Roberto Gobbetti
Delta Institute for Theoretical Physics
Utrecht University

Brane Dynamics and Unwinding Inflation

Cornell University
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with Guido D'Amico, Matthew Kleban, Marjorie Schillo
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Outline of the talk

- Review of unwinding inflation
- Branes scattering
    - annulus diagram
    - particle and string production
    - tachyon condensation
- Reheating in unwinding inflation
- Conclusions and open questions

Unwinding Inflation

If we believe in the string theory landscape, we need a stringy mechanism to generate the universe we see

UI provides a natural way to exit false vacuum eternal inflation, as predicted in the string theory landscape

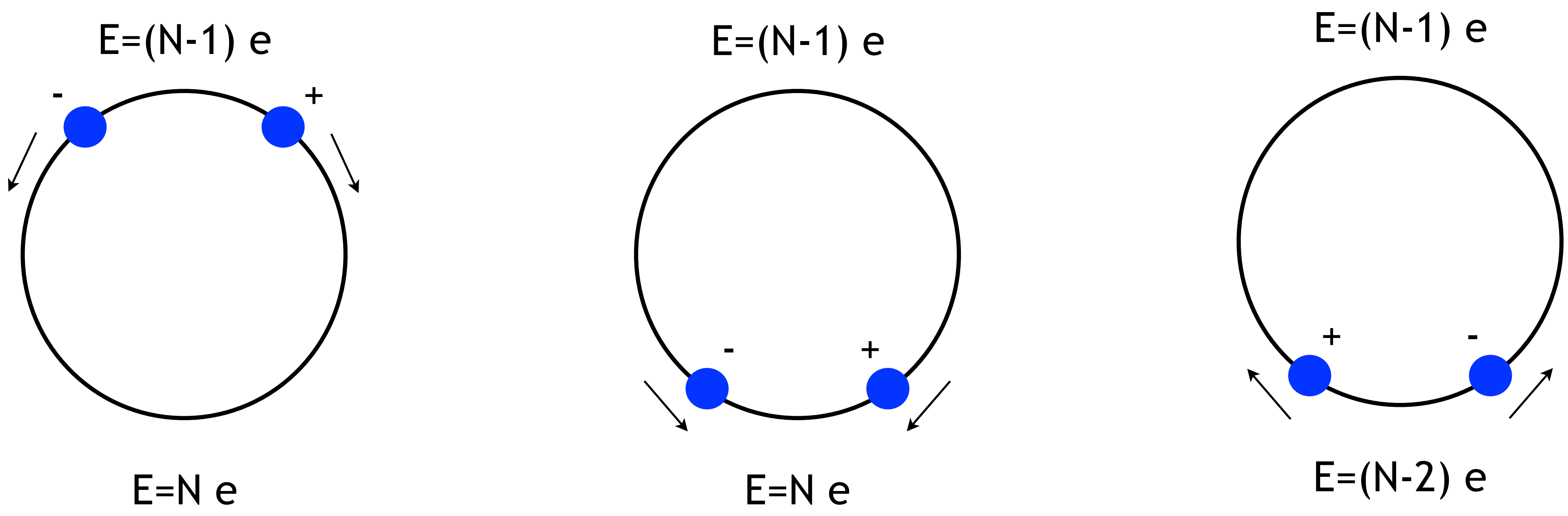
The initial conditions are automatically set by a quantum event and the evolution is completely classical

Reheating is also natural in this framework (but more on this later)

Schwinger on a circle

The energy density $\rho = F^2 = (Qe)^2$ is a vacuum energy. No fermions to begin with, but they can nucleate through quantum pair production

The electron and positron will start accelerating around the circle driven by the difference in the field across them, discharging the field and decreasing the vacuum energy



Kleban, Krishnaiyengar, Porrati (2011)

