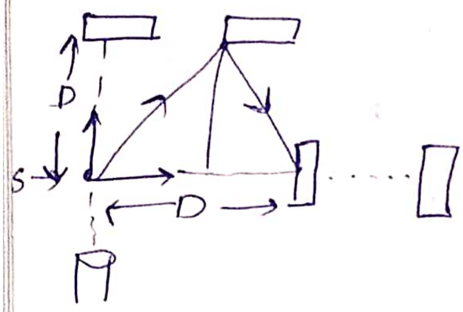
S (x, y, z, t) S' (x', y', z', t)

$x' = x - vt$. $y' = y$ $z' = z$ $t' = t$.

$\frac{dx'^2}{dt^2} = \frac{dx^2}{dt}$ $a' = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$.

$a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$.

Michelson Morley experiment.



$t_2 = \frac{D}{c-v} + \frac{D}{c+v}$

$t_2 = \frac{2Dc}{c^2 - v^2}$

$c^2 t_1^2 = v^2 t_1^2 + D^2$.

$T_1 = \frac{2D_1}{c(1 - \frac{v^2}{c^2})^{\frac{1}{2}}}$.

From Lorentz.

$t_1 = \frac{2L_1}{c(1 - v^2/c^2)}$ $t_2 = \frac{2L_2}{c(1 - v^2/c^2)}$

$= \frac{2L_2(\sqrt{1 - v^2/c^2})}{c(1 - v^2/c^2)} = \frac{2L_2}{c\sqrt{1 - v^2/c^2}}$

$c' = c\sqrt{1 - v^2/c^2}$

Two postulates proposed by Einstein (based on MMExp).

→ The principle of relativity
it means "All inertial frames are equivalent for the performance all physical experiments and for laws".
In other words.
one can say there is no ether or any

Galilean transformation.

$$\frac{dx'}{dt} = \frac{dx}{dt} - v \qquad \frac{dz'}{dt} = \frac{dz}{dt}$$

$$\frac{dy'}{dt} = \frac{dy}{dt}$$

→ The universal speed of light

Lorentz Transformation

S S' (x, y, z, t) (x', y', z', t') $\xrightarrow{v}$

According to Galilean

$$x' = x - vt$$

But acc. to Lorentz

$$x' \propto x - vt \Rightarrow x' = k(x - vt)$$

$$x = k'(x' + vt')$$

$$x' = k\left[k'(x' + vt') - vt\right]$$

$$t' = kt + \left(\frac{1 - kk'}{k'v}\right)x$$

From second postulate

$$x = ct \quad , x' = ct'$$

$$k(x - vt) = c\left[kt + \left(\frac{1-kk'}{k'v}\right)x\right]$$

$$x = ct\left[\frac{1 + v/c}{1 - \left\{\frac{1}{kk'} - 1\right\} c/v}\right]$$

$$\frac{1 + v/c}{1 - \left\{\frac{1}{kk'} - 1\right\} c/v} = 1$$

$$\sqrt{kk'} = \frac{1}{\sqrt{1 - v^2/c^2}} = \sqrt{\frac{1}{\sqrt{1 - v^2/c^2}} \cdot \frac{1}{\sqrt{1 - v^2/c^2}}}$$

$$k = k' = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$t' = kt + \left(\frac{1 - kk'}{k'v}\right)x.$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y \quad , \quad z' = z$$

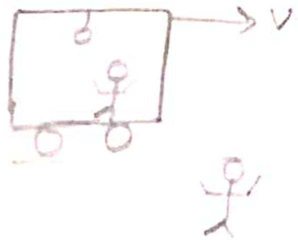
$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

Geometrical consequence of Einstein's postulates or LT.

Three

(i) The relativity of simultaneity.

"Two events that are simultaneous in one inertial frame are not simultaneous in another".



$t_1, t_2 \Rightarrow$ initial time that is equal

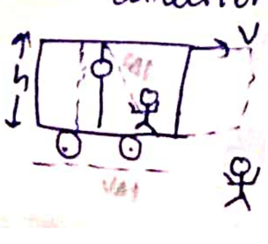
$$t_1' = \frac{t_1 - vx_1/c}{\sqrt{1 - v^2/c^2}}$$

$$t_2' = \frac{t_2 - vx_2/c}{\sqrt{1 - v^2/c^2}}$$

$$t_2' - t_1' = \frac{t_2 - vx_2/c^2 - t_1 + vx_1/c^2}{\sqrt{1 - v^2/c^2}}$$

Absolute space: It is a space that is affecting us, but is not getting affected from our motion.

(ii) Time dilation



$$\Delta t' = \frac{h}{c}$$

$$\Delta t = \frac{\sqrt{h^2 + v^2 \Delta t^2}}{c}$$

$$\Delta t^2 = \frac{h^2 + v^2 \Delta t^2}{c^2}$$

$$\Delta t = \frac{h}{c} \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

$$v = 0.5c$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - 0.25}} = \frac{\Delta t'}{\sqrt{0.75}}$$

S frame

$\Delta t' < \Delta t$ ← S' frame

using Lorentz transformation.

