B. Tech.-I (Semester-I) Branch-All
Subject: Mathematics-I (MA 10 S1)
Tutorial-10

1. Evaluate the following:

   a. $\int_0^a \int_0^{\sqrt{a^2-y^2}} (a^2-x^2-y^2)^{\frac{1}{2}} dxdy$ Ans: $\frac{\pi a^3}{6}$ b. $\int_0^1 \int_0^{\sqrt{1+x^2}} (1+x^2+y^2)^{-1} dxdy$ Ans $\frac{\pi}{4}\log\left[1+\sqrt{2}\right]$

2. Evaluate integral by changing to polar form, $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{1}{\sqrt{a^2-(x^2+y^2)}} dydx$. Ans. a

3. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing to polar coordinates. Hence, deduce that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. Ans. $\frac{\pi}{4}$

4. Evaluate $\iint x^2y^2 dxdy$ over the region bounded by x = 0, y = 0, and $x^2 + y^2 = 1$.

   Ans: $\int_0^1 \int_0^{\sqrt{1-y^2}} x^2y^2 dxdy = \frac{\pi}{96}$

5. Evaluate $\iint \sqrt{\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}} dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (Hint: put $\frac{x}{a}=X, \frac{y}{b}=Y$ Ans. $\frac{\pi ab}{8}(\pi-2)$

6. Transform to the Cartesian form and hence, evaluate the integral $\int_0^\pi \int_0^a r^3 \sin\theta\cos\theta dr d\theta$. Ans. 0

7. Find the area bounded by the parabola $y = x^2$ and the line y = 2x + 3. Ans.32/3

8. Find the area outside the circle r=a and inside the cardioid $r = a(1+cos\theta)$. Ans. $\frac{a^2(\pi+8)}{4} sq$

9. Calculate the area included between the curve $r = a(\sec\theta + cos\theta)\ \pi$ and its asymptote. Ans. $\frac{5\pi a^2}{4}$

10. Use the transformation $x+y=u$ and $y=uv$ to show that $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dydx = \frac{e-1}{2}$.

11. Use the transformation $x+y=u$ and $y=uv$, to show that $\iint \sqrt{xy(1-x-y)} dxdy$, taken over the area of triangle bounded by the lines $x=0,\ y=0,\ x+y=1$ is $\frac{2\pi}{105}$. (Hint: use Beta Gamma function)

12. Find the mass contained in a thin plate of the shape: plane region R bounded by the parabola $x = y - y^2$ and the straight line x + y = 0, having mass density x + y. Ans. 8/15

13. Using double integration prove that the volume, enclosed between $(x^2+y^2)=2ax$ and $z^2=2ax$ is $\frac{128a^3}{15}$.

14. Using double integration find the volume bounded by the paraboloid $(x^2+y^2)=az$, the cylinder $(x^2+y^2)=2ay$, and the plane z=0. Ans. $\frac{3\pi a^3}{2}$