Frequency Modulation : FM

The other type of modulation in continuous-wave modulation is **Angle Modulation**. Angle Modulation is the process in which the frequency or the phase of the carrier signal varies according to the message signal.

The standard equation of the angle modulated wave is

$$s(t) = A_c \cos\theta_i(t)$$

Where,

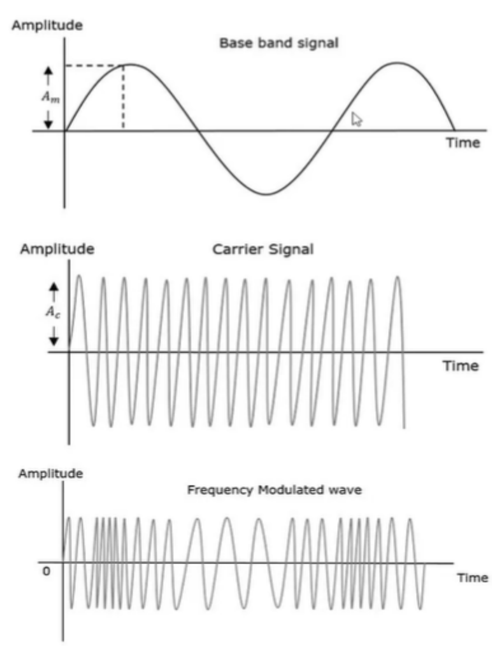
A_c is the amplitude of the modulated wave, which is the same as the amplitude of the carrier signal

$\theta_i(t)$ is the angle of the modulated wave

Angle modulation is further divided into frequency modulation and phase modulation.

- **Frequency Modulation** is the process of varying the frequency of the carrier signal linearly with the message signal.
- **Phase Modulation** is the process of varying the phase of the carrier signal linearly with the message signal.

Frequency Modulated Waveform



The frequency of the modulated wave increases, when the amplitude of the modulating or message signal increases. Similarly, the frequency of the modulated wave decreases, when the amplitude of the modulating signal decreases. Note that, the frequency of the modulated wave remains constant and it is equal to the frequency of the carrier signal, when the amplitude of the modulating signal is zero.

Mathematical Representation

The equation for instantaneous frequency f_i in FM modulation is

$$f_i = f_c + k_f m(t)$$

Where,

f_c is the carrier frequency

k_f is the frequency sensitivity

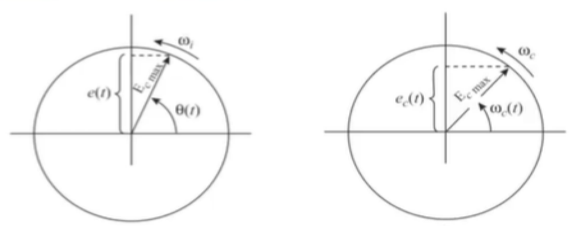
$m(t)$ is the message signal

We know the relationship between angular frequency ω_i and angle $\theta_i(t)$ as

$$\omega_i = \frac{d\theta_i(t)}{dt}$$

$$\Rightarrow 2\pi f_i = \frac{d\theta_i(t)}{dt}$$

$$\Rightarrow \theta_i(t) = 2\pi \int f_i \, dt$$



(a) (b)

Rotating phasor representation of a carrier of amplitude $E_{c\ max}$ rotating
(a) at instantaneous angluar velocity $\omega_i(t)$
(b) at constant angular velocity ω_c.

Substitute, f_i value in the above equation.

$$\theta_i(t) = 2\pi \int (f_c + k_f m(t))\, dt$$

$$\Rightarrow \theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(t)\, dt$$

Substitute, $\theta_i(t)$ value in the standard equation of angle modulated wave.

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)\, dt\right)$$

This is the **equation of FM wave**.

If the modulating signal is $m(t) = A_m \cos(2\pi f_m t)$, then the equation of FM wave will be

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

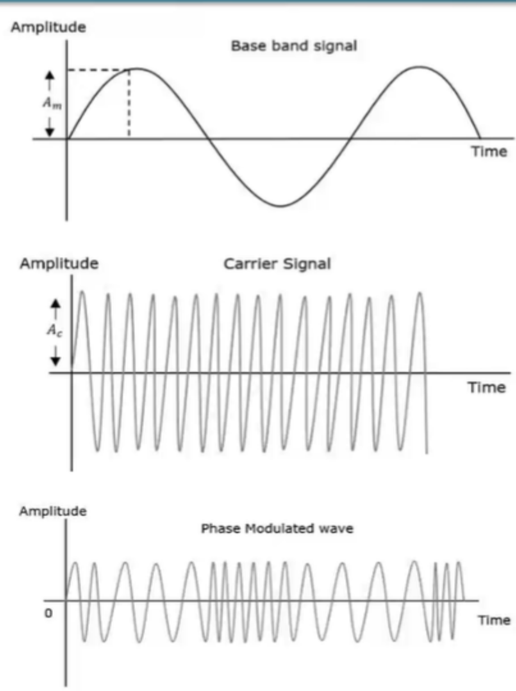
$$\boxed{\beta \ = \textbf{\textit{modulation index}} \ = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}}$$

The difference between FM modulated frequency (instantaneous frequency) and normal carrier frequency is termed as **Frequency Deviation**. It is denoted by Δf , which is equal to the product of k_f and A_m .

