S. V. National Institute of Technology, Surat

Applied Mathematics and Humanities Department

B.Tech-I	Sem-2nd	Branch-All	Subject-Mathematics-II (MA 114 S2)

Tutorial - 2 : Solution of Differential Equation of Higher Order

1. Solve the following Differential Equations

   (a) $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$.
     Ans : $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{\frac{3}{2}x} + c_4 e^{-\frac{1}{2}x}$

   (b) $(D^3 + 6D^2 + 12D + 8)y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 2$.
     Ans $y = x^2 e^{-2x}$

   (c) $y''' + 6y'' + 11y' + 6y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = -1$.
     Ans $y = 2e^{-x} - 3e^{-2x} + e^{-3x}$

   (d) $y'''' - 4y''' + 14y'' - 20y' + 25y = 0$.
     Ans $y = e^x((c_1 + c_2 x)\cos(2x) + (c_3 + c_4 x)\sin(2x))$

   (e) $y''''' - 2y''' + y = 0$.
     Ans $y = (c_1 + c_2 x)e^x + e^{-\frac{x}{2}}[(c_3 + c_4 x)\cos(\frac{\sqrt{3}}{2}x) + (c_5 + c_6 x)\sin(\frac{\sqrt{3}}{2}x)]$

2. Write particular Integral for the following Differential equations

   (a) $(D^3 - 2D^2 - 5D + 6)y = 2e^x + 4e^{3x} + 7e^{-2x} + 8e^{2x} + 15$.
     Ans $y = -\frac{1}{3}xe^x + \frac{2}{5}xe^{3x} + \frac{7}{15}xe^{-2x} - 2e^{2x} + \frac{15}{6}$

   (b) $(D^2 - a^2)y = \sinh ax$
     Ans $y = \frac{x}{2a}\cosh ax$

   (c) $(D^4 + 4)y = x^4$
     Ans $y = \frac{1}{4}(x^4 - 6)$

   (d) $(D^2 - 4D - 5)y = e^{2x} + 3\cos(4x + 3)$
     Ans $y = -\frac{1}{9}e^{2x} - \frac{1}{697}[48\sin(4x + 3) + 63\cos(4x + 3)]$

   (e) $(D^2 + 4)y = \sin 3x + \cos 2x$
     Ans $y = -\frac{1}{5}\sin 3x + \frac{x}{4}\sin 2x$

   (f) $(D^2 + 2)y = x^3 + x^2 + e^{-2x} + \cos 3x$
     Ans $y = \frac{1}{2}(x^3 + x^2 - 3x - 1) + \frac{1}{6}e^{-2x} - \frac{1}{7}\cos 3x$

   (g) $(D^2 - 4D + 4)y = e^{2x}\cos^2 x$
     Ans $y = e^{2x}(\frac{x^2}{2} - \frac{1}{8}\cos 2x)$

   (h) $(D^2 + 6D + 9)y = \frac{e^{-3x}}{x^3}$
     Ans $y = \frac{e^{-3x}}{2x}$

   (i) $(D^2 + 9)y = \sec 3x$
     Ans $y = \frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x \cdot \log(\cos 3x)$

   (j) $(D^2 + 3D + 2)y = x\sin 2x$
     Ans $y = -\frac{(30x-7)}{200}\cos 2x - \frac{5x-12}{100}\sin 2x$

3. Solve the following Differential equations

   (a) $(D^2 + 5D - 6)y = \sin x \sin 4x$
     Ans $y = c_1 e^x + c_2 e^{-6x} + \frac{1}{2}[\frac{\sin 3x - \cos 3x}{30} + \frac{31\cos 5x - 25\sin 5x}{1586}]$

   (b) $(D^2 - 4D + 3)y = e^x \cos 2x + \cos 3x$
     Ans $y = c_1 e^x + c_2 e^{3x} - \frac{e^x}{8}(\sin 2x + \cos 2x) - \frac{1}{30}(2\sin 3x + \cos 3x)$

(c) $(D^2 - 2D + +1)y = xe^x \sin x.$

   Ans $y = (c_1 + c_2 x)e^x - e^x(x \sin x + 2 \cos x)$

(d) $(D^2 + 5D + 6)y = e^{-2x} \sec^2 x(1 + 2 \tan x)$

   Ans $y = c_1 e^{-2x} + c_2 e^{-3x} + e^{-2x} \tan x$

(e) $(D^2 - 1)y = x^2 \cos x$

   Ans $y = c_1 e^x + c_2 e^{-x} + x \sin x + \frac{(1-x^2)\cos x}{2}$

(f) $(D^2 + 3D + 2)y = e^{e^x}$

   Ans $y = c_1 e^{-x} + c_2 e^{-2x} + e^{-2x} e^{e^x}$

(g) $(D^2 + a^2)y = \tan ax$

   Ans $y = c_1 \cos ax + c_2 \sin ax - \frac{1}{a^2} \cos ax \log(\sec ax + \tan ax)$

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