Chap. 3 Data Representation

3-1 Data Types

- Binary information is stored in ***memory*** or ***processor registers***
- Registers contain either ***data*** or ***control information***
  - *Data* are numbers and other binary-coded information
  - *Control information* is a bit or a group of bits used to specify the sequence of command signals
- Data types found in the registers of digital computers
  - *Numbers* used in arithmetic computations
  - *Letters* of the alphabet used in data processing
  - *Other discrete symbols* used for specific purpose
- Number Systems
  - *Base* or *Radix r system* : uses distinct symbols for *r digits*
  - Most common number system :Decimal, Binary, Octal, Hexadecimal
  - Positional-value(weight) System : $r^2\ r^1 r^0 . r^{-1} r^{-2} r^{-3}$
    - » Multiply each digit by an integer power of r and then form he sum of all weighted digits

- Decimal System/Base-10 System
  - Composed of 10 symbols or numerals(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0)
- Binary System/Base-2 System
  - Composed of 10 symbols or numerals(0, 1)
  - Bit = Binary digit
- Hexadecimal System/Base-16 System : Tab. 3-2
  - Composed of 16 symbols or numerals(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)
- Binary-to-Decimal Conversions

$1011.101_2 = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$

$= 8_{10} + 0 + 2_{10} + 1_{10} + 0.5_{10} + 0 + 0.125_{10}$

$= 11.625_{10}$

- Decimal-to-Binary Conversions

Repeated division(See p. 69, ***Fig. 3-1***)

37 / 2 = 18 remainder 1 (binary number will end with 1) : LSB
18 / 2 = 9 remainder 0
9 / 2 = 4 remainder 1
4 / 2 = 2 remainder 0
2 / 2 = 1 remainder 0
1 / 2 = 0 remainder 1 (binary number will start with 1) : MSB

Read the result upward to give an answer of $37_{10} = 100101_2$

0.375 x 2 = 0.750 integer 0 MSB
0.750 x 2 = 1.500 integer 1
0.500 x 2 = 1.000 integer 1 LSB
Read the result downward $.375_{10} = .011$

Hex-to-Decimal Conversion

$2AF_{16} = (2 \times 16^2) + (10 \times 16^1) + (15 \times 16^0)$

$= 512_{10} + 160_{10} + 15_{10}$

$= 687_{10}$

Decimal-to-Hex Conversion

423_{10} / 16 = 26 remainder 7 (Hex number will end with 7) : LSI

26_{10} / 16 = 1 remainder 10

1_{10} / 16 = 0 remainder 1 (Hex number will start with 1) : MSI

Read the result upward to give an answer of $423_{10} = 1A7_{16}$

Table 3-2

Hex	Binary	Decimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

Hex-to-Binary Conversion

$9F2_{16}$ = 9 F 2

= 1001 1111 0010

= 100111110010_2

Binary-to-Hex Conversion

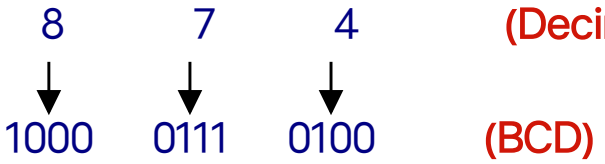
$11101001 10_2$ = 0011 1010 0110

3 A 6

= $3A6_{16}$

Binary-Coded-Decimal Code

- Each digit of a decimal number is represented by its binary equivalent



- Only the four bit binary numbers from 0000 through 1001 are used
- Comparison of BCD and Binary

$137_{10} = 10001001_2$ (Binary) - *require only 8 bits*

$137_{10} = 0001\ 0011\ 0111_{BCD}$ (BCD) - *require 12 bits*

Alphanumeric Representation

- Alphanumeric character set
  - » 10 decimal digits, 26 letters, special character($, +, =,⋯.)
- ASCII :
  - » Standard alphanumeric binary code.

- Complements
  - ***Complements*** are used in digital computers for simplifying the ***subtraction operation*** and for logical manipulation.
  - There are two types of complements for base r system
    - 1) r's complement 2) (r-1)'s complement
      - » Binary number : 2's or 1's complement
      - » Decimal number : 10's or 9's complement
  - (r-1)'s Complement
    - (r-1)'s Complement of N = $(r^n-1)-N$
      - » 9's complement of N=***546700***

        $(10^6-1)-546700= (1000000-1)-546700= 999999-546700$

        $= 453299$
      - » 1's complement of N=***101101***

        $(2^6-1)-101101= (1000000-1)-101101= 111111-101101$

        $= 010010$
  - r's Complement
    - r's Complement of N = r^n-N
      - » 10's complement of ***2389***= 7610+1= ***7611*** r^n-N
      - » 2's complement of ***1101100***= 0010011+1= ***0010100***

N : given number
r : base
n : digit number

546700(N) + 453299(9's com)
=999999

101101(N) + 010010(1's com)
=111111

* *r's Complement*
(r-1)'s Complement +1 =$(r^n-1)-N+1=$ r^n-N

Subtraction of Unsigned Numbers

(M-N), N≠0

- 1) M + (r^n-N)
- 2) M ≥ N : Discard end carry, Result = M-N
- 3) M < N : No end carry, Result = - r's complement of (N-M)

