Tables of Integral Transforms

CALIFORNIA INSTITUTE OF TECHNOLOGY
BATEMAN MANUSCRIPT PROJECT

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Higher Transcendental Functions, 3 volumes.
Tables of Integral Transforms, 2 volumes.

TABLES OF INTEGRAL TRANSFORMS

Volume I

Based, in part, on notes left by

Harry Bateman

Late Professor of Mathematics, Theoretical Physics, and Aeronautics at the California Institute of Technology

and compiled by the

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This work is dedicated to the
memory of

HARRY BATEMAN

as a tribute to the imagination which led him to undertake a project of this magnitude, and the scholarly dedication which inspired him to carry it so far toward completion.

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INTRODUCTION

The present volume is the first of two which are intended as companions and sequel to our *Higher Transcendental Functions*. Volume I of that work contains the Preface and Foreword to the whole series, describing the history and the aims of the so-called Bateman Manuscript Project.

A considerable proportion of the tremendous amount of material collected by the late Professor Harry Bateman concerns definite integrals. The organization and presentation of this material is a very difficult task to which Bateman devoted considerable attention. It is fairly clear that the arrangement used in shorter tables of integrals is not very suitable for a collection about three times the size of *Bierens de Haan*, and the circumstance that a considerable proportion of these integrals involves higher transcendental functions with their manifold and not always highly standardized notations, does not make this task easier. Eventually, Bateman decided to break up his integral tables into several more or less self-contained parts, classifying integrals according to their fields of application. A collection of integrals occurring in the theory of axially symmetric potentials was prepared, and other similar collections were to follow. Clearly such a plan involves a generous amount of duplication if the resulting tables are to be self-contained, but it also has great advantages from the user's point of view.

In planning our work on definite integrals, we were in the fortunate position of being able to restrict its scope. In recent years several excellent tables of integrals of elementary functions appeared, the most easily available ones being those by W. Meyer zur Capellen, and by W. Gröbner and N. Hofreiter. We also learned through the courtesy of the authors that a handbook of elliptic integrals by P.F. Byrd and M.D. Friedman is in preparation (and will have been published before this volume appears). In the assumption that our tables would be used in conjunction with other existing tables, we decided to concentrate mostly on integrals involving higher transcendental functions. We list no double integrals, and, except in the case of inverse transforms no contour integrals.

We adopted Bateman's idea of breaking up the tables into several, more or less self contained, parts; but we modified his principle of subdivision. We found that much of our material could be organized in tables of integral transforms, and accordingly the present volume, and about one half of volume II of our tables, consists of tables of integral transforms; those of our integrals which have not been classified as integral transforms being contained in the second half of volume II. We hope that this division will be found useful. Integral transforms have become an extensively used tool, and their practical application depends largely on tables of transform pairs. Laplace transforms are almost unique in that several up-to-date and thoroughly satisfactory tables of such transforms are available. For Fourier transforms there is an excellent collection of integrals, but it was compiled in 1931, and newer editions do not include additional material. For Hankel and Mellin transforms, and other integral transforms, we know of no extensive tables. In addition to the well-known transforms we give tables of integral transforms whose kernel is a Bessel function of the second kind, a modified Bessel function, Struve function, and the like, partly because some of these transforms are useful in solving certain boundary value problems, or certain integral equations, and partly because they afford a convenient classification of integrals.

Writing integrals as integral transforms helps avoiding one of the greatest difficulties of all integral tables. By a change in the variable of integration, every definite integral may be written in a number of ways, and given such an integral, it is sometimes not at all clear whether one should look for it under integrals with an algebraic integrand, under trigonometric integrals, or perhaps under infinite integrals involving exponential functions. In integral transforms, the variable of integration is standardized usually up to a constant factor (and, in the case of Mellin transforms, a constant exponent). The advantage thus gained is, of course, offset by the circumstance that an integral such as

$$\int_0^\infty x^{3/2} e^{-ax} J_\nu(bx)\, dx$$

might be found under Laplace transforms, Mellin transforms, or Hankel transforms. We attempted to cope with this difficulty by repeating many of our integrals (especially the more basic ones) under several transforms.

As in the case of *Higher Transcendental Functions*, we made only limited use of Bateman's notes, supplementing them by the use of practically all available integral tables, consulting the periodical literature, and textbooks, and evaluating some integrals not found in the literature. Much of the work on these tables was done by the Research Assistants whose names appear on p. vii. Professor Oberhettinger collected most of the integrals which appear in the second half of volume II of our tables, and he continued this work after he joined the staff of the American University. The vari-typing of such a conglomeration of complicated formulas presents very serious difficulties indeed, and we were very fortunate in having with us Miss Stampfel for whom difficulties are an attraction as well as a challenge.

ORGANIZATION AND USE OF THE TABLES

Most of the integrals in this work are arranged in tables of integral transforms. The present volume contains Fourier, Laplace, and Mellin transforms and their inversions. Further transforms will be given in volume II which will also contain, under the heading *Integrals of higher transcendental functions*, a number of integrals which are not found in the transform tables. The transform tables themselves include integrals whose integrands are elementary functions.

For each of the integral transforms we have adopted a standard form; a list of the standard forms of Fourier, Laplace, and Mellin transforms is given on p. xv, and a corresponding list will appear in volume II. In order to find the value of a definite integral, one has to transform it to one of these standard forms, and then look in the corresponding table. In many cases an integral may be subsumed under several standard forms. In the case of important or simple integrals we give the result in several or all tables, in the case of more complicated, or infrequently used, integrals, in the first table into which it fits. For instance, if an integral may be written either as a Fourier or as a Laplace transform, than it is either repeated in both tables, or else more likely to be found in the Fourier transform tables. An exception is made in the case of integrals which appear more "naturally" as Laplace transforms and may be listed accordingly. Integrals involving higher transcendental functions which are not contained in one of the integral transform tables may be found in the second half of volume II. This is true not only of integrals which cannot be written as integral transforms, but also of integrals which, for one reason or another, were not included in the table of the appropriate integral transforms.

From the integrals given in the tables, a further large number of integrals may be derived by a number of devices. One of the most fruitful of these is specializing parameters. Thus, by specializing parameters in an integral involving confluent hypergeometric functions, one may derive integrals with Bessel functions, Laguerre polynomials, parabolic cylinder functions, and many other functions. We hope to give an extensive list of special cases of higher transcendental functions in an Appendix to volume II of this work; alternatively, *Higher Transcendental Functions*, or some similar work on special functions, may be consulted. We mention in particular that the G-function introduced by C.S. Meijer includes all functions of hypergeometric type. We have given several integrals involving this function, and a list of some of its special cases (mostly those involving Bessel functions and related functions, and confluent hypergeometric functions) is given on p. 374 ff. of this volume. Other devices are: differentiation or integration with respect to parameters contained in the integral, integration by parts, substitution of integral representations for one or the other function contained in the integral, and, in the case of integral transforms, use of an inversion formula. In the case of some of the integral transforms there are additional devices listed in the brief description, and in the collection of "general formulas", which are given for each of the transforms.

Conditions of validity are stated for each entry. These are usually not the most general ones. In particular, it may happen that for some special values of, or under additional condition on, some of the parameters the domain of convergence is considerably larger than the one stated. Again, in the case of Fourier and Hankel transforms we take the variable y to be real, although many of the integrals converge also for some complex values of y. Generally speaking, we expect the user of these tables to be sufficiently familiar with the functions he encounters to be able to determine the region of convergence in each case.

Each transform has a chapter to itself. In the tables of transforms, the entries are arranged in tabular form, and the standard form is repeated at the head of every page. In each chapter we first list general formulas valid for the transform involved (usually without giving conditions of validity), and then transform pairs arranged according to the entries in the left column. First come elementary functions, proceeding from rational functions to algebraic functions, functions containing powers with arbitrary (not necessarily rational) indices, exponential functions, logarithms, trigonometric, inverse trigonometric, hyperbolic, and inverse hyperbolic functions. Higher transcendental functions follow, the order being: orthogonal polynomials, the gamma function and related functions, Legendre functions, Bessel functions and related functions, parabolic cylinder functions, hypergeometric functions and their generalizations, elliptic functions, and miscellaneous other functions. Each chapter is subdivided in an appropriate number of sections, the number of these sections and their grouping being different in different chapters. Composite functions are classified according to the "highest" function occurring in them, so that $\log(e^x - 1)$ would be found under logarithms. In some cases there is a degree of ambiguity, for instance in the case of

$$\left(\frac{x+ia}{x-ia}\right)^{\lambda} + \left(\frac{x-ia}{x+ia}\right)^{\lambda} = 2\cos[2\lambda\tan^{-1}(a/x)]$$

which may be found in one form or another under powers with arbitrary index or under inverse trigonometric functions. Since we could not devise a thoroughly systematic and foolproof arrangement, we did not attempt to be very systematic, and hope that a person using these tables frequently will soon find his way in the maze.

The notation adopted for the special functions in these tables is by and large the same as that used in our *Higher Transcendental Functions*, although a few deviations occur. The most important of these are the notations for error functions, Hermite polynomials, and confluent hypergeometric functions. A complete list of the notations used throughout this volume is given on p. 367 ff., and an index to this list on p. 389 ff.

In the case of more complicated results we sometimes avoid repetition by giving cross-references to other parts of the tables. In giving references to specific entries, the section number is followed by the number of the equation; thus 3.2(5) refers to entry (5) in section 3.2. Occasionally when a result seemed too complicated to be given in full, we refer to a book or paper where the result may be found. Similarly, we occasionally refer to sources for additional integrals, not contained in these tables. Apart from cross-references, each chapter may be used independently of the others, although in the case of Fourier transforms, sine, cosine, and exponential transforms, and in the case of Laplace and Mellin transforms, tables of direct and inverse transforms, complement each other to some extent. The various connections between the transforms tabulated in this work may be used, and such possibilities are pointed out in the descriptions preceding the tables of transforms.

ACKNOWLEDGMENTS

It is a pleasant duty to express the thanks of the California Institute of Technology to the family of the late Professor Bateman for the gift of his notes and library, and to the Office of Naval Research for the generous support they have given to this work, and for the understanding they have constantly shown for the difficulties encountered. The Institute also wishes to record its appreciation and thanks to the late Professor A.D. Michal for his preliminary survey of Bateman's notes; to the

University of Edinburgh for granting leave of absence to the undersigned; to the Rockefeller Foundation for making a grant towards traveling expenses; and to the McGraw-Hill Company for technical advice and publication.

The Editor wishes to acknowledge his indebtedness to practically all the published integral tables which have been consulted in the course of the work on the present tables; and he wishes to express his thanks to his colleagues, especially the Research Assistants who played such an important part in the compilation.

Corrections of errors, additions, and suggestion for improvement will be gratefully received by the Editor.

A. ERDÉLYI

STANDARD FORMS

Fourier sine transform

$$\int_0^\infty f(x) \sin(xy)\, dx.$$

Fourier cosine transform

$$\int_0^\infty f(x) \cos(xy)\, dx.$$

Exponential Fourier transform

$$\int_{-\infty}^\infty f(x)\, e^{-ixy}\, dx.$$

Laplace transform

$$\int_0^\infty f(t)\, e^{-pt}\, dt.$$

Inverse Laplace transform

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(p)\, e^{pt}\, dp.$$

Mellin transform

$$\int_0^\infty f(x)\, x^{s-1}\, dx.$$

Inverse Mellin transform

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(s)\, x^{-s}\, ds.$$

ERRATA

TABLES OF INTEGRAL TRANSFORMS, VOL. I.

P. 56, equation (42): On the right-hand side read $H^{(2)}_{\nu-\frac{1}{2}}$ instead of $H^{(2)}_{\nu}$.

P. 57, equation (48): On the left-hand side read $(x^2 - a^2)^{\frac{1}{2}}$ instead of $(x^2 - a^2)$.

P. 59, equations (68) and (69): Insert factor 2 on the right-hand side.

P. 84, equation (14): On the right-hand side read a^{ν} instead of b^{ν}.

P. 122, equation (1): Read -1 instead of $--1$.

P. 153, equation (30): The right-hand side should read

$$(\tfrac{1}{2}\pi)^{\frac{1}{2}}\{\cos(\tfrac{1}{4}p^2)[\tfrac{1}{2} - C(\tfrac{1}{4}p^2)] + \sin(\tfrac{1}{4}p^2)[\tfrac{1}{2} - S(\tfrac{1}{4}p^2)]\} \qquad \operatorname{Re} p > 0.$$

P. 158, equation (64): The right-hand side should read

$$(\tfrac{1}{2}\pi)^{\frac{1}{2}}\{\cos(\tfrac{1}{4}p^2)[\tfrac{1}{2} - S(\tfrac{1}{4}p^2)] - \sin(\tfrac{1}{4}p^2)[\tfrac{1}{2} - C(\tfrac{1}{4}p^2)]\} \qquad \operatorname{Re} p > 0.$$

P. 278, equation (20): On the right-hand side read $\frac{1}{2}$ instead of $-\frac{1}{2}$.

P. 319, equation (18): Read $\exp\left(\frac{1}{4a}\right)$ instead of $e^{-2a^{-1}}$.

P. 320, equation (25): On the right-hand side, change $-$ into $+$ in front of the second ${}_1F_1$.

P. 321, equation (37): Read $\exp\left(\frac{1}{4a}\right)$ instead of $e^{-2a^{-1}}$.

P. 332, equation (36): Insert factor $\Gamma(1-s)$ on the right-hand side.

P. 333, equation (41): On the right-hand side read 2^{s-3} instead of 2^{s-2}.

P. 338, equation (15), line 3: Read

$$0 \qquad 1 < x < \infty$$

instead of $\qquad 1 < x < \infty$.

P. 367, line 7 up: Read $\int_{c+\epsilon}^{b}$ instead of $\int_{c-\epsilon}^{b}$.

P. 378, line 7: Read $k = \frac{1}{2} + 2b - 2a$ for $k = \frac{1}{2} + 2b - 2c$.

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FOURIER TRANSFORMS

We distinguish between *Fourier cosine transforms*

$$\mathfrak{F}_c\{f(x); y\} = \int_0^\infty f(x) \cos(xy)\, dx,$$

Fourier sine transforms

$$\mathfrak{F}_s\{f(x); y\} = \int_0^\infty f(x) \sin(xy)\, dx,$$

and *exponential* (or complex) *Fourier transforms*

$$\mathfrak{F}_e\{f(x); y\} = \int_{-\infty}^\infty f(x)\, e^{-ixy}\, dx.$$

In the first two we assume y to be positive, in the last y is real. For the sake of convenience we omitted a factor $(2/\pi)^{1/2}$ in the definition of the cosine and sine transforms, and a factor $(2\pi)^{-1/2}$ in the definition of the exponential transform. This omission makes the general formulas, notably the inversion formulas, unsymmetrical, but it simplifies somewhat many of the transform pairs.

The principal works describing the theory and application of Fourier transforms are listed on p. 5. The most extensive published list of Fourier transforms is Campbell and Foster (1948). Fourier transforms are closely connected with Laplace transforms and Mellin transforms, and references given for these may be consulted. In addition, many textbooks and works of reference contain material on Fourier transforms.

From the transform pairs given in chapters I to III additional transform pairs may be obtained by means of the general formulas (rules) given in sections 1.1, 2.1, 3.1, or using some of the methods mentioned in the introduction to this volume. Furthermore, Fourier transforms are connected with each other, and with Laplace, Mellin, and Hankel transforms, as described in the formulas given below, and these connections may be used to evaluate further Fourier transforms as combinations of those in the tables, or by means of the tables of Laplace, Mellin, and Hankel transforms given in chapters IV, VI, and VIII.

$$\mathfrak{F}_c\{f(x); y\} = \int_0^\infty f(x)\,dx - y\,\mathfrak{F}_s\{\int_x^\infty f(t)\,dt; y\}$$
$$= \tfrac{1}{2}\mathfrak{F}_e\{f(|x|); y\}$$
$$= \tfrac{1}{2}\mathfrak{L}\{f(x); iy\} + \tfrac{1}{2}\mathfrak{L}\{f(x); -iy\}$$
$$= \tfrac{1}{2}\mathfrak{M}\{f(|\log x|); iy\}$$
$$= (\tfrac{1}{2}\pi)^{\frac{1}{2}}\,\mathfrak{H}_{-\frac{1}{2}}\{f(x); y\}$$

$$\mathfrak{F}_s\{f(x); y\} = y\,\mathfrak{F}_c\{\int_x^\infty f(t)\,dt; y\}$$
$$= \tfrac{1}{2}i\;\mathfrak{F}_e\{\operatorname{sgn} x\; f(|x|); y\}$$
$$= \tfrac{1}{2}i\;\mathfrak{L}\{f(x); iy\} - \tfrac{1}{2}i\;\mathfrak{L}\{f(x); -iy\}$$
$$= \tfrac{1}{2}i\;\mathfrak{M}\{\operatorname{sgn}(\log x)\, f(|\log x|); -iy\}$$
$$= (\tfrac{1}{2}\pi)^{\frac{1}{2}}\,\mathfrak{H}_{\frac{1}{2}}\{f(x); y\}$$

$$\mathfrak{F}_e\{f(x); y\} = \mathfrak{F}_c\{f(x) + f(-x); y\} - i\;\mathfrak{F}_s\{f(x) - f(-x); y\}$$
$$= \mathfrak{L}\{f(x); iy\} + \mathfrak{L}\{f(-x); -iy\}$$
$$= \mathfrak{M}\{f(\log x); -iy\}$$

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See also under Laplace transforms.

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CHAPTER I

FOURIER COSINE TRANSFORMS

1.1. General formulas

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)\,dx \qquad y > 0$
(1)	$g(x)$	$\frac{1}{2}\pi f(y)$
(2)	$f(ax) \qquad a > 0$	$a^{-1} g(a^{-1} y)$
(3)	$f(ax)\cos(bx) \qquad a, b > 0$	$\frac{1}{2a}\left[g\left(\frac{y+b}{a}\right) + g\left(\frac{y-b}{a}\right)\right]$
(4)	$f(ax)\sin(bx) \qquad a, b > 0$	$\frac{1}{2a}\int_0^\infty f(x)\sin\left(\frac{y+b}{a}x\right)dx$ $-\frac{1}{2a}\int_0^\infty f(x)\sin\left(\frac{y-b}{a}x\right)dx$
(5)	$x^{2n} f(x)$	$(-1)^n \frac{d^{2n} g(y)}{dy^{2n}}$
(6)	$x^{2n+1} f(x)$	$(-1)^n \frac{d^{2n+1}}{dy^{2n+1}} \int_0^\infty f(x)\sin(xy)\,dx$

1.2. Algebraic functions

(1)	$1 \qquad 0 < x < a$ $0 \qquad a < x < \infty$	$y^{-1}\sin(ay)$
(2)	$x \qquad 0 < x < 1$ $2 - x \qquad 1 < x < 2$ $0 \qquad 2 < x < \infty$	$y^{-2}(2\cos y - 1 - \cos 2y)$

Algebraic functions (cont'd)

	$f(x)$		$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(3)	0 $1/x$	$0<x<a$ $a<x<\infty$	$-\operatorname{Ci}(ay)$
(4)	$x^{-1/2}$		$\pi^{1/2}(2y)^{-1/2}$
(5)	$x^{-1/2}$ 0	$0<x<1$ $1<x<\infty$	$(2\pi)^{1/2}\,y^{-1/2}\,C(y)$
(6)	0 $x^{-1/2}$	$0<x<1$ $1<x<\infty$	$(2\pi)^{1/2}\,y^{-1/2}\,[1/2-C(y)]$
(7)	$(a+x)^{-1}$	$\lvert\arg a\rvert<\pi$	$-\operatorname{si}(ay)\sin(ay)-\operatorname{Ci}(ay)\cos(ay)$
(8)	$(a-x)^{-1}$	$a>0$	$\cos(ay)\operatorname{Ci}(ay)$ $+\sin(ay)[\tfrac{1}{2}\pi+\operatorname{Si}(ay)]$ The integral is a Cauchy Principal Value
(9)	$(x+a)^{-1/2}$	$\lvert\arg a\rvert<\pi$	$\pi^{1/2}(2y)^{-1/2}[\cos(ay)+\sin(ay)$ $-2C(ay)\cos(ay)-2S(ay)\sin(ay)]$
(10)	0 $(x-a)^{-1/2}$	$0<x<a$ $a<x<\infty$	$\pi^{1/2}(2y)^{-1/2}[\cos(ay)-\sin(ay)]$
(11)	$(x^2+a^2)^{-1}$	$\operatorname{Re} a>0$	$\tfrac{1}{2}\pi\,a^{-1}\,e^{-ay}$
(12)	$x(x^2+a^2)^{-1}$	$a>0$	$-\tfrac{1}{2}[e^{-ay}\,\overline{\operatorname{Ei}}(ay)+e^{ay}\operatorname{Ei}(-ay)]$
(13)	$\dfrac{\beta}{\beta^2+(a-x)^2}+\dfrac{\beta}{\beta^2+(a+x)^2}$	$\lvert\operatorname{Im} a\rvert<\operatorname{Re}\beta$	$\pi\cos(ay)\,e^{-\beta y}$

Algebraic functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\cos(xy)dx \qquad y>0$
(14)	$\dfrac{a+x}{\beta^2+(a+x)^2} + \dfrac{a-x}{\beta^2+(a-x)^2}$ $\quad \lvert \mathrm{Im}\, a\rvert < \mathrm{Re}\,\beta$	$\pi e^{-\beta y}\sin(ay)$
(15)	$(a^2-x^2)^{-1} \qquad a>0$	$\tfrac{1}{2}\pi a^{-1}\sin(ay)$ The integral is a Cauchy Principal Value
(16)	$x(a^2-x^2)^{-1} \qquad a>0$	$\cos(ay)\,\mathrm{Ci}(ay)+\sin(ay)\,\mathrm{Si}(ay)$ The integral is a Cauchy Principal Value
(17)	$(x^2+a^2)^{-1/2} \qquad \mathrm{Re}\, a>0$	$K_0(ay)$
(18)	$[(a^2+x^2)(\beta^2+x^2)]^{-1}$ $\quad \mathrm{Re}\,a>0, \quad \mathrm{Re}\,\beta>0$	$\tfrac{1}{2}\pi(\beta^{-1}e^{-\beta y}-a^{-1}e^{-ay})(a^2-\beta^2)^{-1}$
(19)	$(x^4+a^4)^{-1} \qquad \lvert\arg a\rvert<\pi/4$	$\tfrac{1}{2}\pi a^{-3}\exp(-2^{-1/2}ay)$ $\times\sin(\pi/4+2^{-1/2}ay)$
(20)	$[x^4+2a^2x^2\cos(2\theta)+a^4]^{-1}$ $\quad a>0, \quad -\tfrac{1}{2}\pi<\theta<\tfrac{1}{2}\pi$	$\tfrac{1}{2}\pi a^{-3}e^{-ay\cos\theta}$ $\times\sin(\theta+ay\sin\theta)\csc(2\theta)$
(21)	$x^2[x^4+2a^2x^2\cos(2\theta)+a^4]^{-1}$ $\quad a>0, \quad -\tfrac{1}{2}\pi<\theta<\tfrac{1}{2}\pi$	$\tfrac{1}{2}\pi a^{-1}\csc(2\theta)e^{-ay\cos\theta}$ $\times\sin(\theta-ay\sin\theta)$
(22)	$x^{-1/2}(x^2+a^2)^{-1/2} \qquad a>0$	$(\tfrac{1}{2}\pi y)^{1/2} I_{-1/4}(\tfrac{1}{2}ay)K_{1/4}(\tfrac{1}{2}ay)$
(23)	$x^{-1/2}(a^2-x^2)^{-1/2} \qquad 0<x<a$ $0 \qquad a<x<\infty$	$2^{-3/2}\pi^{3/2}y^{1/2}[J_{-1/4}(\tfrac{1}{2}ay)]^2$

Algebraic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)dx \qquad y>0$
(24)	$0 \qquad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2} \qquad a<x<\infty$	$-2^{-3/2}\pi^{3/2}y^{1/2}J_{-1/4}(\tfrac{1}{2}ay)$ $\times Y_{-1/4}(\tfrac{1}{2}ay)$
(25)	$(a^2+x^2)^{-1/2}[(a^2+x^2)^{1/2}+a]^{1/2}$ $\mathrm{Re}\, a>0$	$(2y/\pi)^{-1/2}e^{-ay}$
(26)	$\dfrac{x^{1/2}}{R_1R_2}\left(\dfrac{R_2+R_1}{R_2-R_1}\right)^{1/2}$ $R_1=[a^2+(b-x)^2]^{1/2}$ $R_2=[a^2+(b+x)^2]^{1/2}$ $a>0$	$b^{-1/2}K_0(ay)\cos(by)$
(27)	$x^{-1/2}(x^2+a^2)^{-1/2}$ $\times[x+(x^2+a^2)^{1/2}]^{-3/2}$ $\mathrm{Re}\, a>0$	$2^{-1/2}a^{-2}\sinh(\tfrac{1}{2}ay)K_1(\tfrac{1}{2}ay)$
(28)	$x^{2m}(x^2+z)^{-n-1}$ $n+1>m\geq 0, \quad \lvert\arg z\rvert<\pi$	$(-1)^{m+n}\tfrac{1}{2}\pi(n!)^{-1}$ $\times\dfrac{d^n}{dz^n}(z^{m-1/2}e^{-yz^{1/2}})$
(29)	$\dfrac{x^{m-1}}{x^{2n}+a^{2n}} \qquad 2n+1>m>0$	$0 \qquad m$ even $\dfrac{\pi}{2na^{2n-m}}\sum_{k=1}^{n}e^{-ay\sin[(2k-1)\pi/(2n)]}$ $\times\sin\left[\dfrac{(2k-1)m\pi}{2n}+ay\cos\dfrac{(2k-1)\pi}{2n}\right]$ m odd

1.3. Powers with arbitrary index

(1)	$x^{-\nu} \qquad 0<\mathrm{Re}\,\nu<1$	$\tfrac{1}{2}\pi[\Gamma(\nu)]^{-1}\sec(\tfrac{1}{2}\nu\pi)\,y^{\nu-1}$

Arbitrary powers (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(2)	$x^{\nu-1}$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$ $\operatorname{Re}\nu>0$	$(2\nu)^{-1}[{}_1F_1(\nu;\nu+1;iy)$ $+{}_1F_1(\nu;\nu+1;-iy)]$
(3)	$(1-x)^{\nu}$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$ $\operatorname{Re}\nu>-1$	$\tfrac{1}{2}iy^{-\nu-1}[e^{i(\frac{1}{2}\nu\pi-y)}\gamma(\nu+1,-iy)$ $+e^{-i(\frac{1}{2}\nu\pi-y)}\gamma(\nu+1,iy)]$
(4)	$x^{\nu}(1-x)^{\nu}$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$ $\operatorname{Re}\nu>-1$	$\pi^{\frac{1}{2}}\Gamma(\nu+1)(2y)^{-\nu-\frac{1}{2}}$ $\times\cos y\,J_{\nu+\frac{1}{2}}(y)$
(5)	$x^{\nu-1}(1-x)^{\mu-1}$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$ $\operatorname{Re}\nu>0,\quad \operatorname{Re}\mu>0$	$\tfrac{1}{2}B(\nu,\mu)[{}_1F_1(\nu;\nu+\mu;iy)$ $+{}_1F_1(\nu;\nu+\mu;-iy)]$
(6)	$x^{\nu}(1+x^2)^{-1}$ $-1<\operatorname{Re}\nu<2$	$\tfrac{1}{2}\pi\cosh y\csc[\tfrac{1}{2}\pi(\nu+1)]$ $+\tfrac{1}{2}\Gamma(\nu+1)\cos[\tfrac{1}{2}\pi(\nu+1)]$ $\times[e^{-y-i\nu\pi}\gamma(-\nu,-y)$ $-e^{y}\gamma(-\nu,y)]$
(7)	$(x^2+a^2)^{-\nu-\frac{1}{2}}$ $\operatorname{Re}a>0,\quad \operatorname{Re}\nu>-\tfrac{1}{2}$	$(\tfrac{1}{2}y/a)^{\nu}\pi^{\frac{1}{2}}[\Gamma(\nu+\tfrac{1}{2})]^{-1}K_{\nu}(ay)$
(8)	$(a^2-x^2)^{\nu-\frac{1}{2}}$ $\quad 0<x<a$ 0 $\quad a<x<\infty$ $\operatorname{Re}\nu>-\tfrac{1}{2}$	$2^{\nu-1}\Gamma(\nu+\tfrac{1}{2})\pi^{\frac{1}{2}}a^{\nu}y^{-\nu}J_{\nu}(ay)$
(9)	0 $\quad 0<x<a$ $(x^2-a^2)^{-\nu-\frac{1}{2}}$ $\quad a<x<\infty$ $\lvert\operatorname{Re}\nu\rvert<\tfrac{1}{2}$	$-2^{-\nu-1}\pi^{\frac{1}{2}}y^{\nu}a^{-\nu}\Gamma(\tfrac{1}{2}-\nu)Y_{\nu}(ay)$

Arbitrary powers (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)dx \qquad y>0$
(10)	$x(a^2-x^2)^{\nu-\frac{1}{2}} \quad 0<x<a$ $0 \quad a<x<\infty$ $\operatorname{Re}\nu>-\frac{1}{2}$	$-a^{\nu+1}y^{-\nu}s_{\nu,\nu+1}(ay)$ $=\frac{1}{2}(\nu+\frac{1}{2})a^{2\nu+1}-2^{\nu-1}\pi^{\frac{1}{2}}a^{\nu+1}y^{-\nu}$ $\times\Gamma(\nu+\frac{1}{2})\,\mathbf{H}_{\nu+1}(ay)$
(11)	$0 \quad 0<x<a$ $x(x^2-a^2)^{-\nu-\frac{1}{2}} \quad a<x<\infty$ $0<\operatorname{Re}\nu<\frac{1}{2}$	$2^{-\nu-1}\pi^{\frac{1}{2}}a^{-\nu+1}\Gamma(\frac{1}{2}-\nu)y^{\nu}J_{\nu-1}(ay)$
(12)	$0 \quad 0<x<2a$ $(x^2-2ax)^{-\nu-\frac{1}{2}} \quad 2a<x<\infty$ $\lvert\operatorname{Re}\nu\rvert<\frac{1}{2}$	$-\frac{1}{2}\pi^{\frac{1}{2}}\Gamma(\frac{1}{2}-\nu)(2a)^{-\nu}y^{\nu}$ $\times[J_{\nu}(ay)\sin(ay)+Y_{\nu}(ay)\cos(ay)]$
(13)	$(x^2+2ax)^{-\nu-\frac{1}{2}} \quad \lvert\operatorname{Re}\nu\rvert<\frac{1}{2}$	$-y^{\nu}a^{-\nu}\pi^{\frac{1}{2}}2^{-\nu-1}\Gamma(\frac{1}{2}-\nu)$ $\times[Y_{\nu}(ay)\cos(ay)-J_{\nu}(ay)\sin(ay)]$
(14)	$(2ax-x^2)^{\nu-\frac{1}{2}} \quad 0<x<2a$ $0 \quad 2a<x<\infty$ $\operatorname{Re}\nu>-\frac{1}{2}$	$\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})(2a)^{\nu}y^{-\nu}J_{\nu}(ay)\cos(ay)$
(15)	$(a^2+x^2)^{-\frac{1}{2}}[x+(a^2+x^2)^{\frac{1}{2}}]^{-\nu}$ $\operatorname{Re}a>0, \quad \operatorname{Re}\nu>-1$	$\csc(\nu\pi)\,\pi a^{-\nu}[\frac{1}{2}\mathbf{J}_{\nu}(iay)+\frac{1}{2}\mathbf{J}_{\nu}(-iay)$ $-I_{\nu}(ay)\cos(\frac{1}{2}\pi\nu)]$
(16)	$x^{-\frac{1}{2}}(a^2+x^2)^{-\frac{1}{2}}$ $\times[(a^2+x^2)^{\frac{1}{2}}+x]^{\nu}$ $\operatorname{Re}a>0, \quad \operatorname{Re}\nu<3/2$	$a^{\nu}2^{-\frac{1}{2}}(\pi y)^{\frac{1}{2}}I_{-\frac{1}{4}-\frac{1}{2}\nu}(\frac{1}{2}ay)$ $\times K_{-\frac{1}{4}+\frac{1}{2}\nu}(\frac{1}{2}ay)$
(17)	$x^{-\frac{1}{2}}(a^2+x^2)^{-\frac{1}{2}}$ $\times[(a^2+x^2)^{\frac{1}{2}}-x]^{\nu}$ $\operatorname{Re}a>0, \quad \operatorname{Re}\nu>-3/2$	$a^{\nu}2^{-\frac{1}{2}}(\pi y)^{\frac{1}{2}}I_{-\frac{1}{4}+\frac{1}{2}\nu}(\frac{1}{2}ay)$ $\times K_{-\frac{1}{4}-\frac{1}{2}\nu}(\frac{1}{2}ay)$
(18)	$x^{-\nu-\frac{1}{2}}(x^2+a^2)^{-\frac{1}{2}}$ $\times[a+(x^2+a^2)^{\frac{1}{2}}]^{\nu}$ $\operatorname{Re}a>0, \quad \operatorname{Re}\nu<\frac{1}{2}$	$a^{-1}(2y)^{-\frac{1}{2}}\Gamma(\frac{1}{4}-\frac{1}{2}\nu)$ $\times W_{\frac{1}{2}\nu,-\frac{1}{4}}(ay)M_{-\frac{1}{2}\nu,-\frac{1}{4}}(ay)$

Arbitrary powers (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(19)	$[(a+ix)^{-\nu}+(a-ix)^{-\nu}]$ $\operatorname{Re} a>0, \quad \operatorname{Re}\nu>0$	$\pi[\Gamma(\nu)]^{-1}y^{\nu-1}e^{-ay}$
(20)	$x^{2n}[(a+ix)^{-\nu}+(a-ix)^{-\nu}]$ $0\le 2n<\operatorname{Re}\nu, \quad \operatorname{Re} a>0$	$(-1)^n\pi[\Gamma(\nu)]^{-1}(2n)!\,e^{-ay}$ $\times y^{\nu-1-2n}L_{2n}^{\nu-1-2n}(ay)$
(21)	$x^{2n-1}[(a-ix)^{-\nu}-(a+ix)^{-\nu}]$ $2\le 2n<\operatorname{Re}\nu+1$ $\operatorname{Re} a>0$	$(-1)^n\pi i[\Gamma(\nu)]^{-1}(2n-1)!\,e^{-ay}$ $\times y^{\nu-2n}L_{2n-1}^{\nu-2n}(ay)$
(22)	$(a^2+x^2)^{-1/2}\{[(a^2+x^2)^{1/2}+x]^\nu$ $+[(a^2+x^2)^{1/2}-x]^\nu\}$ $\operatorname{Re} a>0, \quad \lvert\operatorname{Re}\nu\rvert<1$	$2a^\nu K_\nu(ay)\cos(\tfrac{1}{2}\nu\pi)$
(23)	$\{[x+a+(x^2+2ax)^{1/2}]^\nu$ $+[x+a-(x^2+2ax)^{1/2}]^\nu\}$ $\times(x^2+2ax)^{-1/2}$ $\lvert\operatorname{Re}\nu\rvert<1$	$\pi a^\nu[J_\nu(ay)\sin(ay-\tfrac{1}{2}\pi\nu)$ $-Y_\nu(ay)\cos(ay-\tfrac{1}{2}\pi\nu)]$
(24)	$(a^2-x^2)^{-1/2}\{[x+i(a^2-x^2)^{1/2}]^\nu$ $+[x-i(a^2-x^2)^{1/2}]^\nu\}$ $0<x<a$ $0 \qquad a<x<\infty$ $\lvert\operatorname{Re}\nu\rvert<1$	$\tfrac{1}{2}\pi a^\nu\sec(\tfrac{1}{2}\pi\nu)[\mathbf{J}_\nu(ay)+\mathbf{J}_{-\nu}(ay)]$
(25)	$0 \qquad 0<x<a$ $(x^2-a^2)^{-1/2}\{[x+(x^2-a^2)^{1/2}]^\nu$ $+[x-(x^2-a^2)^{1/2}]^\nu\}$ $a<x<\infty$ $\lvert\operatorname{Re}\nu\rvert<1$	$-\pi a^\nu[Y_\nu(ay)\cos(\tfrac{1}{2}\pi\nu)$ $+J_\nu(ay)\sin(\tfrac{1}{2}\pi\nu)]$
(26)	$0 \qquad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}\{[x+(x^2-a^2)^{1/2}]^\nu$ $+[x-(x^2-a^2)^{1/2}]^\nu\}$ $a<x<\infty$ $\operatorname{Re}\nu<3/2$	$-\tfrac{1}{2}\pi(\tfrac{1}{2}\pi y)^{1/2}a^\nu[J_{-1/4+1/2\nu}(\tfrac{1}{2}ay)$ $\times Y_{-1/4-1/2\nu}(\tfrac{1}{2}ay)$ $+J_{-1/4-1/2\nu}(\tfrac{1}{2}ay)Y_{-1/4+1/2\nu}(\tfrac{1}{2}ay)]$

Arbitrary powers (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(27)	$(4b^2x-x^3)^{-1/2}$ $\times\{[(2b+x)^{1/2}+i(2b-x)^{1/2}]^{4\nu}$ $+[(2b+x)^{1/2}-i(2b-x)^{1/2}]^{4\nu}\}$ $0<x<2b$; 0 $2b<x<\infty$	$(4b)^{2\nu}\pi^{3/2}y^{1/2}2^{-1/2}J_{\nu-1/4}(by)$ $\times J_{-\nu-1/4}(by)$
(28)	$\dfrac{x^{2m}}{(x^2+a^2)^{\nu+1/2}}$ $\operatorname{Re} a>0$ $0\le m<\operatorname{Re}\nu+1/2$	$\dfrac{(-1)^m a^{-\nu}\pi^{1/2}}{2^\nu\Gamma(\nu+1/2)}\dfrac{d^{2m}}{dy^{2m}}[y^\nu K_\nu(ay)]$
(29)	$x^\nu(x^2+a^2)^{-\mu-1}$ $\operatorname{Re} a>0$ $-1<\operatorname{Re}\nu<2\operatorname{Re}\mu+2$	$\tfrac{1}{2}a^{\nu-2\mu-1}[\mathrm{B}(\tfrac{1}{2}+\tfrac{1}{2}\nu,\mu-\tfrac{1}{2}\nu+\tfrac{1}{2})$ $\times{}_1F_2[\tfrac{1}{2}(\nu+1);\tfrac{1}{2}(\nu+1)-\mu,\tfrac{1}{2};\tfrac{1}{4}a^2y^2]$ $+\pi^{1/2}2^{-2\mu+\nu-2}[\Gamma(\mu-\tfrac{1}{2}\nu)]^{-1}$ $\times y^{2\mu-\nu+1}\Gamma(\tfrac{1}{2}\nu-\mu-\tfrac{1}{2})$ $\times{}_1F_2(\mu+1-\nu/2,\mu-\nu/2+3/2;a^2y^2/4)$
(30)	$x^\mu(1-x^2)^\lambda$ $0<x<1$; 0 $1<x<\infty$	see under Hankel-transforms

1.4. Exponential functions

For other integrals with exponential functions see table of Laplace transforms

(1)	e^{-ax} $\operatorname{Re} a>0$	$a(a^2+y^2)^{-1}$
(2)	$x^{-1}(e^{-\beta x}-e^{-ax})$ $\operatorname{Re} a, \operatorname{Re}\beta>0$	$\tfrac{1}{2}\log\left(\dfrac{a^2+y^2}{\beta^2+y^2}\right)$
(3)	$x^{1/2}e^{-ax}$ $\operatorname{Re} a>0$	$\pi^{1/2}/2\,(a^2+y^2)^{-3/4}\cos[3/2\tan^{-1}(y/a)]$
(4)	$x^{-1/2}e^{-ax}$ $\operatorname{Re} a>0$	$\pi^{1/2}2^{-1/2}(a^2+y^2)^{-1/2}[(a^2+y^2)^{1/2}+a]^{1/2}$
(5)	$x^n e^{-ax}$ $\operatorname{Re} a>0$	$n!\left(\dfrac{a}{a^2+y^2}\right)^{n+1}$ $\times\sum_{0\le 2m\le n+1}(-1)^m\binom{n+1}{2m}\left(\dfrac{y}{a}\right)^{2m}$

Exponential functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(6)	$x^{n-\frac{1}{2}}e^{-ax}$ $\qquad \operatorname{Re} a>0$	$(-1)^n\pi^{\frac{1}{2}}2^{-\frac{1}{2}}y$ $\times\dfrac{d^n}{da^n}\{(a^2+y^2)^{-\frac{1}{2}}[(a^2+y^2)^{\frac{1}{2}}-a]^{-\frac{1}{2}}\}$
(7)	$x^{\nu-1}e^{-ax}$ $\qquad \operatorname{Re} a>0, \quad \operatorname{Re}\nu>0$	$\Gamma(\nu)(a^2+y^2)^{-\frac{1}{2}\nu}\cos[\nu\tan^{-1}(y/a)]$
(8)	$x(e^{ax}-1)^{-1}$ $\qquad \operatorname{Re} a>0$	$\frac{1}{2}y^{-2}-\frac{1}{2}\pi^2a^{-2}[\operatorname{csch}(\pi a^{-1}y)]^2$
(9)	$(e^x-1)^{-1}-x^{-1}$	$\log y-\frac{1}{2}[\psi(iy)+\psi(-iy)]$
(10)	$e^{-ax}(1-e^{-\beta x})^{\nu-1}$ $\qquad \operatorname{Re} a>0$, $\operatorname{Re}\beta>0, \quad \operatorname{Re}\nu>0$	$\dfrac{1}{2\beta}\left[\mathrm{B}\left(\nu,\dfrac{a-iy}{\beta}\right)+\mathrm{B}\left(\nu,\dfrac{a+iy}{\beta}\right)\right]$
(11)	e^{-ax^2} $\qquad \operatorname{Re} a>0$	$\frac{1}{2}\pi^{\frac{1}{2}}a^{-\frac{1}{2}}e^{-y^2/(4a)}$
(12)	$x^{-\frac{1}{2}}e^{-ax^2}$ $\qquad \operatorname{Re} a>0$	$\dfrac{\pi y^{\frac{1}{2}}}{(8a)^{\frac{1}{2}}}\exp\left(-\dfrac{y^2}{8a}\right)I_{-\frac{1}{4}}\left(\dfrac{y^2}{8a}\right)$
(13)	$x^{2n}e^{-a^2x^2}$ $\qquad \lvert\arg a\rvert<\pi/4$	$(-1)^n\pi^{\frac{1}{2}}2^{-n-1}a^{-2n-1}e^{-y^2/(8a^2)}$ $\times D_{2n}(2^{-\frac{1}{2}}y/a)$ $=(-1)^n\pi^{\frac{1}{2}}2^{-n-1}a^{-2n-1}e^{-y^2/(4a^2)}$ $\times\mathrm{He}_{2n}(2^{-\frac{1}{2}}y/a)$
(14)	$x^{\nu}e^{-ax^2}$ $\qquad \operatorname{Re} a>0, \quad \operatorname{Re}\nu>-1$	$\frac{1}{2}a^{-\frac{1}{2}(1+\nu)}\Gamma(\frac{1}{2}+\frac{1}{2}\nu)$ $\times{}_1F_1[\frac{1}{2}+\frac{1}{2}\nu;\frac{1}{2};-y^2/(4a)]$
(15)	$(\beta^2+x^2)^{-1}e^{-ax^2}$ $\qquad \operatorname{Re} a>0, \quad \operatorname{Re}\beta>0$	$\frac{1}{4}\pi\beta^{-1}e^{a\beta^2}[e^{-\beta y}\operatorname{Erfc}(a^{\frac{1}{2}}\beta-\frac{1}{2}a^{-\frac{1}{2}}y)$ $+e^{\beta y}\operatorname{Erfc}(a^{\frac{1}{2}}\beta+\frac{1}{2}a^{-\frac{1}{2}}y)]$
(16)	$e^{-ax-\beta x^2}$ $\qquad \operatorname{Re}\beta>0$	$\frac{1}{4}\pi^{\frac{1}{2}}\beta^{-\frac{1}{2}}\{e^{\frac{1}{4}\beta^{-1}(a-iy)^2}\operatorname{Erfc}[\frac{1}{2}\beta^{-\frac{1}{2}}(a-iy)]$ $+e^{\frac{1}{4}\beta^{-1}(a+iy)^2}\operatorname{Erfc}[\frac{1}{2}\beta^{-\frac{1}{2}}(a+iy)]\}$

