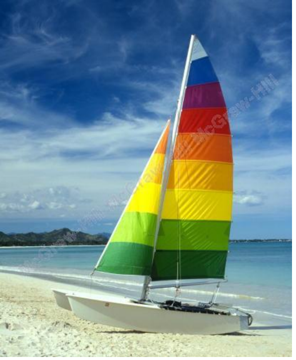


Chapter 6: Formal Relational Query Languages

Database System Concepts, 6th Ed.

Chapter 6: Formal Relational Query Languages

These are three formal languages.

- Relational Algebra
- Tuple Relational Calculus
- Domain Relational Calculus

Relational Algebra

- Procedural language
- Six basic operators
  - select: σ (Sigma)
  - project: $\prod$ (Pi)
  - union: $\cup$
  - set difference: –
  - Cartesian product: x
  - rename: ρ (Rho)

Relational Algebra

- The operators take one or two relations as inputs and produce a new relation as a result
- **Fundamental and Unary operators**
  - select: σ (Sigma)
  - project: ∏ (Pi)
  - rename: ρ (Rho)
- **Fundamental and Binary operators**
  - union: ∪
  - set difference: –
  - Cartesian product: x

Select Operation – Example

- Relation r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

- σ

Select the rows where

A and B are same
And
Also D > 5

Select Operation – Example

- Relation r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

- $\sigma_{A=B \wedge D > 5}(r)$

A	B	C	D
α	α	1	7
β	β	23	10

Select Operation

- σ Unary operator
- Notation: $\sigma_p(r)$
- p is called the **selection predicate**
- Defined as:

  $$\sigma_p(\mathbf{r}) = \{t \mid t \in r \textbf{ and } p(t)\}$$

  Where p is a formula in propositional calculus consisting of **terms** connected by : $\wedge$ (**and**), $\vee$ (**or**), $\neg$ (**not**)
  Each **term** is one of:

  <attribute> *op* <attribute> or <constant>

  where *op* is one of: =, ≠, >, ≥. <. ≤

- Example of selection:

  $$\boldsymbol{\sigma}_{\textit{dept\_name}=\text{“}\textit{Physics}\text{”}}(\textit{instructor})$$

Select Operation – Example

ID	name	dept_name	salary
10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

Figure 6.1 The *instructor* relation.

$$\sigma_{dept\_name = \text{“Physics”}} (instructor)$$

ID	name	dept_name	salary
22222	Einstein	Physics	95000
33456	Gold	Physics	87000

Project Operation – Example

- Relation *r*:

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- $\prod_{A,C}(r)$

Project Operation – Example

- Relation *r*:

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- $\Pi_{A,C}\,(r)$

A	C
α	1
α	1
β	1
β	2

=

A	C
α	1
β	1
β	2

Project Operation

- ∏ Unary Operator
- Notation:

$$\prod_{A_1, A_2, \ldots, A_k} (r)$$

  where A_1, A_2 are attribute names and *r* is a relation name.
- The result is defined as the relation of *k* columns obtained by erasing the columns that are not listed
- **Duplicate rows are removed from result, since relations are sets**
- Example: To eliminate the *dept_name* attribute of *instructor*

$$\prod_{ID,\ name,\ salary} (instructor)$$

Project Operation

ID	name	dept_name	salary
10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

Figure 6.1 The *instructor* relation.

$$\Pi_{ID,\ name,\ salary}(instructor)$$

ID	name	salary
10101	Srinivasan	65000
12121	Wu	90000
15151	Mozart	40000
22222	Einstein	95000
32343	El Said	60000
33456	Gold	87000
45565	Katz	75000
58583	Califieri	62000
76543	Singh	80000
76766	Crick	72000
83821	Brandt	92000
98345	Kim	80000

Union Operation – Example

- Relations *r, s:*

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r ∪ s:

Union Operation – Example

- Relations *r, s:*

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r ∪ s:

A	B
α	1
α	2
β	1
β	3

Union Operation

- $\cup$ Binary Operator
- Notation: $r \cup s$
- Defined as:

  $r \cup s = \{t \mid t \in r \text{ or } t \in s\}$

- For $r \cup s$ to be valid.

  1. r, s must have the *same* **arity** (same number of attributes)
  2. The attribute domains must be **compatible** (example: 2nd column of r deals with the same type of values as does the 2nd column of s)

- Example: to find all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or in both

$$\Pi_{course\_id}(\sigma_{semester=\text{“Fall”} \ \wedge \ year=2009}(section)) \ \cup$$

$$\Pi_{course\_id}(\sigma_{semester=\text{“Spring”} \ \wedge \ year=2010}(section))$$

Union Operation

course_id	*sec_id*	*semester*	*year*	*building*	*room_number*	*time_slot_id*
BIO-101	1	Summer	2009	Painter	514	B
BIO-301	1	Summer	2010	Painter	514	A
CS-101	1	Fall	2009	Packard	101	H
CS-101	1	Spring	2010	Packard	101	F
CS-190	1	Spring	2009	Taylor	3128	E
CS-190	2	Spring	2009	Taylor	3128	A
CS-315	1	Spring	2010	Watson	120	D
CS-319	1	Spring	2010	Watson	100	B
CS-319	2	Spring	2010	Taylor	3128	C
CS-347	1	Fall	2009	Taylor	3128	A
EE-181	1	Spring	2009	Taylor	3128	C
FIN-201	1	Spring	2010	Packard	101	B
HIS-351	1	Spring	2010	Painter	514	C
MU-199	1	Spring	2010	Packard	101	D
PHY-101	1	Fall	2009	Watson	100	A

Figure 6.4 The *section* relation.

