Exercise 3.27

Maximise:

$$Z = 3x_1 + 2x_2$$

s.t.

$$2x_1 + x_2 \leq 100,$$
$$x_1 + x_2 \leq 80$$
$$x_1 \leq 40$$
$$x_1, x_2 \geq 0.$$

to normal form:

$$z - 3x_1 - 2x_2 = 0$$

s.t.

$$2x_1 + x_2 + s_1 = 100,$$
$$x_1 + x_2 + s_2 = 80,$$
$$x_1 + s_3 = 40$$
$$x_1, x_2 \geq 0$$
$$s_1, s_2, s_3 \geq 0.$$

to matrix form / Simplex Tableau

z	x_1	x_2	s_1	s_2	s_3	b
1	-3	-2	0	0	0	0
0	2	1	1	0	0	100
0	1	1	0	1	0	80
0	1	0	0	0	1	40

column 1 ↓

$$\Rightarrow \left[\begin{array}{cccccc|c} 1 & \boxed{-3} & -2 & 0 & 0 & 0 & 0 \\ 0 & (2) & 1 & 1 & 0 & 0 & 100 \\ 0 & (1) & 1 & 0 & 1 & 0 & 80 \\ 0 & (1) & 0 & 0 & 0 & 1 & 40 \end{array}\right] \begin{array}{l} \\ \leftarrow \frac{100}{2} = 50 \\ \leftarrow \frac{80}{1} = 80 \\ \leftarrow \frac{40}{1} = 40. \end{array}$$

smallest +ve value: PIVOT: (R4)

First iteration

$$\left[\begin{array}{cccccc|c} 1 & 0 & -2 & 0 & 0 & 3 & 120 \\ 0 & 0 & 1 & 1 & 0 & -2 & 20 \\ 0 & 0 & 1 & 0 & 1 & -1 & 40 \\ 0 & 1 & 0 & 0 & 0 & 1 & 40 \end{array}\right] \begin{array}{l} R_1 + 3R_4 \\ : R_2 - 2R_4 \\ \\ R_3 - R_4 \end{array}$$

$$2R_4: \quad 0 \quad 2 \quad 0 \quad 0 \quad 0 \quad 2 \quad | \quad 80$$

$$3R_4: \quad 0 \quad 3 \quad 0 \quad 0 \quad 0 \quad 3 \quad | \quad 120$$

Basic Feasible Solution

		x_1	x_2	s_1	s_2	s_3	
R_1	1	0	-2	0	0	3	120
R_2	0	0	1	1	0	-2	20
R_3	0	0	1	0	1	-1	40
R_4	0	1	0	0	0	1	40

basic variables : (x_1, s_1, s_2) , nonbasic variable : $(x_2, s_3) = (0,0)$

R_4 : $x_1 + s_3^{=0} = 40 \Rightarrow x_1 = 40$ } both yields $Z = 120$.

~~R_3 : $x_2 + \cdots$~~ nonbasic $\Rightarrow x_2 = 0$ }

$\Rightarrow$ check $Z = 3x_1 + 2x_2$

$= 3(40) + 2(0)$

$= 120.$

still there is an entry -ve in row 1, so we do another iteration.

column 2

1	0	-2	0	0	+3	120
0	0	1	1	0	-2	20 ← $\frac{20}{1} = 20$
0	0	1	0	1	-1	40 ← $\frac{40}{1} = 40$
0	1	0	0	0	1	40 ← $\frac{40}{0} = !$

+ve

Smallest $\frac{x}{y}$ value : PIVOT (R_2).

$$\left[\begin{array}{cccccc|c} 1 & 0 & -2 & 0 & 0 & 3 & 120 \\ 0 & 0 & \boxed{1} & 1 & 0 & -2 & 20 \\ 0 & 0 & 1 & 0 & 1 & -1 & 40 \\ 0 & 1 & 0 & 0 & 0 & 1 & 40 \end{array}\right] \leftarrow \text{PIVOT : } R_2$$

Second iteration

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 2 & 0 & -1 & 160 \\ 0 & 0 & 1 & 1 & 0 & -2 & 20 \\ 0 & 0 & 0 & -1 & 1 & 1 & 20 \\ 0 & 1 & 0 & 0 & 0 & 1 & 40 \end{array}\right] \begin{array}{l} \leftarrow R_1 + 2R_2 \\ \\ \leftarrow R_3 - R_2 \\ \leftarrow \text{no changes, pivot entry already zero.} \end{array}$$

$2R_2$: 0 0 2 2 0 −4 40

Basic Feasible solution

	x_1	x_2	s_1	s_2	s_3	
	1	0	0	2	0	−1	160
	0	0	1	1	0	−2	20
	0	0	0	−1	1	−3	20
	0	1	0	0	0	1	40
			$s_1=0$		$s_3=0$		

R_4 : $x_1 + s_3^{//0} = 40 \Rightarrow x_1 = 40$.

R_2 : $x_2 + s_1^{//0} + s_3^{//0} = 20 \Rightarrow x_2 = 20$.

} both yields 160

Check : $Z = 3x_1 + 2x_2$

$= 3(40) + 2(20)$

$= 120 + 40$.

First row has $s_3 = -1$: negative entry! once more!

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 2 & 0 & 160 \\ 0 & 0 & 1 & 1 & 0 & 20 \\ 0 & 0 & 0 & -1 & 1 & 20 \\ 0 & 1 & 0 & 0 & 0 & 40 \end{array}\right] \begin{array}{l} \\ \frac{20}{-2} = -10 \leftarrow \text{-ve value!} \\ \frac{20}{1} = 20 \\ \frac{40}{1} = 40 \end{array}$$

Smallest the value : PIVOT R_3

Third iteration

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 1 & 180 \\ 0 & 0 & 1 & -1 & 2 & 60 \\ 0 & 0 & 0 & -1 & 1 & 20 \\ 0 & 1 & 0 & 1 & -1 & 20 \\ 2R_3: 0 & 0 & 0 & -2 & 2 & 40 \end{array}\right] \begin{array}{l} R_1 + R_3 \\ 2R_3 + R_2 \\ \\ R_4 - R_3 \\ \end{array}$$

Basic Feasible Solution

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 1 & 180 \\ 0 & 0 & 1 & -1 & 2 & 60 \\ 0 & 0 & 0 & -1 & 1 & 20 \\ 0 & 1 & 0 & 1 & -1 & 40 \end{array}\right]$$

x_1 x_2 s_1 s_2 s_3

basic variables

non basic $s_1 = 0$, $s_2 = 0$

R_4 : $x_1 + s_1^{=0} + s_2^{=0} = 40 \Rightarrow x_1 = 40$

R_2 : $x_2 + s_1 + s_2 = 60 \Rightarrow x_2 = 60$

} both yields (180)

Check: $Z = 3x_1 + 2x_2$

$= 3(40) + 2(60)$

$= 60 + 120 = 180$ #