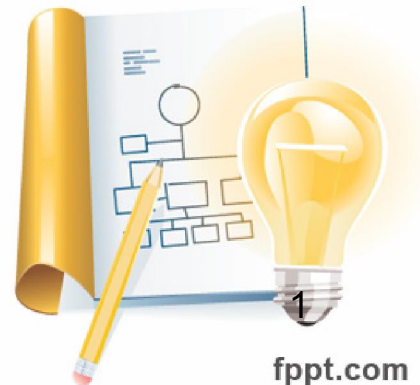


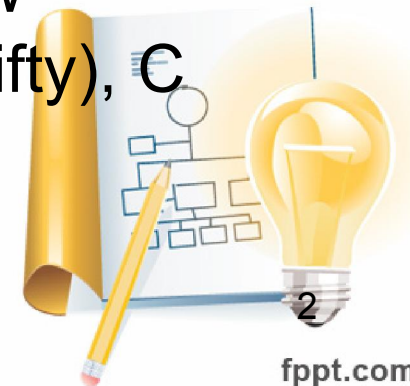
2. Number Systems



Number Systems

- A number system defines a set of values used to represent quantity.
- It can be categorized in two broad categories:
 - 1. Non – Positional Number Systems**
 - Stones or sticks were used to indicate values
 - Difficult to perform arithmetic because it has no symbol for zero.

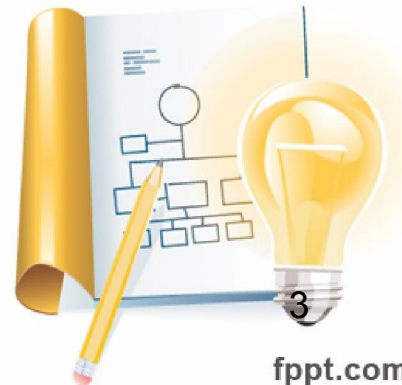
Example: Roman number systems – Few characters are used such as I, V, X, L (fifty), C (hundred)



Number System

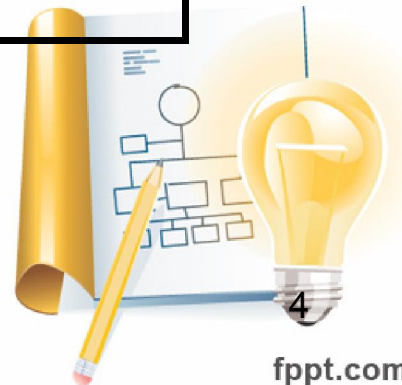
2. Positional Number Systems

- The value of each digit in number is defined not only by the symbol but also by symbol's position.
- Positional number systems have base or radix.



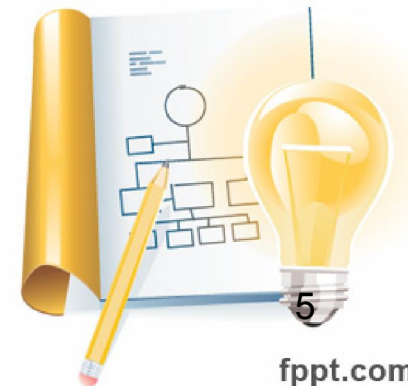
Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, ... 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, ... 7	No	No
Hexa-decimal	16	0, 1, ... 9, A, B, ... F	No	No



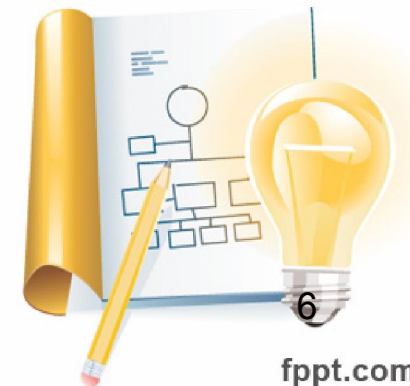
Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa- decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7



Quantities/Counting (2 of 3)

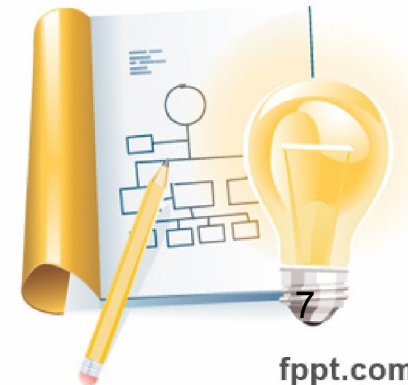
Decimal	Binary	Octal	Hexa- decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F



Quantities/Counting (3 of 3)

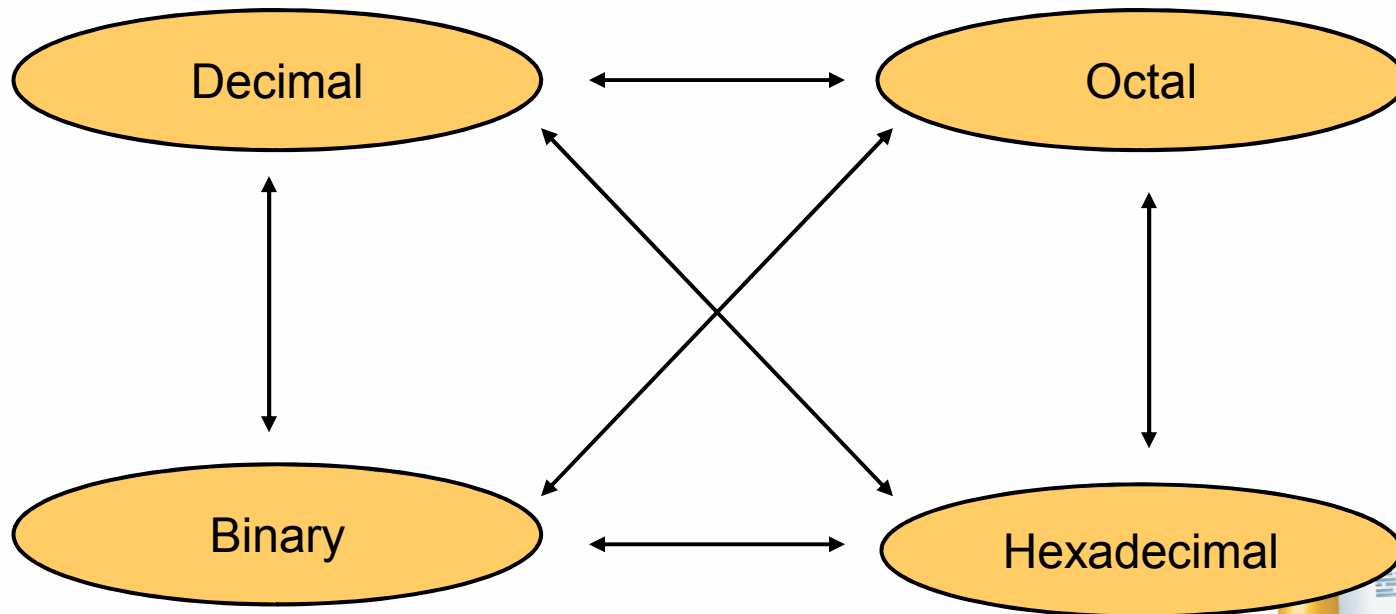
Decimal	Binary	Octal	Hexa- decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Etc.



Conversion Among Bases

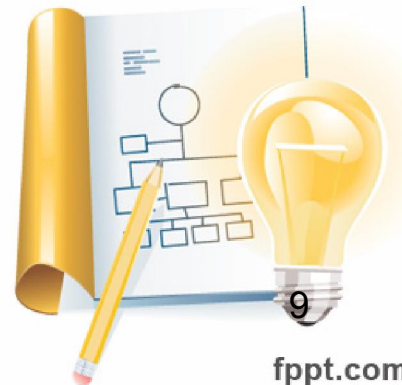
- The possibilities:



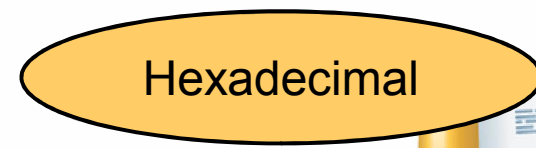
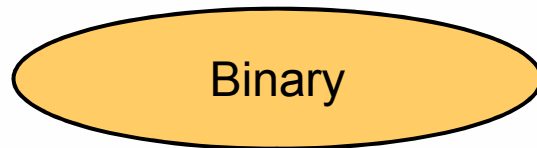
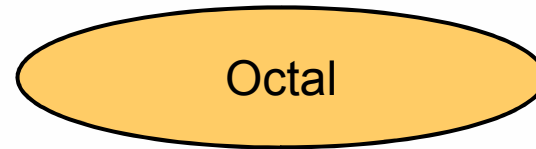
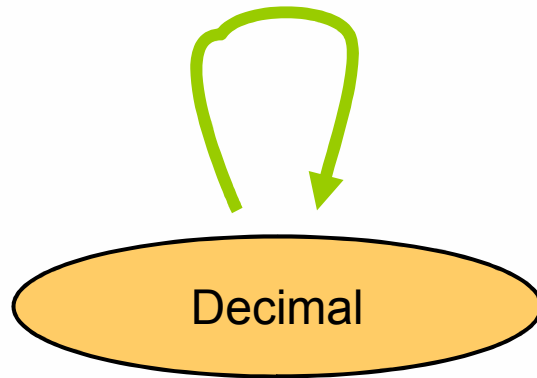
Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

Base



Decimal to Decimal (just for fun)

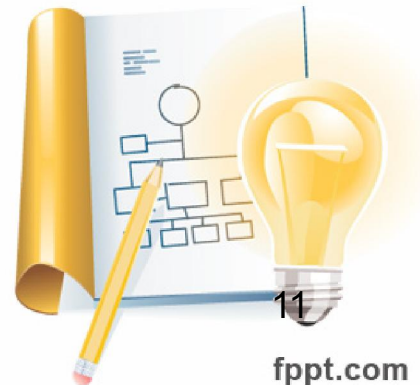


$125_{10} \Rightarrow$

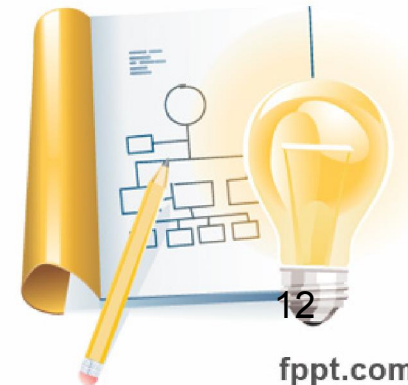
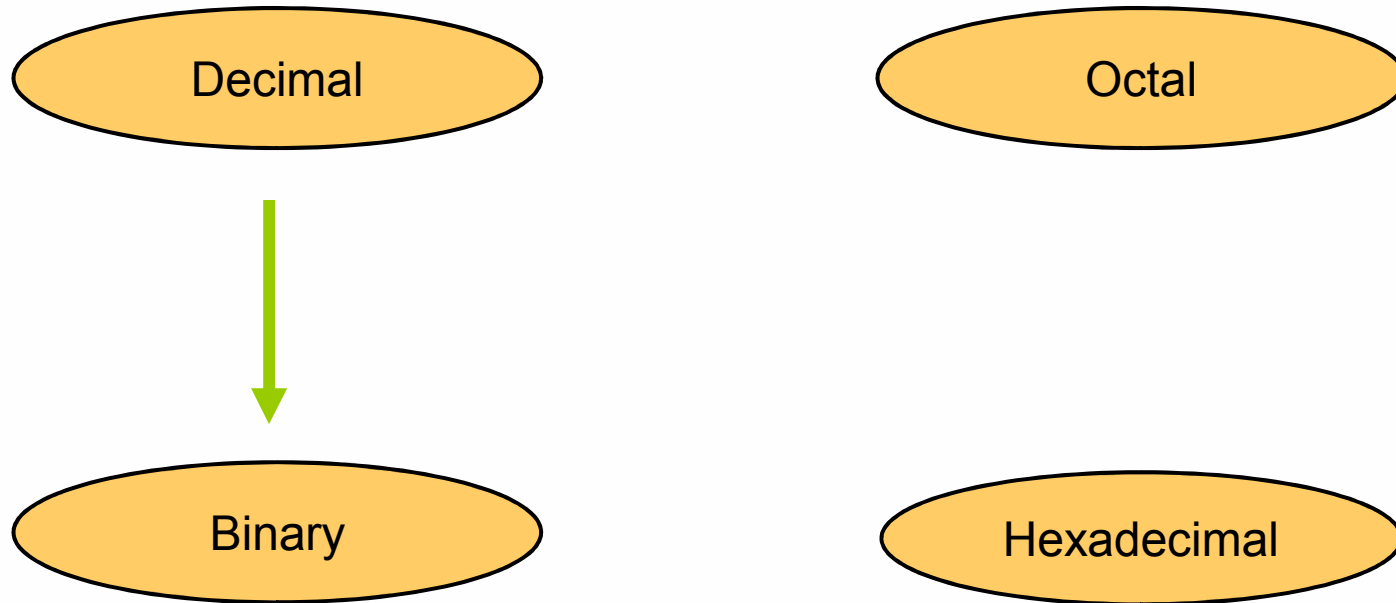
$$\begin{array}{r} 5 \times 10^0 = 5 \\ 2 \times 10^1 = 20 \\ 1 \times 10^2 = 100 \\ \hline 125 \end{array}$$

Weight

Base

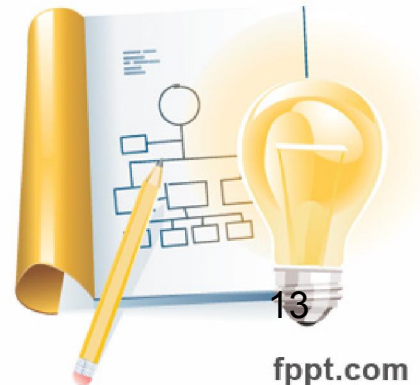


Decimal to Binary



Decimal to Binary

- Technique
 - Divide by two, keep track of the remainder
 - First remainder is bit 0 (LSB, least-significant bit)
 - Second remainder is bit 1
 - Etc.

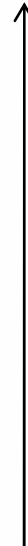


Conversion of Decimal to Binary

1. Determine the Binary equivalent of $(125)_{10}$

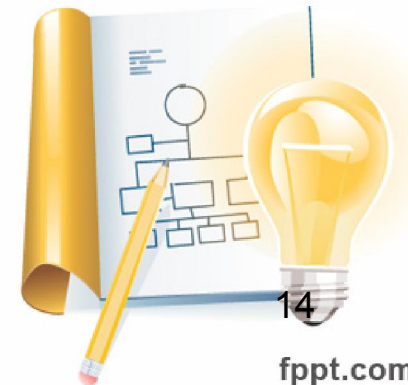
2	125	Remainder
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$125_{10} = 1111101_2$$

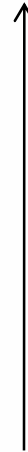


Conversion of Decimal to Binary

2. Determine the Binary equivalent of $(36)_{10}$

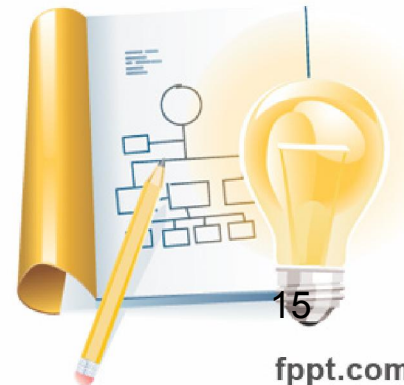
2	36	Remainder
2	18	0
2	9	0
2	4	1
2	2	0
2	1	0
	0	1

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$36_{10} = 100100_2$$

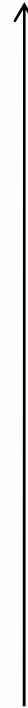


Conversion of Decimal to Binary

3. Determine the Binary equivalent of $(671)_{10}$

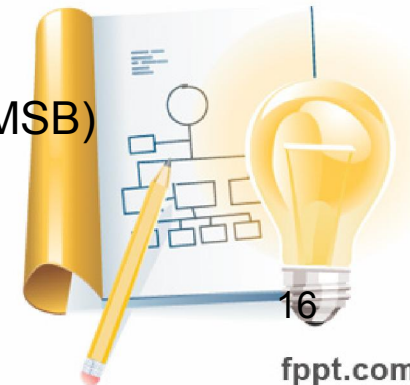
2	671	Remainder
2	335	1
2	167	1
2	83	1
2	41	1
2	20	1
2	10	0
2	5	0
2	2	1
2	1	0
	0	1

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$671_{10} = 1010011111_2$$



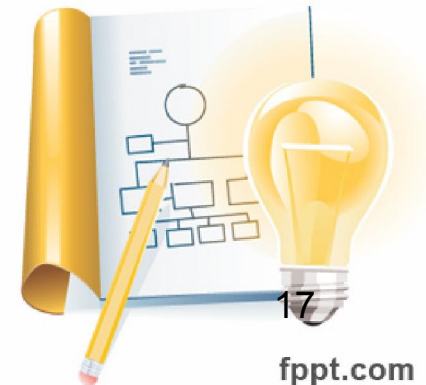
Conversion of Decimal Fraction to Binary Fraction

1. Determine the Binary equivalent of $(0.375)_{10}$

0.375	X	2	= 0.750	0
0.750	X	2	= 1.500	1
0.500	X	2	= 1.000	1



$$0.375_{10} = 011_2$$



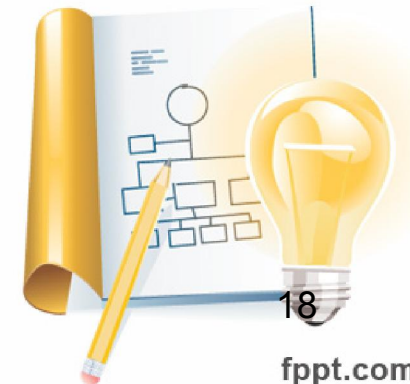
Conversion of Decimal Fraction to Binary Fraction