$t_1' \quad t_2'$

$t_0 = t_2' - t_1'$

$$t_1 = \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$t_2 = \frac{t_2' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

$$t_2 - t_1 = \frac{t_2' + vx'/c^2 - t_1' - vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

$$t_2 - t_1 = \frac{t_2' - t_1'}{\sqrt{1 - v^2/c^2}} = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$t_2 - t_1 = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

S frame

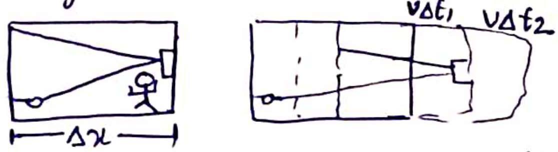
$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}.$$

S' frame

$\Delta t' < \Delta t$ ← S frame

Moving clock runs slow.

(iii) Length contraction.



$$\Delta t' = \sqrt{1 - v^2/c^2}\, \Delta t.$$

$$\Delta t' = \frac{2\Delta x'}{c}.$$

$$\Delta t_1 = \frac{\Delta x + v\Delta t_1}{c} \Rightarrow \Delta t_1 = \frac{\Delta x}{c - v}$$

$$\Delta t_2 = \frac{\Delta x - v\Delta t_2}{c}$$

$$\Rightarrow \Delta t_2 = \frac{\Delta x}{c + v}$$

$$\Delta t = \Delta t_1 + \Delta t_2$$

$$\Delta t = \frac{\Delta x}{c - v} + \frac{\Delta x}{c + v} = \frac{2\Delta x}{c(1 - v^2/c^2)}$$

$$\frac{2\Delta x'}{c} = \sqrt{1-v^2/c^2}\,\frac{2\Delta x}{c(1-v^2/c^2)}$$

$$\Delta x' = \frac{\Delta x}{\sqrt{1-v^2/c^2}}$$

(iii) Length contraction (through LT)

S S' x_1' —— x_2'.

$x_2' - x_1' = l.$

$$x_1' = \frac{x_1 - vt_1}{\sqrt{1-v^2/c^2}} \qquad x_2' = \frac{x_2 - vt_2}{\sqrt{1-v^2/c^2}}$$

$$x_2' - x_1' = \frac{(x_2 - x_1) - v\underbrace{(t_2 - t_1)}_{\to 0}}{\sqrt{1-v^2/c^2}}$$

$\to 0$ becoz measuring at same time.

$$l' = \frac{l_0}{\sqrt{1-v^2/c^2}}$$

The geometric representation of space time.

According to Lorentz transform.

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-v^2/c^2}}$$

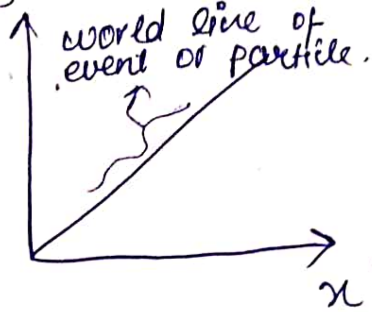
$$ct' = \frac{ct - (v/c)x}{\sqrt{1-v^2/c^2}} \qquad w = ct. \quad \beta = v/c.$$

$$w' = \frac{w - \beta x}{\sqrt{1-\beta^2}} \qquad w = \frac{w' + \beta x'}{\sqrt{1-\beta^2}}$$

$$x' = \frac{x - vt}{\sqrt{1-v^2/c^2}} = \frac{x - (v/c)(ct)}{\sqrt{1-(v/c)^2}}$$

$$= \frac{x - \beta w}{\sqrt{1-\beta^2}}$$

$$x' = \frac{x - \beta w}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + \beta w'}{\sqrt{1-\beta^2}}$$



classical mechanics by Pathari...

$x = \beta w \quad \beta = v/c = 1$

$$\tan\theta = \frac{dx}{dw} = \frac{dx}{c\,dt} = \frac{u}{c}.$$

geometric representation of space-time.