Unwinding Inflation

$dS_4 \times M$ and a constant flux that wraps all dS dimensions

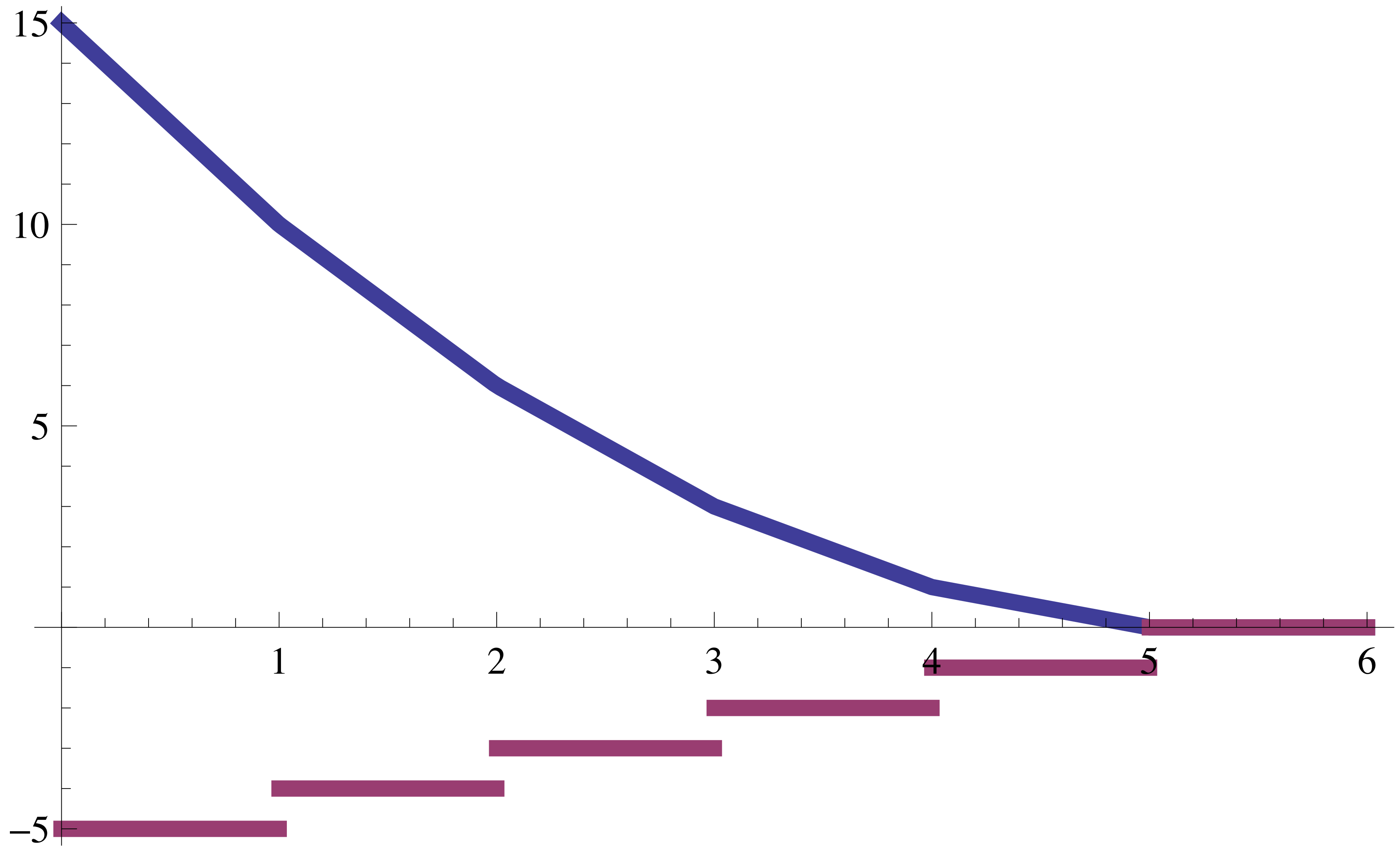
A brane bubble forms and start expanding in the compact and dS directions, colliding with itself and discharging the flux, which decreases steadily if I can not resolve the size of the compact dimensions

The branes move at approximately constant ultra-relativistic velocity because of dS friction and after a few e-folds the system can be treated as a brane colliding with antibrane

Reheating happens when brane and antibrane annihilate

Unwinding Inflation

From an effective point of view the model is described by a piecewise linear potential that mimics a parabola.



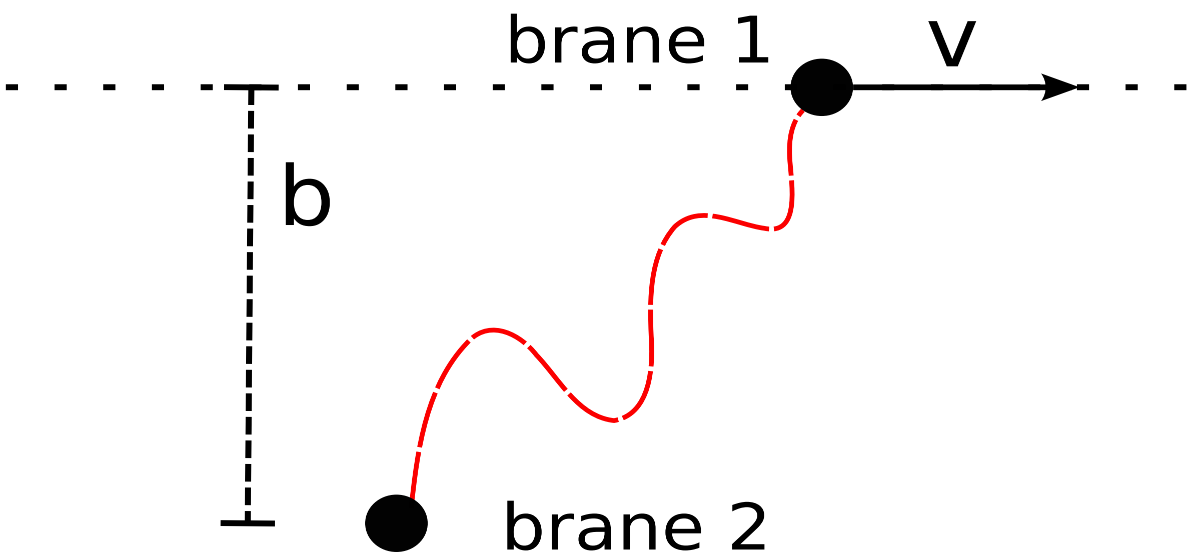
Unwinding Inflation

The power spectrum receives two contributions:
- de Sitter perturbations
- Particles/Strings production: fluctuation in the density of produced particles/strings between colliding branes

