ILLUSTRATIVE PROBLEMS

PROBLEM 7.1 (i) Calculate the present value of Rs 600 (a) received one year from now; (b) received at the end of five years; (c) received at the end of fifteen years. Assume a 5 per cent time preference rate.

(ii) Determine the present value of Rs 700 each paid at the end of each of the next six years. Assume a 8 per cent of interest.

(iii) Assume a 10 per cent discount rate. Compute the present value of Rs 1,100; Rs 900; Rs 1,500 and Rs 700 received at the end of one through four years. For calculations, use the tables given at the end of the book.

Solution

(i) Table C will be used to compute the present value.

(a) The present value factor at 5 per cent for one year is: 0.952. Therefore, the present value of Rs 600 at the end of one year will be: Rs 600 × 0.952 = Rs 571.20.

(b) The present value factor at 5 per cent at the end of five years is: 0.784. Therefore, present value of Rs 600 will be: Rs 600 × 0.784 = Rs 470.40.

(c) The present value factor at 5 per cent at the end of fifteen years is 0.481. Therefore, present value of Rs 600 will be: Rs 600 × 0.481 = Rs 288.60.

(*ii*) As the present value of an annuity of Rs 700 has to be computed. Table D will be used. The present value factor of an annuity of Re 1 at 8 per cent for 6 years is 4.623. Therefore, the present value of an annuity of Rs 700 will be: 4.623 × Rs 700 = Rs 3,236.10.

(*iii*) Table C will be used to compute the present value of the uneven series of cash flows. The computation is shown as follows:

$$P = \text{Rs } 1,100 \times 0.909 + \text{Rs } 900 \times 0.826 + \text{Rs } 1,500 \times 0.751 + \text{Rs } 700 \times 0.683$$

$$= \text{Rs } 999.90 + \text{Rs } 743.40 + \text{Rs } 1,126.50 + \text{Rs } 478.10 = \text{Rs } 3,347.90.$$

PROBLEM 7.2   Exactly ten years from now Sri Chand will start receiving a pension of Rs 3,000 a year. The payment will continue for sixteen years. How much is the pension worth now, if Sri Chand's interest rate is 10 per cent?

Solution

Sri Chand will receive first payment at the end of 10th year, and last payment at the end of 25th year. That provides him 16 payments of pension money. This can be shown on time scale as follows:

Year end   10   25
          0——
                           First payment   Last payment

The discounted value of the annuity of Rs 3,000 starting from the end of year 10 until the end of year 25 is the present value of pension received by Sri Chand. Assuming an annuity for 25 years, PVAF is 9.077. But we know that Sri Chand will not receive anything till the end of year 9. Therefore, if we subtract PVAF at ten per cent for 9 years, *viz.*, 5.759 from PVAF at 10 per cent for 25 years, 9.077, we shall be left with 9.077 – 5.759 = 3.318, which is a PVAF for the annuity starting from the end of year 10 and ending at the end of year 25. Thus, the present value of pension will be equal to:

$$(9.077 - 5.759) \times \text{Rs } 3,000 = 3.318 \times \text{Rs } 3,000 = \text{Rs } 9,954$$

Alternatively, the present value of pension can be found in two steps. First, find out present value of the 16-year annuity at 10 per cent interest rate at the end of year 9.

$$P_9 = \text{Rs } 3,000 \times 7.824 = \text{Rs } 23,472$$

Then find out present value now of the lump sum of Rs 23,472:

$$P_0 = \text{Rs } 23,472 \times 0.424 = \text{Rs } 9,954.$$

PROBLEM 7.3   Your father has promised to give you Rs 1,00,000 in cash on your 25th birthday. Today is your 16th birthday. He wants to know two things: (*a*) If he decides to make annual payments into a fund after one year, how much will each have to be if the fund pays 8 per cent? (*b*) If he decides to invest a lump sum in the account after one year and let it compound annually, how much will the lump sum be? (*c*) If in (*a*) the payments are made in the beginning of the year, how much will be the value of annuity?

Solution

(*a*) Rs 1,00,000 = $A(\text{CVAF}_{9,0.08})$ = Rs 1,00,000 = $A(12.488)$

$$A = \frac{\text{Rs } 1,00,000}{12.488} = \text{Rs } 8,007.69$$

(*b*) Rs 1,00,000 = $P(\text{CVF}_{9,0.08})$ = Rs 1,00,000 = $P(1.999)$

$$P = \frac{\text{Rs } 1,00,000}{1.999} = \text{Rs } 50,025$$

(*c*) This is a problem of an annuity due since payment is made at the beginning of the year.

$$Rs\ 1.00.000 = A(CVAF_{9,0.08}(1.08)$$

$$Rs\ 1.00.000 = A(13.487)$$

$$A = \frac{Rs\ 1.00,000}{13.478} = Rs\ 7,414.55.$$

PROBLEM 7.4 XYZ Bank pays 12 per cent and compounds interest quarterly. If Rs 1.000 are deposited initially, how much shall it grow at the end of 5 years?

