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TUTORIAL 2

A.1 (i) $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

[A is diagonizable iff AM = GM $\forall \lambda$]

$(A - \lambda I) = 0$

$\Rightarrow \begin{vmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow$ ~~$-\lambda\ (\lambda-2)(\lambda-3) - (-2)(2-\lambda) = 0$~~

$\Rightarrow$ ~~$+(\lambda-2)(\lambda(\lambda-3)-2) = 0$~~

$\Rightarrow$ ~~$(\lambda-2)(\lambda^2 - 3\lambda - 2) = 0$~~

$\Rightarrow$ ~~$-(\lambda-2)(\lambda-2)(\lambda-1)$~~

$(-\lambda)[(2-\lambda)(3-\lambda)] + (-2)[-(2-\lambda)] = 0$

$\Rightarrow (2-\lambda)[-\lambda(3-\lambda)+2] = 0$

$\Rightarrow (\lambda-2)[\lambda(3-\lambda)-2] = 0$

$\Rightarrow (\lambda-2)(\lambda-1)(\lambda-2) = 0$

$\therefore \lambda = 1, 2, 2$

For $\lambda = 1$:

$AX = X \Rightarrow (A-I)X = 0$

$\Rightarrow \begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$; $\Rightarrow \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

~~$\Rightarrow x + 2z = 0$~~

~~$\Rightarrow x + y + z = 0$~~

~~$\Rightarrow x + 2z = 0$~~

$\therefore x + 2z = 0$

$y - z = 0$

~~Let $z = t$; $\therefore x = -2t,\ y = -3t$~~

if $z = t$

$\therefore x = -2t,\ y = t$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2t \\ t \\ t \end{bmatrix} = t\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$\therefore$ Eigen vector corresponding to $\lambda = 1$ is $(-2, 1, 1)$

Eigen Space = span $(-2, 1, 1)$

$\because$ one basis $\therefore$ GM = 1 & AM = 1

- For $\lambda = 2$:

$\rightarrow [A - 2\lambda]\, x = 0$

$\Rightarrow \begin{bmatrix} -2 & 0 & -2 \\ 1 & \cancel{2-2}\,0 & 1 \\ 1 & 0 & \cancel{3-2}\,1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} -2 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$+x + z = 0 \quad \therefore x = -z$

Let $y = k,\ z = t \quad \therefore x = -t$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -t \\ k \\ t \end{bmatrix} = k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ Dimension = 2, Basis $((0,1,0), (-1,0,1))$

$\therefore$ GM = 2 and AM = 2

$\therefore$ AM = GM $\forall\ \lambda$

$\therefore$ A is diagonizable.

(iii) Finding ~~Checking~~ P | $P^{-1}AP = D$

$\rightarrow$ $P^{-1}AP = D$

Let $D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

$P^{-1}AP = D$

$\Rightarrow AP = PD$

$\Rightarrow \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix} P = \begin{bmatrix} x_{11} & x_{21} & x_{31} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} D$

$\Leftrightarrow \begin{bmatrix} 0 & 0 & -2 \\ 0 & 2 & -2 \\ 1 & 0 & 3 \end{bmatrix} P = \begin{bmatrix} x_{11} & x_{21} & x_{31} \\ x_{21}-x_{31} & x_{22}-x_{32} & x_{23}-x_{33} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} D$

$\Rightarrow \begin{bmatrix} 0 & 0 & -2 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} P = \begin{bmatrix} x_{11} & x_{21} & x_{31} \\ x_{21}-x_{31}-x_{11} & x_{22}-x_{32}-x_{21} & x_{23}-x_{33}-x_{31} \\ x_{31}+\frac{3x_{11}}{2} & x_{32}+\frac{3x_{21}}{2} & x_{33}+\frac{3x_{31}}{2} \end{bmatrix}$

$\Rightarrow x_{11} = 0 \quad x_{21} = 0 \quad x_{31} = -2$

$\therefore x_{31} = 1 \quad x_{32} = 0 \quad x_{33} = 3$

$x_{21} = 1$

$P^{-1}AP = D$

$\rightarrow D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

ii) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$

$\rightarrow \quad [A - \lambda I] = 0$ $\quad \rightarrow$ Finding eigen values using ~~by~~ Cayley's Theorem

$$\Rightarrow \begin{bmatrix} 1-\lambda & 0 & 0 \\ 1 & 2-\lambda & 0 \\ -3 & 5 & 2-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)\left[(2-\lambda)^2\right] = 0$$

