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MLFO - Assignment
Unit II - Quantum Mech

Q.1 What are matter waves? Mention any 3 properties of matter waves. Describe the experiment Davisson & Germer to demonstrate the wave character of e^-.

A.1 Matter waves consists of group of wave or a wave packet each having the wavelength λ, is associated with the particle. This group travels with the particle velocity v.

Properties of matter waves -

1. Each wave of the group travels with phase velocity of wave, $v_{phase} = \frac{c^2}{v}$
2. Lighter the particle, greater the wavelength.
3. The group travel with particle velocity v.

Davission & Germer Experiment -

- A beam of e^- emitted by an e^- gun is made to fall on Nickel Crystal cut along cubical axis at a particular angle.
- The scattered beam ~~is~~ of e^- is received by the detector which can be rotated at any angle.
- The energy of the incident beam of e^- can be varied by changing the applied voltage ~~to~~ the e^- gun
- Intensity of scattered beam of e^- is found to be maximum when angle ~~of~~ of scattering is 50° & the accelerating potential is 54V.
- De Broglie wavelength of moving e^- at V = 54V is 1.67 Å which is in close agreement with experimentally obtained $\lambda = 1.65$ Å.

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Q.2. Discuss the failure of Classical Physics & How does Quantum Mechanics overcome these failure?

A.2. → Inadequacies of Classical Mechanics:

- It doesn't hold good for atomic dimensions (non-relativistic speeds) i.e. e^-, p^+, n^0 etc.
- Could not explain the observed spectrum of black body radiation
- Could not explain stability of atoms.

→ Quantum Mechanics overcome these problems by introducing following outcomes –

- Dual nature of light & matter
- Planck's Radiation Law
  Quantization of energy was a great leap in understanding atomic level physics.
- It gave the Heisenberg's Uncertainty principle
- Wave mechanics, wave functions & probability were developed to give details about particle behaviour & properties.

Q.3. Explain Heisenberg's Uncertainty Principle. Apply uncertainty principle to obtain
(i) Minimum energy of Harmonic oscillator
(ii) Energy of particle in dimensional box.

A.3 → Heisenberg's Uncertainty Principle states, "It is impossible to determine simultaneously both position & momentum of a moving particle accurately at same time. The product of uncertainty in these quantities is always greater than or equal to $h/4\pi$".

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If Δx & Δp are the uncertainties in the measurement of position & momentum of a particle, then,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

Similarly the other uncertainty relations for other physical variable pair are,

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

$$\Delta L \cdot \Delta \theta \geq \frac{h}{4\pi}$$

i) Minimum energy of Harmonic Oscillator :

Let a particle of mass 'm' execute SHM along X-axis

$\therefore$ Uncertainty in position $= \Delta x$

$\therefore$ From uncertainty principle –

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Rightarrow \quad \Delta p \geq \frac{h}{4\pi \Delta x}$$

$$\therefore \quad \Delta p_{min} = \frac{h}{4\pi \Delta x}$$

Total energy of the oscillator –

$$E = \frac{(\Delta p)^2}{2m} + \frac{1}{2} k (\Delta x)^2$$

$$E = \frac{h^2}{32\pi^2 m (\Delta x)^2} + \frac{1}{2} k (\Delta x)^2$$

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Minimising E wrt Δx $\rightarrow$ $\frac{dE}{d(\Delta x)} = 0$

$$\therefore \frac{-2h^2}{32\pi^2 m(\Delta x)^3} + k(\Delta x) = 0$$

$$\Rightarrow \Delta x = \left[\frac{h^2}{16\pi^2 mk}\right]^{1/4}$$ Put in E

$$\therefore E_{min} = \frac{h^2}{32\pi^2}\left(\frac{k}{m}\right)^{1/2}$$ $\sqrt{\frac{k}{m}} = \omega$ angular freq.

$$\therefore E_{min} = \frac{h^2}{32\pi^2}\omega = \frac{1}{2}\hbar^2\omega$$

(ii) Energy of particle in one dimension :

Consider a particle of mass 'm' in one dimension box of length 'l'. The max. uncertainty in the position of the particle will be -

$$(\Delta x)_{max} = l$$

$\therefore$ From uncertainty principle -

$$\Delta x \cdot \Delta p = \hbar$$

$$\therefore \Delta p = \frac{\hbar}{l}$$

As uncertainty in momentum must be less than momentum itself, therefor min. momentum of particle $= \frac{\hbar}{l}$

$$\therefore \text{KE particle} = T = \frac{p^2}{2m} = \frac{\hbar^2}{2ml^2}$$

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Q.4 Give physical significance of wave f^n. Derive time independent Schrödinger eq^n.

A.4. Significance of wave f^n (ψ)

- ψ is the wave f^n of e^- and it is also called Amplitude f^n of e^- in the coordinates X, Y, Z.
- If we use ψ to determine the probability of e^- then the value can be +ve or -ve but the probability can never be negative so we use ψ^2 to get the probability.
- ψ^2 represents the probability of finding an e^- in a small ~~sq~~ space. It ~~is~~ also represents the probability density at a particular pt.

Time independent Schrödinger eq^n:

Let a wave ~~eq~~ f^n ψ describing the de-broglie wave travelling in the X-dirⁿ is given by-

$$\psi = A e^{i(kx - \omega t)}$$

where ψ is total wave f^n.
A - constant
ω - angular freq. of wave

Differentiate ψ twice wrt x -

$$\frac{d\psi}{dx} = ikAe^{i(kx-\omega t)} \quad ; \quad \frac{d^2\psi}{dx^2} = -k^2Ae^{i(kx-\omega t)}$$

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~~Put $\frac{d^2\psi}{dx^2} = 0$~~ ~~$\therefore$~~

$\therefore \frac{d^2\psi}{dx^2} = -k^2\psi \Rightarrow \frac{d^2\psi}{dx^2} + k^2\psi = 0$

we already know,

$k = \frac{2\pi}{\lambda}$ & $\lambda = \frac{h}{mv}$

$\therefore k = \frac{2\pi mv}{h}$

$\Rightarrow k^2 = \frac{4\pi^2 m^2 v^2}{h^2}$ $\therefore \frac{d^2\psi}{dx^2} + \left[\frac{4\pi^2 m^2 v^2}{h^2}\right]\psi = 0$

Total energy of the particle is the sum of KE ~~of~~ T & PE V

$T = \frac{1}{2}mv^2$ ~~&~~ $\therefore mv^2 = 2(E-V)$

$$\Rightarrow \boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m(E-V)}{h^2}\psi = 0}$$

Time Independent Schrödinger Eq^n