S S' $\to$ v.

$$x' = \frac{x - vt}{\sqrt{1-v^2/c^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-v^2/c^2}}.$$

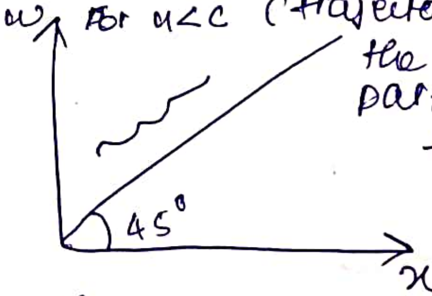
$$x' = \frac{x - \cancel{(v/c)}\beta w}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + \beta w'}{\sqrt{1-\beta^2}}$$

$$w' = \frac{w - \beta x}{\sqrt{1-\beta^2}} \qquad w = \frac{w' + \beta x'}{\sqrt{1-\beta^2}}$$

$v \simeq c.$

$\underbrace{w'\sqrt{1-\beta^2}}_{0} = w - \beta x. \Rightarrow w = \beta x \Rightarrow w = x$

For $u < c$ (trajectory will lie above the world line of particle).



$$\tan\theta = \frac{dx}{dw} = \frac{1}{c}\frac{dx}{dt} = \frac{u}{c}$$

$u < c$

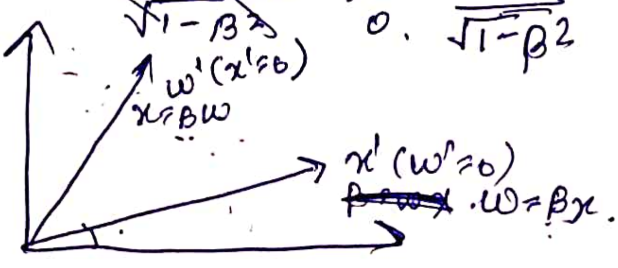
at x, $w = 0$.

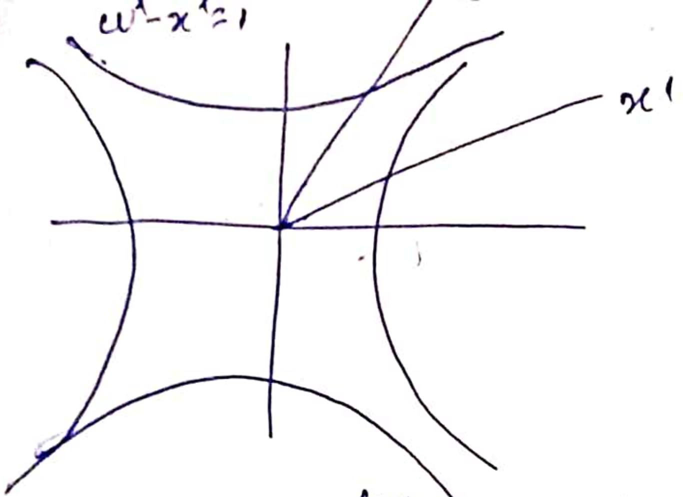
Hence at x', $w' = 0$,

$$\underbrace{w'}_{0} = \frac{w - \beta x}{\sqrt{1-\beta^2}} \Rightarrow w = \beta x.$$

at w', $x' = 0$.

$$\cancel{w' = \frac{w - \beta x}{\sqrt{1-\beta^2}}} \qquad \underbrace{x'}_{0} = \frac{x - \beta w}{\sqrt{1-\beta^2}} \Rightarrow x = \beta w$$





Equation for ~~parabola~~ hyperbola. $x^2-w^2=1$

$x^2-w^2=1$

$(\beta/w)^2-w^2=1.$

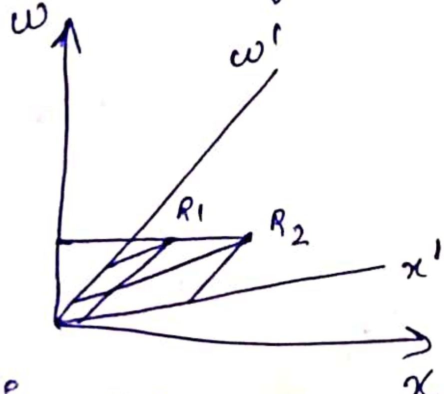
$w=\frac{\beta}{\sqrt{1-\beta^2}}$ $\quad x=\frac{1}{\sqrt{1-\beta^2}}$

$w^2-x^2=1$

$w^2-\beta^2w^2=1$

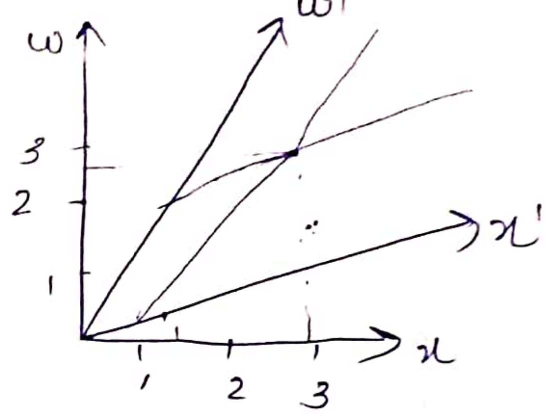
$w=\frac{1}{\sqrt{1-\beta^2}}$ $\quad x=\frac{\beta}{\sqrt{1-\beta^2}}$

Simultaneity.



In one frame if the two objects are happening at one time, then in s' frame it will not occur at the same time.

