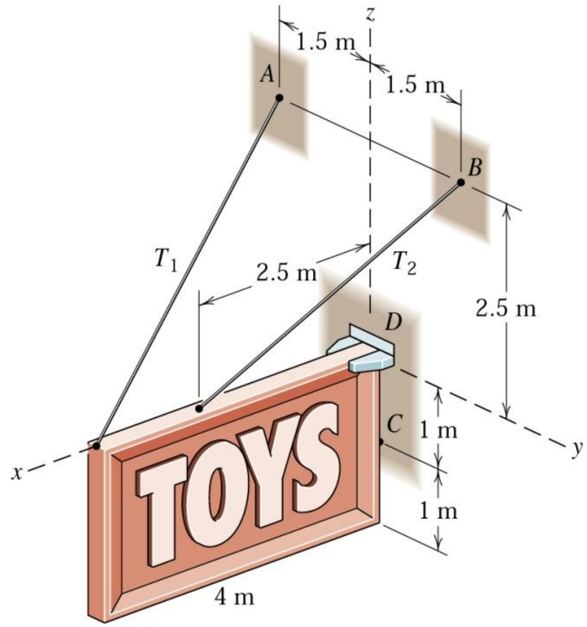
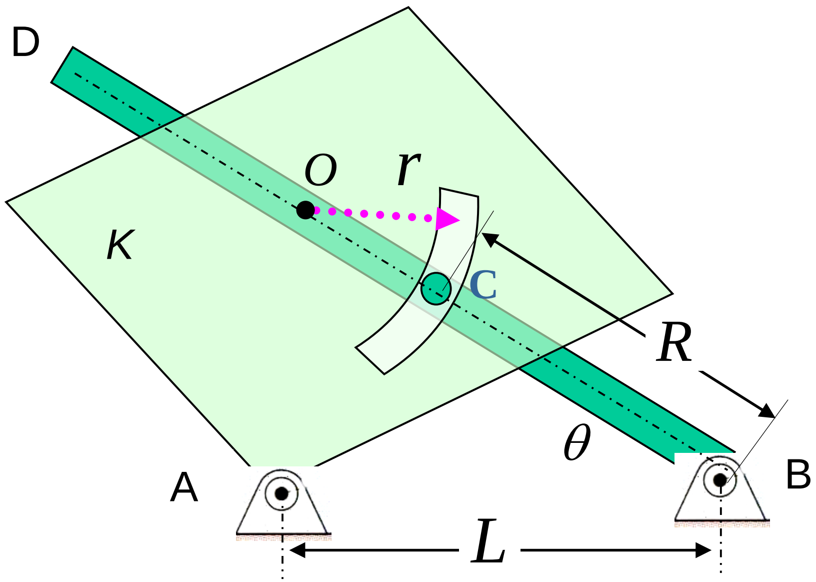
AM 108 Engineering Mechanics





Course Syllabus

Engineering Mechanics

AM 108 S1
AM 108 S2

	L	T	P	Credit
Scheme	3	0	2	04

- **INTRODUCTION TO FORCES/EQUILIBRIUM OF RIGID BODY** **(08 Hours)**
  - Scalar and vector, system of forces, resultant force
  - Statics of particle. Free-body diagram. Equilibrium of particle in two dimensions.
  - Resultant of three or more concurrent forces, Resolution of a force into components. Rectangular components of a force. Resultant by rectangular components.
  - Concurrent force system in space: Resolution of a force into rectangular components in space.
  - Coplanar Non-Concurrent Force Systems, Moments about Point and Axis. Equilibrium of Non-coplanar Non-concurrent Forces

- **CENTROID AND MOMENT OF INERTIA** **(08 Hours)**
  - Distributed forces: Centroid and centre of gravity. Determination of centroid of lines and areas using integral technique.
  - Determination of centroid of composite wires and areas.
  - Centroid of volumes. Theorems of Pappus - Guldinus and its applications.
  - Second moment of areas. Definition of moment of inertia. Determination of moment of area by integration.
  - Parallel axis theorem for Moment of Inertia. MI of composite area. Concept of Mass moment of inertia of body.

Course Syllabus

- **TRUSS** **(06 Hours)**
  - Types of structure in Engineering. Trusses and beams: definition, stability and determinacy.
  - Determination of reactions at supports for planar trusses. Basic assumption for analysis of trusses. Procedures for analysis of trusses.
  - Analysis of plane trusses by method of joint. Concept of zero force member. Analysis of plane trusses by method of section.

- **BEAMS AND CABLES** **(06 Hours)**
  - ***Beams***
    Definitions, types of beam, types of loading, types of support. Determination of reactions for simply supported, overhanging beams and compound beam.
  - ***Cables***
    Cables with Concentrated Loads
- **FRICTION** **(05 Hours)**
  - The Law of Dry Friction. Coefficient of Friction, Angle of Friction.
  - Analysis of systems involving dry frictions such as ladder spheres etc.
  - Belt Friction, Analysis of flat belt, wedge friction.

- **KINETICS OF PARTICLES** **(08 Hours)**
  - Force and acceleration. Newton's laws of motion. D'Alembert's principle.
  - Dependent motion of particles. Analysis for dependent motion of particles.
  - Impulse and Momentum: Concept, Definition, Principle of linear momentum and impulse
  - Work Energy Principle.

Course Syllabus

- **VIBRATIONS** **(03 Hours)**
  - Definitions, Equation of motion for single degree of freedom.
  - Introduction to free and forced vibrations.
  - Procedure for analysis of system involving free and forced vibrations.
  - Example on free vibration.
  - Example on forced vibration.

(Total Lecture Hours: 42)

PRACTICAL:

1. Plane force Polygone
2. Forces in space
3. Simple Plane roof truss
4. Coplanar Parallel foces
5. “E” by searle’s apparatus
6. Belt Friction
7. Static Surface Friction
8. Gravitational acceleration
9. Mass M.I. of flywheel

Course Syllabus

REFERENCES:

1. Beer, F.P. and Johnston, E.R., Vector mechanics for engineers: Statics and Dynamics, Tata McGraw-Hill, New Delhi.
2. Desai,J.A. and Mistry, B.B., Engineering Mechanics: Statics and Dynamics, Popular Prakashan, Surat.
3. Hibbeler, R.C., Engineering Mechanics: Statics and Dynamics, Prentice Hall of India, New Delhi.
4. Meriam, J.L. and Kraige, L.G., Engineering Mechanics: Statics and Dynamics, John Wiley and sons, New York.
5. Rajsekaran s, Engineering Mechanics: Statics and Dynamics, Vikas Publication, New Delhi.
6. Shah H. J. and Junarkar S. B., Applied Mechanics, Charotar publication, Anand.
7. Bhavikatti S. S. and Rajashekarappa KG., Engineering Mechanics, Wiley 'Eastern Ltd

