PAGE NO:
DATE / /

Q.6 Solve Recurrence Relation $a_n - 2a_{n-1} = 3^n$; $a_1 = 5$

Ans. degree of R.R = $n-(n-1)=1$.

→ C.F.: let $a_n = r^n$, $n \geq 1$

$r^n - 2r^{n-1} = 0$

$r-2=0 \Rightarrow \underline{r=2}$

— complimentary solution

$C.F. a_n = C_1 2^n$

→ Particular solution: $a_n = A \cdot 3^n$

$\therefore a_n - 2a_{n-1} = 3^n$

$\therefore A\cdot 3^n - 2\cdot A\, 3^{n-1} = 3^n$

$A \cdot 3^{n-1} = 3^n$

$\underline{A=3}$

$\therefore a_n = A\cdot 3^n = 3\cdot 3^n$

$a_n = 3^{n+1}$

→ General solution

$a_n = C_1 2^n + 3^{n+1}$

$a_1 = 5 \Rightarrow a_1 = 5 = C_1 \cdot 2^1 + 3^2$

$\underline{C_1 = 2}$

$\therefore a_n = 2\cdot 2^n + 3^{n+1}$

$a_n = 2^{n+1} + 3^{n+1}$.

Q.7 Find a general term of Fibonacci sequ. 0, 1, 1, 2, 3, 5, -

Ans. Fibonacci series can be written as,

$F_{n+2} = F_{n+1} + F_n$, $n \geq 0$

$\therefore$ Let $a_n = r^n$

$\therefore r^{n+2} = r^{n+1} + r^n$

$r^2 - r - 1 = 0$

$r = + \frac{1 \pm \sqrt{5}}{2}$

Q.

(1)

(2)

→ So, if k is chromatic number of graph, we will need at least k timeperiods / slots for schedule.

→ So, if $k > m$, we cannot make the schedule

28. Let G be graph for marriage (where vertices (V) are marriageable people.
- if there is edge b/w two vertices, people corresponding to the vertices had dated.

→ Clearly this will be a bi-partite graph with two sets - one with boys-vertices & other with girls vertices.

→ Let M be function, which maps every vertex with its adjacent vertex, i.e every person with its dates.

→ ~~The above graph~~
Condition: "Every boy and girl gets married to one of their dates"

→ For above condition to be true, the graph should follow "Halls marriage theorem"

→ According to Halls theorem:
"Let G be bipartite graph with partite sets U & W where $|U| \leq |W|$. Then G satisfies Hall's condition iff, for every $S \leq |U|$, $|S| \leq |M(S)|$".

→ Here, ~~U & W are set~~ For this problems, we can take U & W as sets of boys & girls, M = Map of dates;

→ $\langle 2 \rangle = \{2, 4, 1, \}$

↳ $2^1 = 2$, $2^2 = (2*2)\cancel{\%7} = (2\times 2)\%7 = 4$

$2^3 = (2*2)*2 = 4*2 = (4\times 2)\%7 = 1$

$2^4 = (2*2)*(2*2) = 4*4 = (4\times 4)\%7 = 2$ ↳ repeating again

→ $\langle 3 \rangle = \{3, 2, 6, 4, 5, 1\} = G$

$3^1 = 3$, $3^2 = 9\%7 = 2$, $3^3 = 27\%7 = 6$

$3^4 = 81\%7 = 4$; $3^5 = 243\%7 = 5$; $3^6 = 729\%7 = 1$

→ $\langle 4 \rangle = \{4, 2, 1\}$

$4^1 = 4$, $4^2 = 16\%7 = 2$, $4^3 = 64\%7 = 1$

→ $\langle 5 \rangle = \{5, 4, 6, 2, 3, 1\} = G$

→ $\langle 6 \rangle = \{6, 1\}$

∴ So, we can see that 3 & 5 are generating G (i.e, $\langle 3 \rangle = \langle 5 \rangle = G$), G is cyclic & 3 & 5 are generators.

Q.10 Let $M = \left\{ \begin{bmatrix} a & a \\ b & b \end{bmatrix} \mid a, b \in \mathbb{R}, a+b \neq 0 \right\}$. is M a group under Multiplication?

Ans. let $M_1 = \begin{bmatrix} a & a \\ b & b \end{bmatrix}$, $M_2 = \begin{bmatrix} c & c \\ d & d \end{bmatrix}$

PAGE NO.:
DATE / /

21. P.T. every region in maximal Planar graph is triangle.

Ans. We can ~~see~~ observe that for n (no. of vertices) $=1$ & $n=2$, every graph is ~~max~~ planar graph.

→ For, $n \geq$ so, let's assume $n \geq 3$ and there is a region in max. planar graph, that is not triangle.

→ We can ~~ot~~ say that graph is connected.

→ Let's ~~assume~~ consider that boundary is acyclic, i.e. it is tree with at least 3 vertices. But we can add a edge in Tree, so it will not be max. planar graph.

∴ Boundary of each region is cycle.

→ Also boundary is solely cycle and does not have leaf branching off the cycle, since it will contradict maximality.

→ ∴ ~~Let~~ So, the region that is not a triangle must have three consecutive vertices on its boundary, i.e. v_1, v_2 & v_3 where $(v_1, v_2), (v_2, v_3) \in E$ but $(v_1, v_3) \notin E$. So we can safely add edge (v_1, v_3), which contradicts maximality.

→ So, our assumption is wrong.

∴ Every region in maximal planar graph is triangle

22. P.T. the chromatic no. of graph will not exceed by more than one the maximum degree of vertices in graph.

PAGE NO:

DATE / /

Circular arrangement are considered the same when one can be obtained from another by rotation ?

Ans no. of people $= n = 6$.

