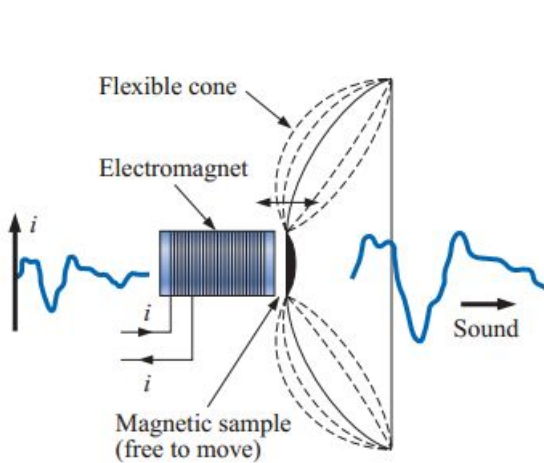


# CONTENTS

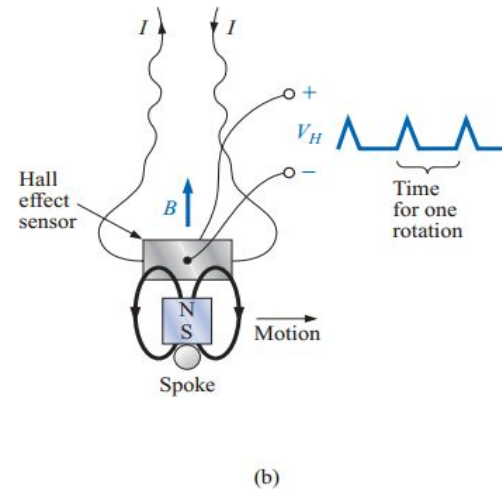
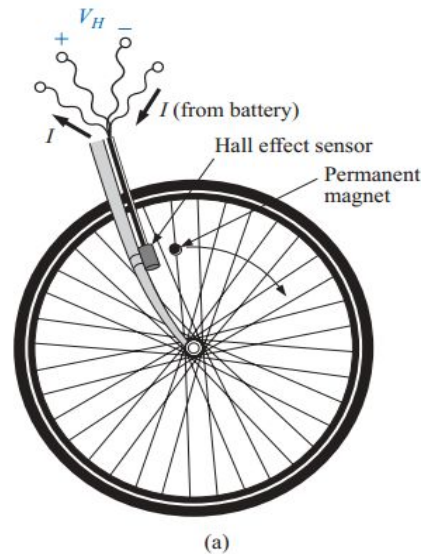
- Magnetic Fields
- Flux Density
- Permeability
- Reluctance
- OHM'S LAW for Magnetic Circuits
- Magnetizing Force
- Hysteresis
- AMPERE'S Circuital Law
- Series Magnetic Circuits
  - Air Gaps
- Series-parallel Magnetic Circuits

# INTRODUCTION

- ❑ Magnetism plays an integral part in almost every electrical device used today in industry, research, or the home.
- ❑ Generators, motors, transformers, circuit breakers, televisions, computers, tape recorders, and telephones all employ magnetic effects to perform a variety of important tasks.



*Fig: Speaker*

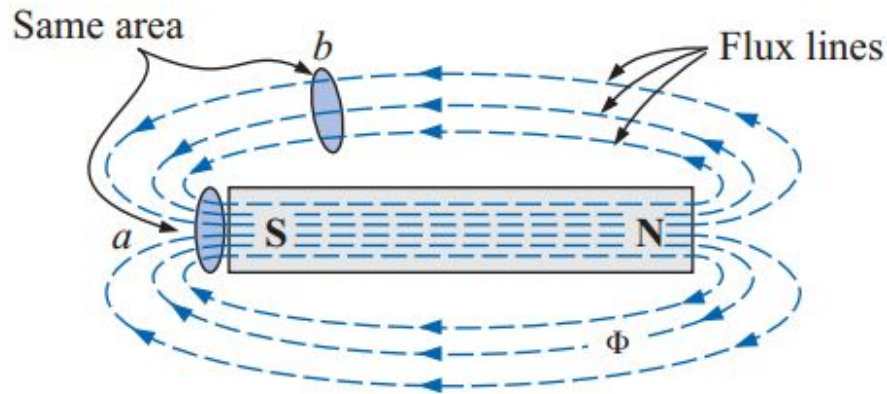


*Fig: Obtaining a speed indication for a bicycle using a Hall effect sensor: (a) mounting the components; (b) Hall effect response.*

For the first time it was demonstrated that electricity and magnetism were related, and the French physicist André-Marie Ampère performed experiments in this area and developed what is presently known as **Ampère's circuital law**.

# MAGNETIC FIELDS

- In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by **magnetic flux lines**
- Magnetic flux lines, do not have origins or terminating points. The symbol for magnetic flux is the Greek letter  $\phi$  (phi).

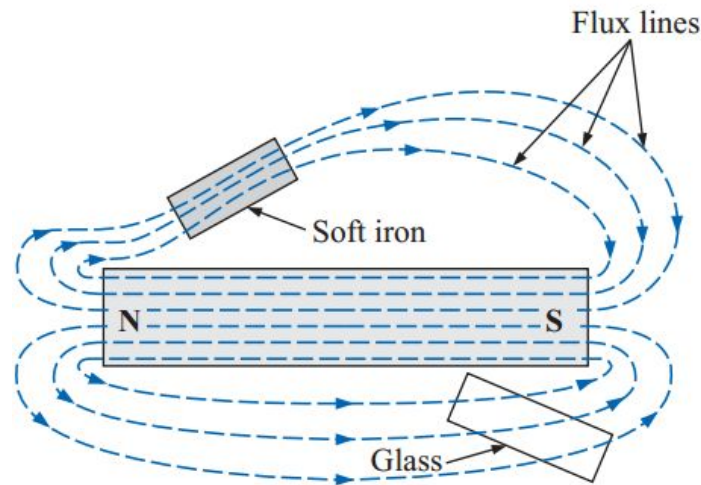


*Fig. Flux distribution for a permanent magnet.*

# MAGNETIC FIELDS

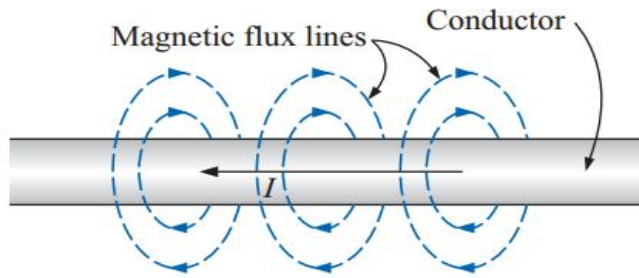
□ When the nonmagnetic material (glass or copper), and magnetic material (soft iron) are placed in the flux paths surrounding a permanent magnet,

there will be an almost unnoticeable change in the flux distribution in non magnetic materials as seen in Fig.

