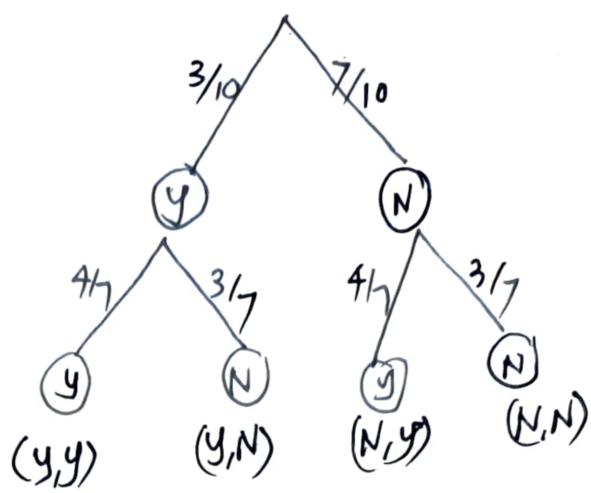
Quick Check 1.4 (pg: 46)

(18) draw tree



← Dina

← Hafiz

(a) Both attend : $P(y,y) = P(y) \times P(y)$

$= \frac{3}{10} \times \frac{4}{7} = 12/70$

(b) No attendence : $P(N,N) = P(N) \times P(N)$

$= \frac{7}{10} \times \frac{3}{7} = \frac{21}{70}$

(c) only one attend : $P(y,N) + P(N,y)$

$P(y,N) = P(y) \times P(N)$

$\frac{3}{10} \times \frac{3}{7} = \frac{9}{70}$

$P(N,y) = P(N) \times P(y)$

$= \frac{7}{10} \times \frac{4}{7} = \frac{28}{70}$

$\therefore \; P(y,N) + P(N,y) = \frac{9}{70} + \frac{28}{70} = \frac{37}{70}.$

(d) at least one attend : $P(y,y) + P(y,N) + P(N,y)$

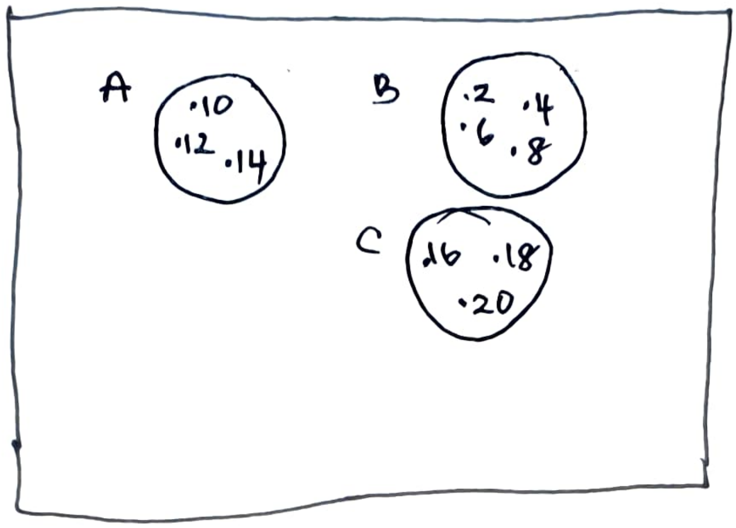
$= \frac{12}{70} + \frac{37}{70}$

$= \frac{49}{70}.$

Quick check 1·4 (pg·47)

20 $S = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$A = \{10, 12, 14\}$ $B = \{2, 4, 6, 8\}$ $C = \{16, 18, 20\}$.



a) set A or B : $A \cup B = \{2, 4, 6, 8, 10, 12, 14\}$

b) set A or B or C : $A \cup B \cup C = S$

Review Questions

13. $n(S) = 40$ students.

A : picking male

B : picking female

$P(A) = \frac{3}{10}$, we know $P(A) = \frac{n(A)}{n(S)}$.

$\Rightarrow \frac{3}{10} = \frac{n(A)}{40}$

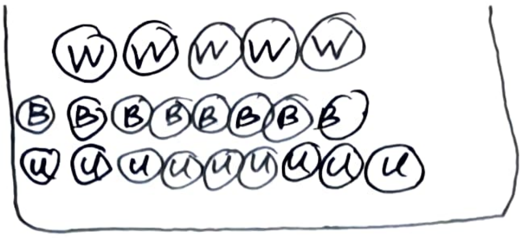
$\therefore n(A) = \frac{120}{10} = 12$ male students.