course_id
CS-101
CS-315
CS-319
CS-347
FIN-201
HIS-351
MU-199
PHY-101

$\Pi_{course\_id}\ (\sigma_{semester = \text{"Fall"} \wedge year=2009}\ (section))$

$\Pi_{course\_id}\ (\sigma_{semester = \text{"Spring"} \wedge year=2010}\ (section))$

Set difference of two relations

- Relations *r*, *s*:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- *r* – *s*:

Set difference of two relations

- Relations *r*, *s*:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- *r* – *s:*

A	B
α	1
β	1

Set Difference Operation

- '-' Binary Operator
- Notation $r - s$
- Defined as:

  $r - s = \{t \mid t \in r$ **and** $t \notin s\}$

- Set differences must be taken between **compatible** relations.
  - r and s must have the same arity
  - attribute domains of r and s must be compatible
- Example: to find all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$\Pi_{course\_id}(\sigma_{semester=\text{“Fall”} \wedge year=2009}(section)) -$$
$$\Pi_{course\_id}(\sigma_{semester=\text{“Spring”} \wedge year=2010}(section))$$

course_id	*sec_id*	*semester*	*year*	*building*	*room_number*	*time_slot_id*
BIO-101	1	Summer	2009	Painter	514	B
BIO-301	1	Summer	2010	Painter	514	A
CS-101	1	Fall	2009	Packard	101	H
CS-101	1	Spring	2010	Packard	101	F
CS-190	1	Spring	2009	Taylor	3128	E
CS-190	2	Spring	2009	Taylor	3128	A
CS-315	1	Spring	2010	Watson	120	D
CS-319	1	Spring	2010	Watson	100	B
CS-319	2	Spring	2010	Taylor	3128	C
CS-347	1	Fall	2009	Taylor	3128	A
EE-181	1	Spring	2009	Taylor	3128	C
FIN-201	1	Spring	2010	Packard	101	B
HIS-351	1	Spring	2010	Painter	514	C
MU-199	1	Spring	2010	Packard	101	D
PHY-101	1	Fall	2009	Watson	100	A

Figure 6.4 The *section* relation.

$$\Pi_{course\_id} (\sigma_{semester = \text{"Fall"} \wedge year=2009} (section)) -$$
$$\Pi_{course\_id} (\sigma_{semester = \text{"Spring"} \wedge year=2010} (section))$$

course_id
CS-347
PHY-101

Cartesian-Product Operation – Example

- Relations *r, s*:

A	B
α	1
β	2

r

C	D	E
α	10	a
β	10	a
β	20	b
γ	10	b

s

- *r* x *s*:

Cartesian-Product Operation – Example

- Relations *r, s*:

A	B
α	1
β	2

r

C	D	E
α	10	a
β	10	a
β	20	b
γ	10	b

s

- *r* x *s*:

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

Cartesian-Product Operation

- 'X' Binary Operator
- Notation *r* x *s*
- Defined as:

  $r \times s = \{t\,q \mid t \in r \textbf{ and } q \in s\}$

- Assume that **attributes of r(R) and s(S) are disjoint (i.e., *R* ∩ *S* = ∅**)
- If attributes of *r(R)* and *s(S)* are not disjoint, then renaming must be used

Composition of Operations

- Can build expressions using multiple operations
- Example: $\sigma_{A=C}(r \; x \; s)$
- $r \; x \; s$

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

- $\sigma_{A=C}(r \; x \; s)$

A	B	C	D	E
α	1	α	10	a
β	2	β	10	a
β	2	β	20	b

Rename Operation

- 'ρ ' Unary Operator
- Allows us to name, and therefore to refer to, the results of relational-algebra expressions
- Allows us to refer to a relation by more than one name
- Example:

$$\rho_X(E)$$

  returns the expression *E* under the name *X*
- If a relational-algebra expression *E* has arity *n*, then

$$\rho_{x(A_1, A_2, \ldots, A_n)}(E)$$

  returns the result of expression *E* under the name *X*, and with the attributes renamed to $A_1, A_2, \ldots, A_n$

Instructor and Salary

ID	*name*	*salary*
10101	Srinivasan	65000
12121	Wu	90000
15151	Mozart	40000
22222	Einstein	95000
32343	El Said	60000
33456	Gold	87000
45565	Katz	75000
58583	Califieri	62000
76543	Singh	80000
76766	Crick	72000
83821	Brandt	92000
98345	Kim	80000

Example Query

- **Find the largest salary in the university**
  - Step 1: find instructor salaries that are less than some other instructor salary (i.e. not maximum)
    - using a copy of *instructor* under a new name *d*
  - $\Pi_{instructor.salary}(\sigma_{instructor.salary < d.salary}(instructor \times \rho_d(instructor)))$

salary
65000
90000
40000
60000
87000
75000
62000
72000
80000
92000

Example Query

- **Find the largest salary in the university**

  - Step 2: Find the largest salary

    - Π_{salary} *(instructor)* –

      $\Pi_{instructor.salary}\,(\sigma_{\,instructor.salary\,<\,d.salary}$
      $(instructor \times \rho_{d}\,(instructor)))$

salary
95000

Example Queries

- **Find the names of all instructors in the Physics department, along with the *course_id* of all courses they have taught**
  - Query 1

$$\Pi_{instructor.ID,course\_id}(\sigma_{dept\_name=\text{“Physics”}}(\sigma_{instructor.ID=teaches.ID}(instructor \times teaches)))$$

  - Query 2

$$\Pi_{instructor.ID,course\_id}(\sigma_{instructor.ID=teaches.ID}(\sigma_{dept\_name=\text{“Physics”}}(instructor) \times teaches))$$

