Mathematica: The Basics

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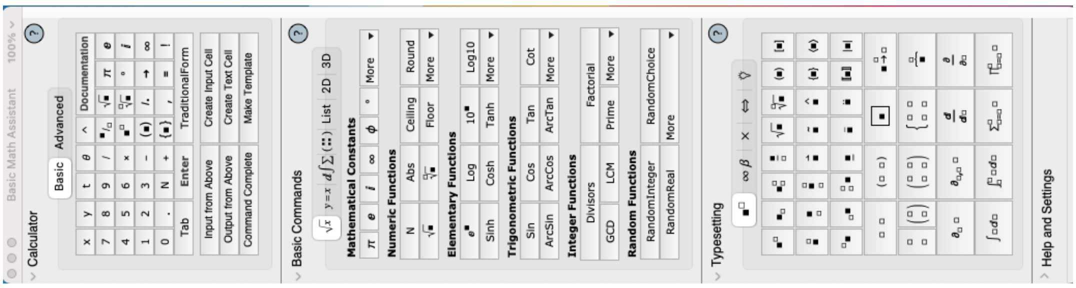
References:

1. Michael Trott , The Mathematica GuideBook for Graphics (2004, Springer-Verlag New York)
2. Michael Trott - The Mathematica Guidebook for Programming (2005, Springer)
3. Michael Trott - The Mathematica GuideBook for Symbolics (2005, Springer)

Mathematica Syntax

Basic operation:
- Shift+Enter   to run/evaluate
- Enter to move to a new line in the same cell (a cell in indicated by a single vertical - bar/bracket on the right hand side)
- Click on an empty space or down button on your keyboard to create a new cell (a horizontal line will appear, just type when you see that line)
- Double click the RHS bracket to collapse the cell
- To quickly evaluate the entire notebook, at the top menu, click Evaluation>Evaluate notebook

Palettes > BAsic Math Assistant



Clear all stored values

```
In[•]:= ClearAll["Global`*"]
```

Rules to remember

The first letter of ALL built-in functions and symbols is **ALWAYS CAPITAL LETTER.**

In[•]:=
```
Abs[-10]
Integrate[y, y]
```
Out[•]= 10

Out[•]= $\frac{y^2}{2}$

Brackets usage:
round - parenthesis
square - functions
curly - sets, range, lists, matrices

Documentation

In[•]:=
```
? Integrate
```

Out[•]=

Symbol

Integrate[f, x] gives the indefinite integral $\int f\,dx$.

Integrate[f, {x, x_{min}, x_{max}}] gives the definite integral $\int_{x_{min}}^{x_{max}} f\,dx$.

Integrate[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, …] gives the multiple integral $\int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \ldots f$.

Integrate[f, {x, y, …} ∈ reg] integrates over the geometric region reg.

In[•]:=
```
?? Integrate
```

Out[•]=

Symbol

Integrate[f, x] gives the indefinite integral $\int f\,dx$.

Integrate[f, {x, x_{min}, x_{max}}] gives the definite integral $\int_{x_{min}}^{x_{max}} f\,dx$.

Integrate[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, …] gives the multiple integral $\int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \ldots f$.

Integrate[f, {x, y, …} ∈ reg] integrates over the geometric region reg.

Documentation Local » | Web »

Options › Assumptions :→ $Assumptions … (4 total)

Attributes {Protected, ReadProtected}

Full Name System`Integrate

In[•]:=
```
Information[Integrate]
```

Out[•]=

Symbol

Integrate[f, x] gives the indefinite integral $\int f\,dx$.

Integrate[f, {x, x_{min}, x_{max}}] gives the definite integral $\int_{x_{min}}^{x_{max}} f\,dx$.

Integrate[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, …] gives the multiple integral $\int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \ldots f$.

Integrate[f, {x, y, …} ∈ reg] integrates over the geometric region reg.

Documentation Local » | Web »

Options › Assumptions :→ $Assumptions … (4 total)

Attributes {Protected, ReadProtected}

Full Name System`Integrate

Numerical Computation

In[•]:=
```
Head[3]
Head [3.3]
Head[3 + 2 I]
Head[x]
```

Out[•]= Integer

Out[•]= Real

Out[•]= Complex

Out[•]= Symbol

In[•]:=
```
Cos[Pi / 5]
```

Out[•]= $\frac{1}{4}\left(1+\sqrt{5}\right)$

In[•]:=
```
N[Cos[Pi / 5], 100]
```

Out[•]=
```
0.80901699437494742410229341718281905886015458990288143106772431135263023140945122485360360209469556 87
```

In[•]:=
```
NIntegrate[y^5, {y, 0, 3}]
```

Out[•]= 121.5

Define Variable

```
a = 20;
```

```
a
```

20

```
a^-1/2
a * 2
```

$$\frac{1}{2\sqrt{5}}$$

```
40
```

```
b = Cos[3] + Log[2.6] + a
19.9655
```

```
a + b
39.9655
```

A defined variable will continue to hold its value (globally: even in a different notebook) until you change or clear it.