$\therefore n(B) = n(S) - n(A)$

$= 40 - 12$

$= 28$ female students.

(14)



$n(W) = 5$: white

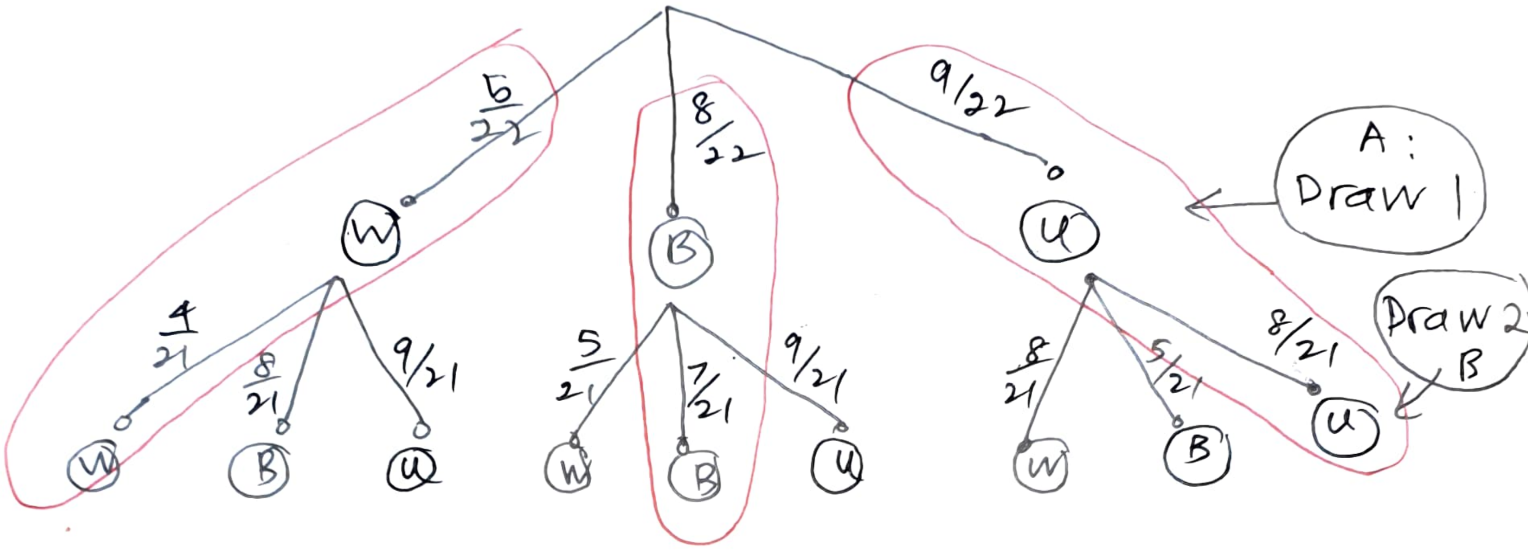
$n(B) = 8$: black

$n(u) = 9$: blue.

$n(S) = 22.$

dependent event

Tree graph approach



$$P_W(A \cap B) = P_W(A) \times P_W(B|A)$$

$$= \frac{5}{22} \cdot \frac{4}{21} = \frac{20}{462}$$

$$P_B(A \cap B) = P_B(A) \times P_B(B|A)$$

$$= \frac{8}{22} \times \frac{7}{21} = \frac{56}{462}.$$

$$P_u(A \cap B) = P_u(A) \times P_u(B|A)$$

$$= \frac{9}{22} \cdot \frac{8}{21} = \frac{72}{462}$$

mutually exclusive.

$$P_W(A \cap B) + P_B(A \cap B) + P_u(A \cup B)$$

$$= \frac{20 + 56 + 72}{462}$$

$$= \frac{148}{462}$$

$$= \frac{74}{231}.$$

(15) $n(B) = 10$: Blue marbles.

$n(Y) = x$: yellow marbles

$n(R) = y$: red marbles.