$\therefore$ no. of arrangement on circular table

$= (n-1)! = (6-1)!$

$= 5!$

$= 120$

Q.4 If a man hiked for 10 hours and covered distance of 45 km. It is known that he hiked 6 km in the first ~~four~~ hour and only 3 km in last hour. S.T. he must have hiked at least 9 km within 2 consecutive hrs.

Ans. - distance covered in remaining 8 hrs =

$n = 45 - 6 - 3 = 36$ km

→ no. of hours $m = 8$

$\therefore$ By pigeonhole principle et he must cover distance of $(\lfloor 36/8 \rfloor + 1) = 5$ ~~hrs~~ km in at least one hour

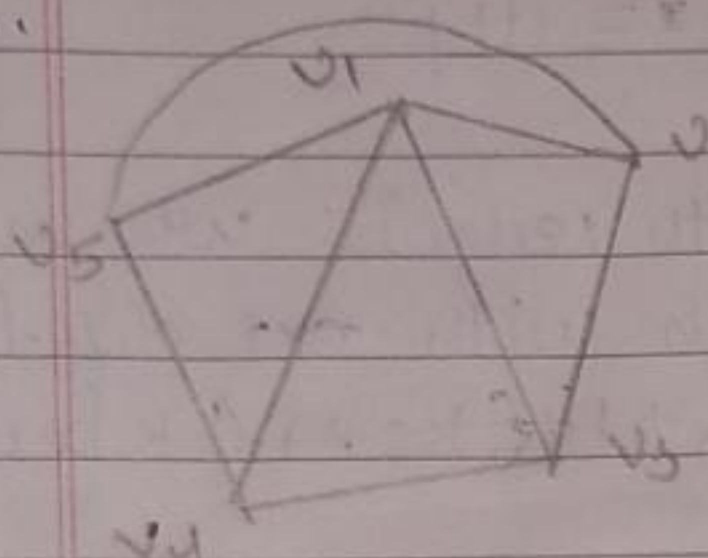
→ And if we divide equal distance / hr

⇒ it will be 4.5 ~~hr~~ km/hr

→ So, he must ~~cov~~ have covered 9 km distance in two consecutive hours else he can not cover given distance

23. Find maximal independent set for the graph. Also, show that graph has only one chromatic partition. What it is?

Ans.



→ Maximal independent set:

$\{v_5, v_3\}$ or $\{v_4, v_2\}$

→ ~~By obs~~ For Independent sets [ϕ : be boolean fun. for it]

$$\phi = v_1v_2 + v_1v_3 + v_1v_4 + v_1v_5 + v_2v_3 + v_2v_5 + v_5v_4 + v_4v_3$$

→ $\phi' = (v_1'+v_2')(v_1'+v_3')(v_1'+v_4')(v_1'+v_5')(v_2'+v_3')(v_2'+v_5')(v_4'+v_5')(v_4'+v_3')$

$$\phi' = (v_1' + v_2'v_3'v_4'v_5')(v_2' + v_3'v_5')(v_4' + v_3'v_5')$$

$$\phi' = (v_1' + v_2'v_3'v_4'v_5')(v_2'v_4' + v_3'v_5')$$

$$\phi' = v_1'v_2'v_4' + v_1'v_3'v_5' + v_2'v_3'v_4'v_5'$$

∴ Independent sets = $\{3, 5\}, \{2, 4\}, \{1\}$

→ above are only independent sets for given graph: let $S_1 = \{3, 5\}$ & $S_2 = \{2, 4\}$ & $S_3 = \{1\}$

→ $S_1 \cup S_2 \cup S_3 = V(G)$

also $(S_1 \cap S_2) = (S_2 \cap S_3) = (S_3 \cap S_1) = \phi$.

∴ there exists only one partition for given graph: $\{\{2, 4\}, \{3, 5\}, \{1\}\}$.

PAGE NO.:
DATE / /

→ ∴ real and diff. roots:

$$F_n = c_1\left(\frac{1+\sqrt{5}}{2}\right)^n + c_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

→ $F_0 = 0$:

$$c_1 + c_2 = 0$$

→ $F_1 = 1$:

$$c_1\left(\frac{1+\sqrt{5}}{2}\right) + c_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$$

$$\frac{(c_1+c_2)}{2} + \frac{\sqrt{5}}{2}(c_1 - c_2) = 1$$

$$\therefore (c_1 - c_2) = 2/\sqrt{5} \qquad \{c_1 + c_2 = 0\}$$

$$c_1 + c_2 = 0$$

$$c_1 = 1/\sqrt{5},\ c_2 = -1/\sqrt{5}$$

$$\therefore F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{-1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

Q.8

(1) Algebraic structure:

A non empty set S is called an algebraic structure w.r.t. binary operation (*), if it follows "closure" [i.e. $a*b \in S \ \forall a,b \in S$].

0,1,1,2,3,5,-

(2) semi-group: for a non empty set S, (S,*) is called a semigroup, if it follows following axiom:

→ closure: $(a*b) \in S \ \forall a,b \in S$

Associativity: $a*(b*c) = (a*b)*c \ \forall a,b,c \in S$.

$4^1 = 4$, $4^2 = 16 \% 7 = 2$, $4^3 = 64 \% 7 = 1$

$\rightarrow \langle 5 \rangle = \{5, 4, 6, 2, 3, 1\} = G$

$\rightarrow \langle 6 \rangle = \{6, 1\}$

$\therefore$ So, we can see that 3 & 5 are generating G (i.e, $\langle 3 \rangle = \langle 5 \rangle = G$), G is cyclic & 3 & 5 are generators.

Q.10 Let $M = \left\{ \begin{bmatrix} a & a \\ b & b \end{bmatrix} \middle| a, b \in R, a+b \neq 0 \right\}$. is M a group under Multiplication?

