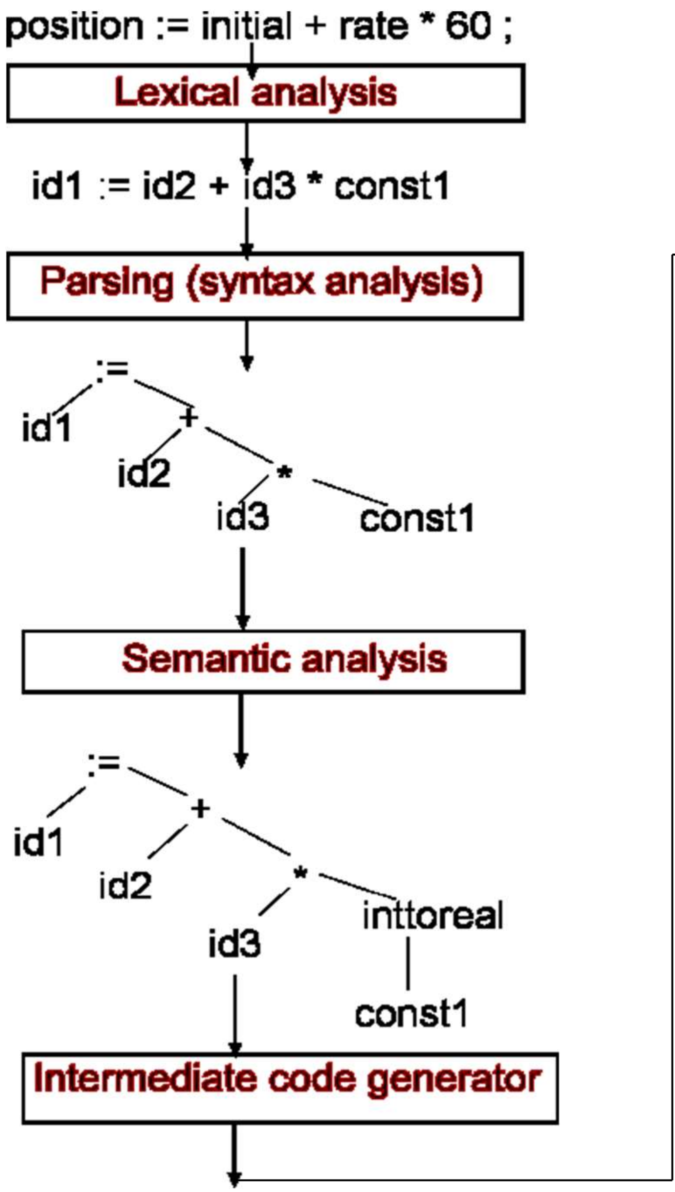
Syntax Analyzer (Parser)

Input: list of tokens produced by scanner/LA
Output: tree(syntax) which shows structure of program

Recap: Overview



temp1 := inttoreal(60)
temp2 := id3 * temp1
temp3 := id2 + temp2
id1 := temp3

Code optimization

temp1 := id3 * 60.0
id1 := id2 + temp1

Code generator()

MOVF ID3, R2
MULF #60.0, R2
MOVF ID3, R1
ADDF R2, R1
MOVF R1, ID1

Introduction

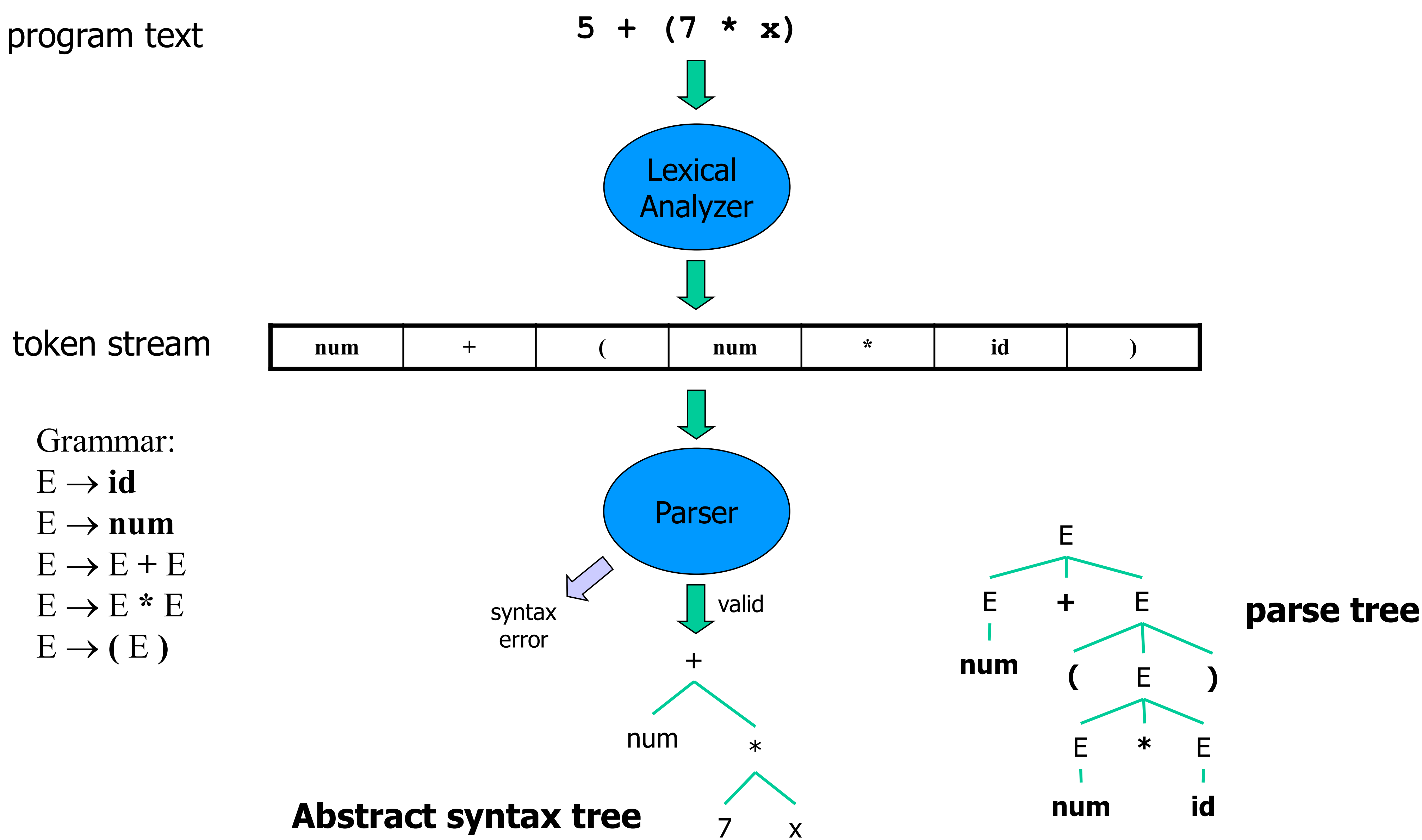
A program represented by a sequence of tokens → Parser → If it is a legal program, then output some abstract representation of the program

- Abstract representations of the input program:

- abstract-syntax tree + symbol table
- intermediate code
- object code

- **Context free grammar (CFG)** is used to specify the structure of legal programs

From text to abstract syntax



Grammar:

E → **id**

E → **num**

E → E + E

E → E * E

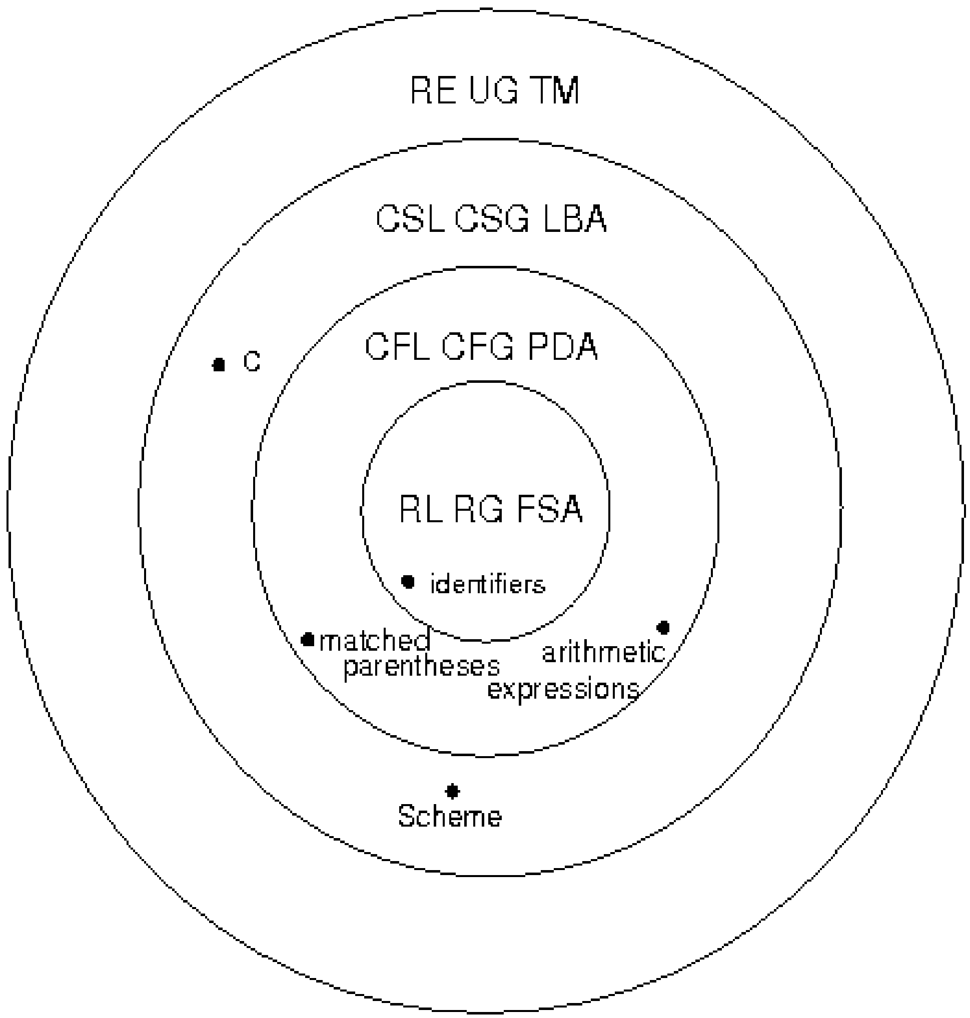
E → **(** E **)**

Goals of parsing

- Programming language has syntactic rules
  - Context-Free Grammars
- Decide whether program satisfies syntactic structure
  - Error detection
  - Error recovery
  - Simplification: rules on tokens
- Build Abstract Syntax Tree

Classes of Grammars
(The Chomsky Hierarchy)

- Type-0: Phrase structured (unrestricted) grammars
  - generate recursively enumerable (unrestricted) languages
  - include all formal grammars
  - implemented with Turing machines
- Type-1 : Context-sensitive grammars
  - generate context-sensitive languages
  - implemented with linear-bounded automata
- Type-2 : Context-free grammars
  - generate context-free languages
  - single non-terminal on left
  - non-terminals & terminals on right
  - implemented with pushdown automata
- Type-3 : Regular grammars
  - generate regular languages
  - no terminals or non-terminals here
  - implemented with finite state automata



Classes of Grammars
(The Chomsky Hierarchy)

Type 0, Phrase Structure (same as basic grammar definition)

Type 1, Context Sensitive

(1) $\alpha \rightarrow \beta$ where α is in $(N \cup \Sigma)^* N (N \cup \Sigma)^*$,

β is in $(N \cup \Sigma)+$, and length(α) $\leq$ length(β)

(2) $\gamma A \delta \rightarrow \gamma \beta \delta$ where A is in N, β is in $(N \cup \Sigma)^+$, and

γ and δ are in $(N \cup \Sigma)^*$

Type 2, Context Free

$A \rightarrow \beta$ where A is in N, β is in $(N \cup \Sigma)^*$

Linear

$A \rightarrow x$ or $A \rightarrow x B y$, where A and B are in N and x and y are in Σ^*

Type 3, Regular Expressions

(1) left linear $A \rightarrow B a$ or $A \rightarrow a$, where A and B are in N and a is in Σ

(2) right linear $A \rightarrow a B$ or $A \rightarrow a$, where A and B are in N and a is in Σ

Type 3 grammer

A grammar is said to be type 3 grammar or regular grammar if all productions in grammar are of the form $A \rightarrow a$ then $A \rightarrow aB$ or equivalent of the form $A \rightarrow a$ or $A \rightarrow Ba$.

In other words in any production (set of rules) the left hand string is single nonterminal and the right hand string is either a terminal or a terminal followed by non-terminal.

Type 2 grammer

A grammar is said to be type 2 grammar or context free grammar if every production in grammar is of the form $A \rightarrow \alpha$.

In other words in any production left hand string is always a non-terminal and a right hand string is any string on T U N .

- Example : $A \rightarrow aBc$

Type 1 grammer

A grammar is said to type 1 grammar or context sensitive grammar if for every production $\alpha \rightarrow ß$. The length of ß is larger than or equal to the length of α.

for example:

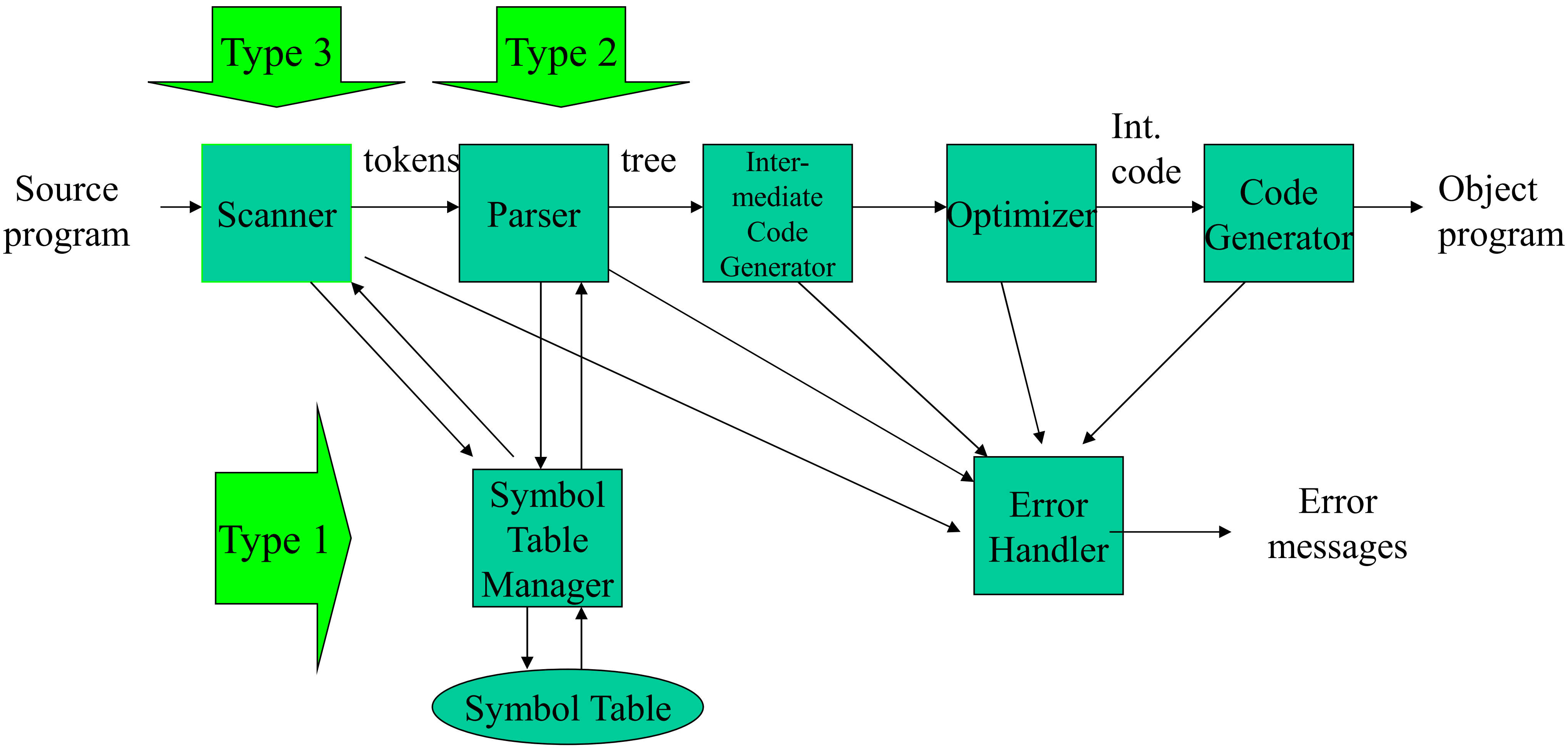
- $A \rightarrow ab$
- $A \rightarrow aA$
- $aAb \rightarrow aBCb$

Type 0 grammer

A grammar with no restriction is referred to as type 0 grammar . They generate exactly all languages that can be recognized by a Turing machine. These languages are also known as the recursively enumerable languages.