2 Class Test + 2 Assignments

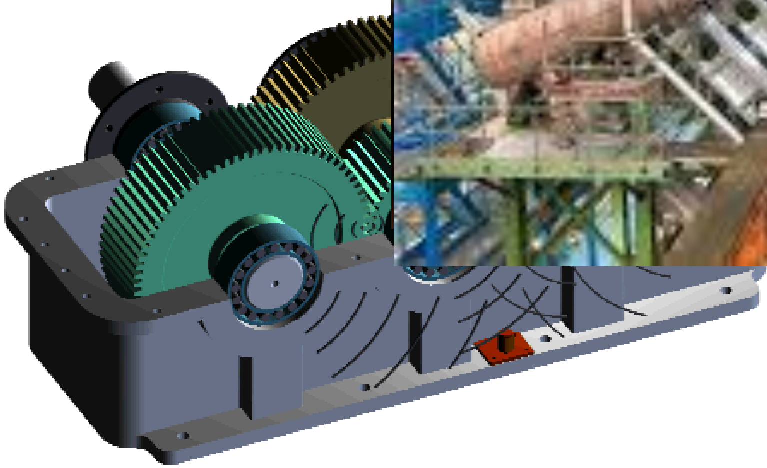
Mid Sem Exam

End Sem Exam

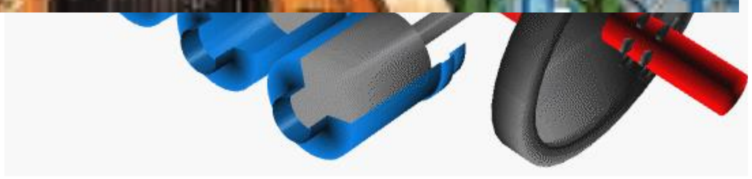
Mechanical Vibrations

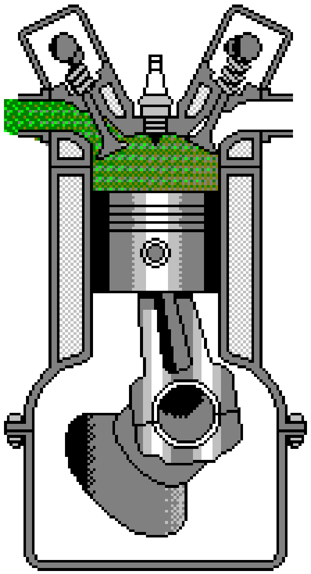
Centrifugal Pump
on Base Plate

Tacoma Bridge Collapse



Parallel shafts gear box





Cylinder in-line engine

Mechanical Vibrations

- *Mechanical vibration* is the motion of a particle or body which oscillates about a position of equilibrium. Most vibrations in machines and structures are undesirable due to increased stresses and energy losses.

- Time interval required for a system to complete a full cycle of the motion is the *period* of the vibration.

- Number of cycles per unit time defines the *frequency* of the vibrations.

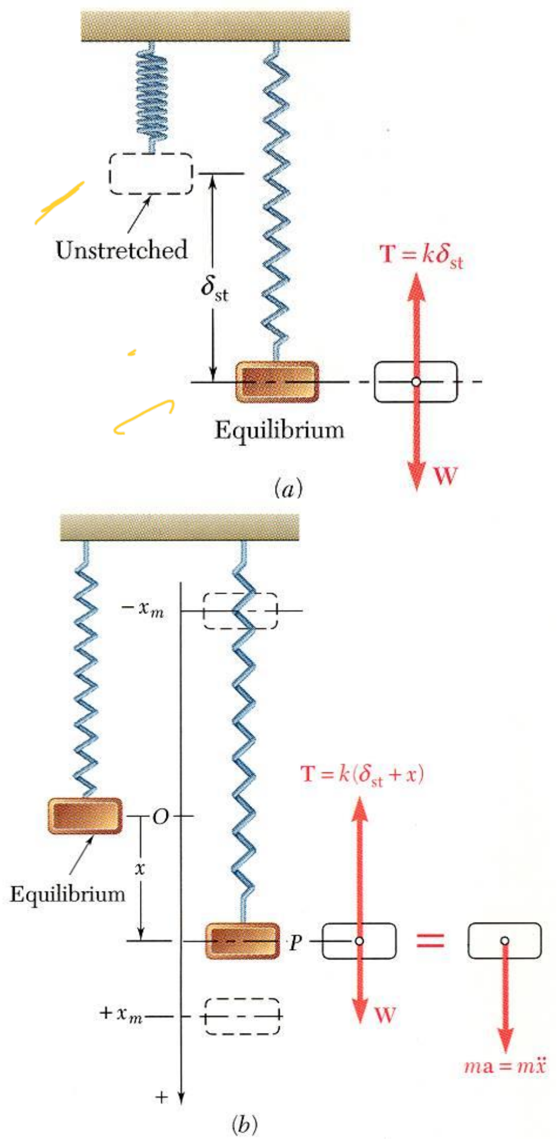
- Maximum displacement of the system from the equilibrium position is the *amplitude* of the vibration.

- When the motion is maintained by the restoring forces only, the vibration is described as *free vibration*. When a periodic force is applied to the system, the motion is described as *forced vibration*.

- When the frictional dissipation of energy is neglected, the motion is said to be *undamped*. Actually, all vibrations are *damped* to some degree.

Mechanical Vibrations

Free Vibrations of Particles. Simple Harmonic Motion



- If a particle is displaced through a distance x_m from its equilibrium position and released with no velocity, the particle will undergo *simple harmonic motion*,

$$ma = F = W - k(\delta_{st} + x) = -kx$$

$$m\ddot{x} + kx = 0$$

- General solution is the sum of two *particular solutions*,

$$x = C_1 \sin\left(\sqrt{\frac{k}{m}}t\right) + C_2 \cos\left(\sqrt{\frac{k}{m}}t\right)$$

