Light has a dual nature

- Wave (electromagnetic) – Interference
  – Diffraction
- Particle (photons) – Photoelectric effect
  – Compton effect

Wave – Particle Duality for light

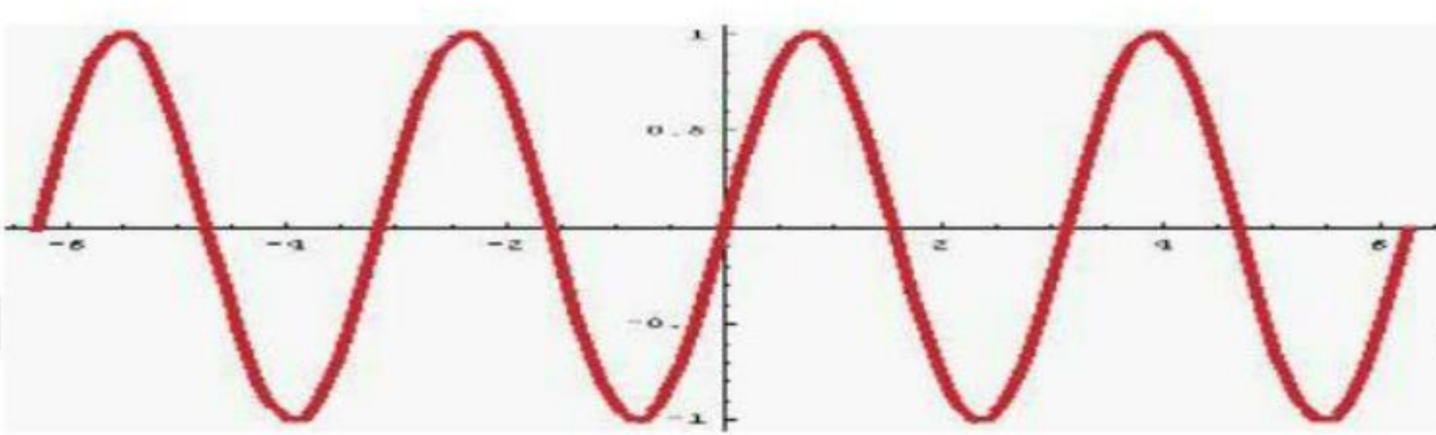
Particle

Our traditional understanding of a particle…

"Localized" – definite position, momentum, confined in space

Wave

Our traditional understanding of a wave....



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e

Waves and Particles:

Wavelength

Frequency

Waves

Spread in space and time

Can be superposed – show interference effects

Pass through each other

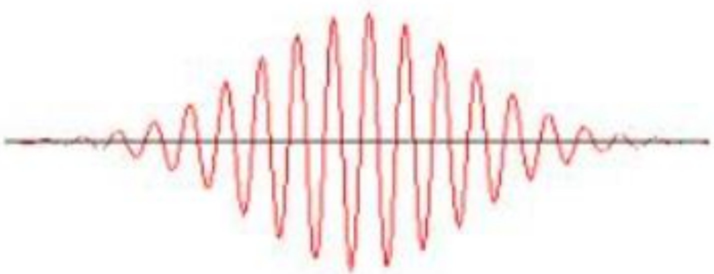
Particles

Localized in space and time

Cannot pass through each other - they bounce or shatter.

Constructing a wave packet by adding up several waves …………

If several waves of different wavelengths (frequencies) and phases are superposed together, one would get a resultant which is a localized wave packet



A wave packet describes a particle

- A wave packet is a group of waves with slightly different wavelengths interfering with one another in a way that the amplitude of the group (envelope) is non-zero only in the neighbourhood of the particle

- A wave packet is localized – a good representation for a particle!

de-Broglie concept of matter waves

- Electromagnetic radiation displays a dual character, behaving as a wave and a particle.
- Louis de Broglie in 1923 extended the wave-particle dualism to all fundamental particles such as electrons, protons, neutrons, atoms and molecules etc..
- According to de Broglie hypothesis, a moving particle is associated with a wave which is known as de Broglie wave or a matter wave. These waves are associated with particles like electrons , protons, neutrons etc.