Calling previous values

```
In[•]:= Clear[a, b, c, d]
In[•]:= a = 4 + b;
In[•]:= c = 5 + b;
In[•]:= d = b;
In[•]:= {%, %%}
Out[•]= {b, 5 + b}
In[•]:= %218
Out[•]= 4 + b
```

To clear a variable or all variables

```
ClearAll[a]
a
a
```

```
a = 20
ClearAll[a, b]
a
b
20
a
b
```

```
a = 2; b = -3; c = 9;(*use ";" to suppress outputs*)
a
b
c
```

```
2
-3
9
```

```
ClearAll["Global`*"](*Clear all the values*)
a
b
c
```

```
a
b
c
```

Arithmetic Operations

```
1 + 2/3 - 8 * 9
```

$-\frac{211}{3}$

```
9.7^1.2
```

```
15.2801
```

Some notes:

1) multiplication can be written in a few different ways: */space/()

```
a b
a * b
a (b)
```

```
a b
a b
a b
```

2) ways to write power: Shift+6 or Ctrl+6

3) ways to write divide: / or Ctrl+/

4) to write square root: use function “Sqrt[]” or Ctrl+2

Some basic built in functions

Three ways calling built-in function

In[•]:=
```
{Abs[-10], Abs@-10, -10 // Abs}
```
Out[•]=
```
{10, 10, 10}
```

In[•]:=
```
N[1/3] (*default significant figures = 6*)
N[π, 8] (*Shows π value with 8 significant figures*)
```
Out[•]=
```
0.333333
```
Out[•]=
```
3.1415927
```

```
Exp[3] (*Exponent*)
Log[3]
Log[3] // N
N[Log[3]]
```
e^3

```
Log[3]
```
```
1.09861
```
```
1.09861
```

Trigonometry

Mathematica uses radians by default

```
Cos[30] // N
```
```
0.154251
```

use “Degree” to convert values to radians, i.e. 30 Degree to radians:

```
Cos[30 Degree] // N
```
```
0.866025
```

In[•]:=
```
{Cos[30 Degree], Sin[30 Degree], Tan[30 Degree]}
```
Out[•]=
$$\left\{\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{1}{\sqrt{3}}\right\}$$

Symbolic Computations

Algebraic operation

In[•]:=
```
Head[x]
```
Out[•]=
```
Symbol
```

In[•]:=
```
Clear[a, b]
b + a + a
```
Out[•]= $2 a + b$

In[•]:=
```
ReplaceAll[2 a + b, {a → 1, b → 0}]
```
Out[•]= 2

In[•]:=
```
2 a + b /. {a → 1, b → 0}
```
Out[•]= 2

In[•]:= $3 a + 1.3 b + \frac{6}{7} b$

Out[•]= $3 a + 2.15714 b$

In[•]:= $g = \text{Factor}\left[\frac{1}{1024} \left(-768 t \text{Cos}[2 t] + 48 t \text{Cos}[4 t] + 384 \left(-1 + 2 t^2\right) \text{Sin}[2 t] - 12 \left(-1 + 8 t^2\right) \text{Sin}[4 t] + 4 t \left(72 t^2 + 2 \left(96 - 64 t^2\right) \text{Cos}[2 t] + 4 \left(-3 + 8 t^2\right) \text{Cos}[4 t] - 128 t \text{Sin}[2 t] + 16 t \text{Sin}[4 t]\right) + 4 \left(24 t^3 + \left(96 - 64 t^2\right) \text{Sin}[2 t] + \left(-3 + 8 t^2\right) \text{Sin}[4 t]\right)\right)\right]$

Out[•]= $-\frac{1}{8} t^3 \left(-3 + 4 \text{Cos}[2 t] - \text{Cos}[4 t]\right)$

In[•]:=
```
Expand[g]
```
Out[•]= $\frac{3 t^3}{8} - \frac{1}{2} t^3 \text{Cos}[2 t] + \frac{1}{8} t^3 \text{Cos}[4 t]$

In[•]:=
```
? FullSimplify
```
Out[•]=

Symbol

System`FullSimplify

Documentation Local » | Web »

Options › Assumptions :→ $Assumptions … (6 total)

Attributes {Protected}

Full Name System`FullSimplify

In[•]:=
```
FullSimplify[g]
```
Out[•]= $t^3 \text{Sin}[t]^4$

In[∘]:= FullSimplify[25 q^2 + 67 q + q s + s^2]

Out[∘]= $25\,q^2 + s^2 + q\,(67 + s)$

In[∘]:= Reduce[x^2 - y^3 == 1, {y}]

Out[∘]= $y == \left(-1 + x^2\right)^{1/3} \,||\, y == -(-1)^{1/3}\left(-1 + x^2\right)^{1/3} \,||\, y == (-1)^{2/3}\left(-1 + x^2\right)^{1/3}$

In[∘]:= Trace[%]

Out[∘]= {%, Out[$Line - 1], {{$Line, 334}, 334 - 1, 333}, %333, $y == \left(-1 + x^2\right)^{1/3} \,||\, y == -(-1)^{1/3}\left(-1 + x^2\right)^{1/3} \,||\, y == (-1)^{2/3}\left(-1 + x^2\right)^{1/3}$}

In[∘]:= Solve[x^2 + a x + 1 == 0, x]

Out[∘]= $\left\{\left\{x \to \frac{1}{2}\left(-a - \sqrt{-4 + a^2}\right)\right\}, \left\{x \to \frac{1}{2}\left(-a + \sqrt{-4 + a^2}\right)\right\}\right\}$

Functions like FullSimplify and Factor are called “Built-in Functions”. They are predefined functions that are written in the *Mathematica* programme.