$\Rightarrow \quad \lambda = 1, 2, 2$ $\quad \rightarrow$ Eigen values

For $\lambda_1 = 1$: $\qquad$ AM=1

$[A - I][X] = 0$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 8 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\Rightarrow \quad x + y = 0$

$-3x + \cancel{5}y + z = 0$

$\therefore$ Let $y = t$

$\therefore x = -t, \quad z = -8t$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t\begin{bmatrix} -1 \\ 1 \\ -8 \end{bmatrix}$$

$\therefore$ Eigen vectors $= (-1, 1, -8)$

Basis $= (-1, 1, -8)$

Dimension $= 1$

$\therefore$ GM $= 1$

$\therefore \boxed{AM = GM}$

AM = 2

For $\lambda = 2$:

$\rightarrow$ $[A - 2I][X] = 0$

$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ -3 & 5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -3 & 5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\Rightarrow$ ~~Let~~ $x = 0$ Let $z = t$

$-3x + 5y = 0$

$y = 0$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} = t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ Eigenvector = (0, 0, 1)

Bases = (0, 0, 1)

Dimension = 1

$\therefore$ AM $\neq$ GM

$\therefore$ A is Not diagonizable matrix

A.2 Eigenspace : For a square matrix $n \times n$ A, eigenspace is span of eigenvectors associated with eigenvalues λ; defined as -

$$E_\lambda(A) = N(A - \lambda I) \rightarrow \text{Null space}$$

λ - eigenvalue associated with eigenvector v

→ The dimension of eigenspace corresponding to an eigenvalue λ is '<u>geometric multiplicity</u>' of value λ where no. of times of values of λ repeats is '<u>algebraic multiplicity</u>' of value λ.

→ A matrix is diagonizable if and only if AM = GM for every λ. Also GM $\leq$ AM $\forall \lambda$.

A.3 $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

$T(x, y, z) = (x + 2y - 3z, x, 2y + z)$

to find matrix represention of T.

→ $T(X) = AX$

$$a_1 x + b_1 y + c_1 z = x + 2y - 3z$$
$$a_2 x + b_2 y + c_2 z = x$$
$$a_3 x + b_3 y + c_3 z = 2y + z$$

$$\Rightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + 2y - 3z \\ x \\ 2y + z \end{bmatrix}$$

$\Rightarrow$

$a_1 = 1$, $b_1 = 2$, $c_1 = -3$

$a_2 = 1$

$a_3 = 0$, $b_2 = 2$, $c_3 = 1$

$$\therefore T(X) = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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A-4 $T: \mathbb{R}^3 \to \mathbb{R}^3$

$$T(x,y,z) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

To Prove T is isomorphism

$\rightarrow$ $T(x,y,z) = (z,\ x-z,\ y+2z)$

[T is isomor... if T^{-1} exists, T is linear]

For point (a,b,c)

~~$T(a,b,c) = (z,$~~

$(a,b,c) = (z,\ x-z,\ y+2z)$

$a = z, \quad b = x-z, \quad c = y+2z$

$\therefore$ $a+b = x$

$c-2a = y$

and, $a = z$

$\therefore$ T is onto and one-one

$\therefore$ $(x,y,z) \rightarrow$ unique solⁿ $\forall$ pts. in $\mathbb{R}^3$

$\therefore$ T^{-1} exists.

Checkin for linear transformation -

(i) $T(v+w) = T(v) + T(w)$

such that $v \in V$, $w \in W$

$\rightarrow$ here V, W is $\mathbb{R}^3$

$T(v+w) = T(x_1+x_2,\ y_1+y_2,\ z_1+z_2)$

$= (z_1+z_2,\ x_1+x_2-z_1-z_2,\ y_1+y_2+2z_1+2z_2)$ — ①

$T(v) = (z_1,\ x_1-z_1,\ y_1+2z_1)$

$T(w) = (z_2,\ x_2-z_2,\ y_2+2z_2)$

$\rightarrow$ $T(v)+T(w) = (z_1+z_2,\ x_1+x_2-z_1-z_2,\ y_1+y_2+2z_1+2z_2)$ — ②

$\therefore$ From ① & ② we can say

$T(v+w) = T(v) + T(w)$

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(ii) $T(kv) = k\,T(v)$

let $v \equiv (x_1, y_1, z_1)$

$T(kx_1, ky_1, kz_1) = (kz_1,\ k(x_1 - z_1),\ k(y_1 + 2z_1))$

$= k\,(z_1, (x_1, z_1), (y_1 + 2z_1))$

$\underline{T(kv) = k\,T(v)}$

$\therefore$ T is a linear transformation.