$\Rightarrow n(S) = 10 + x + y$.

Given $P(B) = \frac{1}{6}$

$$P(B) = \frac{n(B)}{n(S)}$$

$$\frac{1}{6} = \frac{10}{(10 + x + y)}$$

$$10 + x + y = 60 \quad : n(S) = 60$$

$$x + y = 50 \quad \text{—— } (1).$$

Given $P(Y) = \frac{2}{3}$

$$P(Y) = \frac{n(Y)}{n(S)}$$

$$\frac{2}{3} = \frac{x}{60}$$

$$x = \frac{120}{3} = 40. \quad \text{—— } (2).$$

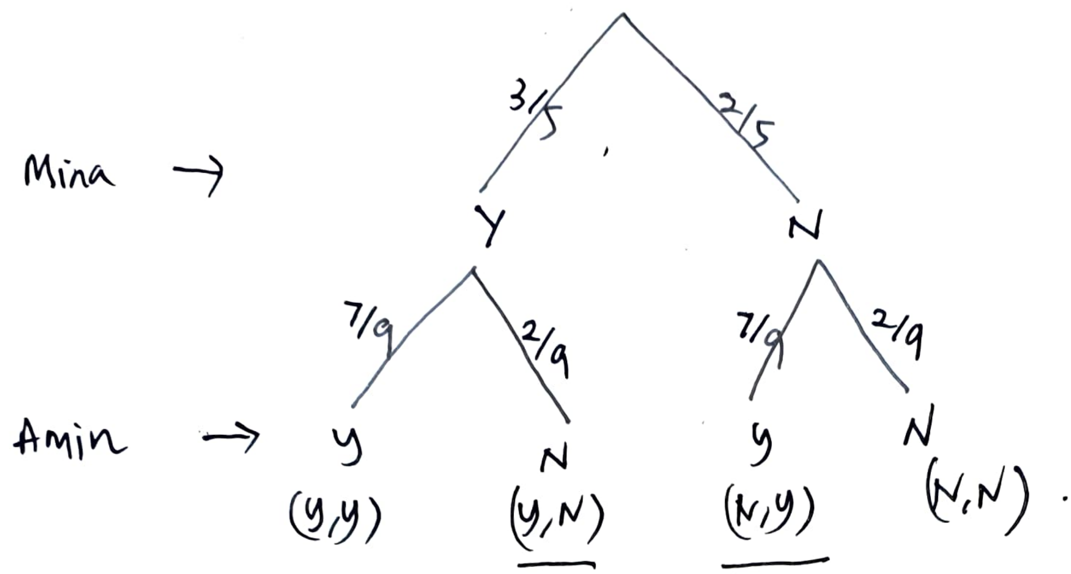
$\therefore$ from (1) and (2)

$$x + y = 50$$

$$y = 50 - x$$

$$= 10 \text{ red marbles.}$$

(16)



only one of them chosen : $P(y,N) + P(N,y)$

$P(y,N) = P(y) \times P(N)$

$= \frac{3}{5} \times \frac{2}{9}$

$= \frac{6}{45}$

$P(N,y) = P(N) \times P(y)$

$= \frac{2}{5} \times \frac{7}{9}$

$= \frac{14}{45}$

$\therefore P(y,N) + P(N,y) = \frac{6+14}{45}$

$= \frac{20}{45}$

$= \frac{4}{9}.$

17 $n(S) = 30$

$n(O) = 10$
$n(A) = 5$
$n(B) = 8$
$n(AB) = 7$

$n(S) = 30$

(a) $P(A \cup B) = \dfrac{n(A \cup B)}{n(S)} = \dfrac{5+8}{30}$

$= \dfrac{13}{30}$

(b) $P(A \cup O) = \dfrac{n(A \cup O)}{n(S)} = \dfrac{15}{30}$.