Class 0 grammars are too general to describe the syntax of programming languages and natural languages.

The Chomsky Hierarchy and the Block Diagram of a Compiler



CFG vs. Regular Expressions

A regular grammar puts the following restrictions on the productions:

- The LHS can only be a single non terminal
- The RHS can be any number of terminals, with (at most) a single non terminal as its last symbol.

A CFG puts the following restrictions on the productions:

- The LHS can only be a single non terminal (just like the regular grammar)
- The RHS can be any combination of terminals and non terminals (this is the new part).

CFG is more expressive than RE

- Every language that can be described by regular expressions can also be described by a CFG

Example : languages that are CFG but not RE

- if-then-else statement, $\{a^n b^n \mid n>=1\}$

Non-CFG

- L1={wcw | w is in (a|b)*}
- $L2=\{a^n b^m c^n d^m \mid n>=1 \text{ and } m>=1\}$

Context Free Grammars

- CFGs
  - Add recursion to regular expressions
    - Nested constructions
  - Notation

    expression → *identifier* | *number* | **-** *expression*
    | **(** *expression* **)**
    | *expression operator expression*

    operator → **+** | **-** | ***** | **/**

    - **`Terminal symbols`**
    - *Non-terminal symbols*
    - Production rule (i.e. substitution rule)

      terminal symbol → terminal and non-terminal symbols

Derivations

- A derivation shows how to generate a syntactically valid string
  - Given a CFG
  - Example:
    - CFG

$$
\begin{aligned}
\textit{expression} \rightarrow\ & \textit{identifier} \\
& \mid \textit{number} \\
& \mid \texttt{-}\ \textit{expression} \\
& \mid \texttt{(}\ \textit{expression}\ \texttt{)} \\
& \mid \textit{expression operator expression} \\
\textit{operator} \rightarrow\ & \texttt{+} \mid \texttt{-} \mid \texttt{*} \mid \texttt{/}
\end{aligned}
$$

- Derivation of

```
slope * x + intercept
```

Derivation Example

- Derivation of `slope * x + intercept`

$$
\begin{aligned}
\textit{expression} &\Rightarrow \textit{expression operator } \underline{\textit{expression}} \\
&\Rightarrow \textit{expression } \underline{\textit{operator}} \texttt{ intercept} \\
&\Rightarrow \underline{\textit{expression}} + \texttt{intercept} \\
&\Rightarrow \textit{expression operator } \underline{\textit{expression}} + \texttt{intercept} \\
&\Rightarrow \textit{expression } \underline{\textit{operator}} \texttt{ x } + \texttt{intercept} \\
&\Rightarrow \underline{\textit{expression}} * \texttt{x} + \texttt{intercept} \\
&\Rightarrow \texttt{slope} * \texttt{x} + \texttt{intercept}
\end{aligned}
$$

$$\textit{expression} \Rightarrow^{*} \texttt{slope} * \texttt{x} + \texttt{intercept}$$

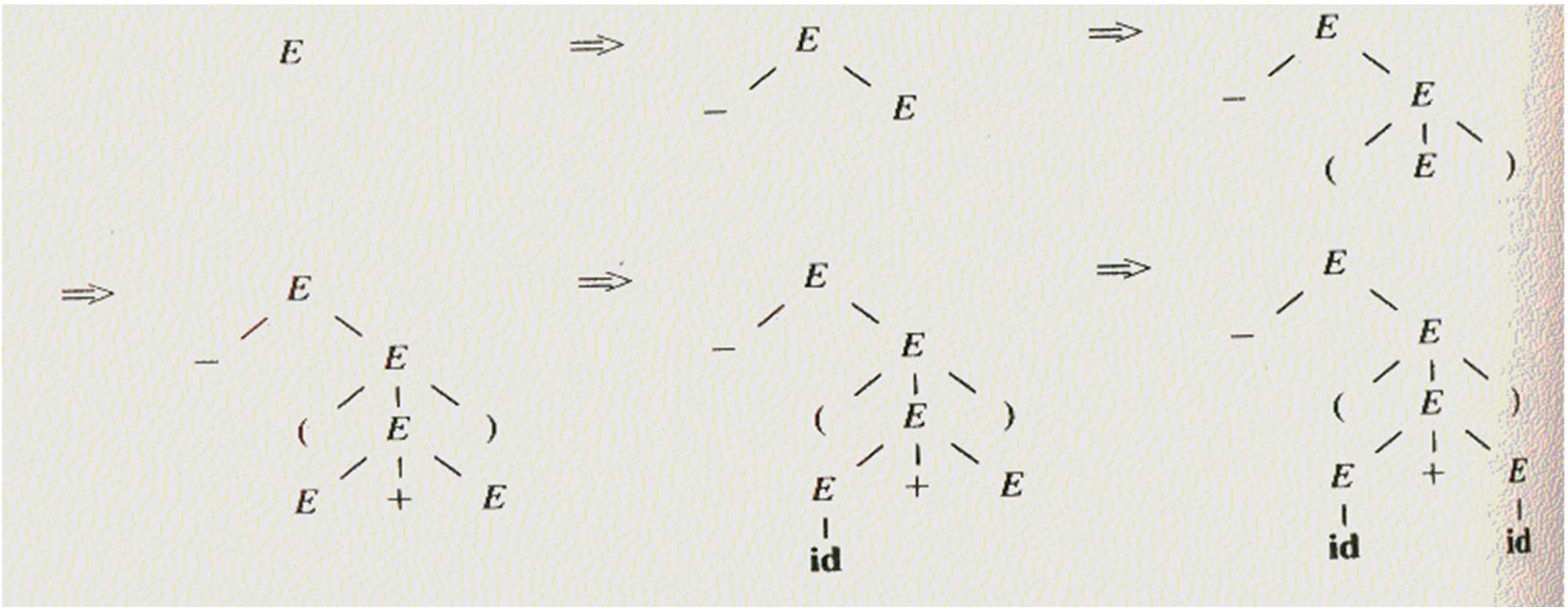
- Identifiers were not derived for simplicity

Parse Trees

- A parse tree is any tree in which
  - The root is labeled with S
  - Each leaf is labeled with a token a or ε
  - Each interior node is labeled by a nonterminal
  - If an interior node is labeled A and has children labeled X1,.. Xn, then A ::= X1...Xn is a production.

