S. V. National Institute of Technology, Surat

Applied Mathematics and Humanities Department

B. Tech.-I (Semester-I) Branch-All

Subject: Mathematics-I (MA 10 S1)

Tutorial 1 :	Hyperbolic Functions, Successive Differentiation and Leibnitz's Theorem

1. Define **hyperbolic cosine function** and **hyperbolic sine function** with their graphs**.** Prove the identities: (i) $\cosh^2 x$ - $\sinh^2 x = 1$ (ii) $\cosh 2x = 2\cosh^2 x - 1$ (ii) $\cosh 2x = 1 + 2\sinh^2 x$.

2. Define exponential formula for sech x, cosech x, tanh x, and coth x. Show that (i) $\text{sech}^2 x = 1 - \tanh^2 x$ (ii) $\coth^2 x = 1 + \text{cosech}^2 x$.

3. Given that $\sinh x = \dfrac{5}{12}$, find the values of (a) cosh x (b) tanh x (c) sech x (d) coth x (e) sinh $2x$ (f) cosh $2x$. Determine the value of x as a natural logarithm.

4. Prove that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right),\ x \in \mathbb{R}$ and $\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)\ x \geq 1$.

5. Find the derivative of (i) $\sinh^{-1}(x^3)$ (ii) $\cosh^{-1}(2x+1)$. **Ans** (i) $\dfrac{3x^2}{\sqrt{x^6+1}}$ (ii) $\dfrac{1}{\sqrt{x^2+x}}$

6. If $y = \dfrac{x^4}{(x-1)(x-2)}$, then find y_n. **Ans**: $y_n = (-1)^n (n)!\left(\dfrac{16}{(x-2)^{n+1}} - \dfrac{1}{(x-1)^{n+1}}\right)$.

7. Find the nth derivative of $y = e^{2x}\cos x \sin^2 2x$

8. Find the nth derivative of $\dfrac{1}{x^2+a^2}$. **Ans**: $\dfrac{(-1)^n n!}{a^{n+2}}\sin(n+1)\theta\sin^{n+1}\theta$.

9. Show that $D^{2n}(x^2-1)^n = (2n)!$.

10. State and prove Leibnitz's Theorem.

11. If $y = x\log\left(\dfrac{x-1}{x+1}\right)$, show that $y_n = (-1)^{n-2}(n-2)!\left(\dfrac{x-n}{(x-1)^n} - \dfrac{x+n}{(x+1)^n}\right)$.

12. If $I_n = \dfrac{d^n}{dx^n}\left(x^n \log x\right)$, prove that $I_n = nI_{n-1} + (n-1)!$.

13. If $y = x^2 e^x$, prove that $y_n = \dfrac{1}{2}n(n-1)y_2 - n(n-2)y_1 + \dfrac{1}{2}(n-1)(n-2)y$.

14. If $y^{1/m} + y^{-1/m} = 2x$, then prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$.

15. Determine $y_n(0)$, where $y = e^{m\cos^{-1}x}$.

Ans.: $y_n(0) = \begin{cases} m^2(2^2+m^2)(4^2+m^2)...((n-2)^2+m^2), n \text{ even} \\ m^2(1^2+m^2)(3^2+m^2)...((n-2)^2+m^2), n \text{ odd} \end{cases}$

16. If $f(x) = \tan x$, prove that

$$f^n(0) - \binom{n}{2} f^{n-2}(0) + \binom{n}{4} f^{n-4}(0) \ldots\ldots = \sin\left(\frac{n\pi}{2}\right).$$

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