Unit cell in s' frame is always larger than that of s frame.

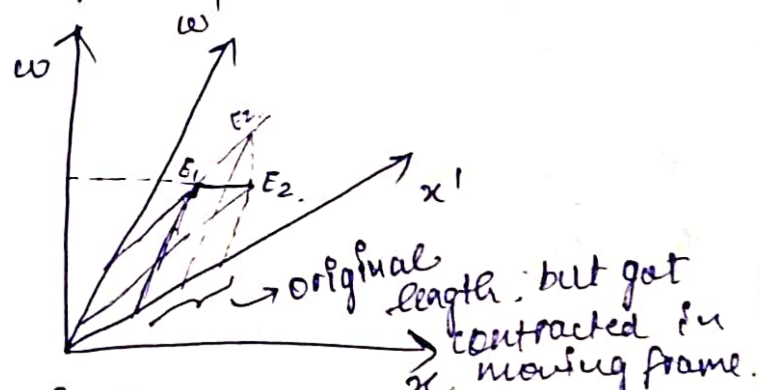


$x=3$
$w=2.1$

$$x'=\frac{x-\beta w}{\sqrt{1-\beta^2}}$$

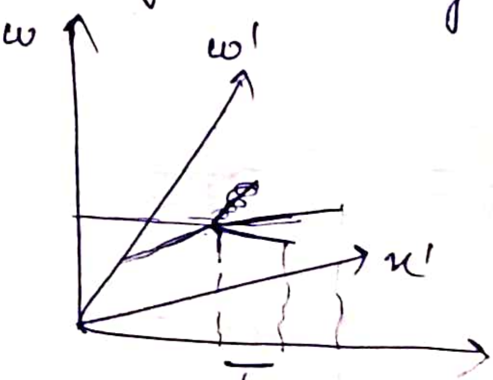
$$w'=\frac{w-\beta x}{\sqrt{1-\beta^2}}.$$

Length contraction.



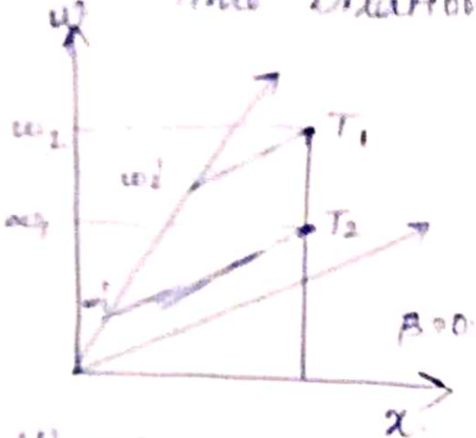
Just to be sure E_1 & E_2 are happening at the same time.

Setting in moving frame.



length got contracted.

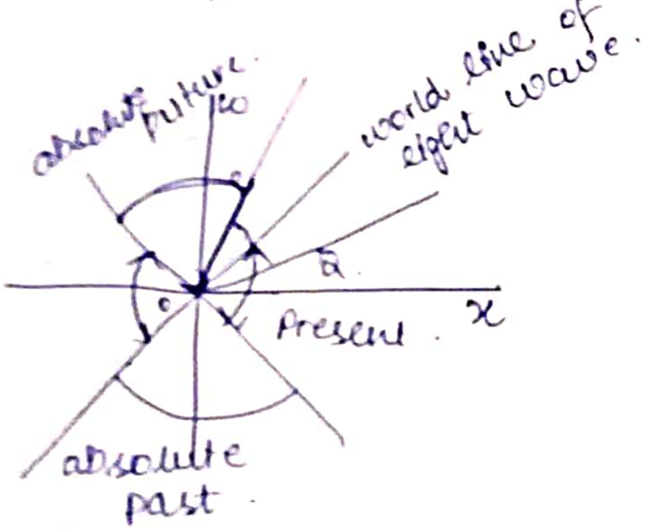
Time Dilation



$w_1 = 2 \qquad w_2 = 3$

$$w_1' = \frac{w_1 - \beta x}{\sqrt{1-\beta^2}} = \frac{2 - 0.5\times 2.1}{\sqrt{1-(0.5)^2}} = 1.09$$

$$w_2' = \frac{w_2 - \beta x}{\sqrt{1-\beta^2}} = 2.25$$



$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

↑ proper time.

$$(\tau_2 - \tau_1)^2 = (t_2 - t_1) - \frac{1}{c^2}[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]$$

