Thus,

$$\boxed{\begin{aligned}\Delta p &\cong p \sin\alpha = \frac{p\lambda}{\Delta x} = \frac{p}{\Delta x}\frac{h}{p} = \frac{h}{\Delta x}\\ (\Delta x)\,(\Delta p) &\cong h\end{aligned}}$$

or

which is *Heisenberg's uncertainty principle.*

One interesting question is whether electrons are present in atomic nuclei or not. As is known, typical nuclei are less than 10^{-14} m in radius. For an electron to be confined within such a nucleus, the uncertainty in its position may not exceed 10^{-14} m. The corresponding uncertainty in the electron's momentum is:

$$\Delta p \geq \frac{h}{4\pi\,(\Delta x)}$$

$$\Delta p \geq \frac{6.62\times10^{-34}}{4\pi\times10^{-4}} = 0.526\times10^{-20}\ \text{kg-m/s}$$

If this is the uncertainty in the electron's momentum, the momentum itself must be atleast comparable in magnitude. An electron whose momentum is 0.5×10^{-20} kg-m/s has a kinetic energy E many times greater than its rest mass energy m_0c^2.

$$\boxed{\begin{aligned}E &= pc = \frac{0.526\times10^{-20}\times3\times10^{8}}{1.6\times10^{-19}}\ \text{eV}\\ E &= 10\ \text{MeV}\end{aligned}}$$

The rest mass energy of the electron m_0c^2 is 0.5 MeV, which is negligible compared to the kinetic energy. Experiments indicate that the electrons associated even with unstable atoms never have more than a fraction of this energy, and we thus conclude that electrons cannot be present within nuclei.

VII. X-RAY DIFFRACTION

X-rays are electromagnetic waves like ordinary light, therefore, they should exhibit interference and diffraction. The wavelength of X-rays is of the order of 0.1 nm, so that ordinary devices such as ruled diffraction grating do not produce observable effects with X-rays. In 1912, German physicist Laue suggested that a crystal which consisted of a three-dimensional array of regularly spaced atoms could serve the purpose of a grating. The crystal differs from the ordinary grating in the sense that the diffracting centres in the crystal are not in one plane. Hence the crystal acts as a space grating rather than a plane grating.

On the suggestion of Laue, his associates, Friedrich and Knipping succeeded in diffracting X-rays by passing them through a thin crystal of zinc blende. The diffraction pattern obtained consists of a central spot and a series of spots arranged in a definite pattern around the central spot. This symmetrical pattern of spots is known as Laue pattern and it proves that X-rays are electromagnetic radiation. A simple interpretation of the diffraction pattern was given by W.L. Bragg. According to him, the spots are produced due to the reflection of some of the incident X-rays from the various sets of parallel crystal planes (called Bragg's planes) which contain a large number of atoms.

VIII. BRAGG'S LAW

Consider a ray PA reflected at atom A in the direction AR from plane 1 and another ray QB reflected at another atom B in the direction BS. Now from the atom A, draw two perpendiculars AC and AD on QB and BS respectively. The two reflected rays will be in phase or out of phase depending on the path difference. When the path difference (CB + BD) is a whole wavelength (λ) or multiple of whole wavelength ($n\lambda$), then the two rays will reinforce each other and produce an intense spot. Thus condition of reinforcement is:

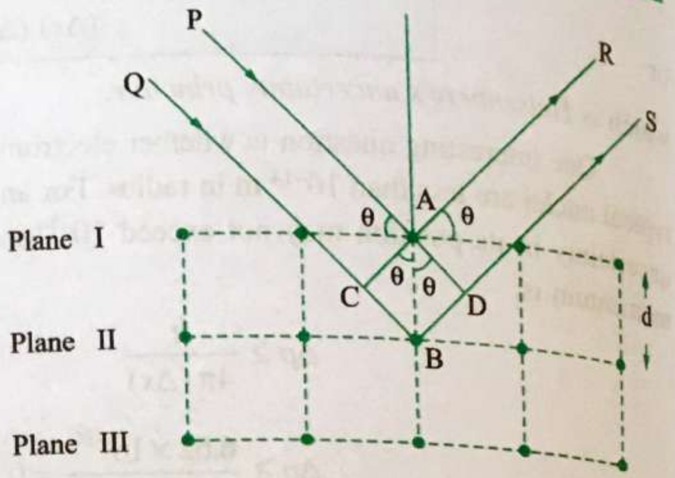

Fig. 5.5 Reflection of X-rays from lattice planes in a crystal

$$CB + BD = n\lambda$$

From Fig. (5.5), we have

$$CB = BD = d \sin \theta$$

where, θ is the angle between the incident ray and the planes of reflection (*glancing angle*). Therefore,

$$2d \sin \theta = n\lambda \tag{5.11}$$

where, d is the interplanar spacing of planes and $n = 1, 2, 3, \ldots$ stands for first order, second order, third order, ... maxima respectively. Equation (5.11) is known as Bragg's law. Different directions in which intense reflections will be produced can be obtained by giving different values to θ, *i.e.*,

for first maximum, $\sin \theta_1 = \dfrac{\lambda}{2d}$

for second maximum, $\sin \theta_2 = \dfrac{2\lambda}{2d}$

for third maximum, $\sin \theta_3 = \dfrac{3\lambda}{2d}$ and so on.

It should be remembered that the intensity goes on decreasing as the order of spectrum increases.

Thus we see that when a beam of monochromatic X-rays falls on a crystal, each atom becomes a source of scattering radiations. It has already been mentioned that in a crystal there are certain planes which are particularly rich in atoms. The combined scattering of X-rays from these planes can be looked upon as reflections from these planes. Generally, the *Bragg scattering* is regarded as *Bragg reflection*, and hence are known as *Bragg planes*. At certain glancing angles, reflections from these set of parallel planes are in phase with each other, and hence they reinforce each other to produce maximum intensity. For other angles, the reflections from different planes are out of phase, and hence they reinforce to produce either zero intensity or extremely feeble intensity.