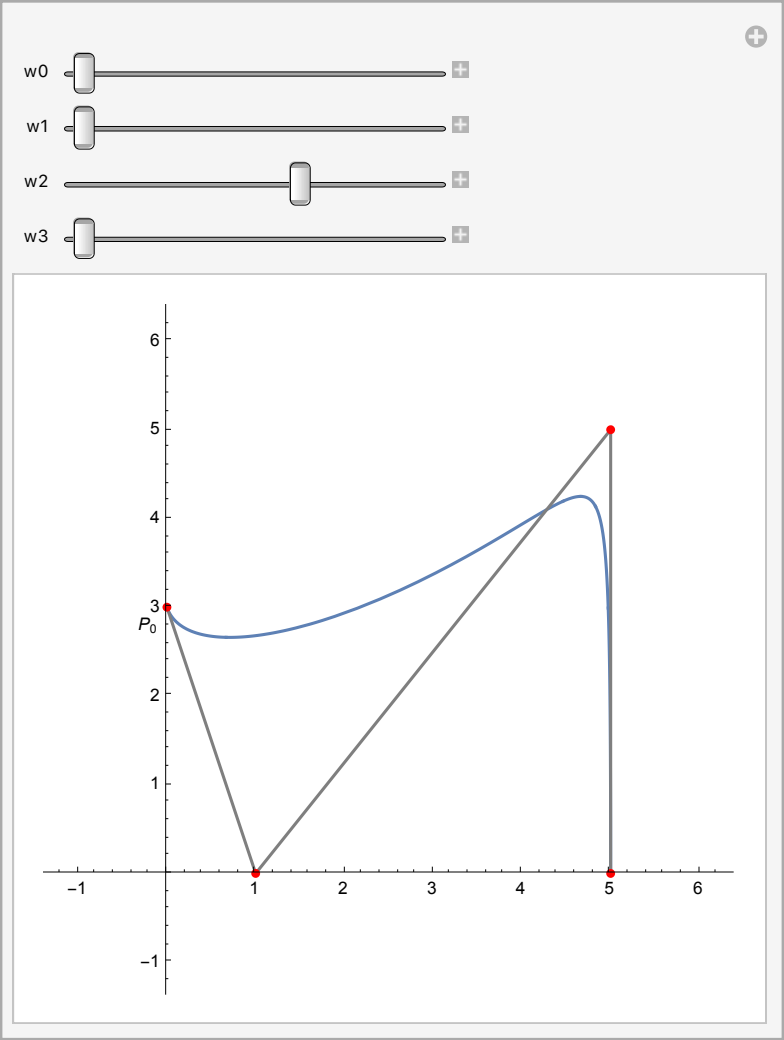
  {P0, P1, P2, P3} = {{0, 3}, {1, 0}, {5, 5}, {5, 0}};
Bez[t, w0, w1, w2, w3]
Manipulate[Show[{ParametricPlot[Bez[t, w0, w1, w2, w3], {t, 0, 1}],
  ListPlot[{{P0, P1, P2, P3}}, Joined → True, PlotStyle → Gray],
  ListPlot[{{P0, P1, P2, P3}}, PlotStyle → Red], Graphics[{{Text[P0, P0 - 0.2]}}],
  PlotRange → {{-1, 6}, {-1, 6}}, AxesOrigin → {0, 0}],
  {w0, 0.1, 1}, {w1, 0.1, 1}, {w2, 0.1, 1}, {w3, 0.1, 1}]

```

Out[270]=

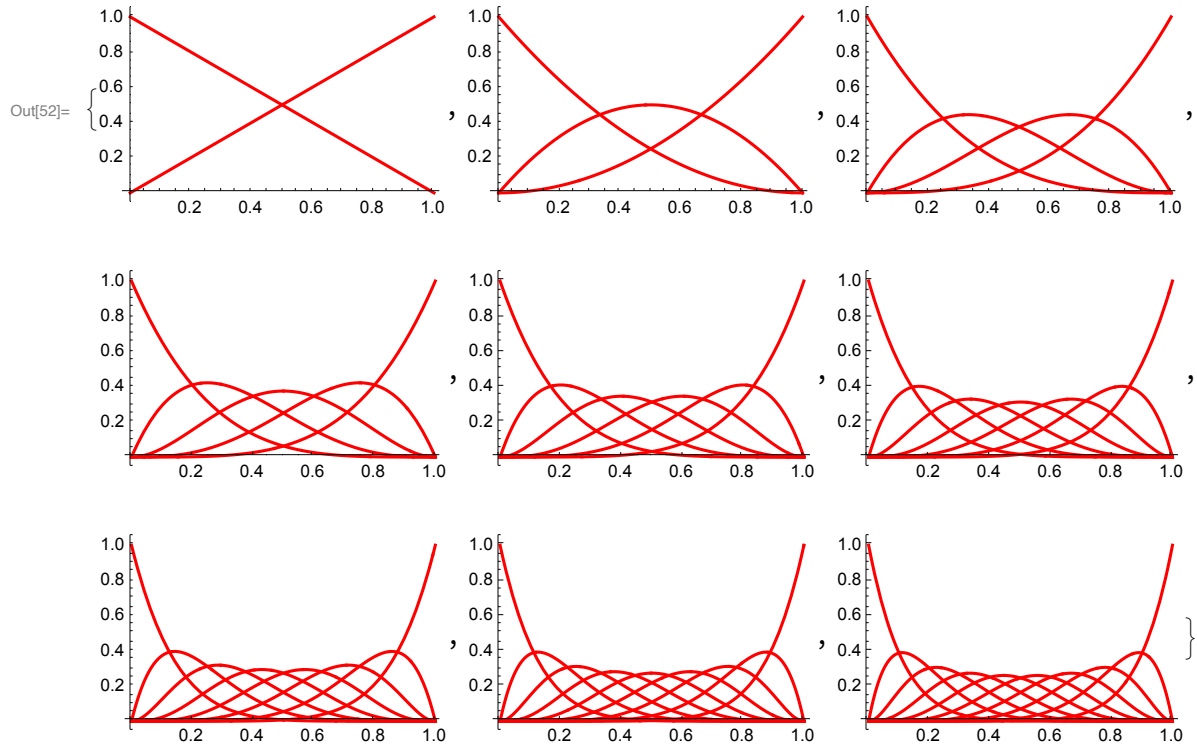
$$\left\{ \frac{3 (1 - t)^2 t w1 + 15 (1 - t) t^2 w2 + 5 t^3 w3}{(1 - t)^3 w0 + 3 (1 - t)^2 t w1 + 3 (1 - t) t^2 w2 + t^3 w3}, \frac{3 (1 - t)^3 w0 + 15 (1 - t) t^2 w2}{(1 - t)^3 w0 + 3 (1 - t)^2 t w1 + 3 (1 - t) t^2 w2 + t^3 w3} \right\}$$

Out[271]=



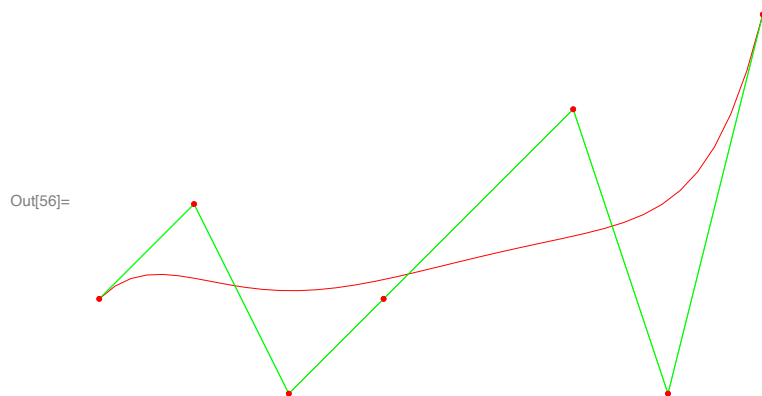
6) General Bezier: basis functions

```
In[52]:= Table[Plot[Evaluate[Table[BernsteinBasis[m, n, t], {n, 0, m}]],
  {t, 0, 1}, PlotStyle -> Red, PlotRange -> All], {m, 1, 9}]
```



```
In[53]:= pts = {{0, 0}, {1, 1}, {2, -1}, {3, 0}, {5, 2}, {6, -1}, {7, 3}};
Pygn = Graphics[{Green, Line[pts], Red, Point[pts]}];
BezStandard = Graphics[
  {Red, BezierCurve[pts, SplineDegree -> 7], Green, Line[pts], Red, Point[pts]};
```

```
Show[Pygn, BezStandard, PlotRange -> All]
```

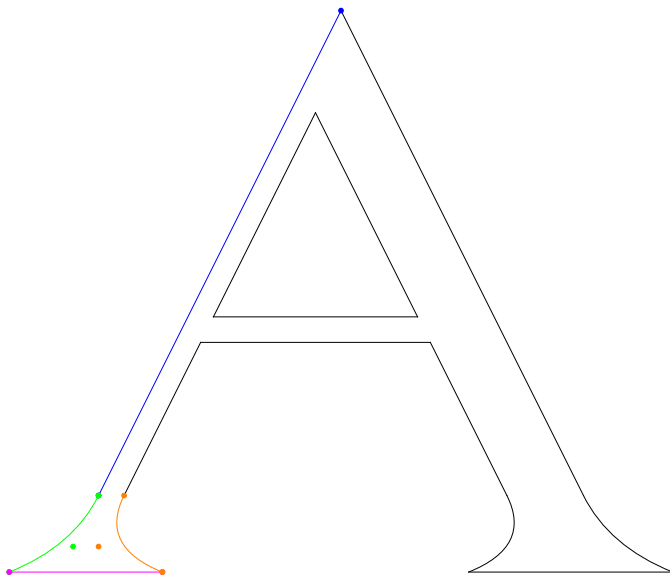


7) Application: Font Design / Morphing

Define the outline of a glyph:

```
In[57]:= curves = Graphics[{
  Blue, BezierCurve[pts1 = {{2, 3}, {0.8125, 0.625}}],
  Green, BezierCurve[pts2 = {{0.8125, 0.625}, {0.6875, 0.375}, {0.375, 0.25}}],
  Magenta, BezierCurve[pts3 = {{0.375, 0.25}, {1.125, 0.25}}],
  Orange, BezierCurve[pts4 = {{1.125, 0.25}, {0.8125, 0.375}, {0.9375, 0.625}}],
  Black, BezierCurve[{{0.9375, 0.625}, {1.3125, 1.375}}],
  BezierCurve[{{1.3125, 1.375}, {2.4375, 1.375}}],
  BezierCurve[{{2.4375, 1.375}, {2.8125, 0.625}}],
  BezierCurve[{{2.8125, 0.625}, {2.9375, 0.375}, {2.625, 0.25}}],
  BezierCurve[{{2.625, 0.25}, {3.625, 0.25}}],
  BezierCurve[{{3.625, 0.25}, {3.3125, 0.375}, {3.1875, 0.625}}],
  BezierCurve[{{3.1875, 0.625}, {2, 3}}],
  BezierCurve[{{1.875, 2.5}, {1.375, 1.5}}],
  BezierCurve[{{1.375, 1.5}, {2.375, 1.5}}],
  BezierCurve[{{2.375, 1.5}, {1.875, 2.5}}]
];
points = Graphics[{Blue, Point[pts1], Green,
  Point[pts2], Magenta, Point[pts3], Orange, Point[pts4]
}];
Show[{curves, points}]
```

Out[59]=



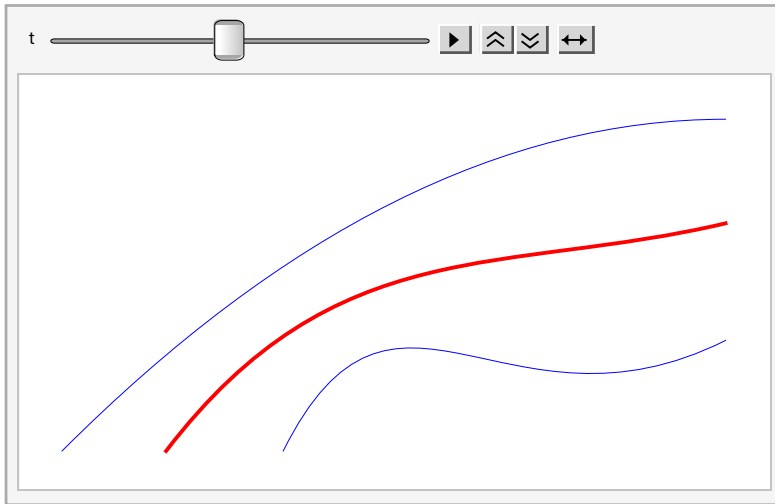
Linear transition from one Bézier curve to another:

```

In[60]:= pts1 = {{0, -1}, {2, 1}, {4, 2}, {6, 2}};
pts2 = {{2, -1}, {3, 1}, {4, -1}, {6, 0}};
g[p1_, p2_, t_] := (1 - t) p1 + t p2
Animate[Graphics[{Blue, BezierCurve[pts1], Blue,
  BezierCurve[pts2], Thick, Red, BezierCurve[g[pts1, pts2, t]]}],
{t, 0, 1}, AnimationDirection → ForwardBackward, SaveDefinitions → True]

```

Out[63]=

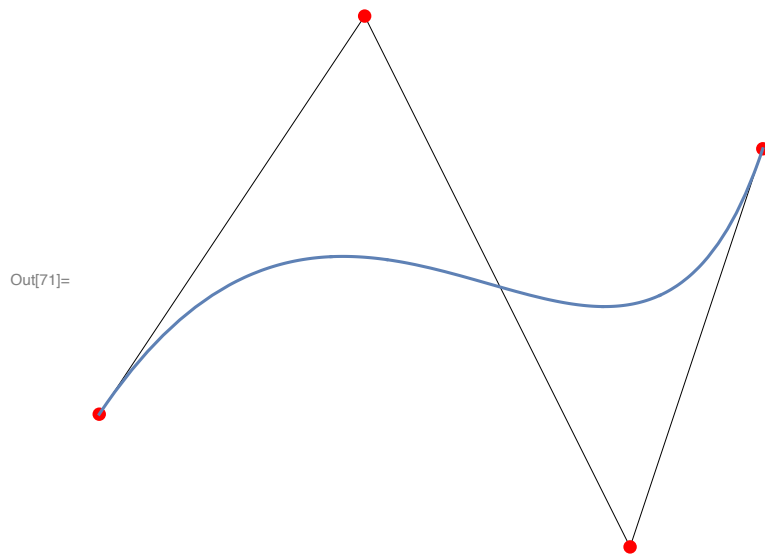


In[64]:=

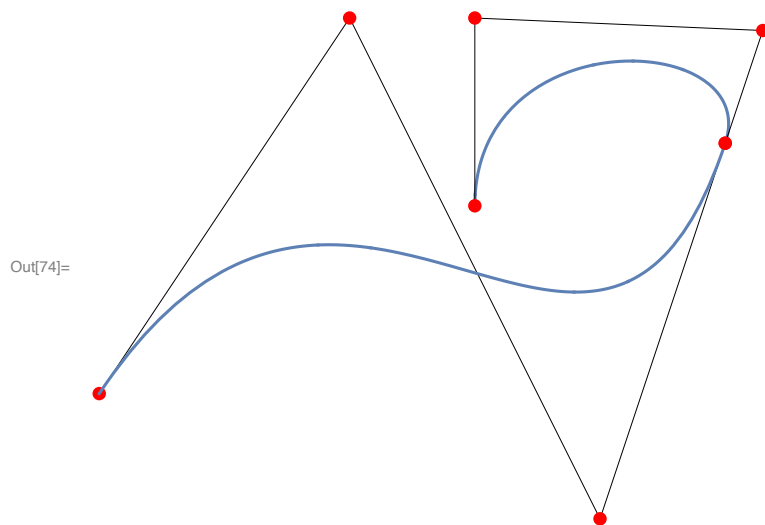
In[65]:=

8) Cubic Bezier spline with G1 continuity

```
In[66]:= B3[t_, p0_, p1_, p2_, p3_] := p0 (1 - t)^3 + 3 t (1 - t)^2 p1 + 3 t^2 (1 - t) p2 + t^3 p3
P3 = {p30, p31, p32, p33} = {{0, 0}, {2, 3}, {4, -1}, {5, 2}};
P4 = {p40, p41, p42, p43} = {{5, 2}, {5.3, 2.9}, {3, 3}, {3, 1.5}};
f31 = ParametricPlot[B3[t, p30, p31, p32, p33], {t, 0, 1}];
f32 = Graphics[{Line[P3], Red, PointSize[Large], Point[P3]}];
Show[f32, f31]
```



```
In[72]:= f42 = Graphics[{Line[P4], Red, PointSize[Large], Point[P4]}];
f41 = ParametricPlot[B3[t, p40, p41, p42, p43], {t, 0, 1}];
Show[f32, f31, f41, f42]
```



BSplines

1) BSplines degree 2

```
In[75]:= ClearAll["Global`*"]
```

```
In[76]:= d = 2;
controlPoints = {b0, b1, b2, b3};
m = (Dimensions[controlPoints][[1]] - 1) + d + 1
e = (m - d);
{td, te}
```

```
Out[78]= 6
```

```
Out[80]= {t2, t4}
```

```
In[81]:= knots = {1, 1, 1, 4, 5, 5, 5};
b0 = {0, 0};
b1 = {1, 1};
b2 = {2, 1};
b3 = {3, 0};
td = knots[[d + 1]];
tmd = knots[[m - d + 1]];
Print[{td, te}, "=", {td, tmd}]

{t2, t4} = {1, 5}
```

```
In[89]:= k = 0;
Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
      "=" PiecewiseExpand[BSplineBasis[{k, knots}, i, x]]}, {i, 0, m - 1}] // TableForm
```

```
Out[90]//TableForm=
```

$N_{0,0}(x)$	0
$N_{1,0}(x)$	0
$N_{2,0}(x)$	$= \left(\begin{array}{cc} 1 & 1 \leq x \leq 4 \\ 0 & \text{True} \end{array} \right)$
$N_{3,0}(x)$	$= \left(\begin{array}{cc} 1 & 4 \leq x \leq 5 \\ 0 & \text{True} \end{array} \right)$
$N_{4,0}(x)$	0
$N_{5,0}(x)$	0

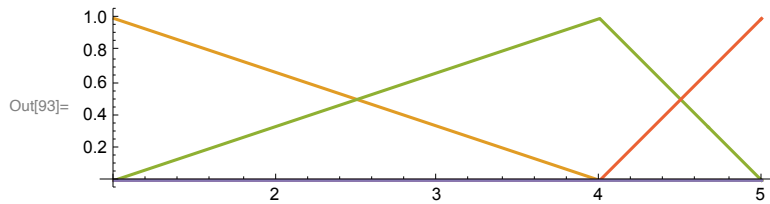

```

In[91]:= k = 1;
Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
      "=" PiecewiseExpand[BSplineBasis[{k, knots}, i, x]], {i, 0, m - 2}] // TableForm
Plot[Evaluate[Table[BSplineBasis[{k, knots}, i, x], {i, 0, m - 2}]],
      {x, 1, 5}, AspectRatio → Automatic, PlotRange → All]

```

Out[92]//TableForm=

$$\begin{aligned}
 N_{0,1}(x) &= 0 \\
 N_{1,1}(x) &= \begin{cases} \frac{4-x}{3} & 1 \leq x \leq 4 \\ 0 & \text{True} \end{cases} \\
 N_{2,1}(x) &= \begin{cases} 5-x & 4 \leq x \leq 5 \\ \frac{1}{3}(-1+x) & 1 \leq x < 4 \\ 0 & \text{True} \end{cases} \\
 N_{3,1}(x) &= \begin{cases} -4+x & 4 \leq x \leq 5 \\ 0 & \text{True} \end{cases} \\
 N_{4,1}(x) &= 0
 \end{aligned}$$



```

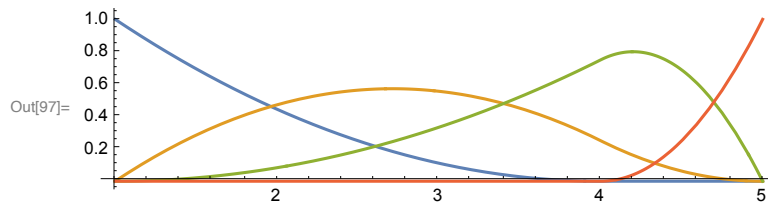
In[94]:= k = 2;
Print[{td, te}, "=", {td, tmd}]
BSpline = Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
  "=" PiecewiseExpand[BSplineBasis[{k, knots}, i, x]]}, {i, 0, m - 3}] // TableForm
Plot[Evaluate[Table[BSplineBasis[{k, knots}, i, x], {i, 0, m - 3}]],
  {x, 1, 5}, AspectRatio → Automatic, PlotRange → All]

```

{t₂, t₄} = {1, 5}

Out[96]//TableForm=

$$\begin{aligned}
 N_{0,2}(x) &= \begin{pmatrix} \begin{cases} \frac{1}{9}(16 - 8x + x^2) & 1 \leq x \leq 4 \\ 0 & \text{True} \end{cases} \end{pmatrix} \\
 N_{1,2}(x) &= \begin{pmatrix} \begin{cases} \frac{1}{36}(-31 + 38x - 7x^2) & 1 \leq x < 4 \\ \frac{1}{4}(25 - 10x + x^2) & 4 \leq x \leq 5 \\ 0 & \text{True} \end{cases} \end{pmatrix} \\
 N_{2,2}(x) &= \begin{pmatrix} \begin{cases} \frac{1}{4}(-85 + 42x - 5x^2) & 4 \leq x \leq 5 \\ \frac{1}{12}(1 - 2x + x^2) & 1 \leq x < 4 \\ 0 & \text{True} \end{cases} \end{pmatrix} \\
 N_{3,2}(x) &= \begin{pmatrix} \begin{cases} 16 - 8x + x^2 & 4 \leq x \leq 5 \\ 0 & \text{True} \end{cases} \end{pmatrix}
 \end{aligned}$$



