S. V. National Institute of Technology, Surat

Applied Mathematics and Humanities Department

B.Tech–I   Sem–1   Branch–All   Subject–Mathematics-I (MA 101 S1)

Tutorial - 2 : Power series, Taylor's series and Maclaurin's series

1. Define Power Series. State and prove Taylor's series theorem.
2. Prove that $\sqrt{1+\sin x} = 1 + \frac{x}{2} - \frac{x^2}{2} - \frac{x^3}{48} + \frac{x^4}{384} + \cdots$
3. Prove that $e^{x \sec x} = 1 + x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \cdots$
4. Find the three terms in the expansion of $\frac{e^x}{e^x+1}$ in powers of x by Maclaurin's theorem.
   Ans : $\frac{1}{2} + \frac{x}{4} - \frac{1}{8}\frac{x^3}{3!} + \cdots$
5. Prove that $\cos^{-1}[\tanh(\log x)] = \pi - 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right)$.
6. Prove that $\log \frac{\sin x}{x} = -\left(\frac{x^2}{6} + \frac{x^4}{180} + \frac{x^6}{2835} + \cdots\right)$
7. Expand $e^{a\sin^{-1}x}$ by Maclaurin's theorem. Hence show that
   $$e^{\theta} = 1 + \sin\theta + \frac{\sin^2\theta}{2!} + \frac{2}{3!}\sin^3\theta + \ldots$$
8. Use Taylor's theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x-2)$.
9. Prove that $\frac{1}{x+h} = \frac{1}{x} - \frac{h}{x^2} + \frac{h^2}{x^3} - \frac{h^3}{x^4} + \cdots$
10. Find value using Taylor's series of
    (i) $\sqrt{25.15}$. **Ans** : 5.32261
    (ii) $\log_{10} 404$, given $\log_{10} 4 = 0.6021$. **Ans** : 2.6121
11. Expand $\tan\left(x + \frac{\pi}{4}\right)$ as far as the term x^4 and evaluate $\tan 44^\circ$ to four significant digits.
    Ans : 0.9657
12. Expand $\sin(a+h)$ as a series of powers of h and hence evaluate $\sin 62^\circ$ correct to four decimal places. **Ans** : 0.88295
13. Prove that $f\left(\frac{x^2}{1+x}\right) = f(x) - \frac{x}{1+x}f'(x) + \frac{x^2}{(1+x)^2}f''(x) + \cdots$
14. Expand $\tan^{-1} x$ in powers of $\left(x - \frac{\pi}{4}\right)$.
    Ans : $\tan^{-1}\frac{\pi}{4} + \left(x - \frac{\pi}{4}\right)\frac{16}{16+\pi^2} - \frac{\pi}{4}\left(x - \frac{\pi}{4}\right)^2 \frac{16^2}{(16+\pi^2)^2} + \cdots$
15. Prove that $\frac{\sin^{-1} x}{\sqrt{1-x^2}} = x + \frac{2}{3}x^3 + \frac{8}{15}x^5 + \cdots$

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