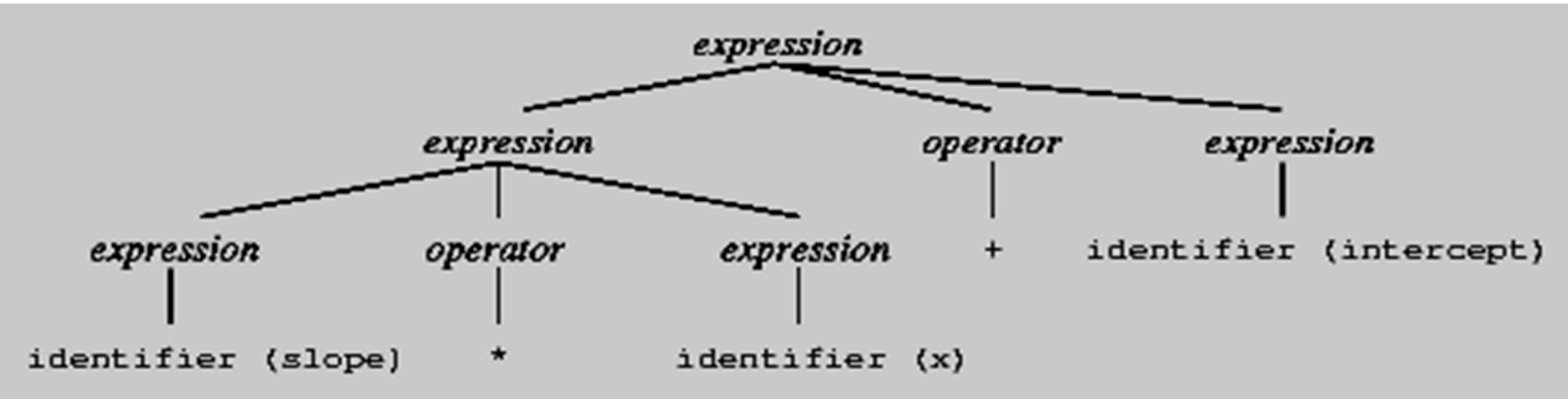
Parse Trees and Derivations

E ::= E + E | E * E | E - E | - E | (E) | id



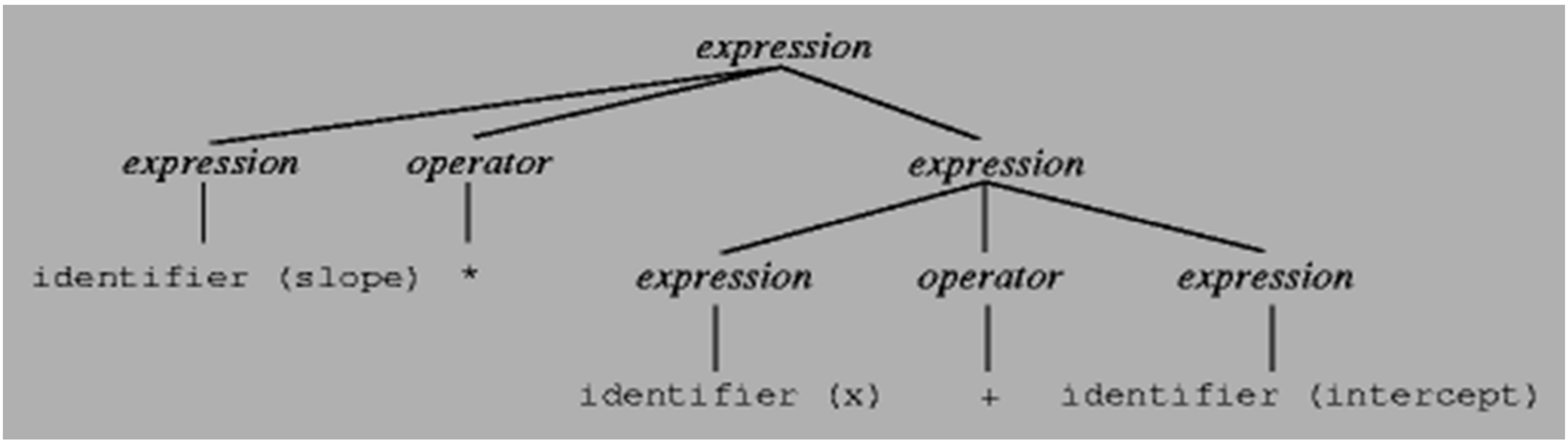
Parse Trees

- A parse is graphical representation of a derivation
- Example



Ambiguous Grammars

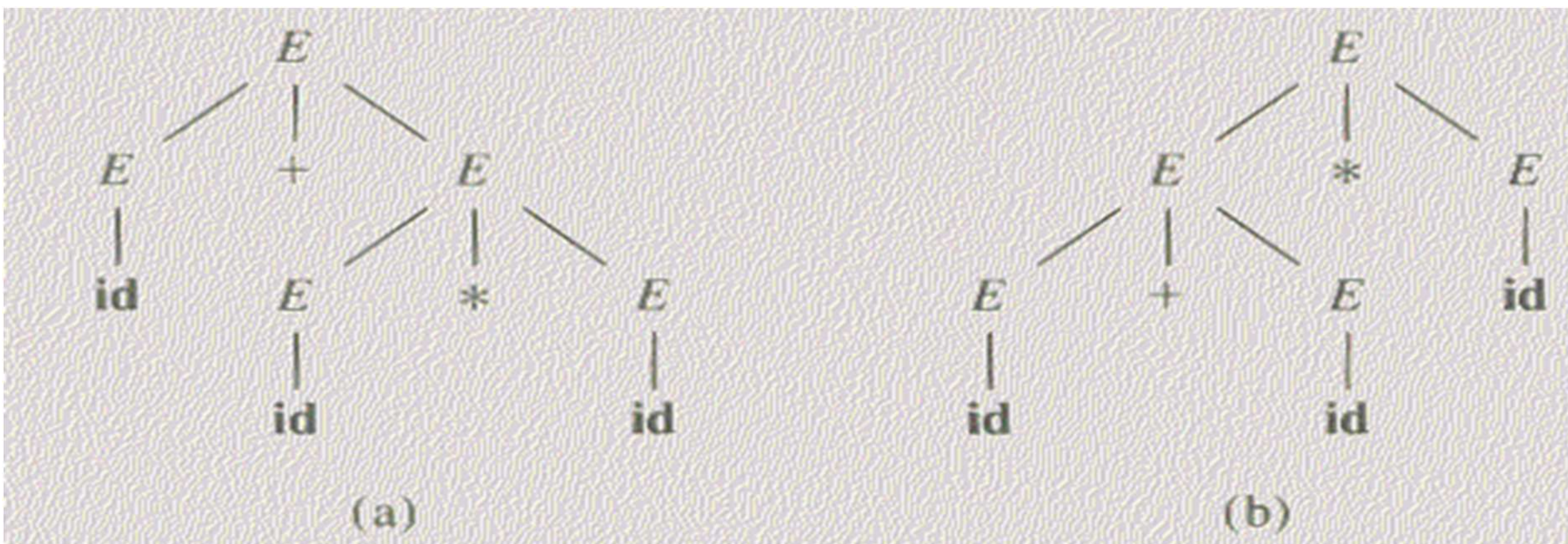
- Alternative parse tree
  - same expression
  - same grammar



  - This grammar is ambiguous

Ambiguity

- A grammar that produces more than one parse tree for some sentence is said to be *ambiguous*.



Eliminating Ambiguity

- There is no deterministic way of finding out whether a grammar is ambiguous and how to fix it. In order to remove ambiguity, we follow some heuristics.
- There are three parts to this:

1. Add a non-terminal for each precedence level
2. Isolate the corresponding part of the grammar
3. Force the parser to recognize the high-precedence sub expressions first

E -> E + E | E − E

| E * E | E / E

| (E) | var

E -> E + T | E − T | T

T -> T * F | T / F | F

F -> (E) | id

E-> TE‘

E‘->+TE‘|-TE‘|eps

T->FT‘

T‘->*FT‘ | /FT‘|eps

Eliminating Left-Recursion

- Direct left-recursion

$$A ::= A\alpha \mid \beta$$

$$\Downarrow$$

$$A ::= \beta A'$$
$$A' ::= \alpha A' \mid \varepsilon$$

$$A ::= A\alpha 1 \mid \ldots \mid A\alpha m \mid \beta 1 \mid \ldots \mid \beta n$$

$$\Downarrow$$

$$A ::= \beta 1 A' \mid \ldots \mid \beta n A'$$
$$A' ::= \alpha 1 A' \mid \ldots \mid \alpha n A' \mid \varepsilon$$