```

In[98]:= Piece1 = ParametricPlot[ $\frac{1}{9}(16 - 8x + x^2) b_0 + \frac{1}{36}(-31 + 38x - 7x^2) b_1 + \frac{1}{12}(1 - 2x + x^2) b_2$ ,
  {x, 1, 4}, Frame → True, Axes → False, PlotRange → All, PlotStyle → Green];

```

```

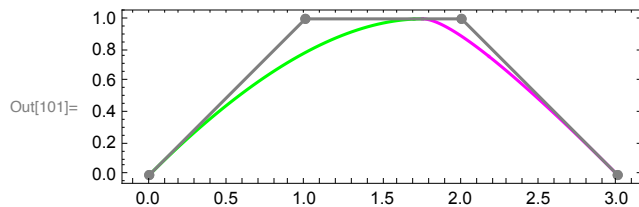
In[99]:= Piece2 = ParametricPlot[ $\frac{1}{4}(25 - 10x + x^2) b_1 + \frac{1}{4}(-85 + 42x - 5x^2) b_2 + (16 - 8x + x^2) b_3$ ,
  {x, 4, 5}, Frame → True, Axes → False, PlotRange → All, PlotStyle → Magenta];

```

```

ctlpts = ListLinePlot[{b0, b1, b2, b3}, Mesh → Full, PlotStyle → Gray];
Show[{Piece1, Piece2, ctlpts}, AspectRatio → Automatic]

```



2) BSplines degree 2

```
In[102]:= ClearAll["Global`*"]
```

```
In[103]:= d = 2;
controlPoints = {b0, b1, b2, b3, b4};
m = (Dimensions[controlPoints][[1]] - 1) + d + 1
e = (m - d);
{td, te}
```

```
Out[105]= 7
```

```
Out[107]= {t2, t5}
```

```
In[108]:= knots = {1, 1, 1, 2, 4, 5, 5, 5};
b0 = {0, 0};
b1 = {1, 1};
b2 = {2, 1};
b3 = {3, 0};
b4 = {4, 3};
td = knots[[d + 1]];
tmd = knots[[m - d + 1]];
Print[{td, te}, "=", {td, tmd}]

{t2, t5} = {1, 5}
```

```
In[117]:= k = 0;
Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
      "=" PiecewiseExpand[BSplineBasis[{k, knots}, i, x]]}, {i, 0, m - 1}] // TableForm
```

```
Out[118]//TableForm=
```

$N_{0,0}(x)$	0
$N_{1,0}(x)$	0
$N_{2,0}(x)$	$= \left(\begin{array}{ll} 1 & 1 \leq x \leq 2 \\ 0 & \text{True} \end{array} \right)$
$N_{3,0}(x)$	$= \left(\begin{array}{ll} 1 & 2 \leq x \leq 4 \\ 0 & \text{True} \end{array} \right)$
$N_{4,0}(x)$	$= \left(\begin{array}{ll} 1 & 4 \leq x \leq 5 \\ 0 & \text{True} \end{array} \right)$
$N_{5,0}(x)$	0
$N_{6,0}(x)$	0

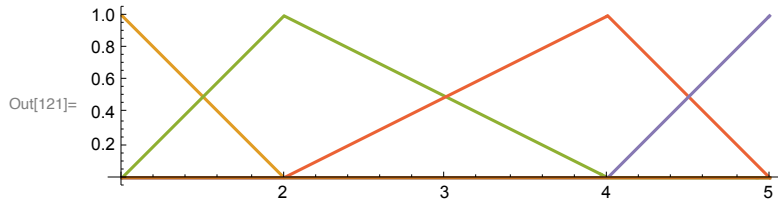
```

In[119]:= k = 1;
Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
      "=" PiecewiseExpand[BSplineBasis[{k, knots}, i, x]]}, {i, 0, m - 2}] // TableForm
Plot[Evaluate[Table[BSplineBasis[{k, knots}, i, x], {i, 0, m - 2}]],
      {x, 1, 5}, AspectRatio → Automatic, PlotRange → All]

```

Out[120]//TableForm=

$$\begin{aligned}
 N_{0,1}(x) &= 0 \\
 N_{1,1}(x) &= \begin{cases} 2-x & 1 \leq x \leq 2 \\ 0 & \text{True} \end{cases} \\
 N_{2,1}(x) &= \begin{cases} \frac{4-x}{2} & 2 \leq x \leq 4 \\ -1+x & 1 \leq x < 2 \\ 0 & \text{True} \end{cases} \\
 N_{3,1}(x) &= \begin{cases} 5-x & 4 \leq x \leq 5 \\ \frac{1}{2}(-2+x) & 2 \leq x < 4 \\ 0 & \text{True} \end{cases} \\
 N_{4,1}(x) &= \begin{cases} -4+x & 4 \leq x \leq 5 \\ 0 & \text{True} \end{cases} \\
 N_{5,1}(x) &= 0
 \end{aligned}$$



```

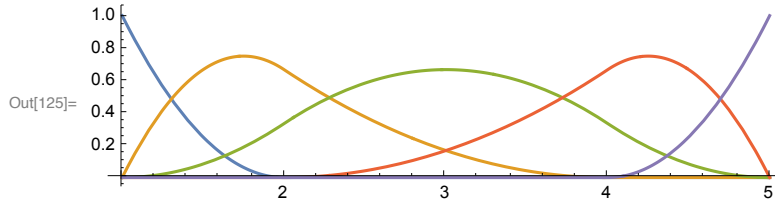
In[122]:= k = 2;
Print[{td, te}, "=", {td, tmd}]
BSpline = Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
  "=" PiecewiseExpand[BSplineBasis[{k, knots}, i, x]]}, {i, 0, m - 3}] // TableForm
Plot[Evaluate[Table[BSplineBasis[{k, knots}, i, x], {i, 0, m - 3}]],
  {x, 1, 5}, AspectRatio → Automatic, PlotRange → All]

```

{t₂, t₅} = {1, 5}

Out[124]//TableForm=

$$\begin{aligned}
 N_{0,2}(x) &= \begin{pmatrix} \begin{cases} 4 - 4x + x^2 & 1 \leq x \leq 2 \\ 0 & \text{True} \end{cases} \end{pmatrix} \\
 N_{1,2}(x) &= \begin{pmatrix} \begin{cases} \frac{1}{6}(16 - 8x + x^2) & 2 \leq x \leq 4 \\ -\frac{2}{3}(5 - 7x + 2x^2) & 1 \leq x < 2 \\ 0 & \text{True} \end{cases} \end{pmatrix} \\
 N_{2,2}(x) &= \begin{pmatrix} \begin{cases} \frac{1}{3}(-7 + 6x - x^2) & 2 \leq x < 4 \\ \frac{1}{3}(25 - 10x + x^2) & 4 \leq x \leq 5 \\ \frac{1}{3}(1 - 2x + x^2) & 1 \leq x < 2 \\ 0 & \text{True} \end{cases} \end{pmatrix} \\
 N_{3,2}(x) &= \begin{pmatrix} \begin{cases} \frac{1}{6}(4 - 4x + x^2) & 2 \leq x < 4 \\ -\frac{2}{3}(35 - 17x + 2x^2) & 4 \leq x \leq 5 \\ 0 & \text{True} \end{cases} \end{pmatrix} \\
 N_{4,2}(x) &= \begin{pmatrix} \begin{cases} 16 - 8x + x^2 & 4 \leq x \leq 5 \\ 0 & \text{True} \end{cases} \end{pmatrix}
 \end{aligned}$$



```

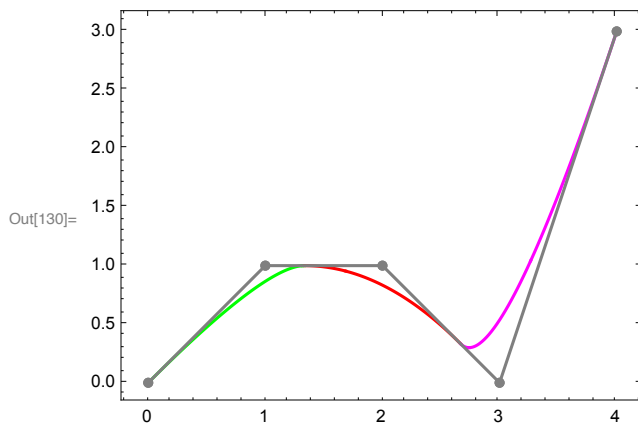
In[126]:= Piece1 = ParametricPlot[(4 - 4x + x2) b0 -  $\frac{2}{3}$  (5 - 7x + 2x2) b1 +  $\frac{1}{3}$  (1 - 2x + x2) b2,
  {x, 1, 2}, Frame → True, Axes → False, PlotRange → All, PlotStyle → Green];

Piece2 = ParametricPlot[ $\frac{1}{6}$  (16 - 8x + x2) b1 +  $\frac{1}{3}$  (-7 + 6x - x2) b2 +  $\frac{1}{6}$  (4 - 4x + x2) b3,
  {x, 2, 4}, Frame → True, Axes → False, PlotRange → All, PlotStyle → Red];

Piece3 = ParametricPlot[ $\frac{1}{3}$  (25 - 10x + x2) b2 -  $\frac{2}{3}$  (35 - 17x + 2x2) b3 + (16 - 8x + x2) b4,
  {x, 4, 5}, Frame → True, Axes → False, PlotRange → All, PlotStyle → Magenta];

```

```
In[129]:= ctlpts = ListLinePlot[{b0, b1, b2, b3, b4}, Mesh → Full, PlotStyle → Gray];
Show[{Piece1, Piece2, Piece3, ctlpts}, AspectRatio → Automatic]
```



3) BSplines degree 3

```
In[131]:= ClearAll["Global`*"]
```

```
In[132]:= d = 3;
controlPoints = {b0, b1, b2, b3, b4};
m = (Dimensions[controlPoints][[1]] - 1) + d + 1
e = (m - d);
{td, te}
```

Out[134]= 8

Out[136]= {t₃, t₅}