Exponential functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(17)	$xe^{-\alpha x-\beta x^2}$ Re $\beta>0$	$-2^{-3}\pi^{1/2}\beta^{-3/2}(\alpha-iy)e^{\frac14\beta^{-1}(\alpha-iy)^2}$ $\times \mathrm{Erfc}[\tfrac12\beta^{-1/2}(\alpha-iy)]$ $-2^{-3}\pi^{1/2}\beta^{-3/2}(\alpha+iy)e^{\frac14\beta^{-1}(\alpha+iy)^2}$ $\times \mathrm{Erfc}[\tfrac12\beta^{-1/2}(\alpha+iy)]+\tfrac12\beta^{-1}$
(18)	$x^{\nu-1}e^{-\alpha x-\beta x^2}$ Re $\beta>0$, Re $\nu>0$	$\tfrac12\Gamma(\nu)(2\beta)^{-\frac12\nu}e^{(\alpha^2-y^2)/(8\beta)}$ $\times\{e^{-\alpha yi/(4\beta)}D_{-\nu}[(\alpha-iy)/(2\beta)^{1/2}]$ $+e^{\alpha yi/(4\beta)}D_{-\nu}[(\alpha+iy)/(2\beta)^{1/2}]\}$
(19)	$x^{-1/2}e^{-\alpha/x}$ Re $\alpha>0$	$\pi^{1/2}(2y)^{-1/2}e^{-(2\alpha y)^{1/2}}[\cos(2\alpha y)^{1/2}$ $-\sin(2\alpha y)^{1/2}]$
(20)	$x^{-3/2}e^{-\alpha/x}$ Re $\alpha>0$	$\pi^{1/2}\alpha^{-1/2}e^{-(2\alpha y)^{1/2}}\cos(2\alpha y)^{1/2}$
(21)	$x^{-\nu-1}e^{-\alpha^2/(4x)}$ $\lvert\arg\alpha\rvert<\tfrac14\pi$, Re $\nu>-1$	$2^{\nu}\alpha^{-\nu}y^{\frac12\nu}[e^{i\pi\nu/4}K_{\nu}(\alpha e^{i\pi/4}y^{1/2})$ $+e^{-i\pi\nu/4}K_{\nu}(\alpha e^{-i\pi/4}y^{1/2})$
(22)	$x^{-1/2}e^{-\alpha x-\beta/x}$ Re $\alpha>0$, Re $\beta>0$	$\pi^{1/2}(y^2+\alpha^2)^{-1/2}e^{-(2\beta^{1/2}u)}$ $\times[u\cos(2\beta^{1/2}v)-v\sin(2\beta^{1/2}v)]$ $u=2^{-1/2}[(y^2+\alpha^2)^{1/2}+\alpha]^{1/2}$ $v=2^{-1/2}[(y^2+\alpha^2)^{1/2}-\alpha]^{1/2}$
(23)	$x^{-3/2}e^{-\alpha x-\beta/x}$ Re $\alpha>0$, Re $\beta>0$	$\pi^{1/2}\beta^{-1/2}e^{-2\beta^{1/2}u}\cos(2\beta^{1/2}v)$ $u=2^{-1/2}[(y^2+\alpha^2)^{1/2}+\alpha]^{1/2}$ $v=2^{-1/2}[(y^2+\alpha^2)^{1/2}-\alpha]^{1/2}$
(24)	$x^{-1/4}e^{-\alpha x^{1/2}}$ Re $\alpha>0$	$\dfrac{\pi\alpha^{1/2}}{2y^{1/2}}\left[J_{1/4}\left(\dfrac{\alpha^2}{8y}\right)\sin\left(\dfrac{\alpha^2}{8y}+\dfrac{\pi}{8}\right)\right.$ $\left.-Y_{1/4}\left(\dfrac{\alpha^2}{8y}\right)\cos\left(\dfrac{\alpha^2}{8y}+\dfrac{\pi}{8}\right)\right]$
(25)	$x^{-1/2}e^{-\alpha x^{-1/2}}$ Re $\alpha>0$	$\pi^{1/2}(2y)^{-1/2}[\cos(2\alpha y^{1/2})-\sin(2\alpha y^{1/2})]$
(26)	$\exp[-\beta(x^2+\alpha^2)^{1/2}]$ Re $\alpha>0$, Re $\beta>0$	$\alpha\beta(y^2+\beta^2)^{-1/2}K_1[\alpha(y^2+\beta^2)^{1/2}]$

Exponential functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(27)	$(x^2+a^2)^{-1/2}\exp[-\beta(x^2+a^2)^{1/2}]$ $\operatorname{Re}a>0, \quad \operatorname{Re}\beta>0$	$K_0[a(\beta^2+y^2)^{1/2}]$
(28)	$x^{-1/2}(\beta^2+x^2)^{-1/2}$ $\times\exp[-a(\beta^2+x^2)^{1/2}]$ $\operatorname{Re}a>0, \quad \operatorname{Re}\beta>0$	$(\pi y)^{1/2}\,2^{-1/2}\,I_{-1/4}\{\tfrac{1}{2}\beta[(a^2+y^2)^{1/2}-a]\}$ $\times K_{1/4}\{\tfrac{1}{2}\beta[(a^2+y^2)^{1/2}+a]\}$
(29)	$x[(\beta^2+x^2)^{1/2}-\beta]^{-1/2}(\beta^2+x^2)^{-1/2}$ $\times\exp[-a(\beta^2+x^2)^{1/2}]$ $\operatorname{Re}a>0, \quad \operatorname{Re}\beta>0$	$\pi^{1/2}\,2^{-1/2}[a+(a^2+y^2)^{1/2}]^{1/2}(a^2+y^2)^{-1/2}$ $\times\exp[-\beta(a^2+y^2)^{1/2}]$
(30)	$(e^{2\pi x^{1/2}}-1)^{-1}$	see Ramanujan, Srinivasa, 1915; *Mess. Math.* (44), p. 75-85

1.5. Logarithmic functions

(1)	$\log x \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$-y^{-1}\,\mathrm{Si}(y)$
(2)	$x^{-1/2}\log x$	$-(2y/\pi)^{-1/2}[\log(4y)+C+\pi/2]$
(3)	$(x^2+a^2)^{-1}\log(\beta x)$ $\operatorname{Re}a>0$	$\tfrac{1}{4}\pi a^{-1}\,[2e^{-ay}\log(a\beta)$ $+e^{ay}\,\mathrm{Ei}(-ay)-e^{-ay}\,\mathrm{Ei}(ay)]$
(4)	$(x^2-a^2)^{-1}\log x \qquad a>0$	$\tfrac{1}{2}\pi a^{-1}\{\sin(ay)[\mathrm{ci}(ay)-\log a]$ $-\cos(ay)[\mathrm{si}(ay)-\tfrac{1}{2}\pi]\}$ The integral is a Cauchy Principal Value
(5)	$x^{\nu-1}\log x$ $0<\operatorname{Re}\nu<1$	$\Gamma(\nu)\,y^{-\nu}\cos(\tfrac{1}{2}\nu\pi)[\psi(\nu)$ $-\tfrac{1}{2}\pi\tan(\tfrac{1}{2}\nu\pi)-\log y]$

Logarithmic functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)\,dx \qquad y > 0$
(6)	$e^{-ax} \log x \qquad \mathrm{Re}\, a > 0$	$-(y^2 + a^2)^{-1} [aC + \tfrac{1}{2} a \log(y^2 + a^2) + y \tan^{-1}(y/a)]$
(7)	$x^{\nu-1} e^{-ax} \log x \qquad \mathrm{Re}\, \nu > 0, \quad \mathrm{Re}\, a > 0$	$\{[\psi(\nu) - \log(a^2 + y^2)^{1/2}] \times \cos[\nu \tan^{-1}(y/a)] - \tan^{-1}(y/a) \sin[\nu \tan^{-1}(y/a)]\} \times \Gamma(\nu)(a^2 + y^2)^{-\frac{1}{2}\nu}$
(8)	$x^{-1} \log(1 + x)$	$\tfrac{1}{2}\{[\mathrm{ci}(y)]^2 + [\mathrm{si}(y)]^2\}$
(9)	$\log \left\| \dfrac{a + x}{a - x} \right\| \qquad a > 0$	$2y^{-1}[\mathrm{si}(ay) \cos(ay) + \mathrm{ci}(ay) \sin(ay)]$
(10)	$\log(1 + a^2/x^2) \qquad \mathrm{Re}\, a > 0$	$\pi y^{-1}(1 - e^{-ay})$
(11)	$x^{-1} \log\left(\dfrac{a + x}{a - x}\right)^2 \qquad a > 0$	$-2\pi\, \mathrm{si}(ay)$
(12)	$\log\left(\dfrac{a^2 + x^2}{\beta^2 + x^2}\right) \qquad \mathrm{Re}\, a > 0, \quad \mathrm{Re}\, \beta > 0$	$(e^{-\beta y} - e^{-ay})\, \pi y^{-1}$
(13)	$\log(1 + e^{-ax}) \qquad \mathrm{Re}\, a > 0$	$\tfrac{1}{2} a y^{-2} - \tfrac{1}{2}\pi y^{-1} \operatorname{csch}(\pi a^{-1} y)$
(14)	$\log(1 - e^{-ax}) \qquad \mathrm{Re}\, a > 0$	$\tfrac{1}{2} a y^{-2} - \tfrac{1}{2}\pi y^{-1} \operatorname{ctnh}(\pi a^{-1} y)$

1.6. Trigonometric functions of argument kx

(1)	$x^{-1} \sin(ax) \qquad a > 0$	$\pi/2 \qquad y < a$ $\pi/4 \qquad y = a$ $0 \qquad y > a$

Trigonometric functions of kx (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)\, dx \qquad y > 0$
(2)	$x^{\nu-1} \sin(ax)$ $a > 0, \quad \lvert\mathrm{Re}\, \nu\rvert < 1$	$\pi \dfrac{(y+a)^{-\nu} - \lvert y-a\rvert^{-\nu} \operatorname{sgn}(y-a)}{4\, \Gamma(1-\nu) \cos(\frac{1}{2}\nu\pi)}$
(3)	$x(x^2+\beta^2)^{-1} \sin(ax)$ $a > 0, \quad \mathrm{Re}\, \beta > 0$	$\frac{1}{2}\pi e^{-a\beta} \cosh(\beta y) \qquad y < a$ $-\frac{1}{2}\pi e^{-\beta y} \sinh(a\beta) \qquad y > a$
(4)	$x^{-1}(x^2+\beta^2)^{-1} \sin(ax)$ $a > 0, \quad \mathrm{Re}\, \beta > 0$	$-\frac{1}{2}\pi\, \beta^{-2} e^{-a\beta} \cosh(\beta y) + \frac{1}{2}\pi \beta^{-2}$ $y < a$ $\frac{1}{2}\pi\, \beta^{-2} e^{-\beta y} \sinh(a\beta) \qquad y > a$
(5)	$x^{-1}(1 - 2a\cos x + a^2)^{-1} \sin x$ $0 < a < 1$	$\frac{1}{2}\pi(1-a)^{-1} a^{[y]}$ $y \neq 0, 1, 2, \ldots,$ $\frac{1}{2}\pi(1-a)^{-1} a^{[y]} + \frac{1}{4}\pi a^{[y]-1}$ $y = 0, 1, 2, \ldots,$
(6)	$e^{-\beta x} \sin(ax)$ $a > 0, \quad \mathrm{Re}\, \beta > 0$	$\dfrac{\frac{1}{2}(a+y)}{\beta^2+(a+y)^2} + \dfrac{\frac{1}{2}(a-y)}{\beta^2+(a-y)^2}$
(7)	$x^{-1} e^{-x} \sin x$	$\frac{1}{2}\tan^{-1}(2/y^2)$
(8)	$x^{-2} \sin^2(ax) \qquad a > 0$	$\frac{1}{2}\pi(a - \frac{1}{2}y) \qquad y < 2a$ $0 \qquad 2a < y$
(9)	$x^{-2} \sin^3(ax) \qquad a > 0$	$2^{-3}(y+3a)\log(y+3a)$ $+2^{-3}(y-3a)\log\lvert y-3a\rvert$ $-2^{-3}(y+a)\log(y+a)$ $-2^{-3}(y-a)\log\lvert y-a\rvert$
(10)	$x^{-3} \sin^3(ax) \qquad a > 0$	$(\pi/8)(3a^2-y^2) \qquad 0 < y < a$ $(\pi/4)y^2 \qquad y = a$ $(\pi/16)(3a-y)^2 \qquad a < y < 3a$ $0 \qquad 3a < y < \infty$

Trigonometric functions of kx (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\cos(xy)\,dx \qquad y > 0$
(11)	$\left(\dfrac{\sin x}{x}\right)^n \qquad n = 2, 3, \ldots$	$\dfrac{n\pi}{2^n} \sum_{0<r<(y+n)/2} \dfrac{(-1)^r (y+n-2r)^{n-1}}{r!\,(n-r)!} \qquad 0 < y < n$ $0 \qquad n \le y < \infty$
(12)	$e^{-ax} (\sin x)^{2n} \qquad \operatorname{Re} a > 0$	$\dfrac{(-1)^n i}{(2n+1)\,2^{2n+2}} \left[\left(\dfrac{y/2 + ia/2 + n}{2n+1}\right)^{-1} - \left(\dfrac{y/2 - ia/2 + n}{2n+1}\right)^{-1}\right]$
(13)	$e^{-ax} (\sin x)^{2n-1} \qquad \operatorname{Re} a > 0$	$\dfrac{(-1)^n}{n\,2^{2n+2}} \left[\left(\dfrac{y/2 - ai/2 - \frac{1}{2} + n}{2n}\right)^{-1} + \left(\dfrac{y/2 + ai/2 - \frac{1}{2} + n}{2n}\right)^{-1}\right]$
(14)	$[\sin(\pi x)]^{\nu-1} \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$ $\operatorname{Re} \nu > 0$	$[2^{1-\nu} \cos(\frac{1}{2}y)\,\Gamma(\nu)]$ $\times \{\Gamma[\frac{1}{2}(\nu + 1 + \pi^{-1} y)]$ $\times \Gamma[\frac{1}{2}(\nu + 1 - \pi^{-1} y)]\}^{-1}$
(15)	$(a^2 + x^2)^{-1} \log[4 \sin^2(\frac{1}{2}x)]$ $0 < a < \pi$	$\pi a^{-1} \cosh(ay) \log(1 - e^{-a})$ $0 < y < 1$
(16)	$x^{-2}(1 - \cos ax) \qquad a > 0$	$\frac{1}{2}\pi(a - y) \qquad y < a$ $0 \qquad a < y$
(17)	$x^{\nu-1} \cos(ax) \qquad 0 < \operatorname{Re} \nu < 1$	$\frac{1}{2}\Gamma(\nu) \cos(\frac{1}{2}\nu\pi)$ $\times [\lvert y - a\rvert^{-\nu} + (y + a)^{-\nu}]$

Trigonometric functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \quad y>0$
(18)	$(x^2+\beta^2)^{-1}\cos(ax)$ $a>0, \quad \mathrm{Re}\,\beta>0$	$\tfrac{1}{2}\pi\beta^{-1}e^{-a\beta}\cosh(\beta y) \qquad y<a$ $\tfrac{1}{2}\pi\beta^{-1}e^{-\beta y}\cosh(a\beta) \qquad a<y$
(19)	$e^{-\beta x}\cos(ax)$ $\mathrm{Re}\,\beta>\lvert\mathrm{Im}\,a\rvert$	$\tfrac{1}{2}\beta\{[\beta^2+(a-y)^2]^{-1}$ $+[\beta^2+(a+y)^2]^{-1}\}$
(20)	$e^{-\beta x^2}\cos(ax) \qquad \mathrm{Re}\,\beta>0$	$\tfrac{1}{2}(\pi/\beta)^{1/2}\exp\left(-\dfrac{a^2+y^2}{4\beta}\right)\cosh\left(\dfrac{ay}{2\beta}\right)$
(21)	$(a^2+x^2)^{-1}(1-2\beta\cos x+\beta^2)^{-1}$ $\mathrm{Re}\,a>0, \quad \lvert\beta\rvert<1$	$\tfrac{1}{2}\pi a^{-1}(1-\beta^2)^{-1}(e^a-\beta)^{-1}$ $\times(e^{a-ay}+\beta e^{ay}) \qquad 0\le y<1$
(22)	$(a^2+x^2)^{-1}(1-2\beta\cos x+\beta^2)^{-1}$ $\mathrm{Re}\,a>0, \quad \lvert\beta\rvert<1$	$\tfrac{1}{2}\pi a^{-1}(1-\beta^2)^{-1}\Big(e^{-ay}$ $+\dfrac{\beta e^{-ay}-\beta^{m+1}e^{-\eta a}}{e^{-a}-\beta}$ $+\dfrac{\beta e^{-a(m+\eta)}+\beta^{m+1}e^{\eta a}}{e^{a}-\beta}\Big)$ $y=m+\eta, \quad m$ integer $0\le\eta<1$
(23)	$(a^2+x^2)^{-1}\dfrac{\cos x-\beta}{1-2\beta\cos x+\beta^2}$ $\mathrm{Re}\,a>0, \quad \lvert\beta\rvert<1$	$\tfrac{1}{2}\pi a^{-1}(e^a-\beta)^{-1}\cosh ay$ $0\le y<1$
(24)	$(a^2+x^2)^{-1}\dfrac{\cos x-\beta}{1-2\beta\cos x+\beta^2}$ $\mathrm{Re}\,a>0, \quad \lvert\beta\rvert<1$	$\dfrac{\pi}{4a}\Big(\dfrac{e^{-a(m+\eta)}-\beta^m e^{\eta a}}{e^{-a}-\beta}$ $+\dfrac{e^{-a(m+\eta)}-\beta^m e^{-\eta a}}{e^{-a}-\beta}\Big)$ $y=m+\eta, \quad m$ integer $0\le\eta<1$

Trigonometric functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(25)	$x^{-n}(\sin ax)^m\ (\cos bx)^p$	Integrals of this type may be computed from entry (11) in this section
(26)	$x^{\frac{1}{2}m-1}\prod_{n=1}^{m}\cos(a_n x)$ $a_n>0$	$0 \qquad y>\sum_{n=1}^{m} a_n$
(27)	$[\cos(\frac{1}{2}\pi x)]^{\nu-1} \qquad 0<x<1$ $0 \qquad 1<x<\infty$ $\operatorname{Re}\nu>0$	$2^{1-\nu}\Gamma(\nu)\,[\Gamma(\frac{1}{2}\nu+\frac{1}{2}+y/\pi)$ $\times\Gamma(\frac{1}{2}\nu+\frac{1}{2}-y/\pi)]^{-1}$
(28)	$(\cos x-\cos\theta)^{\nu-\frac{1}{2}} \qquad 0<x<\theta$ $0 \qquad \theta<x<\infty$ $0<\theta<\pi, \quad \operatorname{Re}\nu>-\frac{1}{2}$	$(\pi/2)^{\frac{1}{2}}(\sin\theta)^{\nu}\Gamma(\nu+\frac{1}{2})\,P_{y-\frac{1}{2}}^{-\nu}(\cos\theta)$
(29)	$x^{-2}\log[\cos^2(ax)]$	$\pi y\log 2-a\pi$
(30)	$(x^2+a^2)^{-1}\log[4\cos^2(\frac{1}{2}bx)]$	$\pi a^{-1}\cosh(ay)\log(1+e^{-ab})$ $0<y<b<\pi/a$
(31)	$x^{-2}(1+x^2)^{-1}\log[\cos^2(ax)]$	$-\pi\log(1+e^{-2a})\cosh y$ $+(y+e^{-y})\,\pi\log 2-a\pi$
(32)	$(x^2+a^2)^{-1}\log(2\pm 2\cos x)$ $a>0$	$\pi a^{-1}\cosh(ay)\log(1\pm e^{-a})$
(33)	$\dfrac{\log(1-2r\cos x+r^2)}{x^2+a^2}$ $\lvert r\rvert<1$	$\pi a^{-1}\log(1-re^{-a})\cosh(ay)$ $+\pi a^{-1}\sum_{1\le n\le y} n^{-1}r^n\sinh[a(y-n)]$

Trigonometric functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(34)	$x(x^2+\beta^2)^{-1}\tan(ax)$ $a>0, \quad \operatorname{Re}\beta>0$	$\pi\cosh(\beta y)\,[e^{2a\beta}+1]^{-1}$ The integral is a Cauchy Principal Value
(35)	$x(x^2+\beta^2)^{-1}\operatorname{ctn}(ax)$ $a>0, \quad \operatorname{Re}\beta>0$	$\pi\cosh(\beta y)\,[e^{2a\beta}-1]^{-1}$ The integral is a Cauchy Principal Value
(36)	$(x^2+a^2)^{-1}\sec(bx)$ $\operatorname{Re}a>0, \quad b>0$	$\tfrac{1}{2}\pi a^{-1}\cosh(ay)/[\cosh(ab)]$ $0<y<b$ The integral is a Cauchy Principal Value
(37)	$x(x^2+a^2)^{-1}\csc(bx)$ $\operatorname{Re}a>0, \quad b>0$	$\tfrac{1}{2}\pi\cosh(ay)/[\sinh(ab)] \qquad 0<y<b$

1.7. Trigonometric functions of other arguments

(1)	$\sin(ax^2) \qquad a>0$	$\dfrac{1}{4}\left(\dfrac{2\pi}{a}\right)^{\frac{1}{2}}\left[\cos\left(\dfrac{y^2}{4a}\right)-\sin\left(\dfrac{y^2}{4a}\right)\right]$
(2)	$\sin[a(1-x^2)] \qquad a>0$	$-\tfrac{1}{2}\pi^{\frac{1}{2}}a^{-\frac{1}{2}}\cos[a+\tfrac{1}{4}\pi+\tfrac{1}{4}a^{-1}y^2]$
(3)	$x^{-2}\sin(ax^2) \qquad a>0$	$\tfrac{1}{2}\pi y\{S[(2\pi a)^{-\frac{1}{2}}y]-C[(2\pi a)^{-\frac{1}{2}}y]\}$ $+(\pi a)^{\frac{1}{2}}\sin(\tfrac{1}{4}a^{-1}y^2+\tfrac{1}{4}\pi)$
(4)	$x^{-\frac{1}{2}}\sin(ax^2) \qquad a>0$	$-\dfrac{\pi}{2}\left(\dfrac{y}{2a}\right)^{\frac{1}{2}}\sin\left(\dfrac{y^2}{8a}-\dfrac{\pi}{8}\right)J_{-\frac{1}{4}}\left(\dfrac{y^2}{8a}\right)$
(5)	$e^{-\alpha x^2}\sin(\beta x^2)$ $\operatorname{Re}\alpha>\lvert\operatorname{Im}\beta\rvert$	$\tfrac{1}{2}\pi^{\frac{1}{2}}(\alpha^2+\beta^2)^{-\frac{1}{4}}e^{-\alpha y^2/[4(\alpha^2+\beta^2)]}$ $\times\sin\left[\tfrac{1}{2}\tan^{-1}\left(\dfrac{\beta}{\alpha}\right)-\dfrac{\beta y^2}{4(\alpha^2+\beta^2)}\right]$

Trigonometric functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)\,dx \qquad y > 0$
(6)	$e^{-\alpha x^2} \cos(\beta x^2)$ $\quad \operatorname{Re} \alpha > \lvert \operatorname{Im} \beta \rvert$	$\frac{1}{2}\pi^{1/2}(\alpha^2+\beta^2)^{-1/4} e^{-\alpha y^2/[4(\alpha^2+\beta^2)]}$ $\times \cos\left[\frac{\beta y^2}{4(\alpha^2+\beta^2)} - \frac{1}{2}\tan^{-1}\left(\frac{\beta}{\alpha}\right)\right]$
(7)	$\cos(ax^2) \qquad a > 0$	$\frac{1}{4}(2\pi/a)^{1/2}[\cos(\frac{1}{4}a^{-1}y^2)$ $+ \sin(\frac{1}{4}a^{-1}y^2)]$
(8)	$\cos(\frac{1}{2}x^2 - \pi/8)$	$\pi^{1/2}2^{-1/2}\cos(\frac{1}{2}y^2 - \pi/8)$
(9)	$x^{-1/2}\cos(ax^2) \qquad a > 0$	$\frac{1}{2}\pi y^{1/2}(2a)^{-1/2}\cos(a^{-1}y^2/8 - \pi/8)$ $\times J_{-1/4}(a^{-1}y^2/8)$
(10)	$\cos[a(1-x^2)] \qquad a > 0$	$\frac{1}{2}\pi^{1/2}a^{-1/2}\sin[a + \frac{1}{4}\pi + \frac{1}{4}y^2/a)]$
(11)	$\cos(a^3x^3) \qquad a > 0$	$\frac{1}{2}(3a)^{-3/2}y^{1/2}\{3^{1/2}K_{1/3}[2(3a)^{-3/2}y^{3/2}]$ $+\pi J_{1/3}[2(3a)^{-3/2}y^{3/2}]$ $+\pi J_{-1/3}[2(3a)^{-3/2}y^{3/2}]\}$
(12)	$x^{-1}\cos(1/x)$	$-\frac{1}{2}\pi Y_0(2y^{1/2}) + K_0(2y^{1/2})$
(13)	$x^{-1/2}\sin(a^2/x) \qquad a > 0$	$2^{-3/2}\pi^{1/2}y^{-1/2}[\sin(2ay^{1/2})$ $+\cos(2ay^{1/2}) + e^{-2ay^{1/2}}]$
(14)	$x^{-1/2}\cos(a^2/x) \qquad a > 0$	$2^{-3/2}\pi^{1/2}y^{-1/2}[\cos(2ay^{1/2})$ $-\sin(2ay^{1/2}) + e^{-2ay^{1/2}}]$
(15)	$x^{-3/2}\sin(a^2/x) \qquad a > 0$	$2^{-3/2}\pi^{1/2}a^{-1}[\sin(2ay^{1/2})$ $+\cos(2ay^{1/2}) + e^{-2ay^{1/2}}]$
(16)	$x^{-3/2}\cos(a^2/x) \qquad a > 0$	$2^{-3/2}\pi^{1/2}a^{-1}[e^{-2ay^{1/2}}$ $+\cos(2ay^{1/2}) - \sin(2ay^{1/2})]$

Trigonometric functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)\, dx \qquad y > 0$
(17)	$x^{\nu-1} \sin(a^2/x)$ $a > 0, \quad \lvert\mathrm{Re}\,\nu\rvert < 1$	$\tfrac{1}{4}\pi a^{\nu} \sec(\tfrac{1}{2}\nu\pi) y^{-\frac{1}{2}\nu}[J_{\nu}(2ay^{\frac{1}{2}})$ $+J_{-\nu}(2ay^{\frac{1}{2}}) + I_{\nu}(2ay^{\frac{1}{2}})$ $-I_{-\nu}(2ay^{\frac{1}{2}})]$
(18)	$x^{\nu-1} \cos(a^2/x)$ $a > 0, \quad \lvert\mathrm{Re}\,\nu\rvert < 1$	$\tfrac{1}{4}\pi a^{\nu} \csc(\tfrac{1}{2}\nu\pi) y^{-\frac{1}{2}\nu}[J_{-\nu}(2ay^{\frac{1}{2}})$ $-J_{\nu}(2ay^{\frac{1}{2}}) + I_{-\nu}(2ay^{\frac{1}{2}})$ $-I_{\nu}(2ay^{\frac{1}{2}})]$
(19)	$x^{-\frac{1}{2}} \sin(ax^{\frac{1}{2}}) \sin(bx^{\frac{1}{2}})$	$\left(\dfrac{\pi}{y}\right)^{\frac{1}{2}} \sin\left(\dfrac{ab}{2y}\right) \sin\left(\dfrac{a^2+b^2}{4y} - \dfrac{\pi}{4}\right)$
(20)	$x^{-\frac{1}{2}} \cos(ax^{\frac{1}{2}})$	$\left(\dfrac{\pi}{y}\right)^{\frac{1}{2}} \cos\left(\dfrac{ab}{2y}\right) \sin\left(\dfrac{a^2+b^2}{4y} + \dfrac{\pi}{4}\right)$
(21)	$x^{-\frac{1}{2}} \cos(ax^{\frac{1}{2}} - \pi/4)$	$\tfrac{1}{2}\pi^{\frac{1}{2}} y^{-\frac{1}{2}} \exp(-\tfrac{1}{2}a^2 y^{-\frac{1}{2}})$
(22)	$x^{-\frac{1}{4}} \sin(2ax^{\frac{1}{2}}) \qquad a > 0$	$-\tfrac{1}{2}\pi(a/y)^{3/2}[\sin(\tfrac{1}{2}a^2y^{-1} - \pi/8)$ $\times J_{-\frac{1}{4}}(\tfrac{1}{2}a^2y^{-1}) + \cos(\tfrac{1}{2}a^2y^{-1} - \pi/8)$ $\times J_{\frac{3}{4}}(\tfrac{1}{2}a^2y^{-1})]$
(23)	$x^{-\frac{1}{4}} \cos(2ax^{\frac{1}{2}}) \qquad a > 0$	$-\tfrac{1}{2}\pi(a/y)^{3/2}[\sin(\tfrac{1}{2}a^2y^{-1} + \pi/8)$ $\times J_{-\frac{3}{4}}(\tfrac{1}{2}a^2y^{-1}) + \cos(\tfrac{1}{2}a^2y^{-1} + \pi/8)$ $\times J_{-\frac{1}{4}}(\tfrac{1}{2}a^2y^{-1})]$
(24)	$x^{-\frac{3}{4}} \sin(ax^{\frac{1}{2}}) \qquad a > 0$	$\pi a^{\frac{1}{2}}(2y)^{-\frac{1}{2}} \cos\left(\dfrac{a^2}{8y} - \dfrac{3\pi}{8}\right) J_{\frac{1}{4}}\left(\dfrac{a^2}{8y}\right)$
(25)	$x^{-\frac{3}{4}} \cos(ax^{\frac{1}{2}}) \qquad a > 0$	$\pi a^{\frac{1}{2}}(2y)^{-\frac{1}{2}} \cos\left(\dfrac{a^2}{8y} - \dfrac{\pi}{8}\right) J_{-\frac{1}{4}}\left(\dfrac{a^2}{8y}\right)$

Trigonometric functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)\, dx \quad y > 0$
(26)	$x^{-1/2} e^{-\alpha x} \cos(\beta x^{1/2})$ $\operatorname{Re} \alpha > 0$	$\pi^{1/2} (\alpha^2+y^2)^{-1/4} e^{-\alpha\beta^2/[4(\alpha^2+\beta^2)]}$ $\times \cos\left[\dfrac{\beta^2 y}{4(\alpha^2+y^2)} - \tfrac{1}{2}\tan^{-1}\left(\dfrac{y}{\alpha}\right)\right]$
(27)	$e^{-ax^{1/2}} \cos(a x^{1/2})$ $\operatorname{Re} a > \lvert \operatorname{Im} a \rvert$	$2^{-3/2} \pi^{1/2} a y^{-3/2} e^{-\frac{1}{2}a^2/y}$
(28)	$x^{-1/2} e^{-ax^{1/2}} [\cos(a x^{1/2})$ $- \sin(a x^{1/2})] \quad \operatorname{Re} a > \lvert \operatorname{Im} a \rvert$	$(\pi/2)^{1/2} y^{-1/2} e^{-\frac{1}{2}a^2/y}$
(29)	$(x^2+a^2)^{-2} \sin[b(x^2+a^2)^{1/2}]$ $a > 0$	$\tfrac{1}{2}\pi a^{-1} b\, e^{-ay} \qquad y > a$
(30)	$(a^2+x^2)^{-1/2} \sin[b(a^2+x^2)^{1/2}]$	$\tfrac{1}{2}\pi J_0[a(b^2-y^2)^{1/2}] \qquad 0 < y < b$ $0 \qquad b < y < \infty$
(31)	$\dfrac{\sin[b(x^2+a^2)^{1/2}]}{(x^2+c^2)(x^2+a^2)^{1/2}}$ $c > 0$	$\dfrac{\pi e^{-cy} \sin[b(a^2-c^2)^{1/2}]}{2c(a^2-c^2)^{1/2}} \qquad c \neq a$ $= 2\pi e^{-cy} b/c \qquad c = a$ $y \geq b$
(32)	$x^{-1/2} (x^2+a^2)^{-1/2} \sin[b(x^2+a^2)^{1/2}]$ $a,\ b > 0$	$(\tfrac{1}{2}\pi)^{3/2} y^{1/2} J_{-1/4}\{\tfrac{1}{2}a[b-(b^2-y^2)^{1/2}]\}$ $\times J_{1/4}\{\tfrac{1}{2}a[b+(b^2-y^2)^{1/2}]\}$ $0 < y < b$
(33)	$\cos[b(x^2+a^2)^{1/2}](x^2+c^2)^{-1}$ $c > 0$	$\tfrac{1}{2}\pi c^{-1} e^{-cy} \cos[b(a^2-c^2)^{1/2}]$ $y \geq b$
(34)	$(x^2+a^2)^{-1/2} \cos[b(x^2+a^2)^{1/2}]$ $a > 0$	$-\tfrac{1}{2}\pi Y_0[a(b^2-y^2)^{1/2}] \qquad 0 < y < b$ $K_0[a(y^2-b^2)^{1/2}] \qquad b < y < \infty$

Trigonometric functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\cos(xy)\,dx \quad y > 0$
(35)	$(x^2+a^2)^{-3/2}\cos[b(x^2+a^2)^{1/2}]$ $a > 0$	$\tfrac{1}{2}\pi a^{-1} e^{-ay}$ $\quad y > a$
(36)	$x^{-1/2}(b^2+x^2)^{-1/2}$ $\times\cos[a(b^2+x^2)^{1/2}]$ $b > 0$	$-\tfrac{1}{2}\pi(\tfrac{1}{2}\pi y)^{1/2} J_{-1/4}\{\tfrac{1}{2}b[a-(a^2-y^2)^{1/2}]\}$ $\times Y_{1/4}\{\tfrac{1}{2}b[a+(a^2-y^2)^{1/2}]\}$ $0 < y < a$
(37)	$\sin[b(a^2-x^2)^{1/2}]$ $\quad 0 < x < a$ 0 $\quad a < x < \infty$ $b > 0$	$\tfrac{1}{2}\pi ab(b^2+y^2)^{-1/2} J_1[a(b^2+y^2)^{1/2}]$
(38)	0 $\quad 0 < x < a$ $\dfrac{x\sin[c(x^2-a^2)^{1/2}]}{b^2+x^2-a^2}$ $\quad a < x < \infty$ $b,\ c > 0$	$\tfrac{1}{2}\pi e^{-bc}\cos[y(a^2-b^2)^{1/2}]$ $0 < y < c$
(39)	0 $\quad 0 < x < a$ $\dfrac{x\sin[c(x^2-c^2)^{1/2}]}{x^2+b^2}$ $\quad a < x < \infty$	$\tfrac{1}{2}\pi e^{-c(b^2+a^2)^{1/2}}\cosh(by)$ $\quad 0 < y < c$
(40)	$\dfrac{\sin[b(a^2-x^2)^{1/2}]}{(a^2-x^2)^{3/4}}$ $\quad 0 < x < a$ 0 $\quad a < x < \infty$ $b > 0$	$(\tfrac{1}{2}\pi)^{3/2} b^{1/2} J_{1/4}\{\tfrac{1}{2}a[(b^2+y^2)^{1/2}-y]\}$ $\times J_{1/4}\{\tfrac{1}{2}a[(b^2+y^2)^{1/2}+y]\}$
(41)	0 $\quad 0 < x < a$ $\dfrac{\sin[b(x^2-a^2)^{1/2}]}{(x^2-a^2)^{-3/4}}$ $\quad a < x < \infty$ $b > 0$	$-(\tfrac{1}{2}\pi)^{3/2} b^{1/2} J_{1/4}\{\tfrac{1}{2}a[y-(y^2-b^2)^{1/2}]\}$ $\times Y_{1/4}\{\tfrac{1}{2}a[y+(y^2-b^2)^{1/2}]\}$ $y > b$

Trigonometric functions of other arguments (cont'd)

	$f(x)$		$g(y) = \int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(42)	$\dfrac{\cos[b(a^2-x^2)^{1/2}]}{(a^2-x^2)^{1/2}}$ 0	$0<x<a$ $a<x<\infty$	$\tfrac{1}{2}\pi J_0[a(b^2+y^2)^{1/2}]$
(43)	0 $\dfrac{\cos[b(x^2-a^2)^{1/2}]}{(x^2-a^2)^{1/2}}$	$0<x<a$ $a<x<\infty$	$K_0[a(b^2-y^2)^{1/2}] \qquad y<\lvert b\rvert$ $-\tfrac{1}{2}\pi Y_0[a(y^2-b^2)^{1/2}] \qquad y>\lvert b\rvert$
(44)	$\dfrac{\cos[b(a^2-x^2)^{1/2}]}{(a^2-x^2)^{3/4}}$ 0	$0<x<a$ $a<x<\infty$	$(\tfrac{1}{2}\pi)^{3/2}b^{1/2}J_{-1/4}\{\tfrac{1}{2}a[(b^2+y^2)^{1/2}-y]\}$ $\times J_{-1/4}\{\tfrac{1}{2}a[(b^2+y^2)^{1/2}+y]\}$
(45)	0 $\dfrac{\cos[b(x^2-a^2)^{1/2}]}{(x^2-a^2)^{3/4}}$	$0<x<a$ $a<x<\infty$	$-(\tfrac{1}{2}\pi)^{3/2}J_{-1/4}\{\tfrac{1}{2}a[y-(y^2-b^2)^{1/2}]\}$ $\times Y_{1/4}\{\tfrac{1}{2}a[y+(y^2-b^2)^{1/2}]\}$ $y>b$
(46)	$\dfrac{\cos[b(a^2-x^2)^{1/2}]}{x^{1/2}(a^2-x^2)^{1/2}}$ 0	$0<x<a$ $a<x<\infty$ $b>0$	$(\tfrac{1}{2}\pi)^{3/2}y^{1/2}J_{-1/4}\{\tfrac{1}{2}a[(b^2+y^2)^{1/2}-b]\}$ $\times J_{-1/4}\{\tfrac{1}{2}a[(b^2+y^2)^{1/2}+b]\}$
(47)	0 $\dfrac{\cos[c(x^2-a^2)^{1/2}]}{x(x^2+b^2)(x^2-a^2)^{1/2}}$	$0<x<a$ $a<x<\infty$	$\tfrac{1}{2}\pi(c^2+b^2)^{-1/2}e^{-c(a^2+b^2)^{1/2}}\cosh(yb)$ $0<y<c$
(48)	$\sin(a\sin x)$ 0	$0<x<\pi$ $\pi<x<\infty$	$(1+\cos\pi y)\,s_{0,y}(a)$
(49)	$\sin(a\cos x)$ 0	$0<x<\tfrac{1}{2}\pi$ $\tfrac{1}{2}\pi<x<\infty$	$\cos(\tfrac{1}{2}\pi y)\,s_{0,y}(a)$

Trigonometric functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(50)	$\cos(a\cos x) \qquad 0<x<\tfrac{1}{2}\pi$ $0 \qquad \tfrac{1}{2}\pi<x<\infty$	$-y\sin(\tfrac{1}{2}\pi y)\, s_{-1,y}(a)$
(51)	$\cos(a\sin x) \qquad 0<x<\pi$ $0 \qquad \pi<x<\infty$	$-y\sin(y\pi)\, s_{-1,y}(a)$

1.8. Inverse trigonometric functions

(1)	$0 \qquad 0<x<a$ $\dfrac{\cos[n\cos^{-1}(\frac{2x-a-b}{b-a})]}{(x-a)^{1/2}(b-x)^{1/2}} \qquad a<x<b$ $0 \qquad b<x<\infty$	$\pi\cos\left[\dfrac{n\pi}{2}-\dfrac{(a+b)y}{2}\right] J_n\left[\dfrac{(b-a)y}{2}\right]$
(2)	$x^{-1/2}(a^2-x^2)^{-1/2}$ $\times\cos[\nu\cos^{-1}(x/a)]$ $0<x<a$ $0 \qquad a<x<\infty$	$(\tfrac{1}{2}\pi)^{3/2}y^{1/2}J_{\nu/2-1/4}(\tfrac{1}{2}ay)$ $\times J_{-\nu/2-1/4}(\tfrac{1}{2}ay)$
(3)	$x^{-1}\tan^{-1}(x/a)$	$-\tfrac{1}{2}\pi\,\mathrm{Ei}(-ay)$
(4)	$(x^2+a^2)^{-\frac{1}{2}\nu}\cos[\nu\tan^{-1}(x/a)]$	$\tfrac{1}{2}\pi y^{\nu-1}e^{-ay}/\Gamma(\nu)$
(5)	$x^\nu(1+x^2)^{\frac{1}{2}\nu}\sin(\nu\,\mathrm{ctn}^{-1}x)$ $-1<\mathrm{Re}\,\nu<0$	$\tfrac{1}{2}\pi^{1/2}\Gamma(\nu+1)y^{-\nu-1/2}[I_{-\nu-1/2}(\tfrac{1}{2}y)$ $\times\sinh(\tfrac{1}{2}y)-I_{\nu+1/2}(\tfrac{1}{2}y)\cosh(\tfrac{1}{2}y)]$
(6)	$x^\nu(1+x^2)^{\frac{1}{2}\nu}\cos(\nu\,\mathrm{ctn}^{-1}x)$ $-1<\mathrm{Re}\,\nu<0$	$-\pi^{-1/2}\Gamma(\nu+1)y^{-\nu-1/2}\sin(\nu\pi)$ $\times\cosh(\tfrac{1}{2}y)\,K_{\nu+1/2}(\tfrac{1}{2}y)$
(7)	$\tan^{-1}(a/x) \qquad a>0$	$\tfrac{1}{2}y^{-1}[e^{-ay}\,\mathrm{Ei}(ay)-e^{ay}\,\mathrm{Ei}(-ay)]$
(8)	$\tan^{-1}(2/x^2)$	$\pi y^{-1}e^{-y}\sin y$