Differentiation (Look up “D” or “differentiation” in documentation center for more info)

? D

D[f, x] gives the partial derivative $\partial f / \partial x$.

D[f, {x, n}] gives the multiple derivative $\partial^n f / \partial x^n$.

D[f, x, y, …] differentiates f successively with respect to x, y, ….

D[f, {{x_1, x_2, …}}] for a scalar f gives the vector derivative $(\partial f / \partial x_1, \partial f / \partial x_2, \ldots)$.

D[f, {*array*}] gives a tensor derivative. >>

In[∘]:= D[Cos[x], x]

Out[∘]= -Sin[x]

In[∘]:= D[x^3, x]

Out[∘]= $3\,x^2$

In[∘]:= D[x^3, {x, 2}] (*second derivative*)

Out[∘]= 6 x

In[∘]:= D[x^3 + y, x] (*partial*)

Out[∘]= $3\,x^2$

Integration (Look up “integrate” in documentation center for more info)

```
? Integrate
```

Integrate[f, x] gives the indefinite integral $\int f\,dx$.

Integrate[f, {x, x_{min}, x_{max}}] gives the definite integral $\int_{x_{min}}^{x_{max}} f\,dx$.

Integrate[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, …] gives the multiple integral $\int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \ldots f$. >>

```
Integrate[x, x]
```

$$\frac{x^2}{2}$$

```
Integrate[x y, x]
```

$$\frac{x^2\,y}{2}$$

In[∘]:= `Integrate[t^3 Sin[t]^4, t]`

Out[∘]= $$\frac{1}{1024}\left(-192\,\left(-1+2\,t^2\right)\,\mathrm{Cos}[2\,t]+3\,\left(-1+8\,t^2\right)\,\mathrm{Cos}[4\,t]+4\,t\,\left(24\,t^3+\left(96-64\,t^2\right)\,\mathrm{Sin}[2\,t]+\left(-3+8\,t^2\right)\,\mathrm{Sin}[4\,t]\right)\right)$$

In[∘]:= `Integrate[Exp[-3 t] * t ^ (2 / 5), {t, 1, Infinity}]`

Out[∘]= $$\mathrm{ExpIntegralE}\left[-\frac{2}{5},\,3\right]$$

Defining functions

$f(x) = \sin(x^2)$

In[∘]:= `f[x_] := Sin[x^2]`

In[∘]:= `? f`

Out[∘]=

Symbol

Global`f

Definitions f[x_] := Sin[x^2]

Full Name Global`f

Calling functions and evaluating functions at certain points

In[∘]:=
```
f[x]
```
Out[∘]= $\text{Sin}[x^2]$

In[∘]:=
```
f[a + 3]
```
Out[∘]= $\text{Sin}[(3 + a)^2]$

In[∘]:=
```
f[b]
```
Out[∘]= $\text{Sin}[b^2]$

In[∘]:=
```
f[π] // N
```
Out[∘]= -0.430301

Multivariable function $g(x, y) = e^x + x\,y$

In[∘]:=
```
Clear[g]
g[x_, y_] := Exp[x] + x y
```

In[∘]:=
```
? g
```

Out[∘]=

Symbol

Global`g

Definitions	g[x_, y_] := Exp[x] + x y
Full Name	Global`g

Calling functions and evaluating functions at certain points

In[∘]:=
```
g[x, y]
```
Out[∘]= $e^x + x\,y$

In[∘]:=
```
g[2, 8]
```
Out[∘]= $16 + e^2$

The underscores behind the variables are compulsory when defining functions, but are not needed when calling them. We CANNOT define a function with g[x]:=x.
The function definitions will remain until you clear or change it.

In[∘]:=
```
Clear[h]
h[x_][y_] := Exp[x] + x y
h[2][8]
```
Out[∘]= $16 + e^2$

setting conditions for the input

```
In[•]:= Clear[f1]
        f1[x_Integer] := x * x

In[•]:= {f1[], f1[3], f1[3.2]}
Out[•]= {f1[], 9, f1[3.2]}
```

More than a variable as input

```
In[•]:= Clear[f2]

In[•]:= f2[x__] := x * x

In[•]:= {f2[], f2[{2, 3}]}
Out[•]= {f2[], {4, 9}}

In[•]:= f2[{4.3, 5.3, 6.3}]
Out[•]= {18.49, 28.09, 39.69}
```

Deleting function definition

```
ClearAll["Global`*"](*will clear all the variables and functions*)
```