```
In[137]:= knots = {1, 1, 1, 1, 3, 4, 5, 6, 7};
b0 = {0, 0};
b1 = {1, 1};
b2 = {2, 1};
b3 = {3, 0};
b4 = {4, 3};
td = knots[[d + 1]];
tmd = knots[[m - d + 1]];
Print[{td, te}, "=", {td, tmd}]

{t3, t5} = {1, 4}
```

```
In[146]:= k = 0;
Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
      "=" PiecewiseExpand[BSplineBasis[{k, knots}, i, x]], {i, 0, m - 1}} // TableForm
```

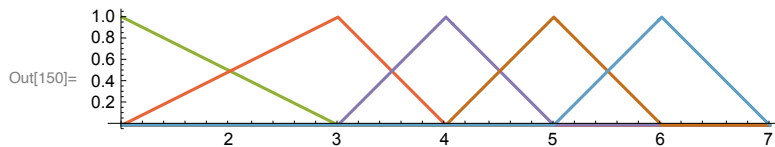
Out[147]//TableForm=

$$\begin{aligned}
 N_{0,0}(x) &= 0 \\
 N_{1,0}(x) &= 0 \\
 N_{2,0}(x) &= 0 \\
 N_{3,0}(x) &= \begin{cases} 1 & 1 \leq x \leq 3 \\ 0 & \text{True} \end{cases} \\
 N_{4,0}(x) &= \begin{cases} 1 & 3 \leq x \leq 4 \\ 0 & \text{True} \end{cases} \\
 N_{5,0}(x) &= \begin{cases} 1 & 4 \leq x \leq 5 \\ 0 & \text{True} \end{cases} \\
 N_{6,0}(x) &= \begin{cases} 1 & 5 \leq x \leq 6 \\ 0 & \text{True} \end{cases} \\
 N_{7,0}(x) &= \begin{cases} 1 & 6 \leq x \leq 7 \\ 0 & \text{True} \end{cases}
 \end{aligned}$$

```
In[148]:= k = 1;
Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
      "=" PiecewiseExpand[BSplineBasis[{k, knots}, i, x]], {i, 0, m - 2}} // TableForm
Plot[Evaluate[Table[BSplineBasis[{k, knots}, i, x], {i, 0, m - 2}]],
      {x, 1, 7}, AspectRatio -> Automatic, PlotRange -> All]
```

Out[149]//TableForm=

$$\begin{aligned}
 N_{0,1}(x) &= 0 \\
 N_{1,1}(x) &= 0 \\
 N_{2,1}(x) &= \begin{cases} \frac{3-x}{2} & 1 \leq x \leq 3 \\ 0 & \text{True} \end{cases} \\
 N_{3,1}(x) &= \begin{cases} 4-x & 3 \leq x \leq 4 \\ \frac{1}{2}(-1+x) & 1 \leq x < 3 \\ 0 & \text{True} \end{cases} \\
 N_{4,1}(x) &= \begin{cases} 5-x & 4 \leq x \leq 5 \\ -3+x & 3 \leq x < 4 \\ 0 & \text{True} \end{cases} \\
 N_{5,1}(x) &= \begin{cases} 6-x & 5 \leq x \leq 6 \\ -4+x & 4 \leq x < 5 \\ 0 & \text{True} \end{cases} \\
 N_{6,1}(x) &= \begin{cases} 7-x & 6 \leq x \leq 7 \\ -5+x & 5 \leq x < 6 \\ 0 & \text{True} \end{cases}
 \end{aligned}$$



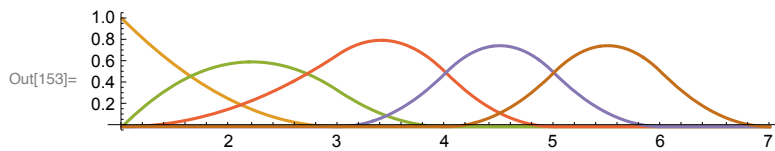
```

In[151]:= k = 2;
BSpline = Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
  " = " PiecewiseExpand[BSplineBasis[{k, knots}, i, x]]}, {i, 0, m - 3}] // TableForm
Plot[Evaluate[Table[BSplineBasis[{k, knots}, i, x], {i, 0, m - 3}]],
  {x, 1, 7}, AspectRatio → Automatic, PlotRange → All]

```

Out[152]//TableForm=

$$\begin{aligned}
 N_{0,2}(x) &= 0 \\
 N_{1,2}(x) &= \begin{cases} \frac{1}{4}(9 - 6x + x^2) & 1 \leq x \leq 3 \\ 0 & \text{True} \end{cases} \\
 N_{2,2}(x) &= \begin{cases} \frac{1}{12}(-17 + 22x - 5x^2) & 1 \leq x < 3 \\ \frac{1}{3}(16 - 8x + x^2) & 3 \leq x \leq 4 \\ 0 & \text{True} \end{cases} \\
 N_{3,2}(x) &= \begin{cases} \frac{1}{6}(-53 + 34x - 5x^2) & 3 \leq x < 4 \\ \frac{1}{2}(25 - 10x + x^2) & 4 \leq x \leq 5 \\ \frac{1}{6}(1 - 2x + x^2) & 1 \leq x < 3 \\ 0 & \text{True} \end{cases} \\
 N_{4,2}(x) &= \begin{cases} \frac{1}{2}(-39 + 18x - 2x^2) & 4 \leq x < 5 \\ \frac{1}{2}(36 - 12x + x^2) & 5 \leq x \leq 6 \\ \frac{1}{2}(9 - 6x + x^2) & 3 \leq x < 4 \\ 0 & \text{True} \end{cases} \\
 N_{5,2}(x) &= \begin{cases} \frac{1}{2}(-59 + 22x - 2x^2) & 5 \leq x < 6 \\ \frac{1}{2}(49 - 14x + x^2) & 6 \leq x \leq 7 \\ \frac{1}{2}(16 - 8x + x^2) & 4 \leq x < 5 \\ 0 & \text{True} \end{cases}
 \end{aligned}$$




```

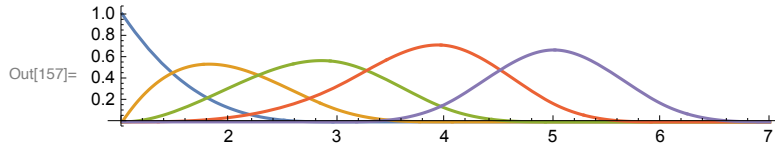
In[154]:= k = 3;
Print[{td, te}, "=", {td, tmd}]
BSpline = Table[{TraditionalForm[BSplineBasis[{k, knots}, i, x]],
  "=" PiecewiseExpand[BSplineBasis[{k, knots}, i, x]]}, {i, 0, m - 4}] // TableForm
Plot[Evaluate[Table[BSplineBasis[{k, knots}, i, x], {i, 0, m - 4}]],
  {x, 1, 7}, AspectRatio → Automatic, PlotRange → All]

```

{t₃, t₅} = {1, 4}

Out[156]//TableForm=

$$\begin{aligned}
 N_{0,3}(x) &= \begin{cases} \frac{1}{8} (27 - 27x + 9x^2 - x^3) & 1 \leq x \leq 3 \\ 0 & \text{True} \end{cases} \\
 N_{1,3}(x) &= \begin{cases} \frac{1}{9} (64 - 48x + 12x^2 - x^3) & 3 \leq x \leq 4 \\ \frac{1}{72} (-217 + 345x - 147x^2 + 19x^3) & 1 \leq x < 3 \\ 0 & \text{True} \end{cases} \\
 N_{2,3}(x) &= \begin{cases} \frac{1}{72} (49 - 111x + 75x^2 - 13x^3) & 1 \leq x < 3 \\ \frac{1}{8} (125 - 75x + 15x^2 - x^3) & 4 \leq x \leq 5 \\ \frac{1}{72} (-923 + 861x - 249x^2 + 23x^3) & 3 \leq x < 4 \\ 0 & \text{True} \end{cases} \\
 N_{3,3}(x) &= \begin{cases} \frac{1}{24} (269 - 267x + 87x^2 - 9x^3) & 3 \leq x < 4 \\ \frac{1}{6} (216 - 108x + 18x^2 - x^3) & 5 \leq x \leq 6 \\ \frac{1}{24} (-1 + 3x - 3x^2 + x^3) & 1 \leq x < 3 \\ \frac{1}{24} (-1011 + 693x - 153x^2 + 11x^3) & 4 \leq x < 5 \\ 0 & \text{True} \end{cases} \\
 N_{4,3}(x) &= \begin{cases} \frac{1}{6} (229 - 165x + 39x^2 - 3x^3) & 4 \leq x < 5 \\ \frac{1}{6} (343 - 147x + 21x^2 - x^3) & 6 \leq x \leq 7 \\ \frac{1}{6} (-27 + 27x - 9x^2 + x^3) & 3 \leq x < 4 \\ \frac{1}{6} (-521 + 285x - 51x^2 + 3x^3) & 5 \leq x < 6 \\ 0 & \text{True} \end{cases}
 \end{aligned}$$



```

In[158]:= Piece1 = ParametricPlot[ $\frac{1}{8} (27 - 27 x + 9 x^2 - x^3) b_0 + \frac{1}{72} (-217 + 345 x - 147 x^2 + 19 x^3) b_1 +$   

 $\frac{1}{72} (49 - 111 x + 75 x^2 - 13 x^3) b_2 + \frac{1}{24} (-1 + 3 x - 3 x^2 + x^3) b_3$ , {x, 1, 3},  

Frame → True, Axes → False, PlotRange → All, PlotStyle → Green];

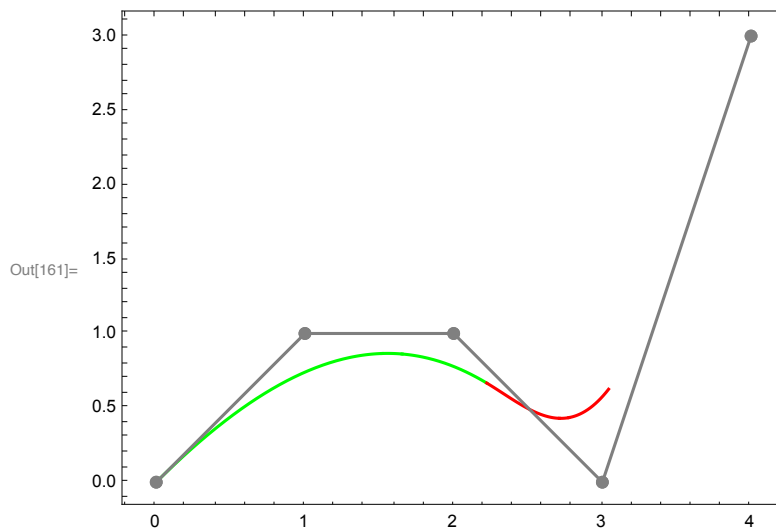
Piece2 = ParametricPlot[ $\frac{1}{9} (64 - 48 x + 12 x^2 - x^3) b_1 + \frac{1}{72} (-923 + 861 x - 249 x^2 + 23 x^3) b_2 +$   