FM can be divided into **Narrowband FM** and **Wideband FM** based on the values of modulation index β .

$$\boxed{\textbf{Modulation index in FM :} \quad m = \frac{\delta}{f_m} = \frac{\text{Maximum frequency deviation}}{\text{Modulating frequency}}}$$

Phase Modulated Wave



The phase of the modulated wave has got infinite points, where the phase shift in a wave can take place. The instantaneous amplitude of the modulating signal changes the phase of the carrier signal. When the amplitude is positive, the phase changes in one direction and if the amplitude is negative, the phase changes in the opposite direction.

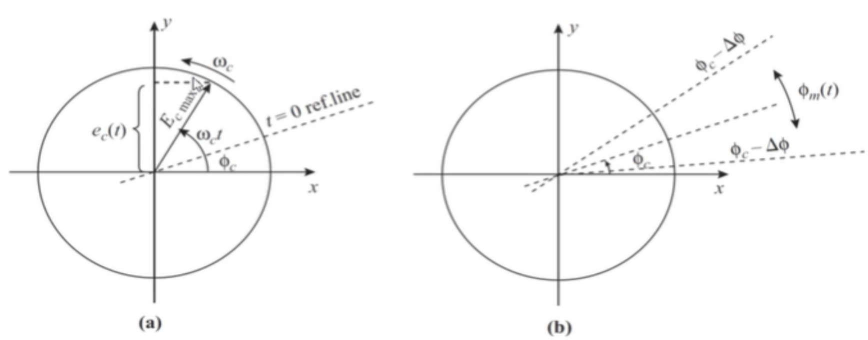
Mathematical Representation

The equation for instantaneous phase ϕ_i in phase modulation is

$$\phi_i = k_p m(t)$$

Where,

- k_p is the phase sensitivity
- $m(t)$ is the message signal



The standard equation of angle modulated wave is

$$s(t) = A_c \cos(2\pi f_c t + \phi_i)$$

Substitute, ϕ_i value in the above equation.

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

This is the **equation of PM wave**.

If the modulating signal, $m(t) = A_m \cos(2\pi f_m t)$, then the equation of PM wave will be

$$s(t) = A_c \cos(2\pi f_c t + \beta \cos(2\pi f_m t))$$

Where,

- β = ***modulation index*** = $\Delta\phi = k_p A_m$
- $\Delta\phi$ is phase deviation

Phase modulation is used in mobile communication systems, while frequency modulation is used mainly for FM broadcasting.

Percentage modulation :

For angle modulation, the percentage modulation is given as the ratio of actual frequency deviation to maximum allowable frequency deviation. i.e.,

$$\% \text{ Modulation} = \frac{\text{Actual frequency deviation}}{\text{Maximum allowable frequency deviation}}$$

Deviation Ratio (DR) :

The deviation ratio is the ratio of maximum frequency deviation to maximum modulating signal frequency. i.e.,

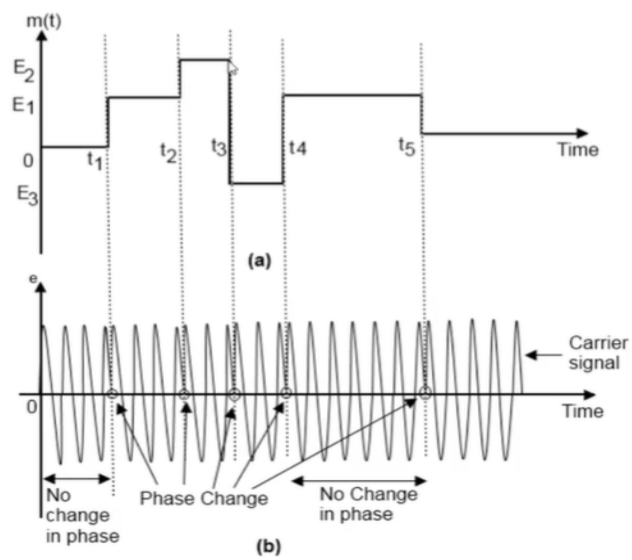
$$\text{Deviation ratio (DR)} = \frac{\text{Maximum frequency deviation}}{f_{m(max)}}$$

Digital Phase Modulation

Phase modulation is very widely used in digital systems. In the simplest system, the voltage levels $\pm V$ representing the binary digits 1 and 0 may be used to multiply the carrier. If the digital signal is represented by $p(t)$ and a balanced modulator is used the modulated signal is essentially a DSBSC wave

$$e(t) = Ap(t)E_{c\,\max} \cos \omega_c t$$

where A is a constant of the multiplier. When $p(t) = +V$, then $e(t) = AV\,E_{c\,\max} \cos \omega_c t$. When $p(t) = -V$, the minus sign can be interpreted as a 180° phase shift, and the modulated wave is $e(t) = AV\,E_{c\,\max} \cos (\omega_c t + 180°)$. Thus the phase of the modulated signal shifts between zero and 180° in accordance with the digital modulating signal. This type of modulation is referred to as *binary phase shift keying* (BPSK), the word "keying" being a relic from the time when a Morse key was the most common method of generating a digitally modulated telegraph signal.