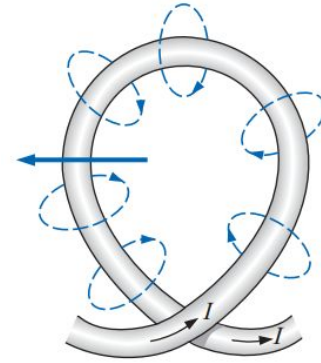


*Fig: Effect of a ferromagnetic sample on the flux distribution of a permanent magnet.*

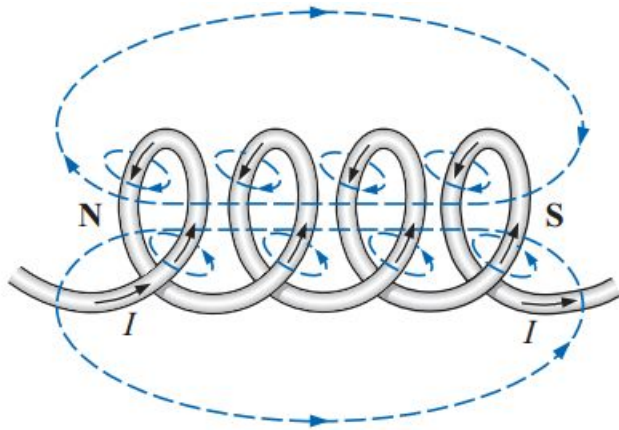
# MAGNETIC FIELDS



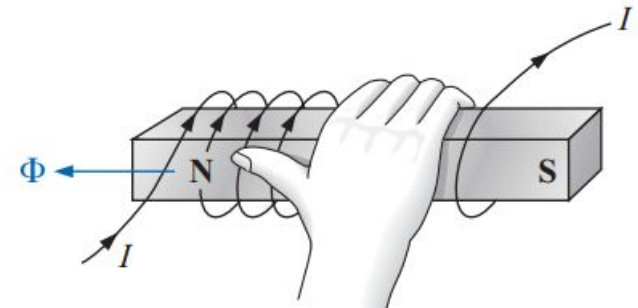
**(a) Magnetic flux lines around a current-carrying conductor**



**(b) Flux distribution of a single-turn coil.**



**(c) Flux distribution of a current-carrying coil.**



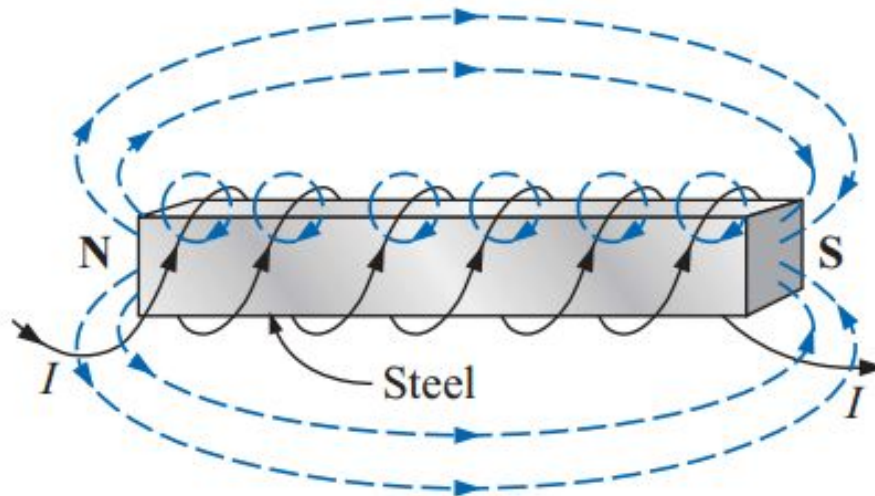
**Right-hand rule**

The direction of the magnetic flux lines can be found simply by placing the thumb of the *right* hand in the direction of *conventional* current flow and noting the direction of the fingers.

# MAGNETIC FIELDS

## Electromagnet

*Electromagnet* has all the properties of a permanent magnet, also has a field strength that can be varied by changing one of the component values (current, turns, and so on).



## FLUX DENSITY

- The number of flux lines per unit area is called the **flux density**, is denoted by the capital letter  $B$ .

$$B = \frac{\Phi}{A}$$

$B$  = teslas (T)

$\Phi$  = webers (Wb)

$A$  = square meters ( $\text{m}^2$ )

**EXAMPLE.1:** For the core of Fig, determine the flux density  $B$  in teslas.



$$\begin{aligned}\Phi &= 6 \times 10^{-5} \text{ Wb} \\ A &= 1.2 \times 10^{-3} \text{ m}^2\end{aligned}$$



## PERMEABILITY

- The **permeability ( $\mu$ )** is a measure of the ease with which magnetic flux lines can be established in the material
- It is similar in many respects to conductivity in electric circuits.
- The ratio of the permeability of a material to that of free space is called its **relative permeability**

$$\mu_r = \frac{\mu}{\mu_o}$$

Permeability of free space  $\mu_o = 4\pi \times 10^{-7}$

## RELUCTANCE

- The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho \frac{l}{A} \quad (\text{ohms, } \Omega)$$

- The **reluctance** of a material to the setting up of magnetic flux lines in the material is determined by the following equation

$$\mathcal{R} = \frac{l}{\mu A} \quad (\text{rels, or At/Wb})$$

Where  $l$  is the length of the magnetic path, and  $A$  is the cross-sectional area

## OHM'S LAW FOR MAGNETIC CIRCUITS

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}}$$

The magnetomotive force  $\mathcal{F}$  is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire

$$\mathcal{F} = NI \quad (\text{ampere-turns, At})$$

## MAGNETIZING FORCE

The magnetomotive force per unit length is called the **magnetizing force** ( $H$ ).

$$H = \frac{\mathcal{F}}{l} \quad (\text{At/m})$$

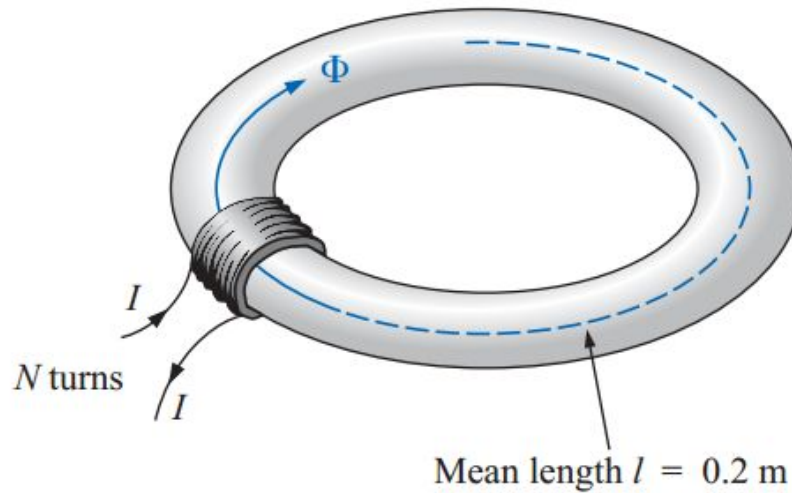
Substituting for the magnetomotive force will result in

$$H = \frac{NI}{l} \quad (\text{At/m})$$

## MAGNETIZING FORCE

### EXAMPLE.2:

For the magnetic circuit of Fig, if  $NI = 40 \text{ At}$  and *length*  $l=0.2 \text{ m}$ , then  $H=?$



The magnetizing force is independent of the type of core material. It is determined solely by the number of turns, the current, and the length of the core.

