Law of addition (two events).
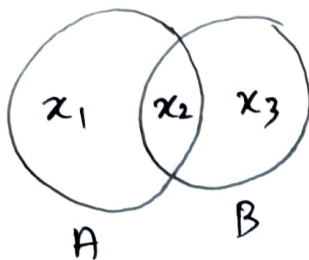

joint :

$$P(A \cup B) = P(A) + P(B) + P(A \cap B)$$

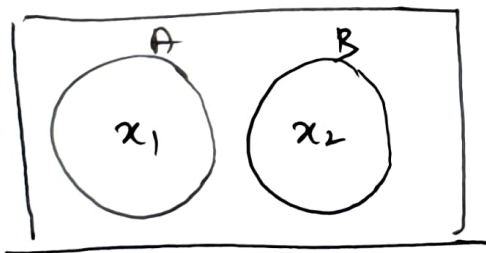$$P(A) = x_1 + x_2$$

$$P(A \cap B) = x_2.$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = (x_1 + \cancel{x_2}) - (\cancel{x_2})$$
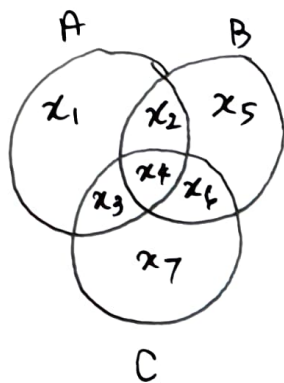$$+ P(B)$$

$$P(B) + x_1 = x_1 + x_2 + x_3$$

$$P(B) = x_2 + x_3 \quad \#.$$

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disjoint = mutual exclusive

$$P(A) + P(B) = P(A \cup B).$$

$$P(A) = x_1 + x_2 + x_3 + x_4$$
$$P(B) = x_2 + x_4 + x_5 + x_6$$
$$P(C) = x_3 + x_4 + x_6 + x_7$$

~~$P(A \cup B) = x_2 + x_4$~~

~~$P(A \cap C) = x_3 + ?$~~

$$P(A \cap B) = x_2 + x_4$$
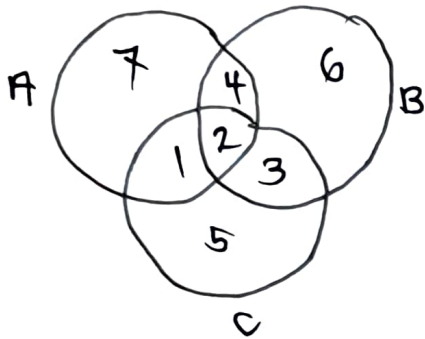$$P(A \cap C) = x_3 + x_4$$
$$P(B \cap C) = x_6 + x_4$$

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
$P(A)$	+	1	1	1	1			
$P(B)$	+		1		1	1	1	
$P(C)$	+			1	1		1	1
$P(A \cap B)$	−		−1		−1			
$P(A \cap C)$	−			−1	−1			
$P(B \cap C)$	−				−1		−1	
Total		1	1	1	0	1	1	1

↑
$P(A \cap B \cap C)$

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C)$$
$$- [P(A \cap B) + P(A \cap C) + P(B \cap C)].$$

Numerical example

$n(S) = 1+2+3+4+5+6+7 = 28$
$\qquad = n(A \cup B \cup C) \; ——①$

$n(A) = 7+4+2+1$
$\qquad = 14$
$n(B) = 6+4+3+2$
$\qquad = 15$
$n(C) = 5+3+2+1$
$\qquad = 11.$

$n(A \cap B) = 4+2 = 6$
$n(A \cap C) = 1+2 = 3$
$n(B \cap C) = 2+3 = 5$

*Based on number of elements.
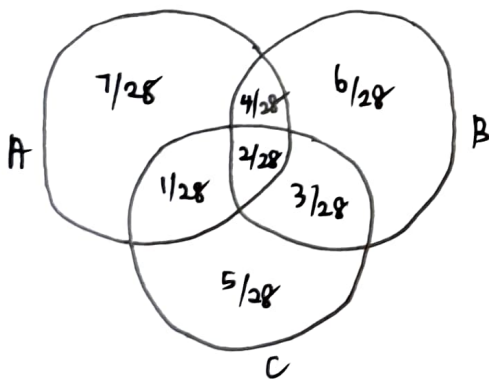
* in probability.

$P(A) = 14/28$
$P(B) = 15/28$
$P(C) = 11/28$
$P(A \cap B) = 6/28$
$P(A \cap C) = 3/28$
$P(B \cap C) = 5/28$
$P(A \cap B \cap C) = 2/28$

FROM ①:

$P(A \cup B \cup C) = \dfrac{n(A \cup B \cup C)}{n(S)}$

$\qquad = \dfrac{28}{28}$

$P(A \cup B \cup C) = 1 \; ——②$

From derived equation of 3 events:

$P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C) - P(A \cap B) - P(A \cap C) - P(B \cap C)$

$\qquad = \dfrac{14}{28} + \dfrac{15}{28} + \dfrac{11}{28} + \dfrac{2}{28} - \dfrac{6}{28} - \dfrac{3}{28} - \dfrac{5}{28}$

$\qquad = \dfrac{42}{28} - \dfrac{14}{28}$

$\qquad = \dfrac{28}{28}$

$\qquad = 1 \quad \# \quad$ same as shown in ②