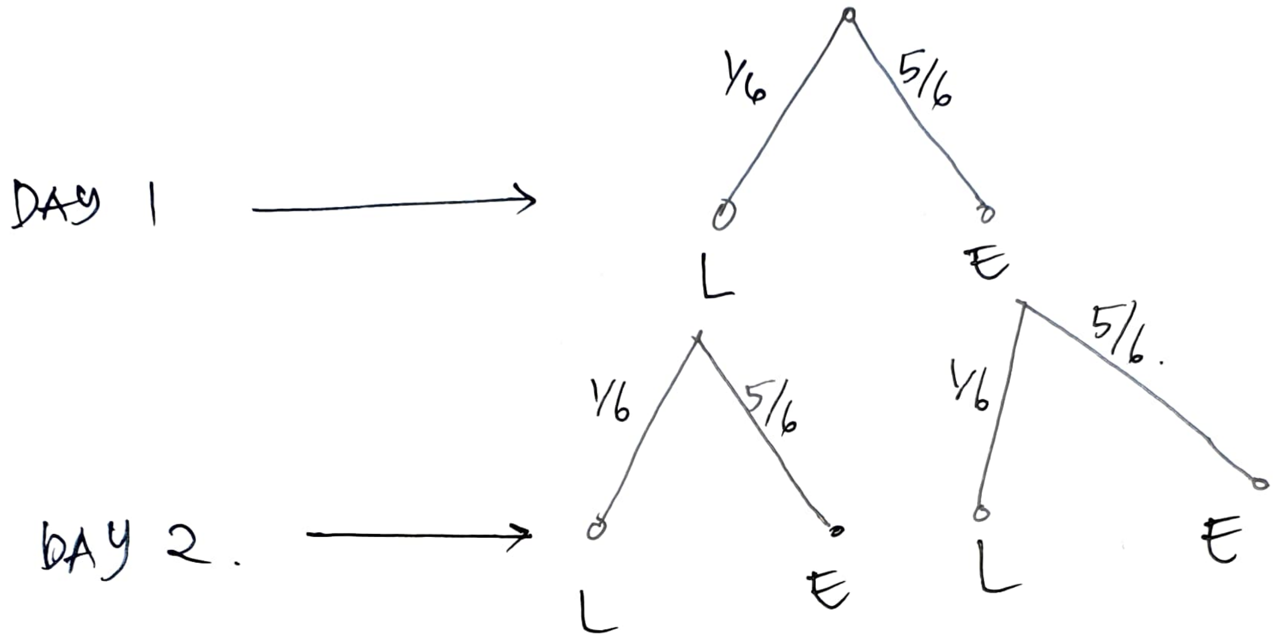
$\Rightarrow P((A \cup O)') = 1 - P(A \cup O)$

$= 1 - 15/30$

$= 15/30$.

(18) $P(L) = 1/6$: late
$P(E) = 5/6$: early

independent events



$S = \{(L,L), (L,E), (E,L), (E,E)\}.$

a)

at least once $= \{(L,L), (L,E), (E,L)\}$

A: $P_1(L) \cdot P_2(L) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}.$

B: $P_1(L) \cdot P(E) = \frac{1}{6} \cdot \frac{5}{6} = \frac{5}{36}.$

C: $P_1(E) \cdot P(L) = \frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}.$

mutually exclusive.

$P(A) + P(B) + P(C) = 1/36 + 5/36 + 5/36$

$= 11/36.$

b). never late in 2 days.

(E,E) : $P_1(E) \cdot P_2(E)$

$= \frac{5}{6} \times \frac{5}{6}$

$= \frac{25}{36}.$

(19) a) dice is thrown

$S = \{1, 2, 3, 4, 5, 6\}$.

b). a card is chosen at random from a set of cards numbered 1 to 12

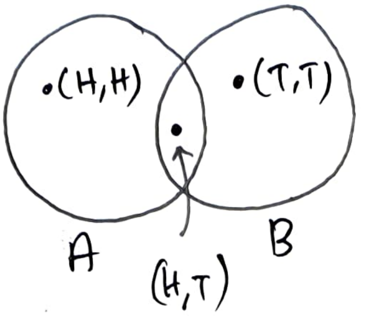
$S = \{1, \ldots, 12\}$.

(20). $S = (A_i, B_i)$, $A_i, B_i \in \{H, T\}$

$\Rightarrow S = \{(H,H), (H,T), (T,H), (T,T)\}$.

$A = \{(H,H), (H,T)\}$

$B = \{(H,T), (T,T)\}$.