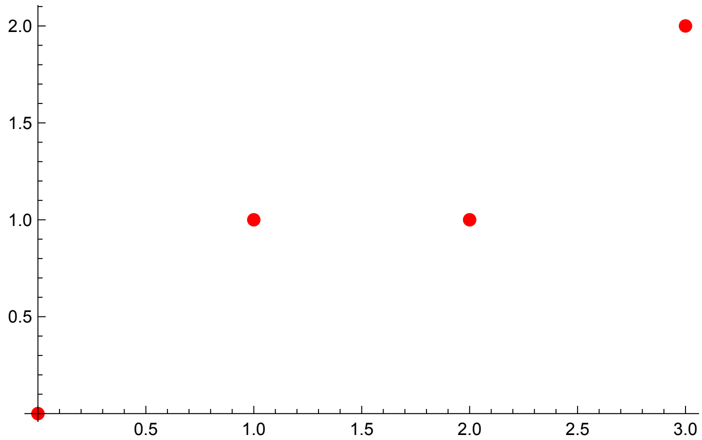
Plotting

Points

Plotting a series of points (Cartesian coordinates)

```
In[•]:= ListPlot[{{0, 0}, {1, 1}, {2, 1}, {3, 2}}, PlotStyle → {PointSize[Large], Red}]
```

Out[•]=

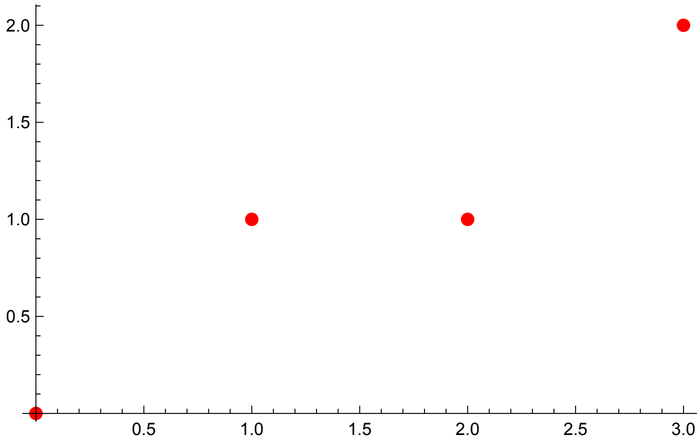


You can “store” all the plot as well

In[•]:=
```
P = {{0, 0}, {1, 1}, {2, 1}, {3, 2}};
fig1 = ListPlot[P, PlotStyle → {PointSize[Large], Red}];
```

In[•]:=
```
fig1
```

Out[•]=

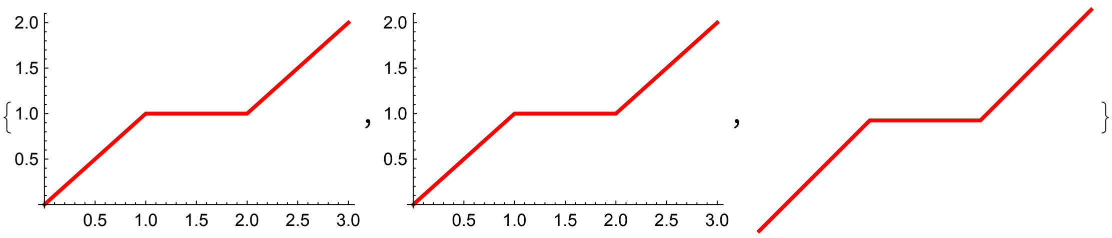


Other than “ListPlot”, we can opt for “Graphics” (“Graphics” do not display axes):

Connecting the points with straight lines

In[•]:=
```
{ListPlot[{{0, 0}, {1, 1}, {2, 1}, {3, 2}}, PlotStyle → {Thick, Red}, Joined → True],
 ListLinePlot[{{0, 0}, {1, 1}, {2, 1}, {3, 2}}, PlotStyle → {Thick, Red}],
 Graphics[{Red, Thick, Line[P]}]}
```