Ans. let $M_1 = \begin{bmatrix} a & a \\ b & b \end{bmatrix}$, $M_2 = \begin{bmatrix} c & c \\ d & d \end{bmatrix}$, $a+b \neq 0$, $c+d \neq 0$
$a, b, c, d \in R$.

$\rightarrow M_1 \cdot M_2 = \begin{bmatrix} a & a \\ b & b \end{bmatrix} \cdot \begin{bmatrix} c & c \\ d & d \end{bmatrix}$

$M_1 \cdot M_2 = \begin{bmatrix} a(c+d) & a(c+d) \\ b(c+d) & b(c+d) \end{bmatrix} \in M$

$\rightarrow$ Identity: let $I = \begin{bmatrix} x & x \\ y & y \end{bmatrix}$ be identity:

$\therefore M * I = \begin{bmatrix} a & a \\ b & b \end{bmatrix} \begin{bmatrix} x & x \\ y & y \end{bmatrix} = \begin{bmatrix} a & a \\ b & b \end{bmatrix} = M$

$\therefore a(x+y) = a$

$\therefore \underline{x+y=1}$

$\rightarrow$ also: $I * M = \begin{bmatrix} x & x \\ y & y \end{bmatrix} \begin{bmatrix} a & a \\ b & b \end{bmatrix} = \begin{bmatrix} a & a \\ b & b \end{bmatrix}$

$\therefore x(a+b) = a$ & $y \cdot (a+b) = b$

$\therefore$ There does not exist an Identity $\therefore$ M is not group.

→ So $|U| = |V| = n$. [Given in Que.]

→ So, if graph follows condition,

$S \subseteq U$ ~~(or V)~~, $|S| \leq |N(S)|$

→ i.e. total no. of girls dates of k boys should be greater than or equal to k, and vice-versa.

→ ~~Here~~

29. Definition:

i) Source: ~~A~~ node is considered a source in a graph, if it has in-degree of 0. (no nodes have a source as their destination)

ii) Sink: A node is considered a sink, if it has out-degree of 0. (no nodes have a sink as their source).

iii) Capacity: The maximum ~~flow~~ amount of flow on given edge is the capacity of the edge

iv) Feasible flow: The value of flow on all edges leaving the source s. A feasible flow is flow, which follows flow conservation for all nodes except source and sink.

v) Maximal flow: Maximal flow is a flow whose value cannot be located without decreasing the flow along some arc.

→ as all vertices ~~fun~~ are connected, Colors for 2^{nd} vertex is $= (\lambda - 1)$

3^{rd} vertex $= (\lambda - 2)$

$\vdots$

n^{th} vertex $= (\lambda - n + 1)$

$\therefore P_n(\lambda) = \lambda(\lambda-1)\cdots(\lambda-n+1)$.

Hence proved.

→

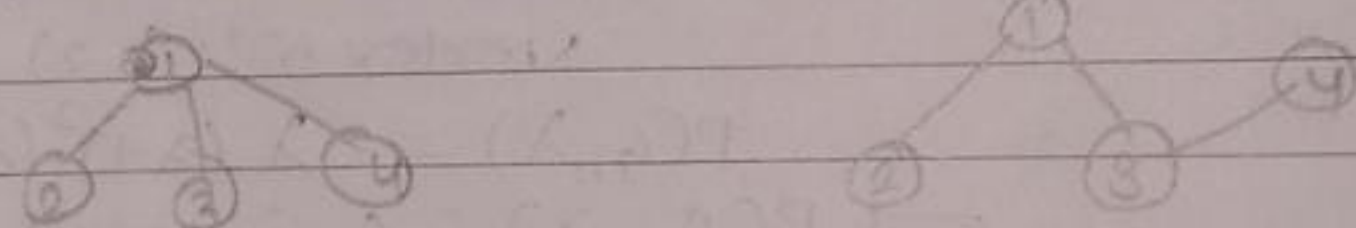
26. Sketch two diff graphs that have same chromatic polynomial.

Ans. Chromatic polynomial for tree with n-vertices

$P_n(\lambda) = \lambda(\lambda-1)^{n-1}$

→ So, any two trees with same no. of vertices (n) will have same C.P.

Ex: Q. $P_n(\lambda) = \lambda(\lambda-1)^3$ $[n=4]$



27. n = no. of courses; m = available hours of week.

→ If we take n verticed-graph G, in ~~with~~ which:

- ~~edo if there are comm~~
- if a student(s) have taken two courses, then the vertices of respective courses will have edge b/w them.
- ~~Chromatic~~ if we color the graph, same Coloured ~~graf~~ vertex (courses) can be taught at same time.

→ i.e. we have m colours [as ~~we~~ colours are ~~free~~ slots]

PAGE NO:

DATE / /

(7) ~~H is~~ $H < G$, $a * b \in G$.

(i) $a * H = H$ iff $a \in H$:

→ let $a * H = H$

let ~~h~~ for $h_1, h_2 \in H$ such that

$a * h_1 = h_2$

$a \cancel{* h_1} = h_2 * h_1^{-1}$ $[h_1 \in H \Rightarrow h_1^{-1} \in H]$

$\cancel{a = h_2}$ → now $h_2 \in H$ & $h_1^{-1} \in H$

$\therefore h_2 * h_1^{-1} \in H$ {H is a group}

$\therefore \underline{a \in H}$

→ let $a \in H$; let $h_1 \in H$

$\therefore \cancel{a * h_1}$ $a * h_1 \in H$ {H is a group}

$\therefore a * \cancel{h_1} = H$.

∴ Hence Proved.

(ii) $a * H = b * H$ iff $a^{-1} * b \in H$.

→ let $h_1, h_2 \in H$ such that,

$a * h_1 = b * h_2$

$\cancel{a = b *}$: $a^{-1} * a * h_1 * h_2^{-1} = a^{-1} * b * h_2 * h_2^{-1}$

$h_1 * h_2^{-1} = a^{-1} * b$

$\therefore h_1 * h_2^{-1} \in H$

$\therefore a^{-1} * b \in H$.