1.9. Hyperbolic functions

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy) dx \qquad y > 0$
(1)	$\operatorname{sech}(ax) \qquad \operatorname{Re} a > 0$	$\frac{1}{2}\pi a^{-1} \operatorname{sech}(\frac{1}{2}\pi a^{-1} y)$
(2)	$[\operatorname{sech}(ax)]^2 \qquad \operatorname{Re} a > 0$	$\frac{1}{2}\pi a^{-2} y \operatorname{csch}(\frac{1}{2}\pi a^{-1} y)$
(3)	$[\operatorname{sech}(ax)]^{2n} \qquad \operatorname{Re} a > 0$	$\frac{4^{n-1}\pi y}{2(2n-1)!\,a^2} \operatorname{csch}\left(\frac{\pi y}{2a}\right) \times \prod_{r=1}^{n-1} \left(\frac{y^2}{4a^2} + r^2\right) \qquad n \geq 2$
(4)	$[\operatorname{sech}(ax)]^{2n+1} \qquad \operatorname{Re} a > 0$	$\frac{\pi 2^{2n-1}}{a(2n)!} \operatorname{sech}\left(\frac{\pi y}{2a}\right) \times \prod_{r=1}^{n} \left[\frac{y^2}{4a^2} + \left(\frac{2r-1}{2}\right)^2\right]$
(5)	$[\operatorname{sech}(ax)]^{\nu} \qquad \operatorname{Re} \nu > 0, \quad \operatorname{Re} a > 0$	$\frac{2^{\nu-2}}{a\Gamma(\nu)} \Gamma\left(\frac{\nu}{2} + \frac{iy}{2a}\right) \Gamma\left(\frac{\nu}{2} - \frac{iy}{2a}\right)$
(6)	$[\cosh(ax) + \cos\beta]^{-1} \qquad \pi \operatorname{Re} a > \lvert \operatorname{Im}(\bar{a}\beta) \rvert$	$\pi a^{-1} \csc\beta \sinh(a^{-1}\beta y) \operatorname{csch}(a^{-1}\pi y)$
(7)	$(\cosh x + \cos a)^{-\frac{1}{2}} \qquad -\pi < a < \pi$	$2^{-\frac{1}{2}} \pi \operatorname{sech}(\pi y) P_{-\frac{1}{2}+iy}(\cos a)$
(8)	$(\cosh x - \cosh a)^{-1} \qquad a > 0$	$-\pi \cosh(\pi y) \sin(ay) \operatorname{csch} a$ The integral is a Cauchy Principal Value
(9)	$\{1 + 2\cosh[(2\pi/3)^{\frac{1}{2}} x]\}^{-1}$	$(\frac{1}{2}\pi)^{\frac{1}{2}} \{1 + 2\cosh[(2\pi/3)^{\frac{1}{2}} y]\}^{-1}$
(10)	$[a + (a^2-1)^{\frac{1}{2}} \cosh x]^{-\nu-1} \qquad \operatorname{Re} \nu > -1, \quad \lvert \arg(a \pm 1) \rvert < \pi$	$\Gamma(\nu - iy + 1) e^{\pi y} Q_{\nu}^{iy}(a)/\Gamma(\nu+1)$
(11)	$\cosh(\frac{1}{2}\pi^{\frac{1}{2}} x) \operatorname{sech}(\pi^{\frac{1}{2}} x)$	$\pi^{\frac{1}{2}} 2^{-\frac{1}{2}} \cosh(\frac{1}{2}\pi^{\frac{1}{2}} y) \operatorname{sech}(\pi^{\frac{1}{2}} y)$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(12)	$\cosh(ax)\operatorname{sech}(\beta x)$ $\lvert\operatorname{Re}a\rvert<\operatorname{Re}\beta$	$\pi\beta^{-1}\left[\dfrac{\cos(\frac12\pi\beta^{-1}a)\cosh(\frac12\pi\beta^{-1}y)}{\cos(\pi\beta^{-1}a)+\cosh(\pi\beta^{-1}y)}\right]$
(13)	$\sinh(ax)\operatorname{sech}(\beta x)$ $\lvert\operatorname{Re}a\rvert<\operatorname{Re}\beta$	$\frac14\beta^{-1}\left\{\psi\left(\dfrac{3\beta-a+iy}{4\beta}\right)+\psi\left(\dfrac{3\beta-a-iy}{4\beta}\right)\right.$ $-\psi\left(\dfrac{3\beta+a-iy}{4\beta}\right)-\psi\left(\dfrac{3\beta+a+iy}{4\beta}\right)$ $+2\pi\sin(\pi a\beta^{-1})[\cos(\pi a\beta^{-1})$ $\left.+\cosh(\pi\beta^{-1}y)]^{-1}\right\}$
(14)	$\sinh(ax)\operatorname{csch}(\beta x)$ $\lvert\operatorname{Re}a\rvert<\operatorname{Re}\beta$	$\frac12\pi\beta^{-1}\sin(\pi\beta^{-1}a)[\cosh(\pi\beta^{-1}y)$ $+\cos(\pi\beta^{-1}a)]^{-1}$
(15)	$\dfrac{\sinh^2(ax)}{\sinh(\beta x)}$ $\quad 2\lvert\operatorname{Re}a\rvert<\operatorname{Re}\beta$	$\frac12\pi\beta^{-1}\sin a\,\sin(\pi a/\beta)\sin(\pi y/\beta)$ $\times[\sin^2 a\sinh^2(\pi y/\beta)$ $+\cosh^2(\pi y/\beta)\cos^2 a+\cos^2(a\pi/\beta)$ $+2\cos a\cos(\pi a/\beta)\cosh(\pi y/\beta)]^{-1}$
(16)	$[\cosh(ax)+\cosh\beta]^{-1}$ $\times\cosh(\frac12 ax)$ $\pi\operatorname{Re}a>\lvert\operatorname{Im}(\bar{a}\beta)\rvert$	$\frac12 a^{-1}\pi\operatorname{sech}(\frac12\beta)\cos(\beta y/a)$ $\times\operatorname{sech}(\pi y/a)$
(17)	$[\cosh(\beta x)+\cos c]^{-1}\cosh(ax)$ $\lvert\operatorname{Re}a\rvert<\operatorname{Re}\beta,\quad 0<c<\pi$	$\pi\beta^{-1}\csc c[\cos(\pi/\beta-ac/\beta)$ $\times\cosh(\pi/\beta+cy/\beta)$ $-\cos(\pi/\beta+ac/\beta)\cosh(\pi/\beta-cy/\beta)]$ $\times[\cosh(2\pi y/\beta)-\cos(2\pi a/\beta)]^{-1}$
(18)	$x\operatorname{csch}(ax)$ $\operatorname{Re}a>0$	$\frac14\pi^2a^{-2}\operatorname{sech}^2(\frac12\pi a^{-1}y)$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(19)	$(x^2+a^2)\,\operatorname{sech}(\tfrac{1}{2}\pi a^{-1}x)$ $\quad \operatorname{Re} a>0$	$2a^3[\operatorname{sech}(ay)]^3$
(20)	$x(x^2+4a^2)\,\operatorname{csch}(\tfrac{1}{2}\pi a^{-1}x)$ $\quad \operatorname{Re} a>0$	$6a^4[\operatorname{sech}(ay)]^4$
(21)	$(1+x^2)^{-1}\operatorname{sech}(\pi x)$	$2\cosh(\tfrac{1}{2}y)-[e^{y}\tan^{-1}(e^{-\frac{1}{2}y}) + e^{-y}\tan^{-1}(e^{\frac{1}{2}y})]$
(22)	$(x^2+1)^{-1}\operatorname{sech}(\tfrac{1}{2}\pi x)$	$(\cosh y)\log(1+e^{-2y})+ye^{-y}$
(23)	$(1+x^2)^{-1}\operatorname{sech}(\tfrac{1}{4}\pi x)$	$2^{-\frac{1}{2}}\pi e^{-y}+2^{\frac{1}{2}}\sinh y\tan^{-1}(2^{-\frac{1}{2}}\operatorname{csch} y) - 2^{-\frac{1}{2}}\cosh y\log[(\cosh y+2^{-\frac{1}{2}}) \times(\cosh y-2^{-\frac{1}{2}})^{-1}]$
(24)	$x(x^2+1)^{-1}\operatorname{csch}(\pi x)$	$-\tfrac{1}{2}+\tfrac{1}{2}ye^{-y}+\cosh y\log(1+e^{-y})$
(25)	$[(m+\tfrac{1}{2})^2+x^2]^{-1}\operatorname{sech}(\pi x)$	$\dfrac{(-1)^m e^{-(m+\frac{1}{2})y}}{2m+1}[y+\log(1+e^{-y})] - \dfrac{e^{-\frac{1}{2}y}}{2m+1}\sum_{n=0}^{m-1}\dfrac{(-1)^n e^{-ny}}{n-m} + \dfrac{e^{-\frac{1}{2}y}}{(2m+1)(m+1)} \times {}_2F_1(1,m+1;m+2;-e^{-y})$
(26)	$(x^2+a^2)^{-1}\operatorname{sech}(\pi x)$ $\quad \operatorname{Re} a>0$	$\sum_{n=0}^{\infty}(-1)^n\dfrac{(n+\frac{1}{2})a^{-1}e^{-ay}-e^{-(n+\frac{1}{2})y}}{(n+\frac{1}{2})^2-a^2}$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)dx \qquad y>0$
(27)	$(x^2+a^2)^{-1}[\cosh(\pi x)+\cos(\pi\beta)]^{-1}$ $\quad \operatorname{Re} a>0, \quad 0<\operatorname{Re}\beta<1$	$\tfrac{1}{2}\pi a^{-1}e^{-ay}[\cos(\pi a)+\cos(\pi\beta)]^{-1}+\csc(\pi\beta)\sum_{n=0}^{\infty}\left[\frac{e^{-(2n+1-\beta)y}}{a^2-(2n+1-\beta)^2}-\frac{e^{-(2n+1+\beta)y}}{a^2-(2n+1+\beta)^2}\right]$
(28)	$x^{-1}\tanh(ax) \qquad \operatorname{Re} a>0$	$\log[\operatorname{ctnh}(\tfrac{1}{4}\pi a^{-1}y)]$
(29)	$x(x^2+1)^{-1}\tanh(\tfrac{1}{2}\pi x)$	$-ye^{-y}-\cosh(y)\log(1-e^{-2y})$
(30)	$x(x^2+1)^{-1}\tanh(\tfrac{1}{4}\pi x)$	$-\tfrac{1}{2}\pi e^{-y}+\cosh y\log(\operatorname{ctnh}\tfrac{1}{2}y)-2\sinh y\tan^{-1}(e^{-y})$
	For similar integrals see Bierens de Haan, D., 1867: *Nouvelles tables d'intégrales définies*, p. 408.	
(31)	$x(x^2+1)^{-1}\operatorname{ctnh}(\pi x)$	$\tfrac{1}{2}ye^{y}+\tfrac{1}{2}\pi e^{-y}-\cosh y\log\lvert 2\sinh(\tfrac{1}{2}y)\rvert-\sinh y(e^{y}-1)^{-1}$
	For similar integrals see Bierens de Haan, D., 1867: *Nouvelles tables d'intégrales définies*, p. 389.	
(32)	$x(x^2+1)^{-1}\operatorname{ctnh}(\tfrac{1}{4}\pi x)$	$-2+\tfrac{1}{2}\pi e^{-y}+\cosh y\log(\operatorname{ctnh}\tfrac{1}{2}y)+2\sinh y\tan^{-1}(e^{-y})$
	For similar integrals see Bierens de Haan, D., 1867: *Nouvelles tables d'intégrales définies*, p. 408.	
(33)	$x^{-1}(x^2+1)^{-1}\operatorname{ctnh}(\pi x)$	$\tfrac{1}{2}(1-y+ye^{-y})+2\sinh^2(\tfrac{1}{2}y)\log(1-e^{-y})$
(34)	$x^{-1}\sinh(ax)\operatorname{sech}(\beta x) \qquad \lvert\operatorname{Re} a\rvert<\operatorname{Re}\beta$	$\tfrac{1}{2}\log\left[\frac{\cosh(\tfrac{1}{2}\pi\beta^{-1}y)+\sin(\tfrac{1}{2}\pi\beta^{-1}a)}{\cosh(\tfrac{1}{2}\pi\beta^{-1}y)-\sin(\tfrac{1}{2}\pi\beta^{-1}a)}\right]$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(35)	$\dfrac{\sinh(ax)}{(x^2+1)\sinh(\pi x)} \qquad \lvert\mathrm{Re}\,a\rvert\le\pi$	$\tfrac{1}{2}e^{-y}(y\sin a-a\cos a)$ $+\tfrac{1}{2}\sin a\cosh y\log(1+2e^{-y}\cos a$ $+e^{-2y})-\cos a\sinh y\tan^{-1}[\sin a(e^{y}$ $+\cos a)^{-1}]$
	For similar integrals see Bierens de Haan, D., 1867: *Nouvelles tables d'intégrales définies*, p. 389.	
(36)	$\dfrac{\sinh(ax)}{(x^2+1)\sinh(\tfrac{1}{2}\pi x)} \qquad \lvert\mathrm{Re}\,a\rvert\le\pi/2$	$\tfrac{1}{2}\pi e^{-y}\sin a$ $-\tfrac{1}{2}\cos a\cosh y\log\left\lvert\dfrac{\cosh y+\sin a}{\cosh y-\sin a}\right\rvert$ $+\sin a\sinh y\tan^{-1}(\cos a/\sinh y)$
(37)	$\dfrac{\cosh(ax)}{(1+x^2)\cosh(\tfrac{1}{2}\pi x)} \qquad \lvert\mathrm{Re}\,a\rvert\le\pi/2$	$ye^{-y}\cos a+ae^{-y}\sin a$ $+\sin a\sinh y\tan^{-1}\left[\dfrac{e^{-2y}\sin(2a)}{1+e^{-2y}\cos(2a)}\right]$ $+\tfrac{1}{2}\cos a\cosh y\log[1+2e^{-2y}$ $\times\cos 2a+e^{-4y}]$
(38)	$(a^2-x^2)^{-1/2}\cosh[\beta(a^2-x^2)^{1/2}] \qquad 0<x<a$ $0 \qquad a<x<\infty$	$\tfrac{1}{2}\pi J_0[a(y^2-\beta^2)^{1/2}]$
(39)	$\dfrac{\sinh(ax)}{e^{\beta x}+1} \qquad \lvert\mathrm{Re}\,a\rvert<\mathrm{Re}\,\beta$	$-\tfrac{1}{2}a(y^2+a^2)^{-1}+\pi\beta^{-1}\sin(\pi a\beta^{-1})$ $\times\cosh(\pi\beta^{-1}y)[\cosh(2\pi\beta^{-1}y)$ $-\cos(2\pi a\beta^{-1})]^{-1}$
(40)	$e^{-ax}[\sinh(\beta x)]^{\nu}$ $\mathrm{Re}\,\nu>-1, \quad \mathrm{Re}\,\beta>0$ $\mathrm{Re}\,\beta\nu<\mathrm{Re}\,a$	$2^{-\nu-2}\beta^{-1}\Gamma(\nu+1)\{\Gamma[\tfrac{1}{2}\beta^{-1}(a-\nu\beta-iy)]$ $\times\Gamma[\tfrac{1}{2}\beta^{-1}(a+\nu\beta-iy)+1]^{-1}$ $+\Gamma[\tfrac{1}{2}\beta^{-1}(a-\nu\beta+iy)]$ $\times\Gamma[\tfrac{1}{2}\beta^{-1}(a+\nu\beta+iy)+1]^{-1}\}$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy) dx \qquad y > 0$
(41)	$e^{-\frac{1}{4}x^2/\beta} \cosh(ax) \qquad \text{Re}\, \beta > 0$	$\pi^{\frac{1}{2}} \beta^{\frac{1}{2}} e^{a^2 \beta} e^{-\beta y^2} \cos(2a\beta y)$
(42)	$\dfrac{\sinh(ax)}{e^{\beta x} - 1} \qquad \lvert\text{Re}\, a\rvert < \text{Re}\, \beta$	$\frac{1}{2}a(y^2 + a^2)^{-1} + \frac{1}{2}\pi\beta^{-1} \sin(2\pi a\beta^{-1})$ $\times [\cosh(2\pi\beta^{-1}y) - \cos(2\pi a\beta^{-1})]^{-1}$
(43)	$\dfrac{e^{-i\pi\beta x^2}}{\cosh(\pi x) + \cos(\pi a)}$ $0 < a < 1, \qquad \text{Im}\, \beta \leq 0$	$\csc(\pi a) \sum_{n=0}^{\infty} [e^{-(2n+1-a)y + (2n+1-a)^2 i\pi\beta}$ $-e^{-(2n+1+a)y + (2n+1+a)^2 i\pi\beta}]$ $+\beta^{-\frac{1}{2}} \csc(\pi a) e^{-\frac{1}{4}i(\pi - y^2\pi^{-1}\beta^{-1})}$ $\times \sum_{n=1}^{\infty} (-1)^{n-1} \sin(n\pi a)$ $\times e^{-(2ny + n^2 i\pi)/(4\beta)}$
(44)	$x^{-2} e^{-x^2} \sinh(x^2)$	$2^{-1/2} \pi^{1/2} e^{-y^2/8}$ $-\frac{1}{4}\pi y \,\text{Erfc}(2^{-3/2} y)$
(45)	$e^{-ax} \,\text{ctnh}(\beta x^{\frac{1}{2}})$	see Mordell, L. J., 1920; *Mess. Math.* 49, 65-72
(46)	$e^{-a\cosh x} \qquad \text{Re}\, a > 0$	$K_{iy}(a)$
(47)	$e^{-ax} \tanh(\beta x^{\frac{1}{2}})$	see Mordell, L. J., 1920; *Mess. Math.* 49, 65-72
(48)	$\log(1 - e^{-2x}) \cosh x$	$(y^2 - 1)(y^2 + 1)^{-2}$ $-\frac{1}{2}\pi y (y^2 + 1)^{-1} \tanh(\frac{1}{2}\pi y)$
(49)	$\log(1 + \cos a \,\text{sech}\, x) \qquad \lvert\text{Re}\, a\rvert < \pi$	$\pi y^{-1} \,\text{csch}(\pi y) [\cosh(\frac{1}{2}\pi y)$ $- \cosh(ay)]$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(50)	$\log\left(\dfrac{\cosh x+\sin a}{\cosh x-\sin a}\right)$ $\quad \lvert\operatorname{Re} a\rvert<\tfrac{1}{2}\pi$	$\pi y^{-1}\sinh(ay)\operatorname{sech}(\tfrac{1}{2}\pi y)$
(51)	$\log\dfrac{\cosh x+\cos a}{\cosh x+\cos\beta}$ $\quad \lvert\operatorname{Re} a\rvert<\pi,\quad \lvert\operatorname{Re}\beta\rvert<\pi$	$\pi y^{-1}\operatorname{csch}(\pi y)[\cosh(\beta y)-\cosh(ay)]$
(52)	$\cos(ax)\operatorname{sech}(\beta x)$ $\quad \lvert\operatorname{Im} a\rvert<\operatorname{Re}\beta$	$\dfrac{\pi\cosh(\tfrac{1}{2}\pi a\beta^{-1})\cosh(\tfrac{1}{2}\pi\beta^{-1}y)}{\beta[\cosh(\pi a\beta^{-1})+\cosh(\pi\beta^{-1}y)]}$
(53)	$\sin(ax)\operatorname{csch}(\beta x)$ $\quad \lvert\operatorname{Im} a\rvert<\operatorname{Re}\beta$	$\dfrac{\pi\sinh(\pi a\beta^{-1})}{2\beta[\cosh(\pi a\beta^{-1})+\cosh(\pi\beta^{-1}y)]}$
(54)	$\sin(\pi^{-1}x^2)\operatorname{sech}x$	$\tfrac{1}{2}\pi[\cos(\tfrac{1}{4}\pi y^2)-2^{-\frac{1}{2}}]\operatorname{sech}(\tfrac{1}{2}\pi y)$
(55)	$\cos(\pi^{-1}x^2)\operatorname{sech}(x)$	$\tfrac{1}{2}\pi[\sin(\tfrac{1}{4}\pi y^2)+2^{-\frac{1}{2}}]\operatorname{sech}(\tfrac{1}{2}\pi y)$
(56)	$\sin(\pi ax^2)\operatorname{sech}(\pi x)$ $\quad a>0$	$-\sum_{k=0}^{\infty}e^{-(k+\frac{1}{2})y}\sin[(k+\tfrac{1}{2})^2\pi a]$ $+a^{-\frac{1}{2}}\sum_{k=0}^{\infty}e^{-a^{-1}(k+\frac{1}{2})y}$ $\times\sin[\tfrac{1}{4}\pi-\tfrac{1}{4}\pi^{-1}a^{-1}y^2+(k+\tfrac{1}{2})^2\pi a^{-1}]$
(57)	$\cos(\pi ax^2)\operatorname{sech}(\pi x)$ $\quad a>0$	$\sum_{k=0}^{\infty}(-1)^k e^{-(k+\frac{1}{2})y}\cos[(k+\tfrac{1}{2})^2\pi a]$ $+a^{-\frac{1}{2}}\sum_{k=0}^{\infty}e^{-a^{-1}(k+\frac{1}{2})y}$ $\times\cos[\tfrac{1}{4}\pi-\tfrac{1}{4}\pi^{-1}a^{-1}y^2+(k+\tfrac{1}{2})^2\pi a^{-1}]$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(58)	$[\cos(\frac{1}{2}x^2)+\sin(\frac{1}{2}x^2)]$ $\times\operatorname{sech}(2^{-\frac{1}{2}}\pi^{\frac{1}{2}}x)$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}[\cos(\frac{1}{2}y^2)+\sin(\frac{1}{2}y^2)]$ $\times\operatorname{sech}(2^{-\frac{1}{2}}\pi^{\frac{1}{2}}y)$
(59)	$\dfrac{\cosh(\frac{1}{2}x^{\frac{1}{2}})\cos(\frac{1}{2}x^{\frac{1}{2}})}{\cosh(x^{\frac{1}{2}})+\cos(x^{\frac{1}{2}})}$	For this and similar integrals see Glaisher, J. W. L., 1871; *Quart. J. Math.. Oxford Series* 11, 328-343
(60)	$\sin(\pi x^2)[1+2\cosh(2\pi 3^{-\frac{1}{2}}x)]^{-1}$	$-3^{\frac{1}{2}}+2\cos(\pi/12-\frac{1}{4}y^2/\pi)$ $\times[8\cosh(3^{-\frac{1}{2}}y)-4]^{-1}$
(61)	$\cos(\pi x^2)[1+2\cosh(2\pi 3^{-\frac{1}{2}}x)]^{-1}$	$1-2\sin(\pi/12-\frac{1}{4}y^2/\pi)$ $\times[8\cosh(3^{-\frac{1}{2}}y)-4]^{-1}$
(62)	$[\operatorname{sech}(x/2)]^{\frac{1}{2}}\sin[2a\cosh(x/2))$ $a>0$	$-\frac{1}{2}\pi^{3/2}a^{1/2}[J_{\frac{1}{4}+iy}(a)\,Y_{\frac{1}{4}-iy}(a)$ $+J_{\frac{1}{4}-iy}(a)\,Y_{\frac{1}{4}+iy}(a)]$
(63)	$[\operatorname{sech}(x/2)]^{\frac{1}{2}}\cos[2a\cosh(x/2)]$ $a>0$	$-\frac{1}{2}\pi^{3/2}a^{1/2}[J_{-\frac{1}{4}+iy}(a)\,Y_{-\frac{1}{4}-iy}(a)$ $+J_{-\frac{1}{4}-iy}(a)\,Y_{-\frac{1}{4}+iy}(a)]$
(64)	$[\operatorname{csch}(x/2)]^{\frac{1}{2}}\sin[2a\sinh(x/2)]$ $a>0$	$(\pi a)^{\frac{1}{2}}[I_{\frac{1}{4}-iy}(a)\,K_{\frac{1}{4}+iy}(a)$ $+I_{\frac{1}{4}+iy}(a)\,K_{\frac{1}{4}-iy}(a)]$
(65)	$[\operatorname{csch}(x/2)]^{\frac{1}{2}}\cos[2a\sinh(x/2)]$ $a>0$	$(\pi a)^{\frac{1}{2}}[I_{-\frac{1}{4}-iy}(a)\,K_{-\frac{1}{4}+iy}(a)$ $+I_{-\frac{1}{4}+iy}(a)\,K_{-\frac{1}{4}-iy}(a)]$
(66)	$(\sec x)^{\frac{1}{2}}\sinh(2a\cos x)$ $0<x<\pi/2$ 0 $\pi/2<x<\infty$	$(\pi/2)^{3/2}(2a)^{1/2}I_{y/2+1/4}(a)$ $\times I_{-y/2+1/4}(a)$
(67)	$(\sec x)^{\frac{1}{2}}\cosh(2a\cos x)$ $0<x<\pi/2$ 0 $\pi/2<x<\infty$	$(\pi/2)^{3/2}(2a)^{1/2}I_{y/2-1/4}(a)$ $\times I_{-y/2-1/4}(a)$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy) dx \qquad y > 0$
(68)	$\operatorname{ctn}^{-1}(e^{2x}) \sinh x$	$\dfrac{\pi \cosh(\frac{1}{4}\pi y) + \pi y \sinh(\frac{1}{4}\pi y)}{2^{3/2}(1+y^2)\cosh(\frac{1}{2}\pi y)} - \dfrac{\frac{1}{4}\pi}{1+y^2}$

1.10. Orthogonal polynomials

(1)	$P_n(1-2x^2) \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$\frac{1}{2}\pi(-1)^n J_{n+\frac{1}{2}}(\frac{1}{2}y) J_{-n-\frac{1}{2}}(\frac{1}{2}y)$
(2)	$(a^2-x^2)^{-\frac{1}{2}} T_{2n}(a^{-1}x)$ $0<x<a$ $0 \qquad a<x<\infty$	$\frac{1}{2}(-1)^n \pi J_{2n}(ay)$
(3)	$(a^2-x^2)^{\nu-\frac{1}{2}} C_{2n}^{\nu}(a^{-1}x)$ $0<x<a$ $0 \qquad a<x<\infty$ $\operatorname{Re}\nu > -\frac{1}{2}$	$\dfrac{(-1)^n \pi a^\nu \Gamma(2n+2\nu) J_{\nu+2n}(ay)}{(2n)!\,\Gamma(\nu)\,(2y)^\nu}$
(4)	$(1-x^2)^\nu P_{2n}^{(\nu,\nu)}(x) \qquad 0<x<1$ $0 \qquad 1<x<\infty$ $\operatorname{Re}\nu > -1$	$\dfrac{(-1)^n 2^{\nu-\frac{1}{2}} \pi^{\frac{1}{2}} \Gamma(2n+\nu-1) J_{2n+\nu+\frac{1}{2}}(y)}{(2n)!\,y^{\nu+\frac{1}{2}}}$
(5)	$[(1-x)^\nu(1+x)^\mu$ $+(1+x)^\nu(1-x)^\mu] P_{2n}^{(\nu,\mu)}(x)$ $0<x<1$ $0 \qquad 1<x<\infty$ $\operatorname{Re}\nu>-1, \quad \operatorname{Re}\mu>-1$	$(-1)^n 2^{2n+\nu+\mu}[(2n)!]^{-1}$ $\times B(2n+\nu+1, 2n+\mu+1) e^{iy} y^{2n}$ $\times[{}_1F_1(2n+\nu+1; 4n+\nu+\mu+2; -2iy)$ $+ {}_1F_1(2n+\mu+1; 4n+\nu+\mu+2; -2iy)]$
(6)	$[(1-x)^\nu(1+x)^\mu$ $-(1+x)^\nu(1-x)^\mu] P_{2n+1}^{(\nu,\nu)}(x)$ $0<x<1$ $0 \qquad 1<x<\infty$ $\operatorname{Re}\nu>-1, \quad \operatorname{Re}\mu>-1$	$(-1)^{n+1} 2^{2n+\nu+\mu+1}[(2n+1)!]^{-1}$ $\times B(2n+\nu+2, 2n+\mu+2) y^{2n+1} ie^{iy}$ $\times[{}_1F_1(2n+\nu+2; 4n+\nu+\mu+4; -2iy)$ $- {}_1F_1(2n+\mu+2; 4n+\nu+\mu+4; -2iy)]$

Orthogonal polynomials (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \quad y>0$
(7)	$e^{-x^2}\,\mathrm{He}_{2n}(2x)$	$\tfrac{1}{2}\pi^{\frac{1}{2}}(-1)^n e^{-\frac{1}{4}y^2}\,\mathrm{He}_{2n}(y)$
(8)	$e^{-\frac{1}{2}x^2/a}\,\mathrm{He}_{2n}[xa^{-\frac{1}{2}}(1-a)^{-\frac{1}{2}}]$ $\mathrm{Re}\,a>0, \quad a\neq 1$	$(-1)^n 2^{-\frac{1}{2}}\pi^{\frac{1}{2}}a^{n+\frac{1}{2}}(1-a)^{-n}$ $\times e^{-\frac{1}{2}ay^2}\,\mathrm{He}_{2n}(y)$
(9)	$e^{-\frac{1}{2}x^2}\,\mathrm{He}_{2n}(x)$	$(-1)^n(\tfrac{1}{2}\pi)^{\frac{1}{2}}y^{2n}e^{-\frac{1}{2}y^2}$
(10)	$e^{-\frac{1}{2}x^2}[\mathrm{He}_n(x)]^2$	$(\tfrac{1}{2}\pi)^{\frac{1}{2}}n!\,e^{-\frac{1}{2}y^2}L_n(y^2)$
(11)	$e^{-\frac{1}{2}x^2}\,\mathrm{He}_n(x)\,\mathrm{He}_{n+2m}(x)$	$(\tfrac{1}{2}\pi)^{\frac{1}{2}}n!(-1)^m y^{2m}e^{-\frac{1}{2}y^2}L_n^{2m}(y^2)$
(12)	$e^{-ax}x^{\nu-2n}L_{2n-1}^{\nu-2n}(ax)$ $\mathrm{Re}\,\nu>2n-1, \quad \mathrm{Re}\,a>0$	$i(-1)^{n+1}\Gamma(\nu)[2(2n-1)!]^{-1}y^{2n-1}$ $\times[(a-iy)^{-\nu}-(a+iy)^{-\nu}]$
(13)	$e^{-ax}x^{\nu-1-2n}L_{2n}^{\nu-1-2n}(ax)$ $\mathrm{Re}\,\nu>2n, \quad \mathrm{Re}\,a>0$	$(-1)^n\Gamma(\nu)[2(2n)!]^{-1}y^{2n}[(a+iy)^{-\nu}$ $+(a-iy)^{-\nu}]$
(14)	$e^{-\frac{1}{2}x^2}L_n(x^2)$	$(\pi/2)^{\frac{1}{2}}(n!)^{-1}e^{-\frac{1}{2}y^2}[\mathrm{He}_n(y)]^2$
(15)	$x^{2m}e^{-\frac{1}{2}x^2}L_n^{2m}(x^2)$	$(-1)^m(\pi/2)^{\frac{1}{2}}(n!)^{-1}e^{-\frac{1}{2}y^2}$ $\times\mathrm{He}_n(y)\,\mathrm{He}_{n+2m}(y)$
(16)	$x^{2n}e^{-\frac{1}{2}x^2}L_n^{n-\frac{1}{2}}(\tfrac{1}{2}x^2)$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}y^{2n}e^{-\frac{1}{2}y^2}L_n^{n+\frac{1}{2}}(\tfrac{1}{2}y^2)$

1.11. Gamma function (including incomplete gamma function) and related functions; Legendre function

(1)	$\lvert\Gamma(a+ix)\rvert^2 \qquad a>0$	$\pi\Gamma(2a)2^{-2a}\operatorname{sech}^{2a}(\tfrac{1}{2}y)$

Gamma function etc. (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)dx \qquad y>0$
(2)	$[B(a+bx, a-bx)]^{-1} \qquad b>0$	$b^{-1}(a-\tfrac{1}{2})[2\cos(\tfrac{1}{2}b^{-1}y)]^{2a-2}$
(3)	$\zeta(\tfrac{1}{2}+ix)$	$2\pi^2\sum_{n=1}^{\infty}(2\pi n^4 e^{-9y/2}-3n^2e^{-5y/2})$ $\times e^{-n^2\pi e^{-2y}}$
(4)	$(1+4x^2)^{-1}\zeta(\tfrac{1}{2}+ix)$	$\tfrac{1}{4}\pi[\cosh(\tfrac{1}{2}y)+\tfrac{1}{4}\theta_3(0\,\vert\, ie^{-2y})$
(5)	$x\,\mathrm{Erfc}(ax) \qquad a>0$	$(2a^2)^{-1}e^{-\frac{1}{4}a^{-2}y^2}-y^{-2}[1-e^{-\frac{1}{4}a^{-2}y^2}]$
(6)	$x^{-1}[\mathrm{Erfc}(ax)-\mathrm{Erfc}(bx)]$ $a,\ b>0$	$\tfrac{1}{2}\mathrm{Ei}(-\tfrac{1}{4}a^{-2}y^2)-\tfrac{1}{2}\mathrm{Ei}(-\tfrac{1}{4}b^{-2}y^2)$
(7)	$\mathrm{si}(ax) \qquad a>0$	$-\dfrac{1}{2y}\log\left\vert\dfrac{y+a}{y-a}\right\vert \qquad y\neq a$
(8)	$e^{-ax}\,\mathrm{si}(bx) \qquad a>0, \quad b>0$	$-\dfrac{1}{2}\dfrac{1}{y^2+a^2}\left\{y\log\left[\dfrac{a^2+(y+b)^2}{a^2+(y-b)^2}\right]^{\frac{1}{2}}\right.$ $\left.+a\tan^{-1}\left(\dfrac{2ab}{b^2-a^2-y^2}\right)\right\}$
(9)	$e^{-ax}\,\mathrm{si}(\beta x) \qquad \mathrm{Re}\,\alpha>\vert\mathrm{Im}\,\beta\vert$	$-\dfrac{\tan^{-1}[(\alpha+iy)/\beta]}{2(\alpha+iy)}$ $-\dfrac{\tan^{-1}[(\alpha-iy)/\beta]}{2(\alpha-iy)}$
(10)	$\mathrm{Si}(bx) \qquad 0<x<a$ $0 \qquad a<x<\infty$	$\tfrac{1}{2}y^{-1}\{2\sin(ay)\,\mathrm{Si}(ab)+\mathrm{Ci}(ay+ab)$ $-\mathrm{Ci}(ay-ab)+\log[(y-b)(y+b)^{-1}]\}$ $y>b$ $\tfrac{1}{2}y^{-1}[2\sin(ab)\,\mathrm{Si}(ab)+\mathrm{Ci}(2ab)$ $-\log(2ab)-C]$ $y=b$

Gamma function etc. (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(11)	$x^{-1}\,\mathrm{Si}(ax) \qquad a>0$	$\tfrac{1}{2}\pi\log(ay^{-1})$
(12)	$x(x^2+b^2)^{-1}\,\mathrm{Si}(ax) \qquad a,\ b>0$	$\tfrac{1}{4}\pi\{e^{-by}[\overline{\mathrm{Ei}}(by)-\mathrm{Ei}(-ab)]$ $-e^{by}[\mathrm{Ei}(-by)-\mathrm{Ei}(-ab)]\}$ $\qquad 0<y<a$ $\tfrac{1}{4}\pi e^{-by}[\mathrm{Ei}(-ab)-\overline{\mathrm{Ei}}(ab)]$ $\qquad a<y<\infty$
(13)	$\mathrm{Ci}(ax) \qquad a>0$	$0 \qquad 0<y<a$ $-\tfrac{1}{2}\pi y^{-1} \qquad a<y<\infty$
(14)	$\mathrm{Ci}(bx) \qquad 0<x<a$ $0 \qquad a<x<\infty$ $b>0$	$\tfrac{1}{2}y^{-1}[2\sin(ay)\,\mathrm{Ci}(ab)-\mathrm{Si}(ay+ab)$ $-\mathrm{Si}(ay-ab)]$
(15)	$(x^2+b^2)^{-1}\,\mathrm{Ci}(ax) \qquad a,\ b>0$	$\tfrac{1}{2}\pi b^{-1}\cosh(by)\,\mathrm{Ei}(-ab)$ $\qquad 0<y\leq a$ $\tfrac{1}{4}\pi b^{-1}\{e^{-by}[\overline{\mathrm{Ei}}(ab)+\mathrm{Ei}(-ab)$ $-\overline{\mathrm{Ei}}(by)]+e^{by}\,\mathrm{Ei}(-by)\}$ $\qquad a\leq y<\infty$
(16)	$e^{-ax}\,\mathrm{Ci}(bx) \qquad a>0, \quad b>0$	$-\dfrac{a\log\{[(a^2+b^2-y^2)^2+4a^2y^2]\,b^{-4}\}}{4(y^2+a^2)}$ $-\dfrac{y\tan^{-1}[2ay(a^2+b^2-y^2)^{-1}]}{2(y^2+a^2)}$
(17)	$e^{-\alpha x}\,\mathrm{Ci}(\beta x) \qquad \mathrm{Re}\,\alpha>\lvert\mathrm{Im}\,\beta\rvert$	$-\dfrac{\log[1+\beta^{-2}(\alpha+iy)^2]}{4(\alpha+iy)}$ $-\dfrac{\log[1+\beta^{-2}(\alpha-iy)^2]}{4(\alpha-iy)}$

Gamma function etc. (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \quad y>0$
(18)	$\mathrm{ci}^2(x)+\mathrm{si}^2(x)$	$\pi y^{-1}\log(1+y)$
(19)	$\mathrm{Ei}(-x)$	$-y^{-1}\tan^{-1}y$
(20)	$e^{x}\,\mathrm{li}(e^{-x})$	$(1+y^2)^{-1}(\log y-\frac{1}{2}\pi y)$
(21)	$e^{-x}\,\mathrm{li}(e^{x})$	$-(1+y^2)^{-1}(\log y+\frac{1}{2}\pi y)$
(22)	$x^{-\frac{1}{2}}C(x)$	$\pi^{1/2}2^{-3/2}y^{-1/2} \quad 0<y<1$ $0 \quad 1<y<\infty$
(23)	$P_\nu(x^2+1) \quad -1<\mathrm{Re}\,\nu<0$	$-2^{\frac{1}{2}}\pi^{-1}\sin\nu\pi\, K^2_{\nu+\frac{1}{2}}(2^{-\frac{1}{2}}y)$
(24)	$Q_\nu(x^2+1) \quad \mathrm{Re}\,\nu>-1$	$2^{-\frac{1}{2}}\pi\, K_{\nu+\frac{1}{2}}(2^{-\frac{1}{2}}y)\, I_{\nu+\frac{1}{2}}(2^{-\frac{1}{2}}y)$
(25)	$\mathrm{P}_\nu(2x^2-1) \quad 0<x<1$ $0 \quad 1<x<\infty$	$\frac{1}{2}\pi\, J_{\nu+\frac{1}{2}}(\frac{1}{2}y)\, J_{-\nu-\frac{1}{2}}(\frac{1}{2}y)$
(26)	$0 \quad 0<x<a$ $(x^2-a^2)^{\frac{1}{2}\nu-\frac{1}{4}}\,\mathrm{P}_0^{\frac{1}{2}-\nu}(ax^{-1})$ $a<x<\infty$ $\lvert\mathrm{Re}\,\nu\rvert<\frac{1}{2}$	$y^{-\nu-\frac{1}{2}}\cos(ay-\frac{1}{2}\nu\pi-\frac{1}{4}\pi)$
(27)	$\mathrm{sech}(\pi x)\,P_{-\frac{1}{2}+ix}(a)$ $-1<a<1$	$[2(a+\cosh y)]^{-\frac{1}{2}}$

Gamma function etc. (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)\, dx \quad y > 0$
(28)	$P^{-\nu}_{x-\frac{1}{2}}(\cos\theta) \quad 0 < \theta < \pi$	$\dfrac{(2\pi)^{\frac{1}{2}}}{2\Gamma(\nu+\frac{1}{2})} \cdot \dfrac{(\cos y - \cos\theta)^{\nu-\frac{1}{2}}}{\sin^{\nu}\theta} \quad 0 < y < \theta$ $\left.\begin{array}{ll} 0 & \nu > \frac{1}{2} \\ \frac{1}{4}(2\pi)^{\frac{1}{2}}(\csc\theta)^{\frac{1}{2}} & \nu = \frac{1}{2} \\ \infty & -1 < \nu < \frac{1}{2} \end{array}\right\} \quad y = \theta$ $0 \quad y > \theta$