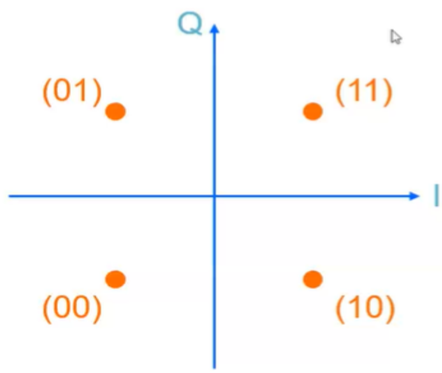
Phase Modulation Waveforms

(a) Non-Sinusoidal Modulating Signal, m(t). (b) Phase-Modulated Carrier Signal e

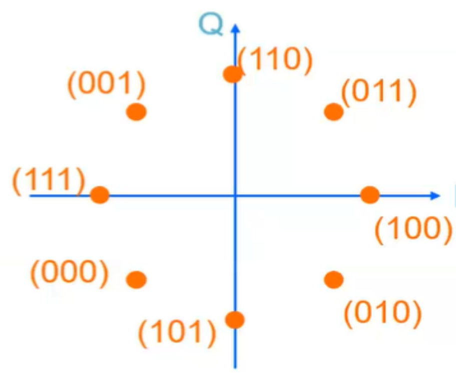
Phase Modulation Waveforms

Differential Modulation

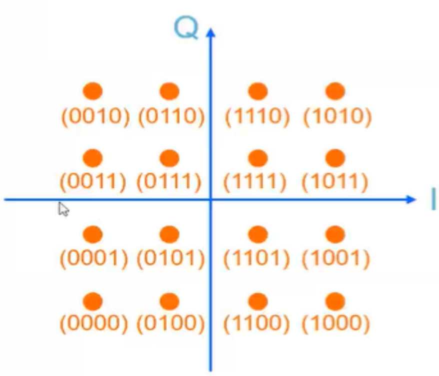
- **QPSK(Quadrature PSK)**
  - Assign the value to points in IQ Space

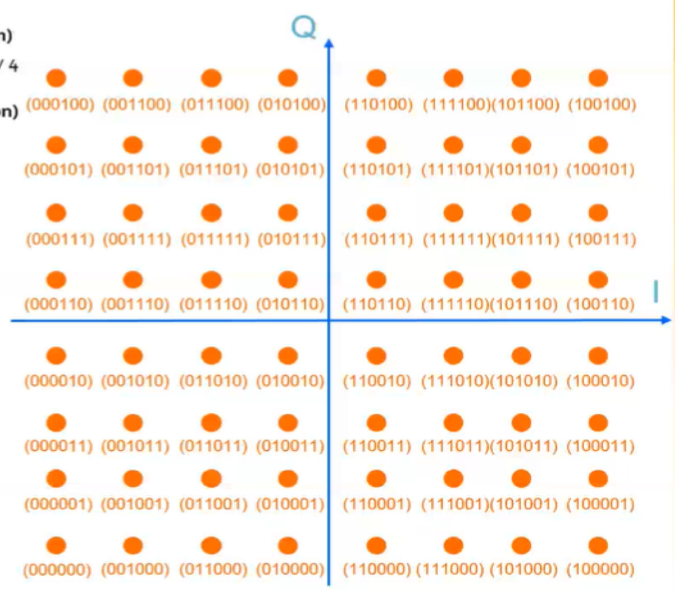


- **8PSK(8-PSK)**
  - Assign the value to points in IQ Space
  - 3 points per symbol



- **16QAM(16-Quadrature Amplitude Modulation)**
  - Each IQ symbol location is represented by 4 data bits
- **64QAM (64-Quadrature Amplitude Modulation)**
  - Each symbol is now worth 6 bits





Symbol Rate and Bit Rate

- **Modulation type determines number of bits per symbol**
  - BPSK 1 bit/symbol
  - DBPSK 1 bit/symbol
  - QPSK 2 bit/symbol
  - p/4 DQPSK 2 bit/symbol
  - DQPSK 2 bit/symbol
  - 8PSK 3 bit/symbol
  - 16QAM 4 bit/symbol
  - 32QAM 5 bit/symbol
  - 64QAM 6 bit/symbol
  - 256QAM 8 bit/symbol
- **For a fixed symbol rate, having more bits will provide a faster transfer rate**

- **32QAM**
  - ADSL etc
- **256QAM**
  - Microwave Communication
  - Some Cable Modem
- **1024QAM**
  - ADSL Technique
- **OQPSK**
  - Offset QPSK
  - Used to avoid zero crossings
- **DQPSK**
- **HPSK**
  - Hybrid Phase Shift Keying
  - Also known as Orthogonal Complex Quadrature Phase Shift Keying (OCQPSK)
  - Used in CDMA2000 (1xRTT) reverse link
- **VSB**
  - Vestigial Side Band
  - 8VSB, 16VSB
  - US Digital Broadcast TV

64-QAM and 256-QAM are often used in digital cable television and cable modem applications

After continuous wave modulation, the next division is Pulse modulation.

In this chapter, let us discuss the following analog pulse modulation techniques.

- Pulse Amplitude Modulation
- Pulse Width Modulation
- Pulse Position Modulation

PAM

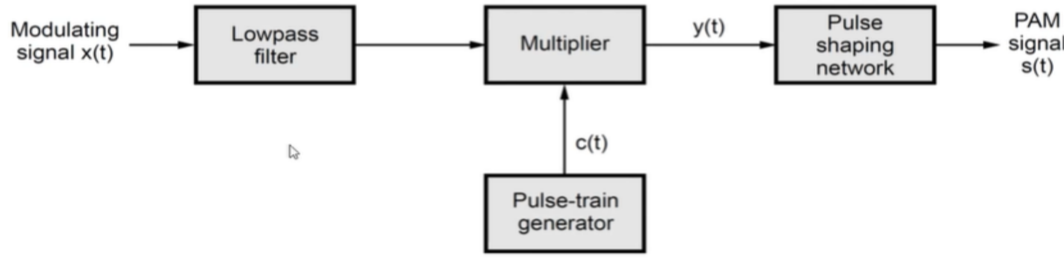
Pulse Amplitude Modulation

In **Pulse Amplitude Modulation (PAM)** technique, the amplitude of the pulse carrier varies, which is proportional to the instantaneous amplitude of the message signal.