Out[•]=



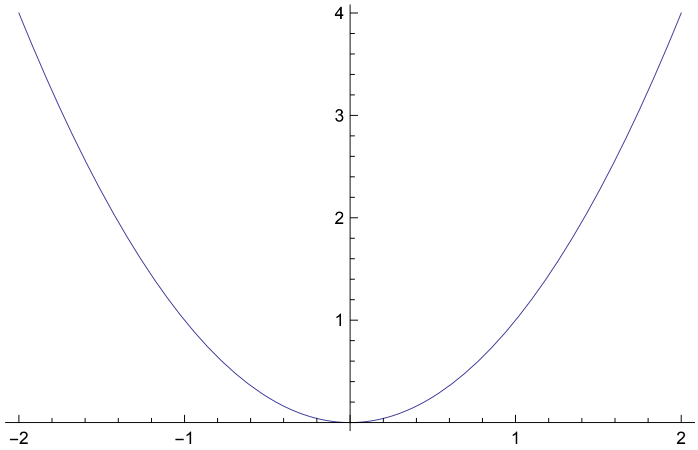
In[•]:=
```
Options[ListPlot]
```

Out[•]=
```
{AlignmentPoint → Center, AspectRatio → 1/GoldenRatio, Axes → Automatic,
 AxesLabel → None, AxesOrigin → Automatic, AxesStyle → {}, Background → None,
 BaselinePosition → Automatic, BaseStyle → {}, ClippingStyle → None,
 ColorFunction → Automatic, ColorFunctionScaling → True, ColorOutput → Automatic,
 ContentSelectable → Automatic, CoordinatesToolOptions → Automatic,
 DataRange → Automatic, DisplayFunction :→ $DisplayFunction, Epilog → {},
 Filling → None, FillingStyle → Automatic, FormatType :→ TraditionalForm,
 Frame → Automatic, FrameLabel → None, FrameStyle → {}, FrameTicks → Automatic,
 FrameTicksStyle → {}, GridLines → None, GridLinesStyle → {}, ImageMargins → 0.,
 ImagePadding → All, ImageSize → Automatic, ImageSizeRaw → Automatic,
 InterpolationOrder → None, IntervalMarkers → Automatic,
 IntervalMarkersStyle → Automatic, Joined → False, LabelingFunction → Automatic,
 LabelingSize → Automatic, LabelStyle → {}, MaxPlotPoints → ∞, Mesh → None,
 MeshFunctions → {#1 &}, MeshShading → None, MeshStyle → Automatic,
 Method → Automatic, PerformanceGoal :→ $PerformanceGoal, PlotLabel → None,
 PlotLabels → None, PlotLayout → Overlaid, PlotLegends → None,
 PlotMarkers → None, PlotRange → Automatic, PlotRangeClipping → True,
 PlotRangePadding → Automatic, PlotRegion → Automatic, PlotStyle → Automatic,
 PlotTheme :→ $PlotTheme, PreserveImageOptions → Automatic,
 Prolog → {}, RotateLabel → True, ScalingFunctions → None,
 TargetUnits → Automatic, Ticks → Automatic, TicksStyle → {}}
```

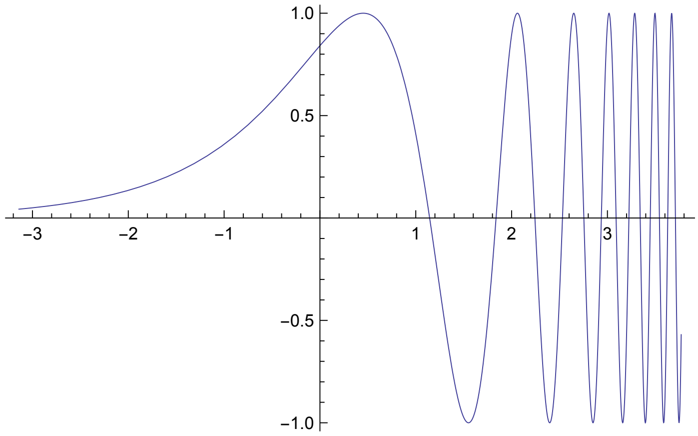
Explicit Function

2D plot

```
Plot[x^2, {x, -2, 2}]
```



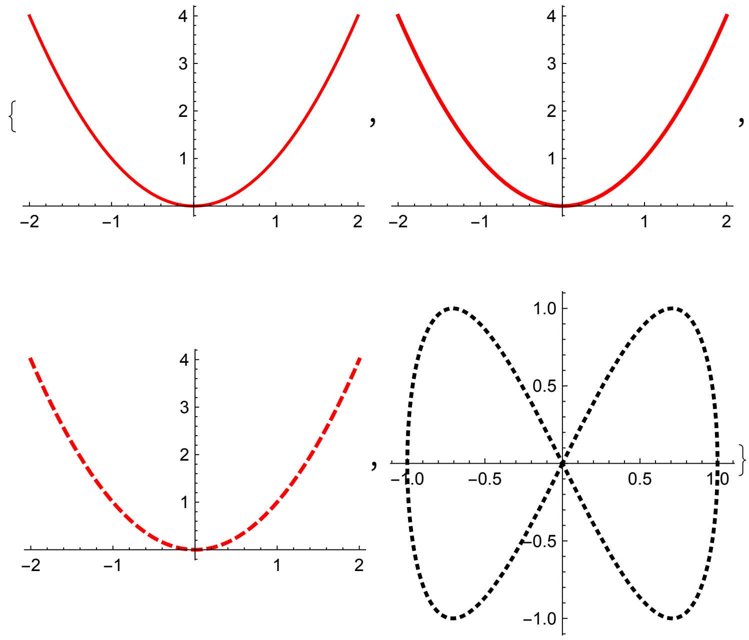
```
f[x_] := Sin[Exp[x]]
Plot[f[x], {x, -π, 1.2 π}]
```



Editing Plot (Colour, Axes, etc.)