 $\frac{1}{24} (269 - 267 x + 87 x^2 - 9 x^3) b_3 + \frac{1}{6} (-27 + 27 x - 9 x^2 + x^3) b_4$ ,  

{x, 3, 4}, Frame → True, Axes → False, PlotRange → All, PlotStyle → Red];

In[160]:= ctrlpts = ListLinePlot[{b0, b1, b2, b3, b4}, Mesh → Full, PlotStyle → Gray];
Show[{Piece1, Piece2, ctrlpts}, AspectRatio → Automatic]

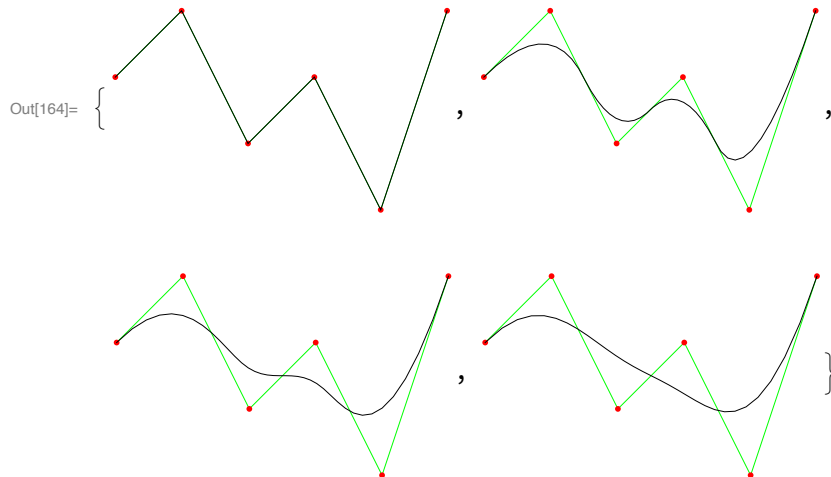
```



4) Built-in BSpline

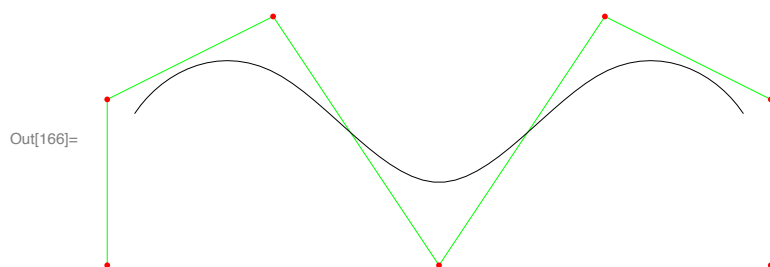
Degree

```
In[163]:= pts = {{0, 0}, {1, 1}, {2, -1}, {3, 0}, {4, -2}, {5, 1}};
Table[Graphics[{Green, Line[pts], Red, Point[pts],
  Black, BSplineCurve[pts, SplineDegree → d]}], {d, 1, 4}]
```



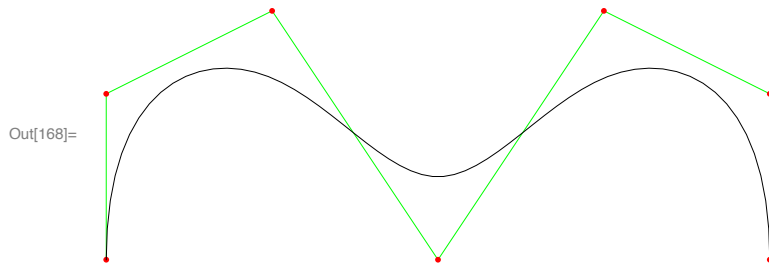
Knot vectors

```
In[165]:= pts = {{0, 0}, {0, 2}, {2, 3}, {4, 0}, {6, 3}, {8, 2}, {8, 0}};
Graphics[{Green, Line[pts], Red, Point[pts], Black,
  BSplineCurve[pts, SplineKnots → {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}]}]
```



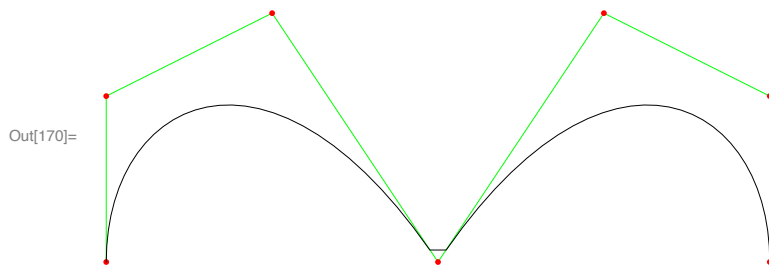
Using SplineKnots as automatic:

```
In[167]:= pts = {{0, 0}, {0, 2}, {2, 3}, {4, 0}, {6, 3}, {8, 2}, {8, 0}};
Graphics[{Green, Line[pts], Red, Point[pts],
  Black, BSplineCurve[pts, SplineKnots → Automatic]]}]
```



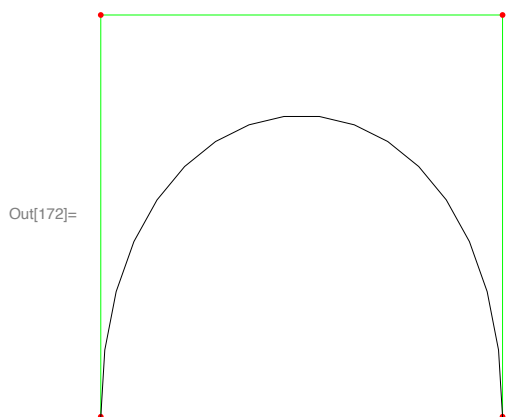
Defining SplineKnots as automatic with $d=3, n=6, m=d+n+1=10$:

```
In[169]:= pts = {{0, 0}, {0, 2}, {2, 3}, {4, 0}, {6, 3}, {8, 2}, {8, 0}};
Graphics[{Green, Line[pts], Red, Point[pts], Black,
  BSplineCurve[pts, SplineKnots → {0, 0, 0, 0, 1, 1, 1, 2, 2, 2, 2}]]}]
```



SplineClosed

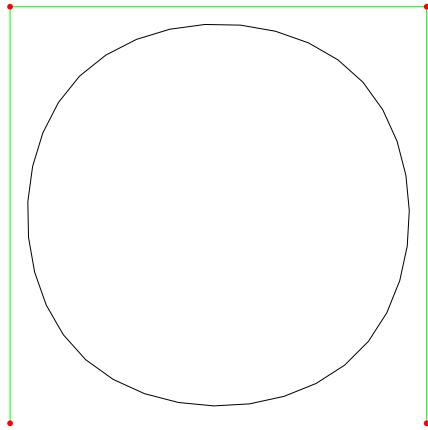
```
In[171]:= pts = {{0, 0}, {0, 1}, {1, 1}, {1, 0}};
Graphics[{Green, Line[pts], Red, Point[pts], Black, BSplineCurve[pts]]}]
```



Smoothly closed B-spline curve with the same control points:

```
In[173]:= Graphics[
  {Green, Line[pts], Red, Point[pts], Black, BSplineCurve[pts, SplineClosed → True]}]
```

Out[173]=

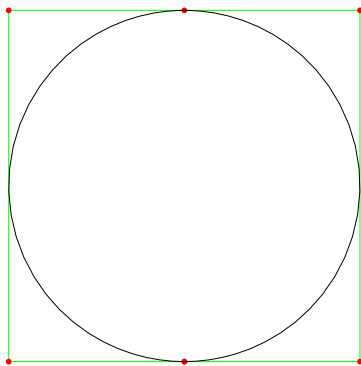


NURBS

Perfect circle using NURBS:

```
In[174]:= pts = {{.5, 0}, {1, 0}, {1, 1}, {.5, 1}, {0, 1}, {0, 0}, {.5, 0}};
w = {1, .5, .5, 1, .5, .5, 1};
k = {0, 0, 0, 1/4, 1/2, 1/2, 3/4, 1, 1, 1};
Graphics[{Green, Line[pts], Red, Point[pts],
  Black, BSplineCurve[pts, SplineWeights → w, SplineKnots → k]}]
```

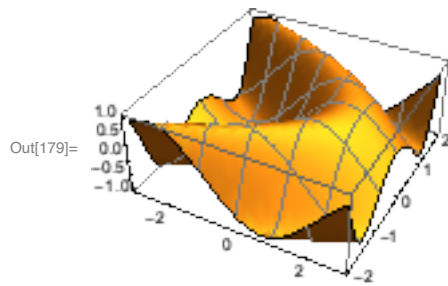
Out[177]=



Surfaces

1) Explicit Surfaces

```
In[178]:= ClearAll["Global`*"]  
Plot3D[Sin[x+y^2], {x, -3, 3}, {y, -2, 2}, MeshStyle → Gray, Mesh → 5]
```



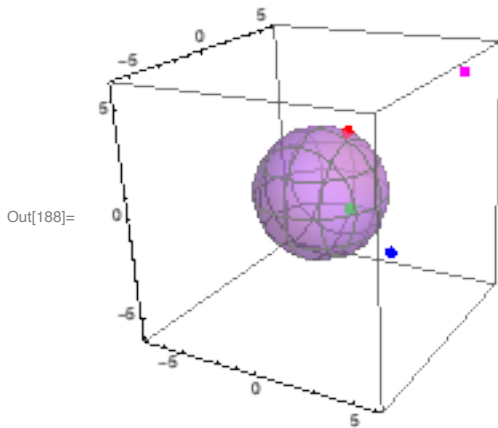
2) Implicit Surfaces

```

In[180]:= ClearAll["Global`*"]
iBall[x_, y_, z_] = x^2 + y^2 + z^2 - 10;
f1 = ContourPlot3D[iBall[x, y, z] == 0,
  {x, -6, 6}, {y, -6, 6}, {z, -6, 6}, MeshStyle -> Gray, Mesh -> 3,
  ContourStyle -> Opacity[0.5], ColorFunction -> "Pastel"];
{iBall[1, 1, 2  $\sqrt{2}$ ], iBall[5, 5, 5], iBall[1, 1, -1], iBall[1, 1, -5]}
f2 = Graphics3D[{PointSize[Large], Red, Point[{1, 1, 2  $\sqrt{2}$ }] }];
f3 = Graphics3D[{PointSize[Large], Magenta, Point[{5, 5, 5}] }];
f4 = Graphics3D[{PointSize[Large], Green, Point[{1, 1, -1}] }];
f5 = Graphics3D[{PointSize[Large], Blue, Point[{1, 5, -5}] }];
Show[{f1, f2, f3, f4, f5}]

```

Out[183]= {0, 65, -7, 17}



3) Parametric Surface

a) Bilinear Patch

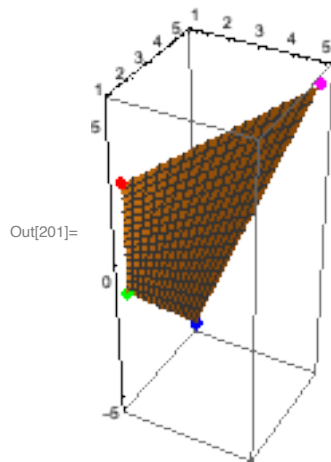
```