→ let

→ let $a^{-1} * b \in H$:

$a^{-1} * b = h = h_1 * h_2$

$\therefore a^{-1} * b = h_1 * h_2$

$\therefore \cancel{a}\, b * h_2^{-1} = a * h_1$

$\therefore a * H = b * H$

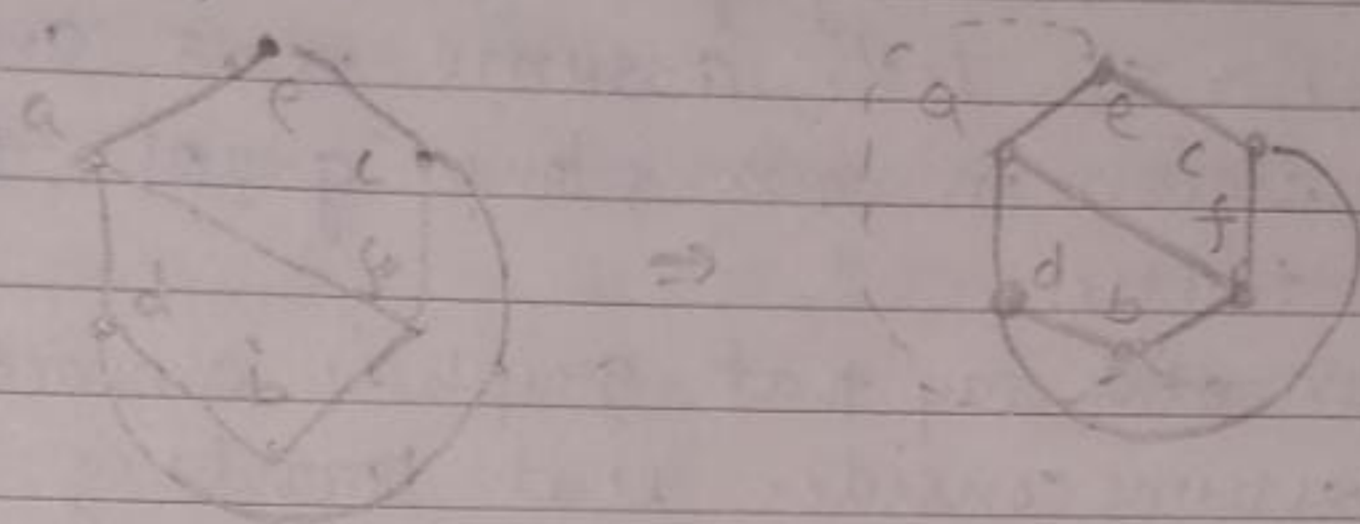
∴ Hence proved.

PAGE NO.:
DATE / /

19. Prove that Kurtowski's second graph is non-planar

Ans Kurtowski's second graph is $K_{3,3}$

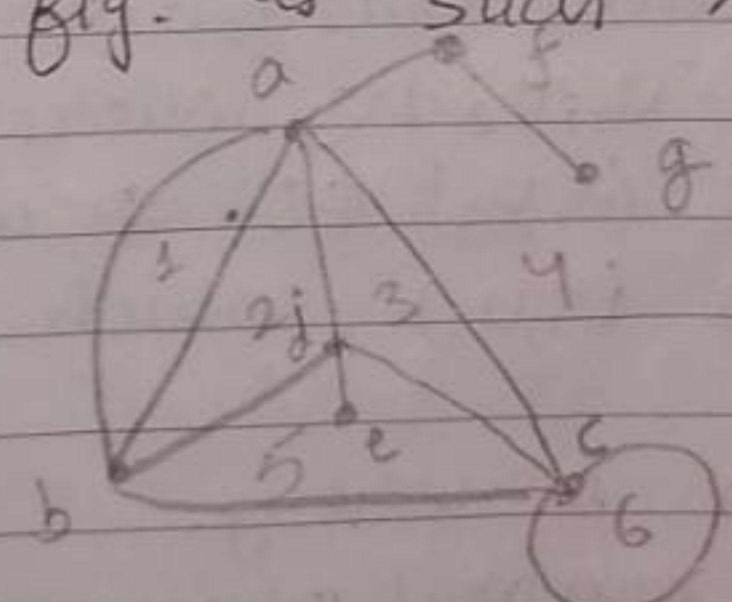
→ let (a,b,c) & (d,e,f) be two sets of vertices of Graph G.



→ We can draw planar graph with 8 edges [without edge (b,e)], But we cannot draw 9th edge (b,e) without crossing at least one edge.

→ So, we cannot draw $K_{3,3}$ planar, i.e. it is non-planar.

20. Verify Euler's formula: $f = e - n + 2$. Redraw fig. (c) such 2 becomes infinite region



→ no. of faces $f = 6$

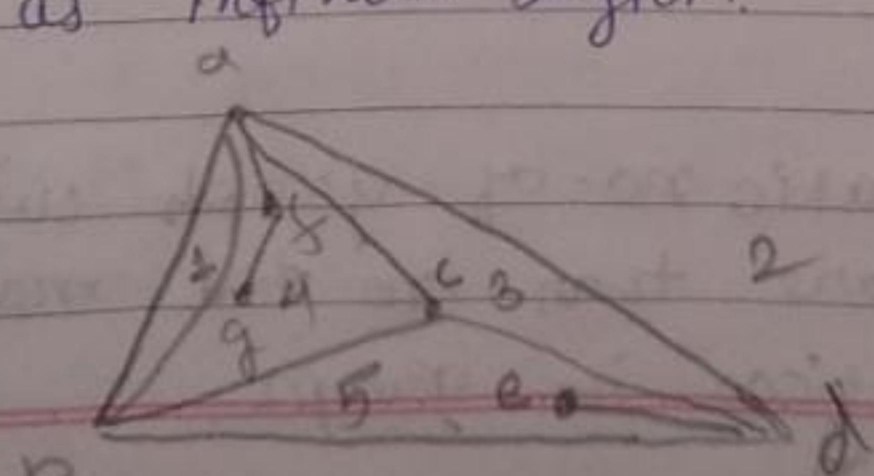
→ no. of edges : $e = 11$

no. of vertices $= 7$

$\therefore e - n + 2 = 11 - 7 + 2 = 6 = f$

∴ Hence verified

→ 2 as infinite region.