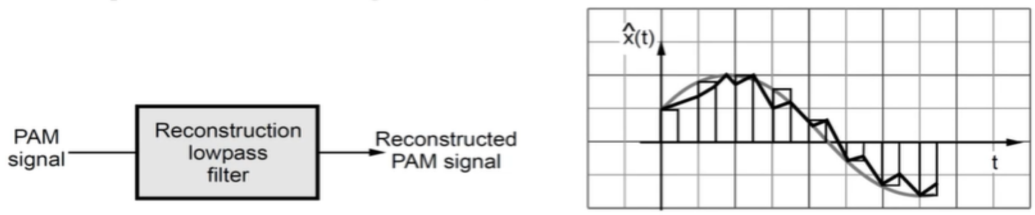
The pulse amplitude modulated signal will follow the amplitude of the original signal, as the signal traces out the path of the whole wave. In natural PAM, a signal sampled at Nyquist rate can be reconstructed, by passing it through an efficient **Low Pass Filter (LPF)** with exact cutoff frequency.

The following figures explain the Pulse Amplitude Modulation.

The amplitude of the pulse is directly proportional to amplitude of the modulating signal at the sampling instant. The width and position of the pulse remains unchanged.



Generation of PAM



(a) Detector **(b) Waveforms**

PAM detector and its waveforms

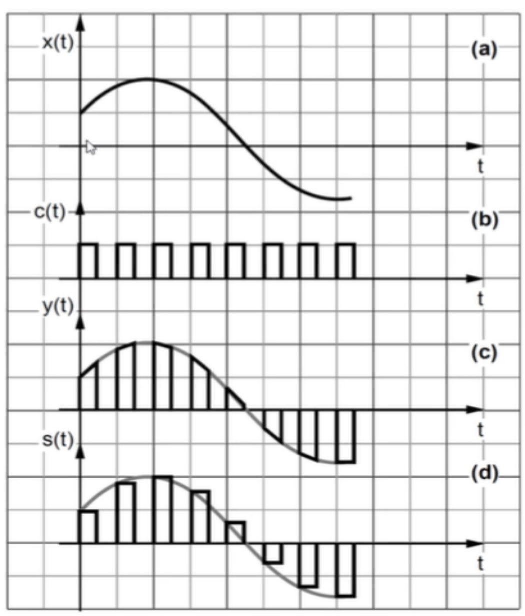
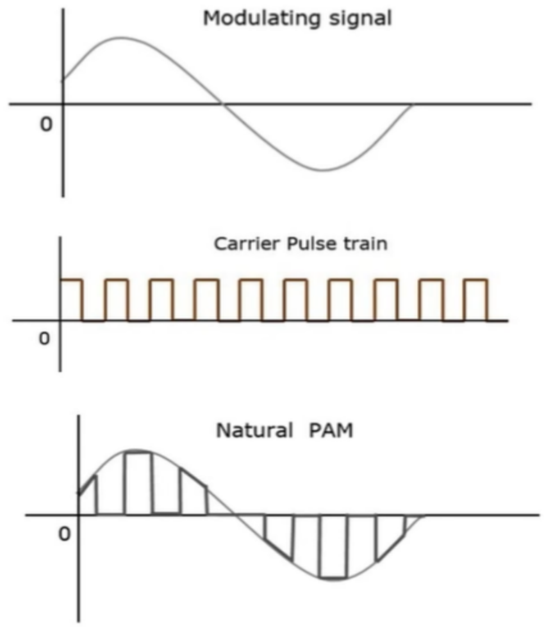
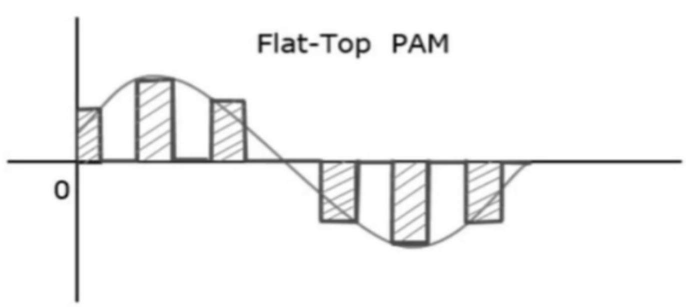


Fig. 2.1.2 PAM waveforms

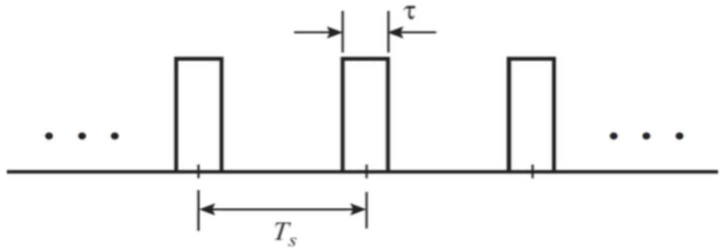
PAM Waveform



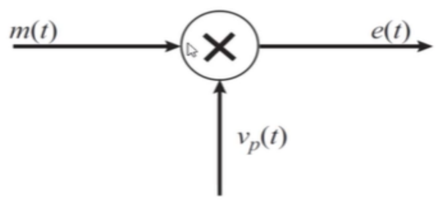
Though the PAM signal is passed through a LPF, it cannot recover the signal without distortion. Hence, to avoid this noise, use flat-top sampling. The flat-top PAM signal is shown in the following figure.