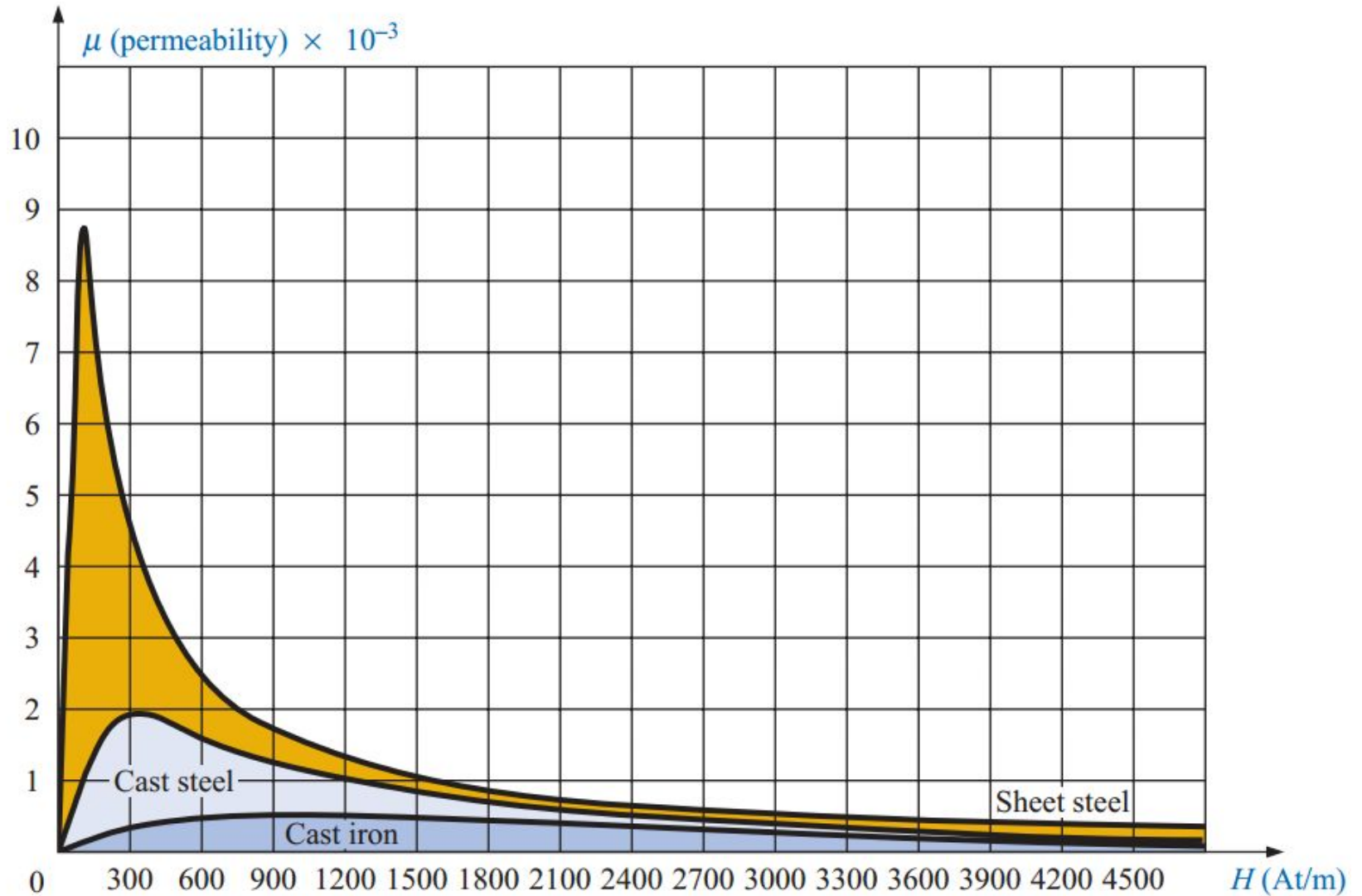
## MAGNETIZING FORCE

- The relation between the flux density and the magnetizing force is

$$B = \mu H$$

- The applied magnetizing force has a pronounced effect on the resulting permeability of a magnetic material.
- As the **magnetizing force increases**, the **permeability rises to a maximum** and then **drops to a minimum**

# MAGNETIZING FORCE



Magnetizing force (H) vs Permeability

## HYSTERESIS

- A curve of the flux density  $B$  versus the magnetizing force  $H$  of a material is of particular importance to the engineer.

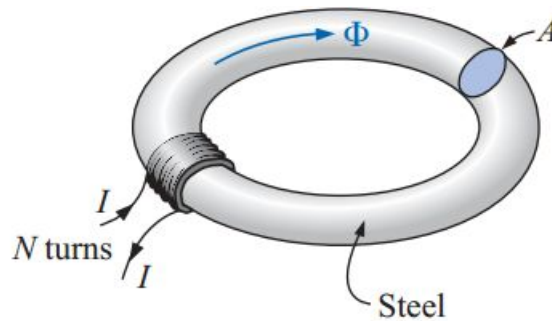


Fig.1

- The core is initially unmagnetized and **the current  $I = 0$** . If the current  $I$  is increased to some value above zero, the **magnetizing force  $H$**  will increase to a value determined by

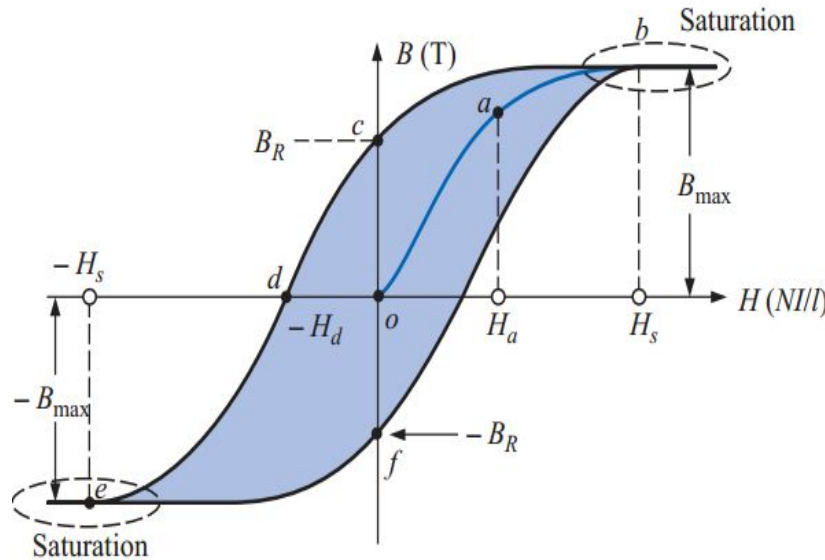
$$H \uparrow = \frac{NI \uparrow}{l}$$

- The flux and the flux density will also increase with the current  $I$  (or  $H$ )



# HYSTERESIS

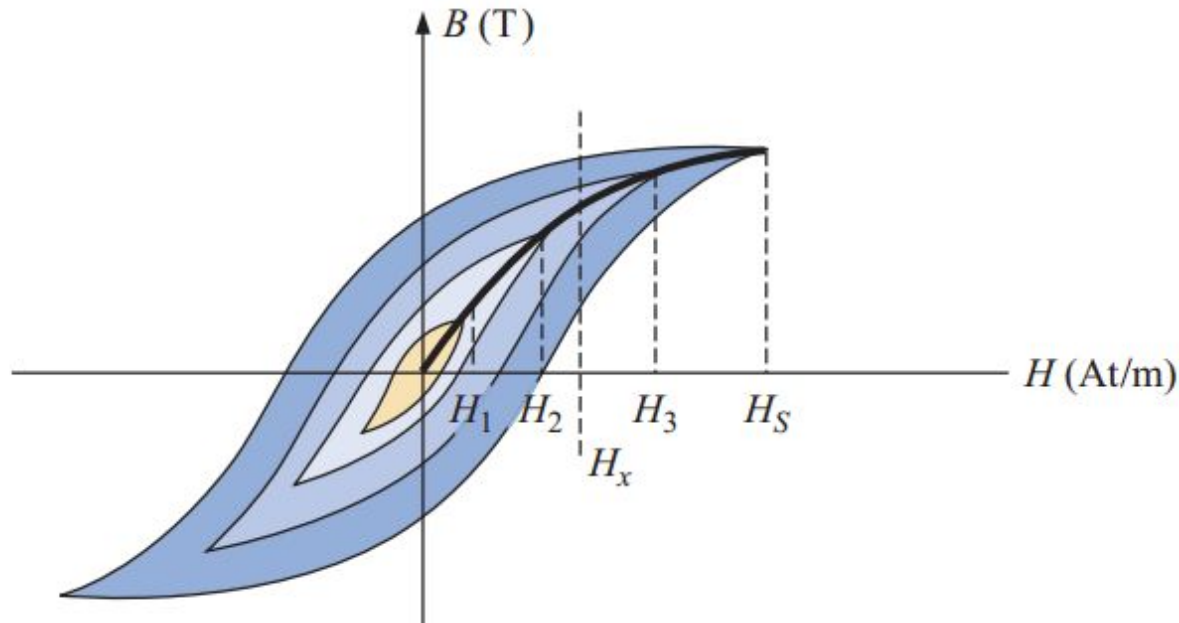
- When saturation occurs, any further increase in current through the coil increasing  $H$  will result in a very small increase in flux density  $B$ .