PAGE NO.:

DATE / /

Q.5 Find the no. of integers b/w 1 and 250 both inclusive that are not divisible by any of integers 2, 3, 5 and 7?

Ans. $N = 250$

A = set of integers divisible by 2

B = set of integers divisible by 3

C = set of integers divisible by 5

D = set of integers divisible by 7

$\rightarrow n(A) = 125 \Rightarrow n(A') = 125$

$n(B) = 83 \Rightarrow n(B') = 167$

$n(C) = 50 \Rightarrow n(C') = 200$

$n(D) = 35 \Rightarrow n(D') = 215$

$\rightarrow n(A \cap B) = 41$; $n(A \cap C) = 25$; $n(A \cap D) = 17$

$n(B \cap C) = 16$; $n(B \cap D) = 11$; $n(C \cap D) = 7$

$\rightarrow n(A \cap B \cap C) = 8$; $(A \cap B \cap D) = 5$; $(A \cap C \cap D) = 3$;

$n(B \cap C \cap D) = 2$

$\rightarrow n(A \cap B \cap C \cap D) = 1$

$\rightarrow n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D)$
$- n(A \cup B) - n(A \cup C) - n(A \cup D) - n(B \cup C)$
$- n(B \cup D) - n(C \cup D) + n(A \cup B \cup C)$
$+ n(B \cup C \cup D) + n(A \cup C \cup D) + n(A \cup B \cup D)$
$- n(A \cup B \cup C \cup D)$

$= \cancel{41+25}\ 125 + 83 + 50 + 35$
$- (41 + 25 + 17 + 16 + 11 + 7) + (8 + 5 + 3 + 2) - 1$

$n(A \cup B \cup C \cup D) = 193$

$\rightarrow$ no.s not divisible by 2, 3, 5 & 7 $= n(A \cup B \cup C \cup D)'$

$= 250 - 193$

$= 57$

Ans let d = max. degree, k = chromatic number

Proof By Induction:

→ let $d=0$: (Trivial graph): we can give ~~any~~ one color to the vertex: $\Rightarrow k=1 \Rightarrow k = d+1$

$\therefore P(1)$ is true

→ let ~~P(d)~~ $d=1$: (Two vertex with 1 edge): we insert one vertex in trivial graph with diff. (new) color, so $k_{(2)}$ become $k_{(1)}+1$ ~~(if k=2)~~ (∵ k becomes 2)

$\therefore k <= d+1$.

→ let $P(d)$ is true: i.e: $k_d <= d+1$

→ For $P(d+1)$:

→ ~~insert an edge~~ Take any graph with max. degree $(d+1)$, remove a vertex ~~from~~ adjacent to max. degree vertex.

→ so, new degree is d.

→ But $P(d)$ is true.

→ So, chromatic no. is $k_d <= d+1$ for modified graph.

→ Now, add the removed vertex again with different new color.

→ It will increase chromatic number by 1.

$\therefore k_{d+1} = k_d + 1$,

$\therefore$ But $k_d <= d+1$

$\therefore k_{d+1} \leq d+2$

$k_{d+1} \leq (d+1)+1$

$\therefore P(d+1)$ is true.

$\therefore P(d)$ is true for all $d \in N$. By Induction

→ Hence Proved.

(1,2), (1,5)   8+7=15

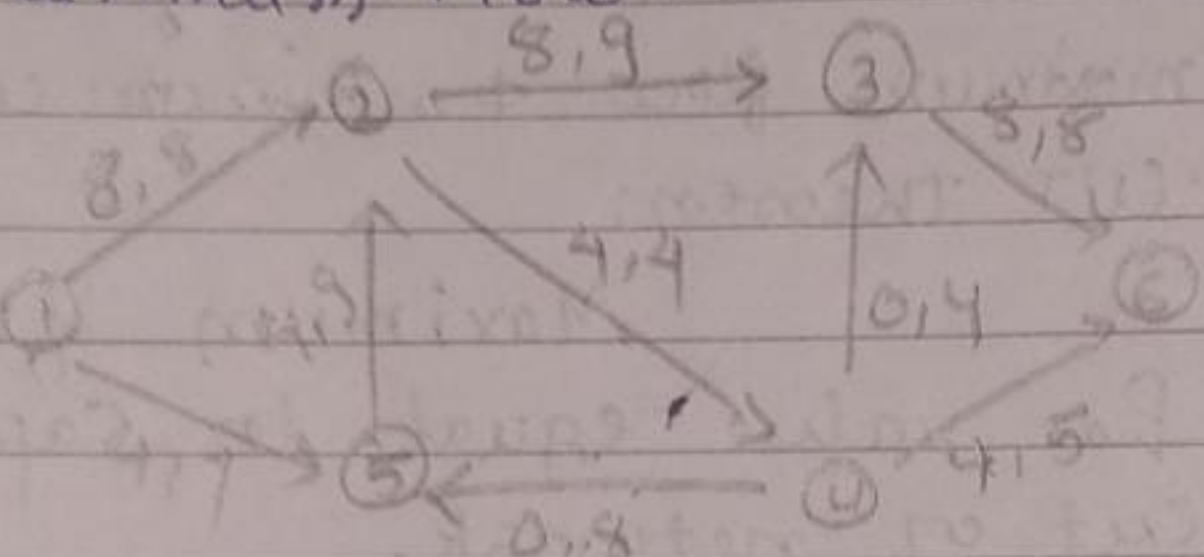
(3,6), (4,3), (2,4), (4,5)   8+4=12

(2,3), (4,6)   9+5=14

(1,2), (5,2)   8+9=17

→ ~~And~~ we can say, by observation that cut {~~(1,2), (1,5)~~} has least capacity. i.e. minimum cut = {(3,6), (4,3), (2,4), (4,5)} = 12

→ Maximum Flow:



→ ~~Maximum~~ flow = flow from source = 8+4=12.
(which is equal to minimum cut)

∴ Maximum flow = 12 = Minimum cut.

