Sardar Vallabhbhai National Institute of Technology, Surat
B.Tech.-I/ M. Sc.-I (Semester-I)
End Semester Examination (March 2022), Online Mode
Sub: MA 101 S1 Mathematics-I

Date: 28-03-2022
Time: 09:30 am to 12:30 pm (including uploading of answer sheets) **M.M.: [50]**
INSTRUCTIONS:
1) There are total **FOUR** questions in the question paper.
2) All questions are compulsory.
3) Figure to the right indicates marks.
4) Follow usual notations.
5) Important Instructions:You must write your Admission Number, Roll Number, Contact Number, Email id on TOP of the first page of the answer sheet and Admissions Number and Page No. with your Signature on all pages.
6) You must upload your answer sheet (single PDF file) on Google classroom or Microsoft team as per your class teacher's instruction latest by**12:30 pm** on the same day.
7) First verify the number of pages in your PDF file and then only upload. Once you upload the file thereafter, we will not consider any updated file.

Q1.Answer the following questions with justification: **[05]**

1) Find the n^{th} differential coefficient of $\dfrac{x^4}{(x-1)(x-2)}$, $n>2$.
2) If $u=\log\left(\dfrac{x^2+y^2}{x+y}\right)$, then find the value of $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$.
3) Show that the curve $x^2y^2=(4-y^2)(1+y)^2$ has a node at the point (0, -1).
4) Evaluate $\int_0^\infty x^{1/4}e^{-\sqrt{x}}\,dx$.
5) Evaluate $\int_0^1\int_0^{\sqrt{(1+x^2)}}\dfrac{dx\,dy}{1+x^2+y^2}$.

Q2. **Answer the following questions.**

(A) If $y=e^{a\sin^{-1}x}$, then show that **[04]**

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0.$$

Hence by Maclaurin's theorem, show that

$$e^{a\sin^{-1}x}=1+a\,x+\frac{a^2}{\lfloor 2}x^2+\frac{a(1^2+a^2)}{\lfloor 3}x^3+\ldots$$

Also deduce that $e^{\theta}=1+\sin\theta+\dfrac{1}{\lfloor 2}\sin^2\theta+\dfrac{1}{\lfloor 3}\,2\sin^3\theta+\ldots$.

OR

(A) Explain the radius of curvature. Also, find the radius of curvature at the point (x,y) of $x^{2/3}+y^{2/3}=a^{2/3}$.

(B) Answer the following questions (Attempt any three) **[09]**

1) Expand $\sin^{-1}(x+h)$ in powers of x as for as the terms in x^3.

2) If u is a homogeneous function of degree n, then prove that

(i) $x\dfrac{\partial^2 u}{\partial x^2}+y\dfrac{\partial^2 u}{\partial x\partial y}=(n-1)\dfrac{\partial u}{\partial x}$,

(ii) $x\dfrac{\partial^2 u}{\partial x\partial y}+y\dfrac{\partial^2 u}{\partial y^2}=(n-1)\dfrac{\partial u}{\partial y}$,

(iii) $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=n(n-1)u.$

3) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2+y^2+z^2=4$.

4) Trace the curve $r=a+b\cos\theta,\quad a>b$.

(C) Compute the approximate value of $(1.04)^{3.01}$ using the theory of approximation. **[02]**

Q3. Answer the following questions.

(A) Change the order of integration in the integral $\int_0^1 \int_x^{\sqrt{(2-x^2)}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}$ and then evaluate it. **[04]**

OR

(A) Change the order of integration in the integral $\int_0^{2a} \int_{\sqrt{(2ax-x^2)}}^{\sqrt{(2ax)}} V\,dx\,dy$.

(B) Answer the following questions (Attempt any three). **[09]**

1) Trace the curve $x^5+y^5=a^2x^2y$.

2) Express $\int_0^1 x^m(1-x^p)^n\,dx$ in terms of Beta function and hence evaluate $\int_0^1 x^5(1-x^3)^{10}\,dx$.

3) Show by double integration that the area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $(16/3)a^2$.

4) Find the volume in the first octant bounded by the ellipsoid $9(x^2+y^2)+4z^2=36$ and the planes $x=\sqrt{3}\,y,\ x=0, z=0$ using double integral.

(C) Prove that $\lceil n = \int_0^1 \left(\log\frac{1}{y}\right)^{n-1} dy$. **[02]**

Q4. Answer the following questions.

(A) Evaluate $\iiint xyz\,dx\,dy\,dz$ over the volume enclosed by the planes $x=0, y=0, z=0,$ and $x+y+z=a$. **[04]**

OR

(A) Find the volume bounded by the sphere $x^2+y^2+z^2=2a^2$ and below by the paraboloid $az=x^2+y^2$.

(B) Answer the following questions (Attempt any three) **[09]**

1) Given that $x+y=u, y=u$, change the variables to u, v in the double integral $\iint [xy(1-x-y)]^{1/2}\,dx\,dy$ taken over the area enclosed by the lines $x=0, y=0, x+y=1$. Hence, show that the value of the integral is $2\pi/105$.

2) Evaluate $\int_0^1 \int_x^{\sqrt{(2x-x^2)}} (x^2+y^2)\,dx\,dy$ by changing to polars.

3) Evaluate $\iint \frac{r\,d\theta\,dr}{\sqrt{a^2+r^2}}$ over one loop of the laminscate $r^2=a^2\cos 2\theta$.

4) Find the value of $\iiint x^2\,dx\,dy\,dz$ over the volume bounded by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(C) Find the value of $\int_0^{\pi/2} \sin^3 x \cos^{5/2} x\,dx$. **[02]**

END