Solution

The quarterly interest rate will be 3 per cent and the number of periods for which it will be compounded will be 20 (i.e., 5 years × 4). Thus:

$$F_5 = P\left[1 + \frac{i}{m}\right]^{n \times m}$$

$$F_5 = Rs\ 1,000\left[1 + \frac{0.12}{4}\right]^{5 \times 4}$$

$$= Rs\ 1,000(1.03)^{20} = Rs\ 1,000 \times 1.806 = Rs\ 1,806.$$

PROBLEM 7.5 How long will it take to double your money if it grows at 12 per cent annually?

Solution

$$F_n = P(CVF_{n,i})$$

$$Rs\ 2 = Re\ 1(CVF_{n,0.12})$$

$$CVF_{n,0.12} = 2.00$$

From Table A, the factor nearest to 2.00 is $CVF_{6,0.12} = 1.974$. Therefore, $n = 6$ years.

PROBLEM 7. 6 Mohan bought a share 15 years ago for Rs 10. It is now selling for Rs 27.60. What is the compound growth rate in the price of the share?

Solution

$$F_n = P(CVF_{n,i})$$

$$27.60 = 10(CVF_{15,i})$$

$$CVF_{15,i} = \frac{27.60}{10} = 2.760$$

From Table A, $i = 7\%$.

PROBLEM 7.7 Sadhulal Bhai is borrowing Rs 50,000 to buy a low-income group house. If he pays equal instalments for 25 years and 4 per cent interest on outstanding balance, what is the amount of instalment? What shall be amount of instalment if quarterly payments are required to be made?

Solution

Annual payment:

$$P = A(PVAF_{n,i})$$

$$Rs\ 50,000 = A(PVAF_{25,0.04})$$

$$Rs\ 50,000 = A(15.622)$$

$$A = \frac{Rs\ 50,000}{15.622} = Rs\ 3,200.61$$

Quarterly payment

The quarterly interest rate will be 0.04/4 = 0.01 and number of compounding periods will be 25 × 4 = 100

$$Rs\ 50,000 = A(PVAF_{100,0.01})$$
$$Rs\ 50,000 = A(63.029)$$
$$A = \frac{Rs\ 50,000}{63.029} = Rs\ 793.28$$

PROBLEM 7.8 A company has issued debentures of Rs 50 lakh to be repaid after 7 years. How much should the company invest in a sinking fund earning 12 per cent in order to be able to repay debentures?

Solution

$$A(FVAF_{0.12,7}) = 50,00,000$$
$$A(10.089) = 50,00,000$$
$$A = \frac{50,00,000}{10.089} = Rs\ 4,95,589$$

PROBLEM 7.9 A bank has offered to you an annuity of Rs 1,800 for 10 years if you invest Rs 12,000 today. What rate of return would you earn?

Solution

$$12,000 = 1,800(PVAF_{r,10})$$
$$PVAF_{r,10} = \frac{12,000}{1,800} = 6.667$$

When you refer to Table D at the end of the book, you obtain a present value factor of an annuity of Re 1 equal to 6.710 at 8 per cent rate of interest for 10 years. At 9 per cent the factor is 6.418. Thus, the rate of return lies between 8–9 per cent. By interpolation, we can obtain the rate of return as follows:

$$Rate\ of\ return = 8\% + \left[\frac{6.710 - 6.667}{6.710 - 6.418}\right] = 8\% + \frac{0.043}{0.292} = 8\% + 0.15\% = 8.15\%.$$

PROBLEM 7.10 A firm purchases a machinery for Rs 8,00,000 by making a down payment of Rs 1,50,000 and remainder in equal instalments of Rs 1,50,000 for six years. What is the rate of interest to the firm?

Solution

$$8,00,000 - 1,50,000 = 1,50,000\ PVAF_{6,r}$$
$$PVAF_{6,r} = 6,50,000/1,50,000 = 4.333$$

For Table B in the appendix, we observe that for 10 per cent rate of interest for 6 years, PVAF is 4.355. Thus the rate of interest which the firm will be paying is approximately 10 per cent.

PROBLEM 7.11 AB Limited is creating a sinking fund to redeem its preference capital of Rs 5 lakh issued on 6 April, 1994 and maturing on 5 April, 2005. The first annual payment will be made on 6 April, 1994. The company will make equal annual payments and expects that the fund will earn 12 per cent per year. How much will be the amount of sinking fund payment?

Solution

AB Co. wants to accumulate a future sum of Rs 5,00,000. Since the annual payments will be made in the beginning of the year, we can use the formula for the compound value of an annuity due to solve the problem:

$$A(\text{CVAF}_{n,\,i})(1 + i) = 5,00,000$$
$$A(\text{CVAF}_{12,\,0.12})(1.12) = 5,00,000$$
$$A(24.133)(1.12) = 5,00,000$$
$$27.029A = 5,00,000$$
$$A = 5,00,000/27.029$$
$$= \text{Rs } 18,498.65.$$