- The flux density  $B_R$ , which remains when the magnetizing force is zero, is called **the residual flux density**.
- The magnetizing force  $-H_d$  required to “coerce” the flux density to reduce its level to zero is called **the coercive force**, a measure of the coercivity of the magnetic sample.

# HYSTERESIS

- Three hysteresis loops for the same material for maximum values of  $H$ , less than the saturation value are shown in Fig.



- Note from the various curves that for a particular value of  $H$ , say,  $H_x$ , the value of  $B$  can vary widely, as determined by the history of the core.

# HYSTERESIS

- A comparison of Figs. (A) and (B) shows that for the same value of  $H$ , the value of  $B$  is higher in Fig. (B) for the materials with the higher  $\mu$  in fig. (A).

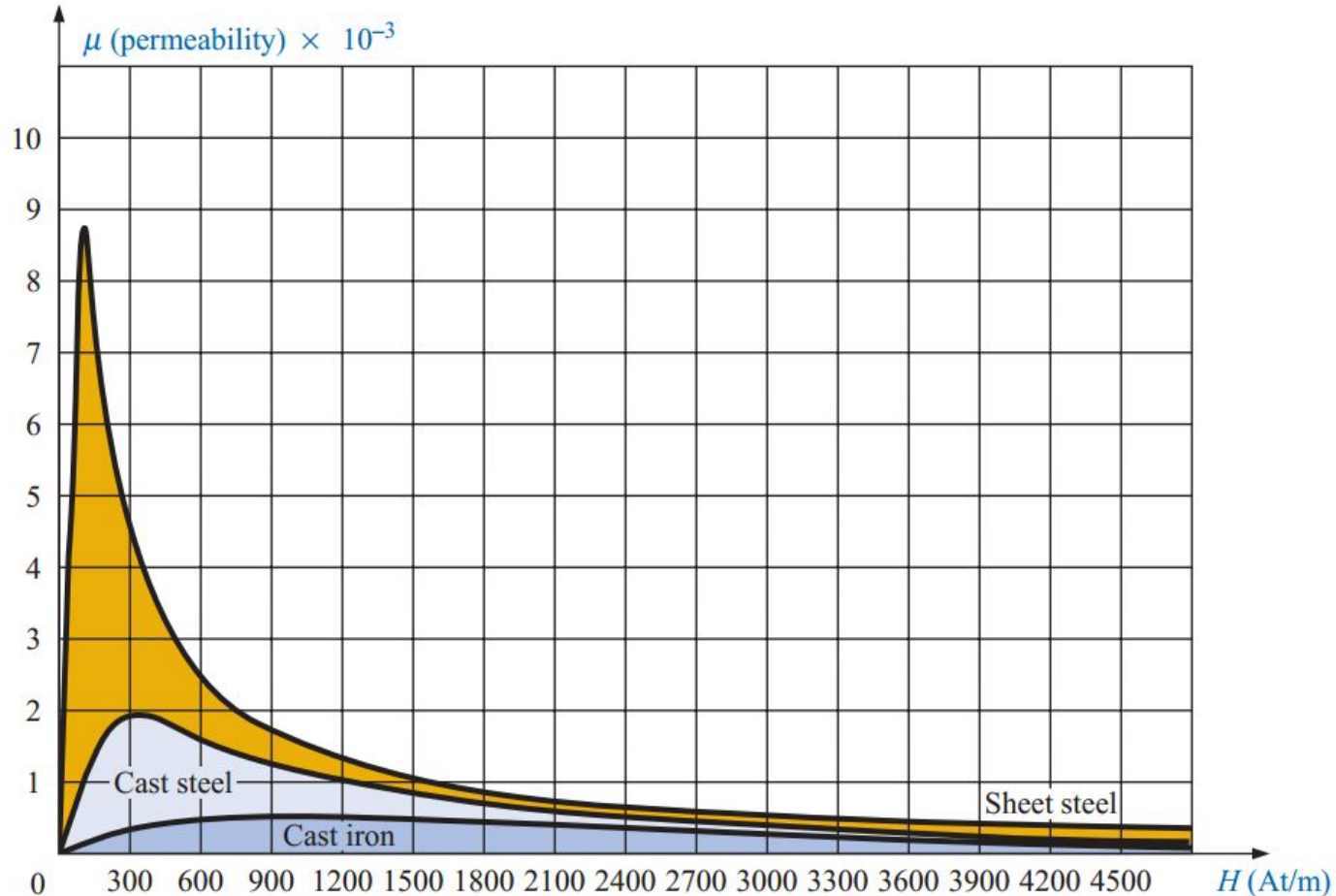


Fig. (A).

