Department of Applied Mathematics and Humanities
S.V. National Institute of Technology, Surat, Gujarat
B.Tech.- I (Semester- I) Branch (All)
Subject: Mathematics -I (MA 101 S1)

Tutorial-06
Error and Approximations, Jacobians, Extreme values of functions of two variables, Lagrange's Method of undetermined multipliers

1. If $D = \frac{a^2}{b} + \frac{c^2}{2}$, find the percentage error in D if error in measuring a is $\frac{1}{2}\%$ and in measuring b and c are 1% each.
   Ans: $D = \frac{2bc^2}{2a^2+bc^2}$.

2. If $z = 2xy^2 - 3x^2y$ and x increases at the rate of $2cm/s$ as it passes through $x = 3\ cm$. Show that if y is passing through $y = 1\ cm$, y must decrease at the rate of $\frac{32}{15}\ cm/s$ in order that z remains constant.

3. A balloon is in the form of a right circular cylinder of radius $1.5\ m$ and height $4\ m$ and is surmounted by hemispherical ends. If the radius is increased by $0.01\ m$ and the height by $0.05\ m$, find the percentage change in the volume of the balloon.
   Ans: 2.389%

4. Evaluate $(1.99)^2(3.01)^3(0.98)^{\frac{1}{10}}$ using approximation.
   Ans: 107.784

5. Find the Jacobian for each of the following functions:
   $(a)\ x = a\ cosh\theta cos\emptyset\ , y = a\ sinh\theta\ sin\emptyset$
   Ans: $\frac{a^2}{2}\ (cosh2\theta - cos2\emptyset)$

   $(b) u = xyz\ , v = x^2 + y^2 + z^2\ , w = x + y + z$
   Ans: $2(y-z)(z-x)(y-x)$

6. Verify $J * J' = 1$ for the function $x = u\ , y = u \tan v\ , z = w$.

7. If $u = e^x \cos y, v = e^x \sin y,\ where\ x = lr + sm\ and\ y = mr - ls$, verify chain rule of Jacobians, l, m being constant.

8. Find the extreme values of the each of the following functions:
   (a) $u = x^3 + y^3 - 63(x+y) + 12xy$.
   Ans: $u_{max} = 2156\ , u_{min} = -216$.

   $(b)\ sinx + siny + \sin(x+y)$
   Ans: $f_{max} = \frac{3\sqrt{3}}{2}$

9. Find the stationary value of $xy(a-x-y)$

Ans: $f_{extreme} = \frac{a^3}{27}$

10. Find the points on the surface $z^2 = xy + 1$ nearest to the origin. Also find that distance.
Ans: The points $(0,0,1)$ and $(0,0,-1)$on the surface are nearest to the origin. Minimum distance = 1.

11. Find the point on the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a minimum value and find this minimum f.

Ans: The minimum value of $f = \frac{p^2}{a^2+b^2+c^2}$

12. Show that the rectangular solid of maximum value that can be inscribed in a sphere is a cube.

13. Find the maximum and minimum distances from the origin to the curve $3x^2 + 4xy + 6y^2 = 140$.
Ans: The maximum and minimum distances are $\sqrt{70}, \sqrt{20}$.

14. Use the method of Lagrange's multipliers to find volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Ans: $V = \frac{8abc}{3\sqrt{3}}$.