» *Decimal Example)*

M ≥ N *72532(M) - 13250(N) = 59282*

```
  72532
+ 86750  (10's complement of 13250)
 159282
```

Discard End Carry

Result = *59282*

M < N *13250(M) - 72532(N) = -59282*

```
  13250
+ 27468  (10's complement of 72532)
  40718
```

No End Carry

Result = -(10's complement of 40718)
= -(59281+1) = *-59282*

» *Binary Example)*

X ≥ Y *1010100(X) - 1000011(Y) = 0010001*

```
  1010100
+ 0111101  (2's complement of 1000011)
 10010001
```

Result = *0010001*

X < Y *1000011(X) - 1010100(Y) = -0010001*

```
  1000011
+ 0101100  (2's complement of 1010100)
  1101111
```

Result = -(2's complement of 1101111)
= -(0010000+1) = *-0010001*

Integer Representation

- Signed-magnitude representation
- Signed-1’s complement representation
- Signed-2’s complement representation

+14	-14
0 0001110 0001110	1
0 0001110 1110001	1
0 0001110 1110010	1

Arithmetic Addition

- Addition Rules of Ordinary Arithmetic
  - » The signs are *same* : sign= *common* sign, result= *add*
  - » The signs are *different* : sign= *larger* sign, result= *larger-smaller*
- Addition Rules of the signed 2's complement
  - » Add the two numbers including their sign bits
  - » Discard any carry out of the sign bit position

(-12) + (-13) = -25
(+12) + (+13) = +25

(+25) + (-37) = 37 - 25 = -12

*Addition Exam)

+ 6	00000110	- 6	11111010
+13	00001101	±13	00001101
+19	00010011	+ 7	00000111
+ 6	00000110	- 6	11111010
-13	11110011	-13	11110011
- 7	11111001	-19	11101101

Arithmetic Subtraction

- Subtraction is changed to an Addition
  - » (± A) - (+ B) = (± A) + (- B)
  - » (± A) - (- B) = (± A) + (+ B)

* Subtraction Exam) (- 6) - (- 13) = +7
11111010 - 11110011 = 11111010 + 2's comp of 11110011
= 11111010 + 00001101
= 1 00000111 = +7

Discard End Carry

Overflow

- Two numbers of n digits each are added and the sum occupies n+1 digits
- n + 1 bit cannot be accommodated in a register with a standard length of n bits.

Overflow

- An overflow may occur if the two numbers added are both positive or both negative
  - » When two unsigned numbers are added
    - an overflow is detected from the end carry out of the MSB position
  - » When two signed numbers are added
    - the MSB always represents the sign
      - *the sign bit is treated as part of the numbe*
      - *the end carry does not indicate an overflo*

*** Overflow Example)**

	out	*in*		*out*	*in*
carries	0	1	carries	1	0
+ 70	0	1000110	- 70	1	
0111010					
+ 80	0	1010000	- 80	1	
0110000					
+ 150	1	0010110	- 150	0	1101010

3-4 Floating-Point Representation

- The floating-point representation of a number
  - 1) Mantissa : signed, fixed-point number
  - 2) Exponent : position of binary(decimal) point

* Decimal + 6132.789

Fraction	*Exponent*
+0.6132789	+4

- Scientific notation : $m \times r^e$ (+0.6132789 x 10...)
  - *m* : mantissa, *r* : radix, *e* : exponent

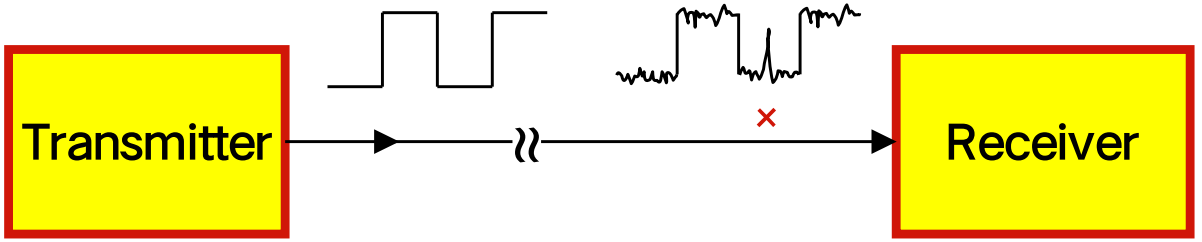
Fraction	*Exponent*
01001110	000100

- Example : $m \times 2^e = +(.1001110)_2 \times 2^{+4}$

- Normalization
  - Most significant digit of mantissa is *nonzero*

3-6 Error Detection Codes

- Binary information transmitted through some form of communication medium is subject to external noise



- Parity Bit
  - An extra bit included with a binary message to make the total number of 1's either odd or even.
- Even-parity method
  - The value of the parity bit is chosen so that the total number of 1s *(including the parity bit)* is an ***even*** number

1 1 0 0 0 0 1 1

Added parity bit

- Odd-parity method
  - Exactly the same way except that the total number of 1s is an ***odd*** number

1 1 0 0 0 0 0 1

Added parity bit