Formal Definition

- A basic expression in the relational algebra consists of either one of the following:
  - A relation in the database
  - A constant relation
- Let E_1 and E_2 be relational-algebra expressions; the following are all relational-algebra expressions:
  - $E_1 \cup E_2$
  - $E_1 - E_2$
  - $E_1 \times E_2$
  - $\sigma_p (E_1)$, P is a predicate on attributes in E_1
  - $\prod_S(E_1)$, S is a list consisting of some of the attributes in E_1
  - $\rho_x (E_1)$, x is the new name for the result of E_1

Additional Operations

- Additional operations that do not add any power to the relational algebra, but that simplify common queries
- Whenever it is used, utilizes basic operations

  - Set intersection
  - Assignment
  - Join

Set-Intersection Operation

- Notation: $r \cap s$
- Defined as:
- $r \cap s = \{ t \mid t \in r \textbf{ and } t \in s \}$
- Assume:
  - *r*, *s* have the *same arity*
  - attributes of *r* and *s* are compatible
- Note: $\boldsymbol{r \cap s = r - (r - s)}$

Set-Intersection Operation – Example

- Relation *r, s*:

r

A	B
α	1
α	2
β	1

s

A	B
α	2
β	3

- $r \cap s$

A	B
α	2

Assignment Operation

- The assignment operation ($\leftarrow$) provides a **convenient way to express complex queries**
  - Write query as a sequential program consisting of
    - a series of assignments
    - followed by an expression whose value is displayed as a result of the query
  - Assignment must always be made to a temporary relation variable

Join Operation

- Cartesian Product of two relation
  - Gives all the possible tuples that are paired together
  - **Also called cross join**
  - But it might **NOT** be feasible for
    - Huge relations where number of tuples are in thousands and
    - Attributes of both relations are considerable large
- **Join Operation**
  - Combination of Cartesian product followed by selection process
  - Join operation pairs two tuples from different relations if and only if the given join condition is satisfied
  - Types
    - Inner Join (Theta Join, Equi Join, Natural Join)
    - Outer Join (Left outer Join, Right outer Join, Full outer Join)

Cartesian-Product Operation – Example

- Relations *r, s*:

A	B
α	1
β	2

r

C	D	E
α	10	a
β	10	a
β	20	b
γ	10	b

s

- *r* x *s*:

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

Inner Join

- An inner-join **process includes only tuples with matching attributes,** rest are discarded in resulting relation
  - Theta Join
  - Equi Join
  - Natural Join

Theta (θ) Join

- θ in Theta join is the join condition
- Combines tuples from different relations provided they satisfy the theta condition
- The **theta join** operation $r \bowtie_{\theta} s$ is defined as
  - $r \bowtie_{\theta} s = \sigma_{\theta} (r \times s)$
- R1 $\bowtie_{\theta}$ R2
  - R1 and R2 are relations with their attributes (A1, A2, .., An) and (B1, B2,.. ,Bn) such that **NO Attribute matches** that is R1 ∩ R2 = Φ
- Theta join can use all kinds of comparison operators

Theta (θ) Join

Student

SID	Name	Std
101	Alex	10
102	Maria	11

Subjects

Class	Subject
10	Math
10	English
11	Music
11	Sports

- STUDENT $\bowtie_{\text{Student.Std = Subject.Class}}$ SUBJECT

SID	Name	Std	Class	Subject
101	Alex	10	10	Math
101	Alex	10	10	English
102	Maria	11	11	Music
102	Maria	11	11	Sports

Equi Join

- When Theta join uses only **equality** comparison operator it is said to be Equi-Join
- The early example conrresponds to equi join

Natural Join

- Does not use any comparison operator
- Does not concatenate the way Cartesian product does
- Instead, Natural Join can only be performed if the there **is at least one common attribute exists between relation**
  - Those attributes must have same name and domain
- Acts on those matching attributes where the values of attributes in both relation is same

Natural Join Operation

- Notation: r ⋈ s
- Let *r* and *s* be relations on schemas *R* and *S* respectively
  Then, r ⋈ s is a relation on schema $R \cup S$ obtained as follows:
  - Consider each pair of tuples t_r from *r* and t_s from *s*
  - If t_r and t_s have the same value on each of the attributes in $R \cap S$, add a tuple *t* to the result, where
    - *t* has the same value as t_r on *r*
    - *t* has the same value as t_s on *s*
- Example:

  $R = (A, \mathbf{B}, C, \mathbf{D})$

  $S = (E, \mathbf{B}, \mathbf{D})$

  - Result schema = $(A, \mathbf{B}, C, \mathbf{D}, E)$
  - $r \bowtie s$ is defined as:

    $$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{\mathbf{r.B = s.B} \wedge \mathbf{r.D = s.D}}\ (r \times s))$$

Natural Join Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	ε

s

- r ⋈ s

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

Natural Join – Example

- Relation *instructor*

ID	*name*	*dept_name*
10101	Srinivasan	Comp. Sci.
12121	Wu	Finance
15151	Mozart	Music

- Relation *teaches*

ID	*course_id*
10101	CS-101
12121	FIN-201
76766	BIO-101

- Join

  instructor ⋈ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201

Natural Join

- Find the names of all instructors in the Comp. Sci. department together with the course titles of all the courses that the instructors teach
  - $\Pi_{name,\ title}(\sigma_{dept\_name=\text{“Comp. Sci.”}}(instructor \bowtie teaches \bowtie course))$
- Natural join is associative
  - $(instructor \bowtie teaches) \bowtie course$ is equivalent to $instructor \bowtie (teaches \bowtie course)$

Exercise

Courses

CID	Course	Dept
CS01	Database	CS
ME01	Mechanics	ME
EE01	Electronics	EE

HoD

Dept	Head
CS	Alex
ME	Maya
EE	Mira

Display all courses with their department Head names.