The latter is subdominant, but string production influences the evolution at late time and eventually causes the end of inflation and reheating

It is fundamental to understand the phenomenon thoroughly

Scattering



I will study the scattering of two p branes approaching each other at constant velocity in a flat background

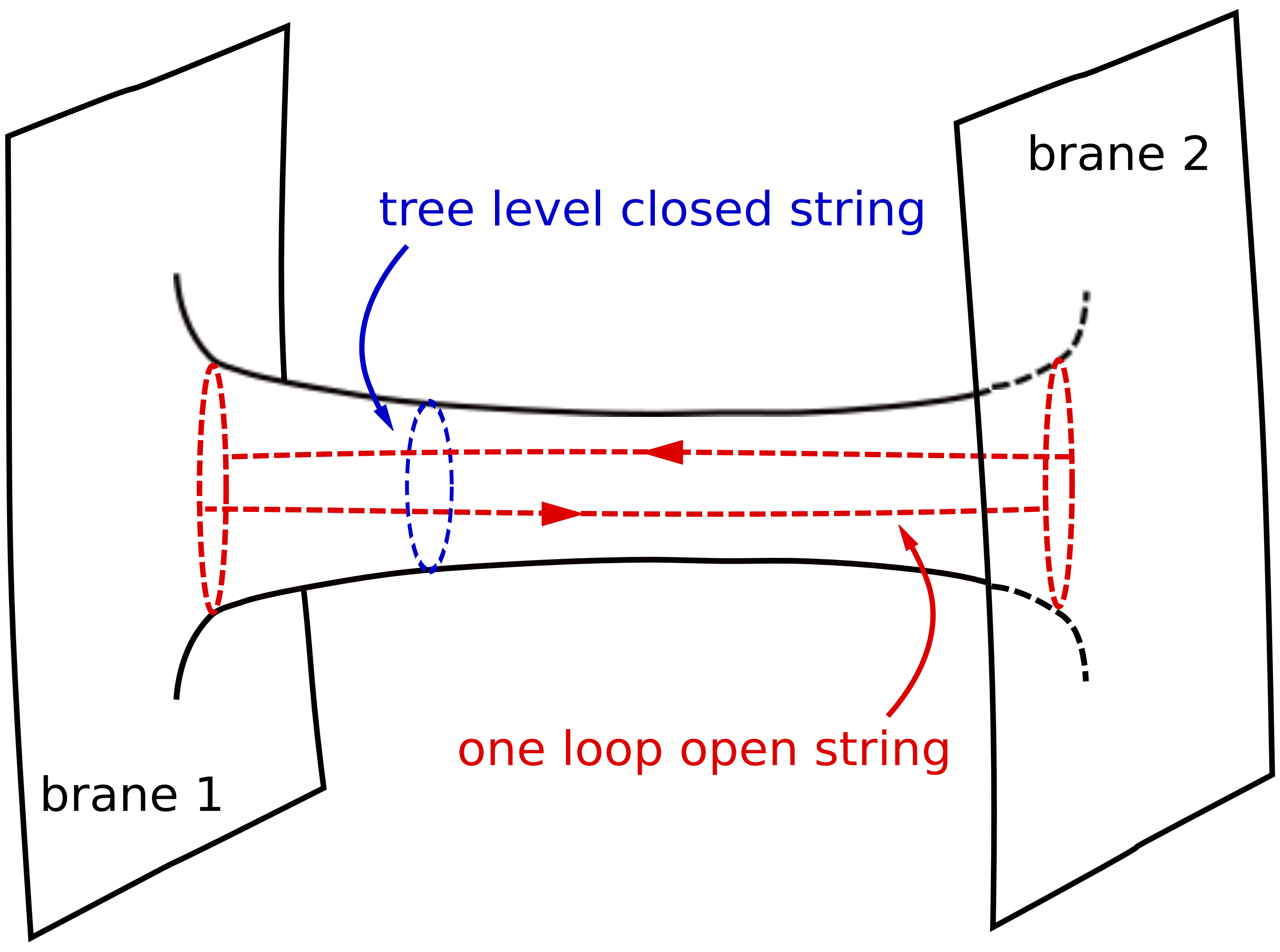
This problem is T-dual to two static p+1 branes extended in the $\vec{v}$ direction with a constant electric field in the $\vec{v}$ direction, with the specification

$$\vec{v}_0 + \vec{v}_\pi \leftrightarrow 2\pi\alpha'(e_\pi - e_0)\vec{E}$$

Scattering

At lowest order in g_s, the relevant diagram is the annulus diagram, which has two equivalent channels.