# HYSTERESIS

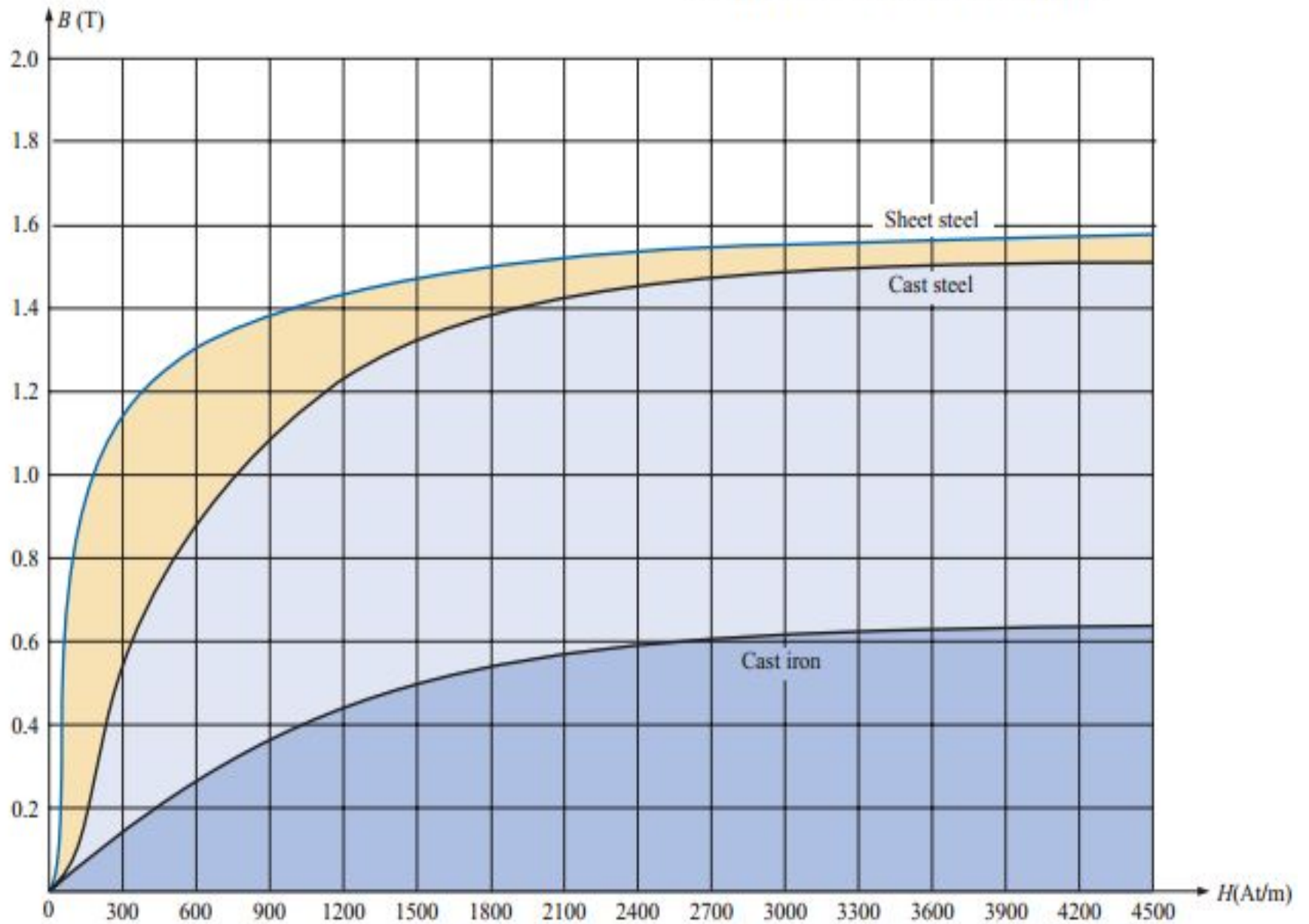


Fig. (B)

## AMPERE'S CIRCUITAL LAW

There is a broad similarity between the analyses of electric and magnetic circuits

	Electric Circuits	Magnetic Circuits
Cause	$E$	$\mathcal{F}$
Effect	$I$	$\Phi$
Opposition	$R$	$\mathcal{R}$

### Kirchhoff's voltage law:

The algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.

$$\boxed{\sum_{\mathcal{C}} \mathcal{F} = 0} \quad (\text{for magnetic circuits})$$

## AMPERE'S CIRCUITAL LAW

$$\boxed{\sum_{\mathcal{C}} \mathcal{F} = 0} \quad (\text{for magnetic circuits})$$

This equation is referred to as **Ampère's circuital law**

When it is applied to magnetic circuits,

□ The sources of mmf are expressed by the equation is

$$\boxed{\mathcal{F} = NI} \quad (\text{At})$$

□ The drop mmf across a portion of a magnetic circuit is expressed by the equation

$$\boxed{\mathcal{F} = \Phi \mathcal{R}}$$

$$(\text{At})$$

or

$$\boxed{\mathcal{F} = Hl}$$

$$(\text{At})$$

## AMPERE'S CIRCUITAL LAW

### EXAMPLE.3:

consider the magnetic circuit appearing in Fig. constructed of three different ferromagnetic materials.

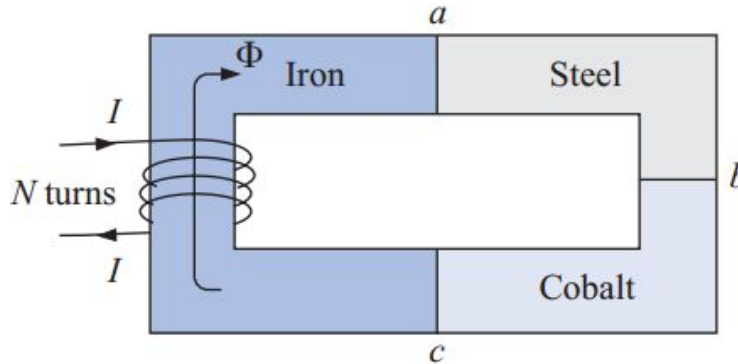


FIG. Series magnetic circuit of three different materials.

□ Applying Ampère's circuital law, we have

$$\sum_C \mathcal{F} = 0$$

$$\underbrace{+NI}_{\text{Rise}} - \underbrace{H_{ab}l_{ab}}_{\text{Drop}} - \underbrace{H_{bc}l_{bc}}_{\text{Drop}} - \underbrace{H_{ca}l_{ca}}_{\text{Drop}} = 0$$

$$\underbrace{NI}_{\text{Impressed mmf}} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{\text{mmf drops}}$$

# AMPERE'S CIRCUITAL LAW

## Kirchhoff's current law:

The sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction

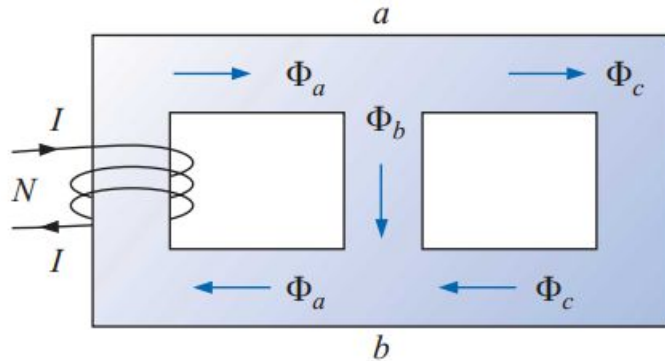


FIG. Flux distribution of a series-parallel magnetic network

$$\Phi_a = \Phi_b + \Phi_c \quad (\text{at junction } a)$$

$$\Phi_b + \Phi_c = \Phi_a \quad (\text{at junction } b)$$

Both of which are equivalent.



# SERIES MAGNETIC CIRCUITS

*Series magnetic circuits in which the flux  $\phi$  is the same throughout*

- An approach frequently employed in the analysis of magnetic circuits is the table method.
- Before a problem is analyzed in detail, a table is prepared listing in the extreme left-hand column the various sections of the magnetic circuit.
- The columns on the right are reserved for the quantities to be found for each section.
- Now we will see some of the series magnetic circuits and find the magnitude of the magnetomotive force of magnetic circuit.

## SERIES MAGNETIC CIRCUITS

### EXAMPLE.4:

For the series magnetic circuit of Fig. (i):

- Find the value of  $I$  required to develop a magnetic flux of  $\Phi = 4 \times 10^{-4}$  Wb.
- Determine  $\mu$  and  $\mu_r$  for the material under these conditions.

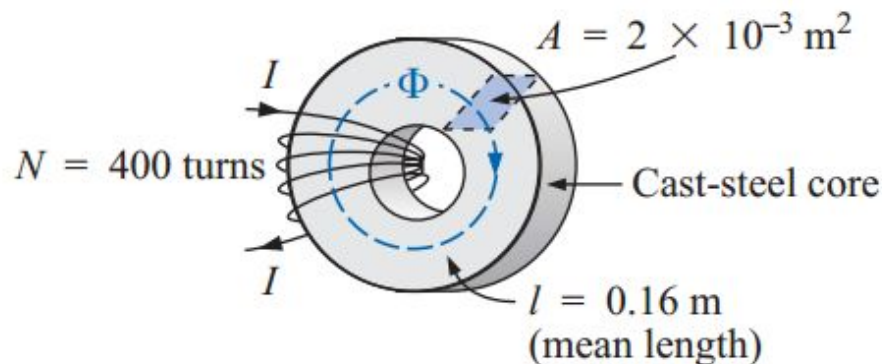
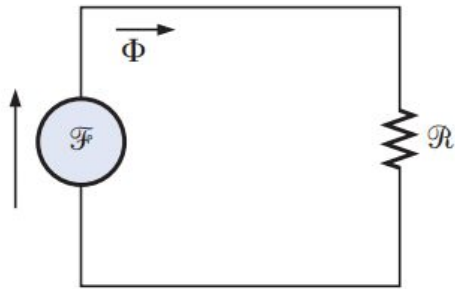