Eliminating Indirect Left-Recursion

- Indirect left-recursion
- Algorithm

$$S ::= Aa \mid b$$
$$A ::= Ac \mid Sd \mid \varepsilon$$

Arrange the nonterminals in some order $A_1,\ldots,A_n$.
for (i in 1..n) {
    for (j in 1..i-1) {
        replace each production of the form $A_i ::= A_j\gamma$ by the
        productions $A_i ::= \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$ where
            $A_j ::= \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$
    }
    eliminate the immediate left recursion among A_i productions
}

Left Factoring

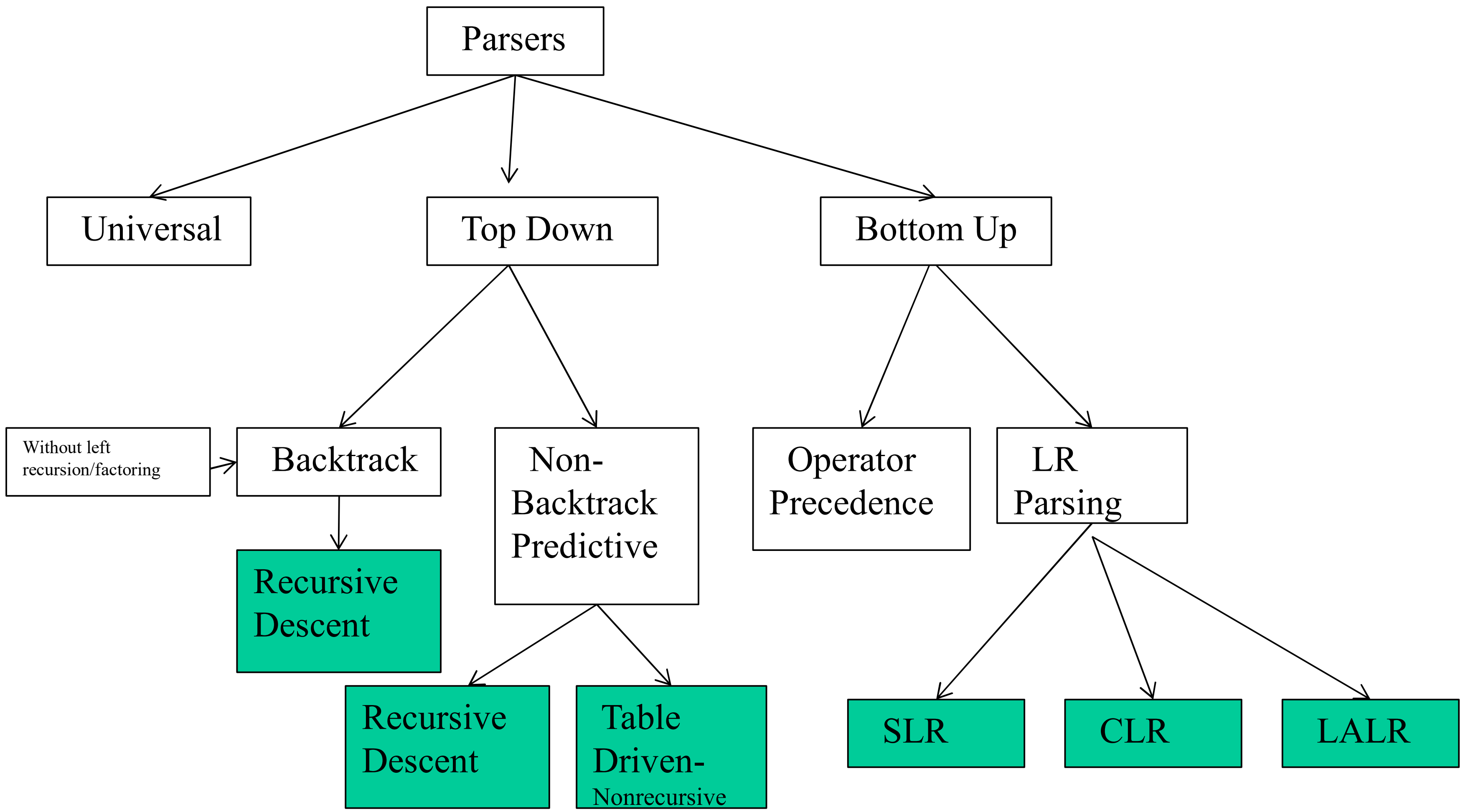
$$A ::= \alpha\beta 1 \mid \ldots \mid \alpha\beta n \mid \gamma$$

$$\downarrow$$

$$A ::= \alpha A' \mid \gamma$$

$$A' ::= \beta 1 \mid \ldots \mid \beta n$$

Types of Parsers



Top-Down Parsing

- Start from the start symbol and build the parse tree top-down
- Apply a production to a nonterminal. The right-hand of the production will be the children of the nonterminal
- Match terminal symbols with the input
- May require backtracking
- Some grammars are backtrack-free (predictive)

TDP

- The parse tree is created top to bottom.
- Top-down parser
  - Recursive-Descent Parsing
    - Backtracking is needed (If a choice of a production rule does not work, we backtrack to try other alternatives.)
    - It is a general parsing technique, but not widely used.
    - Not efficient
  - Predictive Parsing
    - no backtracking
    - efficient
    - needs a special form of grammars (LL(1) grammars).
    - **Recursive Predictive Parsing is a special form of Recursive Descent parsing without backtracking.**
    - **Non-Recursive (Table Driven) Predictive Parser is also known as LL(1) parser.**

Construct Parse Trees Top-Down

– Start with the tree of one node labeled with the start symbol and repeat the following steps until the fringe of the parse tree matches the input string

1. At a node labeled A, select a production with A on its LHS and for each symbol on its RHS, construct the appropriate child
2. When a terminal is added to the fringe that doesn't match the input string, backtrack
3. Find the next node to be expanded

– Minimize the number of backtracks

Example

Left-recursive

```
E ::=   T | E + T | E - T
T ::=   F | T * F | T / F
F ::=   id | number | (E)
```

Right-recursive

```
E ::=    T E'
E'::=    + T E'
         | - T E'
         | e
T::=     F T'
T' ::=    * F T'
         | / F T'
         | e
F ::=    id
         | number
         | (E)
```

x - 2 * y

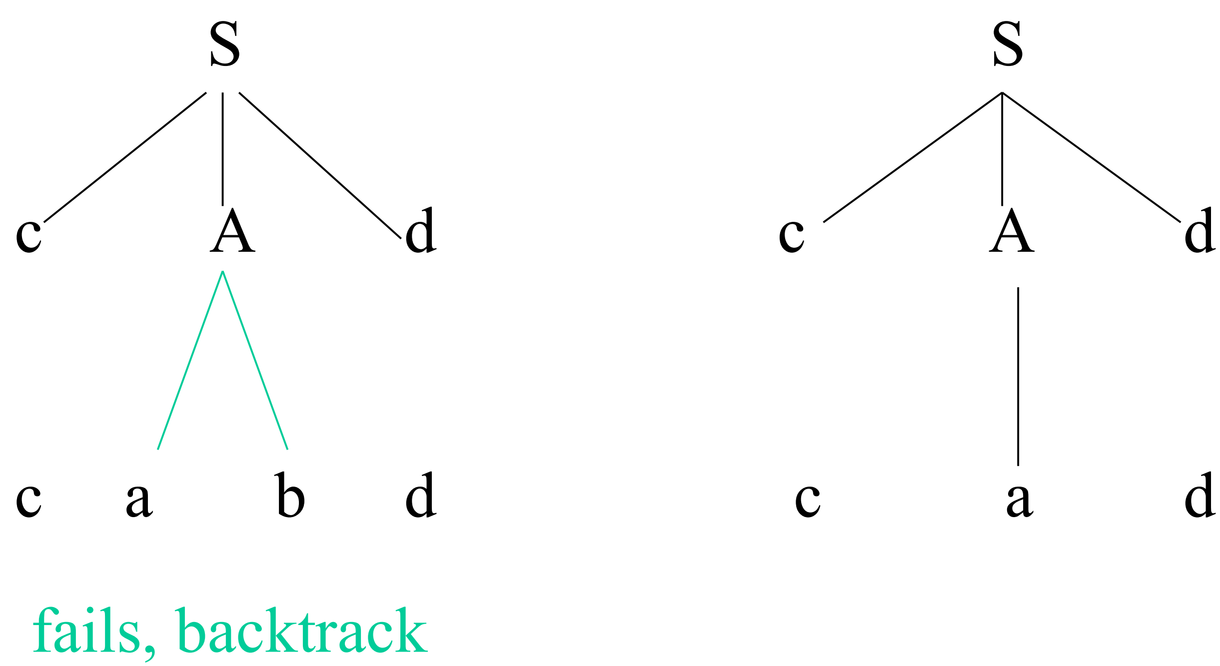
Recursive-Descent Parsing (uses Backtracking)

- Backtracking is needed.
- It tries to find the left-most derivation.
- Grammar rule of a non-terminal "A" is viewed as a definition of a procedure that will recognize "A".