$\therefore$ T is a isomorphism $\therefore$ Proved

A.5 $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$

$T(X) = AX$

$$AX = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & 0 & -1 & 5 \\ 3 & 0 & -2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix}$$

(i) $Ker(T) = \left\{ \begin{array}{l} v \in V; \\ T(v) = 0 \end{array} \right.$

For $v \equiv (1,0,1,0)$ to belong to $Ker(T)$

$T(v) = 0$

$\Rightarrow T(v) = (x - z + 3u,\ 2x - z + 5u,\ 3x - 2z + 8u)$

$T(1,0,1,0) = (1-1,\ 2-1,\ 3-2)$

$= (0, 1, 1) \neq (0, 0, 0)$

$\therefore (1,0,1,0)$ does not belong to $Ker(T)$

(ii) $Im(T) = \{ T(v);\ v \in V \}$

$Im(T) = \{ (x - z + 3u,\ 2x - z + 5u,\ 3x - 2z + 8u)$ where $v \equiv (x, y, z, u) \in V \}$

For (3,5,8) to exist in $Im(T)$, there must exist a soln for the foll -

$$\begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & 0 & -1 & 5 \\ 3 & 0 & -2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}$$

$\Rightarrow \quad AX = B$

$\rightarrow [A \mid B]$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 3 \\ 2 & 0 & -1 & 5 & 5 \\ 3 & 0 & -2 & 8 & 8 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 3 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 3 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ Rank (A) = Rank (A|B)

$\therefore$ there exist atleast one soln.

$\therefore$ (3, 5, 8) belongs to Im (T).

(iii) Nullity (T) and rank (T) ?

$\rightarrow$ Nullity (T) = dim (Ker(T))

Rank (T) = dim (Im(T))

Ker (T) is subspace of V,

V has dimensions = 4

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$\therefore$ Nullity $(T) = 4$

Im (T) is a sub-space of W
where ~~W is space~~ $V \to W$.

$\therefore \dim(\text{Im}(T)) = 3$

$\therefore$ Rank $(T) = 3$

A.6 $T: \mathbb{R}^3 \to \mathbb{R}^2$

$T(1,2,3) = (1,0,0)$

$T(1,2,0) = (0,1,0)$

$T(1,-1,0) = (0,1,0)$

To find $T(x,y,z)$

$\rightarrow$

$T(x,y,z) = (a_1x + b_1y + c_1z,$
$a_2x + b_2y + c_2z, \; a_3x + b_3y + c_3z)$

(i) for $(1,2,3)$

$\Rightarrow (1,0,0) = (a_1 + 2b_1 + 3c_1, \; a_2 + 2b_2 + 3c_2,$
$a_3 + 2b_3 + 3c_3)$

(ii) for $(1,2,0)$

$\Rightarrow (0,1,0) = (a_1 + 2b, \; a_2 + 2b_2, \; a_3 + 2b_3)$

(iii) for $(1,-1,0)$

$\Rightarrow (0,1,0) = (a_1 - b, \; a_2 - b_2, \; a_3 - b_3)$

- $a_3 - b_3 = 0 \Rightarrow a_3 = b_3$ — ①
  $a_3 + 2b_3 = 0 \Rightarrow \boxed{a_3 = b_3 = 0}$ (using ①)
  $\therefore \boxed{c_3 = 0}$
- $a_1 - b_1 = 0 \Rightarrow a_1 = b_1$ — ②
  $a_1 + 2b = 0 \Rightarrow \boxed{a_1 = b_1 = 0}$ (using ②)
  $\therefore \boxed{c_1 = 1/3}$
- $a_2 - b_2 = 1$
  $a_2 + 2b_2 = 1 \quad \therefore \boxed{b_2 = 0} \; \boxed{a_2 = 1} \; \therefore \boxed{c_2 = -1/3}$

$$\boxed{\therefore T(x,y,z) = \left(\frac{z}{3}, \frac{x-z}{3}, 0\right)}$$

A.7 Extend the set $\{(1,-2,0), (0,1,1)\}$ to a basis for $\mathbb{R}^3$.