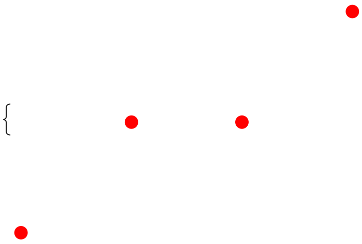
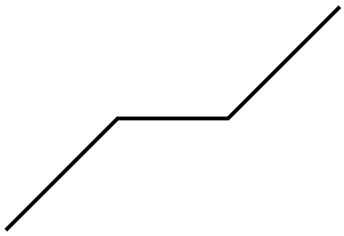
```
In[•]:= {Plot[x^2, {x, -2, 2}, PlotStyle → Red],
  Plot[x^2, {x, -2, 2}, PlotStyle → {Red, Thick}],
  Plot[x^2, {x, -2, 2}, PlotStyle → {Red, Thick, Dashed}], ParametricPlot[
   {Sin[t], Sin[2 t]}, {t, 0, 2 Pi}, PlotStyle → {Black, Dotted, Thick}]}
```

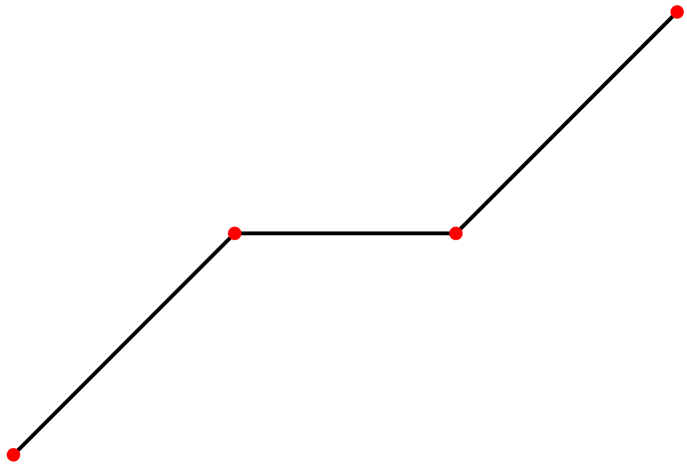
Out[•]=



Combining two graphs

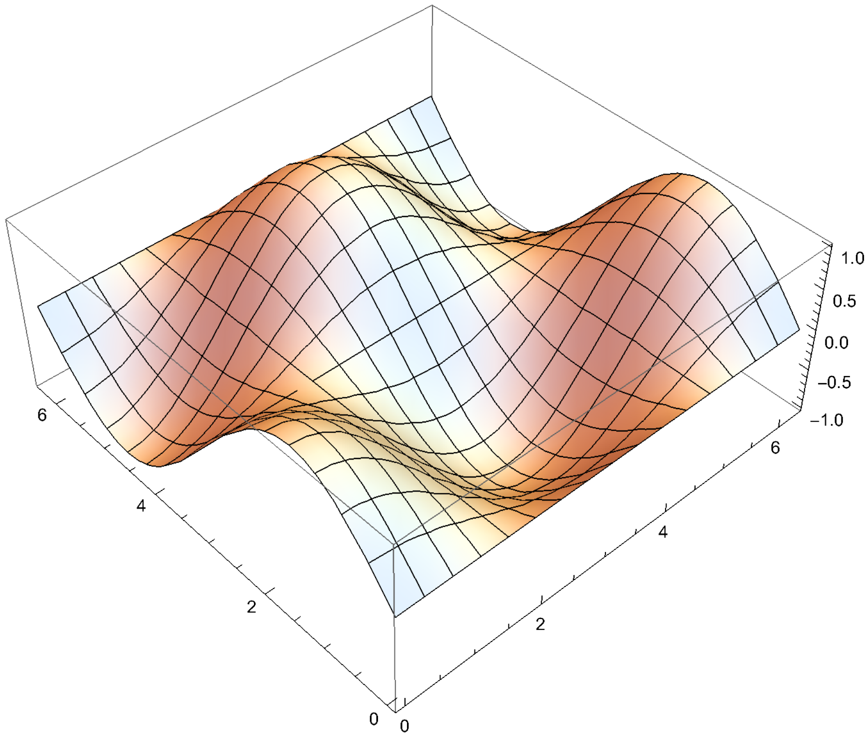
In[ ]:=
```
{figure1 = Graphics[{Red, PointSize[Large], Point[P]}],
 figure2 = Graphics[{Black, Thick, Line[P]}]}
Show[figure2, figure1]
```

Out[ ]= {  ,  }

Out[ ]= 

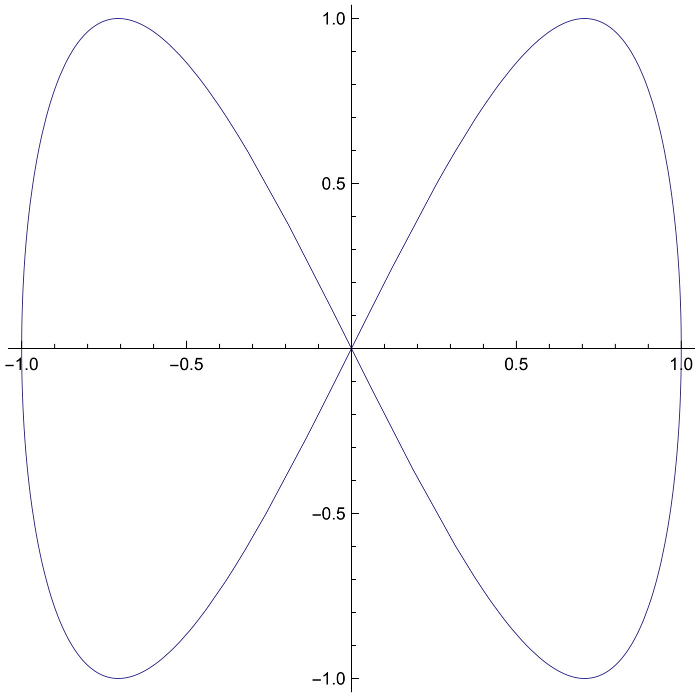
3D Plot

```
g[x_, y_] := Cos[x] Sin[y]
Plot3D[g[x, y], {x, 0, 2 π}, {y, 0, 2 π}]
```