$S \rightarrow cAd$

$A \rightarrow ab \mid a$

input: cad



Recursive Descent Parser- Example

- A separate recursive procedure is written for every non-terminals

```
Procedure S()
{
    if input = 'c'
    {
    Advance();      //procedure that is written to advance the input pointer to next position
    A();
    if input = 'd'
    {
    Advance();
    return true;
    }
    else return false;
    else return false;
}
```

Cont.

```
Procedure A()
{
isave=in-ptr;     // i-save saves the input pointer position before each alternate to facilitate backtracking
If input ='a'
{   Advance();
    if input = 'b'
    {
        Advance();
        return true;
    }
}
In-ptr=isave
If input ='a'
{   Advance();
    return true;
}
return false;
return false;
}
```

Cont.

- Problems??

- Left recursion – ambiguity as how many times to call? Solution – eliminate it
- Backtracking – when more than one alternative in the rule. Solution – left factoring
- Very difficult to identify the position of the errors

Predictive Parser

a grammar ➔ eliminate left recursion ➔ left factor ➔ a grammar suitable for predictive parsing (a LL(1) grammar)

- When re-writing a non-terminal in a derivation step, a predictive parser can **uniquely** choose a production rule by just looking the current symbol in the input string.

$A \rightarrow \alpha_1 \mid \ldots \mid \alpha_n$ input: ... a ....

current token

Predictive Parser (example)

stmt → `if` ...... |
`while` ...... |
`begin` ...... |
`for` .....

- When we are trying to write the non-terminal *stmt*, if the current token is `if` we have to choose first production rule.
- When we are trying to write the non-terminal *stmt*, we can uniquely choose the production rule by just looking the current token.
- We eliminate the left recursion in the grammar, and left factor it. But it may not be suitable for predictive parsing (not LL(1) grammar).

Recursive Predictive Parsing

- Each non-terminal corresponds to a procedure.

Ex: $A \rightarrow aBb$ (This is only the production rule for A)

proc A {
- match the current token with a, and move to the next token;
- call ‘B’;
- match the current token with b, and move to the next token;

}

Recursive Predictive Parsing (cont.)

$A \rightarrow aBb \mid bAB$

```
proc A {
   case of the current token {
         'a':   - match the current token with a, and move to the next token;
                - call 'B';
                - match the current token with b, and move to the next token;
         'b':   - match the current token with b, and move to the next token;
                - call 'A';
                - call 'B';
   }
}
```

Recursive Predictive Parsing (cont.)

- When to apply ε-productions.

  $A \rightarrow aA \mid bB \mid \varepsilon$

- If all other productions fail, we should apply an ε-production. For example, if the current token is not a or b, we may apply the ε-production.
- Most correct choice: We should apply an ε-production for a non-terminal A when the current token is in the follow set of A (which terminals can follow A in the sentential forms).

Recursive Predictive Parsing (Example)

$A \rightarrow aBe \mid cBd \mid C$

$B \rightarrow bB \mid \varepsilon$

$C \rightarrow f$

```
proc A {
    case of the current token {
        a:   - match the current token with a,
               and move to the next token;
             - call B;
             - match the current token with e,
               and move to the next token;
        c:   - match the current token with c,
               and move to the next token;
             - call B;
             - match the current token with d,
               and move to the next token;
        f:   - call C
    }
}
```

first set of C

```
proc C {  match the current token with f,
          and move to the next token; }
```

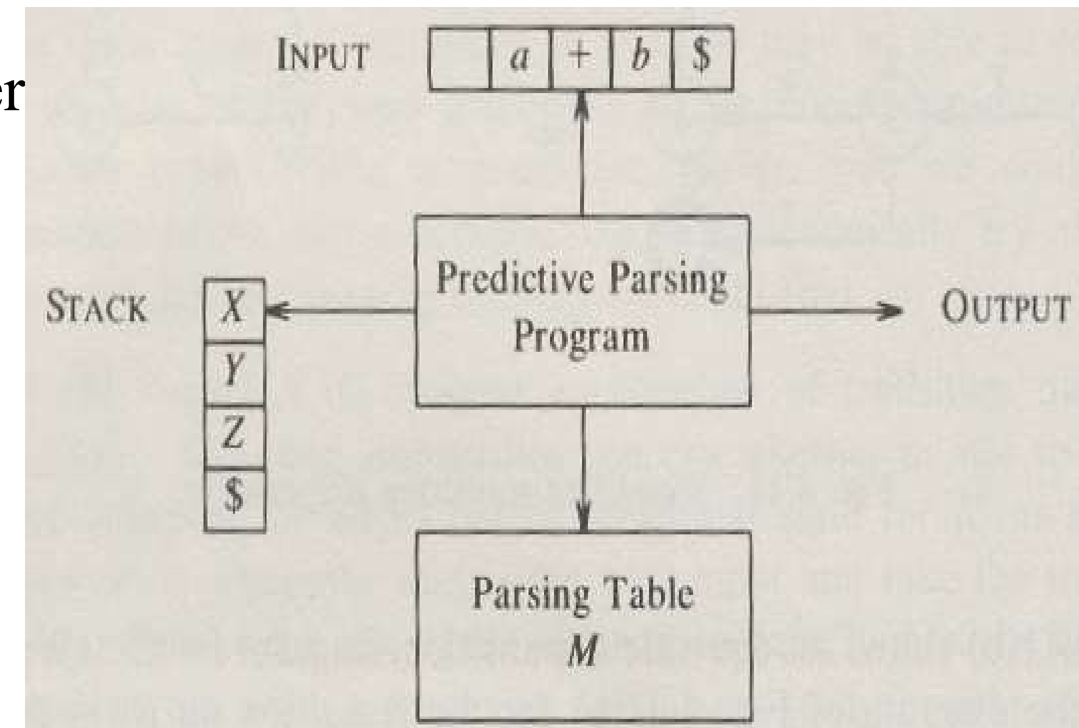
```
proc B {
    case of the current token {
        b:   - match the current token with b,
               and move to the next token;
             - call B
        e,d:  do nothing
    }
}
```

follow set of B

Non-Recursive Predictive Parsing - LL(1) Parser

- An **LL parser** is a top-down parser for a subset of the context-free grammars. It parses the input from **L**eft to right, and constructs a **L**eftmost derivation of the sentence
- Non-Recursive predictive parsing is a table-driven parser.
- It is a top-down parser.
- It is also known as LL(1) Parser

An LL parser is called an LL(*k*) parser if it uses *k* tokens of lookahead when parsing a sentence



LL(1) Parser

input buffer

- our string to be parsed. We will assume that its end is marked with a special symbol $.

output

- a production rule representing a step of the derivation sequence (left-most derivation) of the string in the input buffer.

stack

- contains the grammar symbols
- at the bottom of the stack, there is a special end marker symbol $.
- initially the stack contains only the symbol $ and the starting symbol S. $S ← initial stack
- when the stack is emptied (ie. only $ left in the stack), the parsing is completed.

parsing table

- a two-dimensional array M[A,a]
- each row is a non-terminal symbol
- each column is a terminal symbol or the special symbol $
- each entry holds a production rule.