1.12. Bessel functions of argument kx

(1)	$J_0(ax) \quad a > 0$	$(a^2 - y^2)^{-\frac{1}{2}} \quad 0 < y < a$ $\infty \quad y = a$ $0 \quad a < y < \infty$
(2)	$J_{2n}(ax) \quad a > 0$	$(-1)^n (a^2 - y^2)^{-\frac{1}{2}} T_{2n}(y/a) \quad 0 < y < a$ $0 \quad a < y < \infty$
(3)	$J_{\nu}(ax) \quad \mathrm{Re}\, \nu > -1, \quad a > 0$	$(a^2 - y^2)^{-\frac{1}{2}} \cos[\nu \sin^{-1}(y/a)] \quad 0 < y < a$ $-a^{\nu} \sin(\frac{1}{2}\nu\pi)(y^2 - a^2)^{-\frac{1}{2}} \times [y + (y^2 - a^2)^{\frac{1}{2}}]^{-\nu} \quad a < y < \infty$
(4)	$x^{-1} J_{\nu}(ax) \quad \mathrm{Re}\, \nu > 0, \quad a > 0$	$\nu^{-1} \cos[\nu \sin^{-1}(y/a)] \quad 0 < y < a$ $\nu^{-1} a^{\nu} \cos(\frac{1}{2}\nu\pi)[y + (y^2 - a^2)^{\frac{1}{2}}]^{-\nu} \quad a < y < \infty$
(5)	$x^{-2} J_1(ax) \quad a > 0$	$(4a/3\pi)\{(1 + \frac{1}{4}a^{-2}y^2) \times E[(1 - \frac{1}{4}a^{-2}y^2)^{\frac{1}{2}}] - \frac{1}{2}a^{-2}y^2 K[(1 - \frac{1}{4}a^{-2}y^2)^{\frac{1}{2}}]\}$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(6)	$x^{-2}J_\nu(ax) \qquad \mathrm{Re}\,\nu>1, \quad a>0$	$\dfrac{a\cos[(\nu-1)\sin^{-1}(y/a)]}{2\nu(\nu-1)} + \dfrac{a\cos[(\nu+1)\sin^{-1}(y/a)]}{2\nu(\nu+1)} \qquad 0<y<a$ $\dfrac{a^\nu\sin(\frac{1}{2}\nu\pi)}{2\nu(\nu-1)[y+(y^2-a^2)^{\frac{1}{2}}]^{\nu-1}} - \dfrac{a^{\nu+2}\sin(\frac{1}{2}\nu\pi)}{2\nu(\nu+1)[y+(y^2-a^2)^{\frac{1}{2}}]^{\nu+1}} \qquad a<y<\infty$
(7)	$x^{-\frac{1}{2}}J_{2n+\frac{1}{2}}(ax) \qquad a>0$	$(-1)^n\pi^{\frac{1}{2}}(2a)^{-\frac{1}{2}}P_{2n}(y/a) \qquad 0<y<a$ $0 \qquad a<y<\infty$
(8)	$x^{-\frac{1}{2}}J_\nu(ax) \qquad \mathrm{Re}\,\nu>-\frac{1}{2}, \quad a>0$	$\dfrac{\Gamma(\frac{1}{2}\nu+\frac{1}{4})}{2^{\frac{1}{2}}\Gamma(\frac{1}{2}\nu+\frac{3}{4})a^{\frac{1}{2}}} \times {}_2F_1(\frac{1}{2}\nu+\frac{1}{4}, \frac{1}{4}-\frac{1}{2}\nu; \frac{1}{2}; y^2/a^2) \qquad 0<y<a$ $\dfrac{\Gamma(\frac{1}{2}\nu+\frac{1}{4})\,\pi^{\frac{1}{2}}a^\nu}{2^{\frac{1}{2}}y^{\nu+\frac{1}{2}}\Gamma(\nu+1)\Gamma(\frac{1}{4}-\frac{1}{2}\nu)} \times {}_2F_1(\frac{1}{2}\nu+\frac{1}{4}, \frac{1}{2}\nu+\frac{3}{4}; \nu+1; a^2/y^2) \qquad a<y<\infty$
(9)	$x^{-\nu}J_\nu(ax) \qquad \mathrm{Re}\,\nu>-\frac{1}{2}$	$\pi^{\frac{1}{2}}(2a)^{-\nu}[\Gamma(\nu+\frac{1}{2})]^{-1}(a^2-y^2)^{\nu-\frac{1}{2}} \qquad 0<y<a$ $0 \qquad a<y<\infty$
(10)	$x^{-\nu}J_{\nu+2n}(ax) \qquad \mathrm{Re}\,\nu>-\frac{1}{2}, \quad a>0$	$(-1)^n 2^{\nu-1}a^{-\nu}(2n)! \times\Gamma(\nu)[\Gamma(2\nu+2n)]^{-1} \times(a^2-y^2)^{\nu-\frac{1}{2}}C_{2n}^{\nu}(y/a) \qquad 0<y<a$ $0 \qquad a<y<\infty$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(11)	$x^{1-\nu}J_\nu(ax)$ $\operatorname{Re}\nu>\frac{1}{2}, \quad a>0$	$[2^{\nu-1}a^{2-\nu}\Gamma(\nu)]^{-1}$ $\times {}_2F_1(1,1-\nu;\frac{1}{2};a^{-2}y^2) \quad 0<y<a$ $-(\frac{1}{2}a)^\nu[\Gamma(1+\nu)]^{-1}y^{-2}$ $\times {}_2F_1(1,3/2;\nu+1;a^2y^{-2})$ $a<y<\infty$
(12)	$x^{\nu+1}J_\nu(ax)$ $-1<\operatorname{Re}\nu<-\frac{1}{2}, \quad a>0$	$0 \quad 0<y<a$ $2^{\nu+1}\pi^{1/2}a^\nu[\Gamma(-\frac{1}{2}-\nu)]^{-1}y$ $\times(y^2-a^2)^{-\nu-3/2} \quad a<y<\infty$
(13)	$x^{2\mu-1}J_{2\nu}(ax)$ $-\operatorname{Re}\nu<\operatorname{Re}\mu<\frac{3}{4}$	$\dfrac{2^{2\mu-1}a^{-2\mu}\Gamma(\nu+\mu)}{\Gamma(1+\nu-\mu)}$ $\times {}_2F_1(\nu+\mu,\mu-\nu;\frac{1}{2};a^{-2}y^2)$ $0<y<a$ $\dfrac{(\frac{1}{2}a)^{2\nu}y^{-2\nu-2\mu}\Gamma(2\nu+2\mu)\cos(\nu\pi+\pi\mu)}{\Gamma(2\nu+1)}$ $\times {}_2F_1(\nu+\mu,\nu+\mu+\frac{1}{2};2\nu+1;a^2y^{-2})$ $a<y<\infty$
(14)	$(x^2+\beta^2)^{-1}J_0(ax)$ $a>0, \quad \operatorname{Re}\beta>0$	$\frac{1}{2}\beta^{-1}\pi e^{-\beta y}I_0(a\beta) \quad a<y<\infty$
(15)	$x(x^2+\beta^2)^{-1}J_0(ax) \quad \operatorname{Re}\beta>0$	$\cosh(\beta y)K_0(a\beta) \quad 0<y<a$
(16)	$x^{-\nu}(x^2+\beta^2)^{-1}J_\nu(ax)$ $\operatorname{Re}\nu>-5/2$ $\operatorname{Re}\beta>0, \quad a>0$	$\frac{1}{2}\pi\beta^{-\nu-1}e^{-\beta y}I_\nu(a\beta) \quad a<y<\infty$
(17)	$x^{\nu+1}(x^2+\beta^2)^{-1}J_\nu(ax)$ $-1<\operatorname{Re}\nu<3/2$ if $0<y<1$ $a>0, \quad \operatorname{Re}\beta>0$	$\beta^\nu\cosh(\beta y)K_\nu(a\beta) \quad 0<y<1$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(18)	$x^{\nu}\sin x\, J_{\nu}(x) \qquad -1<\mathrm{Re}\,\nu<\tfrac{1}{2}$	$\pi^{1/2}2^{\nu-1}[\Gamma(\tfrac{1}{2}-\nu)]^{-1}(y^2+2y)^{-\nu-1/2} \qquad 0<y<2$ $\pi^{1/2}2^{\nu-1}[\Gamma(\tfrac{1}{2}-\nu)]^{-1}[(y^2+2y)^{-\nu-1/2}-(y^2-2y)^{-\nu-1/2}] \qquad 2<y<\infty$
(19)	$x^{-\nu}\cos x\, J_{\nu}(x) \qquad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\pi^{1/2}2^{-\nu-1}[\Gamma(\nu+\tfrac{1}{2})]^{-1}(2y-y^2)^{\nu-1/2} \qquad 0<y<2$ $0 \qquad 2<y<\infty$
(20)	$x^{-\nu}\sin x\, J_{\nu+1}(x) \qquad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$2^{-\nu-1}\pi^{1/2}[\Gamma(\nu+\tfrac{1}{2})]^{-1}\times(1-y)(2y-y^2)^{\nu-1/2} \qquad 0<y<2$ $0 \qquad 2<y<\infty$
(21)	$J_{\nu}(x)\,J_{-\nu}(x)$	$\tfrac{1}{2}P_{\nu-1/2}(\tfrac{1}{2}y^2-1) \qquad 0<y<2$ $0 \qquad 2<y<\infty$
(22)	$x^{1/2}[J_{-1/4}(ax)]^2 \qquad a>0$	$(\tfrac{1}{2}\pi y)^{-1/2}(4a^2-y^2)^{-1/2} \qquad 0<y<2a$ $0 \qquad 2a<y<\infty$
(23)	$x^{1/2}J_{\nu-1/4}(ax)\,J_{-\nu-1/4}(ax) \qquad a>0$	$\{[(2a+y)^{1/2}+i(2a-y)^{1/2}]^{4\nu}+[(2a+y)^{1/2}-i(2a-y)^{1/2}]^{4\nu}\}\times(4a)^{-2\nu}(2\pi y)^{-1/2}(4b^2-y^2)^{-1/2} \qquad 0<y<2a$ $0 \qquad 2a<y<\infty$
(24)	$x^{1/2}J_{\nu-1/4}(ax)\,J_{-\nu-1/4}(ax)$	$(\tfrac{1}{2}\pi y)^{-1/2}(4a^2-y^2)^{-1/2}\times\cos[2\nu\cos^{-1}(\tfrac{1}{2}a^{-1}y)] \qquad 0<y<2a$ $0 \qquad 2a<y<\infty$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \quad y>0$
(25)	$x^{\nu-\mu+1} J_\mu(ax)\, J_\nu(bx)$ $b>a>0, \quad -1<\operatorname{Re}\nu<\operatorname{Re}\mu$	$0 \qquad 0<y<b-a$
(26)	$x^{\rho-\mu-1} J_\mu(ax)\, J_\rho(bx)$ $0<\operatorname{Re}\rho<\operatorname{Re}\mu+2$ $b>a>0$	$2^{\rho-\mu-1} b^{-\rho} a^{\mu}\Gamma(\rho)/\Gamma(\mu+1)$ $0<y<b-a$
(27)	$x^{\lambda} J_\mu(ax)\, J_\rho(bx)$	see Bailey, W. N., 1936: *Proc. Lond. Math. Soc.* (2), 40, 37-48
(28)	$Y_0(ax) \qquad a>0$	$0 \qquad 0<y<a$ $-(y^2-a^2)^{-1/2} \qquad a<y<\infty$
(29)	$Y_\nu(ax) \qquad a>0, \quad \lvert\operatorname{Re}\nu\rvert<1$	$\dfrac{-\tan(\frac{1}{2}\nu\pi)}{(a^2-y^2)^{1/2}}\cos[\nu\sin^{-1}(y/a)]$ $0<y<a$ $-\sin(\frac{1}{2}\nu\pi)(y^2-a^2)^{-1/2}$ $\times\{a^{-\nu}[y-(y^2-a^2)^{1/2}]^{\nu}\operatorname{ctn}(\nu\pi)$ $+a^{\nu}[y-(y^2-a^2)^{1/2}]^{-\nu}\csc(\nu\pi)\}$ $a<y<\infty$
(30)	$x^{\nu} Y_\nu(ax) \quad \lvert\operatorname{Re}\nu\rvert<\frac{1}{2}, \quad a>0$	$0 \qquad 0<y<a$ $-2^{\nu}\pi^{1/2}a^{\nu}[\Gamma(\frac{1}{2}-\nu)]^{-1}(y^2-a^2)^{-\nu-1/2}$ $a<y<\infty$
(31)	$x^{\nu}\cos(ax)\, Y_\nu(ax)$ $\lvert\operatorname{Re}\nu\rvert<\frac{1}{2}, \quad a>0$	$-2^{\nu-1}\pi^{1/2}a^{\nu}[\Gamma(\frac{1}{2}-\nu)]^{-1}$ $\times(y^2+2ay)^{-\nu-1/2} \qquad 0<y<2a$ $-2^{\nu-1}\pi^{1/2}a^{\nu}[\Gamma(\frac{1}{2}-\nu)]^{-1}$ $\times[(y^2+2ay)^{-\nu-1/2}+(y^2-2ay)^{-\nu-1/2}\}$ $2a<y<\infty$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(32)	$Y_\nu(ax)\cos(\tfrac{1}{2}\nu\pi)$ $+J_\nu(ax)\sin(\tfrac{1}{2}\nu\pi)$ $\lvert\operatorname{Re}\nu\rvert<1, \quad a>0$	$0 \qquad 0<y<a$ $-\tfrac{1}{2}a^{-\nu}(y^2-a^2)^{-1/2}\{[y+(y^2-a^2)^{1/2}]^\nu$ $+[y-(y^2-a^2)^{1/2}]^\nu\} \qquad y>a$
(33)	$x^\nu[J_\nu(ax)\sin(ax)$ $+Y_\nu(ax)\cos(ax)]$ $\lvert\operatorname{Re}\nu\rvert<\tfrac{1}{2}, \quad a>0$	$0 \qquad 0<y<2a$ $-\pi^{1/2}(2a)^\nu[\Gamma(\tfrac{1}{2}-\nu)]^{-1}(y^2-2ay)^{-\nu-1/2}$ $2a<y<\infty$
(34)	$J_\nu(ax)\sin(ax-\tfrac{1}{2}\nu\pi)$ $-Y_\nu(ax)\cos(ax-\tfrac{1}{2}\nu\pi)$ $\lvert\operatorname{Re}\nu\rvert<1, \quad a>0$	$\tfrac{1}{2}a^{-\nu}(y^2+2ay)^{-1/2}$ $\times\{[y+a+(y^2+2ay)^{1/2}]^\nu$ $+[y+a-(y^2+2ay)^{1/2}]^\nu\}$
(35)	$x^\nu[Y_\nu(ax)\cos(ax)$ $-J_\nu(ax)\sin(ax)]$ $\lvert\operatorname{Re}\nu\rvert<\tfrac{1}{2}, \quad a>0$	$-\pi^{1/2}(2a)^\nu[\Gamma(\tfrac{1}{2}-\nu)]^{-1}(y^2+2ay)^{-\nu-1/2}$
(36)	$x^{1/2}(x^2+\beta^2)^{-1}$ $\times\{J_\nu(ax)\cos[(\tfrac{1}{4}+\tfrac{1}{2}\nu)\pi]$ $-Y_\nu(ax)\sin[(\tfrac{1}{4}+\tfrac{1}{2}\nu)\pi]\}$ $\lvert\operatorname{Re}\nu\rvert<3/2$ $a>0, \quad \operatorname{Re}\beta>0$	$\beta^{-1/2}\cosh(\beta y)\,K_\nu(a\beta) \qquad 0<y<a$
(37)	$x^{1/2}\,J_{-1/4}(\tfrac{1}{2}ax)\,Y_{-1/4}(\tfrac{1}{2}ax)$ $a>0$	$0 \qquad 0<y<a$ $-(\tfrac{1}{2}\pi y)^{-1/2}(y^2-a^2)^{-1/2} \qquad a<y<\infty$
(38)	$e^{-\frac{1}{2}ax}\,I_0(\tfrac{1}{2}ax) \qquad \operatorname{Re}a>0$	$a(2y)^{-1/2}(a^2+y^2)^{-1/2}$ $\times[y+(a^2+y^2)^{1/2}]^{-1/2}$
(39)	$\sin(ax)\,I_1(bx)/I_2(cx)$	For this and related integrands see Timpe, A., 1912: *Math. Ann.* 71, 480-509.

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \quad y>0$
(40)	$K_\nu(ax)$ $\operatorname{Re} a>0, \quad \lvert\operatorname{Re}\nu\rvert<1$	$\tfrac14(y^2+a^2)^{-1/2}\sec(\tfrac12\nu\pi)$ $\times\{\pi a^{-\nu}[y+(y^2+a^2)^{1/2}]^{\nu}$ $+\pi a^{\nu}[y+(y^2+a^2)^{1/2}]^{-\nu}\}$
(41)	$x^{\pm\mu}K_\mu(ax) \quad \operatorname{Re} a>0$ $\operatorname{Re}\mu>-\tfrac12$ if upper signs are used $\operatorname{Re}\mu<\tfrac12$ if lower signs are used	$\tfrac12\pi^{1/2}(2a)^{\pm\mu}\Gamma(\pm\mu+\tfrac12)(y^2+a^2)^{\mp\mu-1/2}$
(42)	$x^{-\lambda}K_\mu(ax)$ $\operatorname{Re}(\lambda\pm\mu)<1, \quad \operatorname{Re} a>0$	$2^{-\lambda-1}a^{\lambda-1}\Gamma[\tfrac12(\mu-\lambda+1)]$ $\times\Gamma[\tfrac12(1-\lambda-\mu)$ $\times{}_2F_1[\tfrac12(\mu-\lambda+1),\tfrac12(1-\mu-\lambda);\tfrac12;-y^2/a^2]$
(43)	$\cos(\beta x)K_0(ax)$ $\operatorname{Re} a>\lvert\operatorname{Im}\beta\rvert$	$\tfrac12\pi(\beta y)^{1/2}R_1^{-1}R_2^{-1}(R_2+R_1)^{1/2}(R_2-R_1)^{-1/2}$ $R_1=[a^2+(\beta-y)^2]^{1/2}$ $R_2=[a^2+(\beta+y)^2]^{1/2}$
(44)	$\sinh(\tfrac12 ax)K_1(\tfrac12 ax)$ $\operatorname{Re} a>0$	$\pi 2^{-1/2}a^2y^{-1/2}(y^2+a^2)^{-1/2}$ $\times[y+(y^2+a^2)^{1/2}]^{-3/2}$
(45)	$x^{-\nu}\cosh(\tfrac12 ax)K_\nu(\tfrac12 ax)$ $\operatorname{Re} a>0, \quad -\tfrac12<\operatorname{Re}\nu<\tfrac12$	$(\pi^{3/2}/2)[\Gamma(\nu+\tfrac12)]^{-1}\sec(\nu\pi)$ $\times a^{-\nu}y^{\nu-1/2}(a^2+y^2)^{\nu/2-1/4}$ $\times\cos[(\nu-\tfrac12)\operatorname{ctn}^{-1}(y/a)]$
(46)	$K_0(ax)\,I_0(\beta x)$ $\operatorname{Re} a>\lvert\operatorname{Re}\beta\rvert$	$[y^2+(a+\beta)^2]^{-1/2}K\left\{\dfrac{(2a\beta)^{1/2}}{[y^2+(a+\beta)^2]^{1/2}}\right\}$
(47)	$K_\nu(ax)\,I_\nu(\beta x)$ $\operatorname{Re} a>\lvert\operatorname{Re}\beta\rvert, \quad \operatorname{Re}\nu>-\tfrac12$	$\tfrac12(a\beta)^{-1/2}Q_{\nu-1/2}(u)$ $u=(y^2+\beta^2+a^2)(2\beta a)^{-1}$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)\, dx \qquad y > 0$
(48)	$x^{1/2} I_{-1/4}(\frac{1}{2}ax) K_{1/4}(\frac{1}{2}ax)$ $\operatorname{Re} a > 0$	$(\pi/2)^{1/2} y^{-1/2} (y^2 + a^2)^{-1/2}$
(49)	$x^{1/2} I_{-1/4 \mp \beta}(\frac{1}{2}ax) K_{-1/4 \pm \beta}(\frac{1}{2}ax)$ $\operatorname{Re} a > 0$ $\operatorname{Re} \beta < 3/4$ if upper signs are used $\operatorname{Re} \beta > -3/4$ if lower signs are used	$(\frac{1}{2}\pi)^{1/2} a^{-2\beta} y^{-1/2} (a^2 + y^2)^{-1/2}$ $\times [\pm y + (a^2 + y^2)^{1/2}]^{2\beta}$
(50)	$x^{-1/2} K_\nu(ax) I_\nu(ax)$ $\operatorname{Re} \nu > -1/4, \quad \operatorname{Re} a > 0$	$2^{-1/2} \pi^{1/2} y^{-1/2} e^{\nu\pi i} \dfrac{\Gamma(1/4 + \nu)}{\Gamma(1/4 - \nu)}$ $\times Q^{-\nu}_{-1/4} [(y^2 + 4a^2)^{1/2} y^{-1}]$ $\times P^{-\nu}_{-1/4} [(y^2 + 4a^2)^{1/2} y^{-1}]$
(51)	$K_\nu(ax) K_\nu(\beta x)$ $\lvert\operatorname{Re} \nu\rvert < 1/2, \quad \operatorname{Re}(a + \beta) > 0$	$\frac{1}{4} \pi^2 (a\beta)^{-1/2} \sec(\nu\pi)$ $\times P_{\nu - 1/2} [(y^2 + \beta^2 + a^2)(2a\beta)^{-1}]$
(52)	$K_0(x) K_0(ax) [K_0(\beta x)]^{-1}$	see Ollendorff, F., 1926: *Arch. Elektrotechnik* 17, 79-101
(53)	$x^{1/2} K_\nu^2(ax)$ $\lvert\operatorname{Re} \nu\rvert < 3/4, \quad \operatorname{Re} a > 0$	$2^{-1/2} \pi^{1/2} e^{2\nu\pi i} y^{1/2} (y^2 + 4a^2)^{-1/2}$ $\times [\Gamma(-1/4 - \nu)]^{-1} \Gamma(3/4 + \nu)$ $\times Q^{-\nu}_{-1/4} [(y^2 + 4a^2)^{1/2} y^{-1}]$ $\times Q^{-\nu}_{-5/4} [(y^2 + 4a^2)^{1/2} y^{-1}]$
(54)	$x^{-1/2} K_\nu^2(ax)$ $\lvert\operatorname{Re} \nu\rvert < 1/4, \quad \operatorname{Re} a > 0$	$2^{-1/2} \pi^{1/2} e^{2\nu\pi i} \Gamma(\nu + 1/4) [\Gamma(-\nu + 1/4)]^{-1}$ $\times y^{-1/2} [Q^{-\nu}_{-1/4} [(y + 4a^2/y)]^2$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)\, dx \qquad y > 0$
(55)	$x^{1/2} K_\nu(ax) K_{\nu+1}(ax)$ $-5/4 < \operatorname{Re} \nu < 1/4, \quad \operatorname{Re} a > 0$	$2^{-1/2} \pi^{1/2} e^{(2\nu+1)\pi i} y^{1/2} (y^2+4a^2)^{-1/2}$ $\times \Gamma(5/4+\nu) [\Gamma(-3/4-\nu)]^{-1}$ $\times Q_{-1/4}^{-\nu}(u)\, Q_{-1/4}^{-\nu-1}(u)$ $u = (y^2 + 4a^2)^{1/2} y^{-1}$
(56)	$\mathbf{J}_\nu(ax) + \mathbf{J}_{-\nu}(ax) \qquad a > 0$	$a^{-\nu} \cos(\tfrac{1}{2}\nu\pi)$ $\times \dfrac{[y + i(a^2-y^2)^{1/2}]^\nu + [y - i(a^2-y^2)^{1/2}]^\nu}{(a^2 - y^2)^{1/2}} \qquad 0 < y < a$ $0 \qquad a < y < \infty$
(57)	$x^{-\nu-1/2} s_{\nu-1/2,\, \nu+3/2}(x)$ $\operatorname{Re} \nu > -1$	$-\tfrac{1}{2}\pi y (1-y^2)^\nu \qquad 0 < y < 1$ $0 \qquad 1 < y < \infty$

1.13. Bessel functions of other arguments

(1)	$x^{1/2} J_{-1/4}(a^2 x^2)$	$2^{-3/2} a^{-2} (\pi y)^{1/2} J_{-1/4}(\tfrac{1}{4} a^{-2} y^2)$
(2)	$x e^{-bx^2} J_n(a^2 x^2)$	see Terazawa, K., 1916: *Proc. Roy. Soc. London* Ser. *A*, 92, 57-81
(3)	$x^{1/2} \cos(a^2 x^2) J_{-1/4}(a^2 x^2)$	$\tfrac{1}{2} a^{-1} y^{-1/2} \cos(2^{-3} a^{-2} y^2 - 2^{-3}\pi)$
(4)	$x^{1/2} \sin(a^2 x^2) J_{-1/4}(a^2 x^2)$	$-2ay^{1/2} \sin(2^{-3} a^{-2} y^2 - 2^{-3}\pi)$
(5)	$x^{1/2} [J_{-\frac{1}{8}}(a^2 x^2)]^2$	$-2^{-5/2} \pi^{1/2} a^{-2} y^{1/2} J_{-\frac{1}{8}}(2^{-4} a^{-2} y^2)$ $\times Y_{\frac{1}{8}}(2^{-4} a^{-2} y^2)$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)dx \qquad y>0$
(6)	$x^{1/2} J_{-\frac{1}{8}-\nu}(a^2x^2)$ $\times J_{-\frac{1}{8}+\nu}(a^2x^2)$	$2^{1/2}\pi^{-1/2}y^{-3/2}[e^{-i\pi/8}$ $\times W_{\nu,-\frac{1}{8}}(2^{-3}y^2a^{-2}e^{-\frac{1}{2}\pi i})$ $\times W_{-\nu,-\frac{1}{8}}(2^{-3}y^2a^{-2}e^{-\frac{1}{2}\pi i})$ $+e^{\pi i/8}W_{\nu,-\frac{1}{8}}(2^{-3}y^2a^{-2}e^{\frac{1}{2}\pi i})$ $\times W_{-\nu,-\frac{1}{8}}(2^{-3}y^2a^{-2}e^{\frac{1}{2}\pi i})]$
(7)	$x^{1/2} Y_{-1/4}(a^2x^2)$	$-2^{-3/2}\pi^{1/2}a^{-2}y^{1/2}\mathbf{H}_{-1/4}(\frac{1}{4}a^{-2}y^2)$
(8)	$x^{1/2} J_{-\frac{1}{8}}(a^2x^2)\,Y_{-\frac{1}{8}}(a^2x^2)$	$-2^{-3/2}\pi^{1/2}a^{-2}y^{1/2}$ $\times[J_{-\frac{1}{8}}(2^{-4}a^{-2}y^2)]^2$
(9)	$x^{1/3}e^{-x^2}I_{-\frac{1}{3}}(x^2)$	$2^{-3/2}\pi^{1/2}y^{1/3}e^{-y^2/8}I_{-\frac{1}{3}}(y^2/8)$
(10)	$x^{1/2} K_{1/4}(a^2x^2)$	$2^{-5/2}\pi^{3/2}a^{-2}y^{1/2}[I_{-1/4}(\frac{1}{4}a^{-2}y^2)$ $-\mathbf{L}_{-1/4}(\frac{1}{4}a^{-2}y^2)]$
(11)	$x^{1/2} K_{\frac{1}{8}}(a^2x^2)\,I_{-\frac{1}{8}}(a^2x^2)$	$2^{-1/2}\pi^{1/2}(2a)^{-2}y^{1/2}K_{\frac{1}{8}}(2^{-4}y^2/a^2)$ $\times I_{-\frac{1}{8}}(2^{-4}y^2/a^2)$
(12)	$x^{1/2} K_{\frac{1}{8}-\nu}(a^2x^2)$ $\times I_{-\frac{1}{8}-\nu}(a^2x^2) \qquad \mathrm{Re}\,\nu<3/8$	$(2\pi)^{1/2}[\Gamma(\frac{3}{4})]^{-1}y^{-3/2}\Gamma(3/8-\nu)$ $\times W_{\nu,-\frac{1}{8}}(2^{-3}y^2/a^2)$ $\times M_{-\nu,-\frac{1}{8}}(2^{-3}y^2/a^2)$
(13)	$x^{1/2}\mathbf{H}_{-1/4}(a^2x^2)$	$-2^{-3/2}\pi^{1/2}a^{-2}y^{1/2}Y_{-1/4}(\frac{1}{4}a^{-2}y^2)$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)dx \quad y>0$
(14)	$x^{2\lambda}J_{2\nu}(ax^{-1})$ $-\frac{3}{4}<\operatorname{Re}\lambda<\operatorname{Re}\nu-\frac{1}{2}, \quad a>0$	$4^{\lambda-2\nu}\pi^{1/2}y^{2\nu-2\lambda-1}\dfrac{\Gamma(\lambda-\nu+\frac{1}{2})}{\Gamma(2\nu+1)\Gamma(\nu-\lambda)}$ $\times{}_0F_3(2\nu+1,\, \nu-\lambda+\frac{1}{2},\, \nu-\lambda;\, 2^{-4}a^2y^2)$ $+4^{-\lambda-1}a^{2\lambda+1}\dfrac{\Gamma(\nu-\lambda-1/2)}{\Gamma(\nu+\lambda+3/2)}$ $\times{}_0F_3(\frac{1}{2},\, \lambda-\nu+3/2;\, \nu+\lambda+3/2;\, 2^{-4}a^2y^2)$
(15)	$x^{-1}\sin(ax^{-1})J_{2n}(bx^{-1})$ $a>0, \quad b>0$	$(-1)^n(\frac{1}{2}\pi)J_{2n}(cy^{1/2})\,J_{2n}(dy^{1/2})$ $c^2+d^2=4a, \quad cd=2b$
(16)	$x^{-1}\cos(ax^{-1})J_{2n-1}(bx^{-1})$ $a>0, \quad b>0$	$(-1)^n(\frac{1}{2}\pi)J_{2n-1}(cy^{1/2})J_{2n-1}(dy^{1/2})$ $c^2+d^2=4a, \quad cd=2b$
(17)	$x^{-1/2}\cos(ax^{-1})\,J_{2n-1/2}(ax^{-1})$ $a>0$	$(-1)^n\frac{1}{2}\pi^{1/2}y^{-1/2}J_{4n-1}(2^{3/2}a^{1/2}y^{1/2})$
(18)	$x^{-1/2}\sin(ax^{-1})J_{2n-3/2}(ax^{-1})$ $a>0$	$(-1)^{n-1}\frac{1}{2}\pi^{1/2}y^{-1/2}$ $\times J_{4n-3}(2^{3/2}a^{1/2}y^{1/2})$
(19)	$x^{-1}K_0(ax^{-1}) \qquad \operatorname{Re}a>0$	$-\pi K_0[(2ay)^{1/2}]\,Y_0[(2ay)^{1/2}]$
(20)	$x^{-1}K_\nu(ax^{-1})$ $\lvert\operatorname{Re}\nu\rvert<1, \quad \operatorname{Re}a>0$	$-\pi K_\nu[(2ay)^{1/2}]\{J_\nu[(2ay)^{1/2}]$ $\times\sin(\frac{1}{2}\nu\pi)+Y_\nu[(2ay)^{1/2}]\cos(\frac{1}{2}\nu\pi)\}$
(21)	$J_0(ax^{1/2}) \qquad a>0$	$y^{-1}\sin(\frac{1}{4}a^2y^{-1})$
(22)	$J_\nu(ax^{1/2}) \quad \operatorname{Re}\nu>-2, \quad a>0$	$-\frac{1}{4}\pi^{1/2}ay^{3/2}[\sin(2^{-3}a^2y^{-1}-\frac{1}{4}\nu\pi)$ $\times J_{\nu/2-1/2}(2^{-3}a^2/y)$ $+\cos(2^{-3}a^2/y-\frac{1}{4}\nu\pi)$ $\times J_{\nu/2+1/2}(2^{-3}a^2y^{-1})]$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\cos(xy)dx \quad y > 0$
(23)	$x^{-1/2} J_1(ax^{1/2}) \qquad a > 0$	$4a^{-1}\sin^2(2^{-3}a^2y^{-1})$
(24)	$x^{\nu-1/2} J_1(ax^{1/2})$ $-1 < \mathrm{Re}\,\nu < 3/4, \quad a > 0$	$\frac{1}{2}\Gamma(\nu+1)\csc(\nu\pi)y^{-\nu}$ $\times[\sin(2^{-3}a^2y^{-1}+\frac{1}{2}\nu\pi)]$ $\times k_{-2\nu}(-2^{-3}a^2iy^{-1})$ $-\sin(2^{-3}a^2y^{-1}-\frac{1}{2}\nu\pi)$ $\times k_{-2\nu}(2^{-3}a^2iy^{-1})$
(25)	$x^{-1/2} J_\nu(ax^{1/2})$ $\mathrm{Re}\,\nu > -1, \quad a > 0$	$(\pi/y)^{1/2}\cos(2^{-3}a^2y^{-1}-\frac{1}{4}\nu\pi-\frac{1}{4}\pi)$ $\times J_{\frac{1}{2}\nu}(2^{-3}a^2y^{-1})$
(26)	$x^{\frac{1}{2}\nu} J_\nu(ax^{1/2})$ $-1 < \mathrm{Re}\,\nu < \frac{1}{2}, \quad a > 0$	$2^{-\nu}y^{-\nu-1}a^\nu\sin(\frac{1}{4}a^2y^{-1}-\frac{1}{2}\nu\pi)$
(27)	$J_\nu(ax^{1/2})\,J_\nu(bx^{1/2})$ $a > 0, \quad b > 0, \quad \mathrm{Re}\,\nu > -1$	$y^{-1}J_\nu(\frac{1}{2}aby^{-1})\sin[\frac{1}{4}(a^2+b^2)y^{-1}$ $-\frac{1}{2}\nu\pi]$
(28)	$x^{-1/2} K_{2\nu}(2a^{1/2}x^{1/2})$ $-\frac{1}{2} < \mathrm{Re}\,\nu < \frac{1}{2}, \quad \mathrm{Re}\,a^{1/2} > 0$	$-\frac{1}{4}\pi^{-3/2}y^{-1/2}\sec(\nu\pi)\,[J_\nu(\frac{1}{2}ay^{-1})$ $\times\sin(\frac{1}{2}\nu\pi-\frac{1}{2}ay-\frac{1}{4}\pi)$ $+Y_\nu(\frac{1}{2}ay^{-1})\cos(\frac{1}{2}\nu\pi-\frac{1}{2}ay^{-1}-\frac{1}{4}\pi)]$
(29)	$J_0(a^{1/2}x^{1/2})\,K_0(a^{1/2}x^{1/2})$ $\mathrm{Re}\,a > 0$	$\frac{1}{4}\pi y^{-1}[I_0(\frac{1}{2}a/y)-\mathbf{L}_0(\frac{1}{2}a/y)]$
(30)	$K_0(a^{1/2}x^{1/2})\,Y_0(a^{1/2}x^{1/2})$ $\mathrm{Re}\,a^{1/2} > 0$	$-\frac{1}{2}y^{-1}K_0(\frac{1}{2}ay^{-1})$
(31)	$K_0(a^{1/2}e^{\frac{1}{4}\pi i}x^{1/2})\,K_0(a^{1/2}e^{-\frac{1}{4}\pi i}x^{1/2})$ $\mathrm{Re}\,a > 0$	$\pi^2 2^{-3}y^{-1}[\mathbf{H}_0(\frac{1}{2}a/y)-Y_0(\frac{1}{2}a/y)]$
(32)	$x^{-1/2} J_{2\nu}(a^{1/2}x^{1/2})K_{2\nu}(a^{1/2}x^{1/2})$ $\mathrm{Re}\,a^{1/2} > 0, \quad \mathrm{Re}\,\nu > -\frac{1}{2}$	$a^{-1}(\frac{1}{2}\pi y)^{1/2}\Gamma(\frac{1}{4}+\nu)[\Gamma(1+2\nu)]^{-1}$ $\times W_{\frac{1}{4},\nu}(\frac{1}{2}ay^{-1})M_{-\frac{1}{4},\nu}(\frac{1}{2}ay^{-1})$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(33)	$K_0(2x^{1/2}) - (\tfrac{1}{2}\pi)Y_0(2x^{1/2})$	$(\tfrac{1}{2}\pi)y^{-1}\cos(y^{-1})$
(34)	$x^{\nu/2}[e^{i\pi\nu/4}K_\nu(a^{1/2}e^{i\pi/4}x^{1/2})$ $+e^{-i\pi\nu/4}K_\nu(a^{1/2}e^{-i\pi/4}x^{1/2})$ $\mathrm{Re}\,a>0, \quad \mathrm{Re}\,\nu>-1$	$\pi 2^{-\nu-1}a^\nu y^{-\nu-1}e^{-\frac{1}{4}a/y}$
(35)	$J_0[a(x^2+b^2)^{1/2}] \qquad b>0$	$(a^2-y^2)^{-1/2}\cos[b(a^2-y^2)^{1/2}] \qquad 0<y<a$ $0 \qquad a<y<\infty$
(36)	$(x^2+b^2)^{-\frac{1}{2}\nu-1}J_{\nu-1}[a(x^2+b^2)^{1/2}]$ $\mathrm{Re}\,\nu>-5/2, \quad a,\,b>0$	$2^{-\nu}\pi b^{-1}a^{\nu-1}[\Gamma(\nu)]^{-1}e^{-by} \qquad a<y$
(37)	$(x^2+b^2)^{-\frac{1}{2}\nu}J_\nu[a(x^2+b^2)^{1/2}]$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}, \quad a,\,b>0$	$(\tfrac{1}{2}\pi)^{1/2}b^{-\nu+\frac{1}{2}}a^{-\nu}(a^2-y^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times J_{\nu-\frac{1}{2}}[b(a^2-y^2)^{1/2}] \qquad 0<y<a$ $0 \qquad a<y<\infty$
(38)	$(x^2+a^2)^{-1}(x^2+b^2)^{-\frac{1}{2}\nu}$ $\times J_\nu[c(x^2+b^2)^{1/2}]$ $y>c, \quad \mathrm{Re}\,\nu>-5/2 \quad a>0$	$\tfrac{1}{2}\pi a^{-1}e^{-ay}(b^2-a^2)^{-\frac{1}{2}\nu}$ $\times J_\nu[c(b^2-a^2)^{1/2}] \qquad c<y<\infty$
	For more integrands of this type see Hankel Transforms.	
(39)	$(x^2+a^2)^{-\frac{1}{2}-\frac{1}{2}\nu}$ $\times C_{2n}^\nu[x(x^2+a^2)^{-1/2}]$ $\times J_{\nu+2n}[(a^2+x^2)^{1/2}]$ $\mathrm{Re}\,\nu>-3/2, \quad a>0$	$(-1)^n 2^{-\frac{1}{2}}\pi^{1/2}a^{\frac{1}{2}-\nu}(1-y^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times C_{2n}^\nu(y)J_{\nu-\frac{1}{2}}[a(1-y^2)^{1/2}] \qquad 0<y<1$ $0 \qquad 1<y<\infty$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(40)	$x^{1/2} J_{-1/4}\{\tfrac{1}{2}a[(x^2+b^2)^{1/2}-b]\}$ $\times J_{-1/4}\{\tfrac{1}{2}a[(x^2+b^2)^{1/2}+b]\}$	$(2/\pi)^{1/2} y^{-1/2}(a^2-y^2)^{-1/2}$ $\times\cos[b(a^2-y^2)^{1/2}] \qquad 0<y<a$ $0 \qquad a<y<\infty$
	See also Trigonometric functions where more results similar to the above may be obtained by inversion.	
(41)	$(x^2+a^2)^{-1/2\nu} Y_\nu[b(x^2+a^2)^{1/2}]$ $\operatorname{Re}\nu>-\tfrac{1}{2}, \quad a,\ b>0$	$(\tfrac{1}{2}\pi a)^{1/2}(ab)^{-\nu}(b^2-y^2)^{1/2\nu-1/4}$ $\times Y_{\nu-1/2}[a(b^2-y^2)^{1/2}] \qquad 0<y<b$ $-(2a/\pi)^{1/2}(ab)^{-\nu}(y^2-b^2)^{1/2\nu-1/4}$ $\times K_{\nu-1/2}[a(y^2-b^2)^{1/2}] \qquad b<y<\infty$
(42)	$(x^2+a^2)^{-1/2\nu} H_\nu^{(2)}[b(a^2+x^2)^{1/2}]$ $\operatorname{Re}\nu>-\tfrac{1}{2}, \quad a,\ b>0$	$(\tfrac{1}{2}\pi a)^{1/2}(ab)^{-\nu}(b^2-y^2)^{1/2\nu-1/4}$ $\times H_\nu^{(2)}[a(b^2-y^2)^{1/2}] \qquad 0<y<b$ $i(2a/\pi)^{1/2}(ab)^{-\nu}(y^2-b^2)^{1/2\nu-1/4}$ $\times K_{\nu-1/2}[a(y^2-b^2)^{1/2}] \qquad 0<y<b$
(43)	$K_0[a(x^2+\beta^2)^{1/2}]$ $\operatorname{Re}a>0, \quad \operatorname{Re}\beta>0$	$(\pi/2)(y^2+a^2)^{-1/2} e^{-\beta(y^2+a^2)^{1/2}}$
(44)	$(x^2+\beta^2)^{-1/2} K_1[a(x^2+\beta^2)^{1/2}]$ $\operatorname{Re}a>0, \quad \operatorname{Re}\beta>0$	$\tfrac{1}{2}\pi a^{-1}\beta^{-1} e^{-\beta(y^2+a^2)^{1/2}}$
(45)	$(x^2+\beta^2)^{\mp 1/2\nu} K_\nu[a(x^2+\beta^2)^{1/2}]$ $\operatorname{Re}a>0, \quad \operatorname{Re}\beta>0$	$2^{-1/2}\pi^{1/2} a^{\mp\nu}\beta^{1/2\mp\nu}(y^2+a^2)^{\pm 1/2\nu-1/4}$ $\times K_{\pm\nu-1/2}[\beta(y^2+a^2)^{1/2}]$
(46)	$x^{1/2} I_{-1/4}\{\tfrac{1}{2}\beta[(x^2+a^2)^{1/2}-a]\}$ $\times K_{1/4}\{\tfrac{1}{2}\beta[(x^2+a^2)^{1/2}+a]\}$ $\operatorname{Re}a>0, \quad \operatorname{Re}\beta>0$	$(\pi/2)^{1/2} y^{-1/2}(y^2+\beta^2)^{-1/2}$ $\times e^{-a(y^2+\beta^2)^{1/2}}$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)dx \quad y>0$
(47)	$J_0[b(a^2-x^2)^{1/2}]$ $\quad 0<x<a$ 0 $\quad a<x<\infty$ $b>0$	$(b^2+y^2)^{-1/2}\sin[a(b^2+y^2)^{1/2}]$
(48)	0 $\quad 0<x<a$ $J_0[b(x^2-a^2)]$ $\quad a<x<\infty$ $b>0$	$(b^2-y^2)^{-1/2}e^{-a(b^2-y^2)^{1/2}}$ $0<y<b$ $-(y^2-b^2)^{-1/2}\sin[a(y^2-b^2)^{1/2}]$ $b<y<\infty$
(49)	$J_0[b(x^2-a^2)^{1/2}]$ $\quad a,\ b>0$	$(b^2-y^2)^{-1/2}\cosh[a(b^2-y^2)^{1/2}]$ $0<y<b$ 0 $\quad b<y<\infty$
(50)	$(a^2-x^2)^{1/2\nu}J_\nu[b(a^2-x^2)^{1/2}]$ $0<x<a$ 0 $\quad a<x<\infty$ $\mathrm{Re}\,\nu>-1,\quad b>0$	$2^{-1/2}\pi^{1/2}a^{\nu+1/2}b^\nu J_{\nu+1/2}[a(b^2+y^2)^{1/2}]$ $\times(b^2+y^2)^{-1/2\nu-1/4}$
(51)	0 $\quad 0<x<a$ $(x^2-a^2)^{1/2\nu}J_\nu[b(x^2-a^2)^{1/2}]$ $a<x<\infty$ $-1<\mathrm{Re}\,\nu<1/2,\quad b>0$	$(2a/\pi)^{1/2}(ab)^\nu(b^2-y^2)^{-1/2\nu-1/4}$ $\times K_{\nu+1/2}[a(b^2-y^2)^{1/2}]\quad 0<y<b$ $-(1/2\pi a)^{1/2}(ab)^\nu(b^2-y^2)^{-1/2\nu-1/4}$ $\times Y_{-\nu-1/2}[a(y^2-b^2)^{1/2}]\quad b<y<\infty$
(52)	$x^{2n}(1-x^2)^{1/2\nu+m}J_\nu[a(1-x^2)^{1/2}]$ $0<x<1$ 0 $\quad 1<x<\infty$ $\mathrm{Re}\,\nu>-1,\quad a>0$	$2^{-1/2}\pi^{1/2}a^{-\nu}y\left(\frac{d}{ada}\right)^m\left(\frac{d}{ydy}\right)^n$ $\times\{a^{2\nu+2m}y^{2n-1}(a^2+y^2)^{-1/2(\nu+m+n+1/2)}$ $\times J_{\nu+m+n+1/2}[(a^2+y^2)^{1/2}]$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)\,dx \qquad y>0$
(53)	$0 \qquad 0<x<1$ $x(x^2-1)^{\frac{1}{2}\nu} J_\nu[a(x^2-1)^{\frac{1}{2}}]$ $1<x<\infty$ $-1<\operatorname{Re}\nu<-\frac{1}{2}, \quad a>0$	$0 \qquad 0<y<a$ $(\frac{1}{2}\pi)^{\frac{1}{2}} a^\nu y^\nu (y^2-a^2)^{-\frac{1}{2}\nu-\frac{3}{4}}$ $\times J_{-\nu-3/2}[(y^2-a^2)^{\frac{1}{2}}] \quad a<y<\infty$
(54)	$0 \qquad 0<x<b$ $(x^2-b^2)^{-\frac{1}{2}} J_\nu[a(x^2-b^2)^{\frac{1}{2}}]$ $b<x<\infty$ $\operatorname{Re}\nu>-1, \quad a>0$	$-\frac{1}{2}\pi J_{\frac{1}{2}\nu}\{\frac{1}{2}b[y-(y^2-a^2)^{\frac{1}{2}}]\}$ $\times Y_{-\frac{1}{2}\nu}\{\frac{1}{2}b[y+(y^2-a^2)^{\frac{1}{2}}]\}$ $a<y<\infty$
(55)	$0 \qquad 0<x<c$ $x(x^2+\beta^2)^{-1}(x^2-c^2)^{\frac{1}{2}\nu}$ $\times J_\nu[a(x^2-c^2)^{\frac{1}{2}}] \quad c<x<\infty$ $-1<\operatorname{Re}\nu<3/2$ $a>0, \quad \operatorname{Re}\beta>0$	$(\beta^2+c^2)^{\frac{1}{2}\nu}\cosh(\beta y)$ $\times K_\nu[a(\beta^2+c^2)^{\frac{1}{2}}] \quad a<y<\infty$
(56)	$(1-x^2)^{\frac{1}{2}\nu-\frac{1}{4}} C^\nu_{2n}(x)$ $\times J_{\nu-\frac{1}{2}}[a(1-x^2)^{\frac{1}{2}}] \quad 0<x<1$ $0 \qquad 1<x<\infty$ $\operatorname{Re}\nu>-\frac{1}{2}$	$(-1)^n 2^{-\frac{1}{2}}\pi^{\frac{1}{2}} a^{-\frac{1}{2}+\nu}(a^2+y^2)^{-\frac{1}{2}-\frac{1}{2}\nu}$ $\times C^\nu_{2n}[y(a^2+y^2)^{-\frac{1}{2}}]$ $\times J_{\nu+2n}[(a^2+y^2)^{\frac{1}{2}}]$
(57)	$0 \qquad 0<x<c$ $x(x^2+\beta^2)^{-1}(x^2-c^2)^{\frac{1}{2}\nu+n-\frac{1}{2}}$ $\times Y_\nu[a(x^2-c^2)^{\frac{1}{2}}] \quad c<x<\infty$ $-1/2-n<\operatorname{Re}\nu<5/2-2n$ $a>0, \quad \operatorname{Re}\beta>0$	$(-1)^{n+1}(\beta^2+c^2)^{\frac{1}{2}\nu+n-\frac{1}{2}}\cosh(by)$ $\times K_\nu[a(\beta^2+c^2)^{\frac{1}{2}}] \quad 0<y<a$
(58)	$H_0^{(2)}[a(\beta^2-x^2)^{\frac{1}{2}}]$ $-\pi<\arg(\beta^2-x^2)^{\frac{1}{2}}\le 0, \quad a>0$	$i(a^2+y^2)^{-\frac{1}{2}} e^{-i\beta(a^2+y^2)^{\frac{1}{2}}}$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)dx \qquad y>0$
(59)	$H_0^{(1)}[a(\beta^2-x^2)^{1/2}]$ $\pi>\arg(\beta^2-x^2)^{1/2}\geq 0, \quad a>0$	$-i(a^2+y^2)^{-1/2}e^{i\beta(a^2+y^2)^{1/2}}$
(60)	$J_{2\nu}[2a\cos(\frac{1}{2}x)] \quad 0<x<\pi$ $0 \quad \pi<x<\infty$ $\operatorname{Re}\nu>-\frac{1}{2}$	$\pi J_{\nu-y}(a)J_{\nu+y}(a)$
(61)	$I_{2\nu}[2a\cos(\frac{1}{2}x)] \quad 0<x<\pi$ $0 \quad \pi/2<x<\infty$ $\operatorname{Re}\nu>-\frac{1}{2}$	$\pi I_{\nu-y}(a)I_{\nu+y}(a)$
(62)	$J_0[2a\sinh(\frac{1}{2}x)] \quad a>0$	$[I_{iy}(a)+I_{-iy}(a)]K_{iy}(a)$
(63)	$J_{2\nu}[2a\cosh(\frac{1}{2}x)] \quad a>0$	$-(\pi/2)[J_{\nu+iy}(a)Y_{\nu-iy}(a)$ $+J_{\nu-iy}(a)Y_{\nu+iy}(a)]$
(64)	$J_{2\nu}[2a\sinh(\frac{1}{2}x)]$ $\operatorname{Re}\nu>-\frac{1}{2}, \quad a>0$	$I_{\nu-iy}(a)K_{\nu+iy}(a)$ $+I_{\nu+iy}(a)K_{\nu-iy}$
(65)	$Y_0[2a\sinh(\frac{1}{2}x)] \quad a>0$	$-(2/\pi)\cosh(\pi y)[K_{iy}(a)]^2$
(66)	$K_0[2a\sinh(\frac{1}{2}x)] \quad \operatorname{Re} a>0$	$\frac{1}{4}\pi^2\{[J_{iy}(a)]^2+[Y_{iy}(a)]^2\}$
(67)	$J_{\nu-x}(a)J_{\nu+x}(a)$	$\frac{1}{2}J_{2\nu}[2a\cos(\frac{1}{2}y)] \quad y<\pi$ $0 \quad y>\pi$
(68)	$\operatorname{sech}(\frac{1}{2}\pi x)[J_{ix}(a)$ $+J_{-ix}(a)] \quad a>0$	$\sin(a\cosh y)$
(69)	$\operatorname{csch}(\frac{1}{2}\pi x)[J_{ix}(a)-J_{-ix}(a)]$ $a>0$	$-i\cos(a\cosh y)$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy)dx \quad y > 0$
(70)	$e^{\frac{1}{2}\pi x} H_{ix}^{(2)}(a) \qquad a > 0$	$ie^{-ia \cosh y}$
(71)	$e^{-\frac{1}{2}\pi x} H_{ix}^{(1)}(a) \qquad a > 0$	$-ie^{ia \cosh y}$
(72)	$H_{ix}^{(2)}(a) H_{ix}^{(2)}(b) e^{\pi x} \qquad a, b > 0$	$iH_0^{(2)}[(a^2+b^2+2ab\cosh y)^{\frac{1}{2}}]$
(73)	$\operatorname{sech}(\pi x)\{[J_{ix}(a)]^2+[Y_{ix}(a)]^2\} \qquad a > 0$	$-Y_0[2a\cosh(\frac{1}{2}y)]-\mathbf{E}_0[2a\cosh(\frac{1}{2}y)]$
(74)	$I_{\nu+x}(a)\ I_{\nu-x}(a)$	$\frac{1}{2} I_{2\nu}[2a\cosh(\frac{1}{2}y)] \qquad 0 < y < \pi$ $0 \qquad \pi < y < \infty$
(75)	$K_{ix}(a) \qquad \operatorname{Re} a > 0$	$\frac{1}{2}\pi e^{-a\cosh y}$
(76)	$\cosh(\frac{1}{2}\pi x) K_{ix}(a) \qquad a > 0$	$(\frac{1}{2}\pi) \cos(a \sinh y)$
(77)	$K_{ix}(a)\ K_{ix}(b) \qquad a, b > 0$	$(\frac{1}{2}\pi) K_0[(a^2+b^2+2ab\cosh y)^{\frac{1}{2}}]$
	See also the table of Hankel Transforms	