$(t_1, x_1, y_1, z_1) = (0,0,0,0)$

$(t_2, x_2, y_2, z_2) = (t, x, y, z)$

$$\tau^2 = t^2 - \frac{x^2+y^2+z^2}{c^2} \approx t^2 - \frac{x^2}{c^2}$$

$c^2\tau^2 = c^2t^2 - x^2$

$c^2\tau^2 = w^2 - x^2$

For $w'^2 - x'^2$

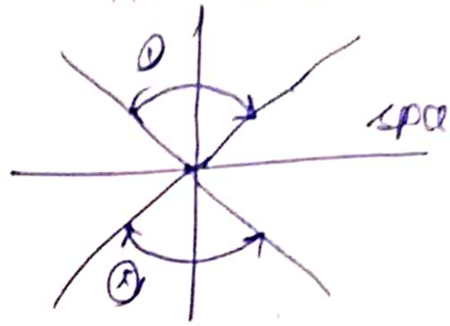
Resnick

$$\left[\frac{w - \beta x}{\sqrt{1-\beta^2}}\right]^2 - \left[\frac{x - \beta w}{\sqrt{1-\beta^2}}\right]^2$$

$\Rightarrow w^2 - x^2$

The only thing which is invariant is $c^2\tau^2$.

Time like



For regions ① & ②.

$c^2\tau^2 = c^2t^2 - x^2$

$-c^2\tau^2 = \sigma^2 \Rightarrow -c^2\tau^2 = -c^2t^2 + x^2$

$\sigma^2 = x^2 - w^2$

For regions ① & ②. $x < w$.

$\sigma^2 < 0$ Time like.

$c^2\tau^2 > 0 \quad c^2\tau^2 = a \quad c\tau = \sqrt{a} \Rightarrow \tau = \frac{\sqrt{a}}{c}$

For region ③.

$x > w$

$\sigma^2 > 0$

$c^2\tau^2 < 0$

$c^2\tau^2 = -a$

$c\tau = \sqrt{-a} = i\sqrt{a}$

$$\boxed{t = \frac{i\sqrt{a}}{c}}$$

$\sigma^2 = x^2 - w^2 = x'^2 - w'^2$ $\to 0$

$\sigma^2 = x'^2$.

Four vectors.

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$y' = y \quad z' = z.$

$x^0 = ct \qquad \beta = v/c$

$x^0 = ct,\ x = x^1,\ y = x^2,\ z = x^3$

$\bar{x}^0 = ct'\ \bar{x} = \bar{x}^1,\ \bar{y} = \bar{x}^2,\ \bar{z} = \bar{x}^3.$

$$ct' = \frac{ct - (v/c)x}{\sqrt{1 - \beta^2}}$$

$\bar{x}^0 = \gamma(x^0 - \beta x^1)$

$\bar{x}^1 = \gamma(x^1 - \beta x^0)$

$\bar{x}^2 = x^2$

$\bar{x}^3 = x^3.$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$s' = (\bar{x}^0, \bar{x}^1, \bar{x}^2, \bar{x}^3)$

$s = (x^0, x^1, x^2, x^3)$

Velocity addition in Relativity.
From galilean transformation.

S, S' → v u'.

$\bar{u} = \bar{u}' + v$

$u_x = u_x' + v$

$u_y = u_y'$

$u_z = u_z'$.

From Lorentz transformation.

S, S' → v ·t ·t'

$x = ut$

$x' = u't'$.

$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$ $t' = \frac{t - v/c^2 x}{\sqrt{1 - v^2/c^2}}$

$x - vt = x'\sqrt{1 - v^2/c^2}$

$= u't'\sqrt{1 - v^2/c^2}$

$= u'\sqrt{1 - v^2/c^2}\,\frac{(t - v/c^2 x)}{\sqrt{1 - v^2/c^2}}$

$x - vt = u'(t - v/c^2 x)$

$x(1 + \frac{u'v}{c^2}) = (u' + v)t$.

$u = \frac{u' + v}{1 + u'v/c^2}$

Speed of light independent from different frames can be proven by this.

$u' = c$ $v = c$

$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$

$v = 0.67c$

$\Delta u_y' = \frac{y_2' - y_1'}{t_2' - t_1'}$

$t_2' - t_1' = \frac{(t_2 - t_1) - v/c^2(x_2 - x_1)}{\sqrt{1 - v^2/c^2}}$

$\Delta u_y' = \frac{\Delta y'}{\Delta t'}$

$= \frac{\Delta y(\sqrt{1 - v^2/c^2})}{\Delta t(1 - v/c^2 \frac{\Delta x}{\Delta t})}$

$= \frac{(\Delta y/\Delta t)(\sqrt{1 - v^2/c^2})}{(1 - v/c^2 \frac{\Delta x}{\Delta t})}$

$\Delta u_y' = \frac{\Delta u_y \sqrt{1 - v^2/c^2}}{1 - v/c^2 \Delta u_x}$

$u_y' = \frac{u_y \sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$ — ①

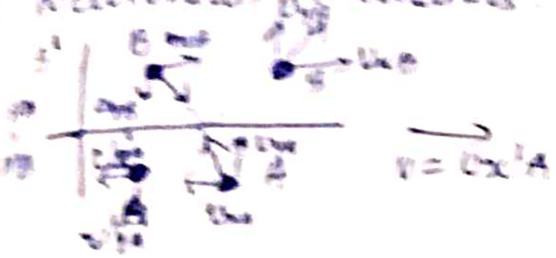
$u_z' = \frac{u_z \sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$ — ②

Ex. Two electrons are ejected in opposite directions from radioactive ~~decay~~ material at rest in the laboratory. Each electron has a speed 0.67c as measured by a laboratory observer. What is the speed of one electron as measured from the other according to relativity?