Louis de Broglie's hypothesis

The dual nature of matter

A particle with momentum p has a *matter wave* associated with it, whose wavelength is given by

$$\lambda = \frac{h}{p}$$

The connecting link – Planck's constant

Dual Nature

Radiation

$$\boxed{E = h\nu}$$

Matter

$$\boxed{\lambda = \frac{h}{p}}$$

Why isn't the wave nature of matter more apparent to us...?

$$\boxed{h = 6.6\text{x}10^{-34}\,\text{J.s}}$$

Planck's constant is so small that we don't observe the wave behaviour of ordinary objects – their de Broglie wavelengths could be many orders of magnitude smaller than the size of a nucleus!

de Broglie wavelength of a material particle moving with momentum p is given by

$$\lambda = h / \sqrt{2mE}$$

In the case of electrons accelerated by a potential V volts from rest to velocity v

$$\lambda = h / \sqrt{2 m_0 V e}$$

Because of the smallness of h, we observe the wave nature only for particles of atomic or nuclear size . For ordinary objects the de Broglie wavelength is very small and so it is not possible to observe wave nature of these macroscopic objects

For electrons, the de-Broglie wavelength $\lambda = 12.26 / \sqrt{V}\ {}^{0}A$

Expression for de Broglie wave length

According to Planck's quantum theory,

$$E = h\upsilon \qquad \text{..............} \; 1$$

Where 'h' is a Planck's constant

According to Einstein's mass energy relation

$$E = mc^2 \qquad \text{...............} \; 2$$

Where 'm' is the mass of the photon and 'c' is the velocity of the photon.

From equations (1) and (2)

$$h\upsilon = mc^2$$

$$h\frac{c}{\lambda} = mc^2 \qquad \left(\because \upsilon = \frac{c}{\lambda}\right)$$

$$\lambda = \frac{hc}{mc^2} = \frac{h}{mc}$$

$$\lambda = \frac{h}{mc}$$

Since, $mc = p$ momentum of a photon

$$\lambda = \frac{h}{p}$$

In case of material particles

Momentum $p = mv$

So the de Broglie wave length of a material particle is

$$\lambda = \frac{h}{mv} \qquad \text{..................} \quad 3$$

de Broglie wavelength in terms of kinetic energy

If 'E' is the kinetic energy of the material particle then

$$E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

$\therefore$ de Broglie wave length

$$\lambda = \frac{h}{\sqrt{2mE}} \qquad \text{.....................} \; 4$$

de Broglie wavelength associated with a particle accelerated by a potential V

When a charged particle carrying a charge q, is accelerated through a potential difference of V volts, then kinetic energy

$$E = qV$$

$\therefore$ de Broglie wave length

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

de Broglie wave length associated with electrons

Let us consider the an electron of rest mass m_0 and charge e being accelerated by a potential V volts. If v is the velocity attained by the electron due to acceleration then

$$\frac{1}{2}m_0 v^2 = eV$$

$$v^2 = \frac{2eV}{m_0}$$

$$v = \sqrt{\frac{2eV}{m_0}}$$

According to de Broglie concept $\lambda = \dfrac{h}{m_0 v}$

$$\therefore \lambda = \frac{h}{m_0 v} = \frac{h}{m_0}\sqrt{\frac{m_0}{2eV}}$$

$$\lambda = \frac{h}{\sqrt{2m_0 eV}} = \frac{6.625\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}V}} = \frac{12.26}{\sqrt{V}}A^0$$

The above equation shows the wave length associated with electron accelerated to V volts.

If $V = 100$ volts. $\lambda = 12.26/\sqrt{100}\,A^0 = 1.226\,A^0$

Properties of de-Broglie waves or matter waves

- Matter waves consists of group of waves or a wave packet each having the wavelength λ , is associated with the particle. This group travels with the particle velocity v.
- Each wave of the group of matter waves travels with a velocity known as phase velocity of the wave $v_{phase} = c^2 / v$
- Lighter is the particle, greater is its wavelength. Smaller is the velocity, greater is the wavelength associated with it. When $v = 0$ then $\lambda = \infty$ which means that the wave becomes indeterminate. This shows that matter waves are generated by the motion of the particles.
- The wave and particle aspects of a moving body can never appear together in the same experiment
- The wave nature of matter introduces an uncertainty in the location of the particle because the wave is spread out in space.

The velocity of matter waves is always greater than the velocity of light.

Proof

We know $E = h\upsilon$ and $E = mc^2$

$h\upsilon = mc^2$ $\qquad$ $\upsilon = \dfrac{mc^2}{h}$

The wave velocity ω is given by

$$\omega = \upsilon\lambda = \frac{mc^2}{h}\lambda$$

Substituting $\lambda = \dfrac{h}{mv}$ we get

$$\omega = \frac{mc^2}{h}\frac{h}{mv} \qquad \omega = \frac{c^2}{v}$$

The particle velocity v cannot exceed velocity of light c , ω is greater than the velocity of light.