I am interested in computing the imaginary part of its amplitude, which is related to string pairs creation



Scattering

$$\begin{aligned}\mathrm{Im}[\mathcal{A}_{\mathcal{B}\mathcal{B}}] =& \frac{C}{4}\left(\frac{L}{2\pi}\right)^{\lambda}\chi^{\lambda/2}\sum_{r=1,3,5}^{\infty}\frac{1}{r^{(\lambda+2)/2}}\exp\left(\frac{-r\pi m_0^2}{\chi}\right)\\ &\times\left\{32+512\exp\left(-\frac{2\pi r}{\chi}\right)+4608\exp\left(-\frac{4\pi r}{\chi}\right)+\cdots\right\}\end{aligned}$$

Bachas, Porrati (1992)
Bachas (1996)

$$\begin{aligned}\mathrm{Im}[\mathcal{A}] =& \frac{C}{4}\left(\frac{L}{2\pi}\right)^{\lambda}\chi^{\lambda/2}\sum_{r=1}^{\infty}\frac{1}{r^{(\lambda+2)/2}}\exp\left(\frac{-r\pi m_0^2}{\chi}\right)\\ &\times\left\{2(-1)^{r+1}\exp\left(\frac{\pi r}{\chi}\right)+16+72(-1)^{r+1}\exp\left(\frac{-\pi r}{\chi}\right)+256\exp\left(\frac{-2\pi r}{\chi}\right)+\cdots\right\}\end{aligned}$$

$$C=\begin{cases}\frac{TL|E(e_0+e_\pi)|}{2\pi}\\ 1\end{cases}\qquad \chi=\begin{cases}\left|\frac{1}{\pi}\left(\tanh^{-1}(\pi e_0 E)+\tanh^{-1}(\pi e_\pi E)\right)\right|\\ \left|\frac{1}{\pi}\left(\tanh^{-1}(v_\pi)-\tanh^{-1}(v_0)\right)\right|\end{cases}\qquad \lambda=\begin{cases}p-1\\ p\end{cases}$$

$$\mathrm{Im}[\mathcal{A}_{\text{field theory}}]=\frac{D}{4}T\left(\frac{L}{2\pi}\right)^{d-1}(eE)^{\frac{d}{2}}\sum_{r=1}^{\infty}\frac{(-1)^{(r+1)(2S+1)}}{r^{d/2}}\exp\left(\frac{-r\pi m^2}{eE}\right)$$

String production

The rate of string production is connected to this amplitude, but it is not the imaginary part of it. To see that, consider a simple situation

$$\Box\phi + (m_0^2 + A^2 t^2)\phi = 0$$

then one has

$$2\mathrm{Im}[\mathcal{A}_{\text{field theory}}] = -\ln(P_0) = \frac{D}{2}\left(\frac{L}{2\pi}\right)^{d-1} A^{\frac{d-1}{2}} \sum_{r=1}^{\infty} \frac{(-1)^{(r+1)}}{r^{(d+1)/2}} \exp\left(\frac{-r\pi m_0^2}{A}\right)$$

I can explicitly compute the rate production for this system by expanding the field in k-modes that satisfy

$$\ddot{u}_k + (m_0^2 + k^2 + A^2 t^2) u_k = 0$$

String production

The general solution is a sum of parabolic cylinder function that can be identified as the in mode and the out mode. Imposing canonical commutation relations to find the Bogolubov coefficients allows to compute

$$\langle in|n_k^{out}|in\rangle = \langle in|a_{\vec{k}}^{out\dagger}a_{\vec{k}}^{out}|in\rangle = \exp\left(\frac{-\pi m_k^2}{A}\right)$$

and then

$$\langle n\rangle = \left(\frac{L}{2\pi}\right)^{d-1}\int d^{d-1}k\langle n_k\rangle = \left(\frac{L}{2\pi}\right)^{d-1} A^{(d-1)/2}\exp\left(\frac{-\pi m_0^2}{A}\right)$$

Keeping the whole sum would imply keeping exponentially suppressed terms in the particle case. In the string case it would imply adding constant or exponentially growing terms.

Euclidean Instanton

I want to explore the reason why string production is enhanced with respect to the particle case. The rate of particle creation can be found using instanton methods. Constant E along X_1

$$S_E = \int d\tau_E \left\{ \frac{1}{2\eta}\delta_{ij}\partial_{\tau_E}X^i\partial_{\tau_E}X^j + \frac{1}{2}m^2\eta + eA_i\partial_{\tau_E}X^i \right\}$$

The solution is a circle in the X_d-X_1 plane with R^{-1}=eE/m and one finds the expected result for the smallest action