1.14. Other higher transcendental functions

(1)	$e^{\frac{1}{4}x^2} D_{-2}(x)$	$\pi^{\frac{1}{2}} 2^{-\frac{1}{2}} e^{\frac{1}{4}y^2} D_{-2}(y)$
(2)	$e^{-\frac{1}{4}x^2} D_{2n}(x)$	$(-1)^n 2^{-\frac{1}{2}} \pi^{\frac{1}{2}} y^{2n} e^{-\frac{1}{2}y^2}$
(3)	$D_{2\nu-\frac{1}{2}}[(2x)^{\frac{1}{2}}]\{D_{-2\nu-\frac{1}{2}}[(2x)^{\frac{1}{2}}]$ $+ D_{-2\nu-\frac{1}{2}}[-(2x)^{\frac{1}{2}}]\}$	$\dfrac{\pi^{\frac{1}{2}} y^{-2\nu-\frac{1}{2}}[(1+y^2)^{\frac{1}{2}}+1]^{2\nu}}{\csc(\frac{1}{4}\pi-\nu\pi)\ (1+y^2)^{\frac{1}{2}}}$

Higher functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\cos(xy)dx \qquad y>0$
(4)	$e^{-\frac{1}{2}x^2}[D_{2\nu-\frac{1}{2}}(x)+D_{2\nu-\frac{1}{2}}(-x)]$ $\operatorname{Re}\nu>\frac{1}{4}$	$\dfrac{2^{\frac{1}{4}-2\nu}\pi^{\frac{1}{2}}y^{2\nu-\frac{1}{2}}e^{-\frac{1}{4}y^2}}{\csc(\frac{1}{4}\pi+\nu\pi)}$
(5)	${}_1F_2(a;\beta,\frac{1}{2};-\frac{1}{4}x^2)$ $\operatorname{Re}\beta>\operatorname{Re}a>0$	$\dfrac{\pi\Gamma(\beta)}{\Gamma(a)\Gamma(\beta-a)}y^{2a-1}(1-y^2)^{\beta-a-1}$ $\qquad 0<y<1$ $0 \qquad 1<y<\infty$
(6)	$x^{-3/2}W_{\mu+\rho,-1/8+\lambda}(\frac{1}{2}x^2)$ $\times M_{\mu-\rho,-1/8-\lambda}(\frac{1}{2}x^2)$ $\operatorname{Re}\rho<1/8, \quad \operatorname{Re}\lambda<3/8$	$(\frac{1}{2}\pi)^{1/2}y^{-3/2}\Gamma(\frac{3}{4}-2\rho)[\Gamma(\frac{3}{4}-2\lambda)]^{-1}$ $\times W_{\mu+\lambda,-1/8+\rho}(\frac{1}{2}y^2)$ $\times M_{\mu-\lambda,-3/8-\rho}(\frac{1}{2}y^2)$
(7)	$x^{-2\nu-1}e^{\frac{1}{4}x^2}W_{3\nu,\nu}(\frac{1}{2}x^2)$ $\operatorname{Re}\nu<\frac{1}{4}$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}y^{-2\nu-1}e^{\frac{1}{4}y^2}W_{3\nu,\nu}(\frac{1}{2}y^2)$
(8)	$x^{-2\mu-1}e^{-\frac{1}{2}x^2}M_{-\kappa,\mu}(x^2)$ $\operatorname{Re}(\kappa-\mu)<\frac{1}{2}$	$2^{\kappa-\mu}\pi^{\frac{1}{2}}\Gamma(1+2\mu)[\Gamma(\frac{1}{2}-\kappa+\mu)]^{-1}$ $\times y^{\mu-\kappa-1}e^{-2^{-3}y^2}$ $\times W_{-\frac{1}{2}\kappa-\frac{3}{4}\mu,-\frac{1}{2}\kappa+\frac{1}{2}\mu}(\frac{1}{4}y^2)$
(9)	${}_2F_1(a,\beta;\frac{1}{2};-c^2x^2)$ $\operatorname{Re}a>0, \quad \operatorname{Re}\beta>0, \quad c>0$	$2^{-a-\beta+1}\pi c^{-a-\beta}[\Gamma(a)\Gamma(\beta)]^{-1}$ $\times y^{a+\beta-1}K_{a-\beta}(y/c)$
(10)	For other cosine transforms of functions of the form $\quad {}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;x^2)$ see the table of Hankel transforms.	
(11)	$[\theta_4(0\vert ie^{2x})+\theta_2(0\vert ie^{2x})$ $-\theta_3(0\vert ie^{2x})]e^{\frac{1}{2}x}$	$\frac{1}{2}(2^{\frac{1}{2}+iy}-1)(1-2^{\frac{1}{2}-iy})\pi^{-\frac{1}{4}-\frac{1}{2}iy}$ $\times\Gamma(\frac{1}{4}+\frac{1}{2}iy)\ \zeta(\frac{1}{2}+iy)$

Higher functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \cos(xy) dx \qquad y > 0$
(12)	$e^{\frac{1}{2}x}[\theta_3(0\vert ie^{2x})-1]$	$2(1+4y^2)^{-1}[1-2\xi(y)]$
(13)	$(x^2+a^2)^{-\frac{1}{2}} K[b(x^2+a^2)^{-\frac{1}{2}}]$ $\quad a > b > 0$	$\frac{1}{2}\pi I_0\{\frac{1}{2}[a-(a^2-b^2)^{\frac{1}{2}}]y\}$ $\times K_0\{\frac{1}{2}[a+(a^2-b^2)^{\frac{1}{2}}]y\}$

CHAPTER II

FOURIER SINE TRANSFORMS

2.1. General formulas

	$f(x)$		$g(y) = \int_0^\infty f(x)\sin(xy)dx \quad y>0$
(1)	$g(x)$		$\frac{1}{2}\pi f(y)$
(2)	$f(ax)$	$a>0$	$a^{-1}g(a^{-1}x)$
(3)	$f(ax)\cos(bx)$	$a,\ b>0$	$\frac{1}{2a}\left[g\left(\frac{y+b}{a}\right)+g\left(\frac{y-b}{a}\right)\right]$
(4)	$f(ax)\sin(bx)$	$a,\ b>0$	$\frac{1}{2a}\int_0^\infty f(x)\cos\left(\frac{y-b}{a}x\right)dx$ $-\frac{1}{2a}\int_0^\infty f(x)\cos\left(\frac{y+b}{a}x\right)dx$
(5)	$x^{2n}f(x)$		$(-1)^n\frac{d^{2n}g(y)}{dy^{2n}}$
(6)	$x^{2n+1}f(x)$		$(-1)^{n+1}\frac{d^{2n+1}}{dy^{2n+1}}\int_0^\infty f(x)\cos(xy)dx$

2.2. Algebraic functions

(1)	1 0	$0<x<a$ $a<x<\infty$	$y^{-1}[1-\cos(ay)]$

Algebraic functions (cont'd)

	$f(x)$		$g(y) = \int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(2)	x $2-x$ 0	$0<x<1$ $1<x<2$ $x>2$	$y^{-2}[2\sin y - \sin(2y)]$
(3)	x^{-1}		$\frac{1}{2}\pi$
(4)	x^{-1} 0	$0<x<a$ $a<x<\infty$	$\mathrm{Si}(ay)$
(5)	0 x^{-1}	$0<x<a$ $a<x<\infty$	$-\mathrm{si}(ay)$
(6)	$x^{-1/2}$		$(\frac{1}{2}\pi)^{1/2} y^{-1/2}$
(7)	$x^{-1/2}$ 0	$0<x<1$ $1<x<\infty$	$(2\pi)^{1/2} y^{-1/2} S(y)$
(8)	0 $x^{-1/2}$	$0<x<1$ $1<x<\infty$	$(2\pi)^{1/2} y^{-1/2}[\frac{1}{2}-S(y)]$
(9)	$x^{-3/2}$		$(2\pi y)^{1/2}$
(10)	$(x+a)^{-1}$	$\lvert\arg a\rvert<\pi$	$\mathrm{Ci}(ay)\sin(ay)-\mathrm{si}(ay)\cos(ay)$
(11)	$(a-x)^{-1}$	$a>0$	$\sin(ay)\,\mathrm{Ci}(ay)$ $-\cos(ay)\,[\frac{1}{2}\pi+\mathrm{Si}(ay)]$ The integral is a Cauchy Principal Value

Algebraic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(12)	$(x+a)^{-1/2}$ $\qquad \lvert\arg a\rvert<\pi$	$\pi^{1/2}(2y)^{-1/2}[\cos(ay)-\sin(ay)$ $+2C(ay)\sin(ay)-2S(ay)\cos(ay)]$
(13)	$0 \qquad 0<x<a$ $(x-a)^{-1/2} \qquad a<x<\infty$	$\pi^{1/2}(2y)^{-1/2}[\sin(ay)+\cos(ay)]$
(14)	$(x^2+a^2)^{-1}$ $\qquad a>0$	$(2a)^{-1}[e^{-ay}\overline{\mathrm{Ei}}(ay)-e^{ay}\mathrm{Ei}(-ay)]$
(15)	$x(x^2+a^2)^{-1}$ $\qquad \mathrm{Re}\,a>0$	$\frac{1}{2}\pi e^{-ay}$
(16)	$\dfrac{\beta}{\beta^2+(a-x)^2}-\dfrac{\beta}{\beta^2+(a+x)^2}$ $a\pm i\beta$ not real, $\quad \mathrm{Re}\,\beta>0$	$\pi e^{-\beta y}\sin(ay)$
(17)	$\dfrac{(a+x)}{\beta^2+(a+x)^2}-\dfrac{(a-x)}{\beta^2+(a-x)^2}$ $a\pm i\beta$ not real, $\quad \mathrm{Re}\,\beta>0$	$\pi e^{-\beta y}\cos(ay)$
(18)	$(a^2-x^2)^{-1}$ $\qquad a>0$	$a^{-1}[\sin(ay)\mathrm{Ci}(ay)-\cos(ay)\mathrm{Si}(ay)]$ The integral is a Cauchy Principal Value
(19)	$x(a^2-x^2)^{-1}$ $\qquad a>0$	$-\frac{1}{2}\pi\cos(ay)$ The integral is a Cauchy Principal Value
(20)	$x^{-1}(x^2+a^2)^{-1}$ $\qquad \mathrm{Re}\,a>0$	$\frac{1}{2}\pi a^{-2}(1-e^{-ay})$
(21)	$x(a^3\pm a^2x+ax^2\pm x^3)^{-1}$ $a>0$	$\pm\frac{1}{4}a^{-1}[e^{-ay}\overline{\mathrm{Ei}}(ay)-e^{ay}\mathrm{Ei}(-ay)$ $-2\,\mathrm{Ci}(ay)\sin(ay)$ $+2\,\mathrm{Si}(ay)\cos(ay)]+\pi e^{-ay}-\pi\cos(ay)$ The integral is a Cauchy Principal Value when lower signs are taken

Algebraic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(22)	$x^2(a^3 \pm a^2x + ax^2 \pm x^3)^{-1}$ $\qquad a>0$	$\frac{1}{4}[e^{ay}\,\mathrm{Ei}(-ay)-e^{-ay}\,\overline{\mathrm{Ei}}(ay)$ $+2\,\mathrm{Ci}(ay)\sin(ay)$ $-2\,\mathrm{Si}(ay)\cos(ay)]\pm\pi e^{-ay}\pm\pi\cos(ay)$ The integral is a Cauchy Principal Value when lower signs are taken
(23)	$x[x^4+2a^2x^2\cos(2\theta)+a^4]^{-1}$ $\qquad a>0, \quad \lvert\theta\rvert<\frac{1}{2}\pi$	$\frac{1}{2}\pi a^{-2}\csc(2\theta)e^{-ay\cos\theta}\sin(ay\sin\theta)$
(24)	$x^3[x^4+2a^2x^2\cos(2\theta)+a^4]^{-1}$ $\qquad a>0, \quad \lvert\theta\rvert<\frac{1}{2}\pi$	$\frac{1}{2}\pi\csc(2\theta)e^{-ay\cos\theta}\sin[2\theta-ay\sin\theta]$
(25)	$x[(a^2+x^2)(\beta^2+x^2)]^{-1}$ $\qquad \mathrm{Re}\,a>0, \quad \mathrm{Re}\,\beta>0$	$\frac{1}{2}\pi(e^{-\beta y}-e^{-ay})(a^2-\beta^2)^{-1}$
(26)	$(x^2+a^2)^{-1/2}$ $\qquad \mathrm{Re}\,a>0$	$\frac{1}{2}\pi[I_0(ay)-\mathbf{L}_0(ay)]$
(27)	$x(x^2+a^2)^{-3/2}$ $\qquad \mathrm{Re}\,a>0$	$yK_0(ay)$
(28)	$x^{-1/2}(x^2+a^2)^{-1/2}$ $\qquad \mathrm{Re}\,a>0$	$(\frac{1}{2}\pi)^{1/2}y^{1/2}I_{1/4}(\frac{1}{2}ay)K_{1/4}(\frac{1}{2}ay)$
(29)	$x^{-1/2}(a^2-x^2)^{-1/2}$ $\qquad 0<x<a$ 0 $\qquad a<x<0$	$(\frac{1}{2}\pi)^{3/2}y^{1/2}[J_{1/4}(\frac{1}{2}ay)]^2$
(30)	0 $\qquad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}$ $\qquad a<x<\infty$	$-\frac{1}{2}\pi(\frac{1}{2}\pi y)^{1/2}J_{1/4}(\frac{1}{2}ay)Y_{1/4}(\frac{1}{2}ay)$
(31)	$(x^2+a^2)^{-1/2}[(x^2+a^2)^{1/2}-a]^{1/2}$	$\pi^{1/2}(2y)^{-1/2}e^{-ay}$
(32)	$x^{-1/2}(x^2+a^2)^{-1/2}[x+(x^2+a^2)^{1/2}]^{-1/2}$ $\qquad \mathrm{Re}\,a>0$	$2^{1/2}a^{-1}\sinh(\frac{1}{2}ay)K_0(\frac{1}{2}ay)$

Algebraic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \qquad y>0$
(33)	$x^{-1/2}(x^2+a^2)^{-1/2}[x+(x^2+a^2)^{1/2}]^{1/2}$ $\operatorname{Re} a>0$	$2^{-1/2}\pi e^{-\frac{1}{2}ay} I_0(\tfrac{1}{2}ay)$
(34)	$\dfrac{x^{1/2}}{R_1R_2}\left(\dfrac{R_2-R_1}{R_2+R_1}\right)^{1/2}$ $R_1=[a^2+(b-x)^2]^{1/2}$ $R_2=[a^2+(b+x)^2]^{1/2}$ $a>0$	$b^{-1/2}K_0(ay)\sin(by)$
(35)	$x(x^2+a^2)^{-n} \qquad \operatorname{Re} a>0$	$\dfrac{\pi y e^{-ay}}{2^{2n-2}(n-1)!\,a^{2n-3}}$ $\times\sum_{m=0}^{n-2}\dfrac{(2n-m-4)!(2ay)^m}{m!(n-m-2)!}$ $n=2,3,\ldots$
(36)	$x^{-1}(a^2+x^2)^{-n}$	$\tfrac{1}{2}\pi a^{-2n}[1-2^{1-n}e^{-ay}$ $\times F_{n-1}(ay)/(n-1)!]$ $F_0(z)=1,$ $F_n(z)=(z+2n)F_{n-1}(z)-zF'_{n-1}(z)$
(37)	$\dfrac{x^{2m+1}}{(a^2+x^2)^{n+1/2}}$ $-2\le 2m\le 2n, \qquad \operatorname{Re} a>0$	$\dfrac{(-1)^{m+1}\pi^{1/2}}{2^n a^n\Gamma(n+\frac{1}{2})}\dfrac{d^{2m+1}}{dy^{2m+1}}[y^nK_n(ay)]$
(38)	$\dfrac{x^{m-1}}{x^{2n}+a^{2n}}$ $0\le m\le 2n, \qquad \lvert\arg a\rvert<\tfrac{1}{2}\pi/n$	$0 \qquad m$ odd $(-\tfrac{1}{2}\pi a^{m-2n}/n)\sum_{k=1}^{n}e^{-ay\sin[(k-\frac{1}{2})\pi/n]}$ $\times\{\cos[(k-\tfrac{1}{2})m\pi/n]$ $+ay\cos[(k-\tfrac{1}{2})\pi/n]\} \qquad m$ even

Algebraic functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \sin(xy)\,dx \qquad y > 0$
(39)	$\dfrac{x^{2m+1}}{(z+x^2)^{n+1}}$ $\lvert \arg z \rvert < \pi, \quad 0 \le 2m \le 2n$	$\dfrac{(-1)^{n+m}}{n!} \dfrac{\pi}{2} \dfrac{d^n}{dz^n} (z^m e^{-z^{1/2} y})$

2.3. Powers with arbitrary index

(1)	$x^{-\nu} \qquad 0 < \mathrm{Re}\,\nu < 2$	$y^{\nu-1}\Gamma(1-\nu)\cos(\frac{1}{2}\nu\pi)$
(2)	$x^{\nu-1} \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$ $\mathrm{Re}\,\nu > -1$	$\frac{1}{2}\nu^{-1}[{}_1F_1(\nu; \nu+1; iy)$ $- {}_1F_1(\nu; \nu+1; -iy)]$
(3)	$(1-x)^{\nu} \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$ $\mathrm{Re}\,\nu > -1$	$y^{-1} - \Gamma(\nu+1) y^{-\nu-1} C_\nu(y)$
(4)	$x^{\nu}(1-x)^{\nu} \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$ $\mathrm{Re}\,\nu > -1$	$\pi^{1/2}\Gamma(\nu+1)(2y)^{-\nu-1/2} \sin y \, J_{\nu+1/2}(y)$
(5)	$x^{\nu-1}(1-x)^{\mu-1} \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$ $\mathrm{Re}\,\nu > 0, \quad \mathrm{Re}\,\mu > 0$	$\frac{1}{2}\mathrm{B}(\nu, \mu)[{}_1F_1(\nu; \nu+\mu; iy)$ $- {}_1F_1(\nu; \nu+\mu; -iy)]$
(6)	$(a^2+x^2)^{\nu-1/2}$ $\mathrm{Re}\,\nu < \frac{1}{2}, \quad \mathrm{Re}\,a > 0$	$2^{\nu-1}\pi^{1/2}\Gamma(\frac{1}{2}+\nu) y^{-\nu} a^{\nu} [I_{-\nu}(ay)$ $- \mathbf{L}_\nu(ay)]$ $\nu \ne -1/2, -3/2, -5/2, \ldots$

Arbitrary powers (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(7)	$(a^2-x^2)^{\nu-1/2} \quad 0<x<a$ $0 \quad a<x<\infty$ $\mathrm{Re}\,\nu>-1/2$	$2^{\nu-1}\pi^{1/2}a^{\nu}\Gamma(\nu+1/2)y^{-\nu}\mathbf{H}_{\nu}(ay)$ $=a^{\nu}y^{-\nu}s_{\nu,\nu}(ay)$
(8)	$0 \quad 0<x<a$ $(x^2-a^2)^{\nu-1/2} \quad a<x<\infty$ $\lvert\mathrm{Re}\,\nu\rvert<1/2$	$2^{\nu-1}\pi^{1/2}a^{\nu}y^{-\nu}\Gamma(\nu+1/2)J_{-\nu}(ay)$
(9)	$x(a^2-x^2)^{\nu-1/2} \quad 0<x<a$ $0 \quad a<x<\infty$ $\mathrm{Re}\,\nu>-1/2$	$2^{\nu-1}\pi^{1/2}a^{\nu+1}\Gamma(\nu+1/2)y^{-\nu}J_{\nu+1}(ay)$
(10)	$0 \quad 0<x<a$ $x(x^2-a^2)^{\nu-1/2} \quad a<x<\infty$ $-1/2<\mathrm{Re}\,\nu<0$	$2^{\nu-1}\pi^{1/2}a^{\nu+1}\Gamma(\nu+1/2)y^{-\nu}Y_{-\nu-1}(ay)$
(11)	$x(x^2+a^2)^{\nu-3/2}$ $\mathrm{Re}\,\nu>-1, \quad \mathrm{Re}\,a>0$	$1/2\,\pi^{1/2}(2a)^{\nu}[\Gamma(3/2-\nu)]^{-1}$ $\times y^{1-\nu}K_{\nu}(ay)$
(12)	$(x^2+2ax)^{-\nu-1/2}$ $-1/2<\mathrm{Re}\,\nu<3/2, \quad \lvert\arg a\rvert<\pi$	$a^{-\nu}\pi^{1/2}2^{-\nu-1}\Gamma(1/2-\nu)y^{\nu}$ $\times[J_{\nu}(ay)\cos(ay)+Y_{\nu}(ay)\sin(ay)]$ $\nu\neq 1/2$
(13)	$(ax-x^2)^{\nu-1/2} \quad 0<x<a$ $0 \quad a<x<\infty$ $\mathrm{Re}\,\nu>-1/2$	$\pi^{1/2}\Gamma(\nu+1/2)a^{\nu}y^{-\nu}\sin(1/2\,ay)J_{\nu}(1/2\,ay)$

Arbitrary powers (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)\,dx \quad y>0$
(14)	$0 \qquad 0<x<a$ $(x^2-ax)^{\nu-\frac{1}{2}} \qquad a<x<\infty$ $\lvert\mathrm{Re}\,\nu\rvert<\frac{1}{2}$	$\frac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})a^{\nu}y^{-\nu}$ $\times[J_{-\nu}(\frac{1}{2}ay)\cos(\frac{1}{2}ay)$ $-Y_{-\nu}(\frac{1}{2}ay)\sin(\frac{1}{2}ay)]$
(15)	$(a+ix)^{-\nu}-(a-ix)^{-\nu}$ $\mathrm{Re}\,\nu>0, \quad \mathrm{Re}\,a>0$	$\pi i\,[\Gamma(\nu)]^{-1}e^{-ay}y^{\nu-1}$
(16)	$x[(a+ix)^{-\nu}+(a-ix)^{-\nu}]$ $\mathrm{Re}\,\nu>0, \quad \mathrm{Re}\,a>0$	$-\pi[\Gamma(\nu)]^{-1}y^{\nu-2}(1-ay)e^{-ay}$
(17)	$x^{2n}[(a-ix)^{-\nu}-(a+ix)^{-\nu}]$ $0\le 2n<\mathrm{Re}\,\nu, \quad \mathrm{Re}\,a>0$	$(-1)^{n+1}\pi i\,[\Gamma(\nu)]^{-1}(2n)!$ $\times e^{-ay}y^{\nu-2n-1}L_{2n}^{\nu-1-2n}(ay)$
(18)	$x^{2n+1}[(a+ix)^{-\nu}+(a-ix)^{-\nu}]$ $-1\le 2n+1<\mathrm{Re}\,\nu$ $\mathrm{Re}\,a>0$	$(-1)^{n+1}\pi[\Gamma(\nu)]^{-1}e^{-ay}y^{\nu-2n-2}$ $\times(2n+1)!\,L_{2n+1}^{\nu-2n-2}(ay)$
(19)	$(x^2+a^2)^{-\frac{1}{2}}[x+(x^2+a^2)^{\frac{1}{2}}]^{-\nu}$ $\mathrm{Re}\,\nu>-1, \quad a>0$	$\pi a^{-\nu}\csc(\nu\pi)[\sin(\frac{1}{2}\nu\pi)I_{\nu}(ay)$ $+\frac{1}{2}i\,\mathbf{J}_{\nu}(iay)-\frac{1}{2}i\,\mathbf{J}_{\nu}(-iay)]$
(20)	$(x^2+a^2)^{-\frac{1}{2}}\{[(x^2+a^2)^{\frac{1}{2}}+x]^{\nu}$ $-[(x^2+a^2)^{\frac{1}{2}}-x]^{\nu}\}$ $\lvert\mathrm{Re}\,\nu\rvert<1, \quad \mathrm{Re}\,a>0$	$2a^{\nu}K_{\nu}(ay)\sin(\frac{1}{2}\nu\pi)$
(21)	$(a^2-x^2)^{-\frac{1}{2}}\{[x+i(a^2-x^2)^{\frac{1}{2}}]^{\nu}$ $+[x-i(a^2-x^2)^{\frac{1}{2}}]^{\nu}\}$ $0<x<a$ $0 \qquad a<x<\infty$	$\frac{1}{2}\pi a^{\nu}\csc(\frac{1}{2}\nu\pi)[\mathbf{J}_{\nu}(ay)-\mathbf{J}_{-\nu}(ay)]$
(22)	$0 \qquad 0<x<a$ $(x^2-a^2)^{-\frac{1}{2}}\{[x+(x^2-a^2)^{\frac{1}{2}}]^{\nu}$ $+[x-(x^2-a^2)^{\frac{1}{2}}]^{\nu}\} \qquad a<x<\infty$ $\lvert\mathrm{Re}\,\nu\rvert<1$	$\pi a^{\nu}[J_{\nu}(ay)\cos(\frac{1}{2}\nu\pi)$ $-Y_{\nu}(ay)\sin(\frac{1}{2}\nu\pi)]$

Arbitrary powers (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(23)	$x^{-1/2}(x^2+a^2)^{-1/2}[(x^2+a^2)^{1/2}+x]^{\nu}$ $\operatorname{Re}\nu<3/2, \quad \operatorname{Re}a>0$	$a^{\nu}(\tfrac12\pi y)^{1/2}\,I_{1/4-1/2\nu}(\tfrac12 ay)$ $\times K_{1/4+1/2\nu}(\tfrac12 ay)$
(24)	$x^{-1/2}(a^2-x^2)^{1/2}\{[(a+x)^{1/2}$ $+i(a-x)^{1/2}]^{4\nu}+[(a+x)^{1/2}$ $-i(a-x)^{1/2}]^{4\nu}\} \qquad 0<x<a$ $0 \qquad a<x<\infty$	$2^{-1/2}(2a)^{2\nu}\pi^{3/2}y^{1/2}J_{\nu+1/4}(\tfrac12 ay)$ $\times J_{-\nu+1/4}(\tfrac12 ay)$
(25)	$0 \qquad 0<x<a$ $x^{-1/2}(x^2-a^2)^{-1/2}\{[x+(x^2-a^2)^{1/2}]^{\nu}$ $+[x-(x^2-a^2)^{1/2}]^{\nu}\} \qquad a<x<\infty$ $\operatorname{Re}\nu<3/2$	$-\tfrac12\pi(\tfrac12\pi y)^{1/2}a^{\nu}[J_{1/4+1/2\nu}(\tfrac12 ay)$ $\times Y_{1/4-1/2\nu}(\tfrac12 ay)$ $+J_{1/4-1/2\nu}(\tfrac12 ay)\,Y_{1/4+1/2\nu}(\tfrac12 ay)]$
(26)	$(x^2+2ax)^{-1/2}\{[x+a+(x^2+2ax)^{1/2}]^{\nu}$ $+[x+a-(x^2+2ax)^{1/2}]^{\nu}\}$ $\lvert\operatorname{Re}\nu\rvert<1, \quad \lvert\arg a\rvert<\pi$	$\pi a^{\nu}[Y_{\nu}(ay)\sin(ay-\tfrac12\nu\pi)$ $+J_{\nu}(ay)\cos(ay-\tfrac12\nu\pi)]$
(27)	$x^{-\nu-1/2}(x^2+a^2)^{-1/2}$ $\times[a+(x^2+a^2)^{1/2}]^{\nu}$ $\operatorname{Re}\nu<3/2, \quad \operatorname{Re}a>0$	$2^{1/2}a^{-1}y^{-1/2}\Gamma(\tfrac34-\tfrac12\nu)\,W_{1/2\nu,1/4}(ay)$ $\times M_{-1/2\nu,1/4}(ay)$
(28)	$x^{2\nu}(x^2+a^2)^{-\mu-1}$ $-1<\operatorname{Re}\nu<\operatorname{Re}\mu+1$ $\operatorname{Re}a>0$	$\tfrac12 a^{2\nu-2\mu}\dfrac{\Gamma(1+\nu)\Gamma(\mu-\nu)}{\Gamma(\mu+1)}\,y$ $\times{}_1F_2(\nu+1;\nu+1-\mu;3/2;\tfrac14a^2y^2)$ $+4^{\nu-\mu-1}\pi^{1/2}\dfrac{\Gamma(\nu-\mu)}{\Gamma(\mu-\nu+3/2)}\,y^{2\mu-2\nu+1}$ $\times{}_1F_2(\mu+1;\mu-\nu+3/2;\mu-\nu+1;\tfrac14a^2y^2)$
(29)	$x^{\mu}(1-x^2)^{\nu} \qquad 0<x<a$ $0 \qquad a<x<\infty$	see under Hankel Transforms

2.4. Exponential functions

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(1)	e^{-ax} $\qquad \operatorname{Re} a>0$	$y(a^2+y^2)^{-1}$
(2)	$x^{-1}e^{-ax}$ $\qquad \operatorname{Re} a>0$	$\tan^{-1}(a^{-1}y)$
(3)	$x^n e^{-ax}$ $\qquad \operatorname{Re} a>0$	$n!\left(\dfrac{a}{a^2+y^2}\right)^{n+1} \times \sum_{m=0}^{[\frac{1}{2}n]}(-1)^m\binom{n+1}{2m+1}\left(\dfrac{y}{a}\right)^{2m+1}$
(4)	$x^{-\frac{1}{2}}e^{-ax}$ $\qquad \operatorname{Re} a>0$	$\dfrac{\pi^{\frac{1}{2}}2^{-\frac{1}{2}}[(a^2+y^2)^{\frac{1}{2}}-a]^{\frac{1}{2}}}{(a^2+y^2)^{\frac{1}{2}}}$
(5)	$x^{-3/2}e^{-ax}$ $\qquad \operatorname{Re} a>0$	$(2\pi)^{\frac{1}{2}}[(a^2+y^2)^{\frac{1}{2}}-a]^{\frac{1}{2}}$
(6)	$x^{n-\frac{1}{2}}e^{-ax}$ $\qquad n=-1,0,1,2,\ldots$ $\qquad \operatorname{Re} a>0$	$(-1)^n\pi^{\frac{1}{2}}2^{-\frac{1}{2}}\dfrac{d^n}{da^n} \times\{(a^2+y^2)^{-\frac{1}{2}}[(a^2+y^2)^{\frac{1}{2}}-a]^{\frac{1}{2}}\}$
(7)	$x^{\nu-1}e^{-ax}$ $\qquad \operatorname{Re}\nu>-1, \quad \operatorname{Re} a>0$	$\Gamma(\nu)(a^2+y^2)^{-\frac{1}{2}\nu}\sin[\nu\tan^{-1}(y/a)]$
(8)	$x^{-2}(e^{-ax}-e^{-\beta x})$ $\qquad \operatorname{Re} a>0, \quad \operatorname{Re}\beta>0$	$\frac{1}{2}y\log[(y^2+\beta^2)/(y^2+a^2)] +\beta\tan^{-1}(\beta^{-1}y)-a\tan^{-1}(a^{-1}y)$
(9)	$x^{-1}(1+\beta x)^{-1}e^{-ax}$ $\qquad \operatorname{Re} a>0, \quad \lvert\arg\beta\rvert<\pi$	$\tan^{-1}(y/a)-\frac{1}{2}i\,\{\exp[(a-iy)/\beta] \times\operatorname{Ei}[-(a-iy)/\beta] -\exp[(a+iy)/\beta]\operatorname{Ei}[-(a+iy)/\beta]\}$
(10)	$(e^{ax}+1)^{-1}$ $\qquad \operatorname{Re} a>0$	$\frac{1}{2}y^{-1}-\frac{1}{2}\pi a^{-1}\operatorname{csch}(\pi a^{-1}y)$
(11)	$(e^{ax}-1)^{-1}$ $\qquad \operatorname{Re} a>0$	$\frac{1}{2}\pi a^{-1}\operatorname{ctnh}(\pi a^{-1}y)-\frac{1}{2}y^{-1}$

Exponential functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(12)	$x^{-1}(e^{ax}+1)^{-1}$	see Fock, V., 1926: *Arch. Elektrotechnik* 16, 331-340
(13)	$e^{-\frac{1}{2}x}(1-e^{-x})^{-1}$	$-\frac{1}{2}\tanh(\pi y)$
(14)	$e^{-nx}(1-e^{-x})^{-1}$	$\frac{\pi}{2}-\frac{1}{2y}+\frac{\pi}{e^{2\pi y}-1}-\sum_{k=1}^{n-1}\frac{y}{y^2+k^2}$
(15)	$e^{-ax}(e^{-x}-1)^{-1} \qquad \operatorname{Re} a>-1$	$\frac{1}{2}i\,\psi(a+iy)-\frac{1}{2}i\,\psi(a-iy)$
(16)	$(e^{ax}-e^{\beta x})^{-1}$ $\operatorname{Re} a>0, \quad \operatorname{Re}\beta>0$	$-\frac{1}{2}(a-\beta)^{-1}i\left[\psi\left(\frac{a+iy}{a-\beta}\right)-\psi\left(\frac{a-iy}{a-\beta}\right)\right]$
(17)	$e^{-ax}(1-e^{-\beta x})^{\nu-1}$ $\operatorname{Re}\nu>-1$ $\operatorname{Re} a>\operatorname{Re}(\nu-1)\beta$	$-\frac{1}{2}i\beta^{-1}\left[\mathrm{B}\left(\nu,\frac{a-iy}{\beta}\right)-\mathrm{B}\left(\nu,\frac{a+iy}{\beta}\right)\right]$
(18)	$e^{-ax^2} \qquad \operatorname{Re} a>0$	$\frac{1}{2}a^{-1}ye^{-\frac{1}{4}y^2/a}\,{}_1F_1(1/2;3/2;\frac{1}{4}a^{-1}y^2)$ $=\frac{1}{2}a^{-1}y\,{}_1F_1(1,3/2;-\frac{1}{4}a^{-1}y^2)$ $=-\frac{1}{2}i\pi^{\frac{1}{2}}a^{-\frac{1}{2}}e^{-\frac{1}{4}y^2/a}\operatorname{Erf}(\frac{1}{2}ia^{-\frac{1}{2}}y)$
(19)	$xe^{-ax^2} \qquad \lvert\arg a\rvert<\frac{1}{2}\pi$	$\frac{1}{4}\pi^{1/2}a^{-3/2}ye^{-\frac{1}{4}a^{-1}y^2}$
(20)	$xe^{iax^2} \qquad -\pi<\arg a<0$	$\frac{1}{4}\pi^{1/2}ya^{-3/2}e^{i(\frac{3}{4}\pi-\frac{1}{4}a^{-1}y^2)}$
(21)	$x^{-1}e^{-ax^2} \qquad \lvert\arg a\rvert<\frac{1}{2}\pi$	$\frac{1}{2}\pi\operatorname{Erf}(\frac{1}{2}a^{-\frac{1}{2}}y)$
(22)	$x^{-\frac{1}{2}}e^{-ax^2} \qquad \lvert\arg a\rvert<\frac{1}{2}\pi$	$2^{-3/2}\pi a^{-1/2}y^{1/2}e^{-2^{-3}a^{-1}y^2}$ $\times I_{\frac{1}{4}}(2^{-3}a^{-1}y^2)$

