Department of Applied Mathematics & Humanities

S. V. National Institute of Technology, Surat

B.Tech- I (Sem.-I) MM 101 S1 [Mathematics - I]

Tutorial – 4 :

Partial Differentiation, Euler's Theorem and Modified Euler's Theorem

1. State and prove Euler's theorem on homogeneous functions. Hence prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u.$
2. State and prove Modified Euler's theorem on homogeneous functions. Hence prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=g(u)[g'(u)-1]$ where $g(u)=n\frac{f(u)}{f'(u)}$.
3. If $u=f(r)$, where $r^2=x^2+y^2$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$.
4. If $z(x+y)=x^2+y^2$, show that $\left[\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right]^2=4\left[1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right]$.
5. If $z=xf\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$, prove that $x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=0.$
6. Prove that if $f(x,y)=\frac{1}{\sqrt{y}}\cdot e^{-\frac{(x-a)^2}{4y}}$ then $f_{xy}=f_{yx}$.
7. If $x=e^{r\cos\theta}\cos(r\sin\theta)$ and $y=e^{r\cos\theta}\sin(r\sin\theta)$ then prove that $\frac{\partial x}{\partial r}=\frac{1}{r}\frac{\partial y}{\partial\theta},\ \frac{\partial y}{\partial r}=-\frac{1}{r}\frac{\partial x}{\partial\theta}$.
8. Let $r^2=x^2+y^2+z^2$ and $V=r^m$, prove that $V_{xx}+V_{yy}+V_{zz}=m(m+1)r^{m-2}$.
9. If $\log u=\frac{x^2y^2}{x+y}$ then show that $xu_x+yu_y=3u\log u$.
10. If $u=F(x-y,y-z,z-x)$, then show that $u_x+u_y+u_z=0$.
11. If $x^xy^yz^z=c$, show that at $x=y=z$, $\frac{\partial^2 z}{\partial x\partial y}=-(x\log ex)^{-1}$.
12. If $u=\log(x^3+y^3+z^3-3xyz)$ then prove that $\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right]^2u=-\frac{9}{(x+y+z)^2}$.
13. Find $\frac{dy}{dx}$ when $y^{x^y}=\sin x$. Ans: $-\frac{yx^{y-1}\log y-\cot x}{x^y\left(\log x\cdot\log y+\frac{1}{y}\right)}$
14. If $z=f(x+ct)+g(x-ct)$, prove that $\frac{\partial^2 z}{\partial t^2}=c^2\frac{\partial^2 z}{\partial x^2}$.
15. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right); x\neq y$ prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\sin 4u-\sin 2u$ $=2\sin u\cos 3u.$