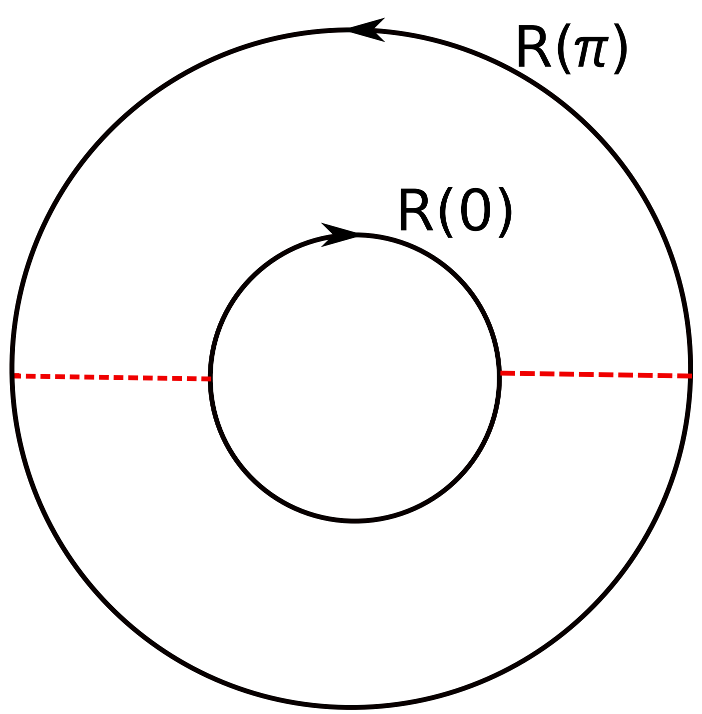
$$S_E = 2\pi Rm - eE\pi R^2 = \pi\frac{m^2}{eE}$$

Euclidean Instanton

String case with constant E along X_1 and string stretching along X_2

$$S_E = \int d\tau_E \int_0^\pi d\sigma \Big\{ \frac{1}{2\pi} \left[\dot{X}^i \dot{X}_i + X'^i X'_i \right] - \frac{E}{2} \left[e_0 \delta_D(\sigma) + e_\pi \delta_D(\sigma - \pi) \right] \left(\dot{X}^d X^1 - \dot{X}^1 X^d \right) \Big\}$$

The solution of the equations of motion is a strip in the X_d-X_1 plane with the radius depending on σ



$$R(\sigma) = \frac{b}{\pi\chi} \cosh(\chi\sigma) \qquad S_E = \frac{b^2}{\pi\chi}$$

Each element of the string contributes to the action like a particle with

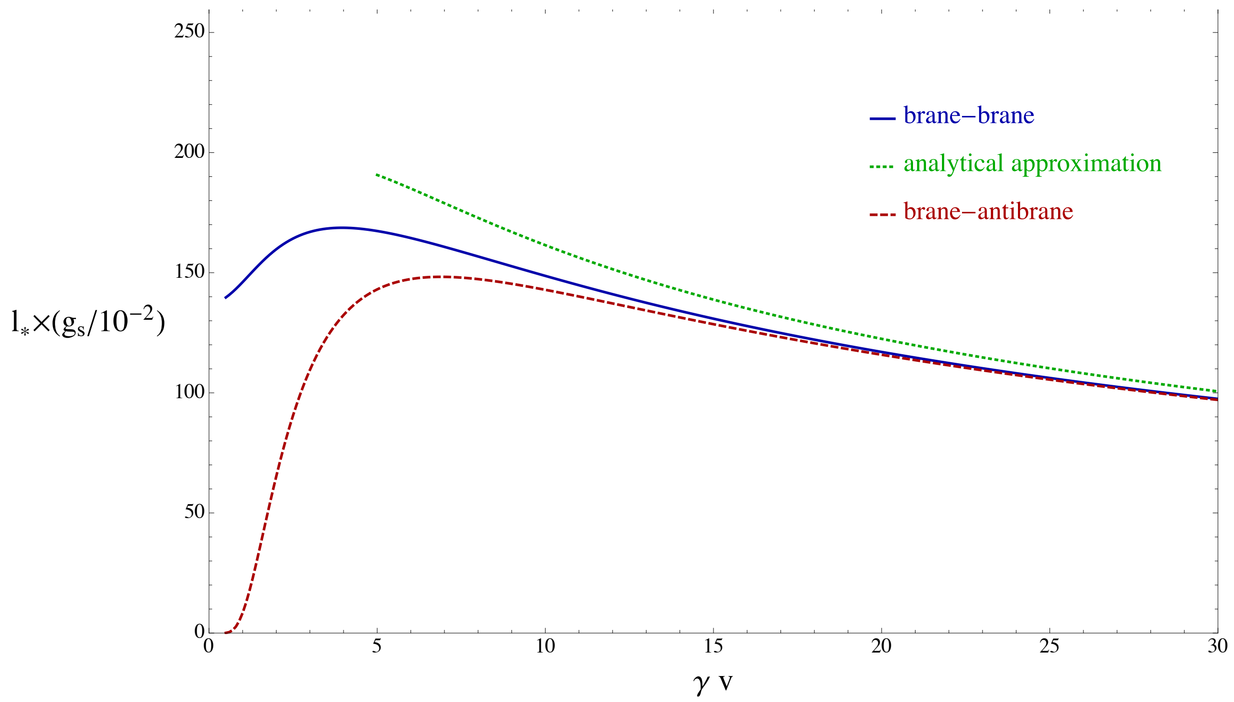
$$\delta S = 2\pi R(\sigma) \delta m \qquad R(\sigma) < R(\pi)$$