$u_x' = \frac{0.67c + 0.67c}{(1 + (0.67)(0.67)c^2/c^2)} = 0.92c$.

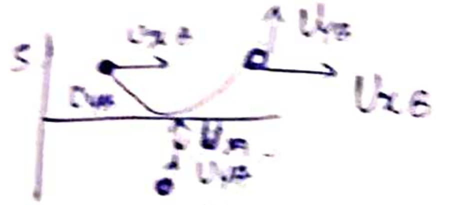
$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$

Newtonian Mechanics



$u'_{yA} = -u'_{yA} = u'_{yB} = -u'_{yB}$

$u'_{xA} = u'_{xA} = -u'_{xB} = -u'_{xB}$



$u_{xA} = 0$

→ From momentum conservation

$m_A u_y - m_A u_{yA}$

$2 m_A u_{yA}$

$-2 m_A u_{yA} = 2 m_B u_{yB}$

$u_{yA} = u_{yB}$

$$u'_{yA} = \frac{u_{yA}\sqrt{1-v^2/c^2}}{1-u_{xA}v/c^2} = u_{yA}\sqrt{1-v^2/c^2}$$

$u_{xA} = 0$ → (Because of S' frame)

$$u'_{yB} = \frac{u_{yB}\sqrt{1-v^2/c^2}}{1-\frac{u_{xB}v}{c^2}}$$

Free to choose inertial frame

$$u_{yA}\sqrt{1-v^2/c^2} = \frac{u_{yB}\sqrt{1-v^2/c^2}}{1-u_{xB}v/c^2}$$

$$m_A = \frac{m_B u_{yB}\sqrt{1-u_{xB}v/c^2}}{u_{yB}}$$

$$m_B = \frac{m_A}{\sqrt{1-u_{xB}v/c^2}}$$

$$m_T = \frac{m_0}{\sqrt{1-u^2/c^2}}$$

$$u_{yA} = \frac{u_{yB}}{1-\frac{u_{xB}v}{c^2}}$$

2m →

$$P_x = \frac{m_0 u_x}{\sqrt{1-u^2/c^2}} \qquad P_y = \frac{m_0 u_y}{\sqrt{1-u^2/c^2}}$$

$$P_z = \frac{m_0 u_z}{\sqrt{1-u^2/c^2}}.$$

Kinematics of the single particle.

$* \; F = m\frac{du}{dt}$

$$F = \frac{d}{dt}(mu) = \frac{d}{dt}\left(\frac{m_0 u}{\sqrt{1-u^2/c^2}}\right)$$

$* \; K.E = \int_0^u F\cdot dl.$

$$= \int_0^u m_0 u\cdot du \Rightarrow \frac{m_0u^2}{2}\Big|_0^u = \frac{m_0u^2}{2}.$$

$$K.E = \int_0^u \vec{F}\cdot d\vec{l} = \int_0^u \frac{d}{dt}(mu)\cdot dl$$

$$= \int_0^u \frac{d}{dt}(mu)\cdot dx$$

$$= \int_0^u u\, d(mu)$$

$$= \int_0^u u(m\,du + u\,dm)$$

$$= \int_0^u c^2 dm$$

$$K = c^2 m\Big|_{m_0}^{m}$$

$$\boxed{K = c^2m - m_0c^2}$$

$$m = \frac{m_0}{\sqrt{1-u^2/c^2}}$$

$$m^2 = \frac{m_0^2c^2}{c^2-u^2}.$$

$$m^2c^2 - m^2u^2 = m_0^2c^2$$

$$2m\,dm\,c^2 - 2m\,dm\,u^2 - 2m^2u\,du = 0$$

$$u^2 dm + mu\,du = c^2 dm.$$

$$K = \frac{m_0 c^2}{\sqrt{1-u^2/c^2}} - m_0 c^2$$

$$K = m_0 c^2 \left[1 - \frac{u^2}{c^2}\right]^{-1/2} - m_0 c^2$$

$$= m_0 c^2 \left[1 + \tfrac{1}{2} u^2/c^2 - 1\right]$$

$$= \tfrac{1}{2} m_0 u^2.$$

$$E = c\sqrt{p^2 + m_0^2 c^2}$$

$$p = mc = \frac{m_0 c}{\sqrt{1-u^2/c^2}}.$$

$$K = mc^2 - m_0 c^2$$

$$= \frac{m_0 c^2}{\sqrt{1-u^2/c^2}} - m_0 c^2$$

$$(K + m_0 c^2)^2 = \frac{m_0^2 c^4}{1-u^2/c^2} + p^2c^2 - p^2c^2$$

$$(K + m_0 c^2)^2 = p^2 c^2 + \frac{m_0^2 c^4}{1-u^2/c^2} - \frac{m_0^2 u^2 c^2}{1-u^2/c^2}$$