LL(1) Parser – Parser Actions

set *ip* to point to the first symbol of $w\$$;

repeat

let X be the top stack symbol and a the symbol pointed to by *ip*;

if X is a terminal or \$ **then**

if $X = a$ **then**

pop X from the stack and advance *ip*

else *error* ()

parsing table

else /* X is a nonterminal */

if $\boxed{M[X, a]} = X \to Y_1 Y_2 \cdots Y_k$ **then begin**

pop X from the stack;

push $Y_k, Y_{k-1}, \ldots, Y_1$ onto the stack, with Y_1 on top;

output the production $X \to Y_1 Y_2 \cdots Y_k$

end

else *error* ()

until $X = \$$ /* stack is empty */

LL(1) Parser – Parser Actions

- The symbol at the top of the stack (say X) and the current symbol in the input string (say a) determine the parser action.
- There are four possible parser actions.

1. If X and a are \$ ➔ parser halts (successful completion)

2. If X and a are the same terminal symbol (different from \$)

   ➔ parser pops X from the stack, and moves the next symbol in the input buffer.

3. If X is a non-terminal

   ➔ parser looks at the parsing table entry M[X,a]. If M[X,a] holds a production rule $X \rightarrow Y_1 Y_2 \ldots Y_k$, it pops X from the stack and pushes $Y_k, Y_{k-1}, \ldots, Y_1$ into the stack. The parser also outputs the production rule $X \rightarrow Y_1 Y_2 \ldots Y_k$ to represent a step of the derivation.

4. none of the above ➔ error
   - all empty entries in the parsing table are errors.
   - If X is a terminal symbol different from a, this is also an error case.

LL(1) Parser – Example1

$S \rightarrow aBa$

$B \rightarrow bB \mid \varepsilon$

	a	**b**	**$**
S	$S \rightarrow aBa$		
B	$B \rightarrow \varepsilon$	$B \rightarrow bB$	

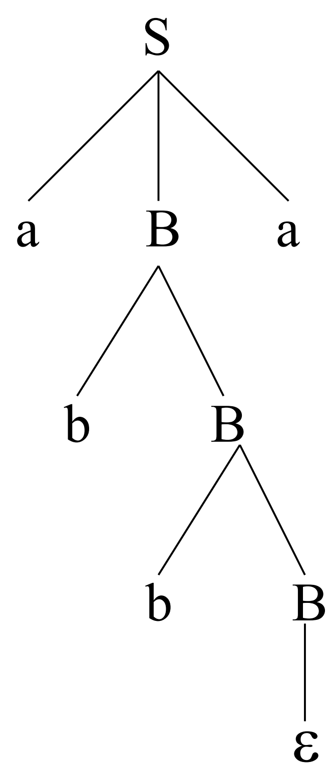
LL(1) Parsing Table

stack	**input**	**output**
$S	abba$	$S \rightarrow aBa$
$aBa	abba$	
$aB	bba$	$B \rightarrow bB$
$aBb	bba$	
$aB	ba$	$B \rightarrow bB$
$aBb	ba$	
$aB	a$	$B \rightarrow \varepsilon$
$a	a$	
$	$	accept, successful completion

LL(1) Parser – Example1 (cont.)

Outputs: $S \rightarrow aBa$ $B \rightarrow bB$ $B \rightarrow bB$ $B \rightarrow \varepsilon$

Derivation(left-most): $S \Rightarrow aBa \Rightarrow abBa \Rightarrow abbBa \Rightarrow abba$



parse tree

LL(1) Parser – Example2

Input= id+id$

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \varepsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \varepsilon$

$F \rightarrow (E) \mid id$

	id	**+**	*****	**(**	**)**	**$**
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \varepsilon$	$E' \rightarrow \varepsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \varepsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \varepsilon$	$T' \rightarrow \varepsilon$
F	$F \rightarrow id$			$F \rightarrow (E)$		

LL(1) Parser – Example2

1. $E \rightarrow TE'$
2. $E' \rightarrow +TE'$
3. $E' \rightarrow \varepsilon$
4. $T \rightarrow FT'$
5. $T' \rightarrow *FT'$
6. $T' \rightarrow \varepsilon$
7. $F \rightarrow (E)$
8. $F \rightarrow id$

FIRST(F) = { (,id}

FIRST(T′) = {*, ε}

FIRST(T) = { (,id}

FIRST(E′) = {+, ε}

FIRST(E) = { (,id}

FOLLOW(E) = { $,) }
FOLLOW(E') = { $,) }
FOLLOW(T) = { +,), $ }
FOLLOW(T') = { +,), $ }
FOLLOW(F) = {+, *,), $ }

	id	+	*	(	)	$
E	1			1		
E'						
T						
T'						
F						

LL(1) Parser – Example2

stack	input	output
E	id+id$	$E \rightarrow TE'$
$E'T	id+id$	$T \rightarrow FT'$
$E' T'F	id+id$	$F \rightarrow id$
$ E' T'id	id+id$	
$ E' T'	+id$	$T' \rightarrow \varepsilon$
$ E'	+id$	$E' \rightarrow +TE'$
$ E' T+	+id$	
$ E' T	id$	$T \rightarrow FT'$
$ E' T' F	id$	$F \rightarrow id$
$ E' T'id	id$	
$ E' T'	$	$T' \rightarrow \varepsilon$
$ E'	$	$E' \rightarrow \varepsilon$
$	$	accept

Constructing LL(1) Parsing Tables

1. Eliminate left recursion in grammar G
2. Perform left factoring on the grammar G
3. Find FIRST and FOLLOW for each NT of grammar G
4. Construct the predictive parse table OR LL(1) parse table
5. Check if the given input string can be accepted by the parser

Compute FIRST

- If α is a terminal symbol 'a' then FIRST(α)={a}

For example, for grammar rule A -> a, FIRST(a)={a}

- If α is a non-terminal symbol 'X' and X -> aα,

then FIRST(X)=FIRST(aα)={a}

For example for grammar rule A->aBC, FIRST(A) = FIRST(aBC) = {a}

- If α is a non-terminal 'X' and X-> Ɛ, then FIRST(X)={Ɛ}

For example for grammar rule A->Ɛ, FIRST(A)={Ɛ}

- If X -> $Y_1 Y_2 .. Y_n$ then add to FIRST($Y_1 Y_2 .. Y_n$) all the non- Ɛ symbols of FIRST(Y_1). Also add the non- Ɛ symbols of FIRST(Y_2) if Ɛ is in FIRST(Y_1), the non- Ɛ symbols of FIRST(Y_3) if Ɛ is in both FIRST(Y_1) and in FIRST(Y_2), and so on. Finally add Ɛ to FIRST($Y_1 Y_2 .. Y_n$) if, for all i, FIRST(Y_i) contains Ɛ.

For example for rules: X -> Yb and Y -> a | Ɛ

FIRST(X)=FIRST(Yb)=FIRST(Y)={a, b}

FIRST Example

$E \rightarrow TE'$
$E' \rightarrow +TE' \mid \varepsilon$
$T \rightarrow FT'$
$T' \rightarrow *FT' \mid \varepsilon$
$F \rightarrow (E) \mid id$

$\text{FIRST}(F) = \{(, id\}$
$\text{FIRST}(T') = \{*, \varepsilon\}$
$\text{FIRST}(T) = \{(, id\}$
$\text{FIRST}(E') = \{+, \varepsilon\}$
$\text{FIRST}(E) = \{(, id\}$

$\text{FIRST}(TE') = \{(, id\}$
$\text{FIRST}(+TE') = \{+\}$
$\text{FIRST}(\varepsilon) = \{\varepsilon\}$
$\text{FIRST}(FT') = \{(, id\}$
$\text{FIRST}(*FT') = \{*\}$
$\text{FIRST}(\varepsilon) = \{\varepsilon\}$
$\text{FIRST}((E)) = \{(\}$
$\text{FIRST}(id) = \{id\}$

Compute FOLLOW (for non-terminals)

FOLLOW of a non-terminal A is a set of terminals that follow or occur to the right of A

- If S is the start symbol ➔ $ is in FOLLOW(S)

- if $A \rightarrow \alpha B \beta$ is a production rule
  ➔ everything in FIRST(β) is FOLLOW(B) except ε

- If ($A \rightarrow \alpha B$ is a production rule) or
  ($A \rightarrow \alpha B \beta$ is a production rule and ε is in FIRST(β))
  ➔ everything in FOLLOW(A) is in FOLLOW(B).

