Sardar Vallabhbhai National Institute of Technology, Surat
B. Tech. - I / M. Sc. - I (Semester-II)
End Semester Examination- July 2021
Sub: MA 114 S2 Mathematics-II

Date: 12-07-2021 **Total Marks: [50]**
Time: 09.30am to 12.30pm (2 hours and 30 minutes for writing 30 minutes for uploading answer sheets)

General Instructions

(i) There are total **Four** questions in the question paper.
(ii) All questions are compulsory.
(iii) Figure to the right indicates marks.
(iv) Follow usual notations.
(v) All must write your Admission Number, Role Number, Mobile Number, email on TOP of first page of answer sheet and admissions number and page no. with your signature on all pages.
(vi) **Important Instructions: Students must upload their answer sheet (single PDF file) on Google classroom or Microsoft team as per your class teacher suggestion latest by 12.30 pm on same day.**
(vii) **First verify the number of pages in your PDF file and then upload. Once you upload the file their after we will not consider any updated file.**

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1 Answer the following questions with Justification **[05]**

(1) Solve $x\,dx + y\,dy = \dfrac{a^2(x\,dy - y\,dx)}{x^2+y^2}$.

(2) Find the directional derivative of $\phi = xy + yz + zx$ in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$ at (1,2,0).

(3) Solve $z\left(\dfrac{\partial z}{\partial x}\right)\left(\dfrac{\partial z}{\partial y}\right) = xy$.

(4) Show that all real numbers are ordinary points of the equation $(x^2+1)y'' + xy' - xy = 0$.

(5) For what values of k the equations $4x + y + 10z = k^2, 2x + y + 4z = k$ have a solutions?

2 Answer the following questions

(A) Solve Equations $10x - 2y - z - 2w = 3, -2x + 10y - z - 5w = 15, -x - 2y + 10z - 2w = 27,$ $-x - y - 2z + 10w = -1$ using Gauss Jacobi method up to 5th iteration and correct up to 5 decimal places. **[04]**

OR

(A) Verify Green's theorem in the plane for $\int_C (xy + y^2)dx + x^2 dy$, C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(B) Answer the following questions (Attempt any three) **[09]**

(1) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

(2) A fluid motion is given by $\vec{V} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$, show that the motion is irrotational and hence find the velocity potential

(3) Evaluate $\iint_S \vec{F} \bullet \hat{n}\, ds$, where $\vec{F} = yz\hat{i} + zx\hat{j} + yx\hat{k}$ and S is that of the surface of the sphere $x^2 + y^2 + z^2 = a^2$ which lies in the first octant.

(4) By transforming to a triple integral, evaluate $I = \iint_S (x^3 dydz + x^2 y\, dzdx + x^2 z\, dxdz)$, where S is the closed surface bounded by the planes $z = 0,\ z = b$ and the cylinder $x^2 + y^2 = a^2$.

(C) Show that the system of equation has no solution unless $a+b+c=0$ **[02]**

$-2x + y + z = a,\ x - 2y + z = b,\ x + y - 2z = c$.

3 **Answer the following questions**

(A) Find the power series solution about $x = 0,\ of\ y'' + x y' + x^2 y = 0.$ **[04]**

OR

(A) Solve in series about $x = 0$ regular singular point of $xy'' + y' - y = 0.$

(B) **Answer the following questions (Attempt any three)** **[09]**

(1) **(a)** Solve $x(1-xy)p - y(1+xy)q = z(1-xy)$. **(b)** Form the partial differential equation by eliminating the arbitrary constant a and b from the relation $ax^2 + by^2 + z^2 = 1.$

(2) Solve **(a)** $zpq = p^{\frac{1}{2}} + q^{\frac{1}{2}}$ **(b)** $\dfrac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0.$

(3) Solve **(a)** $p \tan x + q \tan y = \tan z$ **(b)** $\left(\dfrac{1}{z} - \dfrac{1}{y}\right)p + \left(\dfrac{1}{x} - \dfrac{1}{z}\right)q = \dfrac{1}{y} - \dfrac{1}{x}.$

(4) Find the indicial equation about x = 0 of the differential equation $2x^2 y'' - x y' + (x-5)y = 0.$

(C) What is the significance of series solution technique ? Justify with proper example. **[02]**

4 **Answer the following questions**

(A) In LC circuit, an inductance L of one Henry, resistance of 6 Ohm and a condenser of $\dfrac{1}{9}$ Farad have been connected through a battery of e. m. f. $E = \sin t.\ If\ i = q = 0\ at\ t = 0,$ $find\ the\ chage\ q\ and\ current\ i.$ **[04]**

OR

(A) A beam of length l and of uniform cross-section has the differential equation of its elastic curve as $EI \dfrac{d^2 y}{dx^2} = \dfrac{w}{2}\left(\dfrac{l^2}{4} - x^2\right)$ where E is the modulus of elasticity, I is the moment of inertia of the cross section , w is weight per unit length and x is measured from the center of span. If at $x = 0, \dfrac{dy}{dx} = 0.$ Find the equation of elastic curve.

(B) **Answer the following questions (Attempt any three)** **[09]**

(1) Solve $(D^2 - 4D + 4)y = 3x^2 e^{2x} \sin 2x.$

(2) Solve by method of variation of parameter $(D^2 + 2D + 2)y = e^{-x} \sec^3 x.$

(3) Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0.$

(4) A tank contains 1000 liters of fresh water. Salt water which contains 150 gm of salt per liter runs in to it at the rate of 5 liters/min and well stirred mixture runs out if at the same rate. When will the tank contains 5000 gm of salt ?

(C) What is the main difference between SI, SIS and SIR models ? Explain with proper Justification. **[02]**

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