$$= C_1 \sin(\omega_n t) + C_2 \cos(\omega_n t)$$

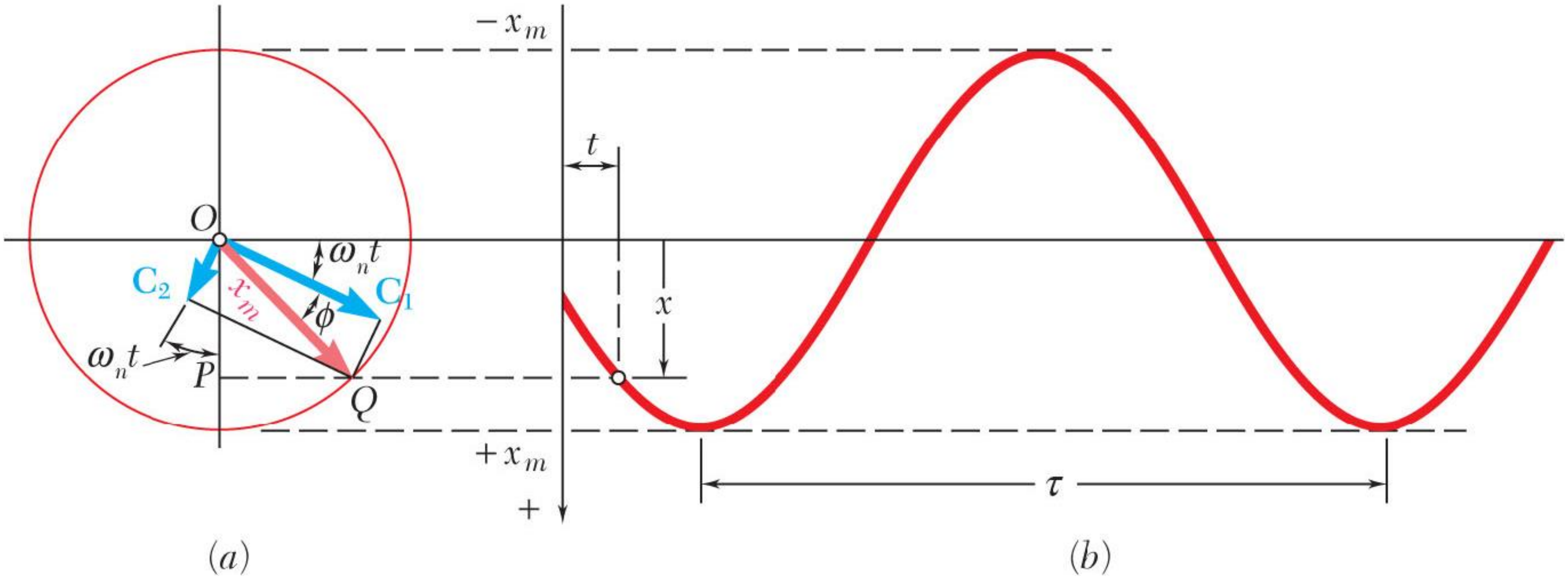
- x is a *periodic function* and ω_n is the *natural circular frequency* of the motion.

- C_1 and C_2 are determined by the initial conditions:

$$x = C_1 \sin(\omega_n t) + C_2 \cos(\omega_n t) \qquad C_2 = x_0$$

$$v = \dot{x} = C_1\omega_n \cos(\omega_n t) - C_2\omega_n \sin(\omega_n t) \qquad C_1 = v_0/\omega_n$$

Mechanical Vibrations



$$x = x_m \sin(\omega_n t + \phi)$$

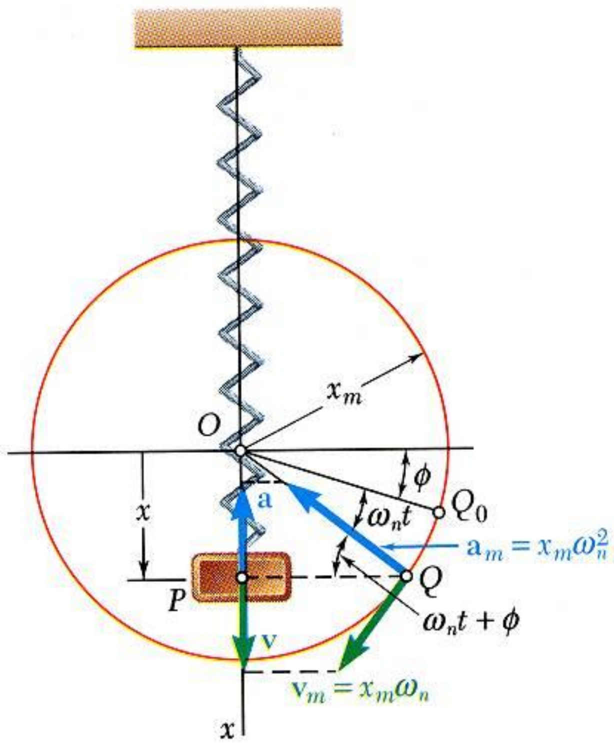
$$x_m = \sqrt{(v_0/\omega_n)^2 + x_0^2} = \textit{amplitude}$$

$$\phi = \tan^{-1}(v_0/x_0\omega_n) = \textit{phase angle}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \textit{period}$$

$$f_n = \frac{1}{\tau_n} = \frac{\omega_n}{2\pi} = \textit{natural frequency}$$

Mechanical Vibrations



- Velocity-time and acceleration-time curves can be represented by sine curves of the same period as the displacement-time curve but different phase angles.

$$x = x_m \sin(\omega_n t + \phi)$$

$$\begin{aligned} v &= \dot{x} \\ &= x_m \omega_n \cos(\omega_n t + \phi) \\ &= x_m \omega_n \sin(\omega_n t + \phi + \pi/2) \end{aligned}$$

$$\begin{aligned} a &= \ddot{x} \\ &= -x_m \omega_n^2 \sin(\omega_n t + \phi) \\ &= x_m \omega_n^2 \sin(\omega_n t + \phi + \pi) \end{aligned}$$

Mechanical Vibrations

Equation of Motion Using Energy Method

- Draw the FBD of the body after giving a small displacement from its equilibrium position.
- Define the location of the body from its equilibrium position using appropriate position coordinate system.
- Formulate equation of energy i.e K.E. + P.E. = constant
- Take time derivative of energy equation and factor out common terms
- Express the resultant differential equation in the following form

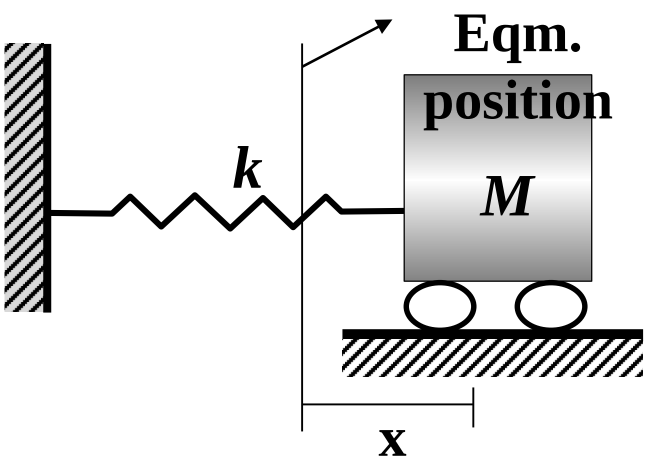
$$\ddot{x} + \omega_n^2 x = 0$$

- Use the expressions developed for simple harmonic motion to find other response quantities

Mechanical Vibrations

Equation of Motion Using Energy Method

- Restoring forces are conservative in nature.
- Therefore, conservation of energy equation can be used to obtain natural frequency.