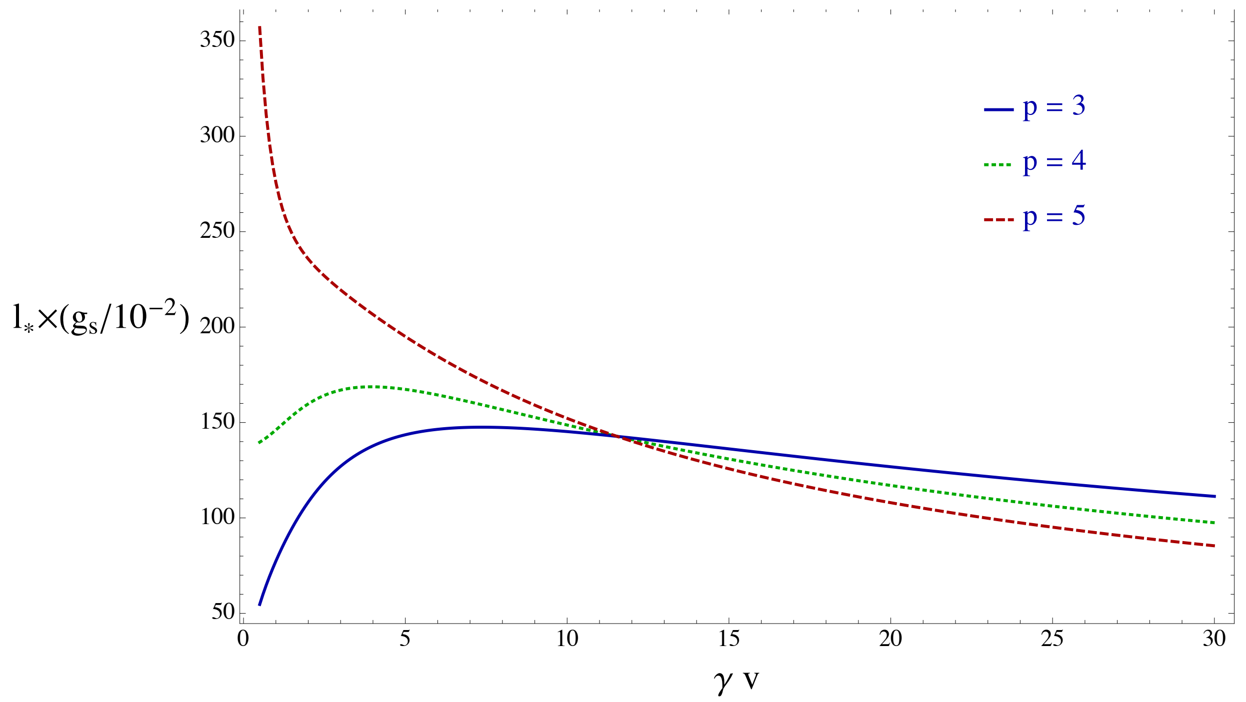
Stopping the branes

The more relativistic the branes, the easier it is to produce strings and therefore convert kinetic energy and stop the branes. The approximate stopping length can be found by equating the kinetic energy of the branes with the string density

McAllister, Mitra (2005)

$$l_* \approx 2^{\frac{p-1}{2}} \pi \frac{\chi^{4-p/2}}{\gamma g_s}$$





Destroying the branes

If a brane and an antibrane are within a string length, a tachyon is present in the open string spectrum. This does not imply that they annihilate since the tachyon might not have time to condense.

At very relativistic velocities, the branes stop quickly and then they possibly annihilate. At small velocities the tachyon has enough time to condense. I expect a range of passing through velocities in between.

Tachyon decay is not well understood, but I will try to capture the physics with an effective action to estimate the decay probability per unit volume.

Destroying the branes

Proposed effective action

$$S = -8\tau_p \int dt d^p x \left[\frac{1}{2} e^{-2|y|^2} |\partial_\mu y|^2 + \frac{1}{4} e^{-2|y|^2} \right]$$

Minahan, Zwiebach (2001)

$$S = -8\tau_p \int dt d^p x \left[\frac{1}{2} e^{-2|y|^2} |\partial_\mu y|^2 + e^{-2|y|^2} \left\{ \frac{1}{4} + \frac{1}{2} \left(\frac{vt}{\pi} \right)^2 |y|^2 \right\} \right]$$

$$S_2 = -\int dt d^p x \left(|\partial_\mu \phi|^2 + 2\tau_p + \left(-1 + \left(\frac{vt}{\pi} \right)^2 \right) |\phi|^2 + \mathcal{O}(g_s |\phi|^4) \right)$$

At t→-∞ the wave function will be concentrated around y=0, I expand around this point. $\varphi^2 \simeq 4\tau y^2$ and non-linearities are at $|\varphi|^2 \sim 1/g_s$ which I will take as definition of decay.