2. Determine the Binary equivalent of $(0.29)_{10}$

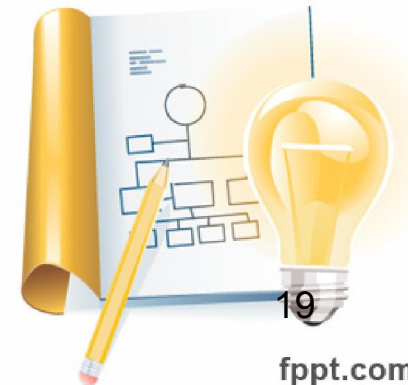
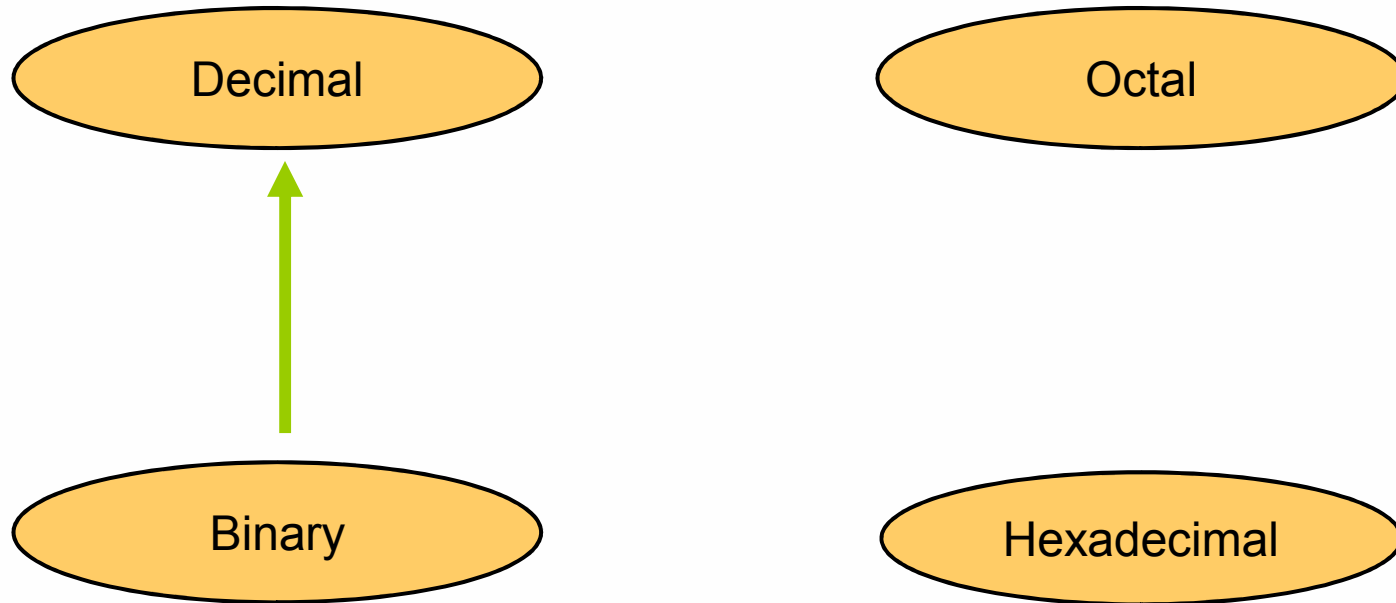
0.29	X	2	= 0.58	0
0.58	X	2	= 1.16	1
0.16	X	2	= 0.32	0
0.32	X	2	= 0.64	0
0.64	X	2	= 1.28	1
0.28	X	2	= 0.56	0
.				
.				
.				
∞				



$$0.29_{10} = (0.010010)_2$$

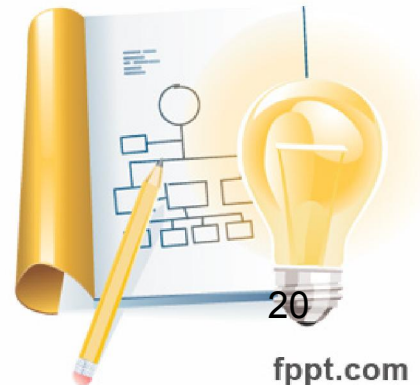


Binary to Decimal



Binary to Decimal

- Technique
 - Multiply each bit by 2^n , where n is the “weight” of the bit
 - The weight is the position of the bit, starting from 0 on the right
 - Add the results

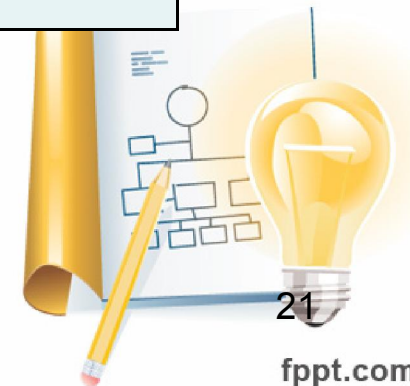


Binary to Decimal

1. Determine the Decimal equivalent of $(101011)_2$

Binary Number	1	0	1	0	1	1
Weight of Each Bit	2^5	2^4	2^3	2^2	2^1	2^0
Weighted Value	1×2^5	0×2^4	1×2^3	0×2^2	1×2^1	1×2^0
Solved Multiplication	32	0	8	0	2	1

$$\begin{aligned}\text{Sum of weight of all bits} &= 32 + 0 + 8 + 2 + 1 \\ &= 43\end{aligned}$$

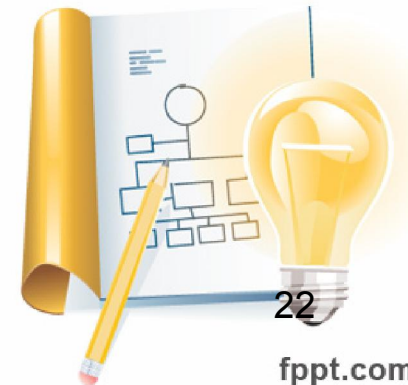


Binary to Decimal

2. Determine the Decimal equivalent of $(11010)_2$

Binary Number	1	1	0	1	0
Weight of Each Bit	2^4	2^3	2^2	2^1	2^0
Weighted Value	1×2^4	1×2^3	0×2^2	1×2^1	0×2^0
Solved Multiplication	16	8	0	2	0

$$\begin{aligned}\text{Sum of weight of all bits} &= 16 + 8 + 0 + 2 + 0 \\ &= 26\end{aligned}$$

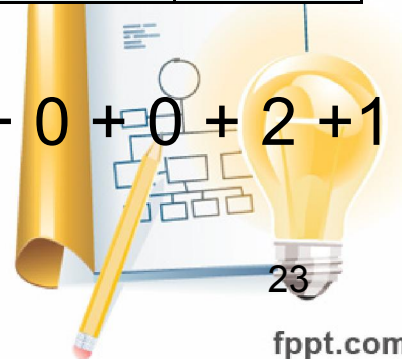


Binary to Decimal

3. Determine the Decimal equivalent of $(10110011)_2$

Binary Number	1	0	1	1	0	0	1	1
Weight of Each Bit	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Weighted Value	1×2^7	0×2^6	1×2^5	1×2^4	0×2^3	0×2^2	1×2^1	1×2^0
Solved Multiplication	128	0	32	16	0	0	2	1

Sum of weight of all bits = $128 + 0 + 32 + 16 + 0 + 0 + 2 + 1$
= 179

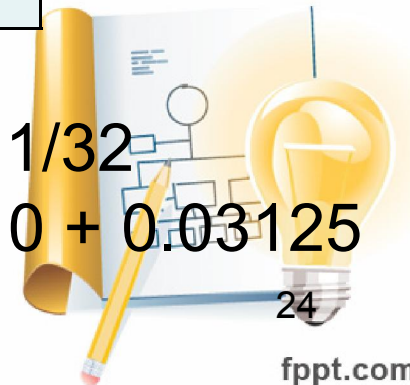


Conversion of Binary Fraction to Decimal Fractions

1. Determine the Decimal equivalent of $(0.01101)_2$

Binary Number	0	1	1	0	1
Weight of Each Bit	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Weighted Value	0×2^{-1}	1×2^{-2}	1×2^{-3}	0×2^{-4}	1×2^{-5}
Solved Multiplication	0	$1/4$	$1/8$	0	$1/32$

$$\begin{aligned}\text{Sum of weight of all bits} &= 0 + 1/4 + 1/8 + 0 + 1/32 \\ &= 0 + 0.25 + 0.125 + 0 + 0.03125 \\ &= 0.40625\end{aligned}$$

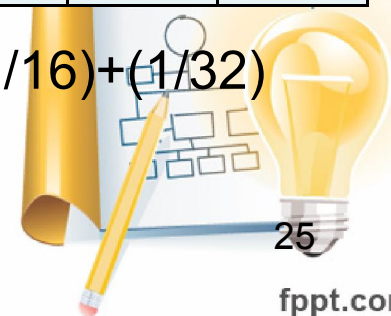


Conversion of Binary Fraction to Decimal Fractions

2. Determine the Decimal equivalent of $(11101.10111)_2$

Binary Number	1	1	1	0	1	1	0	1	1	1
Weight of Each Bit	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Weighted Value	1×2^4	1×2^3	1×2^2	0×2^1	1×1	1×2^{-1}	0×2^{-2}	1×2^{-3}	1×2^{-4}	1×2^{-5}
Solved Multiplication	16	8	4	0	1	1/2	0	1/8	1/16	1/32

Sum of weight of all bits = $16+8+4+0+1+(1/2)+0+(1/8)+(1/16)+(1/32)$
= $16+8+4+0+1+0.5+0+0.125+0.0625+0.03125$
= 29.71875

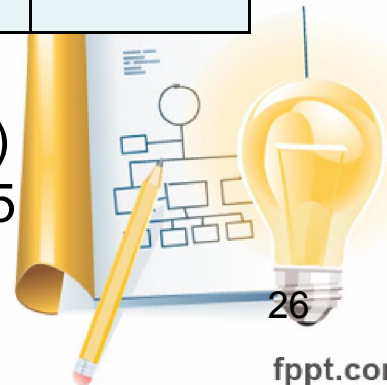


Conversion of Binary Fraction to Decimal Fractions

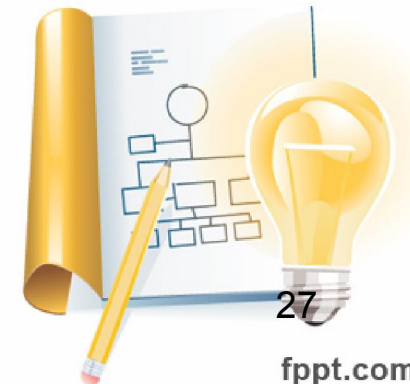
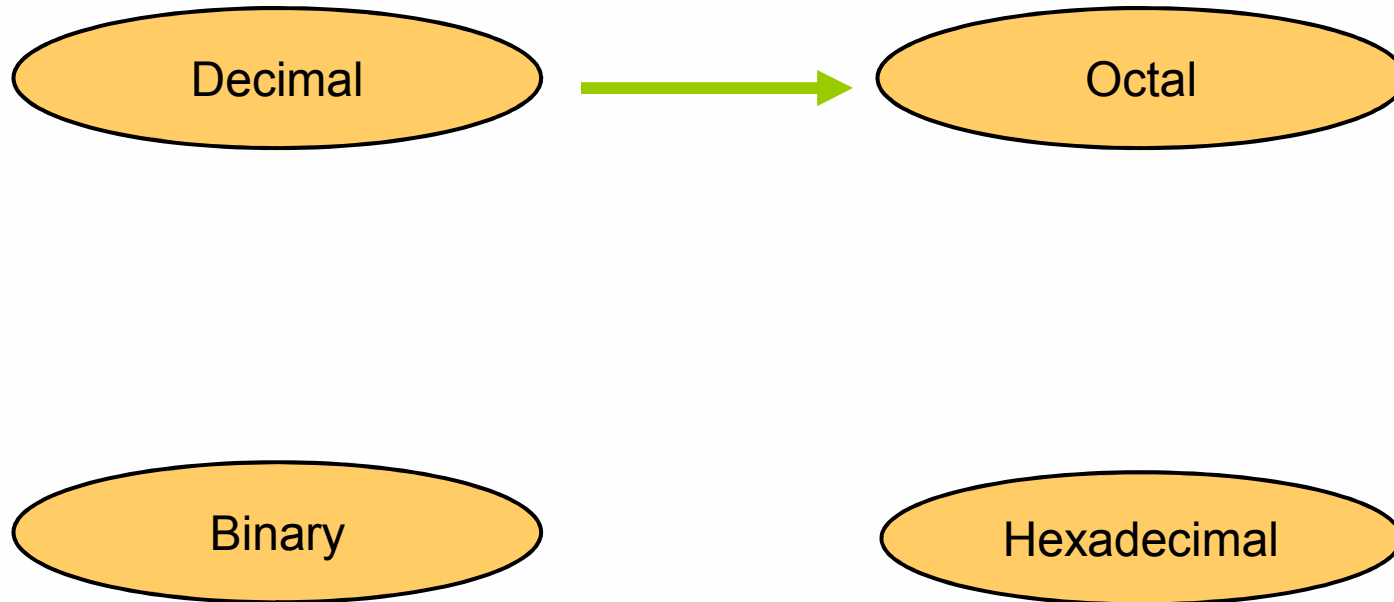
3. Determine the Decimal equivalent of $(10.1011)_2$

Binary Number	1	0	1	0	1	1
Weight of Each Bit	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Weighted Value	1×2	1×2^0	1×2^{-1}	0×2^{-2}	1×2^{-3}	1×2^{-4}
Solved Multiplication	2	1	$1/2$	0	$1/8$	$1/16$

$$\begin{aligned}\text{Sum of weight of all bits} &= 2 + 1 + (1/2) + 0 + (1/8) + (1/16) \\ &= 2 + 1 + 0.5 + 0 + 0.125 + 0.0625 \\ &= 2.6875\end{aligned}$$

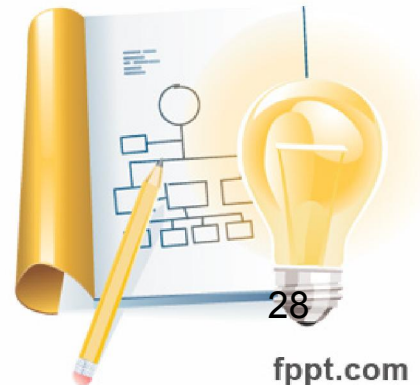


Decimal to Octal



Decimal to Octal

- Technique
 - Divide by 8
 - Keep track of the remainder



Conversion of Decimal to Octal

1. Determine the Octal equivalent of (1234)₁₀

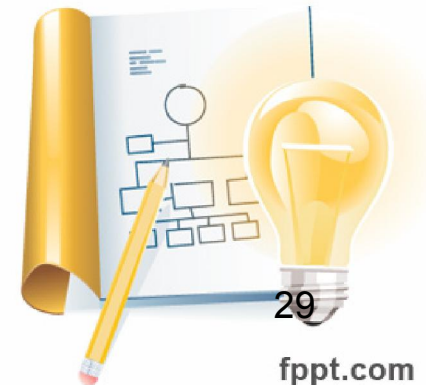
8	1234	Remainder
8	154	2
8	19	2
8	2	3
	0	2

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$1234_{10} = 2322_8$$

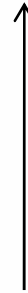


Conversion of Decimal to Octal

2. Determine the Octal equivalent of $(359)_{10}$

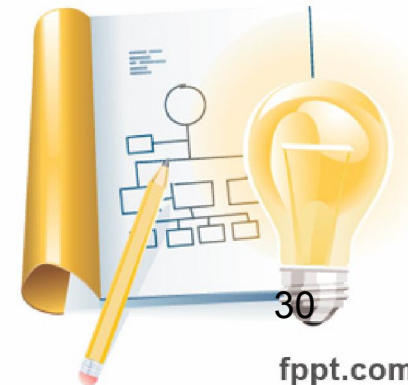
8	359	Remainder
8	44	7
8	5	4
	0	5

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$359_{10} = 547_8$$

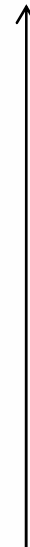


Conversion of Decimal to Octal

3. Determine the Octal equivalent of $(432267)_{10}$

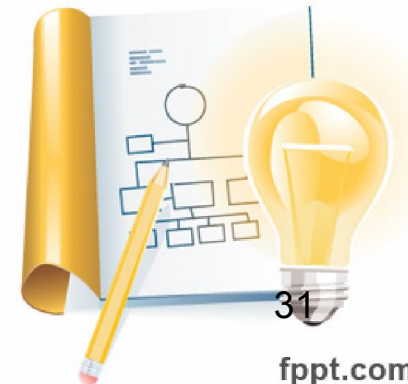
8	432267	Remainder
8	54033	3
8	6754	1
8	844	2
8	105	4
8	13	1
8	1	5
	0	1

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$432267_{10} = 1514213_8$$



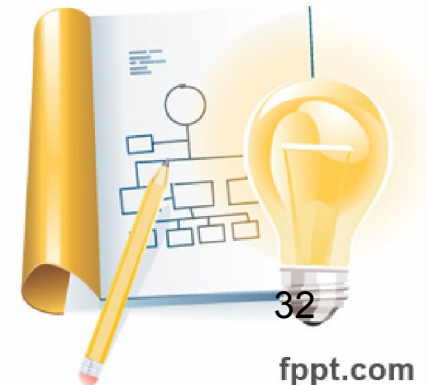
Conversion of Decimal Fraction to Octal Fraction

1. Determine the Octal equivalent of $(0.3125)_{10}$

0.3125	X	8	= 2.5	2
0.50	X	8	= 4.0	4



$$0.375_{10} = 0.24_8$$



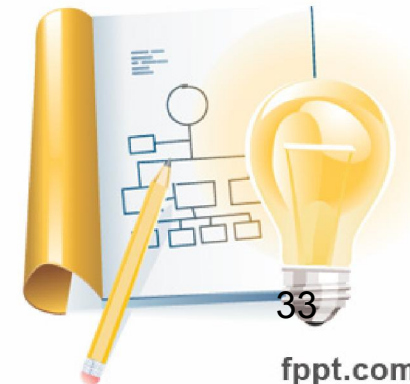
Conversion of Decimal Fraction to Octal Fraction

2. Determine the Octal equivalent of $(0.1325)_{10}$

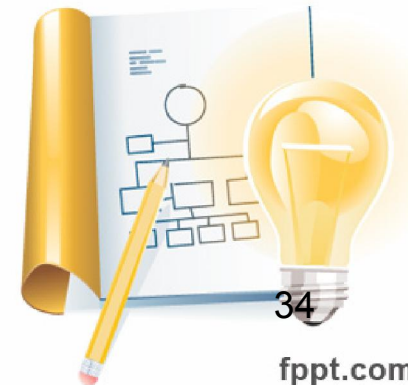
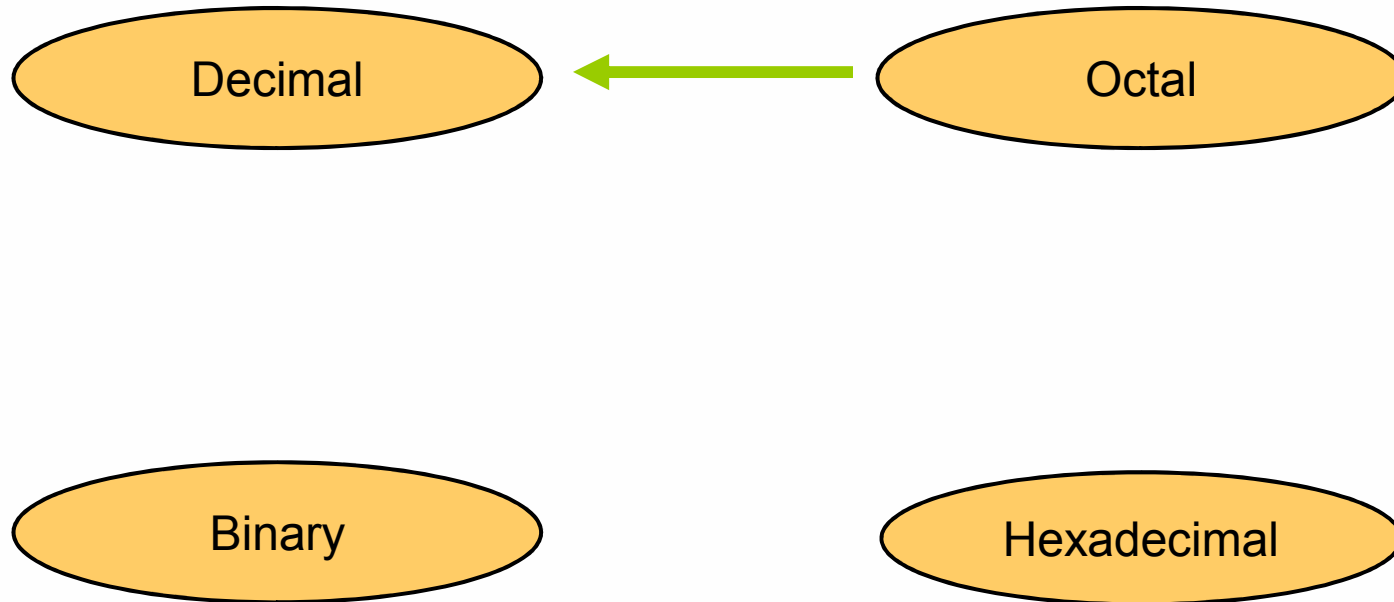
0.1325	X	8	=	1.0600	1
0.0600	X	8	=	0.4800	0
0.4800	X	8	=	3.8400	3
0.8400	X	8	=	6.7200	6
6.7200	X	8	=	5.7600	5
.					
.					
.					
∞					



$$0.1325_{10} = 0.10365_8$$

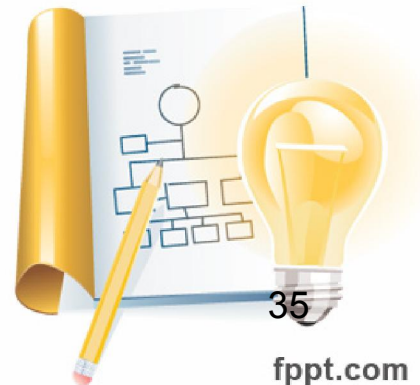


Octal to Decimal



Octal to Decimal

- Technique
 - Multiply each bit by 8^n , where n is the “weight” of the bit
 - The weight is the position of the bit, starting from 0 on the right
 - Add the results