Courses ⋈ HoD

Dept	CID	Course	Head
CS	CS01	Database	Alex
ME	ME01	Mechanics	Maya
EE	EE01	Electronics	Mira

If…

Courses

CID	Course	Dept
CS01	Database	CS
ME01	Mechanics	ME
EE01	Electronics	EE
MU01	**MusicNotes**	**MU**

HoD

Dept	Head
CS	Alex
ME	Maya
EE	Mira
FIN	**XYZ**

Display all courses with their department Head names.

Courses $\bowtie$ HoD

Dept	CID	Course	Head
CS	CS01	Database	Alex
ME	ME01	Mechanics	Maya
EE	EE01	Electronics	Mira

?

Outer Join

- An extension of the join operation that avoids loss of information
- Computes the join and then adds tuples from one relation that does not match tuples in the other relation to the resultant relation of the join
- Uses *null* values:
  - *null* signifies that the value is unknown or does not exist
  - All comparisons involving *null* are (roughly speaking) **false** by definition

- Outer join can be expressed using basic operations
  - e.g. $r ⟕ s$ can be written as

$$(r \bowtie s) \ \text{U} \ (r - \prod_R(r \bowtie s) \ \text{x} \ \{(null, \ldots, null)\}$$

Outer Join – Example

- Relation *instructor*

ID	*name*	*dept_name*
10101	Srinivasan	Comp. Sci.
12121	Wu	Finance
15151	Mozart	Music

- Relation *teaches*

ID	*course_id*
10101	CS-101
12121	FIN-201
76766	BIO-101

Outer Join – Example

- Relation *instructor*

ID	*name*	*dept_name*
10101	Srinivasan	Comp. Sci.
12121	Wu	Finance
15151	Mozart	Music

- Relation *teaches*

ID	*course_id*
10101	CS-101
12121	FIN-201
76766	BIO-101

- Natural Join

instructor $\bowtie$ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201

Outer Join – Example

- Relation *instructor1*

ID	*name*	*dept_name*
10101	Srinivasan	Comp. Sci.
12121	Wu	Finance
15151	Mozart	Music

- Relation *teaches1*

ID	*course_id*
10101	CS-101
12121	FIN-201
76766	BIO-101

- Join

instructor ⋈ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201

- Left Outer Join

instructor ⟕ *teaches*

select * from instructor left outer join teaches on instructor.ID = teaches.ID;

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
15151	**Mozart**	**Music**	***null***

Outer Join – Example

- Right Outer Join

  instructor ⟖ *teaches*

select * from instructor rightt outer join teaches on instructor.ID = teaches.ID;

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
76766	**null**	**null**	**BIO-101**

- Full Outer Join

  instructor ⟗ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
15151	**Mozart**	**Music**	***null***
76766	***null***	***null***	**BIO-101**

Exercise

Courses

A	B
100	Database
101	Mechanics
102	Electronics

HoD

A	C
100	Alex
102	Maya
104	Mira

Courses ⟕ HoD

A	B	C
100	Database	Alex
101	Mechanics	null
102	Electronics	Maya

Courses ⟖ HoD

A	B	C
100	Database	Alex
102	Electronics	Maya
104	null	Mira

Courses ⟗ HoD

Courses.A	Courses.B	HoD.A	HoD.C
100	Database	100	Alex
101	Mechanics	null	null
102	Electronics	102	Maya
null	null	104	Mira

Exercise

Employee

Name	EmpId	DeptName
Harry	3415	Finance
Sally	2241	Sales
George	3401	Finance
Harriet	2202	Sales

Dept

DeptName	Manager
Finance	George
Sales	Harriet
Production	Charles

Employee ⋈ Dept

Name	EmpId	DeptName	Manager
Harry	3415	Finance	George
Sally	2241	Sales	Harriet
George	3401	Finance	George
Harriet	2202	Sales	Harriet

Employee ⟕ Dept

Employee ⟖ Dept

Employee ⟗ Dept

Extended Relational-Algebra-Operations

- Generalized Projection
- Aggregate Functions

Generalized Projection

- Extends the projection operation by **allowing arithmetic functions to be used in the projection list.**

$$\prod_{F_1, F_2, \ldots, F_n}(E)$$

- E is any relational-algebra expression
- Each of F_1, F_2, …, F_n are are arithmetic expressions involving constants and attributes in the schema of E.
- Given relation *instructor(ID, name, dept_name,* salary) where **salary is annual salary, get the same information but with monthly salary**

$$\prod_{ID,\ name,\ dept\_name,\ \mathbf{salary/12}} (instructor)$$

Aggregate Functions and Operations

- **Aggregation function** takes a collection of values and returns a single value as a result

  avg: average value
  min: minimum value
  max: maximum value
  sum: sum of values
  count: number of values

- **Aggregate operation** in relational algebra

$$_{G_1, G_2, \ldots, G_n}\ \mathcal{G}\ _{F_1(A_1), F_2(A_2), \ldots, F_n(A_n)}(E)$$

  E is any relational-algebra expression
  - G_1, G_2 …, G_n is a list of attributes on which to group (can be empty)
  - Each F_i is an aggregate function
  - Each A_i is an attribute name

- Note: Some books/articles use γ instead of $\mathcal{G}$ (Calligraphic G)

Aggregate Operation – Example

- Relation *r*:

A	*B*	*C*
α	α	7
α	β	7
β	β	3
β	β	10

- $\mathcal{G}_{\textbf{sum(c)}}\,(\mathrm{r})$

sum(*c*)
27

Aggregate Operation – Example

- Find the average salary in each department

$$_{dept\_name}\mathcal{G}_{\textbf{avg}(salary)}\ (instructor)$$

ID	*name*	*dept_name*	*salary*
76766	Crick	Biology	72000
45565	Katz	Comp. Sci.	75000
10101	Srinivasan	Comp. Sci.	65000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
22222	Einstein	Physics	95000

dept_name	*salary*
Biology	72000
Comp. Sci.	77333
Elec. Eng.	80000
Finance	85000
History	61000
Music	40000
Physics	91000

Aggregate Functions (Cont.)