Destroying the branes

To estimate the decay probability I compute the change in the variance due to the appearance of the tachyon at t=0 and compare it with the scale of non-linearities 1/g_s

Smearing with a filter function on a volume with characteristic size R

$$\phi_R(x) = \int d^p y W_R(|x-y|)\phi(y) = \int d^p k\, \tilde{W}(kR)\phi_{\vec{k}}$$

$$\langle|\phi_R|^2\rangle \doteq \sigma_R^2 = \int d^p k\, \tilde{W}^2(kR)|u_k(t)|^2$$

The main contribution will be for k→1/R

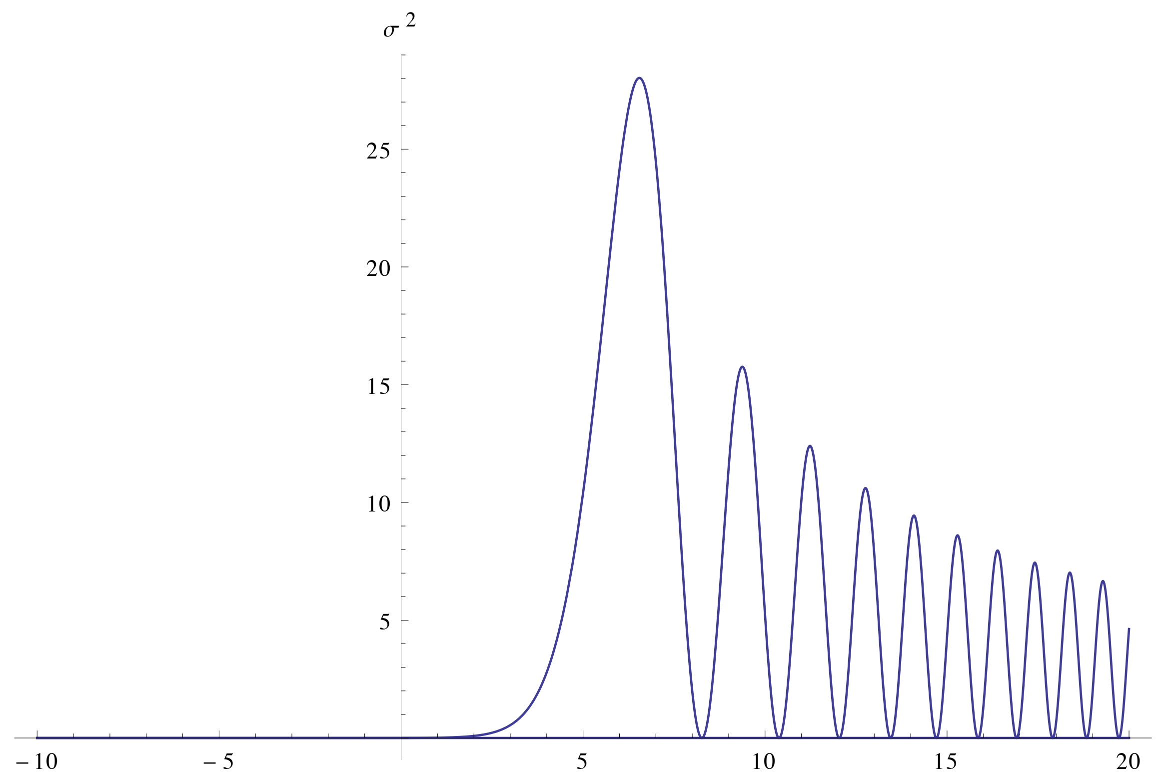
$$\Rightarrow \qquad \sigma_R^2 \approx \frac{1}{R^p}|u_{1/R}|^2$$

Destroying the branes

I can estimate the maximum value of σ

$$\sigma^2_{R,max} \approx R^{-p} \exp(\pi^2/v)$$

and the probability



$$P_R \approx \mathrm{erfc}\left[\frac{1}{\sigma\sqrt{2g_s}}\right] \approx \exp\left[-\frac{1}{2g_s\sigma^2_{R,max}}\right] \approx \exp\left[\frac{-R^p e^{-\pi^2/v}}{2g_s}\right]$$

which implies

$$v \gtrsim \frac{\pi^2}{|\ln g_s|}$$

& requiring $l_{stop} > l_s$

$$\gamma \lesssim \frac{1}{g_s}$$

Reheating

The branes in unwinding inflation will not stop suddenly even if they have large velocities since there is a force driving them apart.

Hubble friction keeps their velocity roughly constant, but will slowly decrease while the flux gets discharged and will eventually fall below the limit for tachyon condensation. This moment is the (beginning of) reheating.

Is reheating uniform? Is there any flux left when the brane annihilates?

Reheating

Scenario #1: all the flux is discharged in most of the universe and some regions undershoot or overshoot. These will have higher energy density and look like black holes that quickly evaporate.

Scenario #2: some flux is leftover.

- If only few Hubble regions overshoot ($P_{OS} \sim 10^{-180}$), then cosmology is highly perturbed.
- If the # of regions that overshoot is high enough so that the scale of perturbation is out of CMB detection ($P_{OS} \sim 10^{-135}$) then there is no problem.

Reheating

It seems reasonable that even more region overshoot they will expand, collide and pass through in 4D, keeping on discharging the flux until there is none left.