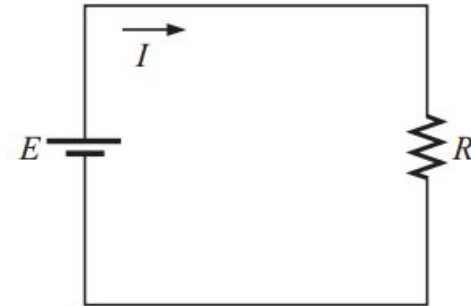
Fig. (i)

# SERIES MAGNETIC CIRCUITS

Solution:



(a)



(b)

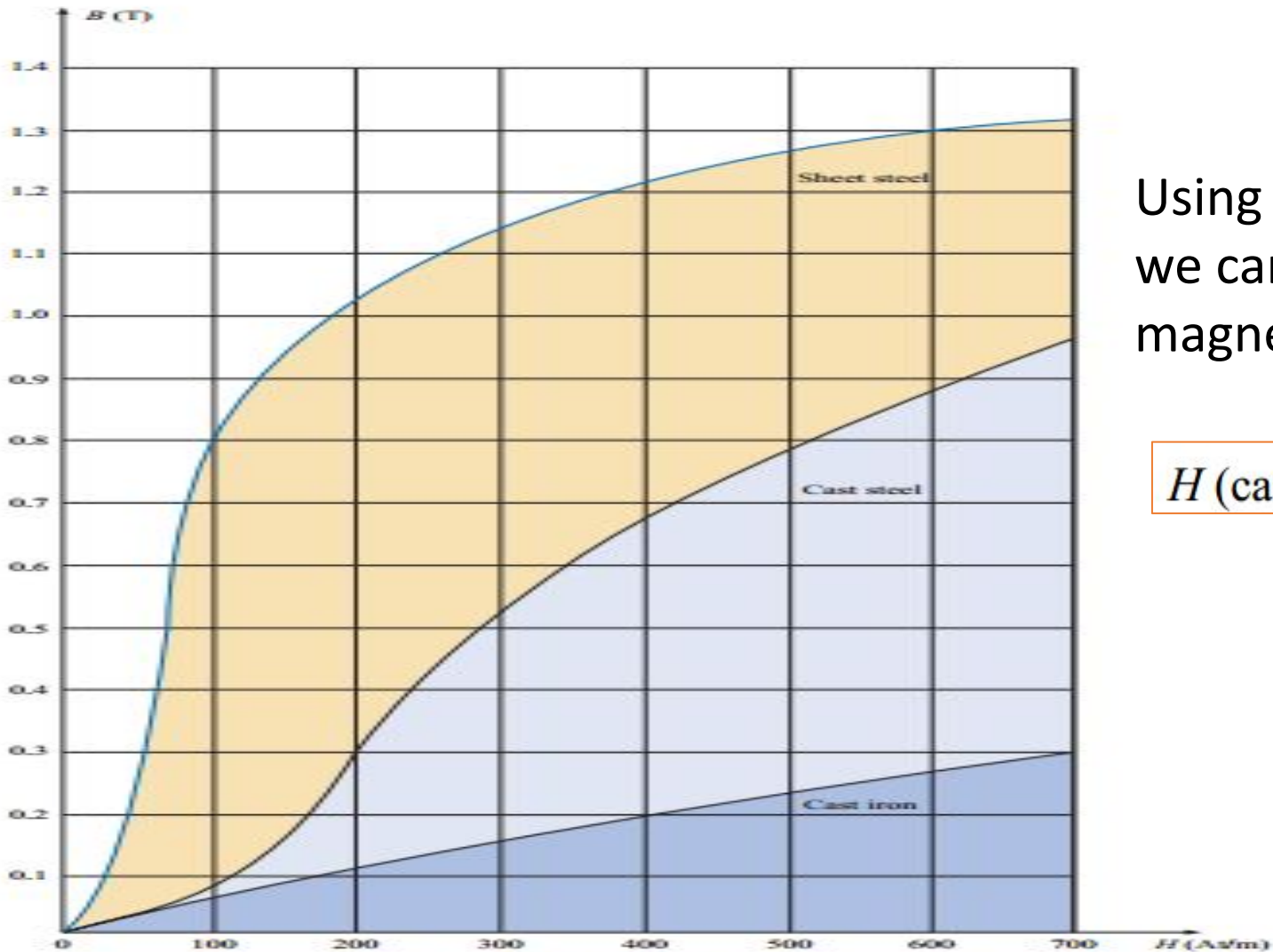
*Fig:(a) Magnetic circuit equivalent and (b) electric circuit analogy.*

Section	$\Phi$ (Wb)	$A$ (m <sup>2</sup> )	$B$ (T)	$H$ (At/m)	$l$ (m)	$HI$ (At)
One continuous section	$4 \times 10^{-4}$	$2 \times 10^{-3}$			0.16	

(a) The flux density  $B$  is

## SERIES MAGNETIC CIRCUITS

$$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4} \text{ Wb}}{2 \times 10^{-3} \text{ m}^2} = 2 \times 10^{-1} \text{ T} = 0.2 \text{ T}$$



Using the  $B$ - $H$  curves, we can determine the magnetizing force  $H$ ;

$$H (\text{cast steel}) = 170 \text{ At/m}$$

Applying Ampère's circuital law yields

$$NI = Hl$$

$$I = \frac{Hl}{N} = \frac{(170 \text{ At/m})(0.16 \text{ m})}{400 \text{ t}} = \mathbf{68 \text{ mA}}$$

(Recall that t represents turns.)

(b) The permeability of the material

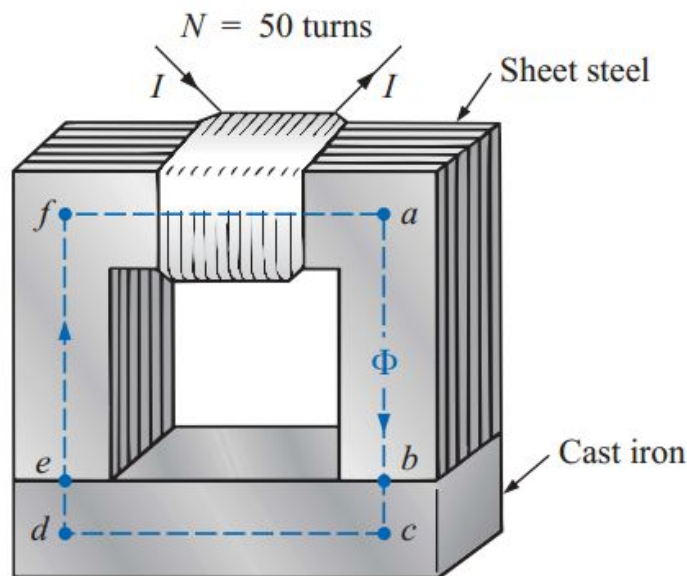
$$\mu = \frac{B}{H} = \frac{0.2 \text{ T}}{170 \text{ At/m}} = \mathbf{1.176 \times 10^{-3} \text{ Wb/A} \cdot \text{m}}$$

the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{935.83}$$

## PROBLEM.1:

The electromagnet of Fig. has picked up a section of cast iron. Determine the current  $I$  required to establish the indicated flux in the core.



$$\begin{aligned}l_{ab} &= l_{cd} = l_{ef} = l_{fa} = 4 \text{ in.} \\l_{bc} &= l_{de} = 0.5 \text{ in.} \\ \text{Area (throughout)} &= 1 \text{ in.}^2 \\ \Phi &= 3.5 \times 10^{-4} \text{ Wb}\end{aligned}$$

**To solve this problem, Use these B-H curve to get H values for corresponding B values**

$$H(\text{sheet steel, Fig. 11.24}) \cong 70 \text{ At/m}$$

$$H(\text{cast iron, Fig. 11.23}) \cong 1600 \text{ At/m}$$

## AIR GAPS

- The spreading of the flux lines outside the common area of the core for the air gap in Fig. (a) is known as *fringing*.
- For our purposes, we shall neglect this effect and assume the flux distribution to be as in Fig.(b).