- Result of aggregation does not have a name
  - Can use rename operation to give it a name
  - For convenience, we permit renaming as part of aggregate operation

$$_{dept\_name}\mathcal{G}_{\textbf{avg}(salary)\ \textbf{as}\ avg\_sal}(instructor)$$

SQL and Relational Algebra

- **select** *A1, A2, .. An*
  from *r1, r2, …, rm*
  where P

  is equivalent to the following relational algebra expression

  $$\prod_{A1,\ ..,\ An} (\sigma_P (r1 \times r2 \times .. \times rm))$$

- **select** *A1, A2,* **sum***(A3)*
  from *r1, r2, …, rm*
  where P
  group by *A1, A2*

  is equivalent to the following relational algebra expression

  $$_{A1,\ A2}\,\mathcal{G}_{\textbf{sum}(A3)} (\sigma_P (r1 \times r2 \times .. \times rm)))$$

SQL and Relational Algebra

- Only selected column to display
- More generally, the non-aggregated attributes in the **select** clause may be a subset of the **group by** attributes, in which case the equivalence is as follows:

  select *A1*, **sum**(*A3*)
  from *r1*, *r2*, …, *rm*
  where P
  group by *A1*, *A2*

  is equivalent to the following relational algebra expression

  $$\Pi_{\cancel{A1, sumA3}}(\ _{A1,A2}\mathcal{G}\ \textbf{sum}(A3)\ \textbf{as}\ sumA3(\sigma_P(r1 \times r2 \times .. \times rm)))$$

Exercise

- For these queries design relational model, write relational algebra and sql queries.

1. Retrieve the name and address of all employees who work for the ‘Physics” department.
2. For the project number 10, retrieve the controlling department number and department manager’s last name and address.
3. Find the names of employees who work on all the projects controlled by the department “Physics”.
4. Retrieve the list of project numbers for projects that involve an employee whose last name is “Patel”, either as a worker or as a manager of the department that controls the project.

Null Values

- It is possible for tuples to have a null value, denoted by *null*, for some of their attributes
- *null* **signifies an unknown value or that a value does not exist**
- The **result of any arithmetic expression involving *null* is *null***
- **Aggregate functions simply ignore null values** (as in SQL)
- For duplicate elimination and grouping, null is treated like any other value, and two nulls are assumed to be the same (as in SQL)

Null Values

- Comparisons with null values return the special truth value: *unknown*
  - If *false* was used instead of *unknown*, then *not (A < 5)* would not be equivalent to *A >= 5*
- Three-valued logic using the truth value *unknown*:
  - OR: (*unknown* **or** *true*) = *true*,
    (*unknown* **or** *false*) = *unknown*
    (*unknown* **or** *unknown*) = *unknown*
  - AND: (*true* **and** *unknown*) = *unknown,*
    (*false* **and** *unknown*) = *false,*
    (*unknown* **and** *unknown*) = *unknown*
  - NOT*:* (**not** *unknown*) = *unknown*
  - In SQL "*P* **is unknown**" evaluates to true if predicate *P* evaluates to *unknown*
- Result of select predicate is treated as *false* if it evaluates to *unknown*

Modification of the Database

- The content of the database may be modified using the following operations:
  - Deletion
  - Insertion
  - Updating
- All these operations can be expressed using the assignment operator

Deletion

- A delete request is expressed similarly to a query, except instead of displaying tuples to the user, the selected tuples are removed from the database
- Can delete only whole tuples; cannot delete values on only particular attributes
- A deletion is expressed in relational algebra by:

  $r \leftarrow r - E$

  where r is a relation and E is a relational algebra query

Deletion Examples

- Delete all account records in the Perryridge branch.

$$account \leftarrow account - \sigma_{branch\_name = \text{"Perryridge"}}(account)$$

- Delete all loan records with amount in the range of 0 to 50

$$loan \leftarrow loan - \sigma_{amount \geq 0 \text{ and } amount \leq 50}(loan)$$

- Delete all accounts at branches located in Needham.

$$r_1 \leftarrow \sigma_{branch\_city = \text{"Needham"}}(account \bowtie branch)$$

$$r_2 \leftarrow \prod_{account\_number,\ branch\_name,\ balance}(r_1)$$

$$r_3 \leftarrow \prod_{customer\_name,\ account\_number}(r_2 \bowtie \text{depositor})$$

$$account \leftarrow \text{account} - r_2$$

$$depositor \leftarrow \text{depositor} - r_3$$

Insertion

- To insert data into a relation, we either:
  - specify a tuple to be inserted
  - write a query whose result is a set of tuples to be inserted
- in relational algebra, an insertion is expressed by:

  $r \leftarrow r \cup E$

  where *r* is a relation and *E* is a relational algebra expression.
- The insertion of a single tuple is expressed by letting *E* be a constant relation containing one tuple.