These are still conjectures, we working out a numerical simulation to understand the phenomenon better.

The preliminary results suggest that it is fairly easy to have units of flux left, but the annihilation occurs uniformly.

Conclusions

- D-brane scattering is T-dual to branes in constant electric field
- Open string production is easier than particle production
- This makes the scattering of branes extremely inelastic
- Brane and antibrane coud annihilate due to tachyon condensation
- But there is a range of velocities for which annihilation does not happen
- Brane annihilation provides a natural way to end unwinding inflation
- It seems easy to produce our cosmology but more work is in order

Production of scalars

$$m^2 = m_0^2 + A^2 t^2$$

$$u_k = C_1 D_{-\nu-1}(z) + C_2 D_\nu(iz)$$

$$u_k^{in} = \frac{e^{-\pi m_k^2/(8A)}}{(2A)^{1/4}} D_\nu(iz)$$

$$a_{\vec{k}}^{out} = \alpha_{kk'} a_{\vec{k}'}^{in} + \beta_{kk'} a_{-\vec{k}'}^{in\dagger}$$

$$u_k^{out} = \frac{e^{-\pi m_k^2/(8A)}}{(2A)^{1/4}} D_{-\nu-1}(z)$$

$$P_n(k) = |\langle in| \frac{(a_{\vec{k}}^{out\dagger} a_{-\vec{k}}^{out\dagger})^n}{n!} |out\rangle|^2$$

$$|in\rangle = C_0 \exp\left(\int \frac{d^{d-1}q}{(2\pi)^{d-1}} \frac{\beta_q}{2\alpha_q^*} a_{\vec{q}}^{out\dagger} a_{-\vec{q}}^{out\dagger} \right) |out\rangle$$

$$P_{vac} = \prod_{\vec{k}/\mathbf{Z}_2} P_0(\vec{k}) = \exp\left[\frac{1}{2} L^{d-1} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \ln(P_0(k)) \right]$$

Euclidean Instanton

String case with constant E along X1 and string stretching along X2 (conformal gauge)

$$S_E = \int d\tau_E \int_0^\pi d\sigma \left\{ \frac{1}{2\pi} \left[\dot{X}^i \dot{X}_i + X'^i X'_i \right] \right.$$
$$\left. - \frac{E}{2} \left[e_0 \delta_D(\sigma) + e_\pi \delta_D(\sigma - \pi) \right] \left(\dot{X}^d X^1 - \dot{X}^1 X^d \right) \right\}$$

The solution of the equations of motion is

$$\begin{aligned} X^d &= R(\sigma) \sin \chi \tau_E \\ X^1 &= R(\sigma) \cos \chi \tau_E \\ X^2 &= b\sigma/\pi \end{aligned} \qquad \begin{aligned} R(\sigma) &\doteq \frac{b}{\pi\chi} \cosh(\chi_0 - \chi\sigma) \\ \chi_0 &\doteq \tanh^{-1}(e_0 E \pi) \end{aligned}$$

$$S_E = \int_0^\pi d\sigma 2R(\sigma)\sqrt{R'(\sigma)^2 + b^2/\pi^2} - E\pi \left(e_0 R(0)^2 + e_\pi R(\pi)^2 \right) = \frac{b^2}{\pi\chi}$$

String Energy

Brane moving in X1 with impact parameter in X2.
Classical solution for string is

$$(X^0, X^1, X^2) = \left(\frac{b}{\pi\chi} \sinh(\chi\tau)\cosh(\chi\sigma),\ \frac{b}{\pi\chi} \sinh(\chi\tau)\sinh(\chi\sigma),\ \frac{b}{\pi}\sigma \right)$$

The force in the X1 direction is

$$F_1 = T_0 \frac{\pi\chi X^0}{\sqrt{b^2 \cosh^2(\chi\sigma) + (\pi\chi X^0)^2}} = T_0 \frac{\pi\chi X^0}{\sqrt{b^2\gamma^2 + (\pi\chi X^0)^2}}$$

therefore the work on the string is

$$W = \int dX^1\, F_1(\sigma = \pi) = \int_0^{X^0} dX^0\, \tanh(\pi\chi) F_1(\sigma = \pi)$$
$$= T_0 \left(-\frac{b}{\pi\chi} \sinh(\pi\chi) + \frac{\tanh(\pi\chi)}{\pi\chi} \sqrt{b^2 \cosh^2(\pi\chi) + (\pi\chi X^0)^2} \right)$$

String Energy

The bulk energy of the string is though

$$\mathcal{E}_{\text{bulk}} = \int dl\, T_0 \gamma_T = \int d\sigma \sqrt{(\partial_\sigma X^1)^2 + \left(\frac{b}{\pi}\right)^2}\, T_0\, \gamma_T$$
$$= T_0 \frac{\tanh(\pi\chi)}{\pi\chi} \sqrt{b^2 \cosh^2(\pi\chi) + (\pi\chi X^0)^2}$$

At late times is is just ~vt = l. The difference of this and the work is constant, but the constant is arbitrary as it is clearer in the electric field picture