Department of Applied Mathematics and Humanities

S. V. National Institute of Technology, Surat, Gujarat

Tutorial-03

B. Tech.-I (Semester-I) Branch-All

Subject: Mathematics-I (MA101 S1)

Topic: Curvature and Radius of curvature

1. Prove that radius of curvature at any point of the Astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is three times the length of the perpendicular from the origin to the tangent at that point.
2. If ρ_1 and ρ_2 are the radii of curvatures at the extremities of a focal chord of the parabola $y^2 = 4ax$, then show that $\rho_1^{-\frac{2}{3}} + \rho_2^{-\frac{2}{3}} = (2a)^{-\frac{2}{3}}$.
3. For the curve $s^2 = 8ay$ Show that $\rho = 4a\sqrt{1 - (\frac{y}{2a})}$.
4. In the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that the radius of curvature at an end of the major axis is equal to the sem-latus rectum of the ellipse.
5. Find the radius of curvature at the origin of the following curves:
   (i) $x^3 + y^3 = 3axy$ (ii) $y^2 = x^2(\frac{a+x}{a-x})$ (iii) $x^4 - y^4 + x^3 - y^3 + x^2 - y^2 + y = 0$.
   Ans. (i) $\frac{3a}{2}$ (ii) $a\sqrt{2}$ (iii) $\rho = \frac{1}{2}$
6. State and prove radius of curvature for cartesian curves.
7. The tangents at two points P, Q on the cycloid x = a(t-sin t), y = a(1-cost) are at right angles; show that if ρ_1, ρ_2 be the radii of curvature at these points, then $\rho_1^2 + \rho_2^2 = 16\,a^2$.
8. For the curve $y = c\,cosh(\frac{x}{c})$, show that $\rho = \dfrac{y^2}{c}$ at $(0, c)$.
9. For the curve $r = a(1 + cos\theta)$, show that $\frac{\rho^2}{r}$ is constant.
10. Find the radius of curvature for the curve $y^2(2 + x) = x(2 - x)$ at a point whose x-coordinate is 1. ~~**Ans**. $\rho = 2\sqrt{3}$~~
11. Find the points on the parabola $y^2 = 8x$ at which the radius of curvature is $7\frac{13}{16}$.
    Ans. $(\frac{9}{8}, 3)$ $(\frac{9}{8}, -3)$
12. For the curve $y = \frac{ax}{a+x}$, if ρ is the radius of curvature at any point (x, y), show that $(\frac{2\rho}{a})^{\frac{2}{3}} = (\frac{y}{x})^2 + (\frac{x}{y})^2$.
13. Show that the radius of curvature at any point of the curve $x = a(\theta + sin\theta),\ y = a(1 - cos\theta)$ is 4a $cos(\frac{\theta}{2})$.
14. Find the point where the radius of curvature of the curve $x^2\,y = a(x^2 + \frac{a^2}{\sqrt{5}})$ is minimum.
    Ans. $x = a$, minimum curvature is $\frac{9a}{10}$