PAGE NO:
DATE / /

(3) Monoid:

for A non-empty-set $S, (S,*)$ is called a monoid, if it follows following axioms.

→ Closure

→ Associativity

→ Identity Element : $\exists e \in S : a*e = e*a = a \; \forall a \in S$

(4) Group: For a non-empty set G, $(G,*)$ is called a group if it follows following axiom

→ Closure

→ Associativity

→ Identity element

→ Inverses : ~~∀a∈G~~ $\exists a^{-1} \in G : a*a^{-1} = a^{-1}*a = e \; \forall a \in G$

(5) Abelian Group: A non-empty set S, $(S,*)$ is called abelian group, if it is a group as well as follows Commutativity (i.e, $a*b = b*a \; \forall a,b \in S$).

Q9. Is the group "$\{1,2,3,4,5,6\}$ under modulo 7 multiplication" cyclic? If yes, what are generators? ~~[...]~~

Ans. Cyclic group: Group G is cyclic if there exists $a \in G$, such that cyclic subgroup generated by a, $\langle a \rangle$, equals to all of G. [a is generator]

~~generat~~

→ Here $a*b = (a)(b) \% 7$.

→ also $\langle a \rangle = \{a^n \mid n \in N, a \in G\}$

→ $\langle 1 \rangle = \{1\}, *)$ → $1^1 = 1$

as $1^2 = 1 * 1 = 1 \% 7 = 1$

$1^3 = 1*1*1 = (1)*1 = 1$ and so on.

∴ ~~let $e = b^{-1} a$,~~

Now, $a, b \in H \Rightarrow a, b, b^{-1} \in H$

~~$a * b^{-1} \in H$~~ {$*$ is associative ⇒ property def}

~~$(a * b^{-1}) * e \in H$~~ {$e \in H$}

∴ $a * (b^{-1})^{-1} \in H$

∴ $a * b \in H$

∴ H is closed.

∴ H is group

∴ H is subset of G

∴ H is subgroup of G.

Hence Proved.

18. What is Coset? Define Normal Subgroup. Let H be subgroup of group $(G, *)$ and $a * b \in G$ then prove.
(i) $a * H = H$ iff $a \in H$ (ii) $a * H = b * H$ iff $a^{-1} * b \in H$

Ans. Coset: Coset is a set composed of all the products obtained by multiplying each element of a subgroup in turn by one particular element of group containing subgroup.

→ Let H be subgroup of $(G, *)$ & $a \in G$, then
$a * H = \{a * h \mid h \in H\}$ is left coset of H &
$H * a = \{h * a \mid h \in H\}$ is right coset of H.

→ Normal subgroup:

A ~~gro~~ subgroup is called normal subgroup, if its left ~~sub~~ coset is equivalent to right subgroup for particular element.
[i.e. $x * H = H * x \;\forall x \in G$]

PAGE NO:
DATE / /

→ Cosets of H:

$0+H = \{2g \mid g \in G\} = H$

$1+H = \{2g+1 \mid g \in G\}$

→ no. of cosets = 2.

14. Permulation:

A permulation is a collection or a combination of objects from a set where the order or the arrangement of chosen objects does matter.

→ Let G be non-empty set, then a one-one onto mapping to itself is called Permulation.

(i) $(1,2,3)(5,6,4,1,8) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 4 & 5 & 6 & 7 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 3 & 1 & 6 & 4 & 7 & 5 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 8 & 1 & 6 & 4 & 7 & 5 \end{pmatrix}$

(ii) $(1,2)(5,7,9) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 7 & 6 & 9 & 8 & 5 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 3 & 4 & 7 & 6 & 9 & 8 & 5 \end{pmatrix}$

15. Langrange's Thearom:

For any group G, the order of its subgroup H is a divisor of order of G.

→ Converse: If a group G has order m, and $n \mid m$, then G has a sub-group of order n.

→ Converse is NOT TRUE.

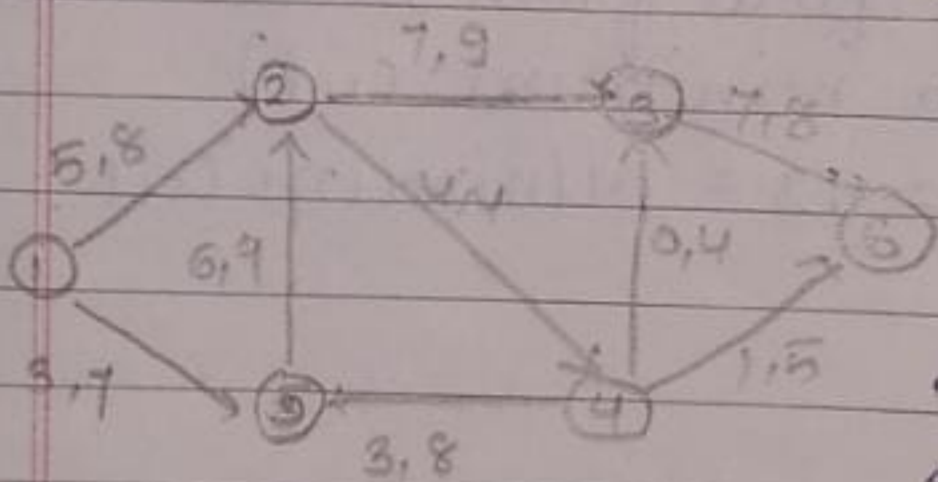
vi) Cut: A ~~cut~~ x-y cut is a set of edges (arcs) whose removal disconnects node x & node y

vii) Cut capacity: Cut-capacity is equal to maximal flow that can cross the cut from the source to sink.