We apply these rules until nothing more can be added to any follow set.

FOLLOW Example

$E \rightarrow TE'$
$E' \rightarrow +TE' \mid \varepsilon$
$T \rightarrow FT'$
$T' \rightarrow *FT' \mid \varepsilon$
$F \rightarrow (E) \mid id$

FOLLOW(E) = { \$,) }

FOLLOW(E') = { \$,) }

FOLLOW(T) = { +,), \$ }

FOLLOW(T') = { +,), \$ }

FOLLOW(F) = {+, *,), \$ }

First(E')={+,ep}

First(T')={*,ep}

Constructing LL(1) Parsing Table -- Algorithm

- for each production rule $A \rightarrow \alpha$ of a grammar G
  - for each terminal a in FIRST(α)
    ➔ add $A \rightarrow \alpha$ to M[A,a]
  - If ε in FIRST(α)
    ➔ for each terminal a in FOLLOW(A) add $A \rightarrow \alpha$ to M[A,a]
  - If ε in FIRST(α) and \$ in FOLLOW(A)
    ➔ add $A \rightarrow \alpha$ to M[A,\$]

- All other undefined entries of the parsing table are error entries.

Constructing LL(1) Parsing Table -- Example

$E \to TE'$	$FIRST(TE')=\{(,id\}$	➔ $E \to TE'$ into $M[E,(]$ and $M[E,id]$
$E' \to +TE'$	$FIRST(+TE')=\{+\}$	➔ $E' \to +TE'$ into $M[E',+]$
$E' \to \varepsilon$	$FIRST(\varepsilon)=\{\varepsilon\}$ but since ε in $FIRST(\varepsilon)$ and $FOLLOW(E')=\{\$,)\}$	➔ none ➔ $E' \to \varepsilon$ into $M[E',\$]$ and $M[E',)]$
$T \to FT'$	$FIRST(FT')=\{(,id\}$	➔ $T \to FT'$ into $M[T,(]$ and $M[T,id]$
$T' \to *FT'$	$FIRST(*FT')=\{*\}$	➔ $T' \to *FT'$ into $M[T',*]$
$T' \to \varepsilon$	$FIRST(\varepsilon)=\{\varepsilon\}$ but since ε in $FIRST(\varepsilon)$ and $FOLLOW(T')=\{\$,),+\}$	➔ none ➔ $T' \to \varepsilon$ into $M[T',\$]$, $M[T',)]$ and $M[T',+]$
$F \to (E)$	$FIRST((E))=\{(\}$	➔ $F \to (E)$ into $M[F,(]$
$F \to id$	$FIRST(id)=\{id\}$	➔ $F \to id$ into $M[F,id]$

LL(1) Grammars

- A grammar whose parsing table has no multiply-defined entries is said to be LL(1) grammar.

one input symbol used as a look-head symbol to determine parser action

LL(1) left most derivation

input scanned from left to right

- ***The parsing table of a grammar may contain more than one production rule. In this case, we say that it is not a LL(1) grammar.***

A Grammar which is not LL(1)

$S \rightarrow i\ C\ t\ S\ E \quad | \quad a$
$E \rightarrow e\ S \quad | \quad \varepsilon$
$C \rightarrow b$

FIRST(iCtSE) = {i}
FIRST(a) = {a}
FIRST(eS) = {e}
FIRST(ε) = {ε}
FIRST(b) = {b}

FOLLOW(S) = { \$,e }
FOLLOW(E) = { \$,e }
FOLLOW(C) = { t }

	a	**b**	**e**	**i**	**t**	**\$**
S	$S \rightarrow a$			$S \rightarrow iCtSE$		
E			$E \rightarrow e\ S$ $E \rightarrow \varepsilon$			$E \rightarrow \varepsilon$
C		$C \rightarrow b$				

two production rules for M[E,e]

Problem ➔ ambiguity

A Grammar which is not LL(1) (cont.)

- What do we have to do if the resulting parsing table contains multiply defined entries?
  - If we didn't eliminate left recursion, eliminate the left recursion in the grammar.
  - If the grammar is not left factored, we have to left factor the grammar.
  - If its (new grammar's) parsing table still contains multiply defined entries, that grammar is ambiguous or it is inherently not a LL(1) grammar.
- A left recursive grammar cannot be a LL(1) grammar.
  - $A \rightarrow A\alpha \mid \beta$
    - ➔ any terminal that appears in FIRST(β) also appears FIRST($A\alpha$) because $A\alpha \Rightarrow \beta\alpha$.
    - ➔ If β is ε, any terminal that appears in FIRST(α) also appears in FIRST($A\alpha$) and FOLLOW(A).
- A grammar is not left factored, it cannot be a LL(1) grammar
  - $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$
    - ➔ any terminal that appears in FIRST($\alpha\beta_1$) also appears in FIRST($\alpha\beta_2$).
- An ambiguous grammar cannot be a LL(1) grammar.

Properties of LL(1) Grammars

- A grammar G is LL(1) if and only if the following conditions hold for two distinctive production rules $A \rightarrow \alpha$ and $A \rightarrow \beta$

1. Both α and β cannot derive strings starting with same terminals.

2. At most one of α and β can derive to ε.

3. If β can derive to ε, then α cannot derive to any string starting with a terminal in FOLLOW(A).

Example

- Construct predictive parse table for the following grammar. Also show parser actions for the input string - (a,a)

  S->a | ↑ | (T)

  T->T,S | S

- Eliminate left recursion
- left factor
- First
- Follow
- Construct parsing table – check multiple entries
- Show Actions

Cont.

- Eliminate left recursion

S-> a | ↑ | (T)

T-> ST’

T’-> ,ST’ | Ɛ

- Its not needed to left factor
- FIRST

FIRST(S)={a, ↑, (}

FIRST(T)=FIRST(ST’)=FIRST(S)={a, ↑,(}

FIRST(T’)={, , Ɛ}

- FOLLOW

FOLLOW(S)={$,) and ,}

FOLLOW(T)={) }

FOLLOW(T’)={) }

- Is following grammar LL(1)? Also trace input string - ibtaea

S -> iCtSS` | a

S` -> eS | Ɛ

C ->b

- Is following grammar LL(1)? Also trace input string – int*int

$E \rightarrow T + E \mid T$

$T \rightarrow int \mid int * T \mid (E)$

$E \rightarrow T X$

$X \rightarrow + E \mid \varepsilon$

$T \rightarrow (E) \mid int\ Y$

$Y \rightarrow * T \mid \varepsilon$

First(T) = {int, (}

First(E) = {int, (}

First(X) = {+, ε }

First(Y) = {*, ε }

Follow(+) = { int, (}   Follow(*) = { int, (}

Follow(( ) = { int, (}   Follow(E) = {), $}

Follow(X) = {$,) }   Follow(T) = {+,) , $}

Follow( )) = {+,) , $}   Follow(Y) = {+,) , $}

Follow(int) = {*, +,) , $}

Motivation Behind First & Follow

First: Is used to help find the appropriate production to follow given the top-of-the-stack non-terminal and the current input symbol.

Example: If $A \rightarrow \alpha$, and a is in First(α), then when a=input, replace A with α (in the stack).

(a is one of first symbols of α, so when A is on the stack and a is input, POP A and PUSH α.

Follow: Is used when First has a conflict, to resolve choices, or when First gives no suggestion. When $\alpha \rightarrow \in$ or $\alpha \stackrel{*}{\Rightarrow} \in$, then what follows A dictates the next choice to be made.

Example: If $A \rightarrow \alpha$, and b is in Follow(A), then when $\alpha \stackrel{*}{\Rightarrow} \in$ and b is an input character, then we expand A with α , which will eventually expand to $\in$, of which b follows!

($\alpha \stackrel{*}{\Rightarrow} \in$: i.e., First(α) contains $\in$.)