Sardar Vallabhbhai National Institute of Technology, Surat
B. Tech.-I/ M. Sc.-I (Semester-I)
Mid Semester Examination (Jan.-Feb. 2022)
Sub: MA 101 S1 Mathematics-I

Date: 31-01-2022
Time: 09:30 am to 11:30 am (including uploading of answer sheets) **M.M.: [30]**

INSTRUCTIONS:
1) There are total **THREE** questions in the question paper.
2) All questions are compulsory.
3) Figure to the right indicates marks.
4) Follow usual notations.
5) Important Instructions: You must write your Admission Number, Role Number, Contact Number, Email id on TOP of first page of answer sheet and Admissions Number and Page No. with your Signature on all pages.
6) You must upload your answer sheet (single PDF file) on Google classroom or Microsoft team as per your class teacher's instruction latest by **11:30 am** on same day.
7) First verify the number of pages in your PDF file and then only upload. Once you upload the file there after we will not consider any updated file.

Q1. Answer the following questions: **[10]**

(a) Solve the equation $2\ cosh2x\ + 10sinh2x\ =\ 5$, giving your answer in terms of a natural logarithm.

(b) Find n[th] derivative of $y = \dfrac{x}{(x+1)^4}$.

(c) If $u = \tan^{-1}\dfrac{x^3+y^3}{x+y}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$.

(d) If $u(x+y) = x^2+y^2$, prove that $\left(\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\right)^2 = 4\left(1 - \dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\right)$.

(e) Trace the curve $y^2(2a-x) = x^3$.

Q2.(A) If $y = e^{m\cos^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)x\,y_{n+1} - (n^2+m^2)y_n = 0$, and hence find the value of $(y_n)_0$. **[3]**

OR

Q2.(A) Apply Maclaurin's theorem to obtain the terms up to x^4 in the expansion of $\log(1+\sin^2 x)$. **[3]**

Q2.(B) Attempt **Any Three:** **[6]**

(i) Find n[th] derivative of $y = \sin^5 x\cos^3 x$.

(ii) Find the radius of curvature of the curve $x = a\left(t - \dfrac{t^3}{3}\right)$, $y = at^2$.

(iii) If $z = f(x,y), x = r\cos\theta, y = r\sin\theta$, then show that $\left(\dfrac{\partial f}{\partial x}\right)^2 + \left(\dfrac{\partial f}{\partial y}\right)^2 = \left(\dfrac{\partial f}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial f}{\partial \theta}\right)^2$.

(iv) If $z = f(x,y)$ where $x = \log u, y = \log v$, show that $\dfrac{\partial^2 z}{\partial x\,\partial y} = uv\dfrac{\partial^2 z}{\partial u\,\partial v}$.

Q2.(C) Prove that $D^{2n}(x^2-1)^n = (2n)!$. **[1]**

Q3.(A) State Euler's Theorem for homogeneous function and verify it for $u = \dfrac{x^2+y^2}{x+y}$. **[3]**

OR

Q3.(A) Trace the curve $y^2(x^2+y^2)+a^2(x^2-y^2)=0$. **[3]**

Q3.(B) Attempt **Any Three:** **[6]**

(i) Expand $f(x,y)=21+x-20y+4x^2+xy+6y^2$ in Taylor series of maximum order about the point (-1, 2).

(ii) Find the minimum value of $x^2+y^2+z^2$ subject to the condition $xyz=a^3$.

(iii) Verify $J.J^{-1}=1$ for the transformation $x=u,\ y=u\tan v,\ z=w$.

(iv) Trace the curve $r=a\sin 3\theta$.

Q3.(C) Find those asymptotes of the curve $x^2y^2=a^2(x^2+y^2)$ which are parallel to the coordinate axes. **[1]**