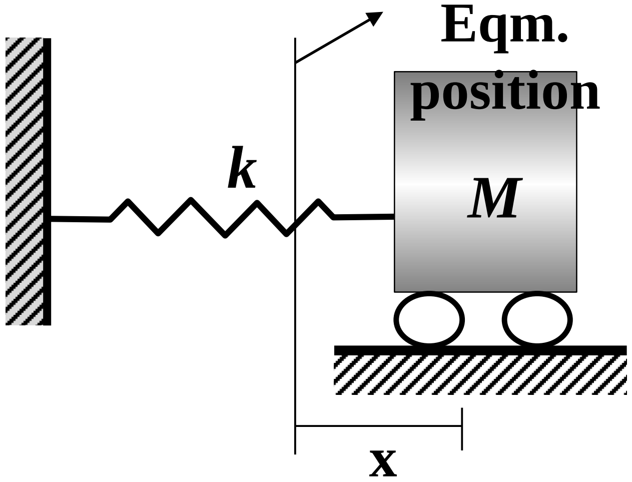
Total energy of system = K.E. + P.E.

$$= \frac{1}{2}M\dot{x}^2 + \frac{1}{2}kx^2$$

= Constant

Mechanical Vibrations

Equation of Motion Using Energy Method



- Differentiating total energy with respect to time, we get,

$$M\ddot{x} + kx = 0$$

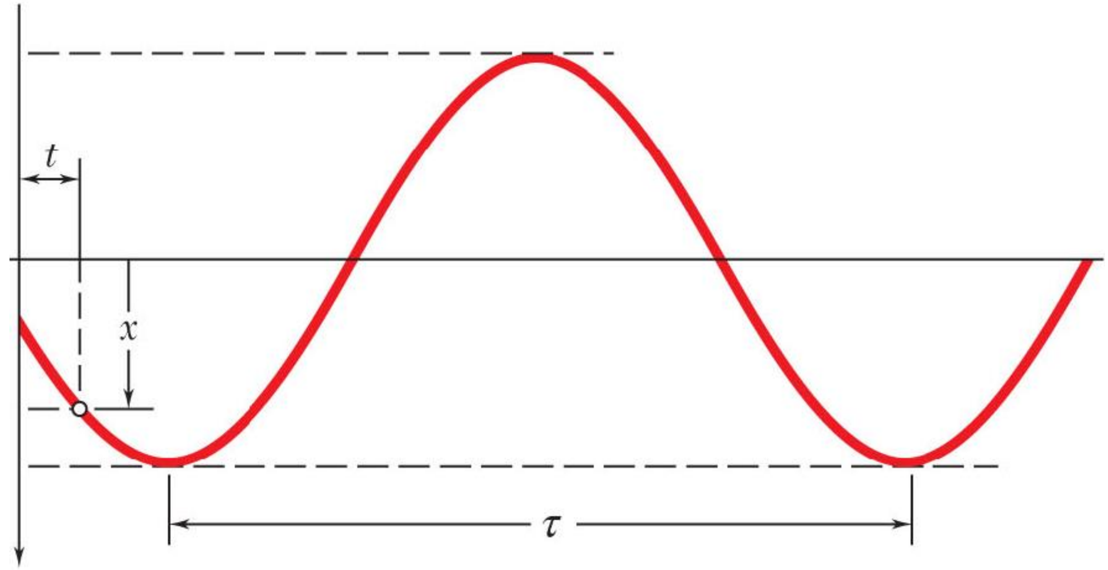
- Max. K.E. = $\frac{1}{2} M v_{max}^2$

- Max. P.E. = $\frac{1}{2} k x_{max}^2$

Mechanical Vibrations

Concept Question

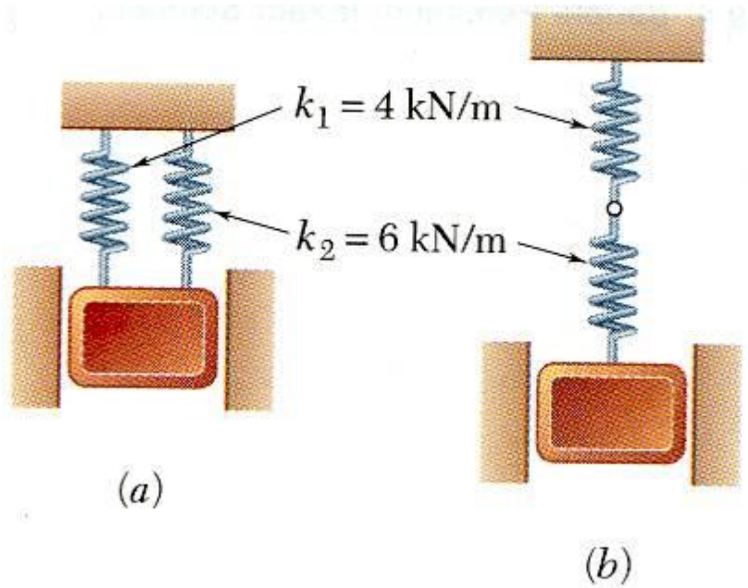
The amplitude of a vibrating system is shown to the right. Which of the following statements is true (choose one)?



(b)

a) The amplitude of the acceleration equals the amplitude of the displacement
b) The amplitude of the velocity is always opposite (negative to) the amplitude of the displacement
c) The maximum displacement occurs when the acceleration amplitude is a minimum
d) The phase angle of the vibration shown is zero

Mechanical Vibrations



A 50-kg block moves between vertical guides as shown. The block is pulled 40mm down from its equilibrium position and released.

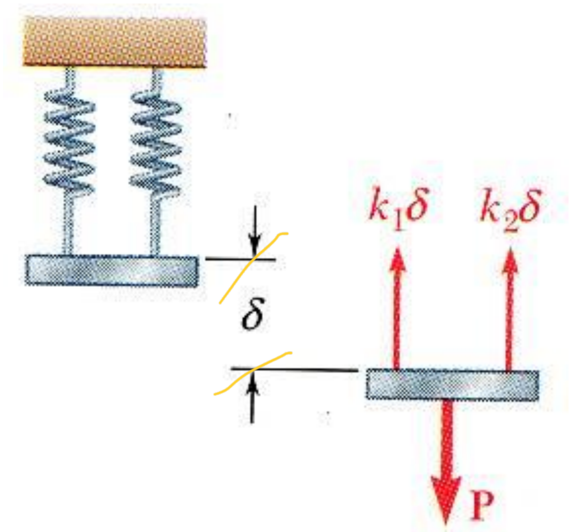
For each spring arrangement, determine *a*) the period of the vibration, *b*) the maximum velocity of the block, and *c*) the maximum acceleration of the block.

<u>SOLUTION</u>:

- For each spring arrangement, determine the spring constant for a single equivalent spring.
- Apply the approximate relations for the harmonic motion of a spring-mass system.

