Tutorial-I

Exact differential equation and Integrating factors & 1st order and higher degree

1. Define Order and Degree of Differential Equation with example.
2. Define Exact Differential Equation. State and prove necessary and sufficient condition for differential equation to be exact.
3. Solve the following differential equations:
   a) $(x^2+y^2-a^2)x\,dx+(x^2-y^2-b^2)y\,dy=0.$ $\quad Ans: x^4+2x^2y^2-2a^2x^2-y^4-2b^2y^2=4c$
   b) $\left(cosxtany+cos\left(x+y\right)\right)dx+(sinxsec^2y+cos\left(x+y\right))dy=0$ $\;Ans: sinxtany+sin\left(x+y\right)=c$
   c) $(2xy+y-tany)dx+(x^2-xtan^2y+sec^2y)dy=0.$ $\;Ans: x^y+xy^2-xtany+tany=c$
   d) $xdy-ydx=(x^2-y^2)^{1/2}.$ $\quad Ans: sin^{-1}\left(x/y\right)=c$
   ~~e) $x^2+x^3y+cosec\left(xy\right)=0$ $\quad Ans: \dfrac{1}{2x^2}+cos\left(xy\right)=c$~~
   f) $\dfrac{dy}{dx}=\dfrac{x^3+y^3}{xy^2}.$ $\quad Ans: 3log\left(x\right)-\left(\dfrac{y}{x}\right)^3=c$
   g) $(xysin\left(xy\right)+cosxy)y\,dx+(xysin\left(xy\right)-cosxy)x\,dy=0$ $\;Ans: log\left(secxy\right)+logx-logy=c$
   h) $ylogy\,dx+(x-logy)dy=0.$ $\quad Ans: xlogy-\dfrac{1}{2}(logy)^2=c$
   i) $(4xy+3y^2-x)dx+x\left(x+2y\right)dy=0.$ $\;Ans: 4x^3y+4x^3y^2-x^4=0$
   j) $(2x^2y^2+y)dx+(3x-x^3y)dy=0.$ $\;Ans: 4x^{\frac{10}{7}}y^{-\frac{5}{7}}-5x^{-\frac{4}{7}}y^{-\frac{12}{7}}=20c$
   k) $(3x+2y^2)y\,dx+2(2x+3y^2)x\,dy=0.$ $\quad Ans: x^3y^4+x^2y^6=c$
4. Solve the following differential equations:
   a. $xyp^3+\left(x^2-2y^2\right)p^2-2xyp=0.$ $\;Ans: \left(y-c\right)\left(y-cx^2\right)\left(x^2+y^2-2c\right)=0$
   b. $p^2+2pcos2x-sin^2x=0.$ Ans: $2y+2x+sin2x+c=0$
   c. $y^2p^2-3xp+y=0.$ $\quad Ans: x=p+\dfrac{1}{p}\;;\; y=\dfrac{p^2}{2}-logp+c$
   d. $x+2\left(xp-y\right)+p^2=0.$ Ans: $y=\dfrac{1}{2}x+xp+\dfrac{1}{2}p^2;\; x=\dfrac{1}{2}e^{2p-c}+1-p$
   e. $xp-y+x^{\frac{3}{2}}=0.$ Ans: $y=cx-2x^{\frac{3}{2}}$
   f. $p=\log(px-y)$ Ans: $y=cx-e^c$