Department of Applied Mathematics and Humanities
S. V. National Institute of Technology, Surat, Gujarat
B. Tech.-I (Semester-I) Branch-All
Subject: Mathematics-I (MA101 S1)

Tutorial - 05
Taylor's Theorem and Maclaurin's Theorem for two variables, Tangent Plane and Normal Line

1. Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree .
   Ans $e^x \log(1+y) = y + xy - \frac{1}{2}y^2 + \frac{1}{2}(x^2y - xy^2) + \frac{1}{3}y^3 + \cdots$

2. Expand $\sin x \cos y$ in powers of x and y upto terms of third degree.
   Ans $\sin x \cos y = x - \frac{1}{6}(x^3 + 3\ x\ y^2)$

3. If $f(x,y) = \tan^{-1} xy$, Compute $f(0.9, -1.2)$ approximately.
   Ans $-\ 0.8232$

4. Expand by Taylor's theorem upto the second degree terms $\sin xy$ in powers of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$.
   Ans $\sin xy = 1 - \frac{1}{8}\pi^2(x-1)^2 - \frac{1}{2}\pi(x-1)(y-\frac{\pi}{2}) - \frac{1}{2}(y-\frac{\pi}{2})^2$

5. Expand $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x+1)$ and $(y-1)$ upto second degree.
   Ans. $\tan^{-1}(\frac{y}{x}) = \frac{3\pi}{4} - \frac{1}{2}(x+1) - \frac{1}{2}(y-1) - \frac{1}{2}(x+1)^2 + \frac{1}{2}(y-1)^2$

6. Find the 3rd-order Taylor polynomial of $f(x,y) = e^{x^2+y}$ about $(x,y) = (0,0)$.
   Ans $e^{x^2+y} = 1 + y + x^2 + \frac{1}{2}y^2 + x^2y + \frac{1}{6}y^3$

7. If $f(x,y) = \sin x \cosh y$, evaluate all the partial derivatives of $f(x,y)$ up to order five at the point $(x,y) = (0,0)$, and, hence, show that $\sin x \cosh y = x - \frac{1}{6}(x^3 - 3xy^2) + \frac{1}{120}(x^5 - 10x^3y^2 + 5xy^4) + \cdots$

8. If $f(x,y) = (x^3 - 3xy^2)$, show that $f(2+h, 1+k) = 2 + 9h - 12k + 6(h^2 - hk - k^2) + h^3 - 3hk^2$

9. Find the tangent plane and normal to the surface $2x^2 + y^2 = 3 - 2z$ at the given point $(2, 1, -3)$
   Ans $\quad 4\ x + y + z = 6; \quad \frac{x-2}{4} = y - 1 = z + 3$

10. Find the tangent plane and normal to the surface $xyz = a^2$ at the given point (x_1, y_1, z_1)
    Ans $\quad \frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 3; \quad x_1(x - x_1) = y_1(y - y_1) = z_1(z - z_1)$

11. Find the equation of the normal to the surface $x^2 + y^2 + z^2 = a^2$
    Ans $\quad \frac{X - x}{x} + \frac{Y - y}{y} + \frac{Z - z}{z}$

12. Find the tangent plane and normal line to $x^2 + y^2 + z^2 = 30$ at the point $(1, -2, 5)$.
    Ans $2(x-1) - 4(y+2) + 10(z-5) = 0$ and $(1 + 2t, -2 - 4t, 5 + 10t)$

13. Find the equation of the tangent plane and equation of the normal line of the surface $z = x^2 + y^2$, at the point $(1, -2, 5)$
Ans. $2x - 4y - z = 5$ and $(1 + 2t, -2 - 4t, 5 + 10t)$

14. Compute equations of the tangent plane and the normal line to the given surface at the indicated point.

(a) $\ln(x + y + z) = 2$ at point $P(-1, e^2, 1)$
Ans $x + y + z = e^2$ and $(-1, e^2, 1) + t(1, 1, 1)$

(b) $x^2 + y^2 + z^2 = 1$ at point $P\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$
Ans $x + y + z = \sqrt{3}$ and $\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) + t(1, 1, 1)$