In[190]:= ClearAll["Global`*"]
p00 = {1, 1, 2  $\sqrt{2}$ };
p01 = {5, 5, 5};
p10 = {1, 1, -1};
p11 = {1, 5, -5};
fp00 = Graphics3D[{PointSize[Large], Red, Point[p00]}];
fp01 = Graphics3D[{PointSize[Large], Magenta, Point[p01]}];
fp10 = Graphics3D[{PointSize[Large], Green, Point[p10]}];
fp11 = Graphics3D[{PointSize[Large], Blue, Point[p11]}];

```

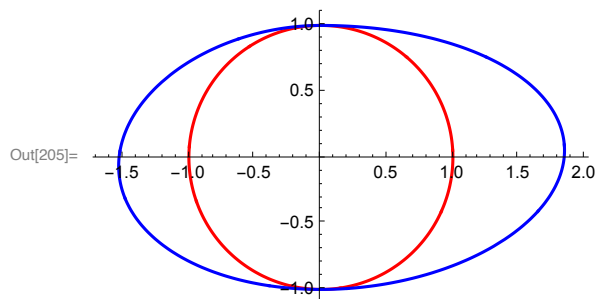
```
In[199]:= Bilinear[u_, v_] = (1 - u) * (1 - v) p00 + (1 - u) v p01 + u (1 - v) p10 + u v p11
fB = ParametricPlot3D[Bilinear[u, v], {u, 0, 1}, {v, 0, 1}];
Show[{fB, fp00, fp01, fp10, fp11}]
```

```
Out[199]:= { (1 - u) (1 - v) + u (1 - v) + 5 (1 - u) v + u v, (1 - u) (1 - v) + u (1 - v) + 5 (1 - u) v + 5 u v,
  2  $\sqrt{2}$  (1 - u) (1 - v) - u (1 - v) + 5 (1 - u) v - 5 u v }
```



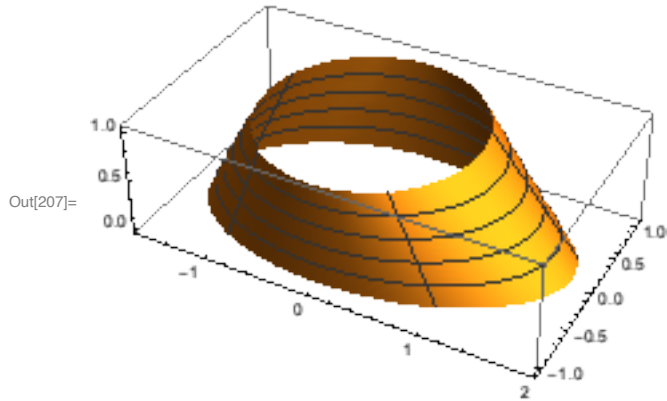
b) Lofting

```
In[202]:= ClearAll["Global`*"]
C1[u_] := {Sin[u], Cos[u]}
C2[u_] := {(2 - 0.1 u) Sin[u], Cos[u]}
f1 = Show[{ParametricPlot[C1[u], {u, 0, 2  $\pi$ }, PlotStyle -> Red],
  ParametricPlot[C2[u], {u, 0, 2  $\pi$ }, PlotStyle -> Blue]}, PlotRange -> All]
```




```
In[206]:= LoftedSurface[u_, v_] = {(1 - v) C2[u] + v C1[u], v}
f2 = ParametricPlot3D[LoftedSurface[u, v], {u, 0, 2 π}, {v, 0, 1}, Mesh → 4]
```

```
Out[206]:= {{(2 - 0.1 u) (1 - v) Sin[u] + v Sin[u], (1 - v) Cos[u] + v Cos[u]}, v}
```

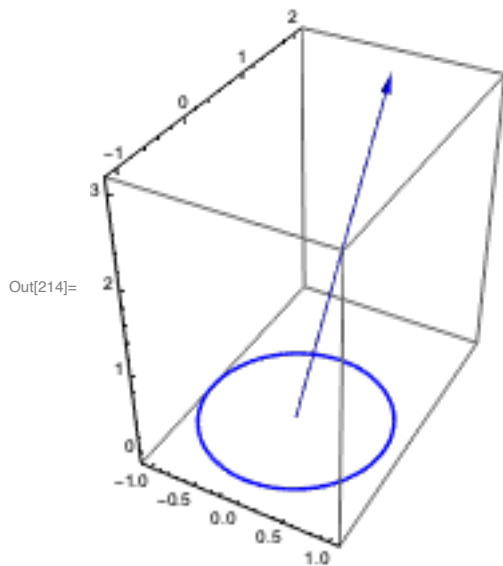


```
In[208]:= {{(2 - 0.1` u) (1 - v) Sin[u] + v Sin[u], (1 - v) Cos[u] + v Cos[u]}, v}
```

```
Out[208]:= {{(2 - 0.1 u) (1 - v) Sin[u] + v Sin[u], (1 - v) Cos[u] + v Cos[u]}, v}
```

c) Extrusion Surface

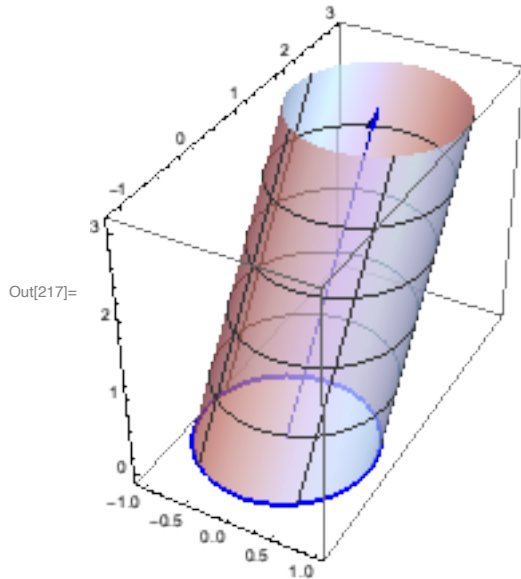
```
In[209]:= ClearAll["Global`*"]
C1[u_] := {Sin[u], Cos[u], 0}
v = {0, 2, 3};
f1 = ParametricPlot3D[C1[u], {u, 0, 2 π}, PlotStyle → Blue];
f2 = Graphics3D[{Blue, Arrow[Tube[{0, 0, 0}, v]]}];
Show[{f1, f2}, PlotRange -> All]
```



```

In[215]:= ExtrudeS[s_, t_] := C1[s] + v t
f3 =
  ParametricPlot3D[ExtrudeS[s, t], {s, 0, 2  $\pi$ }, {t, 0, 1}, AspectRatio  $\rightarrow$  Automatic,
    PlotStyle  $\rightarrow$  { Specularity[White, 50], Opacity[0.7]}, Mesh  $\rightarrow$  4];
Show[{f1, f2, f3}, PlotRange  $\rightarrow$  All]

```



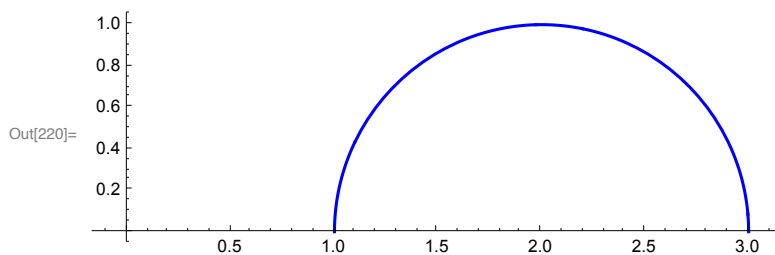
d) Surface of Revolution

x-axis

```

In[218]:= ClearAll["Global`*"]
C1[u_] := {2, 0} + {Sin[u], Cos[u]}
ParametricPlot[C1[u], {u, - $\pi/2$ ,  $\pi/2$ }, PlotStyle  $\rightarrow$  Blue, AxesOrigin  $\rightarrow$  {0, 0}]
m = Graphics3D[{PointSize[Large], Red, Point[{2, 0, 0}]}];

```

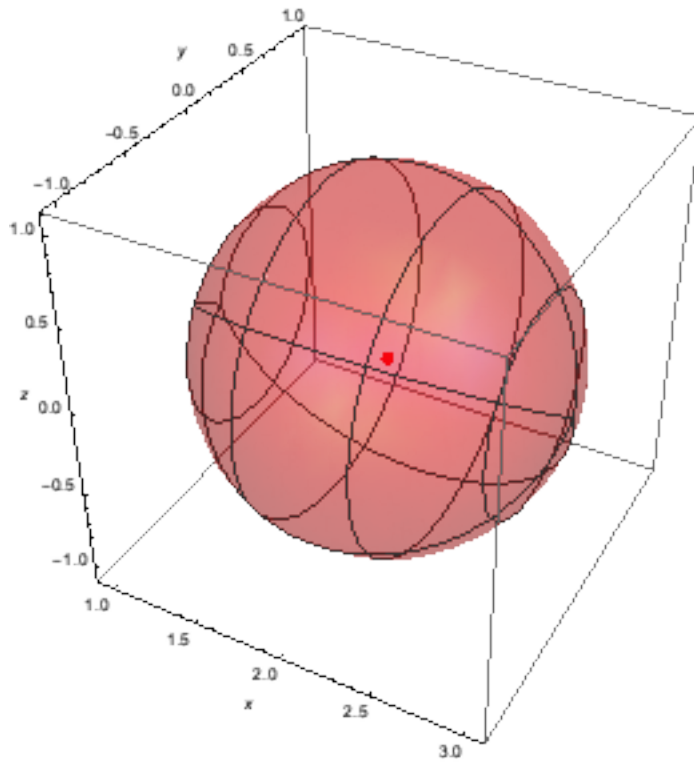


```

In[222]:= gx[u_] = C1[u][[1]];
          gy[u_] = C1[u][[2]];
          RevolveX[u_,  $\theta$ _] := {gx[u], gy[u] Cos[ $\theta$ ], gy[u] Sin[ $\theta$ ]}
          f1 = ParametricPlot3D[RevolveX[u,  $\theta$ ],
            {u,  $-\pi/2$ ,  $\pi/2$ }, { $\theta$ , 0,  $2\pi$ }, AspectRatio  $\rightarrow$  Automatic, Mesh  $\rightarrow$  4,
            PlotStyle  $\rightarrow$  {Red, Specularity[White, 50], Opacity[0.3]}];
          Show[{f1, m}, PlotRange  $\rightarrow$  All, AxesLabel  $\rightarrow$  {x, y, z}]