Mechanical Vibrations

$k_1 = 4\,\mathrm{kN/m} \quad k_2 = 6\,\mathrm{kN/m}$



$$P = k_1\delta + k_2\delta$$

$$k = \frac{P}{\delta} = k_1 + k_2$$

$$= 10\,\mathrm{kN/m} = 10^4\ \mathrm{N/m}$$

SOLUTION:

- Springs in parallel:
  - determine the spring constant for equivalent spring
  - apply the approximate relations for the harmonic motion of a spring-mass system

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10^4\ \mathrm{N/m}}{20\,\mathrm{kg}}} = 14.14\,\mathrm{rad/s}$$

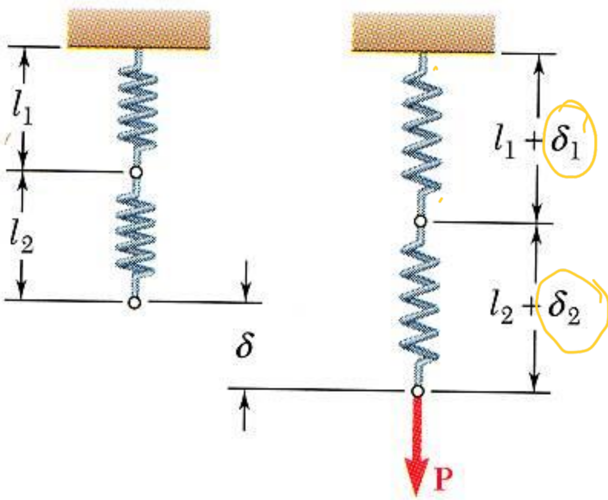
$$\tau_n = \frac{2\pi}{\omega_n} \qquad \boxed{\tau_n = 0.444\,\mathrm{s}}$$

$$v_m = x_m\omega_n$$
$$= (0.040\,\mathrm{m})(14.14\,\mathrm{rad/s}) \qquad \boxed{v_m = 0.566\,\mathrm{m/s}}$$

$$a_m = x_m a_n^2$$
$$= (0.040\,\mathrm{m})(14.14\,\mathrm{rad/s})^2 \qquad \boxed{a_m = 8.00\,\mathrm{m/s^2}}$$

Mechanical Vibrations

$k_1 = 4\,\text{kN/m} \quad k_2 = 6\,\text{kN/m}$



- Springs in series:
  - determine the spring constant for equivalent spring
  - apply the approximate relations for the harmonic motion of a spring-mass system

$$\delta = \delta_1 + \delta_2$$

$$\frac{P}{k} = \frac{P}{k_1} + \frac{P}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$= \_\_\_\_\_\_\_\_\_\_\ \text{N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{\_\_\_\_\_\ \text{N/m}}{20\,\text{kg}}} = \_\_\_\_\_\ \text{rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n}$$

$$\boxed{\tau_n = 0.907\,\text{s}}$$

$$v_m = x_m \omega_n = (0.040\,\text{m})(\_\_\_\_\_\ \text{rad/s})$$

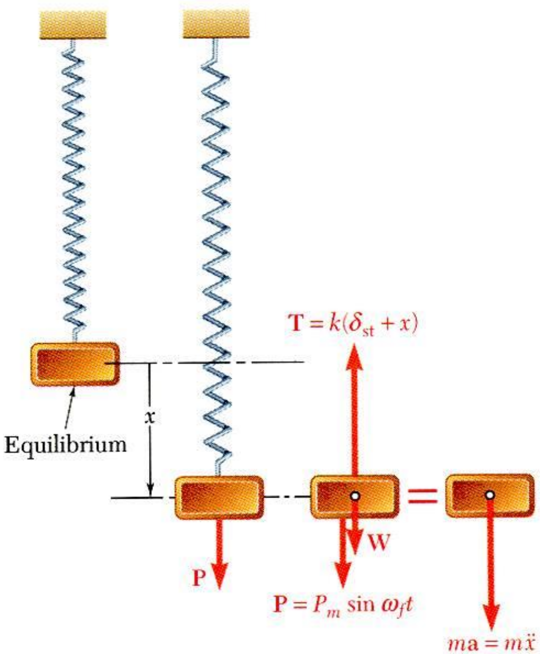
$$\boxed{v_m = 0.277\,\text{m/s}}$$

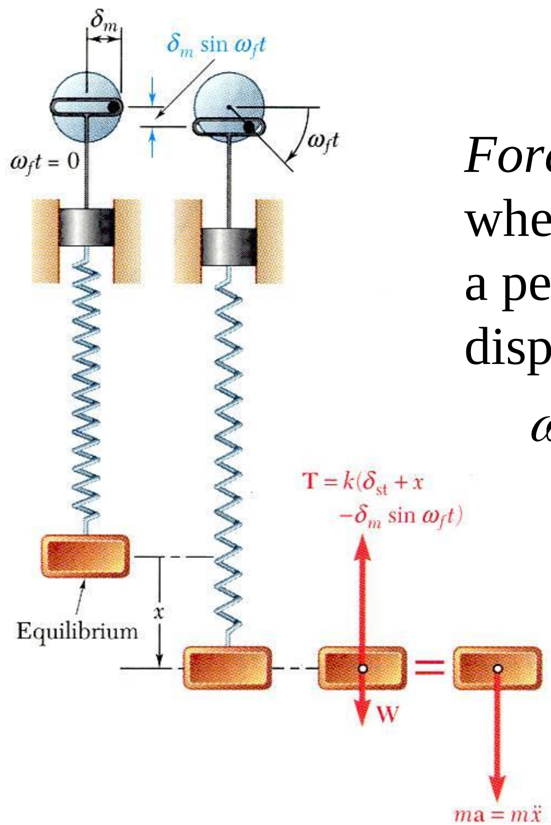
$$a_m = x_m a_n^2 = (0.040\,\text{m})(\_\_\_\_\_\ \text{rad/s})^2$$

$$\boxed{a_m = 1.920\,\text{m/s}^2}$$

Mechanical Vibrations

Forced Vibrations





Forced vibrations - Occur when a system is subjected to a periodic force or a periodic displacement of a support.

$$\omega_f = \textit{forced frequency}$$

$$+\downarrow \sum F = ma:$$

$$P_m \sin \omega_f t + W - k(\delta_{st} + x) = m\ddot{x}$$

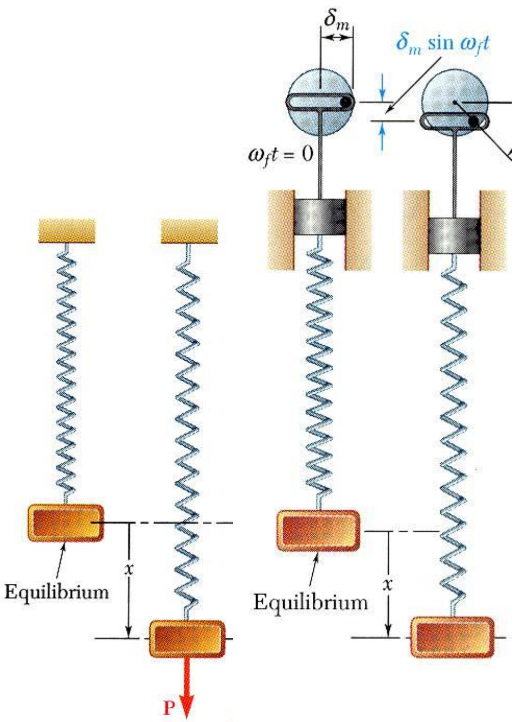
$$m\ddot{x} + kx = P_m \sin \omega_f t$$