Flat-top sampling is the process in which, the sampled signal can be represented in pulses for which the amplitude of the signal cannot be changed with respect to the analog signal, to be sampled. The tops of amplitude remain flat. This process simplifies the circuit design.



Periodic pulse train.



Product modulator used to produce natural PAM.

In pulse modulation the unmodulated carrier is a periodic train of pulses as sketched in Fig. The unmodulated pulse amplitude is shown as A and the pulse width as τ. The periodic time of the pulse train is shown as T_s. The reason for using subscript s will become apparent shortly.

$$v_p(t) = \sum_{k=-\infty}^{\infty} A \; rect\left(\frac{t - kT_s}{\tau}\right)$$

In *pulse amplitude modulation* (PAM) the amplitudes of the pulses are varied in accordance with the modulating signal.

The pulse train acts as a periodic switching signal to the modulator, which when switched on allows *samples* of the modulating signal to pass through to the output. The periodic time of the pulse train is known as the sampling period and hence the use of the subscript s. Note that T_s is the period from the beginning of one sample to the next, not the pulse duration. The sampling frequency is

$$f_s = \frac{1}{T_s}$$

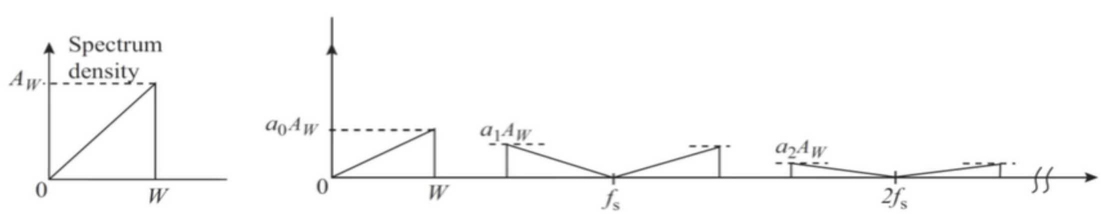
The equation describing natural PAM is found as follows. The Fourier series for the unmodulated pulse train is given by Eq.

$$\begin{aligned} v_p(t) &= a_0 + \sum_{n=1}^{n=\infty} a_n \cos \frac{2\pi nt}{T_s} \\ &= a_0 + a_1 \cos \frac{2\pi t}{T_s} + a_2 \cos \frac{4\pi t}{T_s} + \cdots \end{aligned}$$

The modulated pulse train is then

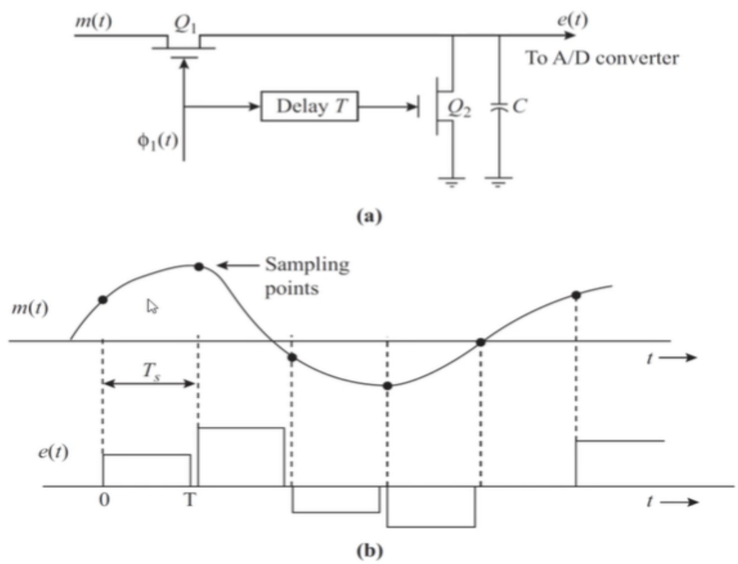
$$\begin{aligned} e(t) &= m(t) \cdot v_p(t) \\ &= a_0 m(t) + a_1 m(t) \cos \frac{2\pi t}{T_s} + a_2 m(t) \cos \frac{4\pi t}{T_s} + \cdots \end{aligned}$$

The right-hand side of this equation shows that the modulated wave consists of the modulating signal, multiplied by the dc term *a0* and a series of DSBSC-type components resulting from the harmonics in the pulse waveform. Denoting the modulating signal spectrum by $M(f)$ and the highest frequency component in this by W, the spectrum for the PAM signal will be as shown in Fig.



(a) (b)

Sample & Hold Circuit for Flat-Top PAM



Advantages

1. PAM can be easily generated and detected.
2. PAM forms the basis for many other pulse modulation techniques such as PCM, DM and ADM.

Disadvantages

1. As we have seen just now, the bandwidth needed for transmission of PAM signal is very very large compared to its maximum frequency content.
2. The amplitude of PAM pulses varies according to modulating signal. Therefore interference of noise is maximum for the PAM signal and this noise cannot be removed very easily.
3. Since amplitude of PAM signal varies, this also varies the peak power required by the transmitter with modulating signal.