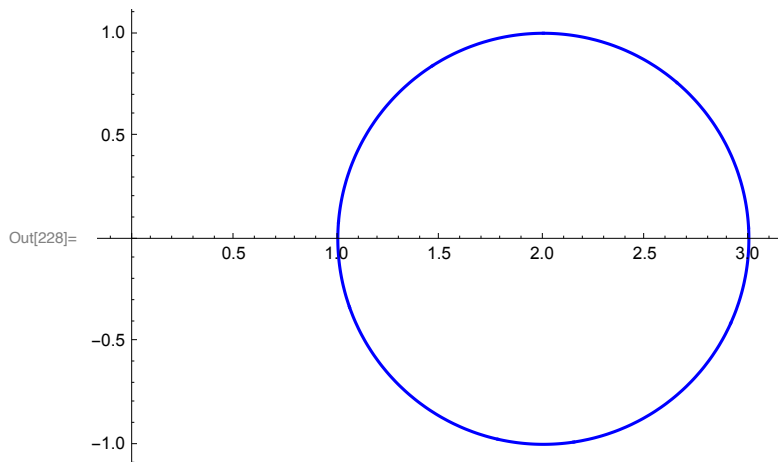
```

Out[226]=

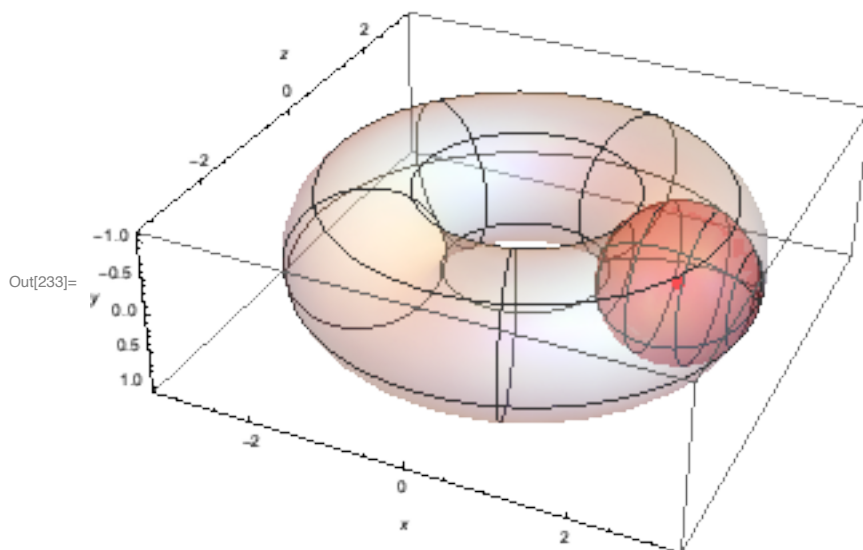


y - axis

```
In[227]:= C1[u_] := {2, 0} + {Sin[u], Cos[u]}
ParametricPlot[C1[u], {u, 0, 2 π}, PlotStyle → Blue, AxesOrigin → {0, 0}]
```



```
In[229]:= gx[u_] = C1[u][[1]];
gy[u_] = C1[u][[2]];
RevolveY[u_, θ_] := {gx[u] Cos[θ], gy[u], gx[u] Sin[θ]}
f2 = ParametricPlot3D[RevolveY[u, θ],
  {u, 0, 2 π}, {θ, 0, 2 π}, AspectRatio → Automatic, Mesh → 4,
  PlotStyle → {White, Specularity[White, 50], Opacity[0.3]}];
Show[{f1, f2, m}, PlotRange → All, AxesLabel → {x, y, z}]
```

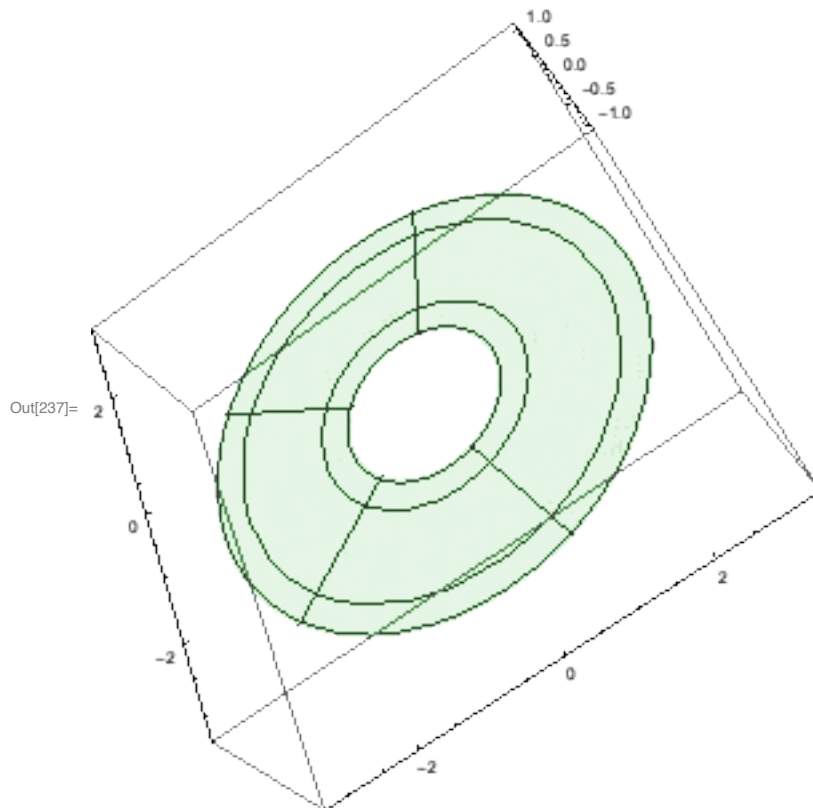


z - axis

```

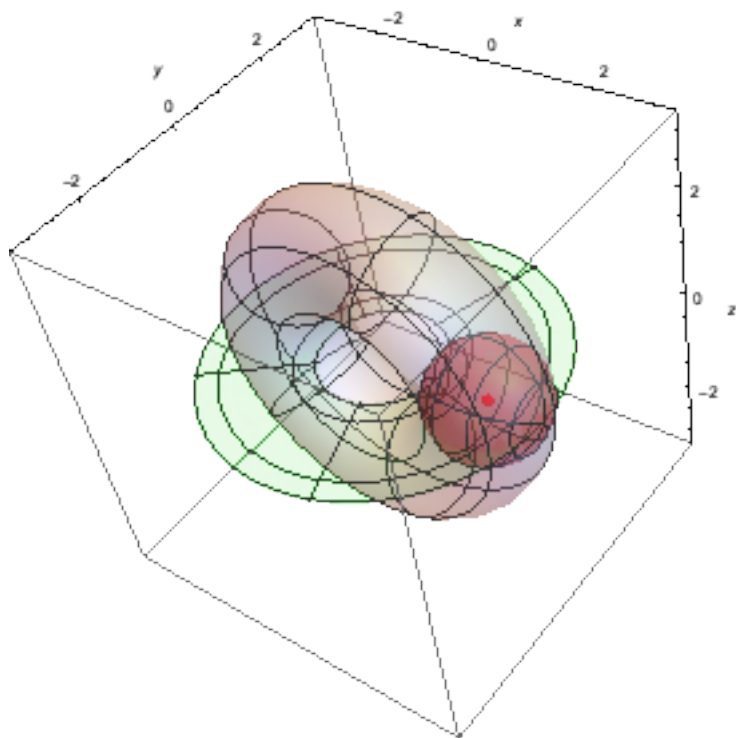
In[234]:= gx[u_] = C1[u][[1]];
          gy[u_] = C1[u][[2]];
          RevolveZ[u_,  $\theta$ _] := {gx[u] Cos[ $\theta$ ], gx[u] Sin[ $\theta$ ], 0}
          f3 = ParametricPlot3D[RevolveZ[u,  $\theta$ ], {u, 0,  $2\pi$ }, { $\theta$ , 0,  $2\pi$ },
            AspectRatio  $\rightarrow$  Automatic, Mesh  $\rightarrow$  4, PlotStyle  $\rightarrow$  {Green, Opacity[0.1]}]

```



```
In[238]:= Show[{f1, f2, f3, m}, PlotRange -> All, AxesLabel -> {x, y, z}]
```

Out[238]=



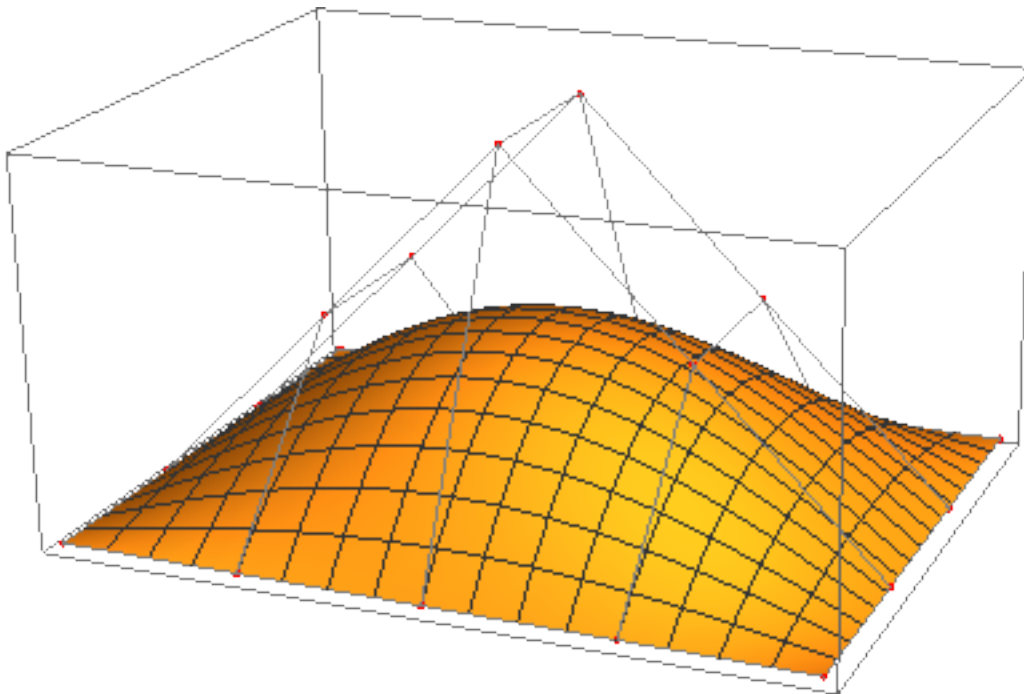
4) Tensor Product

Bezier Surface