Differences between matter wave and electromagnetic waves.

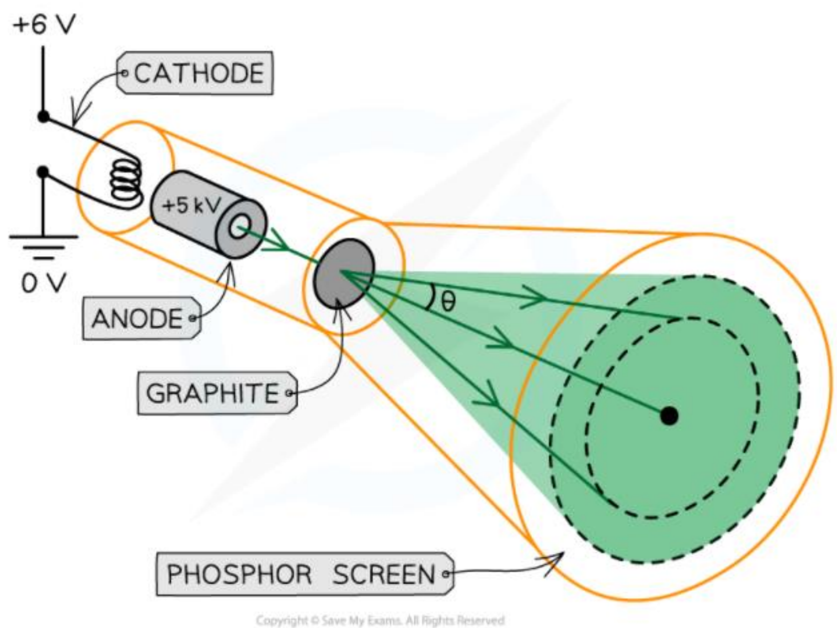
Matter wave	Electromagnetic wave
Matter wave is associated with moving particle	Electromagnetic wave is associated with oscillating charged particle.
Wave length depends on the mass of the particle and its velocity. $\lambda = \frac{h}{mv}$	Wave length depends on the energy of photon. $\lambda = \frac{hc}{E}$
The velocity of matter wave is always greater then the velocity of light.	It travels with the velocity of light C = $3x10^8$
Matter wave is not an electromagnetic wave.	In electromagnetic wave electric and magnetic fields oscillate perpendicular to each other.
These waves are produced whether the particles are charged or uncharged.	These waves are produced by the charged particles.

Electron Diffraction

- Electron diffraction is a collective scattering phenomenon with electrons being (nearly elastically) scattered by atoms in a regular array (crystal).
- Electron diffraction tubes can be used to investigate the wave properties of electrons
- The electrons are accelerated in an electron gun to a high potential, such as 5000 V, and are then directed through a thin film of graphite
- The electrons diffract from the gaps between carbon atoms and produce a circular pattern on a fluorescent screen made from phosphor

- Increasing the voltage between the anode and the cathode causes the energy, and hence speed, of the electrons to increase
- The kinetic energy of the electrons is proportional to the voltage across the anode-cathode:

$$E_k = \frac{1}{2} mv^2 = eV$$



Experimental setup to demonstrate electron diffraction

Diffraction of Electrons through Graphite

- Louis de Broglie discovered that matter, such as electrons, can behave as a wave
- He showed a diffraction pattern is produced when a beam of electrons is directed at a thin graphite film
- Diffraction is a property of waves, and cannot be explained by describing electrons as particles

- In order to observe the diffraction of electrons, they must be focused through a gap similar to their size, such as an atomic lattice
- Graphite film is ideal for this purpose because of its crystalline structure.
- The gaps between neighboring planes of the atoms in the crystals act as slits, allowing the electron waves to spread out and create a diffraction pattern.