Exponential functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(23)	$x^{2n+1}e^{-\alpha x^2}$ $\qquad \lvert\arg\alpha\rvert < \tfrac{1}{2}\pi$	$\dfrac{(-1)^n \pi^{1/2} e^{-2^{-3}\alpha^{-1}y^2}}{2^{n+3/2}\alpha^{n+1}} D_{2n+1}(2^{-1/2}\alpha^{-1/2}y)$ $= \dfrac{(-1)^n \pi^{1/2} e^{-1/4\,\alpha^{-1}y^2}}{2^{n+3/2}\alpha^{n+1}} \mathrm{He}_{2n+1}(2^{-1/2}\alpha^{-1/2}y)$
(24)	$x^{2\nu-2}e^{-\alpha x^2}$ $\qquad \mathrm{Re}\,\nu>0, \quad \lvert\arg\alpha\rvert<\tfrac{1}{2}\pi$	$\tfrac{1}{2}\alpha^{-\nu}\Gamma(\nu)y\,{}_1F_1(\nu;3/2;-\tfrac{1}{4}\alpha^{-1}y^2)$ $=\tfrac{1}{2}\alpha^{-\nu}\Gamma(\nu)ye^{-\frac{1}{4}\alpha^{-1}y^2}$ $\times\,{}_1F_1(3/2-\nu;3/2;\tfrac{1}{4}\alpha^{-1}y^2)$
(25)	$xe^{\alpha(1-x^2)} \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$(\tfrac{1}{2}\pi)^{1/2}y^{-1/2}\sum_{p=0}^{\infty}(2\alpha)^p y^{-p}$ $\times J_{p+3/2}(y)$
(26)	$x(x^2+\beta^2)^{-1}e^{-\alpha^2x^2}$ $\qquad \mathrm{Re}\,\alpha>0, \quad \mathrm{Re}\,\beta>0$	$\tfrac{1}{4}\pi e^{\alpha^2\beta^2}[e^{-\beta y}\mathrm{Erfc}(\alpha\beta-\tfrac{1}{2}\alpha^{-1}y)$ $-e^{\beta y}\mathrm{Erfc}(\alpha\beta+\tfrac{1}{2}\alpha^{-1}y)]$
(27)	$e^{-\alpha x^2-\beta x}$ $\qquad \lvert\arg\alpha\rvert<\tfrac{1}{2}\pi$	$-\tfrac{1}{4}i\pi^{1/2}\alpha^{-1/2}\{e^{\frac{1}{4}\alpha^{-1}(\beta-iy)^2}$ $\times\mathrm{Erfc}[\tfrac{1}{2}\alpha^{-1/2}(\beta-iy)]$ $-e^{\frac{1}{4}\alpha^{-1}(\beta+iy)^2}\mathrm{Erfc}[\tfrac{1}{2}\alpha^{-1/2}(\beta+iy)]\}$
(28)	$xe^{-\alpha x^2-\beta x}$ $\qquad \lvert\arg\alpha\rvert<\tfrac{1}{2}\pi$	$2^{-3}i\pi^{1/2}\alpha^{-3/2}(\beta-iy)e^{-\frac{1}{4}\alpha^{-1}(\beta-iy)^2}$ $\times\mathrm{Erfc}[\tfrac{1}{2}\alpha^{-1/2}(\beta-iy)]-2^{-3}i\pi^{1/2}$ $\times\alpha^{-3/2}(\beta+iy)e^{-\frac{1}{4}\alpha^{-1}(\beta+iy)^2}$ $\times\mathrm{Erfc}[\tfrac{1}{2}\alpha^{-1/2}(\beta+iy)]$
(29)	$x^{-1/2}e^{-\alpha/x}$ $\qquad \lvert\arg\alpha\rvert<\tfrac{1}{2}\pi$	$\pi^{1/2}(2y)^{-1/2}e^{-(2\alpha y)^{1/2}}$ $\times[\cos(2\alpha y)^{1/2}+\sin(2\alpha y)^{1/2}]$

Exponential functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(30)	$x^{-3/2}e^{-\alpha/x} \qquad \lvert\arg\alpha\rvert<\tfrac{1}{2}\pi$	$\pi^{1/2}\alpha^{-1/2}e^{-(2\alpha y)^{1/2}}\sin(2\alpha y)^{1/2}$
(31)	$x^{-\nu-1}e^{-\frac{1}{4}\alpha^2/x}$ $\operatorname{Re}\nu>-1, \qquad \lvert\arg\alpha\rvert<\tfrac{1}{4}\pi$	$i2^{\nu}\alpha^{-\nu}y^{\frac{1}{2}\nu}[e^{\frac{1}{4}\pi i\nu}K_\nu(\alpha e^{\frac{1}{4}\pi i}y^{1/2})$ $-e^{-\frac{1}{4}\pi i\nu}K_\nu(\alpha e^{-\frac{1}{4}\pi i}y^{1/2})]$
(32)	$x^{-1/2}e^{-(\alpha x+\beta x^{-1})}$ $\lvert\arg\alpha\rvert<\tfrac{1}{2}\pi, \qquad \lvert\arg\beta\rvert<\tfrac{1}{2}\pi$	$\pi^{1/2}(y^2+\alpha^2)^{-1/2}e^{-2\beta^{1/2}u}$ $\times[u\sin(2\beta^{1/2}v)+v\cos(2\beta^{1/2}v)]$ $u=2^{-1/2}[\alpha+(y^2+\alpha^2)^{1/2}]^{1/2}$ $v=2^{-1/2}[(y^2+\alpha^2)^{1/2}-\alpha]^{1/2}$
(33)	$x^{-3/2}e^{-\alpha x-\beta x^{-1}}$ $\lvert\arg\alpha\rvert<\tfrac{1}{2}\pi, \qquad \lvert\arg\beta\rvert<\tfrac{1}{2}\pi$	$\pi^{1/2}\beta^{-1/2}e^{-2\alpha v}\sin(2\alpha u)$ $u=2^{-1/2}\alpha^{-1}\beta^{1/2}[(\alpha^2+y^2)^{1/2}-\alpha]^{1/2}$ $v=2^{-1/2}\alpha^{-1}\beta^{1/2}[(\alpha^2+y^2)^{1/2}+\alpha]^{1/2}$
(34)	$x^{-3/4}e^{-\alpha x^{1/2}} \qquad \lvert\arg\alpha\rvert<\tfrac{1}{2}\pi$	$-\tfrac{1}{2}\pi\alpha^{1/2}y^{-1/2}[J_{1/4}(2^{-3}\alpha^2y^{-1})$ $\times\cos(2^{-3}\pi+2^{-3}\alpha^2y^{-1})$ $+Y_{1/4}(2^{-3}\alpha^2y^{-1})\sin(2^{-3}\pi+2^{-3}\alpha^2y^{-1})]$
(35)	$xe^{-\beta(x^2+\alpha^2)^{1/2}}$ $\operatorname{Re}\beta>0, \qquad \operatorname{Re}\alpha>0$	$\beta\alpha^2y(y^2+\beta^2)^{-1}K_2[\alpha(y^2+\beta^2)^{1/2}]$
(36)	$x(\alpha^2+x^2)^{-1/2}e^{-\beta(\alpha^2+x^2)^{1/2}}$ $\operatorname{Re}\alpha>0, \qquad \operatorname{Re}\beta>0$	$\alpha y(y^2+\beta^2)^{-1/2}K_1[\alpha(y^2+\beta^2)^{1/2}]$
(37)	$x^{-1/2}(\beta^2+x^2)^{-1/2}e^{-\alpha(\beta^2+x^2)^{1/2}}$ $\operatorname{Re}\alpha>0, \qquad \operatorname{Re}\beta>0$	$(\tfrac{1}{2}\pi y)^{1/2}I_{1/4}\{\tfrac{1}{2}\beta[(\alpha^2+y^2)^{1/2}-\alpha]\}$ $\times K_{1/4}\{\tfrac{1}{2}\beta[(\alpha^2+y^2)^{1/2}+\alpha]\}$
(38)	$[(\beta^2+x^2)^{1/2}-\beta]^{1/2}(\beta^2+x^2)^{-1/2}$ $\times e^{-\alpha(\beta^2+x^2)^{1/2}}$	$(\tfrac{1}{2}\pi)^{1/2}y[\alpha+(\alpha^2+y^2)^{1/2}]^{-1/2}$ $\times(\alpha^2+y^2)^{-1/2}e^{-\beta(\alpha^2+y^2)^{1/2}}$

Exponential functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(39)	$x^{-1/2}e^{-ax^{-1/2}}$ $\quad \operatorname{Re}a>0, \quad \operatorname{Re}\beta>0$	$\pi^{1/2}(2y)^{-1/2}[\cos(2ay)^{1/2}+\sin(2ay)^{1/2}]$
(40)	$(e^{2\pi x^{1/2}}-1)^{-1}$	see Ramanujan, Srinivasa, 1915: *Mess. Math.* 44, 75-85

2.5. Logarithmic functions

(1)	$\log x \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$-y^{-1}[C+\log y-\operatorname{Ci}(y)]$
(2)	$x^{-1}\log x$	$-\tfrac{1}{2}\pi(C+\log y)$
(3)	$x^{-1/2}\log x$	$-(\tfrac{1}{2}\pi y^{-1})^{1/2}[\log(4y)+C-\tfrac{1}{2}\pi]$
(4)	$x^{\nu-1}\log x \qquad \lvert\operatorname{Re}\nu\rvert<1$	$\dfrac{\pi y^{-\nu}}{2\Gamma(1-\nu)\cos(\tfrac{1}{2}\nu\pi)}$ $\times[\psi(\nu)+\tfrac{1}{2}\pi\operatorname{ctn}(\tfrac{1}{2}\nu\pi)-\log y]$
(5)	$x(x^2+b^2)^{-1}\log(ax)$ $\qquad a,\ b>0$	$\tfrac{1}{2}\pi e^{-by}\log(ab)$ $-\tfrac{1}{4}\pi[e^{by}\operatorname{Ei}(-by)+e^{-by}\overline{\operatorname{Ei}}(by)]$
(6)	$x(x^2-a^2)^{-1}\log x \qquad a>0$	$-\tfrac{1}{2}\pi\{\cos(ay)[\operatorname{Ci}(ay)-\log a]$ $+\sin(ay)[\operatorname{Si}(ay)-\tfrac{1}{2}\pi]\}$ The integral is a Cauchy Principal Value
(7)	$e^{-ax}\log x \qquad \operatorname{Re}a>0$	$(y^2+a^2)^{-1}[a\tan^{-1}(C/a)-\gamma y$ $-\tfrac{1}{2}y\log(y^2+a^2)]$

Logarithmic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(8)	$e^{-ax}x^{\nu-1}\log x$ $\operatorname{Re}\nu>-1, \quad \operatorname{Re}a>0$	$\Gamma(\nu)(a^2+y^2)^{-\frac{1}{2}\nu}\sin[\nu\tan^{-1}(y/a)]$ $\times[\psi(\nu)-\frac{1}{2}\log(a^2+y^2)+\tan^{-1}(y/a)$ $\times\operatorname{ctn}[\nu\tan^{-1}(y/a)]$
(9)	$x^{-1}(\log x)^2$	$\frac{1}{2}\pi C^2+\pi^3/24+\pi C\log y$ $+\frac{1}{2}\pi(\log y)^2$
(10)	$x^{\nu-1}(\log x)^2$ $0<\operatorname{Re}\nu<1$	$\Gamma(\nu)y^{-\nu}\sin(\frac{1}{2}\nu\pi)[\psi'(\nu)+\psi^2(\nu)$ $+\pi\psi(\nu)\operatorname{ctn}(\frac{1}{2}\nu\pi)-2\psi(\nu)\log y$ $-\pi\log y\operatorname{ctn}(\frac{1}{2}\nu\pi)+(\log y)^2-\pi^2]$
(11)	$\log\left\lvert\dfrac{x+a}{x-a}\right\rvert \qquad a>0$	$\dfrac{\pi}{y}\sin(ay)$
(12)	$\log\left(\dfrac{x^2+a^2+x}{x^2+a^2-x}\right)$	$2\pi y^{-1}\exp[-(a^2-\frac{1}{4})^{\frac{1}{2}}y]\sin(\frac{1}{2}y)$
(13)	$\log\left[\dfrac{(x+\beta)^2+a^2}{(x-\beta)^2+a^2}\right]$ $\operatorname{Re}a>0, \quad \lvert\operatorname{Im}\beta\rvert\le\operatorname{Re}a$	$\dfrac{2\pi}{y}e^{-ay}\sin(\beta y)$
(14)	$x^{-1}\log(1+a^2x^2) \qquad a>0$	$-\pi\operatorname{Ei}(-a^{-1}y)$
(15)	$x^{-1}\log\left(\dfrac{a^2x^2+c^2}{b^2x^2+c^2}\right)$ $a,\ b,\ c>0$	$\pi\left[\operatorname{Ei}\left(-\dfrac{cy}{b}\right)-\operatorname{Ei}\left(-\dfrac{cy}{a}\right)\right]$
(16)	$(a^2+x^2)^{-\frac{1}{2}}\log[x+(a^2+x^2)^{\frac{1}{2}}]$ $\operatorname{Re}a>0$	$\frac{1}{2}\pi K_0(ay)+\frac{1}{2}\pi(\log a)[I_0(ay)$ $-\mathbf{L}_0(ay)]$

2.6. Trigonometric functions of argument kx

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)\,dx \qquad y > 0$
(1)	$x^{-1}\sin(ax) \qquad a > 0$	$\tfrac{1}{2}\log\lvert(y+a)(y-a)^{-1}\rvert$
(2)	$x^{-2}\sin(ax) \qquad a > 0$	$\tfrac{1}{2}\pi y \qquad 0 < y < a$ $\tfrac{1}{2}\pi a \qquad a < y < \infty$
(3)	$x^{\nu-1}\sin(ax)$ $-2 < \operatorname{Re}\nu < 1, \quad \nu \neq 0$ $a > 0$	$\pi\dfrac{\lvert y-a\rvert^{-\nu} - \lvert y+a\rvert^{-\nu}}{4\Gamma(1-\nu)\sin(\tfrac{1}{2}\nu\pi)}$
(4)	$(1-x^2)^{-1}\sin(\pi x)$	$\sin y \qquad 0 \le y \le \pi$ $0 \qquad \pi \le y$
(5)	$(x^2+a^2)^{-1}\sin(bx)$ $\operatorname{Re} a > 0, \quad b > 0$	$\tfrac{1}{2}\pi a^{-1} e^{-ab}\sinh(ay) \qquad 0 < y < b$ $\tfrac{1}{2}\pi a^{-1} e^{-ay}\sinh(ab) \qquad b < y < \infty$
(6)	$x^{-1}e^{-ax}\sin(\beta x)$ $\operatorname{Re} a > \lvert\operatorname{Im}\beta\rvert$	$\tfrac{1}{4}\log\{[(y+\beta)^2+a^2]/[(y-\beta)^2+a^2]\}$
(7)	$e^{-ax^2}\sin(\beta x)$ $\operatorname{Re} a > 0$	$\tfrac{1}{2}\pi^{1/2}a^{-1/2}e^{-\frac{1}{4}a^{-1}(y^2+\beta^2)}$ $\times\sinh(\tfrac{1}{2}\beta a^{-1}y)$
(8)	$x^{-1}\sin^2(ax) \qquad a > 0$	$\pi/4 \qquad 0 < y < 2a$ $\pi/8 \qquad y = 2a$ $0 \qquad 2a < y < \infty$
(9)	$x^{-2}\sin^2(ax) \qquad a > 0$	$\tfrac{1}{4}(y+2a)\log\lvert y+2a\rvert$ $+\tfrac{1}{4}(y-2a)\log\lvert y-2a\rvert - \tfrac{1}{2}y\log y$
(10)	$x^{-1}\sin(ax)\sin(bx)$ $a \ge b > 0$	$0 \qquad 0 < y < a-b$ $\tfrac{1}{4}\pi \qquad a-b < y < a+b$ $0 \qquad a+b < y < \infty$

Trigonometric functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \quad y>0$
(11)	$x^{-2}\sin(ax)\sin(bx)$ $a\geq b>0$	$-\frac{1}{4}(y+a-b)\log(y+a-b)$ $+\frac{1}{4}(y+a+b)\log(y+a+b)$ $-\frac{1}{4}\|y-a-b\|\log\|y-a-b\|$ $\times\operatorname{sgn}(a+b-y)$ $+\frac{1}{4}\|y-a+b\|\log\|y-a+b\|$ $\times\operatorname{sgn}(a-b-y)$
(12)	$x^{-3}\sin(ax)\sin(bx)$ $a\geq b>0$	$\frac{1}{2}\pi by \qquad 0<y<a-b$ $\frac{1}{2}\pi by-2^{-3}\pi(a-b-y)^2$ $a-b<y<a+b$ $\frac{1}{2}\pi ab \qquad a+b<y<\infty$
(13)	$x^{-\nu}\sin(ax)\sin(bx)$ $0<\operatorname{Re}\nu<4, \quad \nu\neq 1,2,3$ $a\geq b>0$	$\frac{1}{4}\Gamma(1-\nu)\cos(\frac{1}{2}\nu\pi)$ $\times[(y+a-b)^{\nu-1}-(y+a+b)^{\nu-1}$ $-\|y-a+b\|^{\nu-1}\operatorname{sgn}(a-b-y)$ $+\|y-a-b\|^{\nu-1}\operatorname{sgn}(a+b-y)]$
(14)	$x^{-1}\sin^2(ax)\sin(bx)$ $a>0, \quad b>0$	$\frac{1}{8}\log\|(b+y)^2(2a-b+y)(2a+b-y)\|$ $-\frac{1}{8}\log\|(b-y)^2(2a+b+y)(2a-b-y)\|$
(15)	$x^{-2}\sin^2(ax)\sin(bx)$ $a, b>0$	$\pi 2^{-4}(\|b-2a-y\|+\|b+2a-y\|$ $-2\|b-y\|)$
(16)	$x^{-4}\sin^3(ax)$ $a>0$	$(\pi y/24)(9a^2-y^2) \qquad 0<y\leq a$ $(\pi/48)[24a^3-(3a-y)^3]$ $a\leq y\leq 3a$ $\frac{1}{2}\pi a^3 \qquad a\leq y<\infty$

Trigonometric functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(17)	$x^{-1}\sin^2(ax)\sin^2(bx)$ $\qquad a,\ b>0$	$(\pi/32)[2+\operatorname{sgn}(y-2a+2b)$ $+\operatorname{sgn}(y+2a-2b)-2\operatorname{sgn}(y-2a)$ $-2\operatorname{sgn}(y-2b)]$
(18)	$[\sin(\pi x)]^{\nu-1} \qquad 0<x<1$ $0 \qquad 1<x<\infty$ $\operatorname{Re}\nu>0$	$2^{1-\nu}\sin(\tfrac{1}{2}y)\Gamma(\nu)$ $\times\{\Gamma[\tfrac{1}{2}(\nu+1+y/\pi)]$ $\times\Gamma[\tfrac{1}{2}(\nu+1-y/\pi)]\}^{-1}$
(19)	$e^{-ax}(\sin x)^{2n} \qquad \operatorname{Re} a>0$	$-(-4)^{-n-1}(2n+1)^{-1}$ $\times\left\{\left[\binom{\frac{1}{2}y+\frac{1}{2}ia+n}{2n+1}\right]^{-1}\right.$ $\left.+\left[\binom{\frac{1}{2}y-\frac{1}{2}ia+n}{2n+1}\right]^{-1}\right\}$
(20)	$e^{-ax}(\sin x)^{2n-1} \qquad \operatorname{Re} a>0$	$-(-4)^{-n-1}n^{-1}i$ $\times\left\{\left[\binom{\frac{1}{2}y-\frac{1}{2}ia-\frac{1}{2}+n}{2n}\right]^{-1}\right.$ $\left.-\left[\binom{\frac{1}{2}y+\frac{1}{2}ia+\frac{1}{2}+n}{2n}\right]^{-1}\right\}$
(21)	$(x^2+\beta^2)^{-1}\csc(ax)$ $a>0,\quad \operatorname{Re}\beta>0$	$\tfrac{1}{2}\pi\beta^{-1}\sinh(\beta y)\operatorname{csch}(a\beta) \quad 0<y<a$ Cauchy Principal Value
(22)	$x^{1-\frac{1}{2}m}[\prod_{n=2}^{m}\sin(a_n x)]$ $a_n>0$	$0 \qquad y>\sum_{n=2}^{m}a_n$
(23)	$x^{-1}\cos(ax) \qquad a>0$	$0 \qquad 0<y<a$ $\pi/4 \qquad y=a$ $\pi/2 \qquad a<y<\infty$

Trigonometric functions of kx (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \sin(xy)\, dx \qquad y > 0$
(24)	$x^{\nu-1} \cos(ax) \qquad \lvert \mathrm{Re}\, \nu \rvert < 1$	$\frac{1}{4}\pi \sec(\frac{1}{2}\nu\pi) [\Gamma(1-\nu)]^{-1}$ $\times [(y+a)^{-\nu} - \lvert y-a \rvert^{-\nu} \operatorname{sgn}(a-y)]$
(25)	$x(x^2+a^2)^{-1} \cos(bx)$ $\mathrm{Re}\, a > 0, \quad b > 0$	$-\frac{1}{2}\pi e^{-ab} \sinh(ay) \qquad 0 < y < b$ $\frac{1}{2}\pi e^{-ay} \cosh(ab) \qquad b < y < \infty$
(26)	$x^{-1}(1-2a \cos x + a^2)^{-1}$ $0 < a < 1$	$\frac{1}{2}\pi [(1-a^2)(1-a)]^{-1}$ $\times (1+a-2a^{[y]+1})$ $y \neq 0, 1, 2, \ldots$ $\frac{1}{2}\pi [(1-a^2)(1-a)]^{-1}$ $\times (1+a-a^y-a^{y+1})$ $y = 0, 1, 2, \ldots$
(27)	$(x^2+a^2)^{-1}$ $\times (1-2b \cos x + b^2)^{-1} \sin x$ $0 < b < 1, \quad \mathrm{Re}\, a > 0$	$\frac{1}{2}\pi a^{-1} (e^{a} - b)^{-1} \sinh(ay)$ $0 \leq y < 1$ $\frac{1}{4}\pi a^{-1} (be^{a} - 1)^{-1}$ $\times [b^m e^{(m+1-y)a} - e^{(1-y)a}]$ $-\frac{1}{4}\pi a^{-1} (be^{-a} - 1)^{-1}$ $\times [b^m e^{-(m+1-y)a} - e^{-a(1-y)}]$ $m \leq y < m+1$ $m = 0, 1, 2, \ldots$
(28)	$x^{-n} [\sin(ax)]^m [\cos(bx)]^k$	Integrals of this type may be computed from integrals number (3) and (24) of this section
(29)	$x^{-2}[1 - \cos(ax)] \qquad a > 0$	$\frac{y}{2} \log \left\lvert \frac{y^2-a^2}{y^2} \right\rvert + \frac{a}{2} \log \left\lvert \frac{y+a}{y-a} \right\rvert$
(30)	$x(x^2+\beta^2)^{-1} \sec(ax)$ $a > 0, \quad \mathrm{Re}\, \beta > 0$	$\frac{-\pi \sinh(\beta y)}{2 \sinh(a\beta)} \qquad 0 < y < a$ The integral is a Cauchy Principal Value

Trigonometric functions of kx (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \sin(xy) dx \qquad y > 0$
(31)	$x^{-1}(b^2 - x^2)^{-1} \sec(ax)$ $a, b > 0$	$0 \qquad 0 < y < a$ The integral is a Cauchy Principal Value
(32)	$x^{-1}(x^2 + \beta^2)^{-1} \sec(ax)$ $a > 0, \quad \mathrm{Re}\,\beta > 0$	$\tfrac{1}{2}\pi\beta^{-2} \operatorname{sech}(a\beta) \sinh(\beta y)$ $0 < y < a$ The integral is a Cauchy Principal Value
(33)	$x^{-1}(1 - 2a\cos x + a^2)^{-1}$ $\times (1 - a\cos x) \qquad 0 < a < 1$	$\tfrac{1}{2}\pi(1-a)^{-1}(1 - a^{[y]+1})$ $y \neq 1, 2, \ldots$ $\tfrac{1}{2}\pi(1-a)^{-1}(1-a^n) + \tfrac{1}{4}\pi a^n$ $y = 1, 2, \ldots$
(34)	$x(x^2 + a^2)^{-1} \log(2 \pm 2\cos x)$ $\mathrm{Re}\,a > 0$	$-\pi \sinh(ay) \log(1 \pm e^{-a})$
(35)	$x^{-1} \log(1 + 2a\cos x + a^2)$ $0 < a < 1$	$-\dfrac{\pi}{2} \sum_{n=1}^{[y]} \dfrac{(-a)^n}{n} [1 + \operatorname{sgn}(y - n)]$
(36)	$x^{-1}(1 + x^2)^{-1} \log[\cos^2(ax)]$ $a > 0$	$\pi \log(1 + e^{-2a}) \sinh y - \pi(\log 2)$ $\times (1 - e^{-y})$
(37)	$e^{-ax} \csc(bx)$ $\mathrm{Re}\,a > 0, \quad b > 0$	$-\tfrac{1}{2} i b^{-1} \{ \psi[\tfrac{1}{2} b^{-1}(b + y) - \tfrac{1}{2} i b^{-1} a]$ $- \psi[\tfrac{1}{2} b^{-1}(b - y) - \tfrac{1}{2} i b^{-1} a]\}$ The integral is a Cauchy Principal Value

2.7. Trigonometric functions of other arguments

(1)	$\sin(ax^2) \qquad a > 0$	$\pi^{1/2}(2a)^{-1/2}\{\cos(\tfrac{1}{4}a^{-1}y^2)\, C[(2\pi a)^{-1/2} y]$ $+ \sin(\tfrac{1}{4}a^{-1}y^2)\, S[(2\pi a)^{-1/2} y]\}$

Trigonometric functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \qquad y>0$
(2)	$x^{-1/2}\sin(ax^2) \qquad a>0$	$-\frac{1}{2}\pi(2a)^{-1/2}y^{1/2}\sin(a^{-1}y^2/8-3\pi/8)$ $\times J_{1/4}(a^{-1}y^2/8)$
(3)	$\cos(ax^2) \qquad a>0$	$(2a)^{-1/2}\pi^{1/2}\{\sin(\frac{1}{4}a^{-1}y^2)C[(2\pi a)^{-1/2}y]$ $-\cos(\frac{1}{4}a^{-1}y^2)S[(2\pi a^{-1/2}y]\}$
(4)	$x^{-1/2}\cos(ax^2) \qquad a>0$	$\frac{1}{2}\pi(2a)^{-1/2}y^{1/2}\cos(a^{-1}y^2/8-3\pi/8)$ $\times J_{1/4}(a^{-1}y^2/8)$
(5)	$\sin(a^3x^3) \qquad a>0$	$\frac{1}{2}(3a)^{-3/2}\pi y^{1/2}\{J_{1/3}[2(a^{-1}y/3)^{3/2}]$ $+J_{-1/3}[2(a^{-1}y/3)^{3/2}]$ $-3^{1/2}\pi^{-1}K_{1/3}[2(a^{-1}y/3)^{3/2}]\}$
(6)	$\sin(a^2/x) \qquad a>0$	$(\frac{1}{2}\pi)ay^{-1/2}J_1(2ay^{1/2})$
(7)	$x^{-1}\sin(a^2/x)$	$\frac{1}{2}\pi Y_0(2ay^{1/2})+K_0(2ay^{1/2})$
(8)	$x^{-2}\sin(a^2/x)$	$\frac{1}{2}\pi a^{-1}y^{1/2}J_1(2ay^{1/2})$
(9)	$x^{-1/2}\sin(a^2/x)$	$\pi^{1/2}2^{-3/2}y^{-1/2}[\sin(2ay^{1/2})$ $-\cos(2ay^{1/2})+e^{-2ay^{1/2}}]$
(10)	$x^{-3/2}\sin(a^2/x)$	$-\pi^{1/2}2^{-3/2}a^{-1}[\cos(2ay^{1/2})$ $-\sin(2ay^{1/2})-e^{-2ay^{1/2}}]$
(11)	$x^{\nu-1}\sin(a^2/x) \qquad \lvert\operatorname{Re}\nu\rvert<1$	$\frac{1}{4}\pi a^\nu\csc(\frac{1}{2}\nu\pi)y^{-\frac{1}{2}\nu}$ $\times[J_\nu(2ay^{1/2})-J_{-\nu}(2ay^{1/2})$ $+I_{-\nu}(2ay^{1/2})-I_\nu(2ay^{1/2})]$

Trigonometric functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \quad y>0$
(12)	$x^{-1/2}\cos(a^2/x)$	$\pi^{1/2}2^{-3/2}y^{-1/2}[\sin(2ay^{1/2}) + \cos(2by^{1/2})+e^{-2ay^{1/2}}]$
(13)	$x^{-3/2}\cos(a^2/x)$	$\pi^{1/2}2^{-3/2}a^{-1}[\cos(2ay^{1/2}) + \sin(2ay^{1/2})-e^{-2ay^{1/2}}]$
(14)	$x^{\nu-1}\cos(a^2/x) \qquad \lvert\operatorname{Re}\nu\rvert<1$	$\frac{1}{4}\pi b^{\nu}\sec(\frac{1}{2}\nu\pi)y^{-\frac{1}{2}\nu}[J_{\nu}(2ay^{1/2}) + J_{-\nu}(2ay^{1/2})+I_{-\nu}(2ay^{1/2}) - I_{\nu}(2ay^{1/2})]$
(15)	$x^{-1/2}\sin(ax^{1/2})\sin(bx^{1/2}) \qquad a,\ b>0$	$\pi^{1/2}y^{-1/2}\sin(\frac{1}{2}aby^{-1}) \times\cos[\frac{1}{4}(a^2+b^2)y^{-1}-\frac{1}{4}\pi]$
(16)	$x^{-3/4}\sin(ax^{1/2}) \qquad a>0$	$-2^{-1/2}\pi a^{1/2}y^{-1/2}\sin(a^2y^{-1}/8-3\pi/8) \times J_{1/4}(a^2y^{-1}/8)$
(17)	$e^{-ax^{1/2}}\sin(ax^{1/2}) \qquad \lvert\arg a\rvert<\pi/4$	$\pi^{1/2}2^{-3/2}ay^{-3/2}e^{-\frac{1}{2}a^2/y}$
(18)	$x^{-1/2}\cos(ax^{1/2})$	$(\pi/y)^{1/2}\cos(\frac{1}{4}a^2y+\frac{1}{4}\pi)$
(19)	$x^{-3/4}\cos(ax^{1/2}) \qquad a>0$	$-2^{-1/2}\pi a^{1/2}y^{-1/2}\sin(a^2y^{-1}/8-\pi/8) \times J_{-1/4}(a^2y^{-1}/8)$
(20)	$x^{-1/2}e^{-\beta x}\cos(ax^{1/2}) \qquad \operatorname{Re}\beta>0$	$\pi^{1/2}(y^2+\beta^2)^{-1/4}e^{-\frac{1}{4}a^2\beta/(y^2+\beta^2)} \times\sin[\frac{1}{2}\tan^{-1}(y/\beta) -\frac{1}{4}a^2y/(y^2+\beta^2)]$
(21)	$x^{-1/2}\cos(ax^{1/2})\cos(bx^{1/2}) \qquad a,\ b>0$	$\pi^{1/2}y^{-1/2}\cos(\frac{1}{2}aby^{-1}) \times\cos[\frac{1}{4}(a^2+b^2)y^{-1}+\frac{1}{4}\pi]$

Trigonometric functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(22)	$x^{-1/2}[\cos(ax^{1/2})+\sin(ax^{1/2})]$ $\quad a>0$	$(\tfrac{1}{2}\pi)^{1/2}y^{-1/2}e^{-\frac{1}{2}a^2y^{-1/2}}$
(23)	$x^{-1/2}e^{-ax^{1/2}}[\cos(ax^{1/2})+\sin(ax^{1/2})]$ $\quad \lvert\arg a\rvert<\pi/4$	$(\tfrac{1}{2}\pi)^{1/2}y^{-1/2}e^{-\frac{1}{2}a^2y^{-1}}$
(24)	$x(b^2+x^2)^{-2}\sin[a(x^2+b^2]^{1/2}]$ $\quad a,\ b>0$	$\tfrac{1}{2}\pi a e^{-yb}$ $\quad a<y<\infty$
(25)	$x(x^2+a^2)^{-1}(x^2+b^2)^{-1/2}\times\sin[c(x^2+b^2)^{1/2}]$ $\quad \mathrm{Re}\,a>0,\quad b,\ c>0$	$(\tfrac{1}{2}\pi)(b^2-a^2)^{-1/2}e^{-ay}\times\sin[c(b^2-a^2)^{1/2}]$ $\quad c<y<\infty$
(26)	$x^{-1/2}(x^2+b^2)^{-1/2}\sin[a(b^2+x^2)^{1/2}]$ $\quad a,\ b>0$	$(\tfrac{1}{2}\pi)^{3/2}y^{1/2}J_{1/4}\{\tfrac{1}{2}b[a-(a^2-y^2)^{1/2}]\}\times J_{-1/4}\{\tfrac{1}{2}b[a+(a^2-y^2)^{1/2}]\}$ $\quad 0<y<a$
(27)	$x(x^2+a^2)^{-1}\cos[c(x^2+b^2)^{1/2}]$ $\quad \mathrm{Re}\,a>0,\quad b,\ c>0$	$(\tfrac{1}{2}\pi)e^{-ay}\cos[c(b^2-a^2)^{1/2}]$ $\quad c<y<\infty$
(28)	$x^{-1/2}(b^2+x^2)^{-1/2}\cos[a(b^2+x^2)^{1/2}]$ $\quad a,\ b>0$	$-(\tfrac{1}{2}\pi)^{3/2}y^{1/2}J_{1/4}\{\tfrac{1}{2}b[a-(a^2-y^2)^{1/2}]\}\times Y_{-1/4}\{\tfrac{1}{2}b[a+(a^2-y^2)^{1/2}]\}$ $\quad 0<y<a$
(29)	$x(x^2+b^2)^{-3/2}\cos[a(x^2+b^2)^{1/2}]$ $\quad a,\ b>0$	$\tfrac{1}{2}\pi e^{-by}$ $\quad a<y<\infty$
(30)	$0 \quad 0<x<a$; $(x^2+\gamma^2)^{-1}\sin[b(x^2-a^2)^{1/2}] \quad a<x<\infty$; $a,\ b>0,\quad \mathrm{Re}\,\gamma>0$	$\tfrac{1}{2}\pi\gamma^{-1}e^{-b(a^2+\gamma^2)^{1/2}}\sinh(\gamma y)$ $\quad 0<y<b$

Trigonometric functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \qquad y>0$
(31)	$0 \qquad 0<x<a$ $(x^2-a^2)^{-1/4}\sin[b(x^2-a^2)^{1/2}]$ $a<x<\infty$ $a,\ b>0$	$(\tfrac{1}{2}\pi)^{3/2}b^{1/2}J_{1/4}\{\tfrac{1}{2}a[y-(y^2-b^2)^{1/2}]\}$ $\times J_{-1/4}\{\tfrac{1}{2}a[y+(y^2-b^2)^{1/2}]\}$ $b<y<\infty$
(32)	$x^{-1/2}(a^2-x^2)^{-1/2}$ $\times\cos[\beta(a^2-x^2)^{1/2}]$ $0<x<a$ $0 \qquad a<x<\infty$	$(\tfrac{1}{2}\pi)^{3/2}y^{1/2}J_{1/4}\{\tfrac{1}{2}a[(\beta^2+y^2)^{1/2}-\beta]\}$ $\times J_{1/4}\{\tfrac{1}{2}a[(\beta^2+y^2)^{1/2}+\beta]\}$
(33)	$0 \qquad 0<x<a$ $(x^2-a^2)^{-1/2}\cos[b(x^2-a^2)^{1/2}]$ $a<x<\infty$ $b>0$	$0 \qquad 0<y<b$ $(\tfrac{1}{2}\pi)J_0[a(y^2-b^2)^{1/2}] \qquad b<y>\infty$
(34)	$0 \qquad 0<x<a$ $(x^2-a^2)^{-1/4}\cos[b(x^2-a^2)^{1/2}]$ $a<x<\infty$ $b>0$	$(\tfrac{1}{2}\pi)^{3/2}b^{1/2}J_{-1/4}\{\tfrac{1}{2}a[y-(y^2-b^2)^{1/2}]\}$ $\times J_{1/4}\{\tfrac{1}{2}a[y+(y^2-b^2)^{1/2}]\}$ $b<y<\infty$
(35)	$0 \qquad 0<x<a$ $(x^2+\gamma^2)^{-1}(x^2-a^2)^{-1/2}$ $\times\cos[b(x^2-a^2)^{1/2}]$ $a<x<\infty$ $\mathrm{Re}\ a>0, \quad b>0$	$\tfrac{1}{2}\pi\gamma^{-1}(\gamma^2+a^2)^{-1/2}e^{-b(a^2+\gamma^2)^{1/2}}$ $\times\sinh(\gamma y) \qquad 0<y<b$
(36)	$\sin(a\sin x) \qquad 0<x<\pi$ $0 \qquad \pi<x<\infty$	$\sin(\pi y)s_{0,y}(a)$
(37)	$\cos(a\sin x) \qquad 0<x<\pi$ $0 \qquad \pi<x<\infty$	$-y[1-\cos(\pi y)]s_{-1,y}(a)$

2.8. Inverse Trigonometric functions

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(1)	$0 \qquad 0<x<a$ $\dfrac{\cos[n\cos^{-1}(\frac{2x-a-b}{b-a})]}{(x-a)^{1/2}(b-x)^{1/2}} \qquad a<x<b$ $0 \qquad b<x<\infty$	$\pi\sin[\tfrac{1}{2}(a+b)y-\tfrac{1}{2}n\pi]$ $\times J_n[\tfrac{1}{2}(b-a)y]$
(2)	$(a^2x-x^3)^{-1/2}$ $\times\cos[2\nu\cos^{-1}(x/a)] \qquad 0<x<a$ $0 \qquad a<x<\infty$	$(\tfrac{1}{2}\pi)^{3/2}y^{1/2}J_{\nu+1/4}(\tfrac{1}{2}ay)J_{-\nu+1/4}(\tfrac{1}{2}ay)$
(3)	$\tan^{-1}(x/a) \qquad a>0$	$(\tfrac{1}{2}\pi)y^{-1}e^{-ay}$
(4)	$(x^2+a^2)^{-\frac{1}{2}\nu}\sin[\nu\tan^{-1}(x/a)]$ $\mathrm{Re}\,\nu>0, \quad \mathrm{Re}\,a>0$	$\tfrac{1}{2}\pi[\Gamma(\nu)]^{-1}y^{\nu-1}e^{-ay}$
(5)	$\mathrm{ctn}^{-1}(ax) \qquad \mathrm{Re}\,a>0$	$\tfrac{1}{2}\pi y^{-1}(1-e^{-y/a})$
(6)	$x^{\nu-1/2}(1+x^2)^{\frac{1}{2}\nu-1/4}$ $\times\sin[(\nu-\tfrac{1}{2})\,\mathrm{ctn}^{-1}x]$ $-3/2<\mathrm{Re}\,\nu<1/2$	$-\pi^{-1/2}\Gamma(\nu+\tfrac{1}{2})y^{-\nu}\cos(\nu\pi)$ $\times\sinh(\tfrac{1}{2}y)\,K_\nu(\tfrac{1}{2}y)$
(7)	$x^{\nu-1/2}(1+x^2)^{\frac{1}{2}\nu-1/4}$ $\times\cos[(\nu-\tfrac{1}{2})\,\mathrm{ctn}^{-1}x]$ $-3/2<\mathrm{Re}\,\nu<1/2$	$\tfrac{1}{2}\pi^{1/2}\Gamma(\nu+\tfrac{1}{2})y^{-\nu}[I_{-\nu}(\tfrac{1}{2}y)\cosh(\tfrac{1}{2}y)$ $-I_\nu(\tfrac{1}{2}y)\sinh(\tfrac{1}{2}y)]$
(8)	$\tan^{-1}(2a/x) \qquad \mathrm{Re}\,a>0$	$\pi y^{-1}e^{-ay}\sinh(ay)$
(9)	$\tan^{-1}[2ax/(x^2+b^2)]$	$\pi y^{-1}e^{-(a^2+b^2)^{1/2}y}\sinh(ay)$

2.9. Hyperbolic functions

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(1)	$\operatorname{sech}(ax)$ $\qquad \operatorname{Re} a > 0$	$-\tfrac{1}{2}\pi a^{-1}\tanh(\tfrac{1}{2}\pi a^{-1}y)$ $-\tfrac{1}{2}i a^{-1}[\psi(\tfrac{1}{4}+\tfrac{1}{4}a^{-1}iy)$ $-\psi(\tfrac{1}{4}-\tfrac{1}{4}a^{-1}iy)]$
(2)	$\csc(ax)$ $\qquad \operatorname{Re} a > 0$	$\tfrac{1}{2}\pi a^{-1}\tanh(\tfrac{1}{2}\pi a^{-1}y)$
(3)	$\operatorname{ctnh}(\tfrac{1}{2}ax)-1$ $\qquad \operatorname{Re} a > 0$	$\pi a^{-1}\operatorname{ctnh}(\pi a^{-1}y)-y$
(4)	$1-\tanh(\tfrac{1}{2}ax)$ $\qquad \operatorname{Re} a > 0$	$y-\pi a^{-1}\operatorname{csch}(a^{-1}y)$
(5)	$\sinh(ax)\operatorname{csch}(\beta x)$ $\qquad \lvert\operatorname{Re} a\rvert < \operatorname{Re}\beta$	$\tfrac{1}{2}\pi\beta^{-1}\sinh(\pi\beta^{-1}y)[\cosh(\pi\beta^{-1}y)$ $+\cos(\pi\beta^{-1}a)]^{-1}$ $+\tfrac{1}{2}i\beta^{-1}\{\psi[\tfrac{1}{2}(a+\beta+iy)\beta^{-1}]$ $-\psi[\tfrac{1}{2}(a+\beta-iy)\beta^{-1}]\}$
(6)	$\dfrac{\cosh(ax)}{\cosh(\beta x)}$ $\qquad \lvert\operatorname{Re} a\rvert < \operatorname{Re}\beta$	$\dfrac{i}{4\beta}\Big[\psi\Big(\dfrac{3\beta+a+iy}{4\beta}\Big)-\psi\Big(\dfrac{3\beta+a-iy}{4\beta}\Big)$ $+\psi\Big(\dfrac{3\beta-a+iy}{4\beta}\Big)-\psi\Big(\dfrac{3\beta-a-iy}{4\beta}\Big)$ $-\dfrac{2\pi i\sinh(\beta^{-1}\pi y)}{\cosh(\beta^{-1}\pi y)+\cos(\beta^{-1}\pi a)}\Big]$
(7)	$\dfrac{\cosh(ax)}{\sinh(\beta x)}$ $\qquad \lvert\operatorname{Re} a\rvert < \operatorname{Re}\beta$	$\dfrac{\pi}{2\beta}\Big[\dfrac{\sinh(\beta^{-1}\pi y)}{\cosh(\beta^{-1}\pi y)+\cos(\beta^{-1}\pi a)}\Big]$
(8)	$\sinh(ax)\operatorname{sech}(\beta x)$ $\qquad \lvert\operatorname{Re} a\rvert < \operatorname{Re}\beta$	$\pi\beta^{-1}\sin(\tfrac{1}{2}\pi a\beta^{-1})\sinh(\tfrac{1}{2}\pi\beta^{-1}y)$ $\times[\cosh(\pi\beta^{-1}y)+\cos(\pi a\beta^{-1})]^{-1}$
(9)	$\sinh(ax)[\operatorname{sech}(\beta x)]^2$ $\qquad \lvert\operatorname{Re} a\rvert < 2\operatorname{Re}\beta$	$\pi\beta^{-2}[y\sin(\tfrac{1}{2}\pi\beta^{-1}a)\cosh(\tfrac{1}{2}\pi\beta^{-1}y)$ $-a\cos(\tfrac{1}{2}\pi\beta^{-1}a)\sinh(\tfrac{1}{2}\pi\beta^{-1}y)]$ $\times[\cosh(\pi\beta^{-1}y)-\cos(\pi\beta^{-1}a)]^{-1}$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \qquad y>0$
(10)	$\sinh(\frac{1}{2}x)[\cosh x+\cos a]^{-1}$ $\operatorname{Re} a<\pi$	$\frac{1}{2}\pi\csc(\frac{1}{2}a)\sinh(ay)\operatorname{sech}(\pi y)$
(11)	$\sinh(ax)[\cosh(\gamma x)+\cos\beta]^{-1}$ $\pi\operatorname{Re}\gamma>\lvert\operatorname{Re}(\bar{\gamma}\beta)\rvert$ $\lvert\operatorname{Re} a\rvert<\operatorname{Re}\gamma$	$\pi\gamma^{-1}\csc\beta\{\sin[a\gamma^{-1}(\pi+\beta)]$ $\times\sinh[\gamma^{-1}(\pi-\beta)y]$ $-\sin[a\gamma^{-1}(\pi-\beta)]\sinh[\gamma^{-1}(\pi+\beta)y]\}$ $\times[\cosh(2\pi\gamma^{-1}y)-\cos(2\pi\gamma^{-1}a)]^{-1}$
(12)	$x\operatorname{sech}(ax) \qquad \operatorname{Re} a>0$	$\frac{1}{4}\pi^2a^{-2}\sinh(\frac{1}{2}\pi a^{-1}y)\operatorname{sech}^2(\frac{1}{2}\pi a^{-1}y)$
(13)	$x^{-1}\operatorname{sech}(ax) \qquad \operatorname{Re} a>0$	$2\tan^{-1}(e^{\frac{1}{2}\pi a^{-1}y})$
(14)	$x\operatorname{sech}^2 x$	$-\frac{d}{dy}[\frac{1}{2}\pi y\operatorname{csch}(\frac{1}{2}\pi y)]$
(15)	$(1+x^2)^{-1}\operatorname{csch}(\pi x)$	$-\frac{1}{2}ye^{-y}+(\sinh y)\log(1+e^{-y})$
(16)	$(1+x^2)^{-1}\operatorname{csch}(\frac{1}{2}\pi x)$	$e^y\tan^{-1}(e^{-y})-e^{-y}\tan^{-1}(e^y)$
(17)	$(x^2+m^2)^{-1}\operatorname{csch}(\pi x)$ $m=1,2,3,\ldots$	$\frac{(-1)^m yz^m}{2m}+\frac{1}{2m}\sum_{n=1}^{m-1}\frac{(-1)^n z^n}{m-n}$ $+\frac{(-1)^m z^m}{2m}\log(1+z)$ $+\frac{1}{2m!}\frac{d^{m-1}}{dz^{m-1}}\left[\frac{(1+z)^{m-1}}{z}\log(1+z)\right]$ $z=e^{-y}$

For similar integrals see Titchmarsh, E. C., 1937: *Introduction to the theory of Fourier integrals*, p. 389. Oxford.