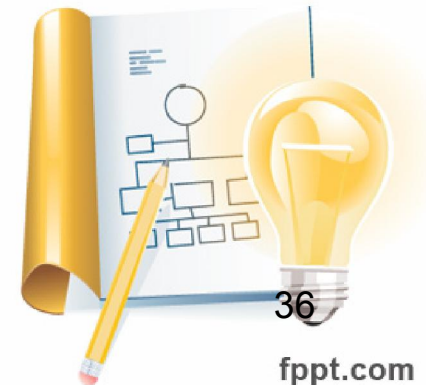


Conversion of Octal to Decimal

1. Determine the Decimal equivalent of (724)₈

Octal Number	7	2	4
Weight of Each Bit	8^2	8^1	8^0
Weighted Value	7×8^2	2×8^1	4×8^0
Solved Multiplication	448	16	4

Sum of weight of all bits = $4 + 16 + 448$
= 468

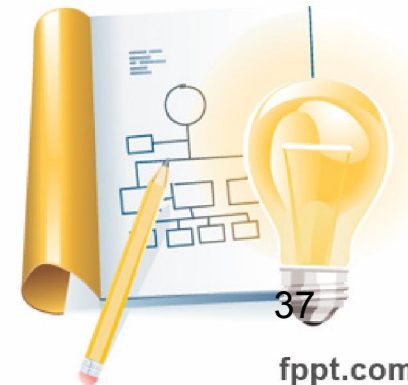


Conversion of Octal to Decimal

2. Determine the Decimal equivalent of $(456)_8$

Octal Number	4	5	6
Weight of Each Bit	8^2	8^1	8^0
Weighted Value	4×8^2	5×8^1	6×8^0
Solved Multiplication	256	40	6

$$\begin{aligned}\text{Sum of weight of all bits} &= 256 + 40 + 6 \\ &= 302\end{aligned}$$

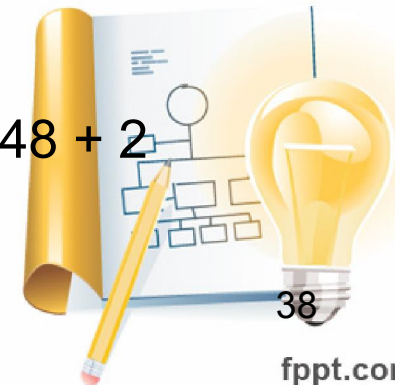


Conversion of Octal to Decimal

3. Determine the Decimal equivalent of (127662)₈

Octal Number	1	2	7	6	6	2
Weight of Each Bit	8^5	8^4	8^3	8^2	8^1	8^0
Weighted Value	1×8^5	2×8^4	7×8^3	6×8^2	6×8^1	2×8^0
Solved Multiplication	32768	8192	3584	384	48	2

$$\begin{aligned}\text{Sum of weight of all bits} &= 32768 + 8192 + 3584 + 384 + 48 + 2 \\ &= 44978\end{aligned}$$

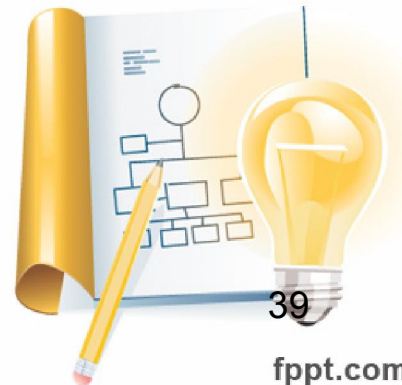


Conversion of Octal Fractions to Decimal Fractions

1. Determine the Decimal equivalent of $(237.04)_8$

Octal Number	2	3	7	0	4
Weight of Each Bit	8^2	8^1	8^0	8^{-1}	8^{-2}
Weighted Value	2×8^2	3×8^1	7×8^0	0×8^{-1}	4×8^{-2}
Solved Multiplication	128	24	7	0	0.0625

$$\begin{aligned}\text{Sum of weight of all bits} &= 128 + 24 + 7 + 0 + 0.0625 \\ &= 159.0625\end{aligned}$$

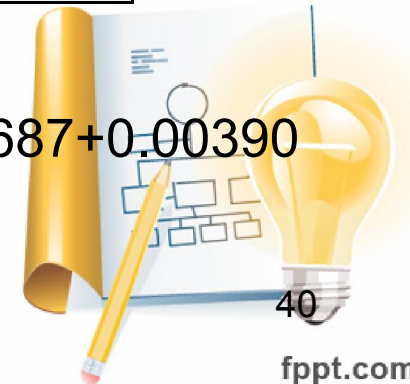


Conversion of Octal Fractions to Decimal Fractions

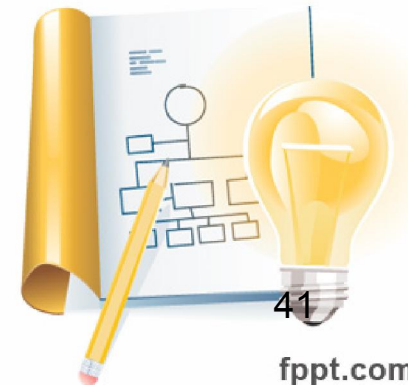
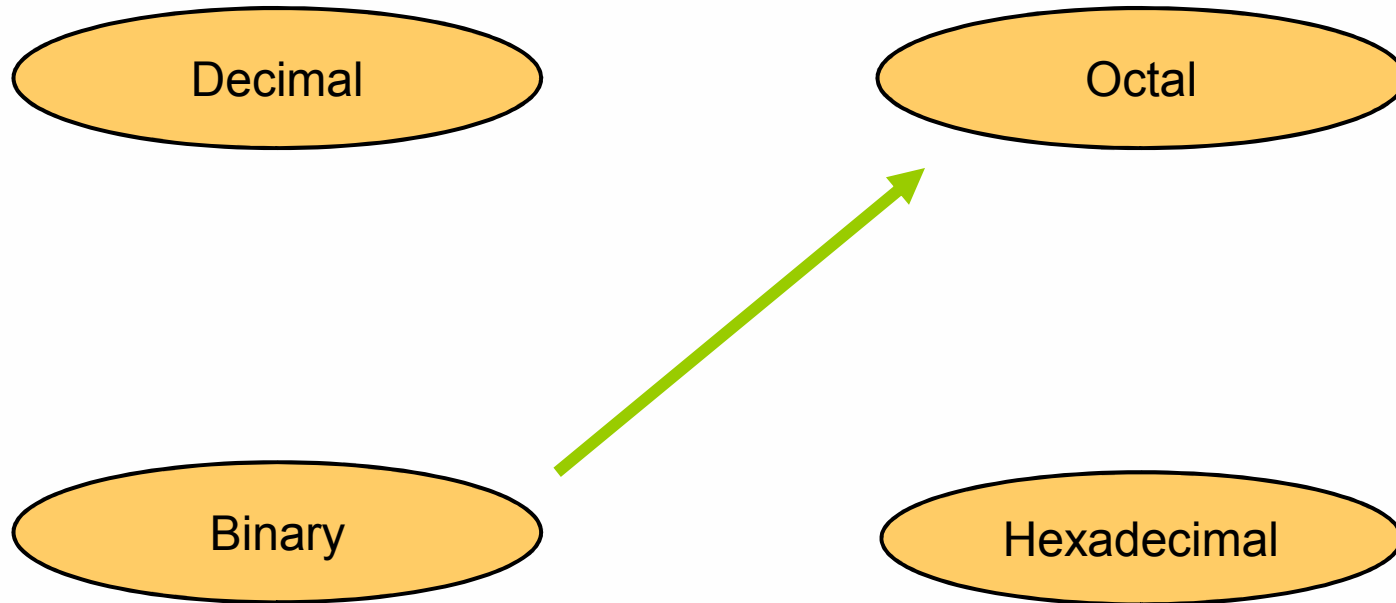
2. Determine the Decimal equivalent of $(6732.032)_8$

Octal Number	6	7	3	2	0	3	2
Weight of Each Bit	8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	8^{-3}
Weighted Value	6×8^3	7×8^2	3×8^1	2×8^0	0×8^{-1}	3×8^{-2}	2×8^{-3}
Solved Multiplication	3072	448	24	2	0	0.046875	0.00390625

$$\begin{aligned}\text{Sum of weight of all bits} &= 3072 + 448 + 24 + 2 + 0 + 0.046875 + 0.00390625 \\ &= 3546.05077\end{aligned}$$

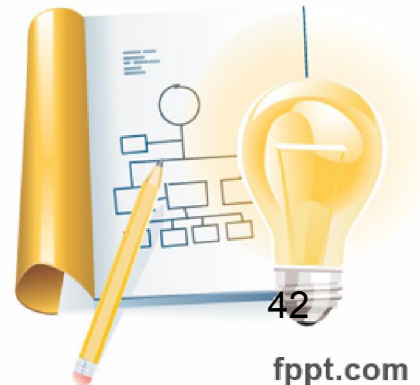


Binary to Octal



Binary to Octal

- Technique
 - Break the binary number into 3 sections starting from LSB to MSB (starting on right)
 - Convert to octal digits

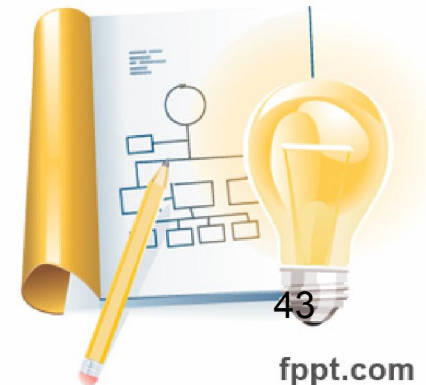


Conversion of Binary to Octal

1. Determine the Octal equivalent of $(1011010111)_2$

Binary Number	001 (MSB)	011	010	111 (LSB)
Octal Number	1	3	2	7

$$1011010111_2 = (1327)_8$$

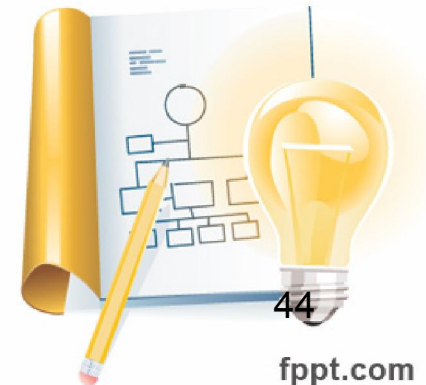


Conversion of Binary to Octal

2. Determine the Octal equivalent of $(010111)_2$

Binary Number	010(MSB)	111 (LSB)
Octal Number	2	7

$$010111_2 = (27)_8$$

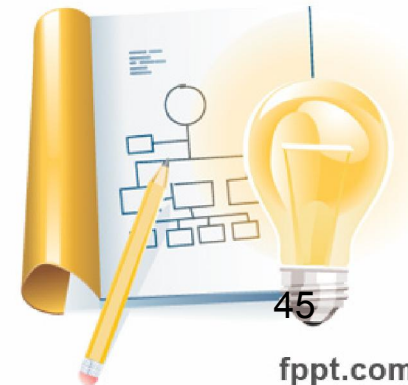


Conversion of Binary to Octal

3. Determine the Octal equivalent of $(1010111110110010)_2$

Binary Number	001(MS B)	010	111	110	110	010(LS B)
Octal Number	1	2	7	6	6	2

$$1010111110110010_2 = (127662)_8$$

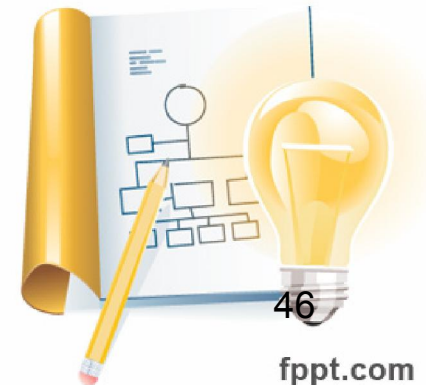


Conversion of Binary Fractions to Octal Fractions

1. Determine the Octal equivalent of $(0.110101)_2$

Binary Number	000	110(MSB)	101 (LSB)
Octal Number	0	6	5

$$0.110101_2 = (0.65)_8$$

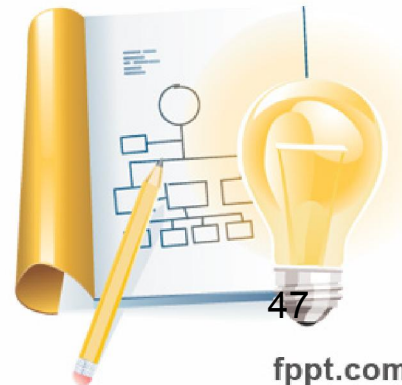


Conversion of Binary Fraction to Octal Fraction

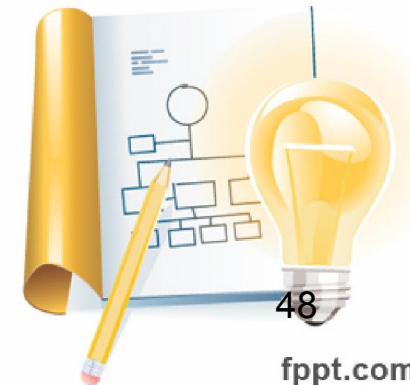
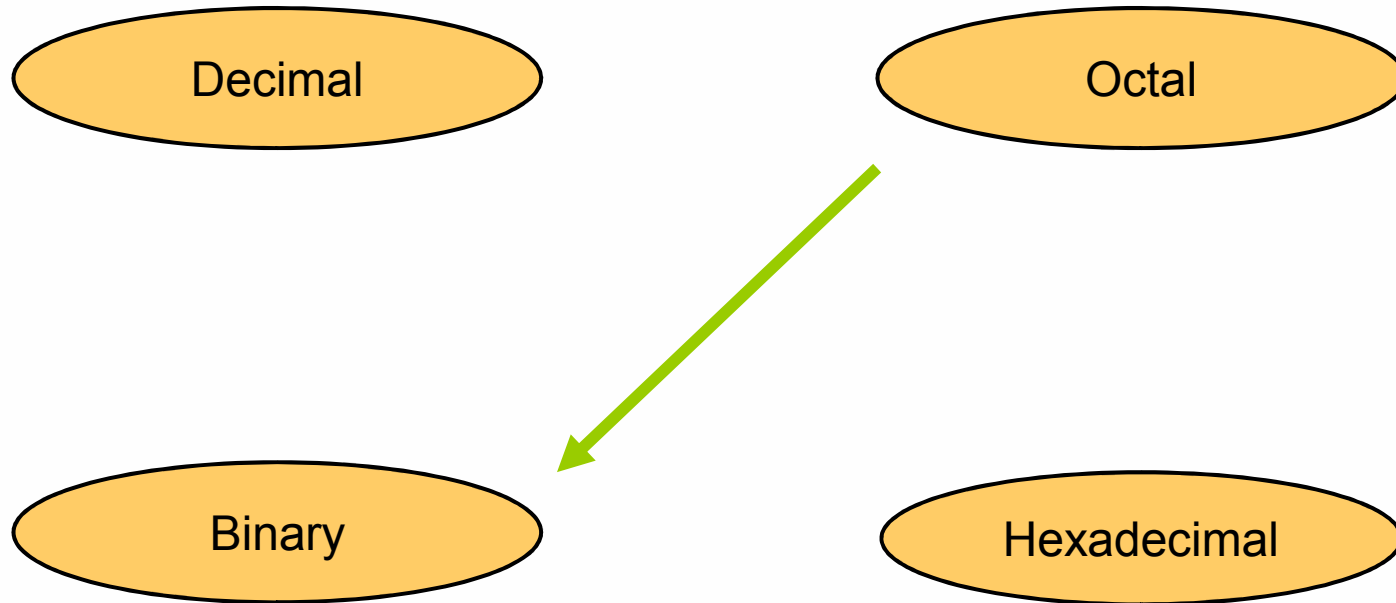
2. Determine the Octal equivalent of $(1100010.1110110)_2$

Binary Number	001	100	010	111	011	000
Octal Number	1	4	2	7	3	0

$$1100010.1110110_2 = (142.730)_8$$

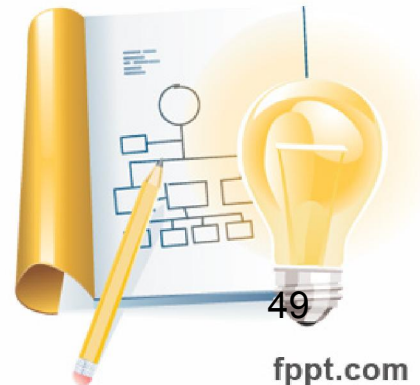


Octal to Binary



Octal to Binary

- Technique
 - Convert each octal digit to a 3-bit equivalent binary representation