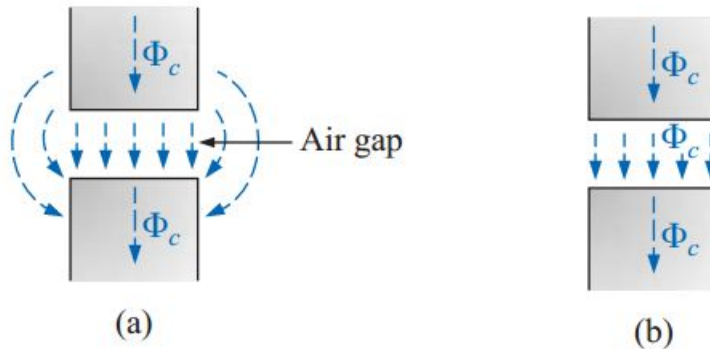


Fig. Air gaps: (a) with fringing; (b) ideal

- The average flux density in the air-gap is slightly less than the flux density in the core i.e.,  $(B_g)_{(av)} < B_c$ .

$$B_g = \frac{\Phi_g}{A_g}$$

## AIR GAPS

- The flux density of the air gap in Fig.(b) is given by

$$\Phi_g = \Phi_{\text{core}}$$

$$A_g = A_{\text{core}}$$

- For most practical applications, **the permeability of air** is taken to be equal to that of **free space**. The magnetizing force of the air gap is then determined by

$$H_g = \frac{B_g}{\mu_o}$$

- The mmf drop across the air gap is equal to  $H_g l_g$ . An equation for  $H_g$  is as follows:

$$H_g = \frac{B_g}{\mu_o} = \frac{B_g}{4\pi \times 10^{-7}}$$

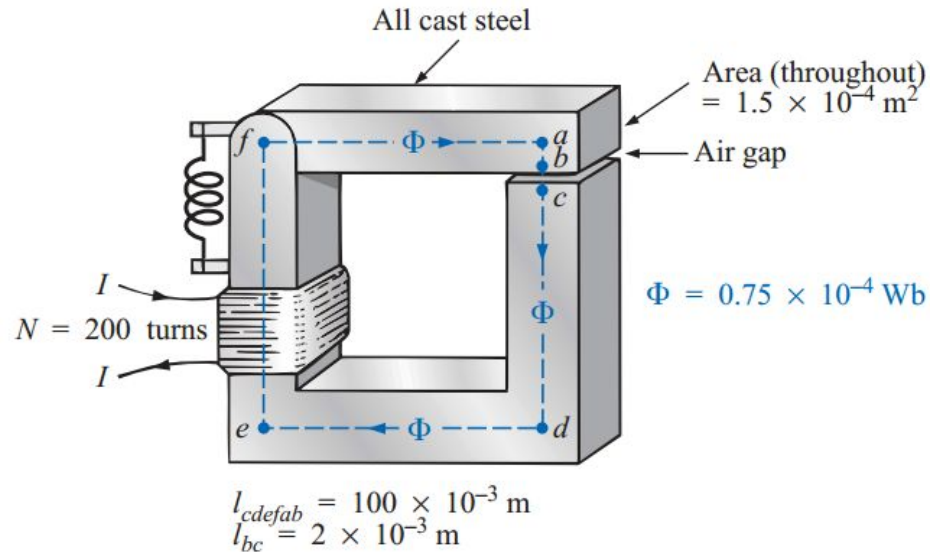
$$H_g = (7.96 \times 10^5) B_g \quad (\text{At/m})$$



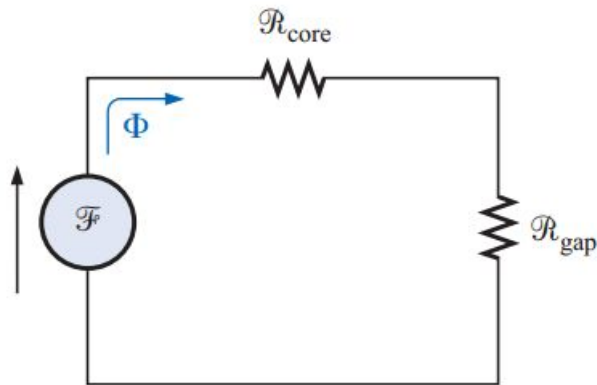
## AIR GAPS

### EXAMPLE.5:

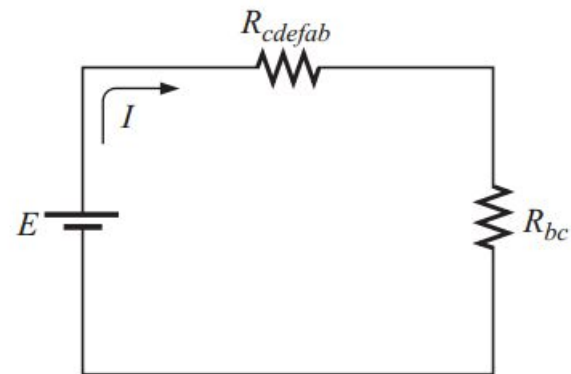
Find the value of  $I$  required to establish a magnetic flux of  $\Phi = 0.75 \times 10^{-4}$  Wb in the series magnetic circuit of Fig.



Solution:



(a)  
Magnetic circuit equivalent



(b)  
Electric circuit analogy

## AIR GAPS

The flux density for each section is  $B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4} \text{ Wb}}{1.5 \times 10^{-4} \text{ m}^2} = 0.5 \text{ T}$

From the  $B$ - $H$  curves,  $H (\text{cast steel}) \cong 280 \text{ At/m}$

An equation for  $H_g$   $H_g = (7.96 \times 10^5)B_g$  (At/m)

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5 \text{ T}) = 3.98 \times 10^5 \text{ At/m}$$

The mmf drops are

$$H_{\text{core}}l_{\text{core}} = (280 \text{ At/m})(100 \times 10^{-3} \text{ m}) = 28 \text{ At}$$

$$H_g l_g = (3.98 \times 10^5 \text{ At/m})(2 \times 10^{-3} \text{ m}) = 796 \text{ At}$$

Applying Ampere's circuital law,

$$NI = H_{\text{core}}l_{\text{core}} + H_g l_g = 28 \text{ At} + 796 \text{ At}$$

$$(200 \text{ t})I = 824 \text{ At}$$

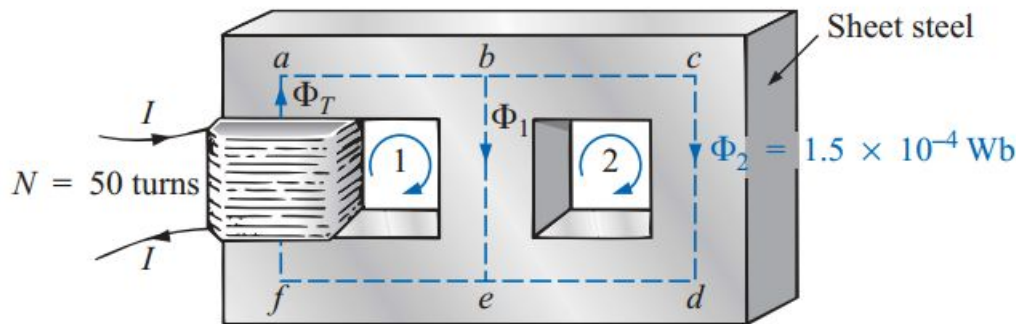
$$I = \mathbf{4.12 \text{ A}}$$

Note from the above that the air gap requires the biggest share (by far) of the impressed  $NI$  due to the fact that air is nonmagnetic.