$$(K + m_0 c^2)^2 = p^2 c^2 + \frac{m_0^2 c^4 (1 - u^2/c^2)}{1-u^2/c^2}.$$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$E^2 = c^2 [p^2 + m_0^2 c^2]$$

$$E = c\sqrt{p^2 + m_0^2 c^2}$$

$$F = \frac{d\vec{p}}{dt}$$

$$\vec{F} = \frac{d}{dt}(mu)$$

$$= \frac{m\,du}{dt} + \frac{u\,dm}{dt}$$

$$= \frac{m\,d\vec{u}}{dt} + \vec{u}\,\frac{d}{dt}\left(\frac{K + mc^2}{c^2}\right)$$

$$\vec{F} = \frac{m\,d\vec{u}}{dt} + \frac{\vec{u}}{c^2}\frac{dK}{dt} \quad \text{— (1)}$$

$$E = K + m_0 c^2.$$

$$mc^2 = K + m_0 c^2$$

$$m = \frac{K + m_0 c^2}{c^2}.$$

$$K = F.dl$$

$$\frac{dK}{dt} = F\cdot\frac{dl}{dt} \Rightarrow F\cdot\vec{u}.$$

From (1)

$$\vec{F} = \frac{m\,d\vec{u}}{dt} + \frac{\vec{u}}{c^2}(\vec{F}\cdot\vec{u})$$

$$\vec{a} = \frac{\vec{F}}{m} - \frac{\vec{u}}{mc^2}(\vec{F}\cdot\vec{u})$$

(i) $a_{\parallel} \sim \vec{F}\cdot\hat{u}$.

$$\vec{a}\cdot\hat{u} = \frac{\vec{F}\cdot\hat{u}}{m} - \frac{\vec{u}\cdot\hat{u}\,(\vec{F}\cdot\vec{u})}{mc^2}$$

$$a_{\parallel} = \frac{F_{\parallel}}{m} - \frac{u^2 F_{\parallel}}{mc^2}$$

$$a_{\parallel} = \frac{F_{\parallel}}{m_0}(1 - u^2/c^2)^{3/2}.$$

$$F_{\parallel} = \frac{m_0\, a_{\parallel}}{(1-u^2/c^2)^{3/2}}$$

↳ longitudinal mass.

(ii) $a_{\perp} \rightarrow \vec{F}\cdot\hat{u} = 0$.

$$a_{\perp} = \frac{\vec{F_{\perp}}}{m}$$

$$F_{\perp} = \frac{m_0\, a_{\perp}}{(1-u^2/c^2)}.$$

↳ Transverse mass.

Transformation properties of momentum and energy.

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u_y' = \frac{u_y\sqrt{1-v^2/c^2}}{1 - u_x v/c^2}$$

$$u_z' = \frac{u_z\sqrt{1-v^2/c^2}}{1 - u_x v/c^2}$$

$$c^2 - u^2 = c^2 - u_x^2 - u_y^2 - u_z^2$$
$$= c^2 - \left[\left(\frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}\right)^2 + (\ )^2 + (\ )^2\right]$$

$$c^2 - u^2 = \frac{c^2(c^2-u'^2)(c^2-v^2)}{(c^2 + u_x' v)^2}$$

$$c^2(1 - u^2/c^2) = \frac{c^6\left(1-\frac{u'^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{c^4(1 + u_x' v/c^2)}$$

$$\frac{1}{\sqrt{1-u^2/c^2}} = \frac{1 + u_x' v/c^2}{\sqrt{\left(1-\frac{u'^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}}$$

Momentum & Energy in S frame.

$$P_x = \frac{m_0 u_x}{\sqrt{1-u^2/c^2}} \quad P_y = \frac{m_0 u_y}{\sqrt{1-u^2/c^2}} \quad P_z = \frac{m_0 u_z}{\sqrt{1-u^2/c^2}}$$

$$E = \frac{m_0 c^2}{\sqrt{1-u^2/c^2}}$$

In s' frame

$$P_x' = \frac{m_0 u_x'}{\sqrt{1-u'^2/c^2}} \quad P_y' = \frac{m_0 u_y'}{\sqrt{1-u'^2/c^2}} \quad P_z' = \frac{m_0 u_z'}{\sqrt{1-u'^2/c^2}}$$