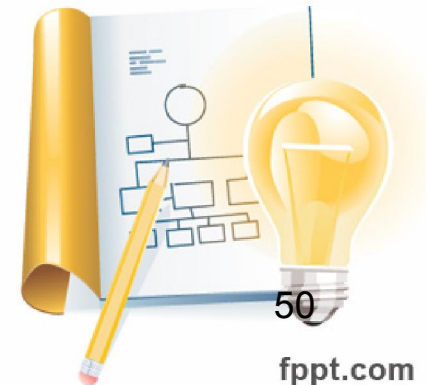


Conversion of Octal to Binary

1. Determine the Binary equivalent of $(705)_8$

Octal Number	7	0	5
Binary Coded Value	111	000	101

$$705_8 = (111000101)_2$$

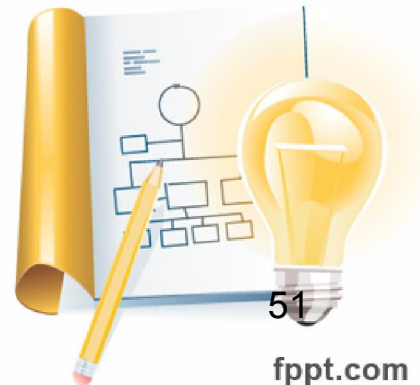


Conversion of Octal to Binary

2. Determine the Binary equivalent of $(231)_8$

Octal Number	2	3	1
Binary Coded Value	010	011	001

$$231_8 = (010011001)_2$$

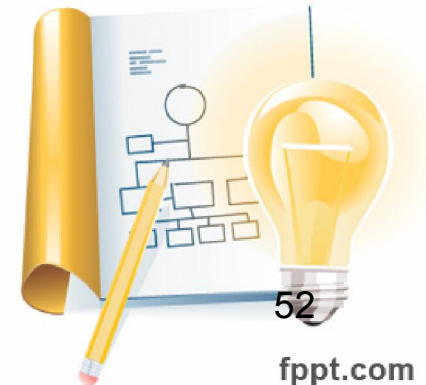


Conversion of Octal to Binary

3. Determine the Binary equivalent of $(453267)_8$

Octal Number	4	5	3	2	6	7
Binary Coded Value	100	101	011	010	110	111

$$453267_8 = (100101011010110111)_2$$

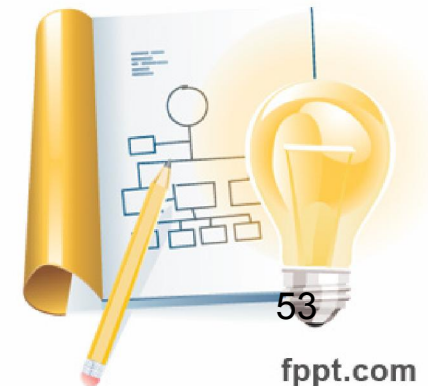


Conversion of Octal Fractions to Binary Fractions

1. Determine the Binary equivalent of $(2.335)_8$

Octal Number	2	3	3	5
Binary Coded Value	010	011	011	101

$$2.335_8 = (010.011011101)_2$$

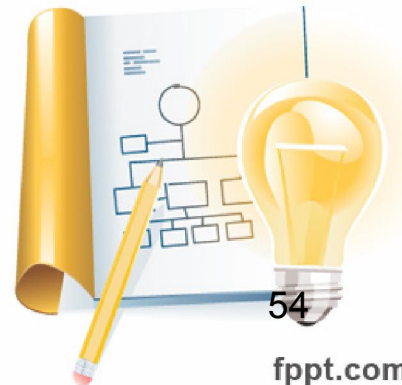


Conversion of Octal Fractions to Binary Fractions

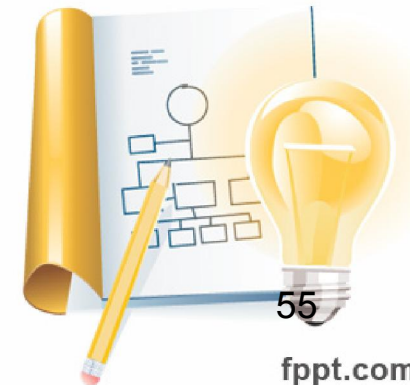
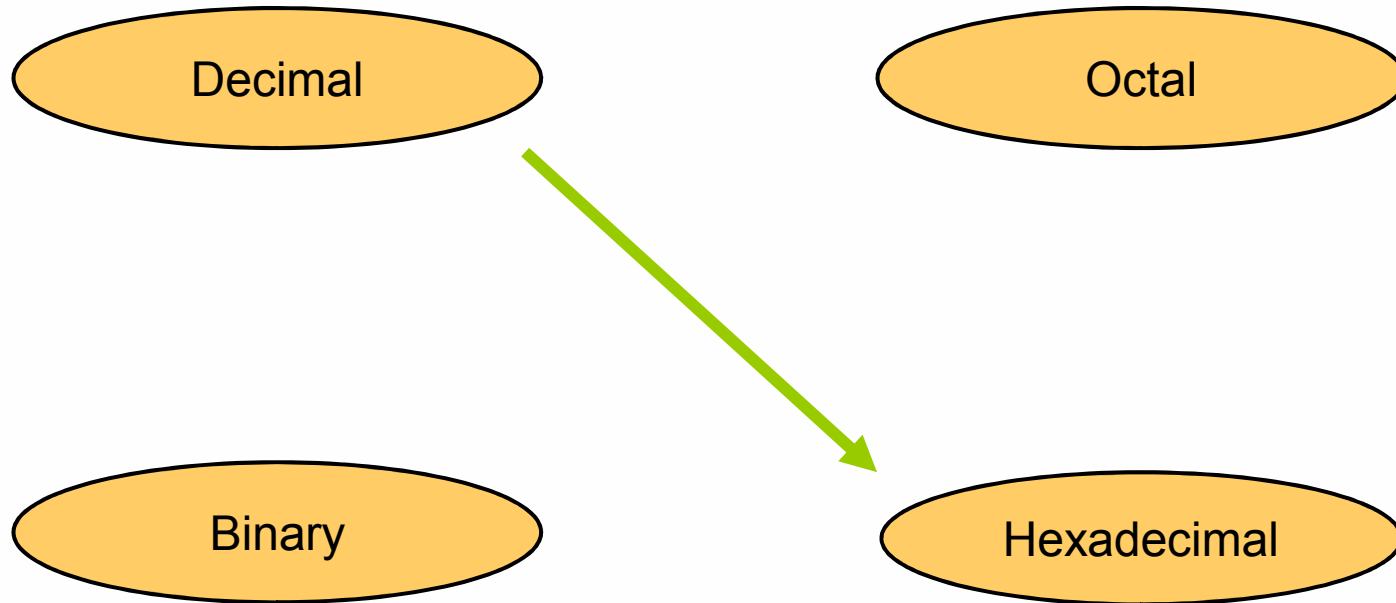
2. Determine the Binary equivalent of $(5667.2411)_8$

Octal Number	5	6	6	7	2	4	1	1
Binary Coded Value	101	110	110	111	010	100	001	001

$$5667.2411_8 = (101110110111.010100001001)_2$$

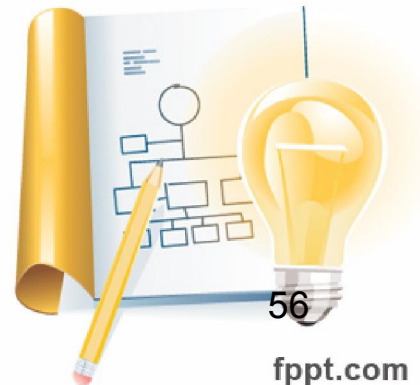


Decimal to Hexadecimal



Decimal to Hexadecimal

- Technique
 - Divide by 16
 - Keep track of the remainder

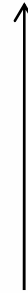


Conversion of Decimal to Hexadecimal

1. Determine the Hexadecimal equivalent of $(1234)_{10}$

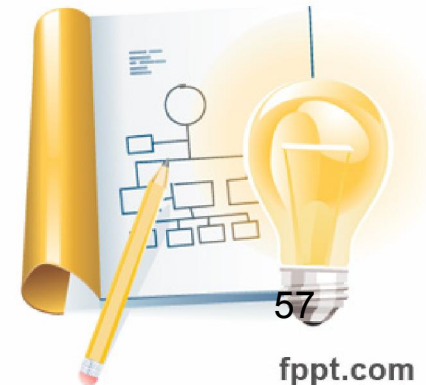
16	1234	Remainder
16	77	2
16	4	13
	0	4

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$1234_{10} = 4D2_{16}$$

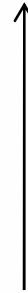


Conversion of Decimal to Hexadecimal

2. Determine the Hexadecimal equivalent of $(5112)_{10}$

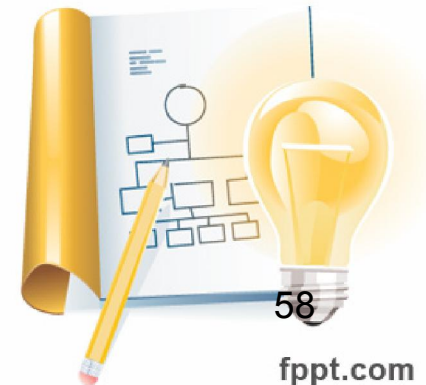
16	511 2	Remainder
16	319	8
16	19	15 = F
16	1	3
	0	1

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$5112_{10} = 13F8_{16}$$

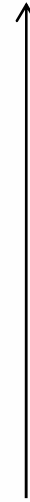


Conversion of Decimal to Hexadecimal

3. Determine the Hexadecimal equivalent of $(584666)_{10}$

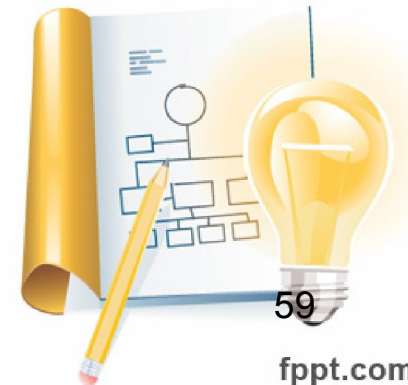
16	584666	Remainder
16	36541	10=A
16	2283	13 = D
16	142	11=B
16	8	14=E
	0	8

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$584666_{10} = 8EBDA_{16}$$



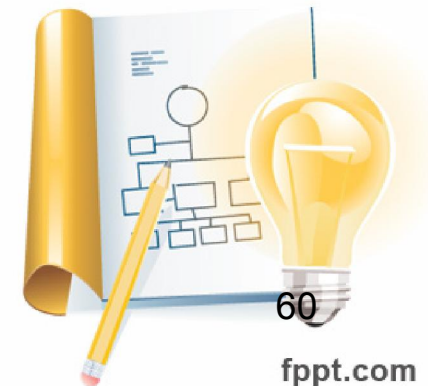
Conversion of Decimal Fraction to Hexadecimal Fraction

1. Determine the Hexadecimal equivalent of $(0.625)_{10}$

0.625	X	16	=	10.000	10 = A
0.000	X	16	=	0.000	0



$$0.1325_{10} = 0.A0_{16}$$



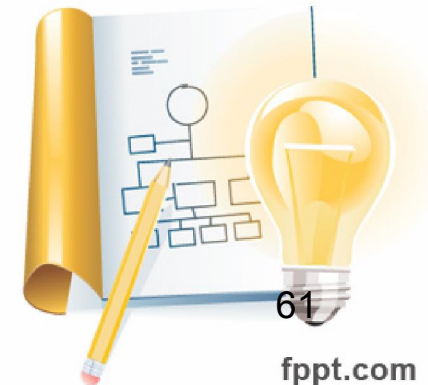
Conversion of Decimal Fraction to Hexadecimal Fraction

2. Determine the Hexadecimal equivalent of $(0.2715)_{10}$

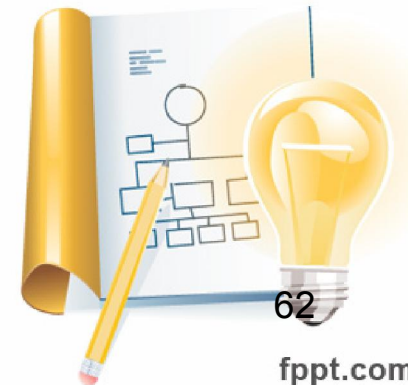
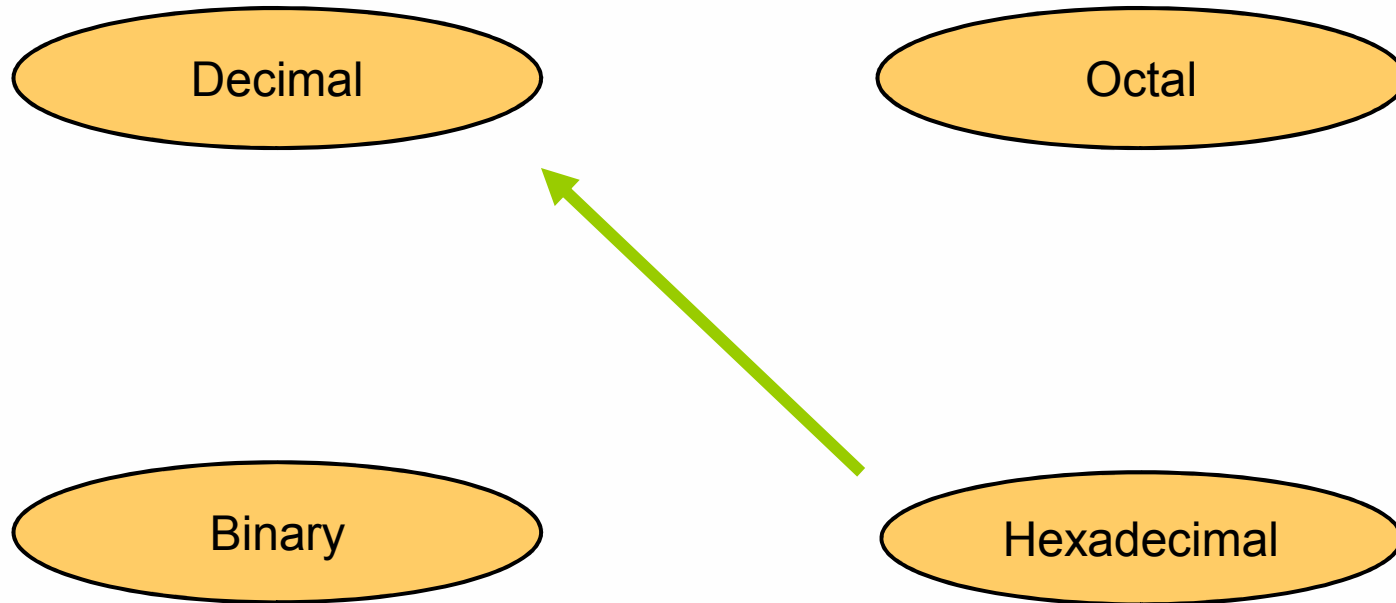
0.2715	X	16	=	4.3440	4
0.3440	X	16	=	5.5040	5
0.5040	X	16	=	8.0640	8
0.0640	X	16	=	1.0240	1
0.0240	X	16	=	0.3840	0
.					
.					
∞					



$$0.2715_{10} = 0.45810_{16}$$

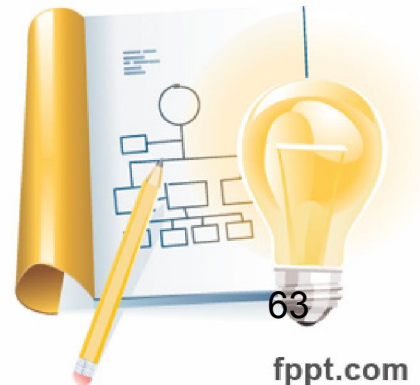


Hexadecimal to Decimal



Hexadecimal to Decimal

- Technique
 - Multiply each bit by 16^n , where n is the “weight” of the bit
 - The weight is the position of the bit, starting from 0 on the right
 - Add the results

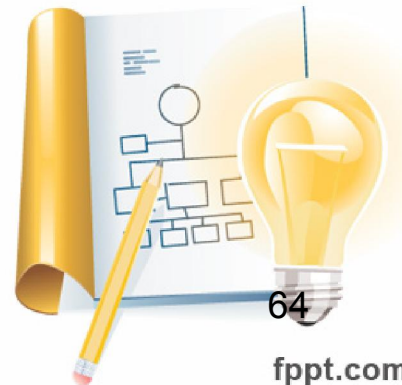


Conversion of Hexadecimal to Decimal

1. Determine the Decimal equivalent of (B14)₁₆

Hexadecimal Number	B=11	1	4
Weight of Each Bit	16^2	16^1	16^0
Weighted Value	11×16^2	1×16^1	4×16^0
Solved Multiplication	2816	16	4

$$\begin{aligned}\text{Sum of weight of all bits} &= 2816 + 16 + 4 \\ &= 2836\end{aligned}$$

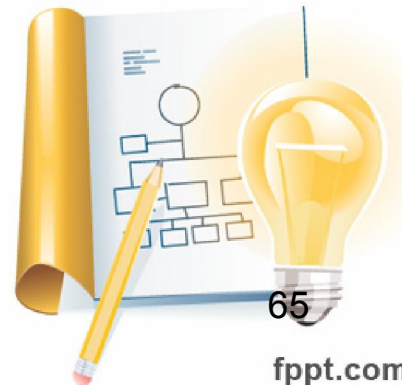


Conversion of Hexadecimal to Decimal

2. Determine the Decimal equivalent of (ABC)₁₆

Hexadecimal Number	A	B	C
Weight of Each Bit	16^2	16^1	16^0
Weighted Value	10×16^2	11×16^1	12×16^0
Solved Multiplication	2560	176	12

Sum of weight of all bits = $2560 + 176 + 12$
= 2748

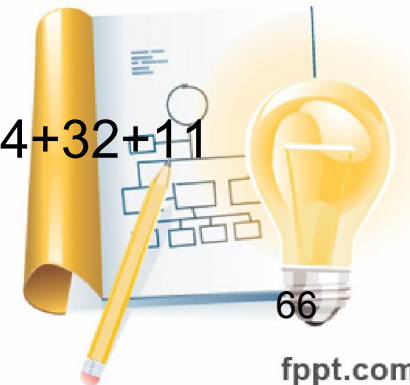


Conversion of Hexadecimal to Decimal

3. Determine the Decimal equivalent of (8AFE2B)₁₆

Hexadecimal Number	8	A	F	E	2	B
Weight of Each Bit	16^5	16^4	16^3	16^2	16^1	16^0
Weighted Value	8×16^5	10×16^4	15×16^3	14×16^2	2×16^1	11×16^0
Solved Multiplication	8388608	655360	61440	3584	32	11

Sum of weight of all bits = $8388608 + 655360 + 61440 + 3584 + 32 + 11$
= 9109035

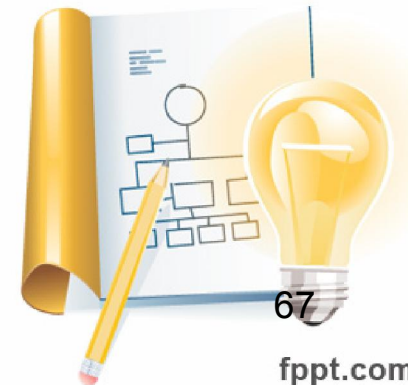


Conversion of Hexadecimal Fractions to Decimal Fractions

1. Determine the Decimal equivalent of (A.23)₁₆

Octal Number	A	2	3
Weight of Each Bit	16^0	16^{-1}	16^{-2}
Weighted Value	10×1	2×16^{-1}	3×16^{-2}
Solved Multiplication	10	0.125	0.01171875

$$\begin{aligned}\text{Sum of weight of all bits} &= 10 + 0.125 + 0.01171875 \\ &= 10.13671875\end{aligned}$$