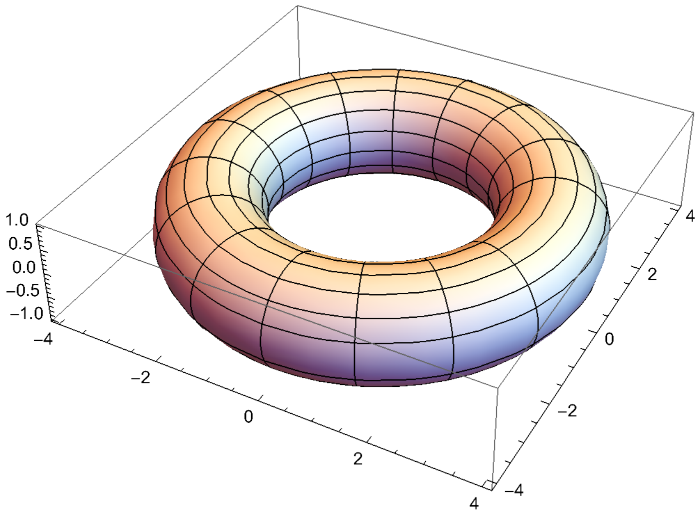


Parametric Plot

```
ParametricPlot[{Sin[t], Sin[2 t]}, {t, 0, 2 Pi}]
```



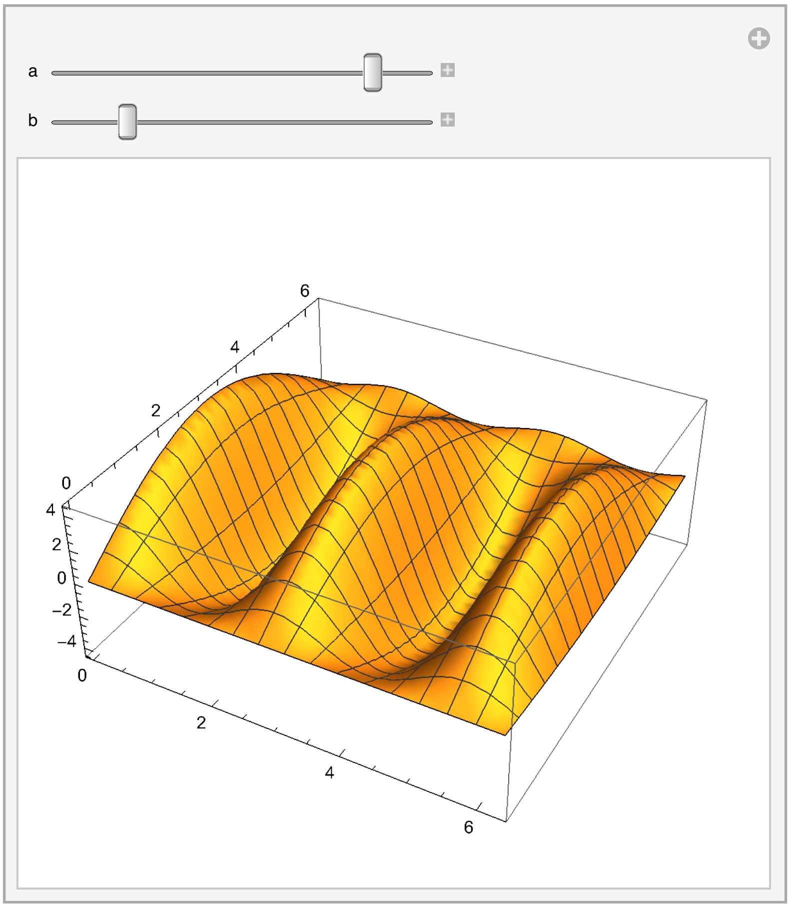
```
ParametricPlot3D[
 {Cos[t] (3 + Cos[u]), Sin[t] (3 + Cos[u]), Sin[u]}, {t, 0, 2 Pi}, {u, 0, 2 Pi}]
```



Simulation and Animation

In[1]:=
```
Clear[g, a, b, x, y]
g[x_, y_, a_, b_] := (a Cos[a x] Sin[b y])/b
Manipulate[Plot3D[g[x, y, a, b], {x, 0, 2 π}, {y, 0, 2 π}], {a, -3, 3}, {b, 0.01, 3}]
```

Out[3]=



Vectors and Matrices

Lists is a n-tuple: order matters

In[27]:=
```
list1 = {1, 2, 3, 4, 5, 6};
list2 = {6, 5, 4, 3, 2, 1};
list3 = {1, 2, 3, 4, 5, 6};
list4 = {6, 2, 3, 4, 5, 1};
```