$$E = \frac{m_0 c^2}{\sqrt{1-u'^2/c^2}}$$

$$P_x = \frac{m_0 u_x(1 + u_x' v/c^2)}{\sqrt{1-u'^2/c^2}\sqrt{1-v^2/c^2}}$$

$$= \frac{1}{\sqrt{1-v^2/c^2}}\left[\frac{m_0 u_x}{\sqrt{1-\frac{u'^2}{c^2}}} + \frac{m_0 u_x' v/c^2}{\sqrt{1-\frac{u'^2}{c^2}}}\right]$$

$$=$$

$$P_x = \frac{1}{\sqrt{1-v^2/c^2}}\left[P_x' + \frac{E'v}{c^2}\right]$$

$$u_x = \frac{u_x' + v}{1 + u_x' v/c^2}.$$

$$P_x = \frac{m_0 u_x}{\sqrt{1-u^2/c^2}} = m_0 u_x \left\{\frac{1 + u_x' v/c^2}{\sqrt{1-u'^2/c^2}\sqrt{1-v^2/c^2}}\right\}$$

$$m_0 \frac{u_x' + v}{\cancel{1 + u_x' v/c^2}}\left\{\frac{\cancel{1 + u_x' v/c^2}}{\sqrt{1-v^2/c^2}\sqrt{1-u'^2/c^2}}\right\}$$

$$= \frac{m_0}{\sqrt{1-v^2/c^2}}\left\{\frac{m_0 u_x'}{\sqrt{1-u'^2/c^2}} + \frac{m_0 v\, c^2/c^2}{\sqrt{1-u'^2/c^2}}\right\}$$

$$P_x = \frac{1}{\sqrt{1-v^2/c^2}}\left\{P_x' + E' v/c^2\right\}$$

$$u_y = \frac{u_y'\sqrt{1-v^2/c^2}}{1 + u_x' v/c^2}.$$

$$P_y = \frac{m_0 u_y}{\sqrt{1-u^2/c^2}} = \frac{m_0}{\sqrt{1-u^2/c^2}}\,\frac{u_y'\sqrt{1-v^2/c^2}}{(1 + u_x' v/c^2)}$$

$$= \frac{m_0(1 + \cancel{u_x' v/c^2})}{\sqrt{1-u'^2/c^2}\,\cancel{\sqrt{1-v^2/c^2}}}\;\frac{u_y'\cancel{\sqrt{1-v^2/c^2}}}{\cancel{(1 + u_x' v/c^2)}}$$

$$\boxed{P_y = P_y'}$$

W.C.V Rossenn Contemporary Physics.

$$E = \frac{m_0 c^2}{\sqrt{1-u^2/c^2}}$$

$$= \frac{m_0 c^2 (1 + u'_x v/c^2)}{\sqrt{1-\frac{u'^2}{c^2}}\sqrt{1-v^2/c^2}}$$

$$= \frac{1}{\sqrt{1-v^2/c^2}}\left[\frac{m_0 c^2}{\sqrt{1-u'^2/c^2}} + \frac{u'_x v m_0}{\sqrt{1-u'^2/c^2}}\right]$$

$$= \frac{1}{\sqrt{1-v^2/c^2}}\left[E' + R' v\right]$$

$$= \frac{1}{\sqrt{1-v^2/c^2}}\left[E' + R'_x v\right]$$

Summary.

$$p'_x = \frac{p_x - Ev/c^2}{\sqrt{1-v^2/c^2}}$$

$$p'_y = p_y$$

$$p'_z = p_z$$

$$E' = \frac{E - vR}{\sqrt{1-v^2/c^2}}$$

$$P^0, P^1, P^2, P^3$$

$$(m, p_x, p_y, p_z) \simeq (ct, x, y, z)$$

For Force.

$S \rightarrow$ frame.

$$F_x = \frac{d}{dt}(m u_x) \; ; \; F_y = \frac{d}{dt}(m u_y)$$

$$F_z = \frac{d}{dt}(m u_z)$$

$S' \longrightarrow$ frame.

$$F'_x = \frac{d}{dt'}(m' u'_x) \qquad F'_y = \frac{d}{dt'}(m' u'_y)$$

$$F_x = \frac{d}{dt}(m u_x) = \frac{d}{dt'}(m u_x)\frac{dt'}{dt}$$

$$m' = \frac{m - p_x v/c^2}{\sqrt{1-v^2/c^2}} \qquad t' = \frac{t - vx/c^2}{\sqrt{1-v^2/c^2}}$$

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2}.$$

$$F_x = F'_x + \frac{u'_y v}{(c^2 + u'_x v)} F'_y + \frac{u'_z v}{(c^2 + u'_x v)} F'_z$$

$$F_y = \frac{\sqrt{1-v^2/c^2}}{1 + u'_x v/c^2} F'_y \qquad F_z = \frac{\sqrt{1-v^2/c^2}}{1 + u'_x v/c^2} F'_z.$$