Applications

1. PAM is used for transmitting signals over a short distance baseband channels and simple communication.
2. PAM is also used in instrumentation systems.
3. It is used in Analog-to-digital converters for computer interfacing.

Pulse Width Modulation

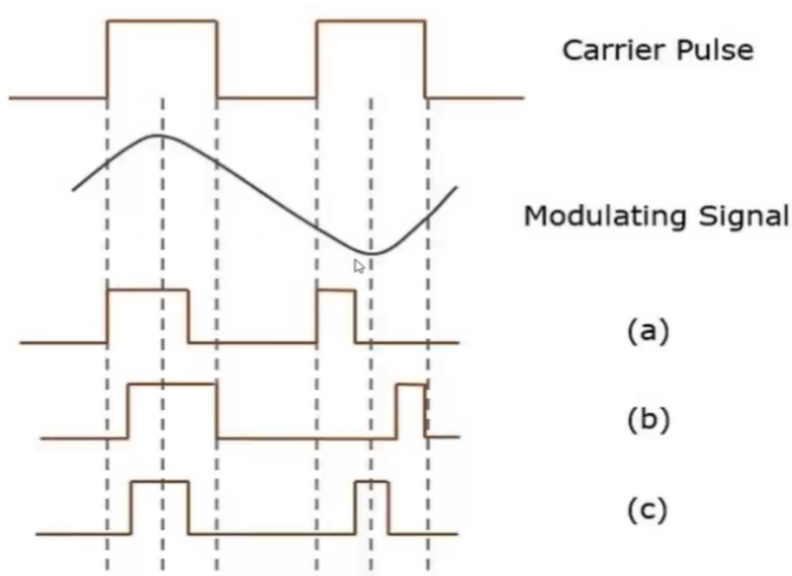
Pulse Width Modulation

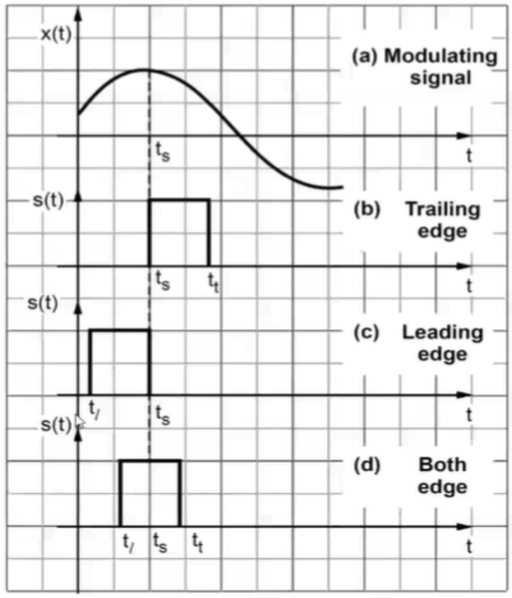
In **Pulse Width Modulation (PWM)** or Pulse Duration Modulation (PDM) or Pulse Time Modulation (PTM) technique, the width or the duration or the time of the pulse carrier varies, which is proportional to the instantaneous amplitude of the message signal.

The width of the pulse varies in this method, but the amplitude of the signal remains constant. Amplitude limiters are used to make the amplitude of the signal constant. These circuits clip off the amplitude to a desired level, and hence the noise is limited.

PWM Signal

The following figure explains the types of Pulse Width Modulations.



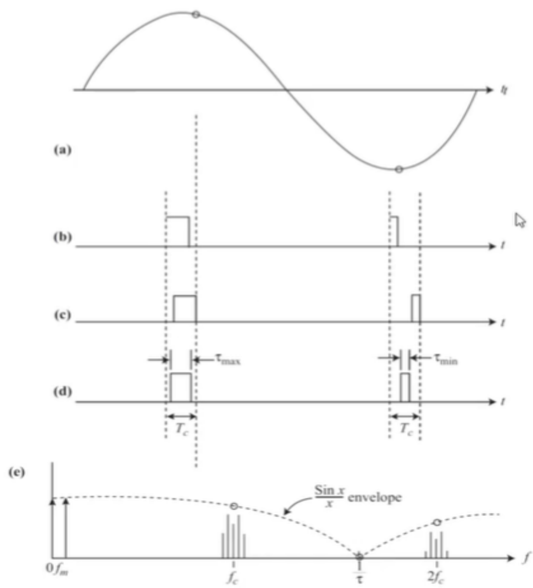


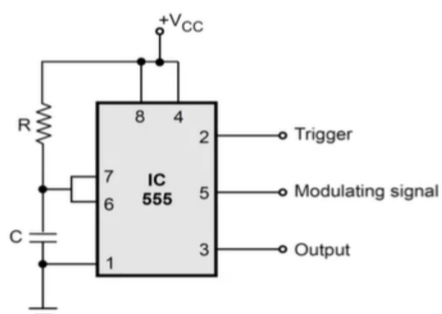
Types of PWM

If the frequency and amplitude of a pulse train are kept constant and the width of the pulses is varied with a modulating signal, then the result is a pulse width modulated (PWM) signal. Three variations are possible, as shown in Fig. (b), (c), and (d).