In[26]:=
```
list1 == list3
```

Out[26]= True

List properties

In[31]:=
```
list1 == list4
```
Out[31]= False

In[46]:=
```
list1[[3]]
```
Out[46]= 3

In[47]:=
```
list1[[1 ;; 3]]
```
Out[47]= {1, 2, 3}

Simple List Operation

In[33]:=
```
Length[list1]
```
Out[33]= 6

In[6]:=
```
2 * list1
```
Out[6]= {2, 4, 6, 8, 10, 12}

In[7]:=
```
list1 + list2
```
Out[7]= {7, 7, 7, 7, 7, 7}

In[8]:= $\sqrt{\text{list2}}$

Out[8]= $\{\sqrt{6}, \sqrt{5}, 2, \sqrt{3}, \sqrt{2}, 1\}$

In[9]:=
```
{1, 2, 3, 4, 5} + {2, 5, 7, 8, 4} / {a, 4, 6, y, 7}
```
Out[9]= $\left\{1+\frac{2}{a}, \frac{13}{4}, \frac{25}{6}, 4+\frac{8}{y}, \frac{39}{7}\right\}$

Generate a list using “Table[]”

```
Table[i, {i, 1, 10}]
```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

```
Table[Cos[2 i], {i, 1, 10}]
```
{Cos[2], Cos[4], Cos[6], Cos[8], Cos[10],
Cos[12], Cos[14], Cos[16], Cos[18], Cos[20]}

```
Table[i * j, {i, 1, 2}, {j, 1, 2}]
```
{{1, 2}, {2, 4}}

Vector

In[19]:=
```
v1 = {1, 3};
v2 = {-2, 6};
v3 = {3, 1}
```
Out[21]= {3, 1}

```
In[22]:= v1 == v3
Out[22]= False
```

Simple Vector Operations

```
In[12]:= q = 2; r = 7;

In[13]:= q v1 - r v2
Out[13]= {16, -36}
```

Matrices

```
In[16]:= A = {{1, 2}, {a, b}}
         B = {{1, 2, 3}, {a, b, c}, {2 a, 3 b, 4 c}}
Out[16]= {{1, 2}, {a, b}}
Out[17]= {{1, 2, 3}, {a, b, c}, {2 a, 3 b, 4 c}}
```

Showing Matrices in Standard Form

```
MatrixForm[A]
```

$$\begin{pmatrix} 1 & 2 \\ a & b \end{pmatrix}$$

```
MatrixForm[B]
```

$$\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \\ 2 a & 3 b & 4 c \end{pmatrix}$$

Size of a Matrix

```
Dimensions[M]
{3, 1}

Dimensions[A]
{2, 2}
```

Simple Matrix Operations

```
In[34]:= A.B(*Example of problem usually encountered*)
         ··· Dot: Tensors {{1, 2}, {a, b}} and {{1, 2, 3}, {a, b, c}, {2 a, 3 b, 4 c}} have incompatible shapes.
Out[34]= {{1, 2}, {a, b}}.{{1, 2, 3}, {a, b, c}, {2 a, 3 b, 4 c}}

In[35]:= M = {{6}, {4}, {2}}
Out[35]= {{6}, {4}, {2}}
```

In[36]:=
```
B * M (*Example of incorrect usage*)
```

Thread: Objects of unequal length in {1, 2, 3} {6} cannot be combined.

Thread: Objects of unequal length in {a, b, c} {4} cannot be combined.

Thread: Objects of unequal length in {2 a, 3 b, 4 c} {2} cannot be combined.

General: Further output of Thread::tdlen will be suppressed during this calculation.

Out[36]= {{6} {1, 2, 3}, {4} {a, b, c}, {2} {2 a, 3 b, 4 c}}

In[37]:=
```
U = B.M
MatrixForm[B.M]
```

Out[37]= {{20}, {6 a + 4 b + 2 c}, {12 a + 12 b + 8 c}}

Out[38]//MatrixForm=

$$\begin{pmatrix} 20 \\ 6a + 4b + 2c \\ 12a + 12b + 8c \end{pmatrix}$$

In[41]:=
```
4 U
MatrixForm[%]
```

Out[41]= {{80}, {4 (6 a + 4 b + 2 c)}, {4 (12 a + 12 b + 8 c)}}

Out[42]//MatrixForm=

$$\begin{pmatrix} 80 \\ 4(6a + 4b + 2c) \\ 4(12a + 12b + 8c) \end{pmatrix}$$

In[43]:=
```
4 U - M
MatrixForm[4 U - M]
```

Out[43]= {{74}, {-4 + 4 (6 a + 4 b + 2 c)}, {-2 + 4 (12 a + 12 b + 8 c)}}

Out[44]//MatrixForm=

$$\begin{pmatrix} 74 \\ -4 + 4(6a + 4b + 2c) \\ -2 + 4(12a + 12b + 8c) \end{pmatrix}$$

GETTING PART OF A LIST/MATRIX

In[60]:=
```
A = {{1, 2, 3}, {a, z, b}};
```

In[61]:=
```
A[[1]]
```

Out[61]= {1, 2, 3}

In[62]:=
```
A[[2]]
```

Out[62]= {a, z, b}

In[66]:=
```
A[[2, 2]]
```

Out[66]= z

In[68]:=
```
A[[2, 2 ;; 3]]
```

Out[68]= {z, b}

END