Tutorial No: 1

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Q.1 Solve the following

(A) Exact differential equation

(1) $(x+2y-2)\,dx + (2x-y+3)\,dy = 0$

(Ans: $x^2+4xy-4x-y^2+6y=c$)

(2) $(x^2+2ye^{2x})\,dy + (2xy+2y^2e^{2x})\,dx = 0$

(Ans: $x^2y+y^2e^{2x}=c$)

(3) $(\sin x\cdot\sin y - xe^y)\,dy = (e^y + \cot x\cdot\cos y)\,dy = 0$

(Ans: $xe^y + \sin x\cdot\cos y = c$)

(4) $(\cos x\cos y - \cot x)\,dx - (\sin x\cdot\sin y)\,dy = 0$

(Ans: $\sin x\cos y = \ln(c\cdot\sin x)$)

(5) Find the value of λ, for the differential equation

$(2xe^y + 3y^2)\frac{dy}{dx} + (3x^2+\lambda e^y) = 0$ is exact. Solve the equation for this value of λ

(Ans: $\lambda=2$, $x^3+2e^x+y^3=c$)

Q.2 Solve the following

Reduce to exact differential equation.

Rule- If $M\,dx+N\,dy=0$ be homogeneous in x and y then $\frac{1}{Mx+Ny}$ is I.F

(1) $y(y^2-2x^2)\,dx + x(2y^2-x^2)\,dy = 0$

(Ans: $x^2y^2(y^2-x^2)=c_1$)

(2) $(x^4+y^4)\,dx - x^3\,dy = 0$ (Ans: $y^4 = 4x^4 \log x + cx^4$)

(3) $y^2\,dx + (x^2 - xy - y^2)\,dy = 0$ (Ans: $(x-y)\,y^2 = c(x+y)$)

Rule If $M\,dx + N\,dy = 0$ is of the form $f_1(xy)\,y\,dx + f_2(xy)\,x\,dy = 0$ then $\frac{1}{Mx-Ny}$ is I.F

(1) $(x^2y^2+2)\,y\,dx + (2 - 2x^2y^2)\,x\,dy = 0$

Ans: $\frac{1}{3}\log x + \frac{1}{3x^2y^2} - \frac{2}{3}\log y = \log c$

(2) $(x^3y^3 + x^2y^2 + xy + 1)\,y\,dx + (x^3y^3 - x^2y^2 - xy + 1)\,x\,dy = 0$

(Ans: $xy - \frac{1}{xy} - 2\log y = c_1$)

Rule: If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ is function of x then $e^{\int f(x)\,dx}$ is I.F

If $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$ is function of y then $e^{\int f(y)\,dy}$ Is I.F.

(1) $(x-y)\,dx - dy = 0$, $y(0) = 2$ Ans: $(x-1)e^x - ye^x = -3$)

(2) $(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$

(Ans: $x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = c$)

(3) $(y\cdot\log y)\,dx + (x - \log y)\,dy = 0$

(Ans: $x\cdot\log y - \frac{1}{2}(\log y)^2 = c$)

Rule: For $x^m y^n (ay\,dx + bx\,dy) + x^{m'} y^{n'} (a'y\,dx + b'x\,dy) = 0$

then. IF $x^h y^k$

① $(2y\,dx + 3x\,dy) + 2xy(3y\,dx + 4x\,dy) = 0$

(Ans: $x^2y^3(1 + 2xy) = c$)

② $(y^2 + 2yx^2)dx + (2x^3 - xy)\,dy = 0$

(Ans: $4(xy)^{1/2} - \frac{2}{3}\left(\frac{y}{x}\right)^{3/2} = c$)

③ $x(3y\,dx + 2x\,dy) + 6y^4(y\,dx + 3x\,dy) = 0$

(Ans: $x^3y^2 + 4x^2y^6 = c$)

Rule: By Inspection:

① $y\,dx - x\,dy + (1 + x^2)\,dx + x^2 \sin y\,dy = 0$

(Ans: $-\frac{y}{x} + x - \frac{1}{x} - \cos y = c$)

② $x\,dy - y\,dx = 0$

(Ans: $\frac{1}{2}\log\left(\frac{x+y}{x-y}\right) + c$ or $\frac{1}{2}\log(x^2+y^2) + c$ or $\frac{y}{x} + c$)

③