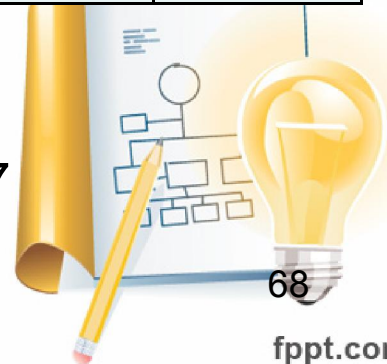


Conversion of Hexadecimal Fractions to Decimal Fractions

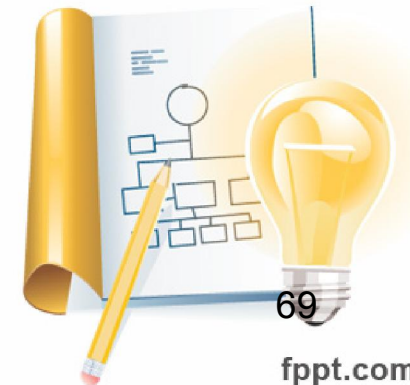
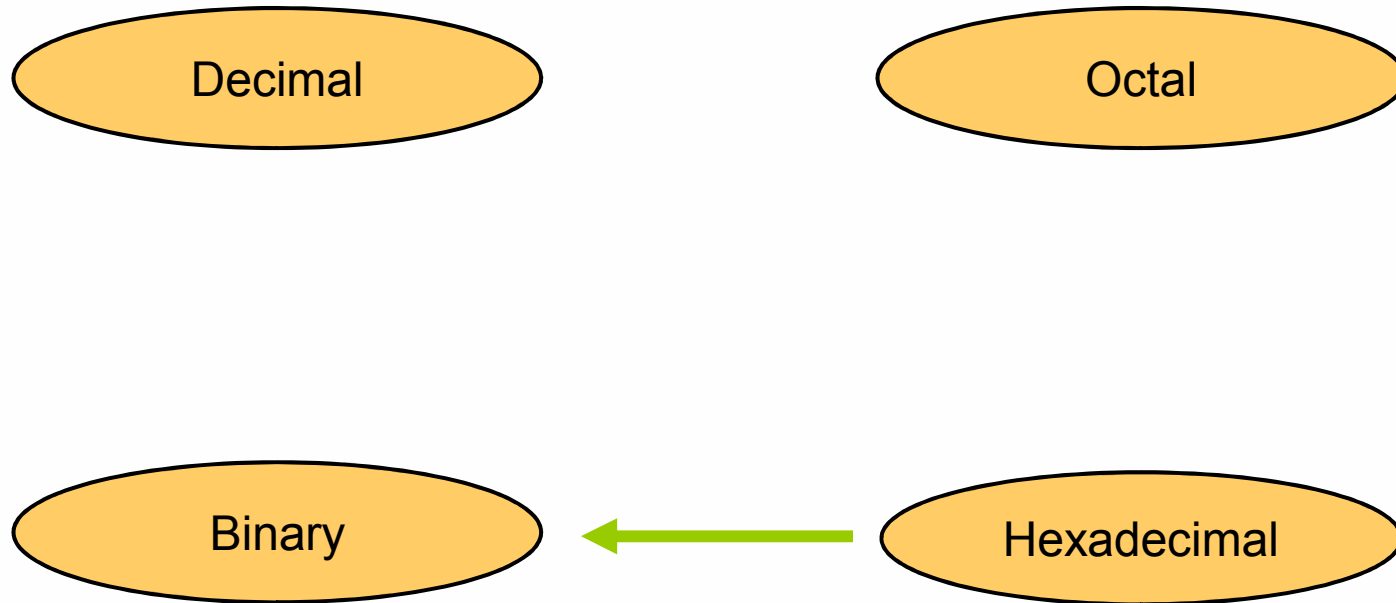
2. Determine the Decimal equivalent of (45C.8BE3)₁₆

Octal Number	4	5	C=12	8	B=11	E=14	3
Weight of Each Bit	16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3}	16^{-4}
Weighted Value	4 X 256	5 X 16	12 X 1	8 X 16^{-1}	11 X 16^{-2}	14 X 16^{-3}	3 X 16^{-4}
Solved Multiplication	1024	80	12	0.5	0.0429687	0.0034179	0.0000457

Sum of weight of all bits =
= 1024 + 80 + 12 + 0.5 + 0.0429687 + 0.0034179 + 0.0000457
= 1116.5464323

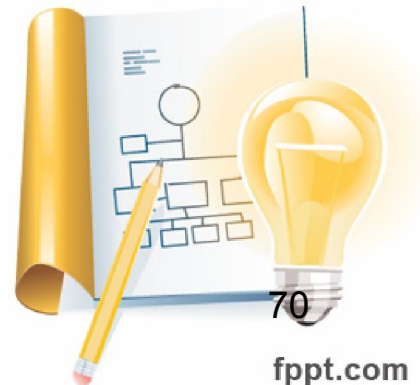


Hexadecimal to Binary



Hexadecimal to Binary

- Technique
 - Convert each hexadecimal digit to a 4-bit equivalent binary representation

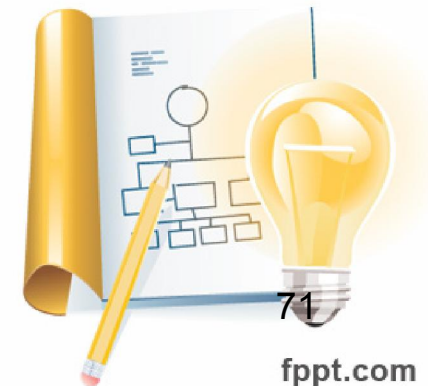


Conversion of Hexadecimal to Binary

1. Determine the Binary equivalent of (10AF)₁₆

Hexadecimal Number	1	0	A=10	F=15
Binary Coded Value	0001	0000	1010	1111

$$10AF_{16} = (0001000010101111)_2$$

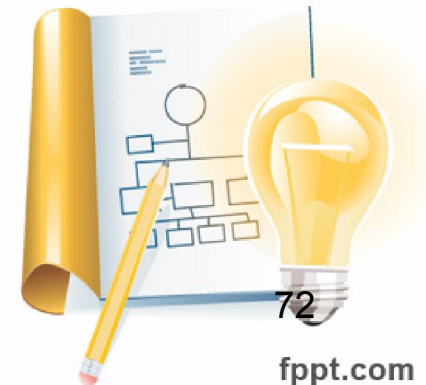


Conversion of Hexadecimal to Binary

1. Determine the Binary equivalent of (5AF)₁₆

Hexadecimal Number	5	A=10	F=15
Binary Coded Value	0101	1010	1111

$$5AF_{16} = (010110101111)_2$$

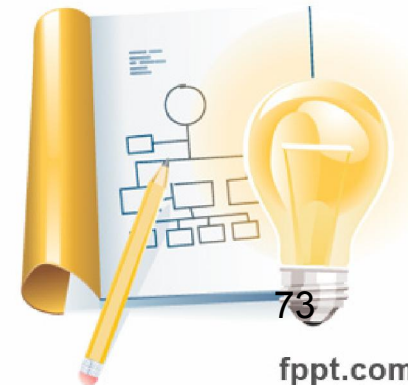


Conversion of Hexadecimal to Binary

1. Determine the Binary equivalent of (86DB45C)₁₆

Hexadecimal Number	8	6	D=13	B=11	4	5	C=12
Binary Coded Value	1000	0110	1101	1011	0100	0101	1100

$$86DB45C_{16} = (1000011011011011010001011100)_2$$

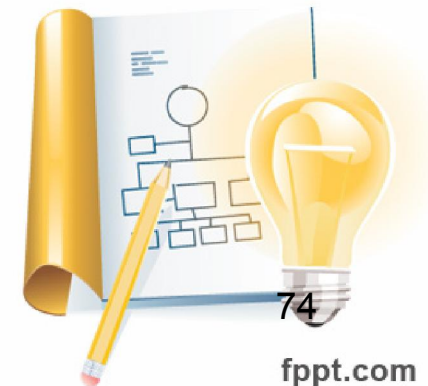


Conversion of Hexadecimal Fractions to Binary Fractions

1. Determine the Binary equivalent of (2B.6C)₁₆

Hexadecimal Number	2	B=11	6	C=12
Binary Coded Value	0010	1011	0110	1100

$$2B.6C_{16} = (00101011.01101100)_2$$

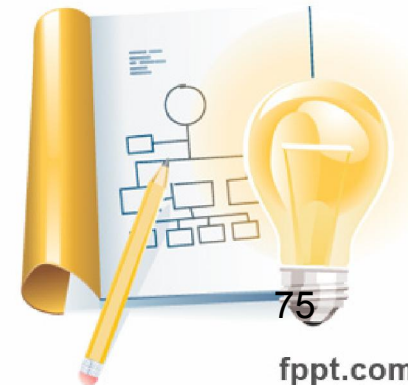


Conversion of Hexadecimal Fractions to Binary Fractions

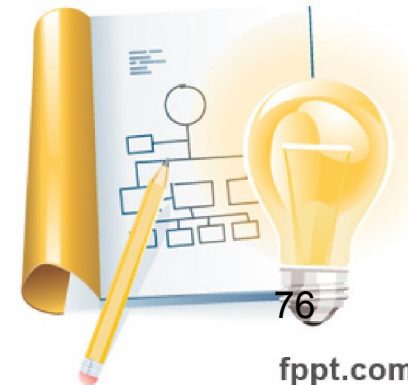
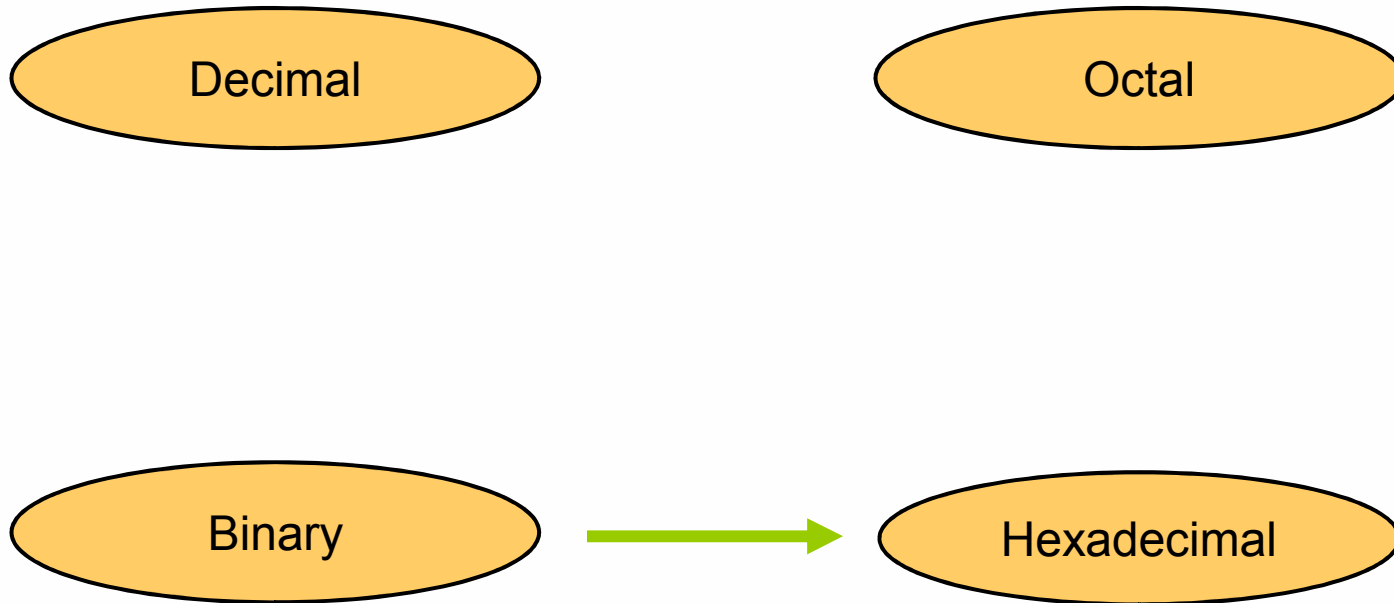
2. Determine the Binary equivalent of $(576E.34DF)_{16}$

Hexadecimal Number	5	7	6	E=14	3	4	D=13	F=15
Binary Coded Value	0101	0111	0110	1110	0011	0100	1101	1111

$$576E.34DF_{16} = (0101011101101110.0011010011011111)_2$$

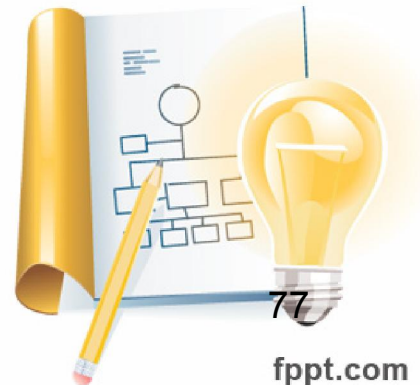


Binary to Hexadecimal



Binary to Hexadecimal

- Technique
 - Group bits in fours, starting on right
 - Convert to hexadecimal digits

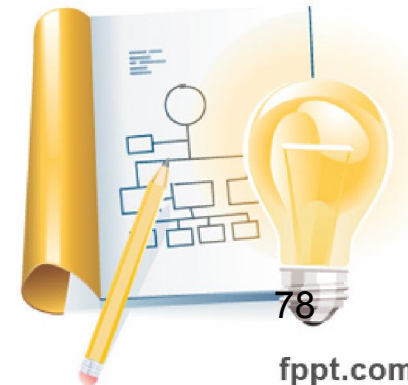


Conversion of Binary to Hexadecimal

1. Determine the Hexadecimal equivalent of $(1010111011)_2$

Binary Number	0010	1011	1011
Decimal Number	2	11	11
Hexadecimal Number	2	B	B

$$1010111011_2 = (2BB)_{16}$$

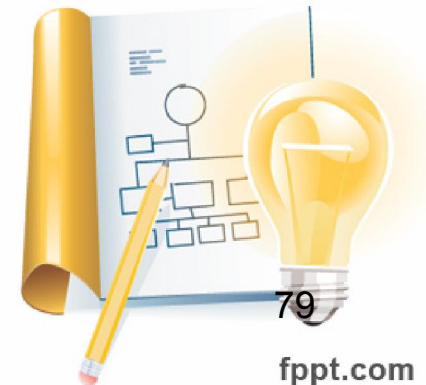


Conversion of Binary to Hexadecimal

2. Determine the Hexadecimal equivalent of $(11001011)_2$

Binary Number	1100	1011
Decimal Number	12	11
Hexadecimal Number	C	B

$$11001011_2 = (CB)_{16}$$

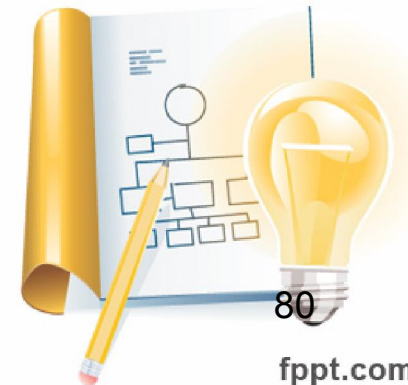


Conversion of Binary to Hexadecimal

3. Determine the Hexadecimal equivalent of $(101011110011011001)_2$

Binary Number	0010	1011	1100	1101	1001
Decimal Number	2	11	12	13	9
Hexadecimal Number	2	B	C	D	9

$$11001011_2 = (2BCD9)_{16}$$

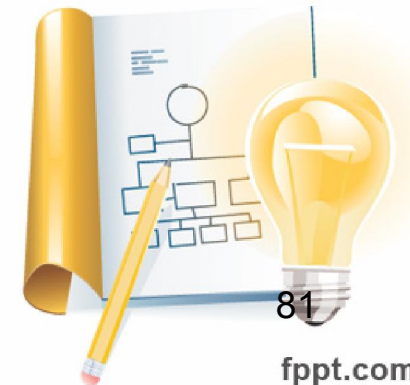


Conversion of Binary Fraction to Hexadecimal Fraction

1. Determine the Hexadecimal equivalent of $(0.11101000)_2$

Binary Number	0000	1110	1000
Decimal Number	0	14	8
Hexadecimal Number	0	E	8

$$0.11101000_2 = (0.E8)_{16}$$

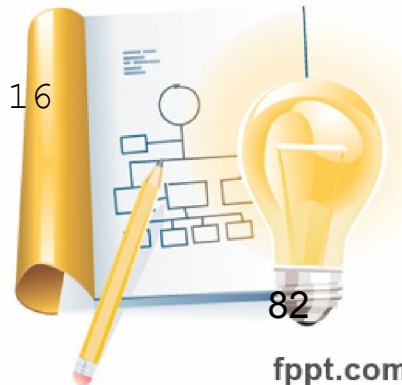


Conversion of Binary Fraction to Hexadecimal Fraction

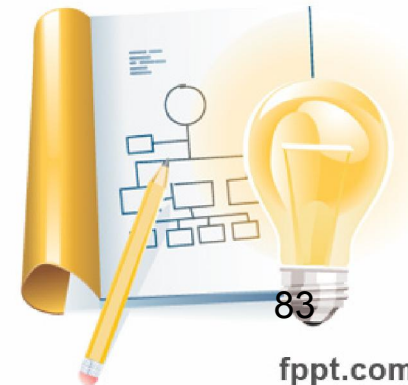
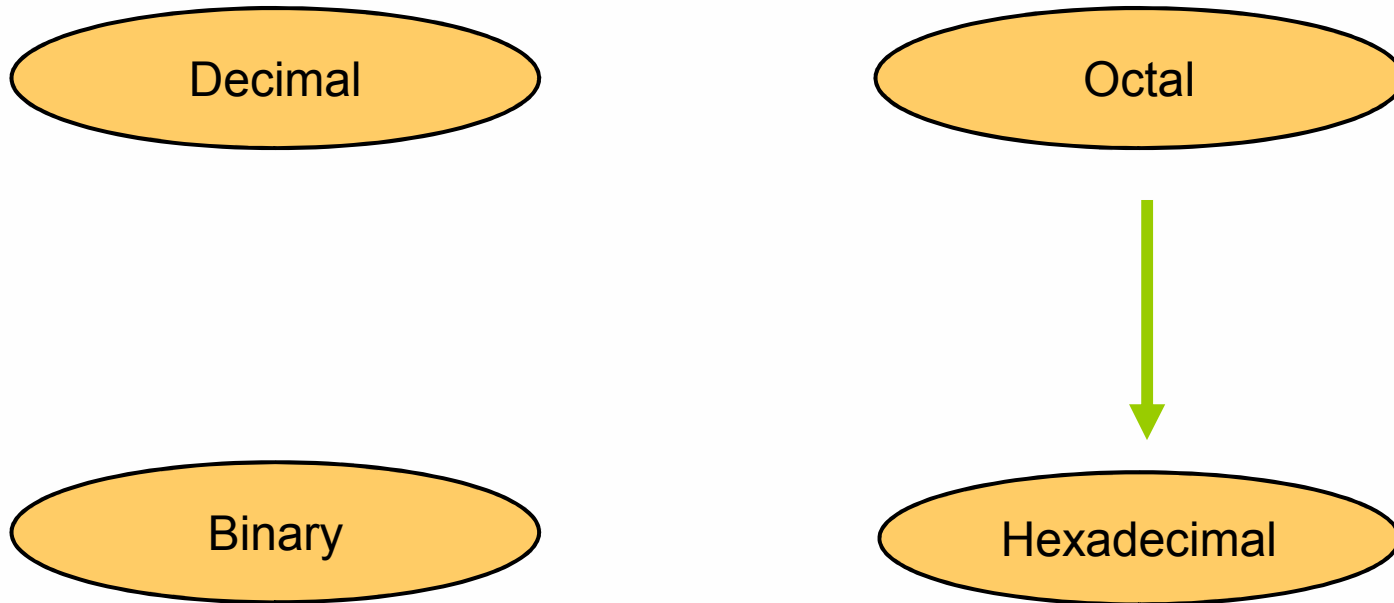
2. Determine the Hexadecimal equivalent of $(1100001.101011110011)_2$