```
In[239]:= pts = {{0, 0, 0}, {0, 1, 0}, {0, 2, 0}, {0, 3, 0}},
             {{1, 0, 0}, {1, 1, 1}, {1, 2, 1}, {1, 3, 0}},
             {{2, 0, 0}, {2, 1, 2}, {2, 2, 2}, {2, 3, 0}},
             {{3, 0, 0}, {3, 1, 1}, {3, 2, 1}, {3, 3, 0}},
             {{4, 0, 0}, {4, 1, 0}, {4, 2, 0}, {4, 3, 0}};
f = BezierFunction[pts]
Show[Graphics3D[{PointSize[Medium], Red, Map[Point, pts]}],
     Graphics3D[{Gray, Line[pts], Line[Transpose[pts]]}],
     ParametricPlot3D[f[u, v], {u, 0, 1}, {v, 0, 1}, Mesh → True]]
```

Out[240]= BezierFunction[ Argument count: 2
Output dimension: 3]

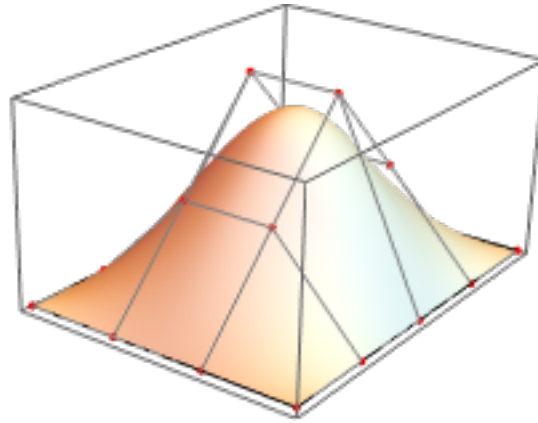
Out[241]=



B-Spline Surface

```
In[242]:= b = Graphics3D[BSplineSurface[pts, SplineDegree → 2]];
Show[Graphics3D[{PointSize[Medium], Red,
  Map[Point, pts], Gray, Line[pts], Line[Transpose[pts]]}], b]
```

Out[243]=



NURBS

```
In[272]:= pts = {{0.5, 0, -0.5}, {0, 0, -0.5}, {0, 1, -0.5},
  {0.5, 1, -0.5}, {1, 1, -0.5}, {1, 0, -0.5}, {0.5, 0, -0.5}},
  {{0.5, 0, 0.7}, {0, 0, 0.7}, {0, 1, 0.7}, {0.5, 1, 0.7},
  {1, 1, 0.7}, {1, 0, 0.7}, {0.5, 0, 0.7}},
  {{0.5, 0, 0.9}, {0, 0, 0.9}, {0, 1, 1.5}, {0.5, 1, 1.5},
  {1, 1, 1.5}, {1, 0, 0.9}, {0.5, 0, 0.9}},
  {{0.5, -0.1, 1}, {0, -0.1, 1}, {0, 0.5, 2}, {0.5, 0.5, 2},
  {1, 0.5, 2}, {1, -0.1, 1}, {0.5, -0.1, 1}},
  {{0.5, -0.3, 1}, {0, -0.3, 1}, {0, -0.3, 2}, {0.5, -0.3, 2},
  {1, -0.3, 2}, {1, -0.3, 1}, {0.5, -0.3, 1}},
  {{0.5, -1.5, 1}, {0, -1.5, 1}, {0, -1.5, 2}, {0.5, -1.5, 2},
  {1, -1.5, 2}, {1, -1.5, 1}, {0.5, -1.5, 1}}};
w = {{1, .5, .5, 1, .5, .5, 1}, {1, .5, .5, 1, .5, .5, 1}, {1, .5, .5, 1, .5, .5, 1},
  {1, .5, .5, 1, .5, .5, 1}, {1, .5, .5, 1, .5, .5, 1}, {1, .5, .5, 1, .5, .5, 1}};
uk = {0, 0, 0, 1/4, 1/2, 3/4, 1, 1, 1};
vk = {0, 0, 0, 1/4, 1/2, 1/2, 3/4, 1, 1, 1};
```



```
In[247]:= pts // MatrixForm
```

```
Out[247]//MatrixForm=
```

$$\begin{pmatrix} \begin{pmatrix} 0.5 \\ 0 \\ -0.5 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ -0.5 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ -0.5 \end{pmatrix} & \begin{pmatrix} 0.5 \\ 1 \\ -0.5 \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \\ -0.5 \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \\ -0.5 \end{pmatrix} & \begin{pmatrix} 0.5 \\ 0 \\ -0.5 \end{pmatrix} \\ \begin{pmatrix} 0.5 \\ 0 \\ 0.7 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0.7 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ 0.7 \end{pmatrix} & \begin{pmatrix} 0.5 \\ 1 \\ 0.7 \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \\ 0.7 \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \\ 0.7 \end{pmatrix} & \begin{pmatrix} 0.5 \\ 0 \\ 0.7 \end{pmatrix} \\ \begin{pmatrix} 0.5 \\ 0 \\ 0.9 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0.9 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ 1.5 \end{pmatrix} & \begin{pmatrix} 0.5 \\ 1 \\ 1.5 \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \\ 1.5 \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \\ 0.9 \end{pmatrix} & \begin{pmatrix} 0.5 \\ 0 \\ 0.9 \end{pmatrix} \\ \begin{pmatrix} 0.5 \\ -0.1 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ -0.1 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ 0.5 \\ 2 \end{pmatrix} & \begin{pmatrix} 0.5 \\ 0.5 \\ 2 \end{pmatrix} & \begin{pmatrix} 1 \\ 0.5 \\ 2 \end{pmatrix} & \begin{pmatrix} 1 \\ -0.1 \\ 1 \end{pmatrix} & \begin{pmatrix} 0.5 \\ -0.1 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 0.5 \\ -0.3 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ -0.3 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ -0.3 \\ 2 \end{pmatrix} & \begin{pmatrix} 0.5 \\ -0.3 \\ 2 \end{pmatrix} & \begin{pmatrix} 1 \\ -0.3 \\ 2 \end{pmatrix} & \begin{pmatrix} 1 \\ -0.3 \\ 1 \end{pmatrix} & \begin{pmatrix} 0.5 \\ -0.3 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 0.5 \\ -1.5 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ -1.5 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ -1.5 \\ 2 \end{pmatrix} & \begin{pmatrix} 0.5 \\ -1.5 \\ 2 \end{pmatrix} & \begin{pmatrix} 1 \\ -1.5 \\ 2 \end{pmatrix} & \begin{pmatrix} 1 \\ -1.5 \\ 1 \end{pmatrix} & \begin{pmatrix} 0.5 \\ -1.5 \\ 1 \end{pmatrix} \end{pmatrix}$$

```

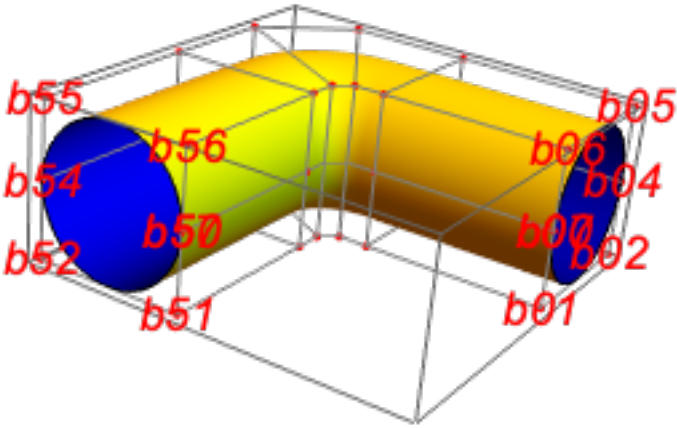
In[275]:= a = Graphics3D[{
  FaceForm[Yellow, Blue],
  BSplineSurface[pts, SplineKnots → {uk, vk}, SplineDegree → 2, SplineWeights → w,
    SplineClosed → {False, True}]], ViewPoint → {Right, Front}, Boxed → False];
Show[Graphics3D[{PointSize[Medium], Red, Map[Point, pts],
  Gray, Line[pts], Line[Transpose[pts]],
  Text[Style["b00", Red, Italic, 24], pts[[1, 1]]],
  Text[Style["b01", Red, Italic, 24], pts[[1, 2]]],
  Text[Style["b02", Red, Italic, 24], pts[[1, 3]]],
  Text[Style["b04", Red, Italic, 24], pts[[1, 4]]],
  Text[Style["b05", Red, Italic, 24], pts[[1, 5]]],
  Text[Style["b06", Red, Italic, 24], pts[[1, 6]]],
  Text[Style["b07", Red, Italic, 24], pts[[1, 7]]],

  (*Text[Style["b10", Red, Italic, 24], pts[[2, 1]]],
  Text[Style["b11", Red, Italic, 24], pts[[2, 2]]],
  Text[Style["b12", Red, Italic, 24], pts[[2, 3]]],
  Text[Style["b14", Red, Italic, 24], pts[[2, 4]]],
  Text[Style["b15", Red, Italic, 24], pts[[2, 5]]],
  Text[Style["b16", Red, Italic, 24], pts[[2, 6]]],
  Text[Style["b17", Red, Italic, 24], pts[[2, 7]]], *)

  Text[Style["b50", Red, Italic, 24], pts[[6, 1]]],
  Text[Style["b51", Red, Italic, 24], pts[[6, 2]]],
  Text[Style["b52", Red, Italic, 24], pts[[6, 3]]],
  Text[Style["b54", Red, Italic, 24], pts[[6, 4]]],
  Text[Style["b55", Red, Italic, 24], pts[[6, 5]]],
  Text[Style["b56", Red, Italic, 24], pts[[6, 6]]],
  Text[Style["b57", Red, Italic, 24], pts[[6, 7]]]
}], a]

```

Out[276]=



```

In[277]:= Manipulate[
  Show[Graphics3D[{PointSize[Medium], Red, Map[Point, pts],
    Gray, Line[pts], Line[Transpose[pts]]}], Graphics3D[{
    FaceForm[Yellow, Blue],
    BSplineSurface[pts, SplineKnots → {uk, vk}, SplineDegree → 2, SplineWeights →
      {{1, a1, a2, a3, a4, .5, 1}, {1, a1, a2, a3, a4, .5, 1}, {1, a1, a2, a3, a4, .5, 1},
      {1, a1, a2, a3, a4, .5, 1}, {1, a1, a2, a3, a4, .5, 1}, {1, a1, a2, a3, a4, .5, 1}},
      SplineClosed → {False, True}]], ViewPoint → {Right, Front}, Boxed → False]],
  {a1, 0.1, 1}, {a2, 0.1, 1}, {a3, 0.1, 1}, {a4, 0.1, 1}]

```

Out[277]=

