# Generalization: Moments

- **Suppose a stream has elements chosen from a set *A* of *N* values**
- **Let $m_i$ be the number of times value *i* occurs in the stream**
- **The $k^{th}$ *moment* is**

$$\sum_{i \in A} (m_i)^k$$

# Special Cases

$$\sum_{i \in A} (m_i)^k$$

- **0th moment =** number of distinct elements
  - The problem just considered
- **1st moment =** count of the numbers of elements = length of the stream
  - Easy to compute
- **2nd moment =** ***surprise number S*** **=** a measure of how uneven the distribution is

# Example: Surprise Number

- **Stream of length 100**
- **11 distinct values**

- Item counts: **10, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9**
  **Surprise *S* = 910**

- Item counts: **90, 1, 1, 1, 1, 1, 1, 1 ,1, 1, 1**
  **Surprise *S* = 8,110**

# AMS Method

[Alon, Matias, and Szegedy]

- **AMS method works for all moments**
- **Gives an unbiased estimate**
- **We will just concentrate on the 2nd moment *S***
- **We keep track of many variables *X*:**
  - For each variable ***X*** we store ***X.el*** and ***X.val***
    - ***X.el*** corresponds to the item ***i***
    - ***X.val*** corresponds to the **count** of item ***i***
  - Note this requires a count in main memory, so number of ***X*****s** is limited
- **Our goal is to compute** $S = \sum_i m_i^2$

# One Random Variable (X)

- **How to set *X.val* and *X.el*?**
  - Assume stream has length $n$ **(we relax this later)**
  - Pick some random time $t$ ($t<n$) to start, so that any time is equally likely
  - Let at time $t$ the stream have item $i$. ***We set X.el = i***
  - Then we maintain count $c$ (***X.val = c***) of the number of $i$s in the stream starting from the chosen time $t$
- **Then the estimate of the 2nd moment ($\sum_i m_i^2$) is:**

$$S = f(X) = n\,(2 \cdot c - 1)$$

  - Note, we keep track of multiple **X**s, $(X_1, X_2, \ldots X_k)$ and our final estimate will be $S = 1/k \sum_j f(X_j)$