30. STATE MAX-FLOW MIN-CUT Theorem. Is given flow feasible? Find maximum flow & minimum cut.

Ans. * MAX-FLOW MIN-CUT Theorem:

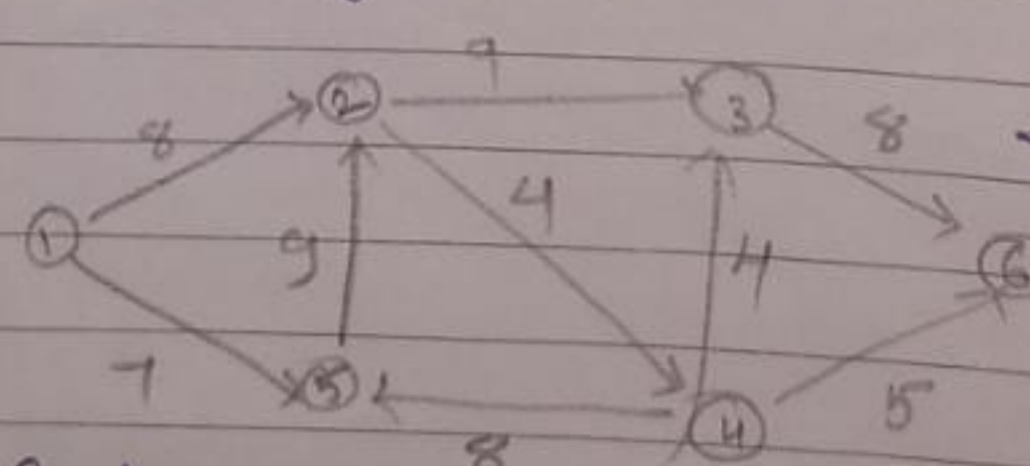
Maximum flow F in a network has value equal to capacity of minimum cut of network.



→ Flow conservation:
+ node 1: source & node 6: sin[k]
node 2 = (7+4) − (6+5) = 0
node 3 = (7) − (7+0) = 0
node 4 = 4 − (0+3+1) = 0
node 5 = (3+3) − 6 = 0

∴ Given flow is feasible.

→ Rewriting Graph with only capacities.



→ let us see capacities of cuts:

Cut	Capacities:
(3,6) (4,6)	8+5 = 13
(2,3) (2,4) (4,5)	9+4 = 13
~~(1,2)~~ (1,	

PAGE NO:
DATE / /

11. $f:(R,+)\to(R-\{0\},\times)$ is homomorphism. $f(2)=5 \Rightarrow f(-8)=?$

Ans. $f(x+y)=f(x)\times f(y) \Rightarrow$

$\rightarrow f(4)=f(2+2)=f(2)\times f(2)=5\times5=25$

$f(8)=f(4+4)=f(4)\times f(4)=25\times25=625$

$f(-8)=f((8)^{-1})=(f(8))^{-1}=(625)^{-1}=1/625=5^{-4}$

$\therefore f(-8)=5^{-4}$

12. When are two group homomorphic? S.T. f is homomorphic: $f(a)=2^a$: $f:(R,+)\to(R^*,\times)$, $R^*=R-\{0\}$

Ans Homomorphism : Two groups $(G,*)$ and $(H,\star)$ are called homomorphic if there exists a function $f:G\to H$ such that $f(x*y)=f(x)\star f(y)$.

$\rightarrow$ ~~f can be defined as~~: $f(a)=2^a$.

~~$f(x+y)=f(x)\times f(y)$~~

$\rightarrow \therefore f(m)=2^m,\ f(n)=2^n,\ m,n\in R \Rightarrow m,n\in G$.

$\therefore f(m+n)=2^{m+n}$ $\quad 2^m,2^n\neq0 \Rightarrow 2^m,2^n\in H$.

$\rightarrow f(m)\times f(n)=2^m\cdot2^n$

$f(m)\times f(n)=2^{m+n}$

$\therefore f(m+n)=f(m)\times f(n)$

$\therefore f$ is a homomorphism

13. $G=\{\begin{bmatrix}a&b\\c&d\end{bmatrix} \mid a,b,c,d\in Z\}$ $(G,+)$ is group.

is $H=\{\begin{bmatrix}a&b\\c&d\end{bmatrix} : a,b,c,d$ are even$\}$ subgroup of G

Ans. YES: $H=\{\begin{bmatrix}2u&2v\\2w&2x\end{bmatrix} : u,v,w,x\in Z\}$ [rewritten]

$\rightarrow$ H is closed: as even + even = even

$\rightarrow$ H is associative as '+' is associative

$\rightarrow$ H has identity : $\begin{bmatrix}0&0\\0&0\end{bmatrix}$ = identity of G

$\rightarrow$ H has inverse = $\begin{bmatrix}-2u&-2v\\-2w&-2x\end{bmatrix}$.

$\leftarrow H=\{2g \mid g\in G\}$

PAGE NO:
DATE / /

16. Prove that a cyclic group with only one generator can have at most ~~two~~ elements.

Ans. Let G be the group & 'g' be generator.

→ If g is generator, then g^{-1} must be a generator

→ But there is only one generator.

