Sardar Vallabhbhai National Institute of Technology, Surat

B.Tech.-I (Semester-I)

Mid Semester Examination- January 2021

Sub: MA 101 S1 Mathematics-I

Date: 18-01-2021

Time: 11.30 am to 01.30 pm (including uploading answer sheets) **Total Marks: [30]**

General Instructions

(i) There are total **THREE** questions in the question paper.
(ii) All questions are compulsory.
(iii) Figure to the right indicates marks.
(iv) Follow usual notations.
(v) All must write your Admission Number, Role Number, Mobile Number, email on TOP of first page of answer sheet and admissions number and page no. with your signature on all pages.
(vi) **Important Instructions**: **Students must upload their answer sheet (single PDF file) on Google classroom or Microsoft team as per your class teacher suggestion latest by 01.30 pm on same day.**
(vii) **First verify the number of pages in your PDF file and then upload. Once you upload the file their after we will not consider any updated file.**

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1 Answer the following questions **[10]**

(1) Find the n^{th} derivative of $\sin^4 x$.

(2) If $\cosh x = \frac{5}{3}$ then find $\tanh x$

(3) If $u = r^2 \cos 2\theta$, then show that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.

(4) Find the equations of tangent plane and normal line to the surface $x^2 + 2y^2 + 3z^2 = 12$ at (1,2, -1).

(5) The power P required to propel a streamer of length l at a speed u is given by $P = \lambda u^3 l^3$ where λ is constant. If u is increased by 3% and l is decreased by 1%, find the corresponding increases in P.

2 **(A)** If $y_n = \frac{d^n}{dx^n}\left(x^n \ln(x)\right)$ then show that $y_n = (n)!\left[\ln x + 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}\right].(x > 0)$ **[03]**

OR

(A) Find the n^{th} derivative of $\frac{x^2}{(x-1)^2(x+2)}$.

(B) Answer the following questions (Attempt any Three) **[06]**

(1) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ then find the value of $y_{n+1}(0)$ in terms of $y_{n-1}(0)$.

(2) Find the Taylor's series for $f(x) = \ln(1+x)$ centered at $x = 0$. By using this expansion find Taylors series for $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ centered at $x = 0$.

(3) Find the points on the parabola $y^2 = 8x$ at which the radius of curvature is $\frac{125}{16}$.

(4) Find the radius of curvature at any point of the curve $y = c\cosh\left(\frac{x}{c}\right)$.

(C) Calculate the approximate value of $\sqrt{24}$ to five decimal places by taking the first four terms of an appropriate Taylor's Expansion. **[01]**

3 **(A)** If $u = \sinh^{-1}\left(\frac{x^3+y^3}{x^2+y^2}\right)$ show that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tanh u$ and **[03]**

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\tanh^3 u$.

OR

(A) Verify Euler's Theorem for the function $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$.

(B) Answer the following questions (Attempt any Three)

[06]

(1) If $u = \frac{y^2}{x}, v = \frac{x^2}{y}$, evaluate $J = \frac{\partial(x,y)}{\partial(u,v)}$ and $J' = \frac{\partial(u,v)}{\partial(x,y)}$ hence verify that $JJ' = 1$.

(2) Expand x^y in powers of *(x-1)* and *(y-1)* up to the third-degree terms.

(3) Find the stationary point, Maximum and Minimum value of $f(x,y) = y^2 + 4xy + 3x^2 + x^3$.

(4) The pressure P at any point *(x, y ,z)* in space is $P = 400xyz^2$ .Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$.

(C) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube **[01]**

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