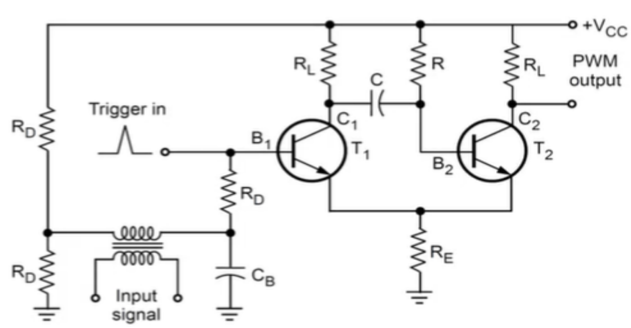
First, the pulse center may be fixed in the center of the repeating time window Tc and both edges of the pulse moved to compress or expand the width. Second, the lead edge can be held at the lead edge of the window and the tail edge modulated. Third, the tail edge can be fixed and the lead edge modulated. The resulting spectra are similar, and, as shown in Fig., they each contain a dc component and a base sideband containing the modulating signal, as well as phase modulated carriers at each harmonic of the pulse frequency. The amplitudes of the harmonic groups are constrained by a (sin (x))/x envelope and extend to infinity.

Since the baseband information appears in the signal and is not distorted by any modulation effects, it may be recovered using a simple low-pass filter to remove the carrier and its harmonics and a high-pass filter to remove the dc component.

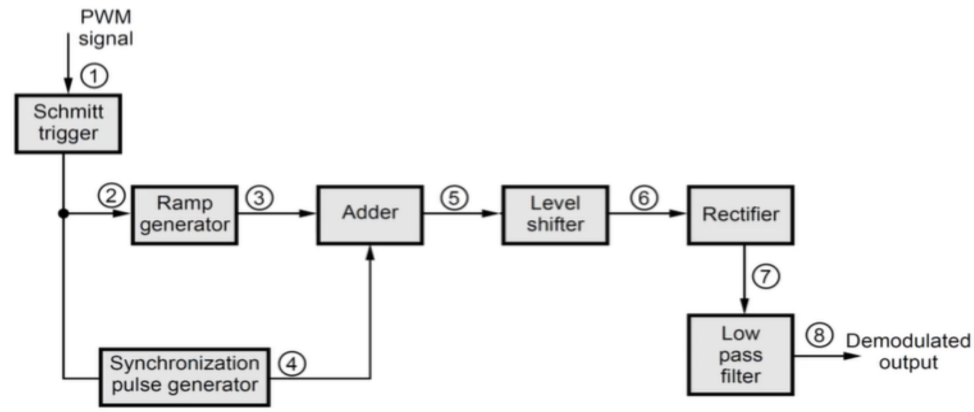




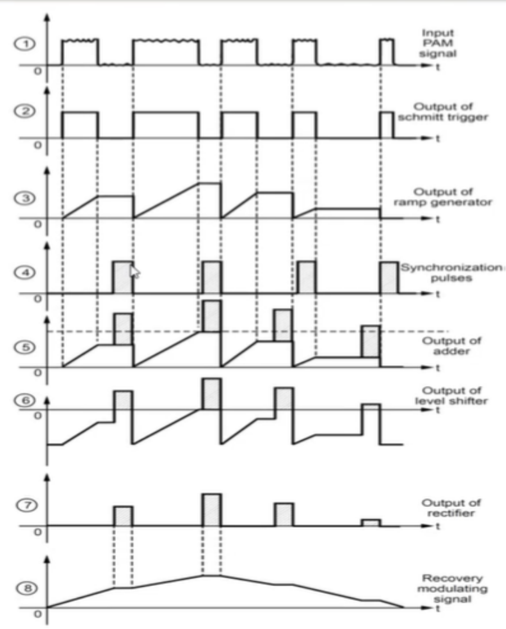
PWM generation circuit



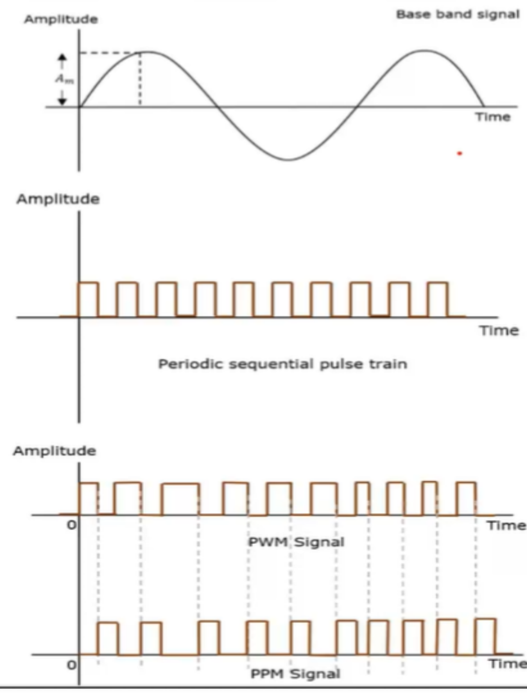
Monostable multivibrator generating pulsewidth modulation