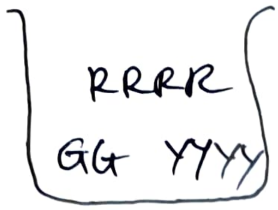


$A \cup B = \{(H,H), (H,T), (T,T)\}$.

$P(A \cup B) = \dfrac{n(A \cup B)}{n(S)}$

$= \dfrac{3}{4}$

(21)



$n(S) = 10$ balls.

A: green drawn

B: Yellow drawn.

C: Red drawn

(a). Probability of

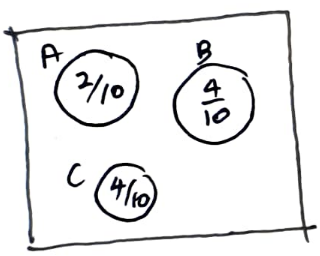
(i) event A : $n(A) = 2$.

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{10}$$

(ii) event B : $n(B) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{10}.$$

(b).



$A \cap B = \phi$

$\therefore$ A and B are mutually exclusive.

(c). $P(A \cup B)$

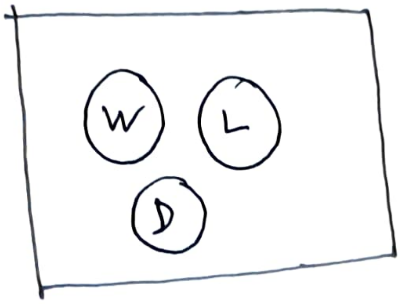
$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{2}{10} + \frac{4}{10}$$

$$= \frac{6}{10} = \frac{3}{5}$$

(22) Games of winning (W), draw (D) or losing (L)

for A:



$$P(S) = 1 = P(W) + P(L) + P(D).$$

given $P_A(W) = 0.2$, $P_B(W) = 0.5$

$\Rightarrow$ means $P_A(L) = 0.5$

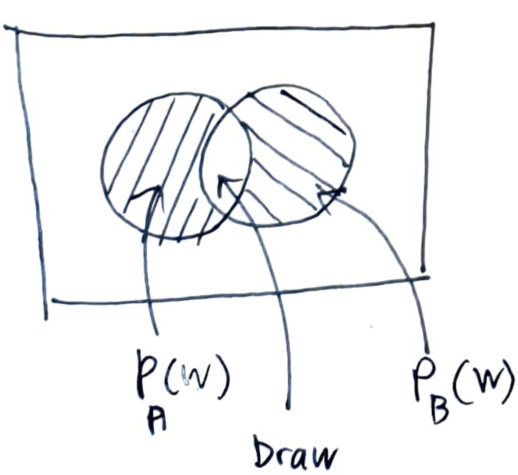
(a) $\therefore P_D = P_A(S) - P_A(W) + P_A(L)$