- The diffraction pattern is observed on the screen as a series of concentric rings.
- This phenomenon is similar to the diffraction pattern produced when light passes through a diffraction grating
- If the electrons acted as particles, a pattern would not be observed, instead, the particles would be distributed uniformly across the screen.
- It is observed that a larger accelerating voltage reduces the diameter of a given ring, while a lower accelerating voltage increases the diameter of the rings

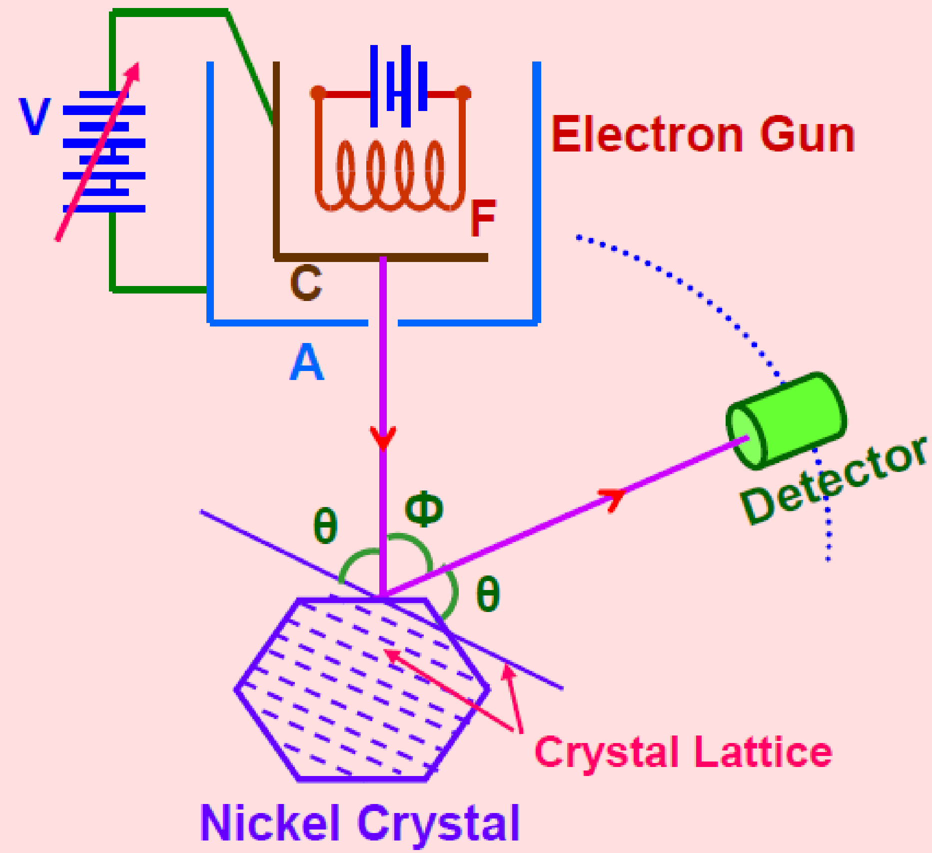
Davisson and Germer Experiment:

A beam of electrons emitted by the electron gun is made to fall on Nickel crystal cut along cubical axis at a particular angle.

The scattered beam of electrons is received by the detector which can be rotated at any angle.

The energy of the incident beam of electrons can be varied by changing the applied voltage to the electron gun.

Intensity of scattered beam of electrons is found to be maximum when angle of scattering is 50° and the accelerating potential is 54 V.



$\theta + 50° + \theta = 180°$ i.e. $\theta = 65°$

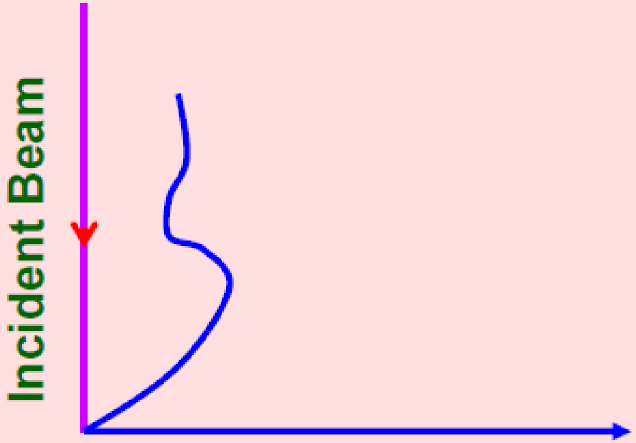
For Ni crystal, lattice spacing $d = 0.91$ Å

For first principal maximum, $n = 1$

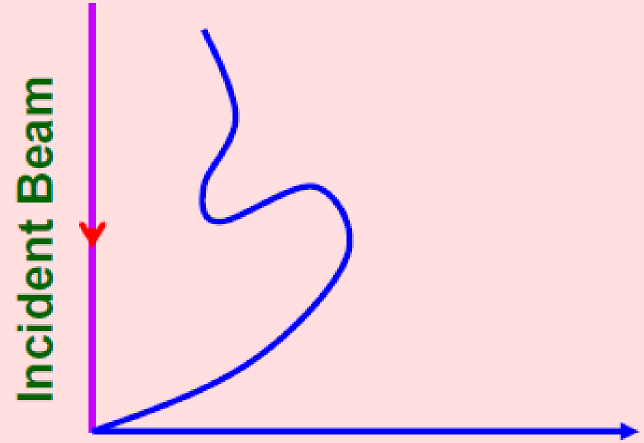
Electron diffraction is similar to X-ray diffraction.