Insertion Examples

- Insert information in the database specifying that Smith has $1200 in account A-973 at the Perryridge branch.

$$account \leftarrow account \cup \{(\text{“A-973”}, \text{“Perryridge”}, 1200)\}$$

$$\text{depositor} \leftarrow depositor \cup \{(\text{“Smith”}, \text{“A-973”})\}$$

Updating

- A mechanism to change a value in a tuple without charging *all* values in the tuple
- Use the generalized projection operator to do this task

$$r \leftarrow \Pi_{F_1, F_2, \ldots, F_l,}(r)$$

- Each F_i is either
  - the I^{th} attribute of *r*, if the I^{th} attribute is not updated, or,
  - if the attribute is to be updated F_i is an expression, involving only constants and the attributes of *r*, which gives the new value for the attribute

Update Examples

- Make interest payments by increasing all balances by 5 percent.

$$account \leftarrow \Pi_{account\_number,\ branch\_name,\ balance\ *\ 1.05}\ (account)$$

- Pay all accounts with balances over $10,000 6 percent interest and pay all others 5 percent

$$account \leftarrow \Pi_{account\_number,\ branch\_name,\ balance\ *\ 1.06}\ (\sigma_{BAL\ >\ 10000}\ (account\ ))$$
$$\cup\ \Pi_{account\_number,\ branch\_name,\ balance\ *\ 1.05}\ (\sigma_{BAL\ \leq\ 10000}\ (account))$$

End

Ch. 10 Storage and File Structure

Division Operator

- Given relations r(R) and s(S), such that $S \subset R$, $r \div s$ is the largest relation t(R-S) such that

  $$t \times s \subseteq r$$

- E.g. let $r(ID, course\_id) = \Pi_{ID,\ course\_id}\ (takes)$ and

  $s(course\_id) = \Pi_{course\_id}\ (\sigma_{dept\_name=\text{“Biology”}}(course))$

  then $r \div s$ gives us students who have taken all courses in the Biology department

- Can write $r \div s$ as

  $temp1 \leftarrow \Pi_{R-S}\ (r)$

  $temp2 \leftarrow \Pi_{R-S}\ ((temp1 \times s) - \Pi_{R-S,S}\ (r))$

  $result = temp1 - temp2$

  - The result to the right of the $\leftarrow$ is assigned to the relation variable on the left of the $\leftarrow$
  - May use variable in subsequent expressions

Tuple Relational Calculus

Tuple Relational Calculus

- A nonprocedural query language, where each query is of the form
  $\{t \mid P(t)\}$
- It is the set of all tuples *t* such that predicate *P* is true for *t*
- *t* is a *tuple variable*, $t[A]$ denotes the value of tuple *t* on attribute *A*
- $t \in r$ denotes that tuple *t* is in relation *r*
- *P* is a *formula* similar to that of the predicate calculus

Predicate Calculus Formula

1. Set of attributes and constants
2. Set of comparison operators: (e.g., $<$, $\leq$, $=$, $\neq$, $>$, $\geq$)
3. Set of connectives: and ($\wedge$), or ($\vee$), not ($\neg$)
4. Implication ($\Rightarrow$): x $\Rightarrow$ y, if x if true, then y is true

   $$x \Rightarrow y \equiv \neg x \vee y$$

5. Set of quantifiers:
   - $\exists\, t \in r\, (Q\,(t\,))$ ≡ ”there exists” a tuple in *t* in relation *r* such that predicate *Q* (*t*) is true
   - $\forall t \in r\, (Q\,(t\,))$ ≡ *Q* is true “for all” tuples *t* in relation *r*

Example Queries

- Find the *ID, name, dept_name, salary* for instructors whose salary is greater than $80,000

$$\{t \mid t \in instructor \land t\,[salary\,] > 80000\}$$

- As in the previous query, but output only the *ID* attribute value

$$\{t \mid \exists\, s \in \text{instructor}\ (t\,[ID\,] = s\,[ID\,] \land s\,[salary\,] > 80000)\}$$

Notice that a relation on schema (*ID*) is implicitly defined by the query

Example Queries

- Find the names of all instructors whose department is in the Watson building

$$\{t \mid \exists s \in instructor\ (t\ [name\ ] = s\ [name\ ]$$
$$\wedge\ \exists u \in department\ (u\ [dept\_name\ ] = s[dept\_name]\ \text{“}$$
$$\wedge\ \ u\ [building] = \text{“Watson”}\ ))\}$$

- Find the set of all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or both

$$\{t \mid \exists s \in section\ (t\ [course\_id\ ] = s\ [course\_id\ ] \wedge$$
$$s\ [semester] = \text{“Fall”} \wedge s\ [\text{year}] = 2009$$
$$\text{v}\ \exists u \in section\ (t\ \ [course\_id\ ] = u\ [course\_id\ ] \wedge$$
$$u\ [semester] = \text{“Spring”} \wedge u\ [\text{year}] = 2010)\}$$

Example Queries

- Find the set of all courses taught in the Fall 2009 semester, and in the Spring 2010 semester

$$
\begin{aligned}
\{t \mid \exists s \in \textit{section}\ (&t\,[\textit{course\_id}\,] = s\,[\textit{course\_id}\,]\ \wedge \\
&s\,[\textit{semester}] = \text{“Fall”} \wedge s\,[\text{year}] = 2009 \\
\wedge\ \exists u \in \textit{section}\ (&t\,[\textit{course\_id}\,] = u\,[\textit{course\_id}\,]\ \wedge \\
&u\,[\textit{semester}] = \text{“Spring”} \wedge u\,[\text{year}] = 2010)\}
\end{aligned}
$$

- Find the set of all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$
\begin{aligned}
\{t \mid \exists s \in \textit{section}\ (&t\,[\textit{course\_id}\,] = s\,[\textit{course\_id}\,]\ \wedge \\
&s\,[\textit{semester}] = \text{“Fall”} \wedge s\,[\text{year}] = 2009 \\
\wedge \neg \exists u \in \textit{section}\ (&t\,[\textit{course\_id}\,] = u\,[\textit{course\_id}\,]\ \wedge \\
&u\,[\textit{semester}] = \text{“Spring”} \wedge u\,[\text{year}] = 2010)\}
\end{aligned}
$$