$$= 1 - (0.2 + 0.5)$$

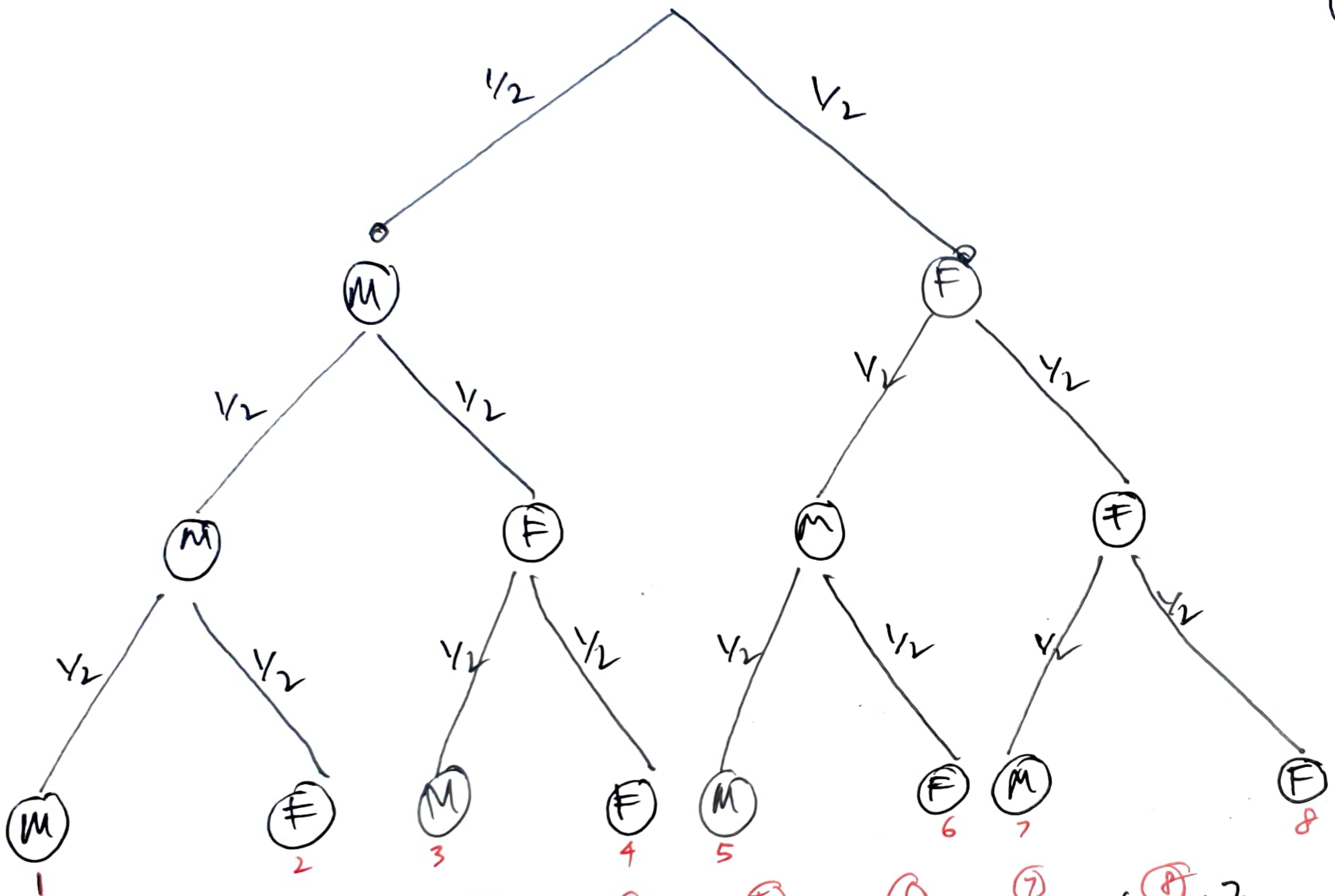
$$= 0.3.$$

(b). $P_A(W) = 0.2$ or $P_B(W) = 0.5$

<u>mutually distinct</u> $\therefore P_A(W) + P_B(W) = 0.7.$



(23)



$$S = \{\overset{1}{(M,M,M)}, \overset{2}{(M,M,F)}, \overset{3}{(M,F,M)}, \overset{4}{(M,F,F)}, \overset{5}{(F,M,M)}, \overset{6}{(F,M,F)}, \overset{7}{(F,F,M)}, \overset{8}{(F,F,F)}\}.$$

(a) Two boys and one girl: $\{2, 3, 5\}$

$$\left.\begin{aligned} P_1(2) &= \tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2} = \tfrac{1}{8} \\ P_2(3) &= \tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2} = \tfrac{1}{8} \\ P_3(5) &= \tfrac{1}{2}\cdot\tfrac{1}{2}\cdot\tfrac{1}{2} = \tfrac{1}{8} \end{aligned}\right\} \begin{aligned} &P_1(2) + P_2(3) + P_3(5) \\ &= \left(\tfrac{3}{8}\right) \end{aligned}$$

(b) at least one boy:

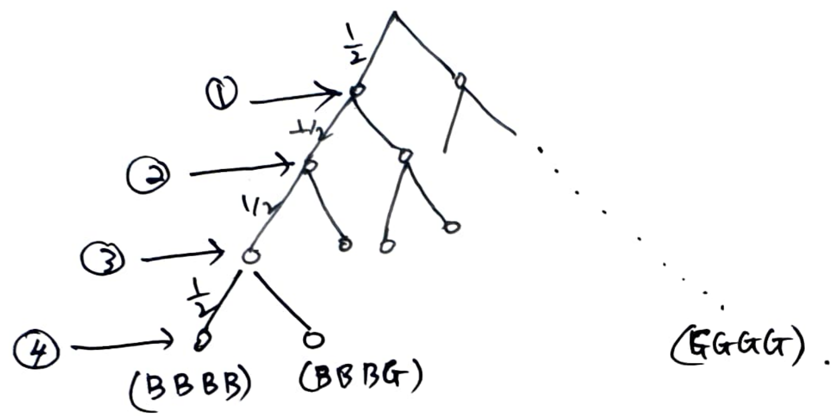
$$\{1, 2, 3, 4, 5, 6, 7\} = B$$

$$P_i(B) = 7 \times \left(\frac{1}{8}\right)$$

$$= \frac{7}{8}$$

(24) $n(S) = 4 \Rightarrow$ (2 choices, 2 choices, 2 choices, 2 choices)

$\Rightarrow 2 \times 2 \times 2 \times 2 = 2^4 = 16$ choices.



(a) P (B,B,G,G) $\Rightarrow$ occur only once.

$\Rightarrow \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{1}{2^4}$

$= \frac{1}{16}$.

(b) P (B B B G) $\Rightarrow$ occur only once too

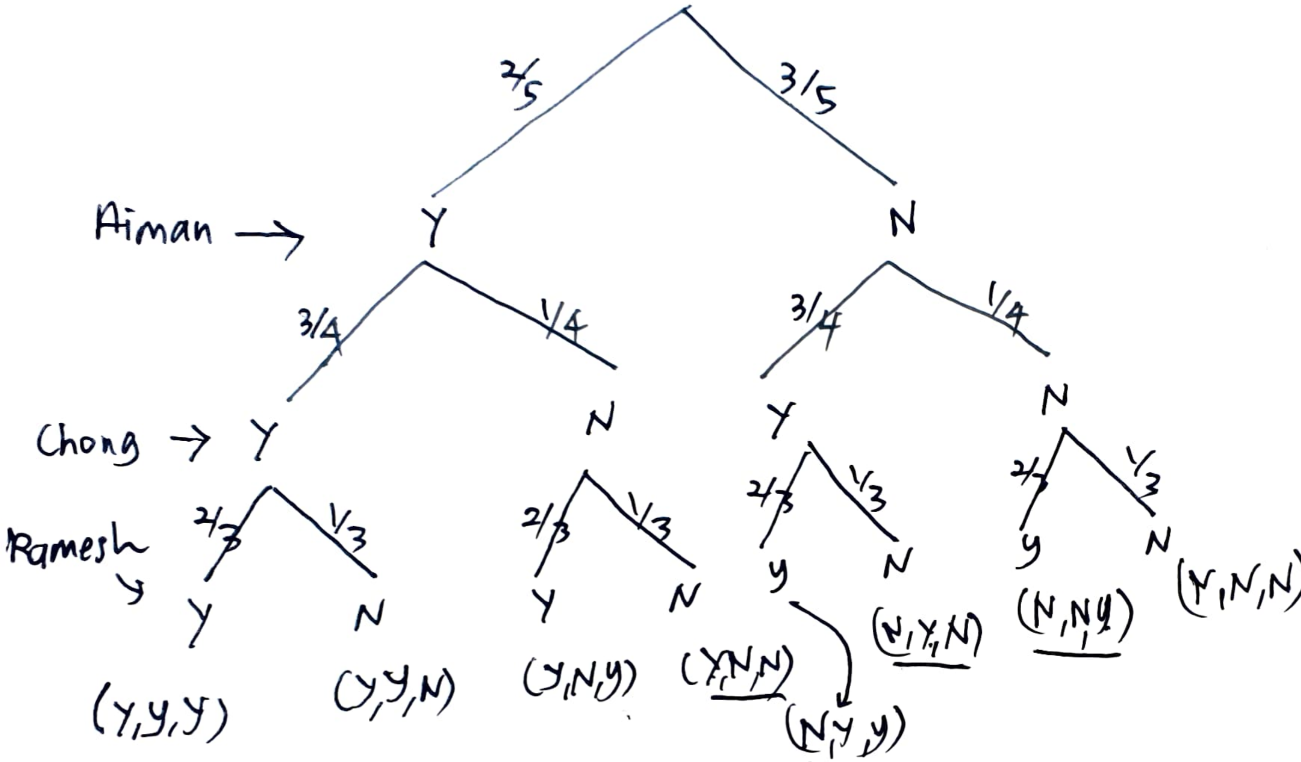
$\Rightarrow \frac{1}{2^4} = \frac{1}{16}$.