$\rightarrow$ ~~Let $(x,y,z) \in \mathbb{R}^3$~~

~~$\Rightarrow (x,y,z) = a(1,-2,0) + b(0,1,1)$~~

~~$\rightarrow x = a + 0 \cdot b$~~

~~$\therefore \boxed{x = a}$~~

~~$\rightarrow y = -2a + b$~~

~~$\rightarrow \boxed{z = b}$~~

~~$\therefore$ To make y independent $(0,1,0)$ can be added.~~

~~$\therefore (x,y,z) = a(1,-2,0) + b(0,1,1) + c(0,1,$~~

Method 2:

Let 3^{rd} vector be (x,y,z) to make set $\{(1,-2,0), (0,1,1)\}$ a basis for $\mathbb{R}^3$.

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ x & y & z & 0 \end{array}\right]$$

3 vectors must be independent

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & y+2x & z & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & z-y-2x & 0 \end{array}\right]$$

$\therefore z - y - 2x \neq 0$

$\Rightarrow y + 2x \neq z$

$\forall\ x, y \in \mathbb{R}$ & $z \in \mathbb{R} - \{2x + 2y\}$

Let $x = 1$, $y = 0$, $z = 1$ can be one of the solutions.

A.8 Using Least square method, to find parabola that best fits the data points : $(-1, \frac{1}{2}), (1, -1), (2, -\frac{1}{2}), (3, 2)$

$\rightarrow \quad A^T A X = A^T B$

Let $f(x) = a + xb + x^2 c$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1/2 \\ -1 \\ -1/2 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 2 & 3 \\ 1 & 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 2 & 3 \\ 1 & 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} -1/2 \\ -1 \\ -1/2 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 5 & 15 \\ 5 & 15 & 35 \\ 15 & 35 & 99 \end{bmatrix} X = \begin{bmatrix} 1 \\ 7/2 \\ 31/2 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 5 & 15 & 1 \\ 5 & 15 & 35 & 7/2 \\ 15 & 35 & 99 & 31/2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 5 & 15 & 1 \\ 1 & 3 & 7 & 7/10 \\ 15 & 35 & 99 & 31/2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 0 & -7 & -13 & -9/5 \\ 1 & 3 & 7 & 7/10 \\ 0 & -10 & -6 & 5 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 7 & 7/10 \\ 0 & 7 & 13 & 9/5 \\ 0 & 10 & 6 & -5 \end{array}\right]$$

$\therefore \quad a = -41/44 \;;\quad b = -379/440 \;;\quad c = 53/88$

$$\boxed{\therefore\ y = \frac{-41}{49} - \frac{379}{440}x + \frac{53x^2}{88}}$$

A.9 If vectors are orthogonal

$\Rightarrow \{(0,2,0),\ (3,0,3),\ (-4,0,4)\}$

$\Rightarrow (0,2,0)\cdot(3,0,3)\cdot(-4,0,4) = (0,0,0)$

$\rightarrow$ Orthogonal

Normalizing vectors we get,

$\Rightarrow \dfrac{(0,2,0)}{2} = (0,1,0)$

$\Rightarrow \dfrac{(3,0,3)}{\sqrt{9+9}} = \dfrac{1}{\sqrt{2}}(1,0,1) = \left(\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$

$\Rightarrow \dfrac{1}{\sqrt{16+16}}(-4,0,4) = \left(-\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$

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A.10 $x_1 = (1,2,3)$ $x_2 = (-2,1,0)$ $x_3 = (3,6,9)$
$x_4 = (2,2,-2)$

(i) Unit vector in direction of $x_1 = \dfrac{(1,2,3)}{\sqrt{1+4+9}} = \left(\dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}\right)$

(ii)
$\langle x_1, x_2 \rangle = -2+2 = 0$
$\Rightarrow$ x_1 & x_2 are orthogonal.

(iii) $\cos\theta = \dfrac{x_2 \cdot x_4}{||x_2||\,||x_4||} = \dfrac{-4+2}{2\sqrt{5}\sqrt{3}} = \dfrac{-2}{2\sqrt{15}} = -\dfrac{1}{\sqrt{15}}$

$\because \theta \neq 0°$ or $180°$
$\therefore$ x_2 & x_4 are not parallel.

(iv) $x_3 = kx_1$ for x_1 & x_3 to be in same dirxn
for $k \in \mathbb{R}^+$
for $k = 3$
$x_3 = kx_1 \Rightarrow x_3 = 3x_1$
$\Rightarrow$ x_1 & x_3 are in same dirxn