Safety of Expressions

- It is possible to write tuple calculus expressions that generate infinite relations.
- For example, $\{ t \mid \neg\, t \in r \}$ results in an infinite relation if the domain of any attribute of relation *r* is infinite
- To guard against the problem, we restrict the set of allowable expressions to safe expressions.
- An expression $\{t \mid P(t)\}$ in the tuple relational calculus is *safe* if every component of *t* appears in one of the relations, tuples, or constants that appear in *P*
  - NOTE: this is more than just a syntax condition.
    - E.g. $\{ t \mid t[A] = 5 \lor \textbf{true} \}$ is not safe --- it defines an infinite set with attribute values that do not appear in any relation or tuples or constants in *P*.

Universal Quantification

- Find all students who have taken all courses offered in the Biology department
  - $\{t \mid \exists\, r \in student\ (t\,[ID] = r\,[ID]) \wedge$
    $(\forall\, u \in course\ (u\,[dept\_name] = \text{“Biology”} \Rightarrow$
    $\exists\, s \in takes\ (t\,[ID] = s\,[ID\,] \wedge$
    $s\,[course\_id] = u\,[course\_id])))\}$
  - Note that without the existential quantification on student, the above query would be unsafe if the Biology department has not offered any courses.

Domain Relational Calculus

Domain Relational Calculus

- A nonprocedural query language equivalent in power to the tuple relational calculus
- Each query is an expression of the form:

$$\{ < x_1, x_2, \ldots, x_n > \mid P(x_1, x_2, \ldots, x_n)\}$$

  - $x_1, x_2, \ldots, x_n$ represent domain variables
  - P represents a formula similar to that of the predicate calculus

Example Queries

- Find the *ID, name, dept_name, salary* for instructors whose salary is greater than \$80,000
  - $\{<i, n, d, s> \mid <i, n, d, s> \in instructor \land s > 80000\}$
- As in the previous query, but output only the *ID* attribute value
  - $\{<i> \mid <i, n, d, s> \in instructor \land s > 80000\}$
- Find the names of all instructors whose department is in the Watson building

$$\{<n> \mid \exists\, i, d, s\, (<i, n, d, s> \in instructor \land \exists\, \text{b, a}\, (<d, b, a> \in department \land b = \text{“Watson”}))\}$$

Example Queries

- Find the set of all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or both

$$
\begin{aligned}
\{<c> \mid\ & \exists\, a, s, y, b, r, t\ (<c, a, s, y, b, t> \in section\ \wedge \\
& \quad s = \text{“Fall”} \wedge y = 2009\ ) \\
& \vee \exists\, a, s, y, b, r, t\,(<c, a, s, y, b, t> \in section\ ] \wedge \\
& \quad s = \text{“Spring”} \wedge y = 2010)\}
\end{aligned}
$$

This case can also be written as

$$
\begin{aligned}
\{<c> \mid\ & \exists\, a, s, y, b, r, t\ (<c, a, s, y, b, t> \in section\ \wedge \\
& \quad (\,(s = \text{“Fall”} \wedge y = 2009\ )\ \vee (s = \text{“Spring”} \wedge y = 2010))\}
\end{aligned}
$$

- Find the set of all courses taught in the Fall 2009 semester, and in the Spring 2010 semester

$$
\begin{aligned}
\{<c> \mid\ & \exists\, a, s, y, b, r, t\ (<c, a, s, y, b, t> \in section\ \wedge \\
& \quad s = \text{“Fall”} \wedge y = 2009\ ) \\
& \wedge \exists\, a, s, y, b, r, t\,(<c, a, s, y, b, t> \in section\ ] \wedge \\
& \quad s = \text{“Spring”} \wedge y = 2010)\}
\end{aligned}
$$

Safety of Expressions

The expression:

$$\{ < x_1, x_2, \ldots, x_n > \mid P(x_1, x_2, \ldots, x_n)\}$$

is safe if all of the following hold:

1. All values that appear in tuples of the expression are values from *dom* (P) (that is, the values appear either in P or in a tuple of a relation mentioned in P).
2. For every "there exists" subformula of the form $\exists\, x\, (P_1(x))$, the subformula is true if and only if there is a value of x in *dom* (P_1) such that $P_1(x)$ is true.
3. For every "for all" subformula of the form $\forall_x\, (P_1\, (x))$, the subformula is true if and only if $P_1(x)$ is true for all values x from *dom* (P_1).

Universal Quantification

- Find all students who have taken all courses offered in the Biology department
  - $\{< i > \mid \exists\ n, d, tc\ (< i, n, d, tc > \in student\ \land$
    $(\forall\ ci, ti, dn, cr\ (< ci, ti, dn, cr > \in course \land dn = \text{“Biology”}$
    $\Rightarrow\ \exists\ si, se, y, g\ (<i, ci, si, se, y, g> \in takes\ ))\}$
  - Note that without the existential quantification on student, the above query would be unsafe if the Biology department has not offered any courses.