$$W - k(\delta_{st} + x - \delta_m \sin \omega_f t) = m\ddot{x}$$

$$m\ddot{x} + kx = k\delta_m \sin \omega_f t$$

Mechanical Vibrations

Forced Vibrations



$$m\ddot{x} + kx = P_m \sin \omega_f t$$

$$m\ddot{x} + kx = k\delta_m \sin \omega_f t$$

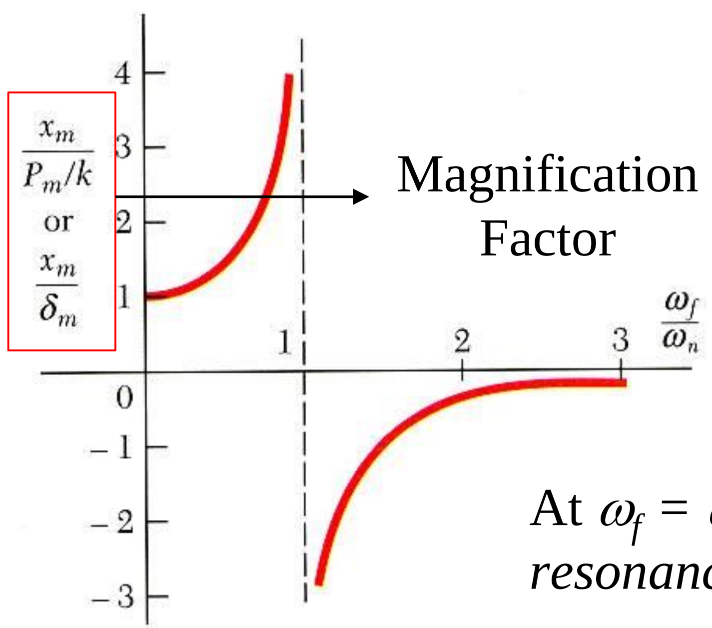
$$x = x_{complementary} + x_{particular}$$

$$= \underbrace{\left[C_1 \sin \omega_n t + C_2 \cos \omega_n t\right]}_{\text{Transient}} + \underbrace{x_m \sin \omega_f t}_{\text{Steady State}}$$

Substituting particular solution into governing equation,

$$-m\omega_f^2 x_m \sin \omega_f t + kx_m \sin \omega_f t = P_m \sin \omega_f t$$

$$x_m = \frac{P_m}{k - m\omega_f^2} = \frac{P_m / k}{1 - (\omega_f / \omega_n)^2} = \frac{\delta_m}{1 - (\omega_f / \omega_n)^2}$$



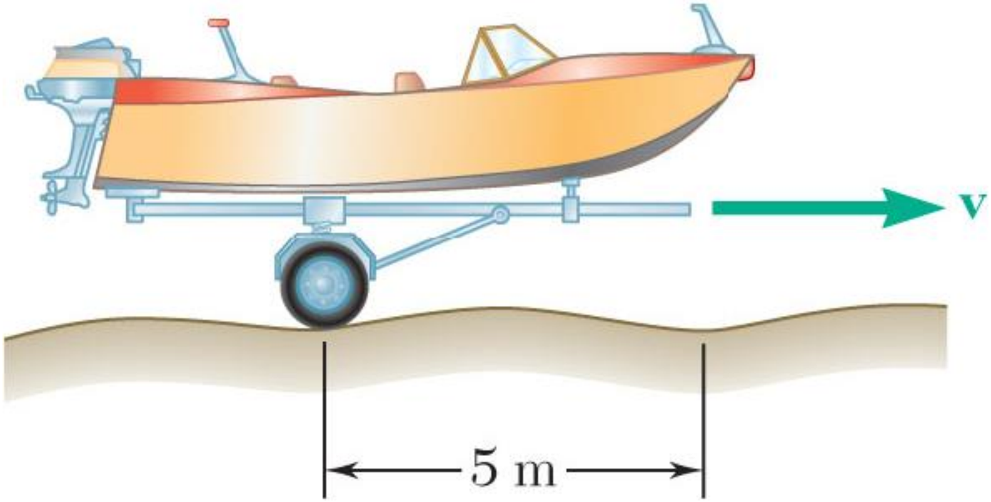
Magnification Factor

At $\omega_f = \omega_n$, forcing input is in *resonance* with the system.

Mechanical Vibrations

Concept Question

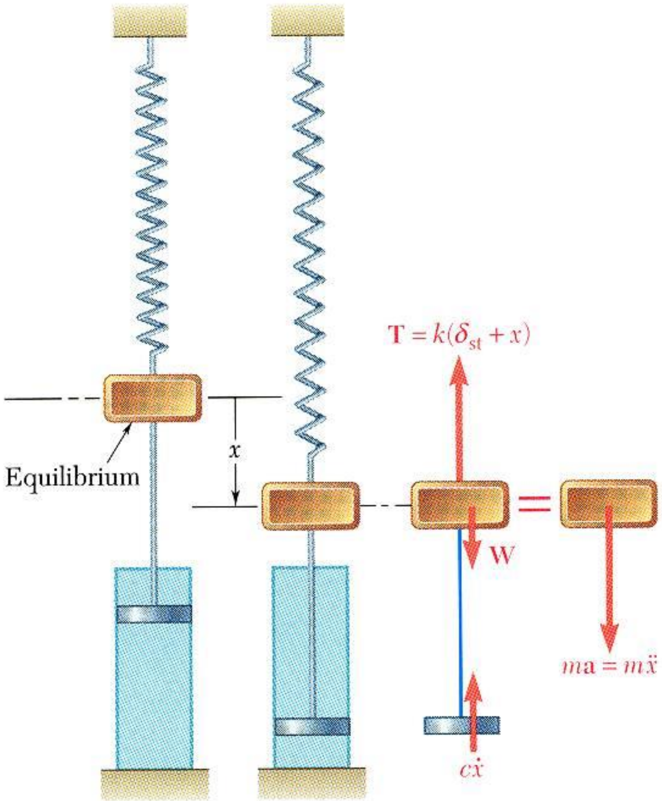
A small trailer and its load have a total mass m. The trailer can be modeled as a spring with constant k. It is pulled over a road, the surface of which can be approximated by a sine curve with an amplitude of 40 mm and a wavelength of 5 m. Maximum vibration amplitude occur at 35 km/hr. What happens if the driver speeds up to 50 km/hr?



a) The vibration amplitude remains the same.
b) The vibration amplitude would increase.
c) The vibration amplitude would decrease.

Mechanical Vibrations

Damped Free Vibrations



- All vibrations are damped to some degree by forces due to *dry friction*, *fluid friction*, or *internal friction*.