(c). $P(B, \_, \_, \_) = 1 - P(GGGG)$ occur once

$= 1 - \frac{1}{16}$

$= 15/16$.

(25)



a) all hit target : $P(y,y,y) = P(y) \times P(y) \times P(y)$

$= \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3}$

$= \frac{1}{5}$.

b) only one hit : $P(y,N,N) + P(NyN) + P(NNy$

$P(y,N,N) = (\frac{2}{5})(\frac{1}{4})(\frac{1}{3}) = \frac{2}{60}$

$P(N,y,N) = (\frac{3}{5})(\frac{3}{4})(\frac{1}{3}) = \frac{9}{60}$

$P(N,N,y) = (\frac{3}{5})(\frac{1}{4})(\frac{2}{3}) = \frac{6}{60}$

Total $= \frac{+9+6}{60} = \frac{17}{60}$

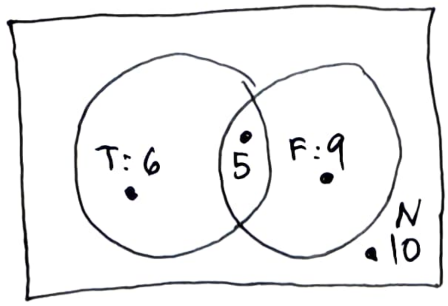
c) at least one hit the target.

$1 - P(N,N,N) = 1 - (\frac{\cancel{3}}{5} \times \frac{1}{4} \times \frac{1}{\cancel{3}})$

$= 1 - \frac{1}{20}$

$= \frac{19}{20}$.

(26)



T : Tennis
F : Football.
N : not T or F

Joint

a) Total nu. of students : $T = 6$, $T \& F = 5$, $F : 9$, $N : 10$

$$\text{Total} = (6 + 5 + 9) + 10$$
$$= 20 \text{ students} + 10 \text{ students}$$
$$S = 30 \ \#$$

b). Students only play tennis : $T = 6$

c) $P(T \cap F) = ?$

$T \cap F = 5$, $S = 30$

$$\Rightarrow P(T \cap F) = 5/30 = \frac{1}{6}$$

d) $P(T \cup F) = ?$

$$T \cup F = 6 + 5 + 9$$
$$= 20$$

$$P(T \cup F) = 20/30 = 2/3.$$

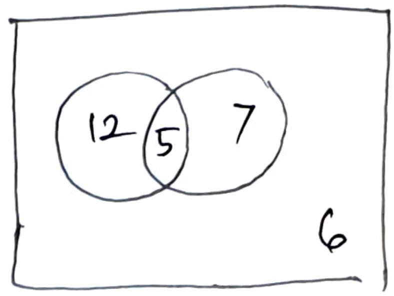
e) $P((T \cup F)') = ?$

$$P((T \cup F)') = 1 - P(T \cup F)$$
$$= 1 - 2/3$$
$$= 1/3.$$

$P(A') = 1 - P(A).$

(27)

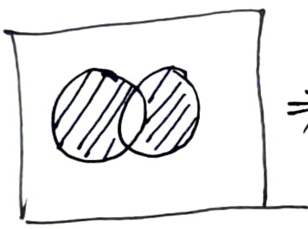
$$\left.\begin{aligned} n(S) &= 30 \\ n(V) &= 17 \\ n(W) &= 12 \\ n(V \cap W) &= 5 \end{aligned}\right\}$$



a) $P(V) = \frac{17}{30}$

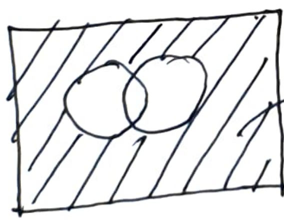
b) $P(W) = \frac{12}{30}$

c)



$\Rightarrow \frac{12+7}{30} = \frac{19}{30}$.

d)



$\frac{6}{30}$