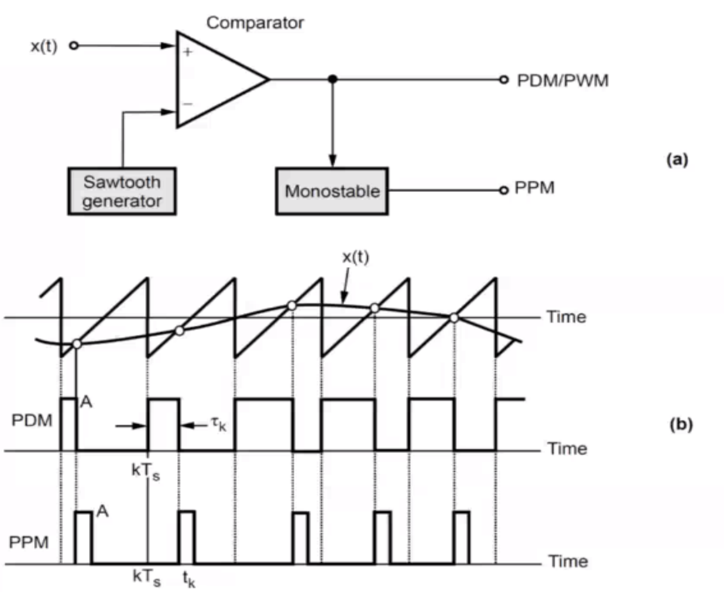


PWM detector

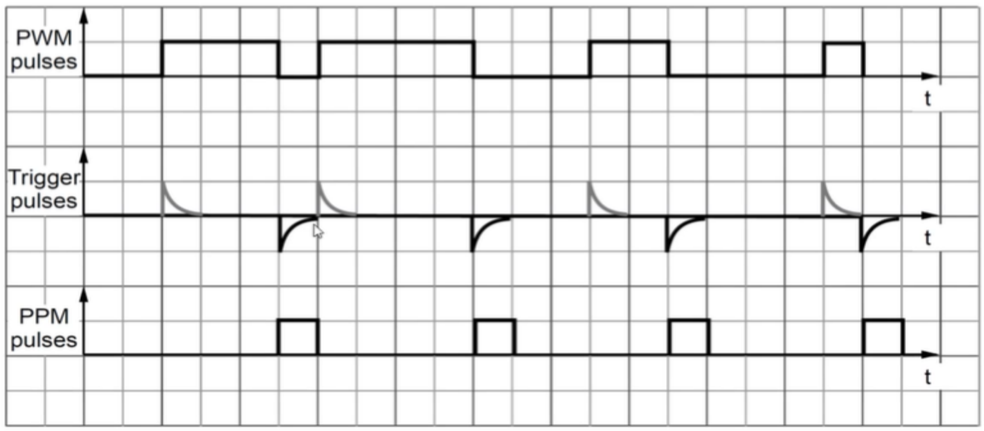


Waveforms for PWM detection circuit

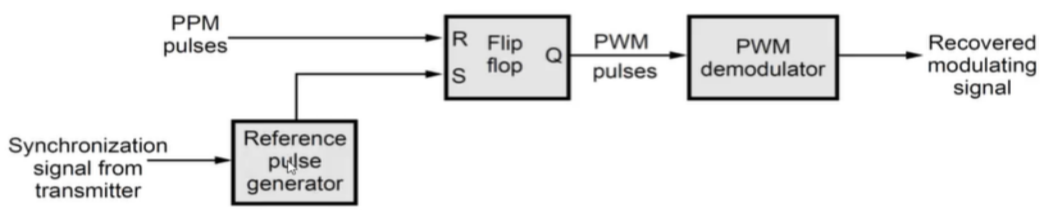




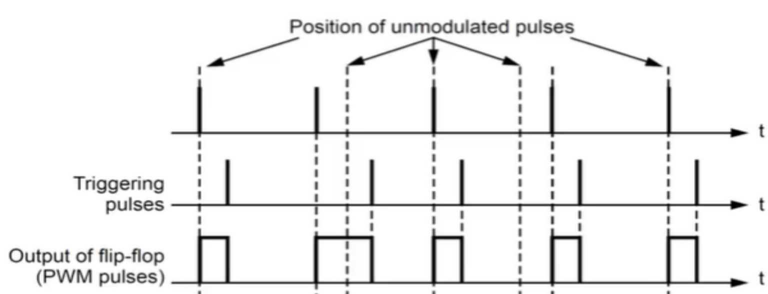
Generator of PPM and PWM (a) Block diagram (b) Waveforms



Waveforms of PPM generator



PPM demodulator



Demodulation waveforms for PPM

Advantages

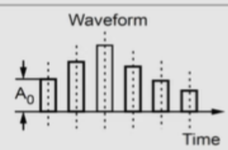
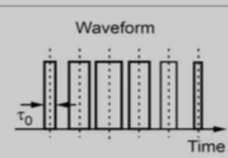
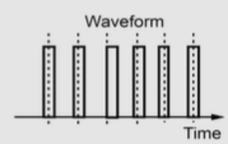
1. Like PWM, in PPM amplitude is held constant thus less noise interference.
2. Like PWM, signal and noise separation is very easy.
3. Due to constant pulse widths and amplitudes, transmission power for each pulse is same.

Disadvantages

1. Synchronization between transmitter and receiver is required.
2. Large bandwidth is required as compared to PAM.

Application

1. Synchronous communication of analog pulses over short distances.

Sr. No.	Pulse Amplitude Modulation	Pulse Width/Duration Modulation	Pulse Position Modulation
1			
2	Amplitude of the pulse is proportional to amplitude of modulating signal.	Width of the pulse is proportional to amplitude of modulating signal.	The relative position of the pulse is proprotional to the amplitude of modulating signal.
3	The bandwidth of the transmission channel depends on width of the pulse.	Bandwidth of transmission channel depends on rise time of the pulse.	Bandwidth of transmission channel depends on rising time of the pulse.
4	The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter remains constant.
5	Noise interference is high.	Noise interference is minimum.	Noise interference is minimum.
6	System is complex.	Simple to implement.	Simple to implement.
7	Similar to amplitude modulation.	Similar to frequency modulation.	Similar to phase modulation.