- With *viscous damping* due to fluid friction,

$$+\downarrow \sum F = ma: \qquad W - k(\delta_{st} + x) - c\dot{x} = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

- Substituting $x = e^{\lambda t}$ and dividing through by $e^{\lambda t}$ yields the *characteristic equation*,

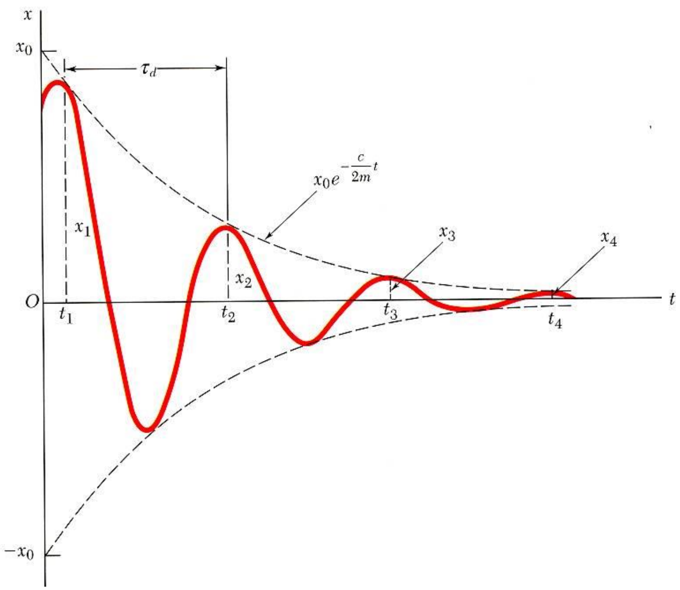
$$m\lambda^2 + c\lambda + k = 0 \qquad \lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

- Define the critical damping coefficient such that

$$\left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} = 0 \qquad c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

Mechanical Vibrations

Damped Free Vibrations



- Characteristic equation,

$$m\lambda^2 + c\lambda + k = 0 \qquad \lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$c_c = 2m\omega_n = \text{critical damping coefficient}$$

- *Heavy damping*: $c > c_c$

$$x = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

  - negative roots
  - nonvibratory motion

- *Critical damping*: $c = c_c$

$$x = (C_1 + C_2 t)e^{-\omega_n t}$$

  - double roots
  - nonvibratory motion

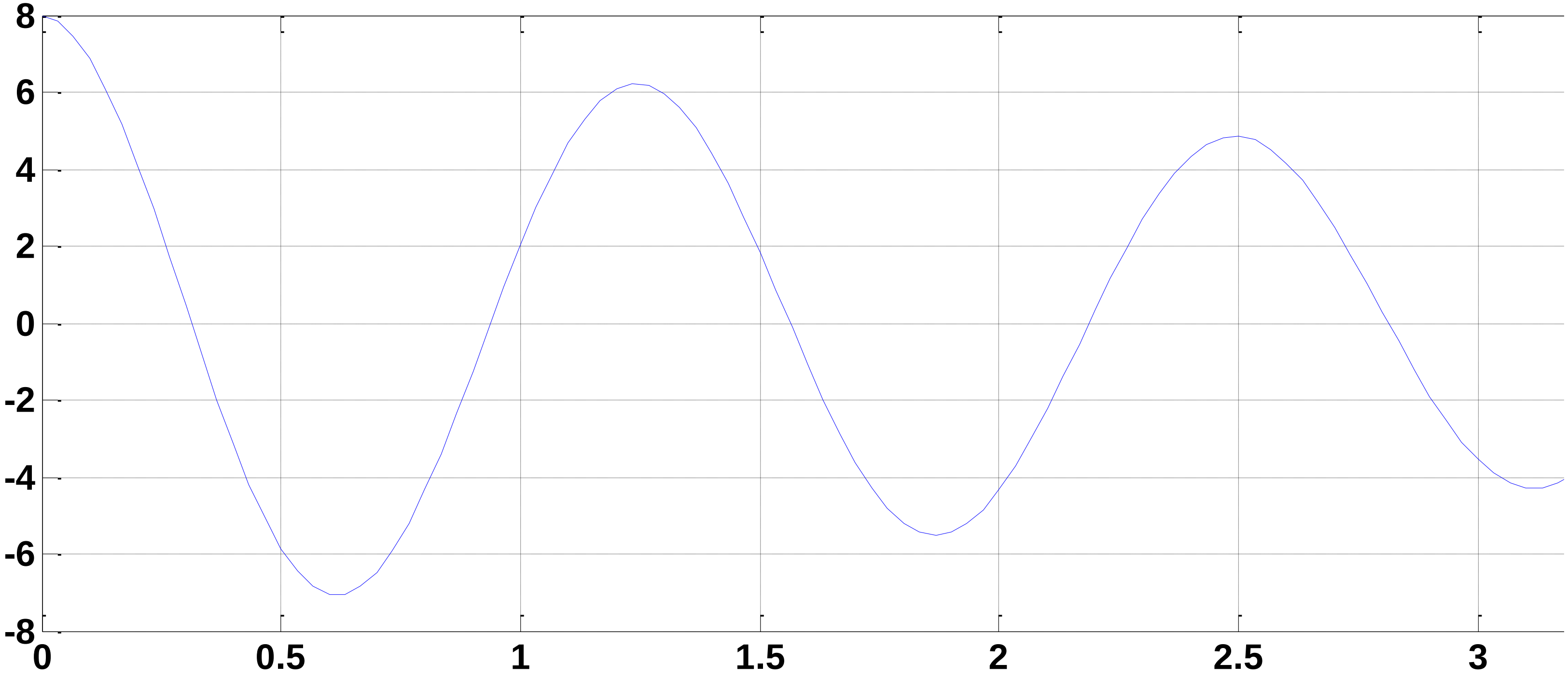
- *Light damping*: $c < c_c$

$$x = e^{-(c/2m)t}\left(C_1 \sin \omega_d t + C_2 \cos \omega_d t\right)$$

$$\omega_d = \omega_n \sqrt{1 - \left(\frac{c}{c_c}\right)^2} = \text{damped frequency}$$