Binary Number	0110	0001	1010	1111	0011
Decimal Number	6	1	10	15	3
Hexadecimal Number	6	1	A	F	3

$$1100001.101011110011_2 = (61.AF3)_{16}$$

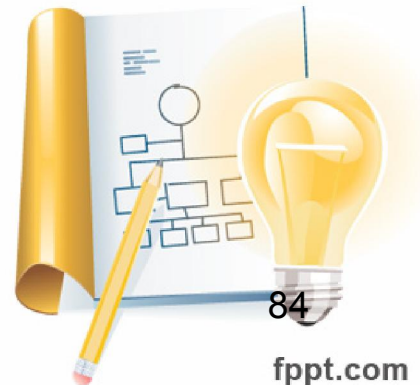


Octal to Hexadecimal



Octal to Hexadecimal

- Technique
 - Use binary as an intermediary



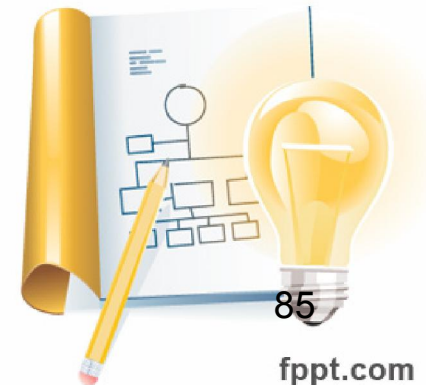
Conversion of Octal to Hexadecimal

1. Determine the Hexadecimal equivalent of $(2327)_8$

Octal Number	2	3	2	7
Binary Coded Value	010	011	010	111

$$2327_8 = (\underbrace{0100}_4 \underbrace{1101}_{13} \underbrace{0111}_7)_2$$

$$2327_8 = (4D7)_{16}$$



Conversion of Octal to Hexadecimal

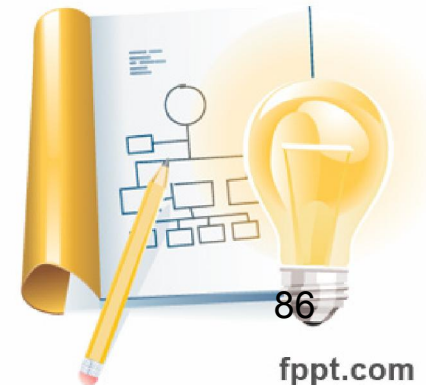
2. Determine the Hexadecimal equivalent of $(1076)_8$

Octal Number	1	0	7	6
Binary Coded Value	001	000	111	110

$$1076_8 = (\mathbf{0010 \ 0011 \ 1110})_2$$

2 3 14

$$1076_8 = (23E)_{16}$$



Conversion of Octal Fraction to Hexadecimal Fraction

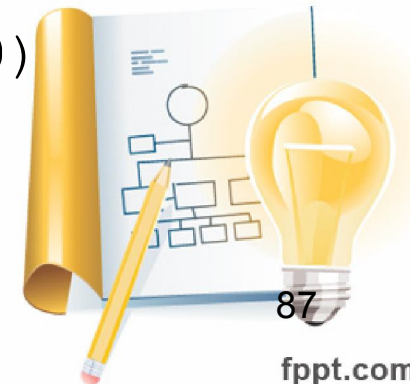
1. Determine the Hexadecimal equivalent of $(31.57)_8$

Octal Number	3	1	5	7
Binary Coded Value	011	001	101	111

$$\begin{aligned} 31.57_8 &= (011001.101111)_2 \\ &= (0001\ 1001.1011\ 1100) \end{aligned}$$

1 9 . 11 12

$$31.57_8 = (19.BC)_{16}$$



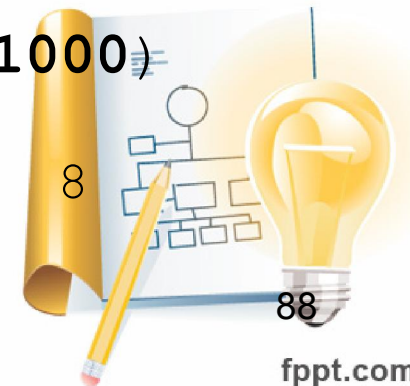
Conversion of Octal Fraction to Hexadecimal Fraction

2. Determine the Hexadecimal equivalent of $(76.665)_8$

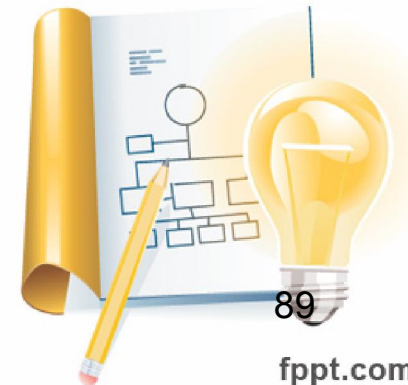
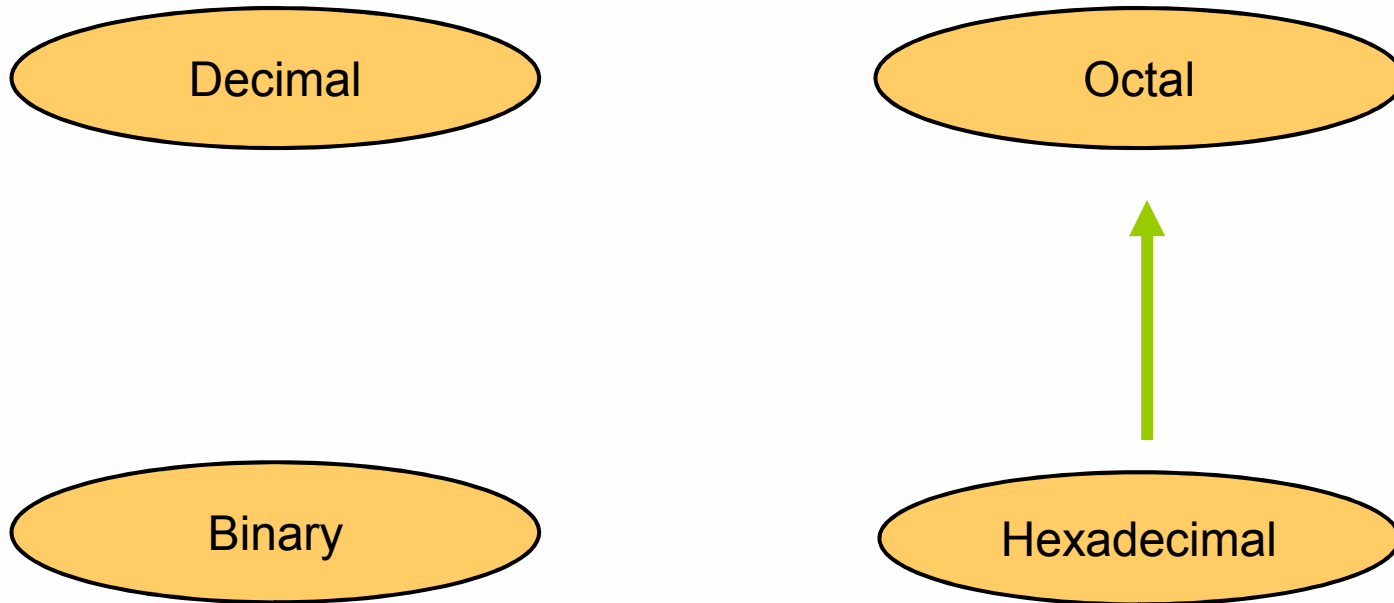
Octal Number	7	6	6	6	5
Binary Coded Value	111	110	110	110	101

$$\begin{aligned} 76.665_8 &= (111110.110110101)_2 \\ &= (0011 \ 1110 \ . \ 1101 \ 1010 \ 1000)_2 \end{aligned}$$

$$\begin{array}{ccccccc} & 3 & & 14 & & & 13 & & 10 \\ & & & & & & & & \\ 76.665_8 &= & (3E.DA8)_{16} \end{array}$$

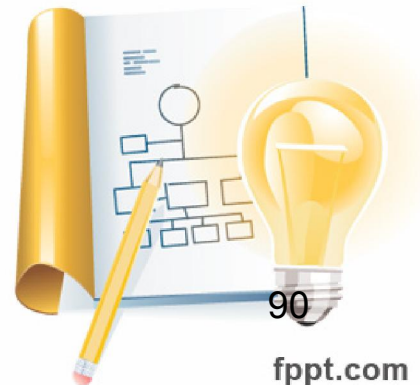


Hexadecimal to Octal



Hexadecimal to Octal

- Technique
 - Use binary as an intermediary



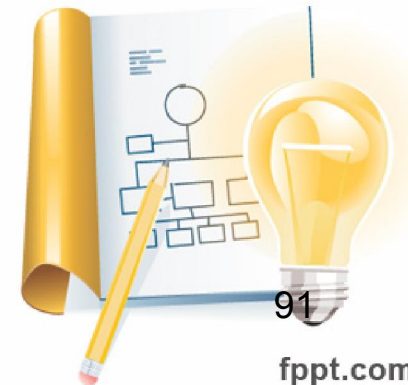
Conversion of Hexadecimal to Octal

1. Determine the Octal equivalent of $(2B6)_{16}$

Hexa decimal Number	2	B=11	6
Binary Coded Value	0010	1011	0110

$$\begin{aligned} 2B6_{16} &= (001010110110)_2 \\ &= (001 \ 010 \ 110 \ 110) \end{aligned}$$

$$\begin{array}{ccccccc} & & 1 & & 2 & & 6 & & 6 \\ 2B6_{16} &= & (1266)_8 \end{array}$$



Conversion of Hexadecimal to Octal

2. Determine the Octal equivalent of $(1F0C)_{16}$

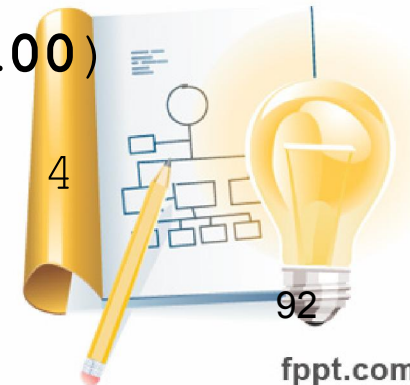
Hexa decimal Number	1	F=15	0	C=12
Binary Coded Value	0001	1111	0000	1100

$$1F0C_{16} = (0001111100001100)_2$$

$$= (000 \ 001 \ 111 \ 100 \ 001 \ 100)$$

0 1 7 4 1 4

$$1F0C_{16} = (017414)_8$$



Conversion of Hexadecimal to Octal

3. Determine the Octal equivalent of $(5DE247)_{16}$

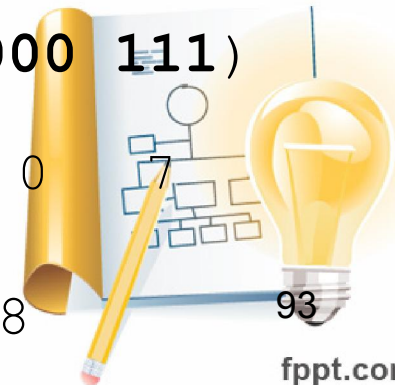
Hexa decimal Number	5	D=13	E=14	2	4	7
Binary Coded Value	0101	1101	1110	0010	0100	0111

$$5DE247_{16} = (010111011110001001000111)_2$$

$$= (010 \ 111 \ 011 \ 110 \ 001 \ 001 \ 000 \ 111)$$

$$\quad \quad \quad 2 \quad \quad 7 \quad \quad 3 \quad \quad 6 \quad \quad 1 \quad \quad 1 \quad \quad 0 \quad \quad 7$$

$$5DE247_{16} = (27361107)_8$$



Conversion of Hexadecimal Fractions to Octal Fractions

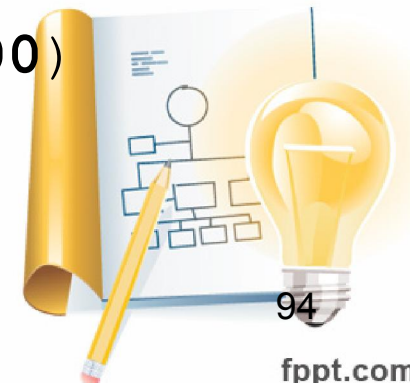
1. Determine the Octal equivalent of $(4.3C)_{16}$

Hexa decimal Number	4	3	C=12
Binary Coded Value	0100	0011	1100

$$\begin{aligned} 4.3C_{16} &= (0100.00111100)_2 \\ &= (000\ 100\ .\ 001\ 111\ 000) \end{aligned}$$

0 4 . 1 7 0

$$4.3C_{16} = (04.170)_8$$



Conversion of Hexadecimal Fractions to Octal Fractions

2. Determine the Octal equivalent of $(7B.64D)_{16}$

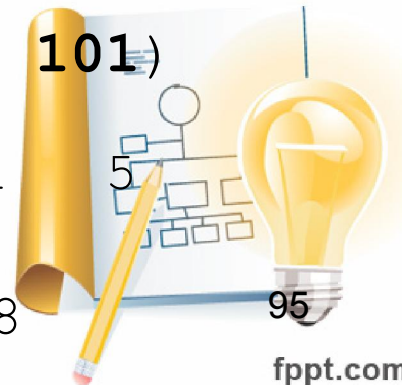
Hexa decimal Number	7	B=11	6	4	D=13
Binary Coded Value	0111	1011	0110	0100	1101

$$7B.64D_{16} = (01111011.011001001101)_2$$

$$= (001\ 111\ 011.011\ 001\ 001\ 101)$$

$$\begin{matrix} 1 & 7 & 3 & . & 3 & 1 & 1 & 5 \end{matrix}$$

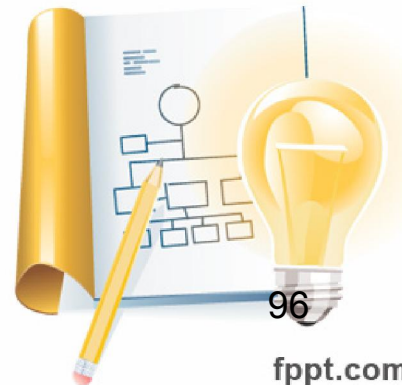
$$7B.64D_{16} = (173.3115)_8$$



Exercise – Convert ...

Decimal	Binary	Octal	Hexa- decimal
33			
	1110101		
		703	
			1AF

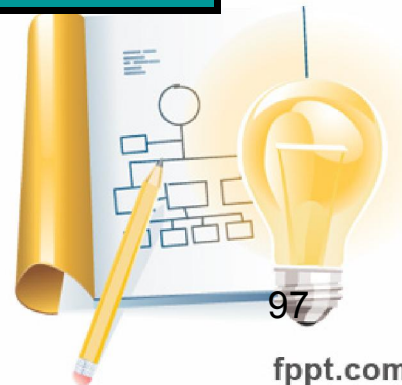
Don't use a calculator!



Exercise – Convert ...

Answer

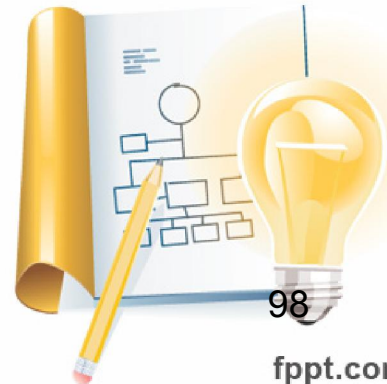
Decimal	Binary	Octal	Hexa- decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF



Exercise – Convert ...

Decimal	Binary	Octal	Hexa- decimal
29.8			
	101.1101		
		3.07	
			C.82

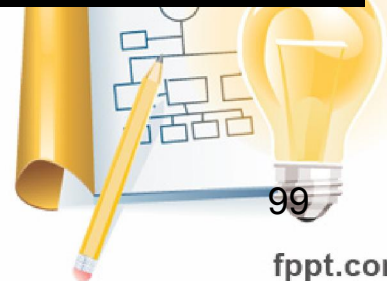
Don't use a calculator!



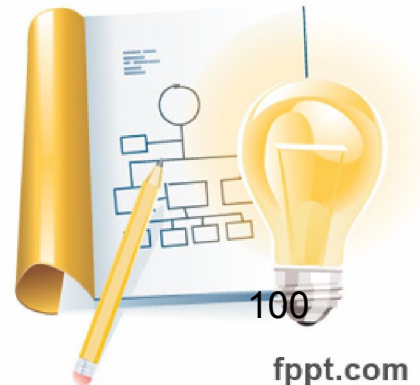
Exercise – Convert ...

