

Tutorial-3

Q.1 Given: $A = \{1, 2\}$, $B = \{x, y, z\}$ and $C = \{3, 4\}$. Find $A \times B \times C$.

Q.2 Find the number of relations from $A = \{a, b, c\}$ to $B = \{1, 2\}$. [Ans: ~~no~~ 64 relations.]

Q.3 Given $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let, R be the following relation from A to B .

$$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}.$$

- Determine the matrix of the relation.
- Draw the arrow diagram of R .
- Find the inverse relation R^{-1} of R .
- Determine the domain and range of R .

Q.4 Let, $A = \{1, 2, 3, 4, 5, 6\}$ and let R be relation on A defined by " x divides y " written $x|y$.

- Write R as a set of ordered pairs.
- Draw its directed graph.
- Find the inverse relation R^{-1} of R . Can R^{-1} be described in words?

Q.5 Let, $A = \{1, 2, 3\}$, $B = \{a, b, c\}$ and $C = \{x, y, z\}$.

Consider the following relations R and S from A to B and from B to C resp.

$$R = \{(1, b), (2, a), (2, c)\} \text{ and } S = \{(a, y), (b, x), (c, y), (c, z)\}.$$

- Find the composition relation $R \circ S$
- Find the matrices M_R , M_S and $M_{R \circ S}$ of the respective relations R , S and $R \circ S$.

Q.6 Let, R and S be the following relations on $A = \{1, 2, 3\}$: $R = \{(1, 1), (1, 2), (2, 3), (3, 1), (3, 3)\}$,

$$S = \{(1,2), (1,3), (2,1), (3,3)\}.$$

Find a) $R \cap S$, $R \cup S$, R' b) $R \circ S$ c) $S^2 = S \circ S$

Q.7 Given $A = \{1, 2, 3, 4\}$. Consider the following relation in A : $R = \{(1,1), (2,2), (2,3), (3,2), (4,2), (4,4)\}$.

a) Draw its directed graph.

b) Is R ① Reflexive, ② Symmetric, ③ transitive or ④ antisymmetric?

Q.8 Give examples of relations R on $A = \{1, 2, 3\}$ having the stated property.

a) R is both symmetric and antisymmetric.

b) R is neither symmetric nor antisymmetric.

~~c) R is transitive but $R \cup R^{-1}$ is not transitive.~~

Q.9 Consider the set \mathbb{Z} of integers and an integer $m > 1$. We say that $x \equiv y \pmod{m}$. i.e. $x - y$ is divisible by m . Show that this defines an equivalence relation on \mathbb{Z} .

Q.10 Let, R be the following equivalence relation on the set $A = \{1, 2, 3, 4, 5, 6\}$; $R = \{(1,1), (1,5), (2,2), (2,3), (2,6), (3,2), (3,3), (3,6), (4,4), (5,1), (5,5), (6,2), (6,3), (6,6)\}$. Find the partition of A induced by R .