Mechanical Vibrations

Concept Question



The graph above represents an oscillation that is…

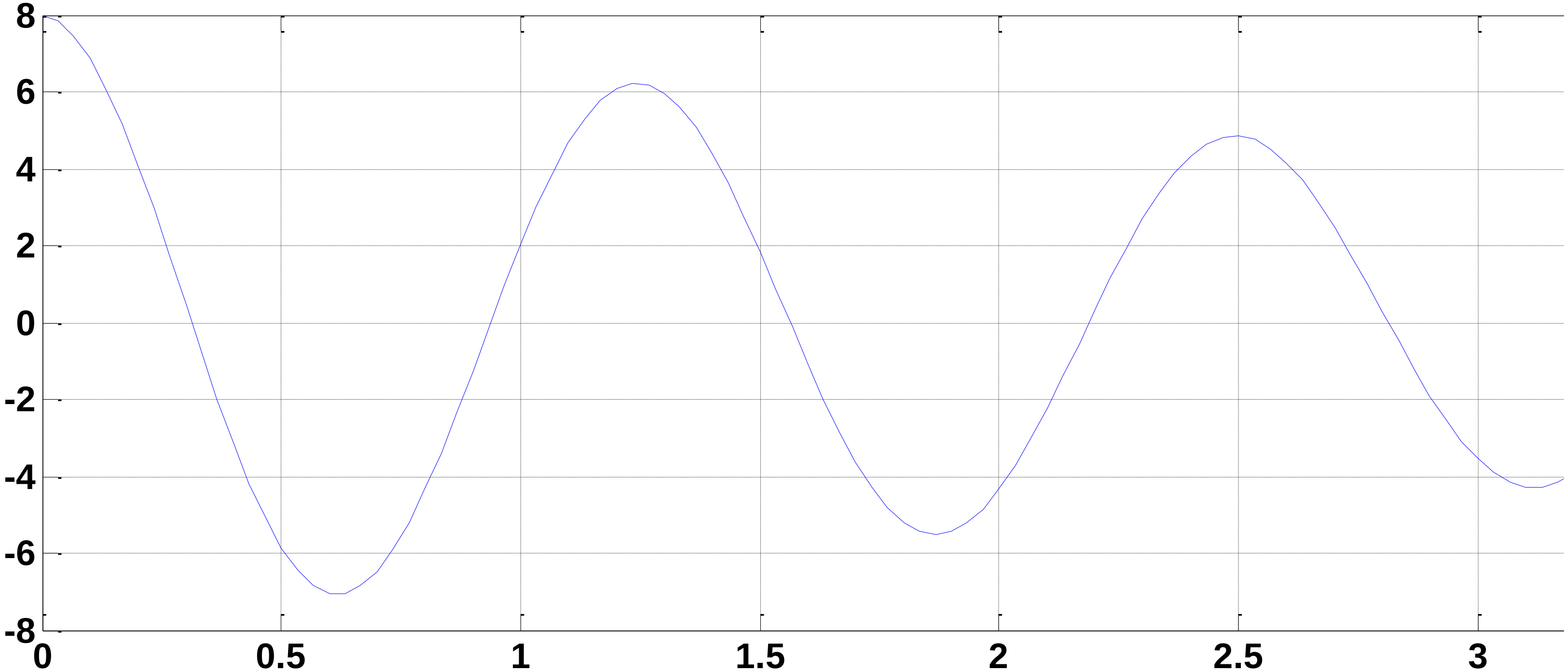
a) Heavily damped

b) critically damped

c) lightly damped

Mechanical Vibrations

Concept Question



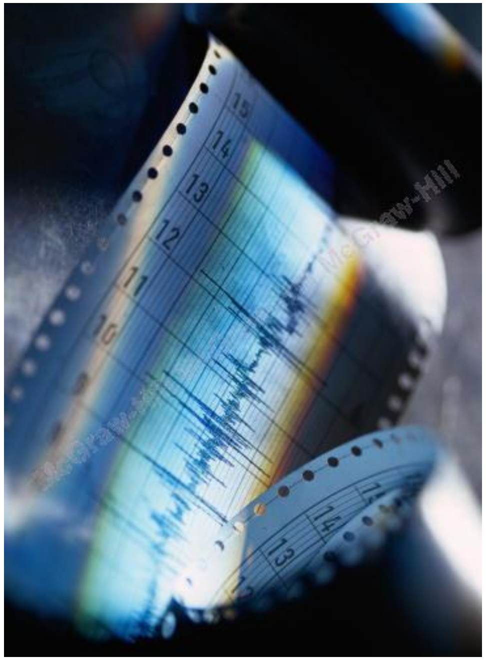
The period for the oscillation above is approximately…

a) 1.25 seconds   b) 2.5 Hz   c) 0.6 seconds

Estimate the phase shift for the oscillation   Zero

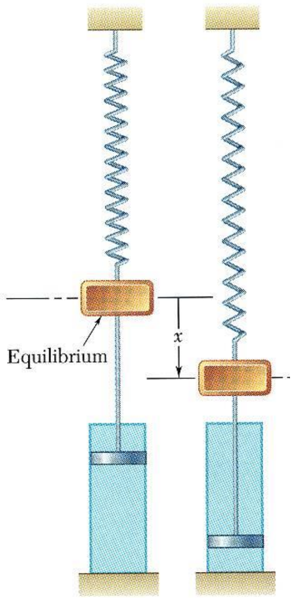
Mechanical Vibrations

Forced vibrations can be caused by a test machine, by rocks on a trail, by rotating machinery, and by earthquakes. Suspension systems, shock absorbers, and other energy-dissipating devices can help to dampen the resulting vibrations.



Mechanical Vibrations

Damped Forced Vibrations



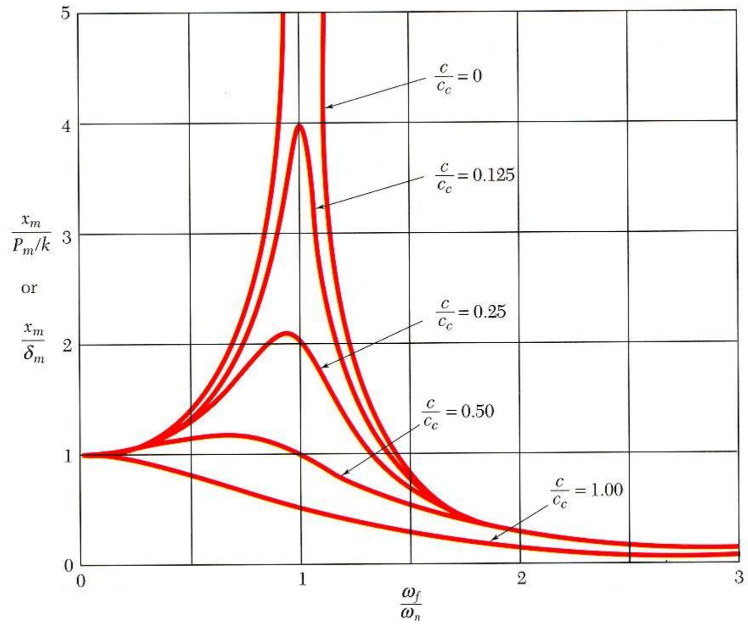


Photo 19.2 The automobile suspension shown consists essentially of a spring and a shock absorber, which will cause the body of the car to undergo *damped forced vibrations* when the car is driven over an uneven road.

$$m\ddot{x} + c\dot{x} + kx = P_m \sin \omega_f t \qquad x = x_{complementary} + x_{particular}$$

$$x_{\text{part}} = x_m \sin\left(\omega_f t - \varphi\right)$$

$$\frac{x_m}{P_m/k} = \frac{x_m}{\delta} = \frac{1}{\sqrt{\left[1-\left(\omega_f/\omega_n\right)^2\right]^2 + \left[2\left(c/c_c\right)\left(\omega_f/\omega_n\right)\right]^2}} = \text{magnification factor}$$

$$\tan \phi = \frac{2\left(c/c_c\right)\left(\omega_f/\omega_n\right)}{1-\left(\omega_f/\omega_n\right)^2} = \text{phase difference between forcing and steady state response}$$