* Above query fixes bug in page 246, last query

Modification of the Database

- The content of the database may be modified using the following operations:
  - Deletion
  - Insertion
  - Updating
- All these operations can be expressed using the assignment operator

Deletion

- A delete request is expressed similarly to a query, except instead of displaying tuples to the user, the selected tuples are removed from the database
- Can delete only whole tuples; cannot delete values on only particular attributes
- A deletion is expressed in relational algebra by:

  $r \leftarrow r - E$

  where r is a relation and E is a relational algebra query

Deletion Examples

- Delete all account records in the Perryridge branch.

$$account \leftarrow account - \sigma_{branch\_name = \text{"Perryridge"}}(account)$$

- Delete all loan records with amount in the range of 0 to 50

$$loan \leftarrow loan - \sigma_{amount \geq 0 \text{ and } amount \leq 50}(loan)$$

- Delete all accounts at branches located in Needham.

$$r_1 \leftarrow \sigma_{branch\_city = \text{"Needham"}}(account \bowtie branch)$$

$$r_2 \leftarrow \Pi_{account\_number,\ branch\_name,\ balance}(r_1)$$

$$r_3 \leftarrow \Pi_{customer\_name,\ account\_number}(r_2 \bowtie \text{depositor})$$

$$account \leftarrow \text{account} - r_2$$

$$depositor \leftarrow \text{depositor} - r_3$$

Insertion

- To insert data into a relation, we either:
  - specify a tuple to be inserted
  - write a query whose result is a set of tuples to be inserted
- in relational algebra, an insertion is expressed by:

  $r \leftarrow r \cup E$

  where *r* is a relation and *E* is a relational algebra expression.
- The insertion of a single tuple is expressed by letting *E* be a constant relation containing one tuple.

Insertion Examples

- Insert information in the database specifying that Smith has $1200 in account A-973 at the Perryridge branch.

$$account \leftarrow account \cup \{(\text{“A-973”}, \text{“Perryridge”}, 1200)\}$$

$$\text{depositor} \leftarrow depositor \cup \{(\text{“Smith”}, \text{“A-973”})\}$$

- Provide as a gift for all loan customers in the Perryridge branch, a $200 savings account. Let the loan number serve as the account number for the new savings account.

$$r_1 \leftarrow (\sigma_{branch\_name\ =\ \text{“Perryridge”}}(borrower \bowtie \text{loan}))$$

$$account \leftarrow account \cup \Pi_{loan\_number,\ branch\_name,\ 200}\ (r_1)$$

$$\text{depositor} \leftarrow depositor \cup \Pi_{customer\_name,\ loan\_number}\ (r_1)$$

Updating

- A mechanism to change a value in a tuple without charging *all* values in the tuple
- Use the generalized projection operator to do this task

$$r \leftarrow \prod_{F_1, F_2, \ldots, F_l,}(r)$$

- Each F_i is either
  - the I^{th} attribute of r, if the I^{th} attribute is not updated, or,
  - if the attribute is to be updated F_i is an expression, involving only constants and the attributes of r, which gives the new value for the attribute

Update Examples

- Make interest payments by increasing all balances by 5 percent.

$$account \leftarrow \prod_{account\_number,\ branch\_name,\ balance\ *\ 1.05} (account)$$

- Pay all accounts with balances over $10,000 6 percent interest and pay all others 5 percent

$$account \leftarrow \prod_{account\_number,\ branch\_name,\ balance\ *\ 1.06} (\sigma_{BAL\ >\ 10000} (account\ )) \cup \prod_{account\_number,\ branch\_name,\ balance\ *\ 1.05} (\sigma_{BAL\ \leq\ 10000} (account))$$

Example Queries

- Find the names of all customers who have a loan and an account at bank.

$$\Pi_{customer\_name}\ (borrower) \cap \Pi_{customer\_name}\ (depositor)$$

- Find the name of all customers who have a loan at the bank and the loan amount

$$\Pi_{customer\_name,\ loan\_number,\ amount}\ (borrower\ \bowtie loan)$$

Example Queries

- Find all customers who have an account from at least the "Downtown" and the Uptown" branches.
  - Query 1

$$\Pi_{customer\_name}\,(\sigma_{branch\_name = \text{"Downtown"}}\,(depositor \bowtie account\,)) \cap$$

$$\Pi_{customer\_name}\,(\sigma_{branch\_name = \text{"Uptown"}}\,(depositor \bowtie account))$$

  - Query 2

$$\Pi_{customer\_name,\, branch\_name}\,(depositor \bowtie account)$$

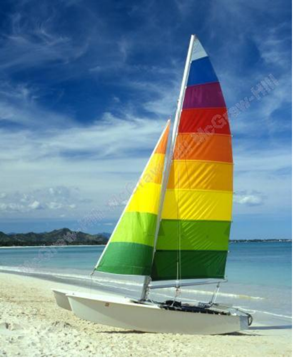
$$\div\ \rho_{temp(branch\_name)}\,(\{(\text{"Downtown"}),\ (\text{"Uptown"})\})$$

Note that Query 2 uses a constant relation.

Bank Example Queries

- Find all customers who have an account at all branches located in Brooklyn city.

$$\Pi_{customer\_name,\ branch\_name}(depositor \bowtie account)$$
$$\div\ \Pi_{branch\_name}(\sigma_{branch\_city\,=\,\text{“Brooklyn”}}(branch))$$



End of Chapter 6

- http://users.cms.caltech.edu/~donnie/cs121/CS121Lec02.pdf
- http://www.inf.unibz.it/~nutt/Teaching/IDBs0910/IDBExercises/4-sol-relAlg.pdf
- http://cir.dcs.uni-pannon.hu/cikkek/DB_relational_algebra_v2.pdf
- https://www.inf.usi.ch/faculty/soule/teaching/2014-spring/04_Relational_Algebra_With_SQL_Equivalents.pdf
- http://users.cms.caltech.edu/~donnie/cs121/CS121Lec03.pdf
- https://www.cs.cornell.edu/projects/btr/bioinformaticsschool/slides/gehrke.pdf
- http://www.cs.toronto.edu/~faye/343/f07/lectures/wk3/03_RAlgebra.pdf