Answer

Decimal	Binary	Octal	Hexa- decimal
29.8	11101.110011...	35.63...	1D.CC...
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82



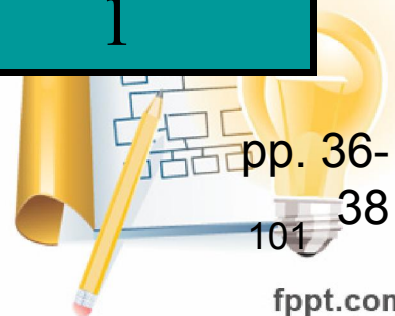
Arithmetic System



Binary Addition

- Addition of Binary Numbers

INPUT		OUTPUT	
X	Y	SUM(S)	CARRY(C)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



Binary Addition

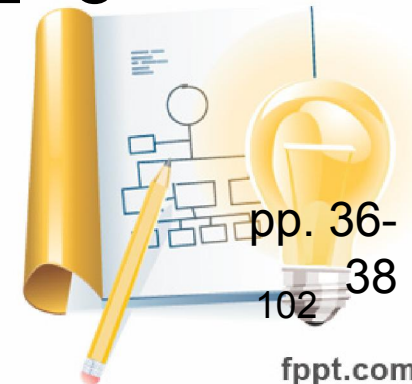
- Add binary numbers 1111 and 1010

Binary

$$\begin{array}{r} 111 \\ + 1111 \\ + 1010 \\ \hline 11001 \end{array}$$

Decimal

$$\begin{array}{r} 15 \\ + 10 \\ \hline 25 \end{array}$$



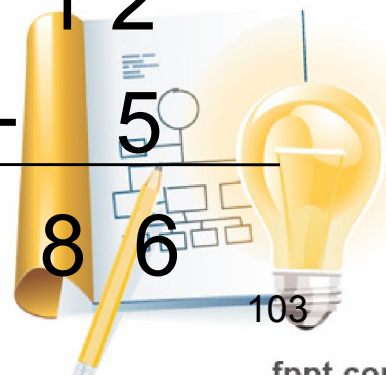
Binary Addition

- Add binary numbers 110011, 10010, 1100 and 101

Binary

$$\begin{array}{r} 111111 \\ 110011 \\ 10010 \\ 1100 \\ + 101 \\ \hline 1010110 \end{array}$$

Decimal

$$\begin{array}{r} 1 \\ 51 \\ 18 \\ 12 \\ + 5 \\ \hline 86 \end{array}$$


103

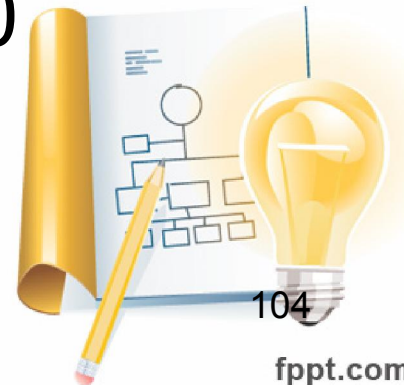
Binary Addition

- Add binary numbers 11.10, 10.10

Binary

$$\begin{array}{r} 111 \\ 11.10 \\ + 10.10 \\ \hline 110.00 \end{array}$$

$$\begin{array}{r} 1 \\ 3.5 \\ 2.5 \\ \hline 6.0 \end{array}$$

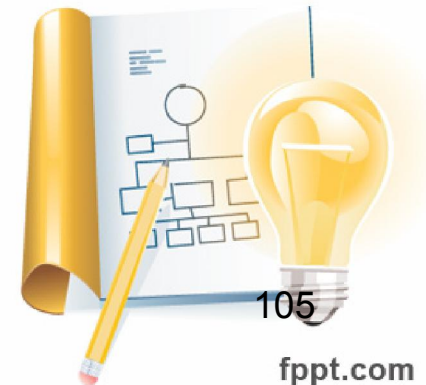


Binary Addition

- Add binary numbers 11010.0100, 1001.01, 001.11 and 10.1010

Binary

$$\begin{array}{r} 11111 \\ 11010.0100 \\ 1001.01 \\ 001.11 \\ + \quad 10.1010 \\ \hline 100111.1110 \end{array}$$



Binary Subtraction

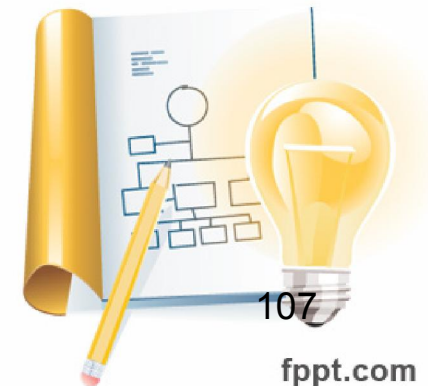
- Subtraction of Binary Numbers

INPUT		OUTPUT	
X	Y	Difference(D)	Borrow(B)
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



Example

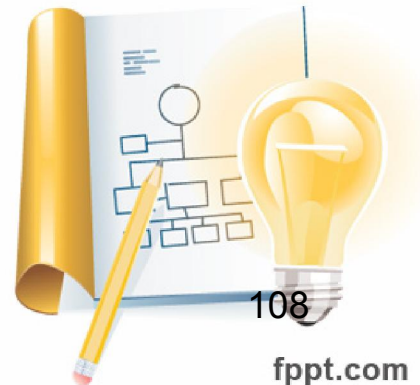
	1				
	10	10	0	10	
1	0	0	1	0	1
0	0	1	0	1	1
<hr/>					
0	1	1	0	1	0



Example

Find the binary difference of 1101 - 10110

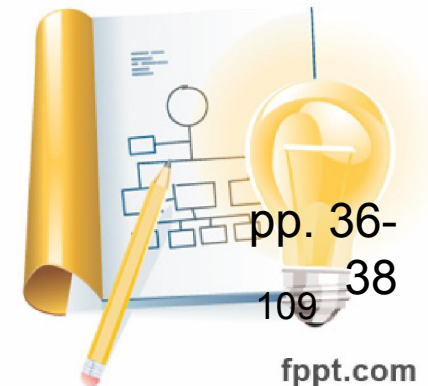
$$\begin{array}{rcccc} & & & 10 & \\ & & & \cancel{0} & \\ 1 & \cancel{1} & & & 1 \\ 1 & 0 & 1 & & 1 \\ \hline 0 & 0 & 1 & & 0 \end{array}$$



Example

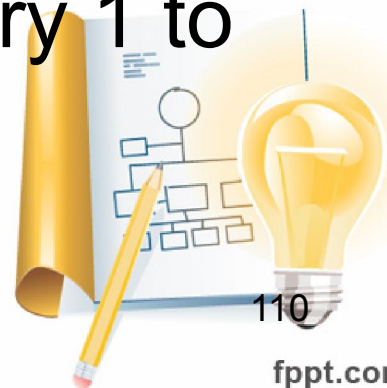
Calculate the binary difference of
11100011 - 10101000

			10	1				
			0	10	10			
1	1	1	0	0	0	1	1	
1	0	1	0	1	0	0	0	
<hr/>								
0	0	1	1	1	0	1	1	



Octal Addition

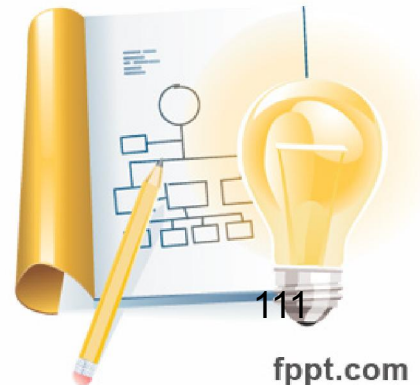
1. First, add the two digits of the unit column of the octal number in decimal.
2. During the process of addition, if the sum is less than or equal to 7, then it can be directly written as octal digit.
3. If the sum is greater than 7, then subtract 8 from that particular digit and carry 1 to the next digit position.



Octal Addition

- Add the octal numbers 26 and 17.

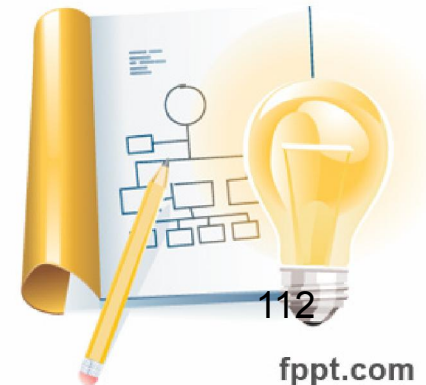
$$\begin{array}{r} 1 \\ 2 6 \\ + 1 7 \\ \hline 4 13 \\ - 8 \\ \hline 4 5 \end{array}$$



Octal Addition

- Add the octal numbers 5647 and 1425

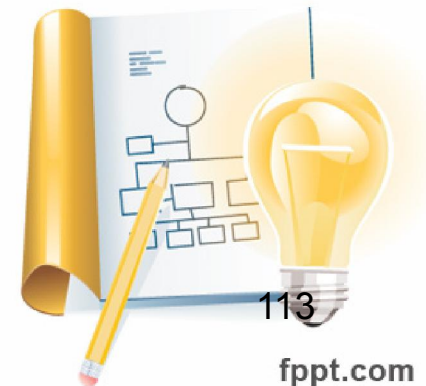
$$\begin{array}{r} 1 1 \\ 5 \ 6 4 \ 7 \\ + 1 \ 4 2 \ 5 \\ \hline 7 \ 10 \ 7 \ 12 \\ -8 -8 \\ \hline 7 \ 2 \ 7 \ 4 \end{array}$$



Octal Subtraction

Subtract the octal numbers 677 from 770

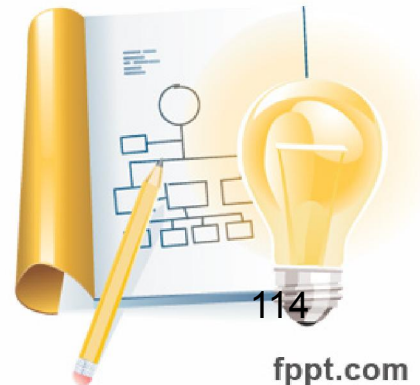
$$\begin{array}{r} 6 \\ \cancel{7} \\ 6 \\ \hline 0 \end{array} \quad \begin{array}{r} 8+6=14 \\ \cancel{6} \\ \cancel{7} \\ 7 \\ \hline 7 \end{array} \quad \begin{array}{r} 8 \\ \cancel{0} \\ 7 \\ \hline 1 \end{array}$$



Octal Subtraction

Subtract the octal numbers 2761 from 6357

5	8+2=10	8+5=13	
6	3	5	7
2	7	6	1
<hr/>			
3	3	7	6



Hexadecimal Addition

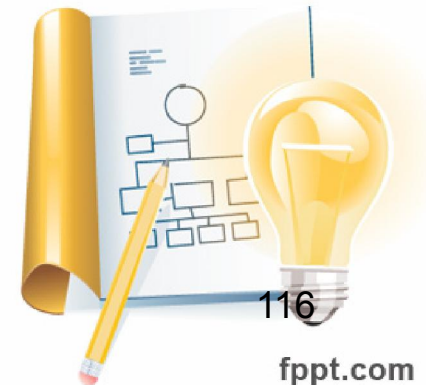
1. First, add the two digits of the unit column of the octal number in decimal.
2. During the process of addition, if the sum is less than or equal to 15, then it can be directly written as a hexadecimal digit.
3. If the sum is greater than 15, then subtract 16 from that particular digit and carry 1 to the next digit position.



Hexadecimal Addition

- Add the hexadecimal numbers 76 and 45.

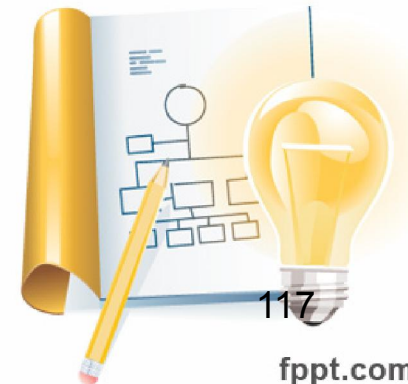
$$\begin{array}{r} 76 \\ + 45 \\ \hline 1111 \\ - - \\ \hline B B \end{array}$$



Hexadecimal Addition

- Add the hexadecimal numbers
A27E9 and 6FB43

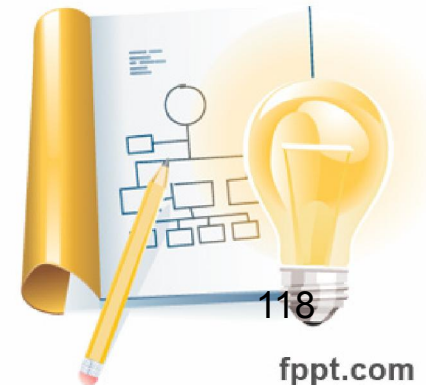
		1	1	1		
		A	2	7	E	9
	+	6	F	B	4	3
1		17	18	19	18	12
		-16	-16	-16	-16	
1		1	2	3	2	12
1		1	2	3	2	C



Hexadecimal Subtraction

Subtract the hexadecimal numbers 75 from 527

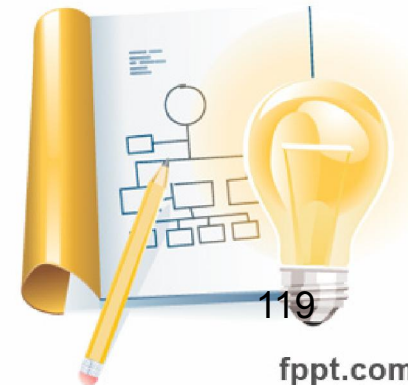
4	16+2=18	
5	2	7
	7	5
<hr/>		
4	11	2
4	B	2



Hexadecimal Subtraction

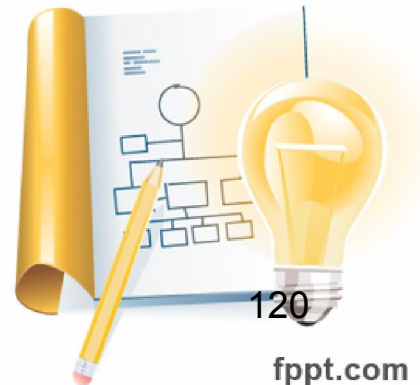
Subtract the hexadecimal numbers 1F65 from 7E2CA

	13	16+2=18		
7	E	2	C	A
	1	F	6	5
<hr/>				
7	12	3	6	5
7	C	3	6	5



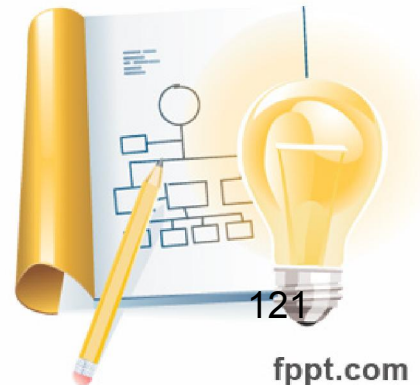
Binary Addition: Example

- $10001 + 11101 = ?$
- $101101 + 11001 = ?$
- $1011001 + 111010 = ?$
- $0011010 + 0001100 = ?$



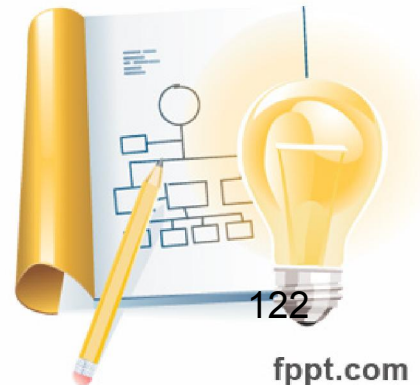
Binary Addition: Example

- $10001 + 11101 = 101110$
- $101101 + 11001 = 1000110$
- $1011001 + 111010 = 10010011$
- $0011010 + 0001100 = 0100110$



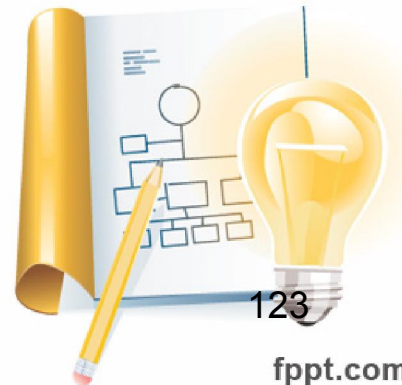
Binary Subtraction: Example

- $1011011 - 10010 = ?$
- $1010110 - 101010 = ?$
- $100010110 - 1111010 = ?$
- $1110110 - 1010111 = ?$



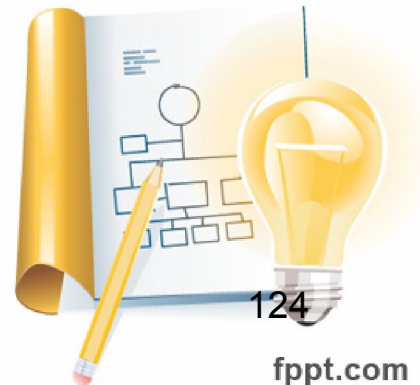
Binary Subtraction: Example

- $1011011 - 10010 = 1001001$
- $1010110 - 101010 = 0101100$
- $100010110 - 1111010 = 010011100$
- $1110110 - 1010111 = 0011111$



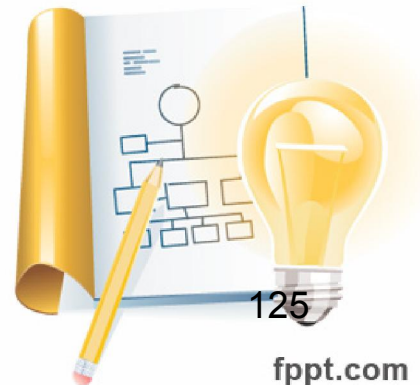
Octal Addition: Example

- $45667 + 2341 = ?$
- $77542 + 16423 = ?$
- $211345 + 456771 = ?$



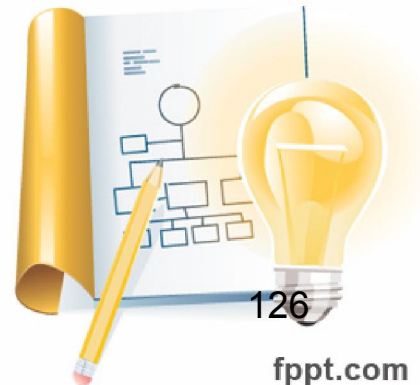
Octal Addition: Example

- $45667 + 2341 = 50230$
- $77542 + 16423 = 116165$
- $211345 + 456771 = 670336$



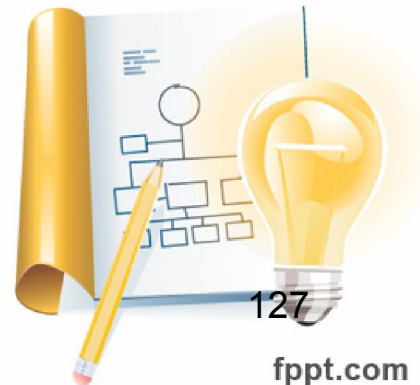
Octal Subtraction: Example

- $76542 - 44367 = ?$
- $123457 - 44663 = ?$
- $456771 - 211345 = ?$



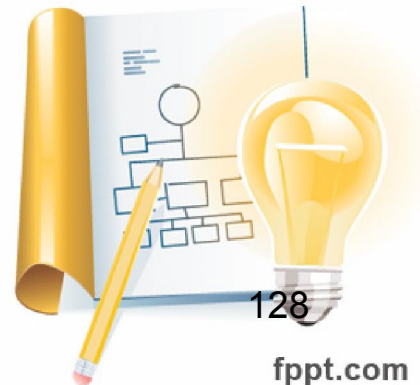
Octal Subtraction: Example

- $76542 - 44367 = 32153$
- $123457 - 44663 = 56574$
- $456771 - 211345 = 245424$