Hyperbolic functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)dx \quad y>0$
(18)	$(x^2+a^2)^{-1}\operatorname{csch}(\pi x)$ $\operatorname{Re} a>0 \qquad a\neq 0, 1, 2, \ldots$	$-\tfrac12 a^{-2} - \pi a^{-1}\csc(\pi a)e^{-ay}$ $+\tfrac12 a^{-2}[{}_2F_1(1,-a;1-a;-e^{-y})$ $+{}_2F_1(1,a;1+a;-e^{-y})]$ $= -\tfrac12 \pi a^{-1}\csc(\pi a)e^{-ay}$ $-\dfrac{1}{2}\sum_{n=0}^{\infty}\dfrac{(-1)^n b_n e^{-ny}}{n^2-a^2}$ $b_0=1, \quad b_n=2, \quad (n\neq 0)$
(19)	$\dfrac{x}{(1+x^2)\cosh(\tfrac14\pi x)}$	$\dfrac{\pi}{2^{1/2}}e^{-y}+\dfrac{1}{2^{1/2}}(\sinh y)\log\left(\dfrac{\cosh y+2^{-1/2}}{\cosh y-2^{-1/2}}\right)$ $-2^{1/2}(\cosh y)\tan^{-1}(2^{-1/2}\operatorname{csch} y)$
(20)	$(1+x^2)^{-1}\tanh(\tfrac12\pi x)$	$ye^{-y}-(\sinh y)\log(1-e^{-2y})$
(21)	$(1+x^2)^{-1}\operatorname{ctnh}(\tfrac12\pi x)$	$(\sinh y)\log(\operatorname{ctnh}\tfrac12 y)$
(22)	$x^{-1}\sinh(ax)\operatorname{csch}(\beta x)$ $\lvert\operatorname{Re} a\rvert<\operatorname{Re}\beta$	$\tan^{-1}[\tan(\tfrac12\pi a\beta^{-1})\tanh(\tfrac12\pi\beta^{-1}y)]$
(23)	$\dfrac{\cosh(ax)}{(x^2+\beta^2)\sinh(\pi x)}$ $\lvert\operatorname{Re} a\rvert<\pi, \quad \operatorname{Re}\beta>0$ $\beta\neq 1, 2, 3, \ldots$	$-\dfrac{\pi}{2}\left[\dfrac{e^{-\beta y}\cos(a\beta)}{\beta\sin(\pi\beta)}\right]$ $-\dfrac{1}{2}\sum_{n=0}^{\infty}\dfrac{(-1)^n\epsilon_n e^{-ny}\cos(an)}{n^2-\beta^2}$ $\epsilon_0=1, \quad \epsilon_n=2, \quad (n\neq 0)$
(24)	$(1+x^2)^{-1}\cosh(ax)\operatorname{csch}(\tfrac12\pi x)$ $\lvert\operatorname{Re} a\rvert<\tfrac12\pi$	$-\tfrac12\pi e^{-y}\cos a$ $+\tfrac12\sin a\sinh y\log\left(\dfrac{\cosh y+\sin a}{\cosh y-\sin a}\right)$ $+\cos a\cosh y\tan^{-1}(\cos a\operatorname{csch} y)$

For similar integrals see Titchmarsh, E. C., 1937: *Introduction to the theory of Fourier integrals*, p. 389.

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \quad y>0$
(25)	$(1+x^2)^{-1}\cosh(ax)\operatorname{csch}(\pi x)$ $\lvert\operatorname{Re} a\rvert<\pi$	$-\frac{1}{2}e^{-y}(y\cos a+a\sin a)$ $+\frac{1}{2}\sinh y\cos a\log(1+2e^{-y}\cos a$ $+e^{-2y})+\cosh y\sin a$ $\times\tan^{-1}[\sin a(e^y+\cos a)^{-1}]$
(26)	$(x^2+\frac{1}{4})^{-1}\sinh(ax)\operatorname{sech}(\pi x)$ $\lvert\operatorname{Re} a\rvert<\pi$	$e^{-\frac{1}{2}y}[y\sin(\frac{1}{2}a)-a\cos(\frac{1}{2}a)]$ $-\sinh(\frac{1}{2}y)\sin(\frac{1}{2}a)\log(1+2e^{-y}\cos a$ $+e^{-2y})+\cosh(\frac{1}{2}y)\cos(\frac{1}{2}a)$ $\times\tan^{-1}[\sin a(1+e^{-y}\cos a)^{-1}]$
(27)	$\dfrac{x\sinh(ax)}{(1+x^2)\sinh(\frac{1}{2}\pi x)}$ $\lvert\operatorname{Re} a\rvert<\pi/2$	$\frac{1}{2}\pi e^{-y}\sin a$ $+\dfrac{1}{2}\cos a\sinh y\log\left(\dfrac{\cosh y+\sin a}{\cosh y-\sin a}\right)$ $-\sin a\cosh y\tan^{-1}(\cos a\operatorname{csch} y)$
	For similar integrals see Titchmarsh, E. C., 1937: *Introduction to the theory of Fourier integrals*, p. 389, Oxford.	
(28)	$\dfrac{e^{-ax}}{\sinh(\beta x)}$ $\quad\lvert\operatorname{Re}\beta\rvert<\operatorname{Re} a$	$\dfrac{1}{2\beta i}\left[\psi\left(\dfrac{a+\beta+iy}{2\beta}\right)-\psi\left(\dfrac{a+\beta-iy}{2\beta}\right)\right]$ $=2y\sum_{n=0}^{\infty}[(a+\beta+2n\beta)^2+y^2]^{-1}$
(29)	$e^{-x}\operatorname{csch} x$	$\frac{1}{2}\pi\operatorname{ctnh}(\frac{1}{2}\pi y)-y^{-1}$
(30)	$e^{-ax}[\sinh(\beta x)]^{\nu}$ $\operatorname{Re}\nu>-2, \quad \operatorname{Re}\beta>0$ $\lvert\operatorname{Re}(\beta\nu)\rvert<\operatorname{Re} a$	$-i\,2^{-\nu-2}\beta^{-1}\Gamma(\nu+1)$ $\times\{\Gamma[\frac{1}{2}\beta^{-1}(a-\nu\beta-iy)]$ $\times\Gamma[\frac{1}{2}\beta^{-1}(a+\nu\beta-iy)+1]^{-1}$ $-\Gamma[\frac{1}{2}\beta^{-1}(a-\nu\beta+iy)]$ $\times\Gamma[\frac{1}{2}\beta^{-1}(a+\nu\beta+iy)+1]^{-1}\}$
(31)	$e^{-ax}\tanh(bx^{\frac{1}{2}})$	see Mordell, L. J., 1920: *Mess. of Math.*, 49, 65-72

Hyperbolic functions (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(32)	$e^{-ax}\operatorname{ctnh}(bx^{1/2})$	see Mordell, L. J., 1920: *Mess. of Math.* 49, 65-72
(33)	$(e^{\beta x}-1)^{-1}\sinh(ax)$ $\operatorname{Re}\beta > \lvert\operatorname{Re}a\rvert$	$-\tfrac{1}{2}y(y^2+a^2)^{-1}+\tfrac{1}{2}\pi\beta^{-1}$ $\times\sinh(2\pi\beta^{-1}y)[\cosh(2\pi\beta^{-1}y)$ $-\cos(2\pi a\beta^{-1})]^{-1}$ $+\tfrac{1}{2}i\beta^{-1}[\psi(a\beta^{-1}+i\beta^{-1}y+1)$ $-\psi(a\beta^{-1}-i\beta^{-1}y+1)]$
(34)	$(e^{\beta x}-1)^{-1}\cosh(ax)$ $\operatorname{Re}\beta > \lvert\operatorname{Re}a\rvert$	$-\tfrac{1}{2}y(y^2+a^2)^{-1}+\tfrac{1}{2}\pi\beta^{-1}\sinh(2\pi\beta^{-1}y)$ $\times[\cosh(2\pi\beta^{-1}y)-\cos(2\pi a\beta^{-1})]^{-1}$
(35)	$(e^{\beta x}+1)^{-1}\cosh(ax)$ $\operatorname{Re}\beta > \lvert\operatorname{Re}a\rvert$	$\tfrac{1}{2}y(y^2+a^2)^{-1}-\pi\beta^{-1}\sinh(\pi\beta^{-1}y)$ $\times\cos(\pi a\beta^{-1})[\cosh(2\pi\beta^{-1}y)$ $-\cos(2\pi a\beta^{-1})]^{-1}$
(36)	$x^{-1}e^{-\beta x}\sinh(ax)$ $\operatorname{Re}\beta > \lvert\operatorname{Re}a\rvert$	$\tfrac{1}{2}\tan^{-1}[2ay/(y^2+\beta^2-a^2)]$
(37)	$e^{-\frac{1}{4}x^2/\beta}\sinh(ax) \qquad \operatorname{Re}\beta>0$	$\pi^{1/2}\beta^{1/2}e^{\beta(a^2-y^2)}\sin(2a\beta y)$
(38)	$e^{-i\pi a x^2}\operatorname{csch}(\pi x)$ $\operatorname{Im}a<0$	$\tfrac{1}{2}-e^{-y+i\pi a}+e^{-2y+4i\pi a}-e^{-3y+9i\pi a}+\dots$ $-\dfrac{1}{a^{1/2}}\exp\left[i\left(\dfrac{\pi}{4}+\dfrac{y^2}{4\pi a}\right)\right]$ $\times[e^{-(\frac{1}{2}y+\frac{1}{4}i\pi)/a}+e^{-(3y/2+9i\pi/4)/a}+\dots]$
(39)	$x^{-1}e^{-x^2}\sinh(x^2)$	$\tfrac{1}{4}\pi\operatorname{Erfc}(2^{-3/2}y)$
(40)	$\sinh x\log(1-e^{-2x})$	$(1+y^2)^{-1}[2y(1+y^2)^{-1}$ $-\tfrac{1}{2}\pi\tanh(\tfrac{1}{2}\pi y)]$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \quad y>0$
(41)	$\cos(ax)\operatorname{csch}(\beta x)$ $\lvert\operatorname{Im} a\rvert<\operatorname{Re}\beta$	$\tfrac{1}{2}\pi\beta^{-1}\sinh(\pi\beta^{-1}y)$ $\times[\cosh(\pi\beta^{-1}y)+\cosh(\pi a\beta^{-1})]^{-1}$
(42)	$\sin(\pi x^2)\operatorname{ctnh}(\pi x)$	$\tfrac{1}{2}\tanh(\tfrac{1}{2}y)\sin(\tfrac{1}{4}\pi+\tfrac{1}{4}y^2/\pi)$
(43)	$\cos(\pi x^2)\operatorname{ctnh}(\pi x)$	$\tfrac{1}{2}\tanh(\tfrac{1}{2}y)[1-\cos(\tfrac{1}{4}\pi+\tfrac{1}{4}y^2/\pi)]$
(44)	$\operatorname{csch}(ax)\sin(\pi^{-1}a^2x^2) \quad a>0$	$\tfrac{1}{2}\pi a^{-1}\sin(\tfrac{1}{4}\pi a^{-2}y^2)\operatorname{csch}(\tfrac{1}{2}\pi a^{-1}y)$
(45)	$\cos(\pi^{-1}a^2x^2)\operatorname{csch}(ax)$ $a>0$	$\tfrac{1}{2}\pi a^{-1}[\cosh(\tfrac{1}{2}\pi a^{-1}y)$ $-\cos(\tfrac{1}{4}\pi a^{-2}y^2)]\operatorname{csch}(\tfrac{1}{2}\pi a^{-1}y)$
(46)	$x^{-1}\cos(ax)\operatorname{sech}(\beta x)$ $\operatorname{Re}\beta>\lvert\operatorname{Im} a\rvert$	$-\tan^{-1}[\cosh(\tfrac{1}{2}\pi\beta^{-1}a)$ $\times\sinh(\tfrac{1}{2}\pi\beta^{-1}y)]$
(47)	$[\sinh(\tfrac{1}{2}x)]^{-\frac{1}{2}}\sin[2a\sinh(\tfrac{1}{2}x)]$ $a>0$	$-i(\pi a)^{\frac{1}{2}}[I_{\frac{1}{4}-iy}(a)K_{\frac{1}{4}+iy}(a)$ $-I_{\frac{1}{4}+iy}(a)K_{\frac{1}{4}-iy}(a)]$
(48)	$[\sinh(\tfrac{1}{2}x)]^{-\frac{1}{2}}\cos[2a\sinh(\tfrac{1}{2}x)]$ $a>0$	$-i(\pi a)^{\frac{1}{2}}[I_{-\frac{1}{4}-iy}(a)K_{-\frac{1}{4}+iy}(a)$ $-I_{-\frac{1}{4}+iy}(a)K_{-\frac{1}{4}-iy}(a)]$
(49)	$\dfrac{\sinh[\tfrac{1}{2}\pi(\tfrac{1}{2}x)^{\frac{1}{2}}]\sin[\tfrac{1}{2}\pi(\tfrac{1}{2}x)^{\frac{1}{2}}]}{\cosh[\pi(\tfrac{1}{2}x)^{\frac{1}{2}}]+\cos[\pi(\tfrac{1}{2}x)^{\frac{1}{2}}]}$	$\left[\dfrac{\partial\theta_1(z\mid q)}{\partial z}\right]_{z=0,\ q=e^{-4y}}$
	For similar integrals see Glaisher, J. W. L., 1871: *Quart. J. Math., Oxford Series* 11, 328-343.	
(50)	$\tan^{-1}(\cosh a\operatorname{csch}x)$ $\lvert\operatorname{Im} a\rvert<\tfrac{1}{2}\pi$	$-(\tfrac{1}{2}\pi)y^{-1}\cosh(ay)\operatorname{sech}(\tfrac{1}{2}\pi y)$
(51)	$(x^2+a^2)^{-\frac{1}{2}}\sinh^{-1}(x/a)$ $\operatorname{Re} a>0$	$\tfrac{1}{2}\pi K_0(ay)$

Hyperbolic functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(52)	$e^{-2\nu\sinh^{-1}(\frac{1}{2}x/a)}$ $\times[x(x^2+4a^2)]^{-\frac{1}{2}} \qquad \mathrm{Re}\,\nu>-\frac{3}{4}$	$(\frac{1}{2}\pi y)^{\frac{1}{2}}\sin(\nu\pi+\frac{1}{4}\pi)$ $\times\{I_{\nu+\frac{1}{4}}(ay)K_{\nu-\frac{1}{4}}(ay)$ $-I_{\nu-\frac{1}{4}}(ay)K_{\nu+\frac{1}{4}}(ay)\}$

2.10. Orthogonal polynomials

(1)	$x^{-\frac{1}{2}}P_{2n+1}(x/a) \qquad 0<x<a$ $0 \qquad a<x<\infty$	$(-1)^{n+1}(\frac{1}{2}\pi)^{\frac{1}{2}}y^{-\frac{1}{2}}J_{2n+3/2}(ay)$
(2)	$P_n(1-2x^2) \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$\frac{1}{2}\pi[J_{n+\frac{1}{2}}(\frac{1}{2}y)]^2$
(3)	$(a^2-x^2)^{-\frac{1}{2}}T_{2n+1}(x/a)$ $0<x<a$ $0 \qquad a<x<\infty$	$(-1)^n\frac{1}{2}\pi\,J_{2n+1}(ay)$
(4)	$(1-x^2)^{\nu-\frac{1}{2}}C^{\nu}_{2n+1}(x) \qquad 0<x<1$ $0 \qquad 1<x<\infty$ $\mathrm{Re}\,\nu>-\frac{1}{2}$	$(-1)^n\pi\dfrac{\Gamma(2n+2\nu+1)J_{2n+\nu+1}(y)}{(2n+1)!\Gamma(\nu)(2y)^{\nu}}$
(5)	$(1-x^2)^{\nu}P^{(\nu,\nu)}_{2n+1}(x) \qquad 0<x<1$ $0 \qquad 1<x<\infty$ $\mathrm{Re}\,\nu>-1$	$\dfrac{(-1)^n\pi^{\frac{1}{2}}\Gamma(2n+\nu+2)J_{2n+\nu+3/2}(y)}{2^{\frac{1}{2}-\nu}(2n+1)!\,y^{\nu+\frac{1}{2}}}$
(6)	$[(1-x)^{\nu}(1+x)^{\mu}-(1+x)^{\nu}(1-x)^{\mu}]$ $\times P^{(\nu,\mu)}_{2n}(x) \qquad 0<x<1$ $0 \qquad 1<x<\infty$ $\mathrm{Re}\,\nu>-1, \quad \mathrm{Re}\,\mu>-1$	$(-1)^{n+1}2^{2n+\nu+\mu}[(2n)!]^{-1}$ $\times\mathrm{B}(2n+\nu+1,\ 2n+\mu+1)\,y^{2n}$ $\times ie^{iy}[{}_1F_1(2n+\mu+1;4n+\nu+\mu+2;-2iy)$ $-{}_1F_1(2n+\nu+1;4n+\nu+\mu+2;-2iy)]$

Orthogonal polynomials (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(7)	$[(1-x)^{\nu}(1+x)^{\mu}+(1+x)^{\nu}(1-x)^{\mu}]$ $\times P_{2n+1}^{(\nu,\mu)}(x) \qquad 0<x<1$ $0 \qquad 1<x<\infty$ $\operatorname{Re}\nu>-1, \quad \operatorname{Re}\mu>-1$	$(-1)^{n+1}\,2^{2n+\nu+\mu+1}\,[(2n+1)!]^{-1}$ $\times \mathrm{B}(2n+\nu+2,\ 2n+\mu+2)\,y^{2n+1}$ $\times e^{iy}[{}_1F_1(2n+\nu+2;4n+\nu+\mu+4;-2iy)$ $+{}_1F_1(2n+\mu+2;4n+\nu+\mu+4;-2iy)]$
(8)	$e^{-\frac{1}{2}x^2}\,\mathrm{He}_{2n+1}(2^{\frac{1}{2}}x)$	$(-1)^n 2^{-\frac{1}{2}}\pi^{\frac{1}{2}}e^{-\frac{1}{2}y^2}\,\mathrm{He}_{2n+1}(2^{\frac{1}{2}}y)$
(9)	$e^{-\frac{1}{2}x^2a^2}\,\mathrm{He}_{2n+1}(ax)$ $\lvert\arg a\rvert<\pi/4$	$(-1)^n(\tfrac{1}{2}\pi)^{\frac{1}{2}}\,a^{-2n-2}\,y^{2n+1}\,e^{-\frac{1}{2}a^{-2}y^2}$
(10)	$e^{-\frac{1}{2}x^2/a}\,\mathrm{He}_{2n+1}[x\,a^{-\frac{1}{2}}(1-a)^{-\frac{1}{2}}]$ $\operatorname{Re}a>0, \quad a\neq 1$	$(-1)^n 2^{-\frac{1}{2}}\pi^{\frac{1}{2}}a^{n+1}(1-a)^{-n-\frac{1}{2}}$ $\times e^{-\frac{1}{2}ay^2}\,\mathrm{He}_{2n+1}(y)$
(11)	$e^{-\frac{1}{2}x^2}\,\mathrm{He}_n(x)\,\mathrm{He}_{n+2m+1}(x)$	$(-1)^m\,2^{-\frac{1}{2}}\pi^{\frac{1}{2}}n!\,y^{2m}\,e^{-\frac{1}{2}y^2}\,L_n^{2m+1}(y^2)$
(12)	$e^{-ax}x^{\nu-2n-1}L_{2n}^{\nu-1-2n}(ax)$ $\operatorname{Re}a>0, \quad \operatorname{Re}\nu>2n$	$(-1)^n i\,\Gamma(\nu)\,[2(2n)!]^{-1}$ $\times y^{2n}[(a-iy)^{-\nu}-(a+iy)^{-\nu}]$
(13)	$e^{-ax}x^{\nu-2n-2}L_{2n+1}^{\nu-2n-2}(ax)$ $\operatorname{Re}a>0, \quad \operatorname{Re}\nu>2n+1$	$(-1)^{n+1}\Gamma(\nu)\,[2(2n+1)!]^{-1}$ $\times y^{2n+1}[(a+iy)^{-\nu}+(a-iy)^{-\nu}]$
(14)	$e^{-\frac{1}{2}x^2}L_n(x^2)$	$(-1)^n\tfrac{1}{2}in!\,(2\pi)^{-\frac{1}{2}}[D_{-n-1}^2(iy)$ $-D_{-n-1}^2(-iy)]$
(15)	$x^{2n+1}e^{-\frac{1}{2}x^2}L_n^{n+\frac{1}{2}}(\tfrac{1}{2}x^2)$	$(\tfrac{1}{2}\pi)^{\frac{1}{2}}y^{2n+1}e^{-\frac{1}{2}y^2}L_n^{n+\frac{1}{2}}(\tfrac{1}{2}y^2)$
(16)	$x^{2m}e^{-\frac{1}{2}x^2}L_n^{2m+1}(x^2)$	$(\tfrac{1}{2}\pi)^{\frac{1}{2}}(-1)^m(n!)^{-1}e^{-\frac{1}{2}y^2}$ $\times \mathrm{He}_n(y)\,\mathrm{He}_{n+2m+1}(y)$

2.11. Gamma functions (including incomplete gamma function) and related functions; Legendre function

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(1)	$[\psi(a+ix)-\psi(a-ix)] \qquad a>0$	$i\pi e^{-ay}(1-e^{-y})^{-1}$
(2)	$e^{-a^2x^2}\operatorname{Erf}(iax)$	$\frac{1}{4}i\pi a^{-1}e^{-\frac{1}{4}a^{-2}y^2}$
(3)	$x^{-\frac{1}{2}}\operatorname{Erf}(ax^{\frac{1}{2}}) \qquad a>0$	$\frac{1}{2(2\pi y)^{\frac{1}{2}}}\left\{\log\left[\frac{y+a(2y)^{\frac{1}{2}}+a^2}{y-a(2y)^{\frac{1}{2}}+a^2}\right] + 2\tan^{-1}\left[\frac{a(2y)^{\frac{1}{2}}}{y-a^2}\right]\right\}$
(4)	$\operatorname{Erfc}(ax) \qquad a>0$	$y^{-1}(1-e^{-\frac{1}{4}a^{-2}y^2})$
(5)	$e^{\frac{1}{2}x^2}\operatorname{Erfc}(2^{-\frac{1}{2}}x)$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}e^{\frac{1}{2}y^2}\operatorname{Erfc}(2^{-\frac{1}{2}}y)$
(6)	$[e^{-bx}\operatorname{Erfc}(ab-\frac{1}{2}x/a)$ $-e^{bx}\operatorname{Erfc}(ab+\frac{1}{2}x/a)]$ $\qquad a,\ b>0$	$2e^{-a^2b^2}y(y^2+b^2)^{-1}e^{-a^2y^2}$
(7)	$\operatorname{si}(ax) \qquad a>0$	$0 \qquad 0<y<a$ $-\frac{1}{2}\pi y^{-1} \qquad a<y<\infty$
(8)	$(x^2+a^2)^{-1}\operatorname{si}(bx) \qquad a,\ b>0$	$\frac{1}{2}\pi a^{-1}\operatorname{Ei}(-ab)\sinh(ay) \qquad 0<y<b$ $\frac{1}{4}\pi a^{-1}e^{-ay}[\operatorname{Ei}(-ay)+\overline{\operatorname{Ei}}(ay) -\operatorname{Ei}(-ab)-\overline{\operatorname{Ei}}(ab)] +\frac{1}{2}\pi a^{-1}\operatorname{Ei}(-ay)\sinh(ay) \qquad b<y<\infty$
(9)	$\operatorname{si}(\frac{1}{4}ax^{-1})$	$-\frac{1}{2}\pi y^{-1}J_0[(ay)^{\frac{1}{2}}]$

Gamma function etc. (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(10)	$(1-2b\cos x+b^2)^{-1}\,\mathrm{si}(ax)$ $a>0, \quad 0<b<1$	$-\dfrac{\pi(b^m+b^{m+1})}{4y(1-b)(1-b^2)} \qquad y=a-m$ $-\dfrac{\pi(2+2b-b^m-b^{m+1})}{4y(1-b)(1-b^2)} \qquad y=a+m$ $-\dfrac{\pi b^{m+1}}{2y(1-b)(1-b^2)}$ $a-m-1<y<a-m$ $-\dfrac{\pi(1+b-b^{m+1})}{2y(1-b)(1-b^2)}$ $a+m<y<a+m+1$ $m=0,1,2,\ldots$
(11)	$\mathrm{Si}(bx) \qquad 0<x<a$ $0 \qquad a<x<\infty$ $b>0$	$\tfrac{1}{2}y^{-1}[\mathrm{Si}(ay+ab)-\mathrm{Si}(ay-ab)$ $-2\cos(ay)\,\mathrm{Si}(ab)]$
(12)	$x^{-1}\,\mathrm{Si}(ax) \qquad a>0$	$\tfrac{1}{2}L_2(ay^{-1})-\tfrac{1}{2}L_2(-ay^{-1})$
(13)	$(x^2+b^2)^{-1}\,\mathrm{Si}(ax) \qquad a,\ b>0$	$\tfrac{1}{4}\pi b^{-1}\{e^{-by}[\overline{\mathrm{Ei}}(by)-\mathrm{Ei}(-ab)]$ $+e^{by}[\mathrm{Ei}(-ab)-\mathrm{Ei}(-by)]\}$ $0<y<a$ $\tfrac{1}{4}\pi b^{-1}e^{-by}[\overline{\mathrm{Ei}}(ab)-\mathrm{Ei}(-ab)]$ $a<y<\infty$
(14)	$\mathrm{ci}(ax) \qquad a>0$	$-\tfrac{1}{2}y^{-1}\log\lvert a^{-2}y^2-1\rvert$
(15)	$(x^2+a^2)^{-1}\,\mathrm{ci}(bx) \qquad a,\ b>0$	$\tfrac{1}{2}\pi\sinh(ay)\,\mathrm{Ei}(-ab) \qquad 0<y<b$ $\tfrac{1}{2}\pi\sinh(ay)\,\mathrm{Ei}(-ay)$ $+\tfrac{1}{4}\pi e^{-ay}[\mathrm{Ei}(-ay)+\overline{\mathrm{Ei}}(ay)$ $-\mathrm{Ei}(-ab)-\overline{\mathrm{Ei}}(ab)] \qquad b<y<\infty$

Gamma function etc. (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(16)	$e^{-ax}\operatorname{ci}(bx)$ $\operatorname{Re} a>0, \quad b>0$	$\frac{1}{2(a^2+y^2)}\left\{a\tan^{-1}\left(\frac{2ay}{a^2+b^2-y^2}\right) - y\log\frac{[(a^2+b^2-y^2)^2+4a^2y^2]^{1/2}}{b^2}\right\}$
(17)	$\operatorname{Ci}(bx) \qquad 0<x<a$ $0 \qquad a<x<\infty$ $b>0$	$\tfrac{1}{2}y^{-1}\{-2\cos(ay)\operatorname{Ci}(ab) + \operatorname{Ci}(ay+ab)+\operatorname{Ci}(ay-ab) + \log[b^2(y^2-b^2)^{-1}]\} \qquad b<y<\infty$
(18)	$\operatorname{Ei}(-ax) \qquad a>0$	$-\tfrac{1}{2}y^{-1}\log(1+a^{-2}y^2)$
(19)	$e^{-\beta x}\operatorname{Ei}(-ax)$ $a>0, \quad \operatorname{Re}\beta>0$	$\frac{1}{y^2+\beta^2}\left\{\tfrac{1}{2}y\log\frac{a^2}{(a+\beta)^2+y^2} + \beta\tan^{-1}[y/(a+b)]\right\}$
(20)	$\operatorname{li}(e^{-ax}) \qquad a>0$	$-\tfrac{1}{2}y^{-1}\log(1+y^2/a^2)$
(21)	$x^{-1/2}S(x)$	$\pi^{1/2}2^{-3/2}y^{-1/2} \qquad 0<y<1$ $0 \qquad 1<y<\infty$
(22)	$(x^2+2)^{-1/2}P_\nu^{-1}(x^2+1)$ $-2<\operatorname{Re}\nu<1$	$2^{-1/2}\pi^{-1}\sin(\nu\pi)yK^2_{\nu+1/2}(2^{-1/2}y)$
(23)	$(x^2+2)^{-1/2}Q_\nu^1(x^2+1)$ $\operatorname{Re}\nu>-3/2$	$-2^{-3/2}\pi y K_{\nu+1/2}(2^{-1/2}y)\,I_{\nu+1/2}(2^{-1/2}y)$
(24)	$0 \qquad 0<x<a$ $(x^2-a^2)^{1/2\nu-1/4}\,\mathrm{P}_0^{1/2-\nu}(ax^{-1})$ $a<x<\infty$ $\lvert\operatorname{Re}\nu\rvert<\tfrac{1}{2}$	$y^{-\nu-1/2}\cos(ay-\tfrac{1}{2}\nu\pi+\tfrac{1}{4}\pi)$