$\therefore g = g^{-1}$

$\therefore g^2 = e$

But as g is generator & $g^2 = e \Rightarrow$ order of g is 2.

$\therefore$ no. of elements in g (as well as in G) = 2

[element = $\{g, e\}$, or $\{e\}$ (if $g = e$)]

$\therefore$ Group can have only ~~two~~ atmost elements.

17. Prove that non-empty subset H of $(G, *)$ is a subgroup iff $a * b^{-1} \in H$ $\forall a, b \in H$.

Ans.

→ let H is ~~$(G,*)$~~ subgroup of $(G, *)$.

$\therefore$ if $b \in H \Rightarrow b^{-1} \in H$ (H is group).

$\therefore$ for ~~$a, b^{-1} \in$~~ $a \in H$,

$a * b^{-1} \in H$ {closure property}

$\therefore a * b^{-1} \in H \; \forall a, b \in H$

$\therefore$ If non-empty subset H of $(G, *)$ is subgroup then $a * b^{-1} \in H \; \forall a, b \in H$

→ Now, let $a * b^{-1} \in H \; \forall a, b \in H$.

let $a = b$ & $a \in H$

$\therefore a * a^{-1} \in H$

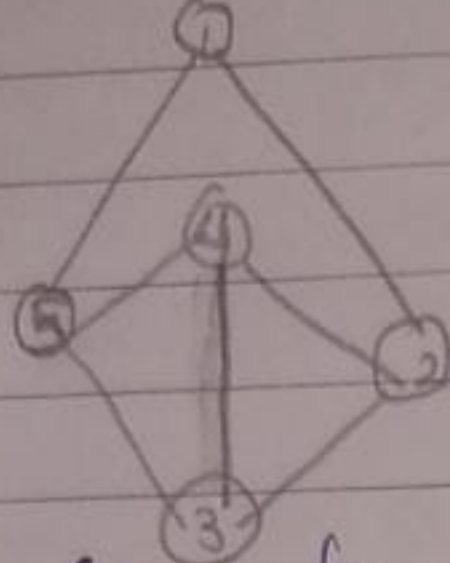
$\therefore e \in H$ {H contains e = identity}

as, $e \in H$ & $a \in H \Rightarrow e * a^{-1} \in H$

$\therefore a^{-1} \in H \; \forall a \in H$. {H contains inverse}

24. Obtain chromatic polynomial of graph shown.

Ans



Graph G

→ λ are total no. of colors.

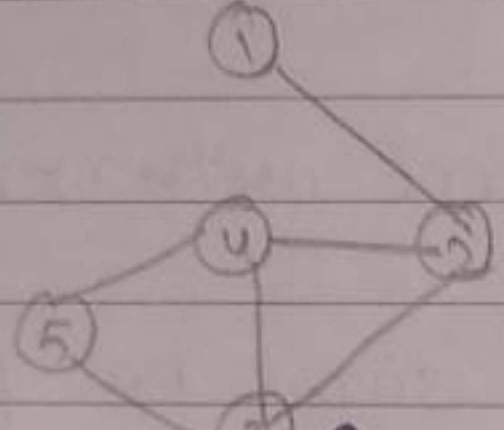
→ ~~For vertex 1 : λ colors~~

→ ~~For vertex 5 : $(\lambda-1)$ colors~~

→ lets take edge $(1,5) = e$

→ G without e ~~[Quotient graph]~~ $= G_1$

→ G after merging of 1, ϕ 5 $= G_2$



G_1

→ $P(G,\lambda) = P(G_1,\lambda) - P(G_2,\lambda)$

→ $P(G_1,\lambda)$: vertex 1 := λ colors

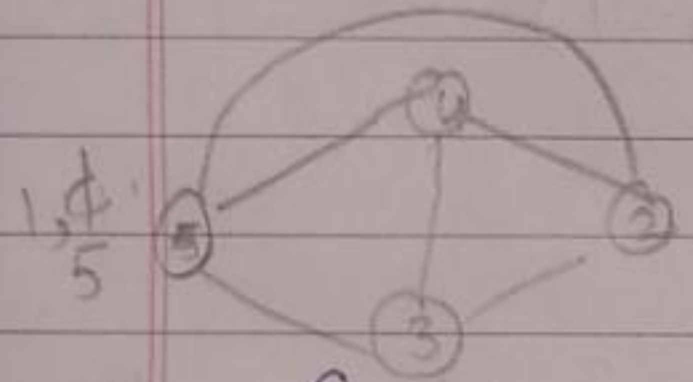
vertex (2) : $(\lambda-1)$ colors

vertex 3 : $(\lambda-1)$ colors

vertex 4 : $(\lambda-2)$ colors

vertex 5 : $(\lambda-2)$ color

$P(G_1,\lambda) = \lambda(\lambda-1)^2(\lambda-2)^2$



G_2

→ $P(G_2,\lambda)$: G_2 is complete graph with 4 vertices.

$\therefore P(G_2,\lambda) = \lambda(\lambda-1)(\lambda-2)(\lambda-3)$

→ $P(G,\lambda) = P(G_1,\lambda) - P(G_2,\lambda)$

$= \lambda(\lambda-1)^2(\lambda-2)^2 - \lambda(\lambda-1)(\lambda-2)(\lambda-3)$

$= \cancel{\lambda(\lambda-1)^2(\lambda-2)(\lambda-5)}$

$P(G,\lambda) = \lambda(\lambda-1)(\lambda-2)(\lambda^2-4\lambda+5)$

25 A graph of n vertices is a complete graph iff its chromatic polynomial $P_n(\lambda) = \lambda(\lambda-1)\cdots(\lambda-n+1)$

Ans Let G be complete graph with n vertices $(G = K_n)$

→ If we take λ color.

$\therefore$ color for 1st vertices $= \lambda$ colors