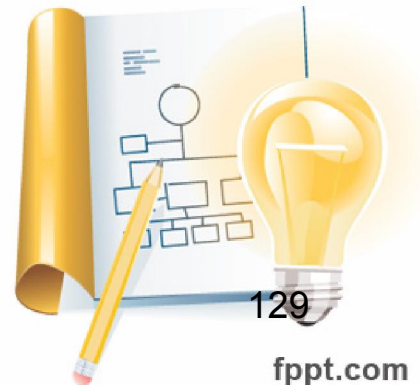
Hexadecimal Addition: Example

- $A89EF + 347C = ?$
- $2467 + 895A = ?$
- $1B59A + 2E3FD = ?$



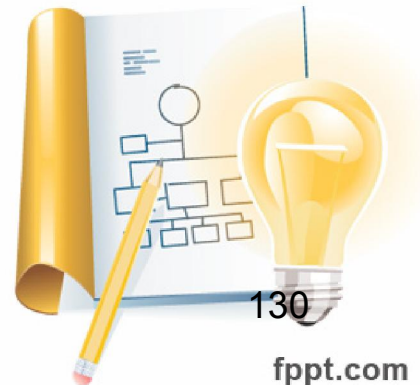
Hexadecimal Addition: Example

- $A89EF + 347C = ABE6B$
- $2467 + 895A = ADC1$
- $1B59A + 2E3FD = 49997$



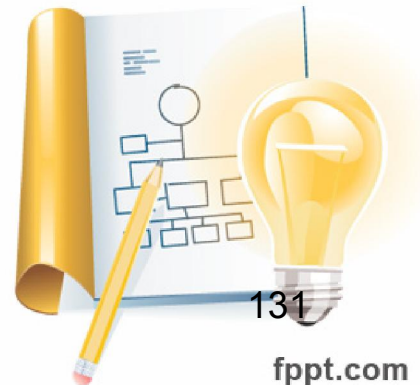
Hexadecimal Subtraction: Example

- $E89B5 - 1FA27 = ?$
- $6B432 - 59876 = ?$
- $1B59A - 2E3D = ?$
- $ABCDEF - FEDCB = ?$



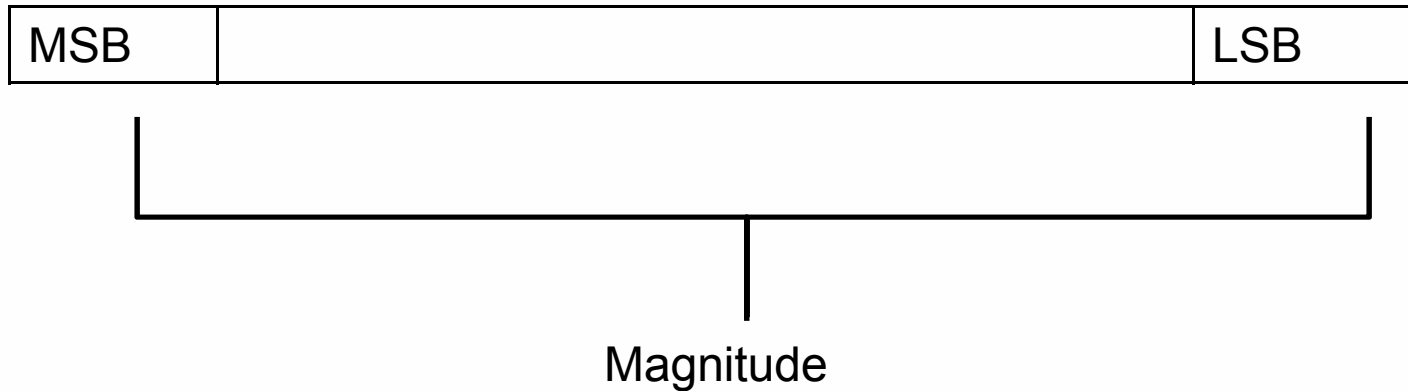
Hexadecimal Subtraction: Example

- $E89B5 - 1FA27 = C8F8E$
- $6B432 - 59876 = 11BBC$
- $1B59A - 2E3D = 1875D$
- $ABCDEF - FEDCB = 9BE024$

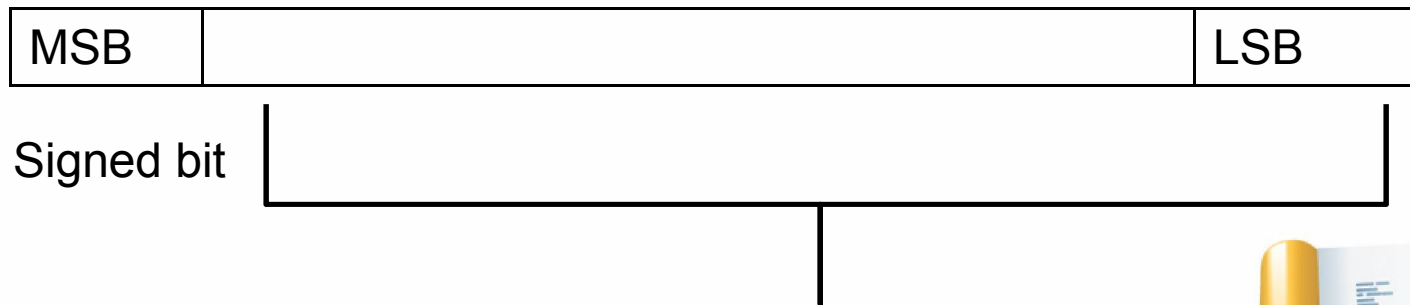


Signed And Unsigned Numbers

UnSigned Number



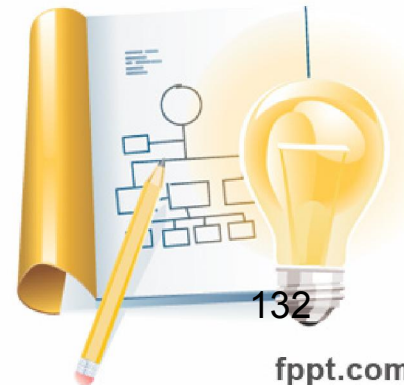
Signed Number



If MSB = 0 → Positive number

Magnitude

If MSB = 1 → Negative number



COMPLEMENT OF NUMBERS

Two types of complements for base R number system:

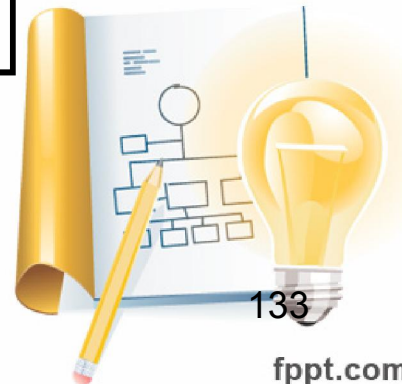
- R's complement and (R-1)'s complement

Example

- Decimal $r=10$, 9's complement and 10's complement
- Binary $r=2$, 1's and 2's complement
- Octal $r=8$, 7's and 8's complement

The R's Complement

Add 1 to the low-order digit of its (R-1)'s complement



SIGNED NUMBERS

Need to be able to represent both *positive* and *negative* numbers

- Following 3 representations

Signed magnitude representation
Signed 1's complement representation
Signed 2's complement representation

Example: Represent +9 and -9 in 7 bit-binary number

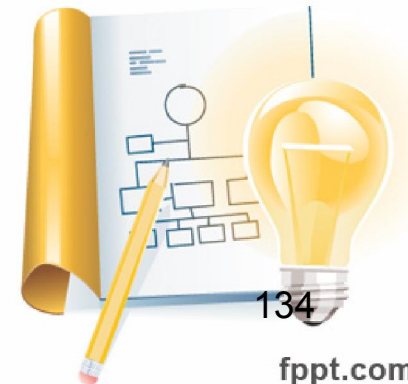
Only one way to represent +9 ==> 0 001001

Three different ways to represent -9:

In signed-magnitude: 1 001001

In signed-1's complement: 1 110110

In signed-2's complement: 1 110111



Given number →



1's complement →



Add 1 +

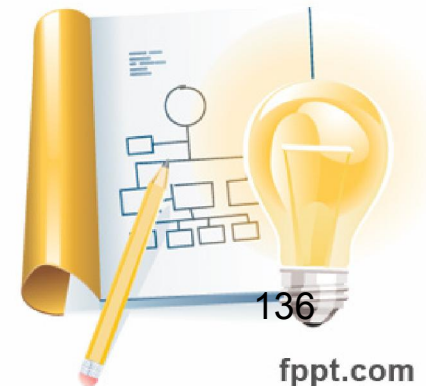


2's complement →



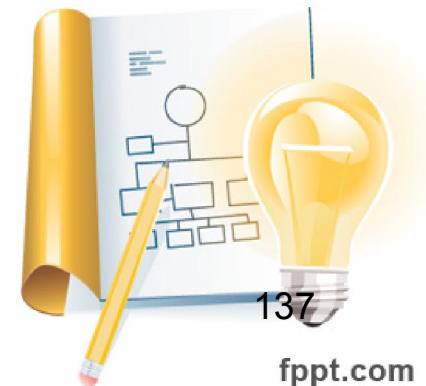
- (14) in 2's complement form

Binary Number		1	1	1	0
1's Complement		0	0	0	1
2's Complement		0	0	1	0
With Sign Bit	1	0	0	1	0



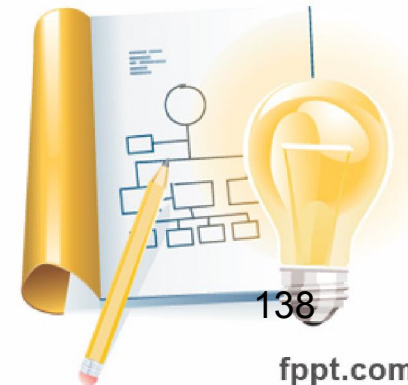
+(12) in 2's complement form

Binary Number		1	1	0	0
1's Complement		0	0	1	1
2's Complement		0	1	0	0
With Sign Bit	0	0	1	0	0



Addition-Subtraction of signed number using 2's complement

1. Convert both numbers to equivalent binary form
2. Find the 2's complement of subtrahend
3. Add this 2's complement number to the minuend
4. If there is carry of 1, ignore it from the result to obtain the correct result.
5. If there is no carry, recompute the result
 - attach the negative sign to the obtained result



Addition-Subtraction of signed number using 2's complement

1. Add $(27)_{10}$ and $(-11)_{10}$ using complementary representation for the negative value.

$$27 = 011011 \quad \text{and} \quad 11 = 001011$$

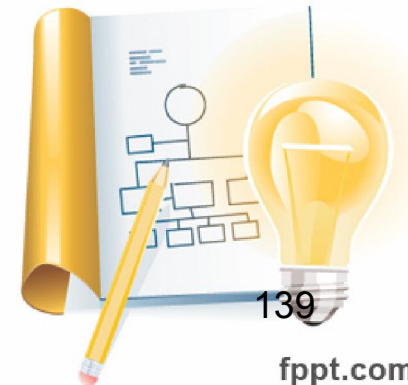
$$\begin{aligned} \text{2's complement of } (001011) &= \text{1's complement of } (001011) + 1 \\ &= 110100 + 1 \\ &= 110101 \end{aligned}$$

Add (011011) and (110101)

$$\begin{array}{r} 1 1 1 1 \\ 0 1 1 1 1 \\ + 1 1 0 1 1 \\ \hline \textcircled{1} 0 1 0 0 0 \end{array}$$

Carry

Hence, result is $(010000)_2$ or $(16)_{10}$



Addition-Subtraction of signed number using 2's complement

2. Subtract $(25)_{10}$ from $(42)_{10}$

$$25 = 011001 \quad \text{and} \quad 42 = 101010$$

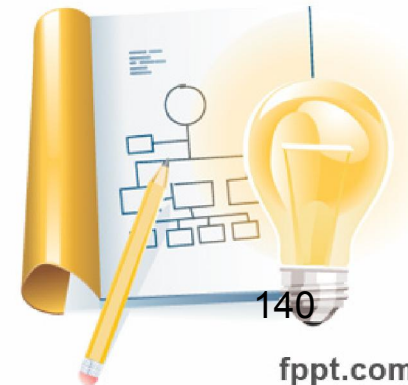
$$\begin{aligned} \text{2's complement of } (011001) &= \text{1's complement of } (011001) + 1 \\ &= 100110 + 1 \\ &= 100111 \end{aligned}$$

Add (101010) and (100111)

$$\begin{array}{r} \\ \\ \\ + \\ \hline \textcircled{1} \end{array}$$

Carry

Hence, Ignore carry and
result is $(010001)_2$ or $(17)_{10}$



Addition-Subtraction of signed number using 2's complement

3. Subtract $(14)_{10}$ from $(46)_{10}$

14 = 00001110 and 46 = 00101110

2's complement of (00001110) = 1's complement of (00001110) + 1

$$= 11110001 + 1$$

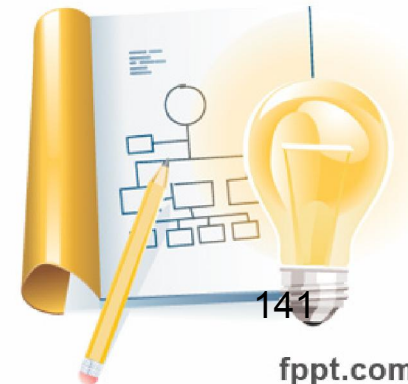
$$= 11110010$$

Add (00101110) and (11110010)

$$\begin{array}{r}
 \\
 \\
 + \\
 \hline
 \textcircled{1}
 \end{array}$$

↓ Carry

Hence, Ignore carry and
result is $(00100000)_2$ or $(32)_{10}$



Addition-Subtraction of signed number using 2's complement

4. Subtract 84 from 68 ($68 - 84$)

binary of 84 = 1010100

binary of 68 = 1000100

1's complement of 84 = 0101011

2's complement of 84 =

$$\begin{array}{r} 0101011 \\ + \quad \quad 1 \\ \hline 0101100 \end{array}$$

Now, add $1000100 + 0101100 = 1110000$

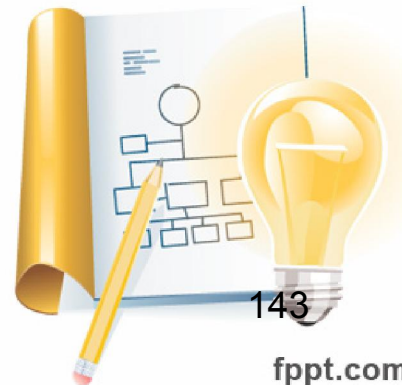
There is no carry, so we will take 2's complement of result

Answer : - 0010000



Two's Complement Overflow Rules

- The rules for detecting overflow in a two's complement sum are simple:
 1. If the sum of two positive numbers yields a negative result, the sum has overflowed.
 2. If the sum of two negative numbers yields a positive result, the sum has underflowed.



Overflow

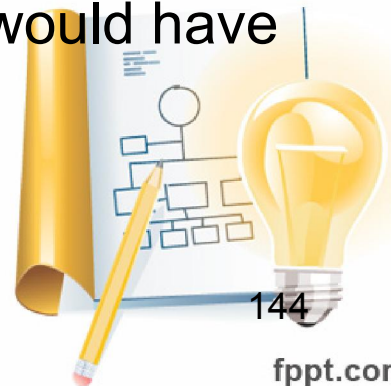
- **Example:** If we add two positive number $7 + 6$ using 4-bit binary number, result should be $+13$

$$\begin{array}{r} 0111 \text{ (7)} \\ + 0110 \text{ (6)} \\ \hline 1101 \end{array}$$

In signed notation, this is a result of -3 , not $+13$

(because in 4 bit binary system, 4th bit represent sign bit and only 3 bits represent magnitude of the number.

Therefore, an overflow has occurred, where result would have more bits than the original numbers.



Underflow

- Example:** If we add two negative numbers -29 and -13 using 8-bit binary number, result should be -42

29 = 00011101

13 = 00001101

1's of 29 = 11100010

1's of 13 = 11110010

2's of 29 = 11100011

2's of 13 = 11110011

now, add 1 1 1 0 0 0 1 1

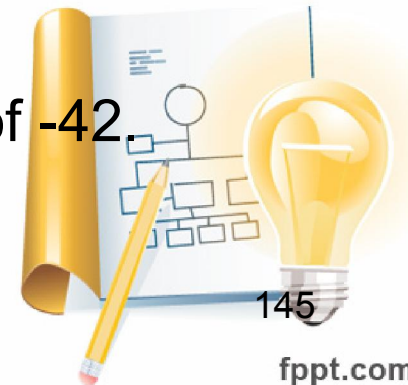
+ 1 1 1 1 0 0 1 1

1 1 1 0 1 0 1 1 0

Result is 11010110

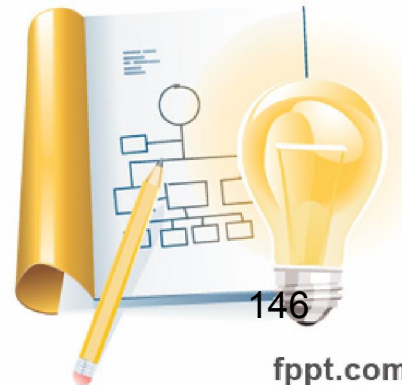
In signed notation, this is a result of +214 and not of -42.

Therefore, an underflow has occurred.



Cont...

- Overflow in two's complement occurs, not when a bit is carried out of the left column, but when there is a carry into the sign.
- A negative and positive added together cannot overflow, because the sum is between the addends.



- $-39 + 92 = 53$:

$$\begin{array}{r}
 \boxed{1} \quad 1 \quad \quad 1 \quad 1 \\
 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \\
 + \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \\
 \hline
 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1
 \end{array}$$

Carryout without overflow. Sum is correct.

- $104 + 45 = 149$:

$$\begin{array}{r}
 \quad 1 \quad 1 \quad \quad 1 \\
 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \\
 + \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \\
 \hline
 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1
 \end{array}$$

Overflow, no carryout. Sum is not correct.

- $10 + -3 = 7$:

$$\begin{array}{r}
 \boxed{1} \quad 1 \quad 1 \quad 1 \quad 1 \\
 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \\
 + \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \\
 \hline
 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1
 \end{array}$$

Carryout without overflow. Sum is correct.



Examples

- $-75 + 59 = -16$:

			1	1	1	1	1	1	
	1	0	1	1	0	1	0	1	
+	0	0	1	1	1	0	1	1	
<hr/>									
	1	1	1	1	0	0	0	0	

No overflow nor carryout.

- $44 + 45 = 89$:

			1			1	1		
	0	0	1	0	1	1	0	0	
+	0	0	1	0	1	1	0	1	
<hr/>									
	0	1	0	1	1	0	0	1	

No overflow nor carryout.

- $-1 + 1 = 0$:

	1	1	1	1	1	1	1		
	1	1	1	1	1	1	1	1	
+	0	0	0	0	0	0	0	1	
<hr/>									
	0	0	0	0	0	0	0	0	

Carryout without overflow. Sum is correct.

- $127 + 1 = 128$:

			1	1	1	1	1	1	
	0	1	1	1	1	1	1	1	
+	0	0	0	0	0	0	0	1	
<hr/>									
	1	0	0	0	0	0	0	0	

Overflow, no carryout. Sum is not correct.

- $-103 + -69 = -172$:

			1	1	1		1	1	
	1	0	0	1	1	0	0	1	
+	1	0	1	1	1	0	1	1	
<hr/>									
	0	1	0	1	0	1	0	0	

Overflow, with incidental carryout. Sum is not correct.

