Sardar Vallabhbhai National Institute of Technology, Surat
B. Tech. - I / M. Sc. - I (Semester-I)
End Semester Examination- March 2021
Sub: MA 101 S1 Mathematics-I

General Instructions
(i)   There are total **THREE** questions in the question paper.
(ii)   All questions are compulsory.
(iii) Figure to the right indicates marks.
(iv)   Follow usual notations.
(v)   All must write your Admission Number, Role Number, Mobile Number, email on TOP of first page of answer sheet and admissions number and page no. with your signature on all pages.
(vi)   **Important Instructions**: **Students must upload their answer sheet (single PDF file) on Google classroom or Microsoft team as per your class teacher suggestion latest by 12.30 pm on same day.**
(vii) **First verify the number of pages in your PDF file and then upload. Once you upload the file their after we will not consider any updated file.**

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1   Answer the following questions with Justification   **[10]**

(1)   If $\cosh x = \dfrac{5}{3}$ calculate $\sinh x$ and $\tanh x$.

(2) If $y = (3x-2)^{15}$ then find y_{12}.

(3) Find the Maclaurin's Series for $f(x) = \dfrac{1}{2+x}$.

(4) Find the radius of curvature for the hyperbola $xy = 4$ at the point $x = 2, y = 2$.

(5) The curve $x^3 + y^3 = 3axy$ is symmetric about y = x line (True or false).

(6) If $u = \sin^{-1}\left(\dfrac{x}{y}\right) + \tan^{-1}\left(\dfrac{y}{x}\right)$, then find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$.

(7) Find the equations of tangent plane and normal to the surface $z^2 = 4x^2 + 4y^2 + 4$ at (2, 2, 6).

(8) Evaluate $\displaystyle\int_{0}^{1}\int_{0}^{x}(x^2 + y^2)\,dA$, where dA indicates small area in xy-plane.

(9) Evaluate $\displaystyle\int_{0}^{1}\int_{0}^{x}\int_{0}^{x+y}(x+y+z)\,dz\,dy\,dx$.

(10) Find the area bounded by the curve $y = x^2$ and $x+ y = 2$.

2   Answer the following questions (Attempt any six)   **[6 X 3 = 18]**

(1) If $y = (x + \sqrt{1+x^2})^m$ then Prove that $(1+x^2)y_{n+2} + x(2n+1)y_{n+1} + (n^2 - m^2)y_n = 0$.

(2) Expand the function $f(x) = (1+x)^x$ in powers of x.

(3) Find the radius of curvature ρ at $x = \dfrac{\pi}{3}$ on the curve $y = 2\ln\left(\sin\left(\dfrac{x}{2}\right)\right)$.

(4) The power P required to propel a streamer of length l at a speed u is given by $P = \lambda u^3 l^3$ where λ is constant. If u is increased by 3% and l is decreased by 1%, find the corresponding increases in P.

(5) If $u = xyz, v = x^2 + y^2 + z^2, w = x + y + z$ then find $J = \dfrac{\partial(x, y, z)}{\partial(u, v, w)}$.

(6) Find the maximum and minimum values of the function $xy + a^3\left(\dfrac{1}{x} + \dfrac{1}{y}\right)$.

(7) Trace the curve $x = (y-1)(y-2)(y-3)$.

(8) If $u = x^2 \tan^{-1}\left(\dfrac{y}{x}\right) - y^2 \tan^{-1}\left(\dfrac{y}{x}\right), x > 0, y > 0$ then evaluate

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$

(9) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, then prove that $\dfrac{1}{2}\dfrac{\partial u}{\partial x} + \dfrac{1}{3}\dfrac{\partial u}{\partial y} + \dfrac{1}{4}\dfrac{\partial u}{\partial z} = 0$.

3   Answer the following questions (Attempt any six)   **[6 X 3 =   18]**

(1) Evaluate $\displaystyle\int_0^\infty e^{-x^2}(x-1)^2\, dx.$

(2) Prove that $\displaystyle\int_0^2 x\left(8 - x^3\right)^{\frac{1}{3}} = \dfrac{16\pi}{9\sqrt{3}}.$

(3) Evaluate $\displaystyle\iint_R (x + y)\, dx\, dy$ where R is the region bounded by $x = 0$, $x = 2$ $y = x$, , $y = x + 2$

(4) Evaluate $\displaystyle\int_0^1 \int_0^{\sqrt{1-y^2}} \left(x^2 + y^2\right) dx\, dy$.

(5) By changing the order of integration find $\displaystyle\int_0^a \int_y^a \dfrac{x}{x^2 + y^2}\, dx\, dy$.

(6) Evaluate $\displaystyle\iiint_V \dfrac{dx\, dy\, dz}{\left(x^2 + y^2 + z^2\right)}$, over the volume of sphere $x^2 + y^2 + z^2 = 2$.

(7) Evaluate $\displaystyle\iiint_S dV$ where S is the region between the plane $x + y + z = 1$ and $x = 0$ where R is bounded by $z = y^2$ and $z = y$.

(8) Evaluate $\displaystyle\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dx\, dy\, dz$

4   Answer the following questions   **[2X 2=   4]**

(1) Find the nth derivative of $\log_{10}\left\{(1 - 2x)^3 (8x + 1)^5\right\}$.

(2) If $u = e^{xyz}$, find the values of $\dfrac{\partial^3 u}{\partial x \partial y \partial z}$.

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