# SERIES-PARALLEL MAGNETIC CIRCUITS

## EXAMPLE.6:

Determine the current  $I$  required to establish a flux of  $\Phi = 1.5 \times 10^{-4} \text{ Wb}$  in the section of the core indicated in Fig.

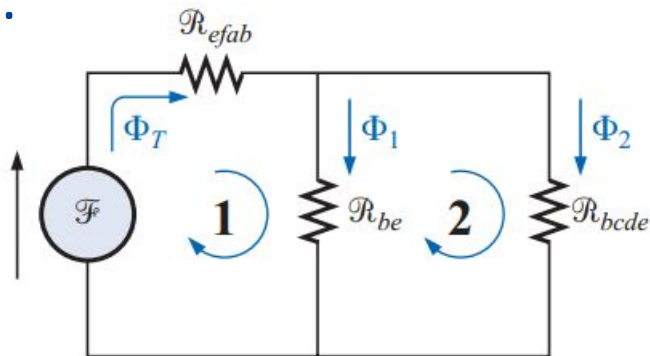


$$l_{bcde} = l_{efab} = 0.2 \text{ m}$$

$$l_{be} = 0.05 \text{ m}$$

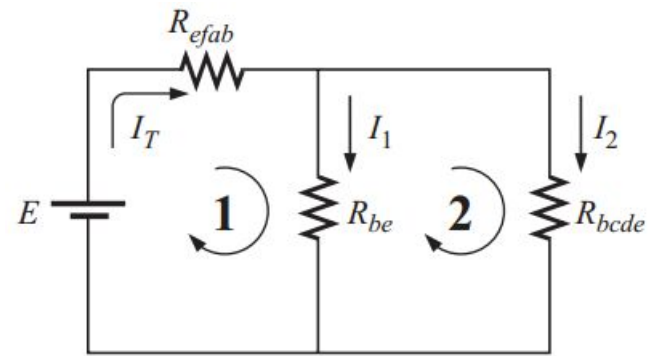
$$\text{Cross-sectional area} = 6 \times 10^{-4} \text{ m}^2 \text{ throughout}$$

Solution:



(a)

*Magnetic circuit equivalent*



(b)

*Electric circuit analogy*

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5 \times 10^{-4} \text{ Wb}}{6 \times 10^{-4} \text{ m}^2} = 0.25 \text{ T}$$

Applying Ampère's circuital law around loop 2

$$\sum_{\mathcal{C}} \mathcal{F} = 0$$

$$H_{be}l_{be} - H_{bcde}l_{bcde} = 0$$

$$H_{be}(0.05 \text{ m}) - (40 \text{ At/m})(0.2 \text{ m}) = 0$$

$$H_{be} = \frac{8 \text{ At}}{0.05 \text{ m}} = 160 \text{ At/m}$$

From BH Curve:

$$B_1 \cong 0.97 \text{ T}$$

$$\Phi_1 = B_1 A = (0.97 \text{ T})(6 \times 10^{-4} \text{ m}^2) = 5.82 \times 10^{-4} \text{ Wb}$$

# Table

Section	$\Phi$ (Wb)	$A$ (m <sup>2</sup> )	$B$ (T)	$H$ (At/m)	$l$ (m)	$HI$ (At)
<i>bcde</i>	$1.5 \times 10^{-4}$	$6 \times 10^{-4}$	0.25	40	0.2	8
<i>be</i>	$5.82 \times 10^{-4}$	$6 \times 10^{-4}$	0.97	160	0.05	8
<i>efab</i>		$6 \times 10^{-4}$			0.2	

The table reveals that we must now turn our attention to section ***efab***:

$$\begin{aligned}\Phi_T &= \Phi_1 + \Phi_2 = 5.82 \times 10^{-4} \text{ Wb} + 1.5 \times 10^{-4} \text{ Wb} \\ &= 7.32 \times 10^{-4} \text{ Wb}\end{aligned}$$

$$\begin{aligned}B &= \frac{\Phi_T}{A} = \frac{7.32 \times 10^{-4} \text{ Wb}}{6 \times 10^{-4} \text{ m}^2} \\ &= 1.22 \text{ T}\end{aligned}$$

From B-H Graph  $H_{efab} \cong 400 \text{ At}$

Applying Ampère's circuital law,

$$+NI - H_{efab}l_{efab} - H_{be}l_{be} = 0$$

$$NI = (400 \text{ At/m})(0.2 \text{ m}) + (160 \text{ At/m})(0.05 \text{ m})$$

$$(50 \text{ t})I = 80 \text{ At} + 8 \text{ At}$$

$$I = \frac{88 \text{ At}}{50 \text{ t}} = \mathbf{1.76 \text{ A}}$$

To demonstrate **that  $\mu$  is sensitive to the magnetizing force  $H$ ,**

□ The permeability of each section is determined as follows.

For section ***bcde***,

$$\mu = \frac{B}{H} = \frac{0.25 \text{ T}}{40 \text{ At/m}} = 6.25 \times 10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.25 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4972.2}$$

For section ***be***,

$$\mu = \frac{B}{H} = \frac{0.97 \text{ T}}{160 \text{ At/m}} = 6.06 \times 10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.06 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4821}$$

For section ***efab***,

$$\mu = \frac{B}{H} = \frac{1.22 \text{ T}}{400 \text{ At/m}} = 3.05 \times 10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{3.05 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{2426.41}$$



## PROBLEMS.2&3:

Calculate the magnetic flux  $\phi$  for the magnetic circuit of Fig. (a) and (b)

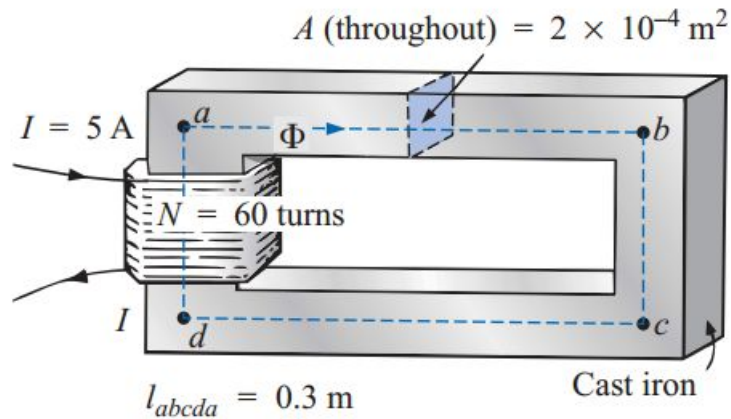


Fig.(a)

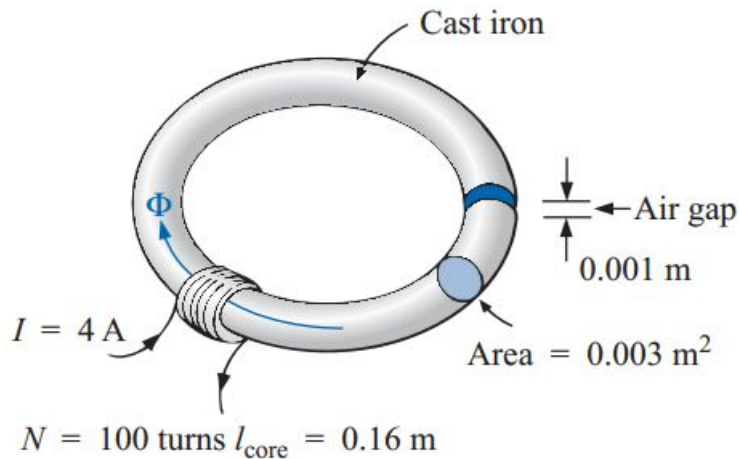


Fig.(b)

Use the B-H Curves to solve this problems