∴ Bragg's equation $2d\sin\theta = n\lambda$ gives

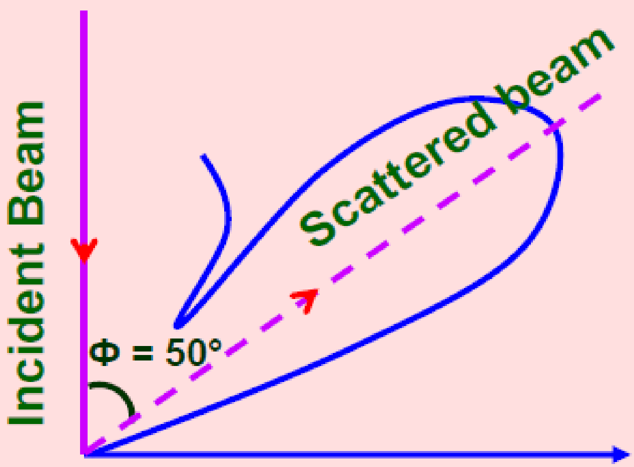
$\boxed{\lambda = 1.65 \text{ Å}}$



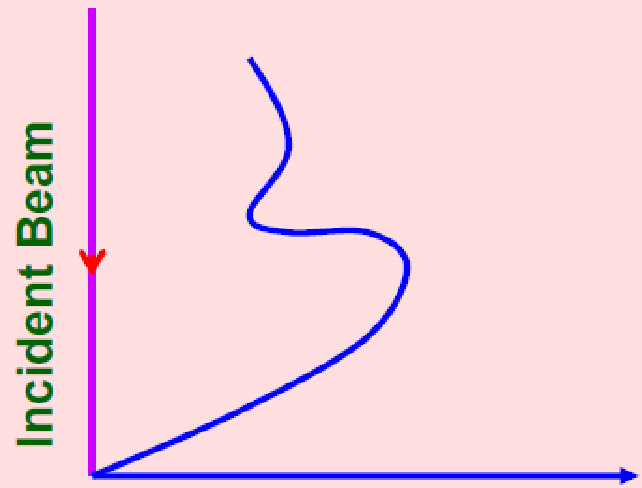
Intensity of scattered beam at 44 V



Intensity of scattered beam at 48 V



Intensity of scattered beam at 54 V



Intensity of scattered beam at 64 V

According to de Broglie's hypothesis, $\boxed{\lambda = \frac{h}{\sqrt{2meV}}}$ or $\boxed{\lambda = \frac{12.27\ \text{Å}}{\sqrt{V}}}$

$\therefore$ de Broglie wavelength of moving electron at V = 54 Volt is 1.67 Å which is in close agreement with 1.65 Å.

De Broglie Wave: Phase velocity (v_{Phase} or v_p)

A point marked on a wave can be regarded as representing a particular phase for the wave at that point. The velocity with which such a point would propagate is known as phase velocity (or) wave velocity.
It is represented by

$$v_{phase} \quad \text{or} \quad v_p = \frac{\omega}{k}$$

where, ω is angular frequency and k is the propagation constant or wave number.

Group Velocity (v_{group} or v_g)
The velocity with which the resultant envelops of the group of waves travels is called group velocity. It is denoted by v_{group} or v_g and is equal to the particle velocity v.

$$v_{group} \quad or \quad v_g = \frac{d\omega}{dk}$$

Relation between Group Velocity v_g and Phase Velocity v_p

$$v_g = v_p - \lambda \frac{dV_p}{d\lambda} \quad or \quad v_{group} = v_{phase} - \lambda\left(\frac{dv_{phase}}{d\lambda}\right)$$

Relation between Group Velocity v_g and Particle Velocity v

$$v_g = v \quad \text{or} \quad v_{group} = v_{particle}$$

Relation between group velocity v_g, phase velocity v_p & velocity of light c

$$v_p \times v_{group} = c^2 \quad \text{or} \quad v_{phase} \times v_{group} = c^2$$