S. V. National Institute of Technology, Surat

Applied Mathematics and Humanities Department

B.Tech–I Sem–1 Branch–All Subject–Mathematics-I (MA 101 S1)

Tutorial - 11 : Triple Integration and its Application

1. Evaluate the following integrals

   (a) $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$ **Ans** : 0

   (b) $\int_{0}^{\frac{\pi}{2}}\int_{x}^{\frac{\pi}{2}}\int_{0}^{xy}\cos(\frac{z}{x})\,dz\,dy\,dx$ **Ans** : $\frac{\pi}{2}-1$

   (c) $\int_{0}^{\log 2}\int_{0}^{x}\int_{0}^{x+\log y}e^{x+y+z}\,dz\,dy\,dx$ **Ans** : $\frac{8}{3}\log 2-\frac{19}{9}$

2. Using the transformation $u=x+y+z, uv=y+z, uvw=z$ evaluate

   (a) $\int\int\int\sqrt{xyz(1-x-y-z)}\,dx\,dy\,dz$ **Ans** : $\frac{\pi^2}{1920}$

   (b) $\int\int\int xyz(x+y+z)^2\,dx\,dy\,dz$ **Ans** : $\frac{1}{960}$

   (c) $\int\int\int e^{(x+y+z)^3}\,dx\,dy\,dz$ **Ans** : $\frac{e-1}{6}$

   taken over the tetrahedral volume enclosed by the planes $x=0, y=0, z=0$ and $x+y+z=1$.

3. Use spherical coordinates to evaluate $\int_{0}^{0}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}}\frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$ **Ans** : $\frac{\pi^2}{8}$

4. Find the volume of the solid surrounded by the surface $(\frac{x}{a})^{\frac{2}{3}}+(\frac{y}{b})^{\frac{2}{3}}+(\frac{z}{c})^{\frac{2}{3}}=1$. **Ans** : $\frac{4\pi abc}{35}$

5. Find the volume of the tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the coordinate planes. **Ans** : $abc/6$

6. Compute the volume of the solid bounded by the plane $2x+3y+4z=12$, xy-plane and the cylinder $x^2+y^2=1$. **Ans** : 3π

7. Find the volume of a solid bounded by the spherical surface $x^2+y^2+z^2=4a^2$ and the cylinder $x^2+y^2-2ay=0$. **Ans** : $\frac{32a^3}{3}(\frac{\pi}{2}-\frac{2}{3})$

8. Find the volume bounded by paraboloid $x^2+y^2=az$, the cylinder $x^2+y^2=2ay$ and the plane $z=0$. **Ans** : $\frac{3\pi a^3}{2}$

9. Find the volume cut from sphere $x^2+y^2+z^2=a^2$ by the cone $z^2=x^2+y^2$. **Ans** : $\frac{\pi a^3(2-\sqrt{2})}{3}$

10. Find the volume enclosed by the cylinders $x^2+y^2=2ax$ and $z^2=2ax$. **Ans** : $\frac{128a^3}{20}$

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