2.12. Bessel functions of argument kx

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(1)	$J_0(ax) \qquad a>0$	$0 \qquad 0<y<a$ $(y^2-a^2)^{-1/2} \qquad a<y<\infty$
(2)	$J_{2n+1}(ax) \qquad a>0$	$(-1)^n(a^2-y^2)^{-1/2}\,T_{2n+1}(y/a) \qquad 0<y<a$ $0 \qquad a<y<\infty$
(3)	$J_\nu(ax) \qquad \operatorname{Re}\nu>-2, \quad a>0$	$(a^2-y^2)^{-1/2}\sin[\nu\sin^{-1}(y/a)] \qquad 0<y<a$ $\dfrac{a^\nu\cos(\frac{1}{2}\nu\pi)}{(y^2-a^2)^{1/2}[y+(y^2-a^2)^{1/2}]^\nu} \qquad a<y<\infty$
(4)	$x^{-1}J_0(ax) \qquad a>0$	$\sin^{-1}(y/a) \qquad 0<y<a$ $\frac{1}{2}\pi \qquad a<y<\infty$
(5)	$x^{-1}J_\nu(ax) \qquad \operatorname{Re}\nu>-1, \quad a>0$	$\nu^{-1}\sin[\nu\sin^{-1}(y/a)] \qquad 0<y<a$ $\dfrac{a^\nu\sin(\frac{1}{2}\nu\pi)}{\nu[y+(y^2-a^2)^{1/2}]^\nu} \qquad a<y<\infty$
(6)	$x^{-2}J_\nu(ax) \qquad \operatorname{Re}\nu>0, \quad a>0$	$\dfrac{(a^2-y^2)^{1/2}\sin[\nu\sin^{-1}(y/a)]}{\nu^2-1}-\dfrac{y\cos[\nu\sin^{-1}(y/a)]}{\nu(\nu^2-1)} \qquad 0<y<a$ $\dfrac{-a^\nu\cos(\frac{1}{2}\nu\pi)[y+\nu(y^2-a^2)^{1/2}]}{\nu(\nu^2-1)[y+(y^2-a^2)^{1/2}]^\nu} \qquad a<y<\infty$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(7)	$x^{-1/2}J_{2n+3/2}(ax) \qquad a>0$	$(-1)^n\pi^{1/2}(2y)^{-1/2}P_{2n+1}(y/a) \qquad 0<y<a$ $0 \qquad a<y<\infty$
(8)	$x^{\nu}J_{\nu}(ax)$ $-1<\operatorname{Re}\nu<1/2, \quad a>0$	$0 \qquad 0<y<a$ $\pi^{1/2}2^{\nu}a^{\nu}[\Gamma(1/2-\nu)]^{-1}(y^2-a^2)^{-\nu-1/2} \qquad a<y<\infty$
(9)	$x^{-\nu}J_{\nu+1}(ax)$ $\operatorname{Re}\nu>-1/2, \quad a>0$	$\pi^{1/2}2^{-\nu}a^{-\nu-1}[\Gamma(\nu+1/2)]^{-1}$ $\times y(a^2-y^2)^{\nu-1/2} \qquad 0<y<a$ $0 \qquad a<y<\infty$
(10)	$x^{-\nu}J_{2n+\nu+1}(ax)$ $\operatorname{Re}\nu>-1/2, \quad a>0$	$(-1)^n2^{\nu-1}a^{-\nu}(2n+1)!\,\Gamma(\nu)$ $\times[\Gamma(2n+2\nu+1)]^{-1}(a^2-y^2)^{\nu-1/2}$ $\times C_{2n+1}^{\nu}(y/a) \qquad 0<y<a$ $0 \qquad y>a$
(11)	$x^{2\mu-1}J_{2\nu}(ax)$ $-\operatorname{Re}\nu-1/2<\operatorname{Re}\mu<3/4$ $a>0$	$4^{\mu}a^{-2\mu-1}y\,\Gamma(1/2+\nu+\mu)[\Gamma(1/2+\nu-\mu)]^{-1}$ $\times{}_2F_1(1/2+\mu+\nu,1/2+\mu-\nu;3/2;a^{-2}y^2)$ $0<y<a$ $(1/2a)^{2\nu}y^{-2\nu-2\mu}\Gamma(2\nu+2\mu)$ $\times[\Gamma(2\nu+1)]^{-1}\sin(\pi\nu+\pi\mu)$ $\times{}_2F_1(1/2+\nu+\mu,\nu+\mu;2\nu+1;a^2y^{-2})$ $a<y<\infty$
(12)	$(x^2+\beta^2)^{-1}J_0(ax)$ $a>0, \quad \operatorname{Re}\beta>0$	$\beta^{-1}\sinh(\beta y)K_0(a\beta) \qquad 0<y<a$
(13)	$x(x^2+\beta^2)^{-1}J_0(ax)$ $a>0, \quad \operatorname{Re}\beta>0$	$1/2\pi e^{-\beta y}I_0(a\beta) \qquad a<y<\infty$
(14)	$x^{1/2}(x^2+\beta^2)^{-1}J_{2n+1/2}(ax)$ $a>0, \quad \operatorname{Re}\beta>0$ $n=0,1,2,\ldots$	$(-1)^n\sinh(\beta y)K_{2n+1/2}(a\beta)$ $0<y<a$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \quad y>0$
(15)	$x^{\nu}(x^2+\beta^2)^{-1}J_{\nu}(ax)$ $-1<\mathrm{Re}\,\nu<5/2$ $a>0, \quad \mathrm{Re}\,\beta>0$	$\beta^{\nu-1}\sinh(\beta y)K_{\nu}(a\beta)$ $0<y<a$
(16)	$x^{1-\nu}(x^2+\beta^2)^{-1}J_{\nu}(ax)$ $\mathrm{Re}\,\nu>-3/2$ $a>0, \quad \mathrm{Re}\,\beta>0$	$\tfrac{1}{2}\pi\beta^{-\nu}e^{-\beta y}I_{\nu}(a\beta) \quad a<y<\infty$
(17)	$x^{-1}e^{-ax}J_0(\beta x)$ $\mathrm{Re}\,a>\lvert\mathrm{Im}\,\beta\rvert$	$\sin^{-1}\left(\dfrac{2y}{r_1+r_2}\right)$ $r_1^2=a^2+(y+\beta)^2$ $r_2^2=a^2+(y-\beta)^2$
(18)	$x^{\nu-1}e^{-ax}J_{\nu}(\beta x)$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}, \quad \mathrm{Re}\,a>\lvert\mathrm{Im}\,\beta\rvert$	$\dfrac{\beta^{\nu}\Gamma(2\nu+1)y}{2^{\nu+1}\Gamma(\nu+1)}$ $\times\displaystyle\int_0^{\pi}\frac{\sin x\,dx}{(a^2+\beta^2+2iay\cos x-y^2\cos^2x)^{\nu+\frac{1}{2}}}$
(19)	$x^{\nu}\cos(x)J_{\nu}(x)$ $-1<\mathrm{Re}\,\nu<\tfrac{1}{2}$	$\pi^{\frac{1}{2}}2^{\nu-1}[\Gamma(\tfrac{1}{2}-\nu)]^{-1}(y^2+2y)^{-\nu-\frac{1}{2}}$ $0<y<2$ $\pi^{\frac{1}{2}}2^{\nu-1}[\Gamma(\tfrac{1}{2}-\nu)]^{-1}[(y^2+2y)^{-\nu-\frac{1}{2}}$ $+(y^2-2y)^{-\nu-\frac{1}{2}}] \quad 2<y<\infty$
(20)	$x^{-\nu}\cos(x)J_{\nu+1}(x)$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$2^{-\nu-1}\pi^{\frac{1}{2}}[\Gamma(\nu+\tfrac{1}{2})]^{-1}$ $\times(y-1)(2y-y^2)^{\nu-\frac{1}{2}} \quad 0<y<2$ $0 \quad 2<y<\infty$
(21)	$x^{-\nu}\sin(x)J_{\nu}(x)$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\pi^{\frac{1}{2}}2^{-\nu-1}[\Gamma(\nu+\tfrac{1}{2})]^{-1}$ $\times(2y-y^2)^{\nu-\frac{1}{2}} \quad 0<y<2$ $0 \quad 2<y<\infty$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(22)	$[x^{-1}J_1(ax)]^2 \qquad a>0$	$\tfrac{1}{2}y-(4a/3\pi)[(1+\tfrac{1}{4}a^{-2}y^2)E(\tfrac{1}{2}a^{-1}y)$ $+(1-\tfrac{1}{4}a^{-2}y^2)K(\tfrac{1}{2}a^{-1}y)$ $\qquad 0<y\le 2a$
(23)	$J_{n+\frac{1}{2}}(ax)\,J_{n+\frac{1}{2}}(bx)$ $\qquad a,\ b>0$	$\tfrac{1}{2}a^{-\frac{1}{2}}b^{-\frac{1}{2}}\mathrm{P}_n\left(\dfrac{a^2+b^2-y^2}{2ab}\right) \qquad 0<y<a+b$ $0 \qquad a+b<y<\infty$
(24)	$x^{\frac{1}{2}}[J_{\frac{1}{4}}(ax)]^2 \qquad a>0$	$(2/\pi)^{\frac{1}{2}}y^{-\frac{1}{2}}(4a^2-y^2)^{-\frac{1}{2}} \qquad 0<y<2a$ $0 \qquad 2a<y<\infty$
(25)	$x^{\frac{1}{2}}J_{\nu+\frac{1}{2}}(ax)\,J_{-\nu+\frac{1}{4}}(ax)$ $\qquad a>0$	$\{[(2a+y)^{\frac{1}{2}}+i(2a-y)^{\frac{1}{2}}]^{4\nu}$ $+[(2a+y)^{\frac{1}{2}}-i(2a-y)^{\frac{1}{2}}]^{4\nu}\}$ $\times(4a)^{-2\nu}\pi^{-\frac{1}{2}}(8a^2y-2y^3)^{-\frac{1}{2}} \qquad 0<y<2a$ $0 \qquad 2a<y<\infty$
(26)	$x^{\frac{1}{2}}J_{\nu+\frac{1}{4}}(ax)\,J_{-\nu+\frac{1}{4}}(ax)$ $\qquad a>0$	$(2/\pi)^{\frac{1}{2}}y^{-\frac{1}{2}}(4a^2-y^2)^{-\frac{1}{2}}$ $\times\cos[2\nu\cos^{-1}(\tfrac{1}{2}a^{-1}y)] \qquad 0<y<2a$ $0 \qquad 2a<y<\infty$
(27)	$J_\nu(ax)\,J_\nu(bx)$ $\qquad 0<a<b, \quad \mathrm{Re}\,\nu>-1$	$0 \qquad 0<y<b-a$ $\tfrac{1}{2}a^{-\frac{1}{2}}b^{-\frac{1}{2}}\mathrm{P}_{\nu-\frac{1}{2}}(A) \qquad b-a<y<b+a$ $-\pi^{-1}a^{-\frac{1}{2}}b^{-\frac{1}{2}}\cos(\nu\pi)\,Q_{\nu-\frac{1}{2}}(-A) \qquad b+a<y<\infty$ $A=(b^2+a^2-y^2)/(2ab)$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \quad y>0$
(28)	$x^{\nu-\mu}J_\mu(ax)\,J_\nu(bx)$ $-1<\mathrm{Re}\,\nu<\mathrm{Re}\,\mu+1$ $0<a<b$	$0 \qquad 0<y<b-a$
(29)	$x^{\nu-\mu-2}J_\mu(ax)\,J_\nu(bx)$ $0<\mathrm{Re}\,\nu<\mathrm{Re}\,\mu+3$ $0<a<b$	$2^{\nu-\mu-1}a^{\mu}b^{-\nu}[\Gamma(\mu+1)]^{-1}\Gamma(\nu)y$ $0<y<b-a$
(30)	$x^{\lambda}J_\mu(ax)\,J_\nu(bx)$	see Bailey, W. N., 1936: *Proc. London Math. Soc.* (2), 40, 37-48
(31)	$Y_0(ax) \qquad a>0$	$2\pi^{-1}(a^2-y^2)^{-1/2}\sin^{-1}(a^{-1}y)$ $0<y<a$ $2\pi^{-1}(y^2-a^2)^{-1/2}\log[a^{-1}y-(a^{-2}y^2-1)^{1/2}]$ $a<y<\infty$
(32)	$Y_1(ax) \qquad a>0$	$0 \qquad 0<y<a$ $-ya^{-1}(y^2-a^2)^{-1/2} \qquad a<y<\infty$
(33)	$Y_\nu(ax) \qquad a>0$ $-2<\mathrm{Re}\,\nu<2$	$\mathrm{ctn}(\tfrac{1}{2}\nu\pi)(a^2-y^2)^{-1/2}\sin[\nu\sin^{-1}(y/a)]$ $0<y<a$ $\tfrac{1}{2}\csc(\tfrac{1}{2}\nu\pi)(y^2-a^2)^{-1/2}$ $\times\{a^{-\nu}\cos(\nu\pi)[y-(y^2-a^2)^{1/2}]^{\nu}$ $-a^{\nu}[y-(y^2-a^2)^{1/2}]^{-\nu}\} \qquad a<y<\infty$
(34)	$x^{-1}Y_0(ax) \qquad a>0$	$0 \qquad 0<y<a$ $\log[a^{-1}y-(a^{-2}y^2-1)^{1/2}]$ $a<y<\infty$
(35)	$x^{-1}Y_\nu(ax)$ $\lvert\mathrm{Re}\,\nu\rvert<1, \quad a>0$	$-\nu^{-1}\tan(\tfrac{1}{2}\nu\pi)\sin[\nu\sin^{-1}(a^{-1}y)]$ $0<y<a$ $(2\nu)^{-1}\sec(\tfrac{1}{2}\nu\pi)\{a^{-\nu}\cos(\nu\pi)$ $\times[y-(y^2-a^2)^{1/2}]^{\nu}-a^{\nu}[y-(y^2-a^2)^{1/2}]^{-\nu}\}$ $a<y<\infty$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)dx \qquad y > 0$
(36)	$x^\nu Y_{\nu-1}(ax)$ $\lvert\mathrm{Re}\,\nu\rvert < \tfrac{1}{2}, \quad a > 0$	$0 \qquad 0 < y < a$ $\dfrac{\pi^{\frac{1}{2}} 2^\nu a^{\nu-1}}{\Gamma(\frac{1}{2}-\nu)} y(y^2-a^2)^{-\nu-\frac{1}{2}}$ $a < y < \infty$
(37)	$x^\nu \sin(ax)\, Y_\nu(ax)$ $-3/2 < \mathrm{Re}\,\nu < 1/2, \quad a > 0$	$2^{\nu-1}\pi^{\frac{1}{2}} a^\nu [\Gamma(\frac{1}{2}-\nu)]^{-1}$ $\times(y^2+2ay)^{-\nu-\frac{1}{2}} \qquad 0 < y < 2a$ $2^{\nu-1}\pi^{\frac{1}{2}} a^\nu [\Gamma(\frac{1}{2}-\nu)]^{-1}$ $\times[(y^2+2ay)^{-\nu-\frac{1}{2}} - (y^2-2ay)^{-\nu-\frac{1}{2}}]$ $2a < y < \infty$
(38)	$[J_\nu(ax)\cos(ax-\nu\pi/2)$ $+ Y_\nu(ax)\sin(ax-\nu\pi/2)]$ $\lvert\mathrm{Re}\,\nu\rvert < 2, \quad a > 0$	$2^{-\nu-1} a^{-\nu} y^{-\frac{1}{2}} (y+2a)^{-\frac{1}{2}}$ $\times\{[(y+2a)^{\frac{1}{2}} + y^{\frac{1}{2}}]^{2\nu}$ $+ [(y+2a)^{\frac{1}{2}} - y^{\frac{1}{2}}]^{2\nu}\}$
(39)	$J_\nu(ax)\cos(\frac{1}{2}\nu\pi)$ $- Y_\nu(ax)\sin(\frac{1}{2}\nu\pi)$ $\lvert\mathrm{Re}\,\nu\rvert < 2$	$0 \qquad 0 < y < a$ $2^{-1} a^{-\nu} (y^2-a^2)^{-\frac{1}{2}}\{[y+(y^2-a^2)^{\frac{1}{2}}]^\nu$ $+ [y-(y^2-a^2)^{\frac{1}{2}}]^\nu\} \qquad a < y < \infty$
(40)	$x^\nu[J_\nu(ax)\cos(ax)$ $+ Y_\nu(ax)\sin(ax)]$ $-1 < \mathrm{Re}\,\nu < \frac{1}{2}$	$\dfrac{\pi^{\frac{1}{2}}(2a)^\nu}{\Gamma(\frac{1}{2}-\nu)} (y^2+2ay)^{-\nu-\frac{1}{2}}$
(41)	$x^\nu[J_\nu(ax)\cos(ax)$ $- Y_\nu(ax)\sin(ax)]$ $-1 < \mathrm{Re}\,\nu < \frac{1}{2}$	$0 \qquad 0 < y < 2a$ $\dfrac{2^\nu \pi^{\frac{1}{2}} b^\nu}{\Gamma(\frac{1}{2}-\nu)} (y^2-2ay)^{-\nu-\frac{1}{2}}$ $2a < y < \infty$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(42)	$x^{1/2}(x^2+\beta^2)^{-1}$ $\times\{J_\mu(ax)\cos[\frac{1}{2}(\frac{1}{2}-\mu)\pi]$ $+Y_\mu(ax)\sin[\frac{1}{2}(\frac{1}{2}-\mu)\pi]\}$ $\lvert\mathrm{Re}\,\mu\rvert<5/2$ $a>0,\quad \mathrm{Re}\,\beta>0$	$\beta^{-1/2}\sinh(\beta y)K_\mu(a\beta) \qquad 0<y<a$
(43)	$x^{1/2}J_{1/4}(\frac{1}{2}ax)\,Y_{1/4}(\frac{1}{2}ax)$	$0 \qquad 0<y<a$ $-(\frac{1}{2}\pi y)^{-1/2}(y^2-a^2)^{-1/2} \qquad y>a$
(44)	$e^{-\frac{1}{2}ax}I_0(\frac{1}{2}ax) \qquad \mathrm{Re}\,a>0$	$(2y)^{-1/2}(y^2+a^2)^{-1/2}$ $\times[y+(y^2+a^2)^{1/2}]^{1/2}$
(45)	$x^{-1}\sin(ax)\,I_1(bx)/I_2(cx)$	For this and related integrals see Timpe, A., 1912: *Math. Ann.* 71, 480-509
(46)	$K_0(ax) \qquad \mathrm{Re}\,a>0$	$(y^2+a^2)^{-1/2}\log[(y/a)+(1+y^2/a^2)^{1/2}]$
(47)	$xK_0(ax) \qquad \mathrm{Re}\,a>0$	$\frac{1}{2}\pi y(a^2+y^2)^{-3/2}$
(48)	$K_\nu(ax)$ $\lvert\mathrm{Re}\,\nu\rvert<2,\quad \nu\neq 0,\quad \mathrm{Re}\,a>0$	$\frac{1}{4}\pi a^{-\nu}\csc(\frac{1}{2}\nu\pi)(a^2+y^2)^{-1/2}$ $\times\{[(y^2+a^2)^{1/2}+y]^\nu-[(y^2+a^2)^{1/2}-y]^\nu\}$
(49)	$x^{1+\nu}K_\nu(ax)$ $\mathrm{Re}\,\nu>-3/2$ $\mathrm{Re}\,a>0$	$\pi^{1/2}(2a)^\nu\Gamma(3/2+\nu)\,y(y^2+a^2)^{-3/2-\nu}$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \sin(xy)\,dx \quad y > 0$
(50)	$x^{-\lambda} K_\mu(ax)$ $\operatorname{Re}(\lambda \pm \mu) < 2, \quad \operatorname{Re} a > 0$	$\dfrac{y\,\Gamma(\frac{1}{2}\mu - \frac{1}{2}\lambda + 1)\,\Gamma(1 - \frac{1}{2}\lambda - \frac{1}{2}\mu)}{2^\lambda a^{2-\lambda}}$ $\times {}_2F_1\left(\dfrac{2+\mu-\lambda}{2}, \dfrac{2-\lambda-\mu}{2}; \dfrac{3}{2}; \dfrac{-y^2}{a^2}\right)$
(51)	$\sinh(\frac{1}{2}ax)\, K_0(\frac{1}{2}ax)$ $\operatorname{Re} a > 0$	$\pi 2^{-3/2} a y^{-1/2} (y^2 + a^2)^{-1/2}$ $\times [y + (y^2 + a^2)^{1/2}]^{-1/2}$
(52)	$\sin(\beta x)\, K_0(ax)$ $\operatorname{Re} a > \lvert \operatorname{Im} \beta \rvert$	$\dfrac{\pi(\beta y)^{1/2}}{2R_1 R_2} \left(\dfrac{R_2 - R_1}{R_2 + R_1}\right)^{1/2}$ $R_1 = [a^2 + (\beta - y)^2]^{1/2}$ $R_2 = [a^2 + (\beta + y)^2]^{1/2}$
(53)	$x^{-\nu} \sinh(\frac{1}{2}x)\, K_\nu(\frac{1}{2}x)$ $-1/2 < \operatorname{Re} \nu < 3/2$	$-\frac{1}{2}\pi^{3/2} [\Gamma(\nu + \frac{1}{2})]^{-1} \sec(\nu\pi)\, y^{\nu - \frac{1}{2}}$ $\times (1 + y^2)^{\frac{1}{2}\nu - \frac{1}{4}} \sin[(\nu - \frac{1}{2}) \operatorname{ctn}^{-1} y]$
(54)	$x K_\nu(ax)\, I_\nu(\beta x)$ $\operatorname{Re} \nu > -3/2, \quad \operatorname{Re} a > \lvert \operatorname{Re} \beta \rvert$	$-\frac{1}{2}(a\beta)^{-3/2} y (u^2 - 1)^{-1/2} Q^1_{\nu - \frac{1}{2}}(u)$ $u = (2a\beta)^{-1}(a^2 + \beta^2 + y^2)$
(55)	$x^{\frac{1}{2}} I_{\frac{1}{4}}(\frac{1}{2}ax)\, K_{\frac{1}{4}}(\frac{1}{2}ax)$ $\operatorname{Re} a > 0$	$\pi^{\frac{1}{2}} (2y)^{-\frac{1}{2}} (y^2 + a^2)^{-\frac{1}{2}}$
(56)	$x^{\frac{1}{2}} I_{\frac{1}{4} - \frac{1}{2}\beta}(\frac{1}{2}ax)\, K_{\frac{1}{4} + \frac{1}{2}\beta}(\frac{1}{2}ax)$ $\operatorname{Re} a > 0, \quad \operatorname{Re} \beta < 5/2$	$(\frac{1}{2}\pi)^{\frac{1}{2}} a^{-\beta} y^{-\frac{1}{2}} (a^2 + y^2)^{-\frac{1}{2}}$ $\times [y + (a^2 + y^2)^{\frac{1}{2}}]^\beta$
(57)	$x^{-\frac{1}{2}} K_\nu(ax)\, I_\nu(ax)$ $\operatorname{Re} \nu > -\frac{3}{4}, \quad \operatorname{Re} a > 0$	$2^{-\frac{1}{2}} \pi^{\frac{1}{2}} y^{-\frac{1}{2}} e^{\nu\pi i} \dfrac{\Gamma(\frac{3}{4} + \nu)}{\Gamma(\frac{3}{4} - \nu)}$ $\times Q^{-\nu}_{-\frac{1}{4}}[(y^2 + 4a^2)^{\frac{1}{2}} y^{-1}]$ $\times P^{-\nu}_{-\frac{1}{4}}[(y^2 + 4a^2)^{\frac{1}{2}} y^{-1}]$

Bessel functions of kx (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(58)	$x^{-1/2}K_\nu^2(ax)$ $\lvert\mathrm{Re}\,\nu\rvert < 3/4, \quad \mathrm{Re}\,a>0$	$(\tfrac{1}{2}\pi)^{1/2} e^{2\nu\pi i} y^{-1/2} \dfrac{\Gamma(3/4+\nu)}{\Gamma(3/4-\nu)}$ $\times\{Q_{-1/4}^{-\nu}[(y^2+4a^2)y^{-1}]\}^2$
(59)	$x^{1/2}K_\nu^2(ax)$ $\lvert\mathrm{Re}\,\nu\rvert < 5/4, \quad \mathrm{Re}\,a>0$	$(\tfrac{1}{2}\pi)^{1/2} e^{2\nu\pi i} y^{1/2}(y^2+4a^2)^{-1/2}\dfrac{\Gamma(5/4+\nu)}{\Gamma(1/4-\nu)}$ $\times Q_{1/4}^{-\nu}[(y^2+4a^2)^{1/2}y^{-1}]$ $\times Q_{-3/4}^{-\nu}[(y^2+4a^2)^{1/2}y^{-1}]$
(60)	$x^{1/2}K_\nu(ax)K_{\nu+1}(ax)$ $-7/4<\mathrm{Re}\,\nu<3/4, \quad \mathrm{Re}\,a>0$	$(\tfrac{1}{2}\pi)^{1/2} e^{(2\nu+1)\pi i} y^{1/2}(y^2+4a^2)^{-1/2}\dfrac{\Gamma(7/4+\nu)}{\Gamma(-1/4-\nu)}$ $\times Q_{-1/4}^{-\nu}[(y^2+4a^2)^{1/2}y^{-1}]$ $\times Q_{-1/4}^{-\nu-1}[(y^2+4a^2)^{1/2}y^{-1}]$
(61)	$x K_\nu(ax)K_\nu(\beta x)$ $\lvert\mathrm{Re}\,\nu\rvert<3/2, \quad \mathrm{Re}\,(a+\beta)>0$	$2^{-2}\pi(a\beta)^{-3/2}y(u^2-1)^{-1/2}$ $\times\Gamma(3/2+\nu)\Gamma(3/2-\nu)P_{\nu-1/2}^{-1}(u)$ $u=(y^2+\beta^2+a^2)(2a\beta)^{-1}$
(62)	$[\mathbf{J}_\nu(ax)-\mathbf{J}_{-\nu}(ax)]$ $a>0$	$\left\{\dfrac{[y+i(a^2-y^2)^{1/2}]^\nu+[y-i(a^2-y^2)^{1/2}]^\nu}{a^\nu\csc(\tfrac{1}{2}\nu\pi)(a^2-y^2)^{1/2}}\right\}$ $\quad 0<y<a$ $0 \quad a<y<\infty$
(63)	$x^{-\nu}\mathbf{H}_\nu(ax)$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}, \quad a>0$	$\pi^{1/2}2^{-\nu}a^{-\nu}[\Gamma(\nu+\tfrac{1}{2})]^{-1}(a^2-y^2)^{\nu-1/2}$ $\quad 0<y<a$ $0 \quad a<y<\infty$
(64)	$U_n(2x,2a)x^{-(n+1)}$	see Mitra, S. C., 1934: *Bull. Calcutta Math. Soc.* 25, 173-178

2.13. Bessel functions of other arguments

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \quad y>0$
(1)	$x^{1/2} J_{1/4}(a^2x^2)$	$2^{-3/2}a^{-2}(\pi y)^{1/2} J_{1/4}(\tfrac{1}{4}a^{-2}y^2)$
(2)	$e^{-ax^2} J_m(ax^2)$	see Terazawa, K., 1916: *Proc. Roy. Soc. London, Ser. A.*, 92, 57-81
(3)	$x^{1/2}\cos(a^2x^2) J_{1/4}(a^2x^2)$	$(4a^2y)^{-1/2}\cos[(a^{-2}y^2-3\pi)/8]$
(4)	$x^{1/2}\sin(a^2x^2) J_{1/4}(a^2x^2)$	$-(4a^2y)^{-1/2}\sin[(a^{-2}y^2-3\pi)/8]$
(5)	$x^{1/2}[J_{1/8}(a^2x^2)]^2$	$-2^{-3/2}\pi^{1/2}y^{1/2}a^{-2}J_{1/8}(2^{-4}a^{-2}y^2) \times Y_{1/8}(2^{-4}a^{-2}y^2)$
(6)	$x^{1/2} J_{1/8-\nu}(a^2x^2) J_{1/8+\nu}(a^2x^2)$	$2^{1/2}\pi^{-1/2}y^{-3/2}[e^{\pi i/8} \times W_{\nu,1/8}(2^{-3}a^{-2}y^2e^{\frac{1}{2}\pi i}) \times W_{-\nu,1/8}(2^{-3}a^{-2}y^2e^{\frac{1}{2}\pi i}) + e^{-\pi i/8}W_{\nu,1/8}(2^{-3}a^{-2}y^2e^{-\frac{1}{2}\pi i}) \times W_{-\nu,1/8}(2^{-3}a^{-2}y^2e^{-\frac{1}{2}\pi i})]$
(7)	$x^{1/2} Y_{1/4}(a^2x^2)$	$-2^{-3/2}\pi^{1/2}a^{-2}y^{1/2}\mathbf{H}_{1/4}(\tfrac{1}{4}a^{-2}y^2)$
(8)	$x^{1/2} J_{1/8}(a^2x^2) Y_{1/8}(a^2x^2)$	$-2^{-3/2}\pi^{1/2}y^{1/2}a^{-2}[J_{1/8}(2^{-4}a^{-2}y^2)]^2$
(9)	$e^{-x^2} I_0(x^2)$	$2^{-3/2}\pi^{1/2}e^{-y^2/8} I_0(y^2/8)$
(10)	$x^{1/2}e^{-x^2} I_{1/4}(x^2)$	$\tfrac{1}{2}y^{-1/2}e^{-y^2/8}$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(11)	$x^{1/2}K_{1/4}(a^2x^2)$ $\lvert\arg a\rvert<\pi/4$	$2^{-5/2}\pi^{3/2}a^{-2}y^{1/2}[I_{1/4}(\tfrac14 a^{-2}y^2)$ $-\mathbf{L}_{1/4}(\tfrac14 a^{-2}y^2)]$
(12)	$x^{1/2}K_{1/8}(a^2x^2)\,I_{1/8}(a^2x^2)$ $\lvert\arg a\rvert<\pi/4$	$2^{-1/2}\pi^{1/2}(2a)^{-2}y^{1/2}K_{1/8}(2^{-4}a^{-2}y^2)$ $\times I_{1/8}(2^{-4}a^{-2}y^2)$
(13)	$x^{1/2}K_{1/8+\nu}(a^2x^2)\,I_{1/8-\nu}(a^2x^2)$ $\operatorname{Re}\nu<5/8, \quad \lvert\arg a\rvert<\pi/4$	$\pi^{1/2}2^{1/2}y^{-3/2}\dfrac{\Gamma(5/8-\nu)}{\Gamma(5/4)}$ $\times W_{\nu,1/8}(2^{-3}a^{-2}y^2)$ $\times M_{-\nu,1/8}(2^{-3}a^{-2}y^2)$
(14)	$x^{1/2}\,\mathbf{H}_{1/4}(a^2x^2)$	$-2^{-3/2}\pi^{1/2}a^{-2}y^{1/2}Y_{1/4}(\tfrac14 a^{-2}y^2)$
(15)	$x^{2\lambda}J_{2\nu}(ax^{-1})$ $-5/4<\operatorname{Re}\lambda<\operatorname{Re}\nu, \quad a>0$	$\dfrac{\pi^{1/2}a^{2\nu}\Gamma(\lambda-\nu+1)y^{2\nu-2\lambda-1}}{4^{2\nu-\lambda}\Gamma(2\nu+1)\,\Gamma(\nu-\lambda+\frac12)}$ $\times {}_0F_3(2\nu+1,\nu-\lambda,\nu-\lambda+\tfrac12;2^{-4}a^2y^2)$ $+\dfrac{a^{2\lambda+2}\Gamma(\nu-\lambda-1)y}{2^{2\lambda+3}\Gamma(\nu+\lambda+2)}$ $\times {}_0F_3(3/2,\lambda-\nu+2,\lambda+\nu+2;2^{-4}a^2y^2)$
(16)	$x^{-1}\sin(ax^{-1})J_{2n+1}(bx^{-1})$ $a>0, \quad b>0$	$(-1)^n(\tfrac12\pi)J_{2n+1}(cy^{1/2})J_{2n+1}(dy^{1/2})$ $c^2+d^2=4a, \quad cd=2b$
(17)	$x^{-1}\cos(ax^{-1})J_{2n}(bx^{-1})$ $a,\ b>0$	$(-1)^n(\tfrac12\pi)J_{2n}(cy^{1/2})J_{2n}(dy^{1/2})$ $c^2+d^2=4a, \quad cd=2b$
(18)	$x^{-1/2}\sin(ax^{-1})J_{2n-1/2}(ax^{-1})$ $a>0$	$(-1)^{n-1}\tfrac12\pi^{1/2}y^{-1/2}J_{4n-1}(2^{3/2}a^{1/2}y^{1/2})$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \sin(xy) dx \quad y > 0$
(19)	$x^{-1/2} \cos(ax^{-1}) J_{2n-3/2}(ax^{-1})$ $\quad a > 0$	$(-1)^{n-1} \tfrac{1}{2} \pi^{1/2} y^{-1/2} \times J_{4n-3}(2^{3/2} a^{1/2} y^{1/2})$
(20)	$x^{-1} K_0(ax^{-1})$ $\quad \mathrm{Re}\, a > 0$	$\pi K_0[(2ay)^{1/2}] J_0[(2ay)^{1/2}]$
(21)	$x^{-1} K_\nu(ax^{-1})$ $\quad -1 < \mathrm{Re}\,\nu < 1, \quad \mathrm{Re}\, a > 0$	$\pi K_\nu[(2ay)^{1/2}]\{J_\nu[(2ay)^{1/2}] \cos(\tfrac{1}{2}\nu\pi) - Y_\nu[(2ay)^{1/2}] \sin(\tfrac{1}{2}\nu\pi)\}$
(22)	$J_0(ax^{1/2})$ $\quad a > 0$	$y^{-1} \cos(\tfrac{1}{4} a^2 y^{-1})$
(23)	$J_\nu(ax^{1/2})$ $\quad \mathrm{Re}\,\nu > -4, \quad a > 0$	$\tfrac{1}{4} a \pi^{1/2} y^{-3/2} [\cos(2^{-3} a^2 y^{-1} - \tfrac{1}{4}\nu\pi) \times J_{\frac{1}{2}\nu-\frac{1}{2}}(2^{-3} a^2 y^{-1}) - \sin(2^{-3} a^2 y^{-1} - \tfrac{1}{4}\nu\pi) \times J_{\frac{1}{2}\nu+\frac{1}{2}}(2^{-3} y^{-1} a^2)]$
(24)	$x^{-1} J_0(ax^{1/2})$ $\quad a > 0$	$\mathrm{si}(\tfrac{1}{4} a^2 y^{-1})$
(25)	$x^{-1/2} J_1(ax^{1/2})$ $\quad a > 0$	$(2a^{-1} \sin(\tfrac{1}{4} a^2 y^{-1})$
(26)	$x^{\nu-1/2} J_1(ax^{1/2})$ $\quad -2 < \mathrm{Re}\,\nu < \tfrac{3}{4}, \quad a > 0$	$\tfrac{1}{2} \csc(\nu\pi) \Gamma(\nu+1) y^{-\nu} \times \cos(2^{-3} a^2 y^{-1} - \tfrac{1}{2}\nu\pi) \times [k_{-2\nu}(2^{-3} a^2 e^{\frac{1}{2} i\pi} y^{-1}) - k_{-2\nu}(2^{-3} a^2 e^{-3i\pi/2} y^{-1})]$
(27)	$x^{-1/2} J_\nu(ax^{1/2})$ $\quad \mathrm{Re}\,\nu > -3, \quad a > 0$	$-\pi^{1/2} y^{-1/2} \sin(2^{-3} a^2 y^{-1} - \tfrac{1}{4}\nu\pi - \tfrac{1}{4}\pi) \times J_{\frac{1}{2}\nu}(2^{-3} a^2 y^{-1})$
(28)	$x^{\frac{1}{2}\nu} J_\nu(ax^{1/2})$ $\quad -2 < \mathrm{Re}\,\nu < \tfrac{1}{2}, \quad a > 0$	$2^{-\nu} a^\nu y^{-\nu-1} \cos(\tfrac{1}{4} a^2 y^{-1} - \tfrac{1}{2}\nu\pi)$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \qquad y>0$
(29)	$J_\nu(ax^{1/2})\,J_\nu(bx^{1/2})$ $\mathrm{Re}\,\nu>-2, \quad a>0$	$y^{-1}\,J_\nu(\tfrac{1}{2}aby^{-1})\cos[\tfrac{1}{4}(a^2+y^2)y^{-1}$ $-\tfrac{1}{2}\nu\pi]$
(30)	$x^{-1/2}\,K_{2\nu}(ax^{1/2})$ $\lvert\mathrm{Re}\,\nu\rvert<3/2, \quad \mathrm{Re}\,a>0$	$-\tfrac{1}{4}\pi^{3/2}y^{-1/2}\sec(\nu\pi)$ $\times[J_\nu(2^{-3}a^2y^{-1})\cos(\tfrac{1}{2}\nu\pi-2^{-3}a^2y^{-1}$ $-\tfrac{1}{4}\pi]-Y_\nu(2^{-3}a^2y^{-1})$ $\times\sin(\tfrac{1}{2}\nu\pi-2^{-3}a^2y^{-1}-\tfrac{1}{4}\pi)]$
(31)	$J_0(ax^{1/2})\,K_0(ax^{1/2}) \qquad \mathrm{Re}\,a>0$	$\tfrac{1}{2}y^{-1}K_0(\tfrac{1}{2}a^2/y)$
(32)	$x^{-1}\,J_2(ax^{1/2})\,K_2(ax^{1/2})$ $\mathrm{Re}\;a>0$	$\tfrac{1}{2}\pi a^{-2}y\,[I_1(\tfrac{1}{2}a^2/y)-\mathbf{L}_1(\tfrac{1}{2}a^2/y)]$
(33)	$x^{-1/2}\,J_\nu(ax^{1/2})\,K_\nu(ax^{1/2})$ $\mathrm{Re}\,\nu>-3/2, \quad \mathrm{Re}\,a>0$	$2^{-1/2}\pi^{1/2}a^{-2}y^{1/2}\dfrac{\Gamma(\tfrac{3}{4}+\tfrac{1}{2}\nu)}{\Gamma(1+\nu)}$ $\times W_{-1/4,\,1/2\nu}(\tfrac{1}{2}a^2/y)\,M_{1/4,\,1/2\nu}(\tfrac{1}{2}a^2/y)$
(34)	$K_0(2x^{1/2})+\tfrac{1}{2}\pi\,Y_0(2x^{1/2})$	$\tfrac{1}{2}\pi y^{-1}\sin(y^{-1})$
(35)	$x\,(x^2+b^2)^{-1/2}\,J_1[a(x^2+b^2)^{1/2}]$ $a>0$	$a^{-1}y\,(a^2-y^2)^{-1/2}\cos[b(a^2-y^2)^{1/2}]$ $0<y<a$ $0 \qquad a<y<\infty$
(36)	$x\,(x^2+b^2)^{-1/2\nu-1}\,J_{\nu-1}[a(x^2+b^2)^{1/2}]$ $\mathrm{Re}\,\nu>-3/2, \quad a,\ b>0$	$2^{-\nu}\pi a^{\nu-1}\,[\Gamma(\nu)]^{-1}\,e^{-by} \qquad a<y<\infty$
(37)	$x\,(x^2+b^2)^{-1/2\nu}J_\nu[a(x^2+b^2)^{1/2}]$ $\mathrm{Re}\,\nu>\tfrac{1}{2}, \quad a>0$	$2^{-1/2}\pi^{1/2}a^{-\nu}b^{-\nu+3/2}y(a^2-y^2)^{\nu/2-3/4}$ $\times J_{\nu-3/2}[b(a^2-y^2)^{1/2}] \quad 0<y<a$ $0 \qquad a<y<\infty$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)dx \qquad y>0$
(38)	$x(x^2+a^2)^{-1}(x^2+b^2)^{-\frac{1}{2}\nu}$ $\times J_\nu[c(x^2+b^2)^{\frac{1}{2}}] \qquad c>0$ $\mathrm{Re}\,\nu>-3/2, \quad \mathrm{Re}\,a>0$	$\frac{1}{2}\pi e^{-ay}(b^2-a^2)^{-\frac{1}{2}\nu}J_\nu[c(b^2-a^2)^{\frac{1}{2}}]$ $c<y<\infty$
(39)	$x^{\frac{1}{2}}J_{\frac{1}{4}}\{\frac{1}{2}a[(b^2+x^2)^{\frac{1}{2}}-b]\}$ $\times J_{\frac{1}{4}}\{\frac{1}{2}a[(b^2+x^2)^{\frac{1}{2}}+b]\}$ $a>0$	$(\frac{1}{2}\pi y)^{-\frac{1}{2}}(a^2-y^2)^{-\frac{1}{2}}$ $\times\cos[b(a^2-y^2)^{\frac{1}{2}}] \qquad 0<y<a$ $0 \qquad a<y<\infty$
	Also see under 'Trigonometric functions' where more results similar to the above may be obtained by inversion.	
(40)	$x^{-1}\{(x^2+b_1^2)^{-\frac{1}{2}\nu_1}$ $\times J_{\nu_1}[a_1(x^2+b_1^2)^{\frac{1}{2}}]\cdots(x^2+b_n^2)^{-\frac{1}{2}\nu_n}$ $\times J_{\nu_n}[a_n(x^2+b_n^2)^{\frac{1}{2}}]\}\, a_i,\ b_i>0$ $\mathrm{Re}(\nu_1+\nu_2+\cdots+\nu_n)>-1-n/2$	$\frac{1}{2}\pi[b_1^{-\nu_1}J_{\nu_1}(a_1b_1)\cdots b_n^{-\nu_n}J_{\nu_n}(a_nb_n)]$ $y>a_1+a_2+\cdots+a_n$
(41)	$(a^2+x^2)^{-\frac{1}{2}-\frac{1}{2}\nu}C^{\nu}_{2n+1}[x(a^2+x^2)^{-\frac{1}{2}}]$ $\times J_{\nu+2n+1}[(a^2+x^2)^{\frac{1}{2}}]$ $\mathrm{Re}\,\nu>-3/2$	$(-1)^n2^{-\frac{1}{2}}\pi^{\frac{1}{2}}a^{\frac{1}{2}-\nu}(1-y^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times C^{\nu}_{2n+1}(y)J_{\nu-\frac{1}{2}}[a(1-y^2)^{\frac{1}{2}}]$ $0<y<1$ $0 \qquad 1<y<\infty$
(42)	$xK_0[a(x^2+\beta^2)^{\frac{1}{2}}]$ $\mathrm{Re}\,a>0, \quad \mathrm{Re}\,\beta>0$	$\frac{1}{2}\pi y\beta(a^2+y^2)^{-1}e^{-\beta(a^2+y^2)^{\frac{1}{2}}}$ $\times[1+\beta^{-1}(a^2+y^2)^{-\frac{1}{2}}]$
(43)	$x(x^2+\beta^2)^{-\frac{1}{2}}K_1[a(x^2+\beta^2)^{\frac{1}{2}}]$ $\mathrm{Re}\,a>0, \quad \mathrm{Re}\,\beta>0$	$\frac{1}{2}\pi a^{-1}y(a^2+y^2)^{-\frac{1}{2}}e^{-\beta(a^2+y^2)^{\frac{1}{2}}}$
(44)	$x(x^2+\beta^2)^{-1}K_2[a(x^2+\beta^2)^{\frac{1}{2}}]$ $\mathrm{Re}\,a>0, \quad \mathrm{Re}\,\beta>0$	$\frac{1}{2}\pi\beta^{-1}a^{-2}ye^{-\beta(y^2+a^2)^{\frac{1}{2}}}$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x)\sin(xy)\,dx \quad y>0$
(45)	$x(x^2+\beta^2)^{\frac{1}{2}\nu}K_{\pm\nu}[a(x^2+\beta^2)^{\frac{1}{2}}]$ $\mathrm{Re}\,a>0, \quad \mathrm{Re}\,\beta>0$	$2^{-1/2}\pi^{1/2}a^{\nu}y\beta^{3/2+\nu}(a^2+y^2)^{-\nu/2-3/4}$ $\times K_{-\nu-3/2}[\beta(a^2+y^2)^{1/2}]$
(46)	$x^{\frac{1}{2}}\, I_{\frac{1}{4}}\{\frac{1}{2}\beta[(a^2+x^2)^{\frac{1}{2}}-a]\}$ $\times K_{\frac{1}{4}}\{\frac{1}{2}\beta[(a^2+x^2)^{\frac{1}{2}}+a]\}$ $\mathrm{Re}\,a>0, \quad \mathrm{Re}\,\beta>0$	$(\frac{1}{2}\pi)^{\frac{1}{2}}y^{-\frac{1}{2}}(\beta^2+y^2)^{-\frac{1}{2}}e^{-a(\beta^2+y^2)^{\frac{1}{2}}}$
(47)	$0 \qquad 0<x<a$ $J_0[b(x^2-a^2)^{\frac{1}{2}}] \qquad a<x<\infty$ $b>0$	$0 \qquad 0<y<b$ $(y^2-b^2)^{-\frac{1}{2}}\cos[a(y^2-b^2)^{\frac{1}{2}}]$ $b<y<\infty$
(48)	$(a^2-x^2)^{-\frac{1}{2}}J_{\nu}[b(a^2-x^2)^{\frac{1}{2}}]$ $0<x<a$ $0 \qquad a<x<\infty$ $\mathrm{Re}\,\nu>-1$	$\frac{1}{2}\pi\, J_{\frac{1}{2}\nu}\{\frac{1}{2}a[(b^2+y^2)^{\frac{1}{2}}-y]\}$ $\times J_{\frac{1}{2}\nu}\{\frac{1}{2}a[(b^2+y^2)^{\frac{1}{2}}+y]\}$
(49)	$0 \qquad 0<x<a$ $(x^2-a^2)^{-\frac{1}{2}}\, J_{\nu}[b(x^2-a^2)^{\frac{1}{2}}]$ $a<x<\infty$ $\mathrm{Re}\,\nu>-1, \quad b>0$	$\frac{1}{2}\pi\, J_{\frac{1}{2}\nu}\{\frac{1}{2}a[y-(y^2-b^2)^{\frac{1}{2}}]\}$ $\times J_{-\frac{1}{2}\nu}\{\frac{1}{2}a[y+(y^2-b^2)^{\frac{1}{2}}]\}$ $b<y<\infty$
(50)	$0 \qquad 0<x<a$ $(x^2-a^2)^{\frac{1}{2}\nu}J_{\nu}[b(x^2-a^2)^{\frac{1}{2}}]$ $a<x<\infty$ $-1<\mathrm{Re}\,\nu<\frac{1}{2}, \quad b>0$	$0 \qquad 0<y<b$ $(\frac{1}{2}\pi a)^{\frac{1}{2}}a^{\nu}b^{\nu}(y^2-b^2)^{-\frac{1}{2}\nu-\frac{1}{4}}$ $\times J_{-\nu-\frac{1}{2}}[a(y^2-b^2)^{\frac{1}{2}}] \qquad b<y<\infty$
(51)	$x(a^2-x^2)^{\frac{1}{2}\nu}\, J_{\nu}[b(a^2-x^2)^{\frac{1}{2}}]$ $0<x<a$ $0 \qquad a<x<\infty$ $\mathrm{Re}\,\nu>-1, \quad b>0$	$2^{-1/2}\pi^{1/2}b^{\nu}a^{\nu+3/2}y(b^2+y^2)^{-\nu/2-3/4}$ $\times J_{\nu+3/2}[a(b^2+y^2)^{\frac{1}{2}}]$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(52)	$0 \qquad 0<x<a$ $x(x^2-a^2)^{\frac{1}{2}\nu}J_\nu[b(x^2-a^2)^{\frac{1}{2}}]$ $a<x<\infty$ $-1<\mathrm{Re}\,\nu<-\frac{1}{2}, \quad b>0$	$-2^{1/2}\pi^{-1/2}a^{\nu+3/2}b^\nu(b^2-y^2)^{-\nu/2-3/4}$ $\times K_{\nu+3/2}[a(b^2-y^2)^{1/2}] \quad 0<y<b$ $2^{-1/2}\pi^{1/2}a^{\nu+3/2}b^\nu(y^2-b^2)^{-\nu/2-3/4}$ $\times Y_{-\nu-3/2}[a(y^2-b^2)^{1/2}] \quad b<y<\infty$
(53)	$0 \qquad 0<x<a$ $(x^2-a^2)^{\frac{1}{2}\nu}(x^2+c^2)^{-1}$ $\times J_\nu[b(x^2-a^2)^{\frac{1}{2}}] \quad a<x<\infty$ $-1<\mathrm{Re}\,\nu<5/2, \quad c>0$	$c^{-1}(a^2+c^2)^{\frac{1}{2}\nu}K_\nu[b(a^2+c^2)^{\frac{1}{2}}]$ $\times \sinh(cy) \qquad 0<y<b$
(54)	$x^{2n+1}(1-x^2)^{\frac{1}{2}\nu+m}J_\nu[a(1-x^2)^{\frac{1}{2}}]$ $0<x<1$ $0 \qquad 1<x<\infty$ $\mathrm{Re}\,\nu>-1$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}a^{-\nu}\left(\frac{d}{ada}\right)^m\left(\frac{d}{ydy}\right)^n$ $\times\{a^{2\nu+2m}y^{2n+1}(a^2+y^2)^{-(\nu+m+n+3/2)/2}$ $\times J_{\nu+m+n+3/2}[(a^2+y^2)^{\frac{1}{2}}]\}$
(55)	$(1-x^2)^{\frac{1}{2}\nu-\frac{1}{4}}C^\nu_{2n+1}(x)$ $\times J_{\nu-\frac{1}{2}}[a(1-x^2)^{\frac{1}{2}}]$ $0<x<1$ $0 \qquad 1<x<\infty$ $\mathrm{Re}\,\nu>-\frac{1}{2}$	$(-1)^n 2^{-\frac{1}{2}}\pi^{\frac{1}{2}}a^{-\frac{1}{2}+\nu}(a^2+y^2)^{-\frac{1}{4}-\frac{1}{2}\nu}$ $\times C^\nu_{2n+1}[y(a^2+y^2)^{-\frac{1}{2}}]$ $\times J_{\nu+2n+1}[(a^2+y^2)^{\frac{1}{2}}]$
(56)	$0 \qquad 0<x<a$ $(x^2+b^2)^{-1}(x^2-a^2)^{\frac{1}{2}\nu+n-\frac{1}{2}}$ $\times Y_\nu[c(x^2-a^2)^{\frac{1}{2}}] \quad a<x<\infty$ $-1/2-n<\mathrm{Re}\,\nu<7/2-2n$ $b>0$	$(-1)^{n+1}b^{-1}(a^2+b^2)^{\frac{1}{2}\nu+n-\frac{1}{2}}\sinh(by)$ $\times K_\nu[c(a^2+b^2)^{\frac{1}{2}}] \qquad 0<y<c$
(57)	$x(x^2-b^2)^{-\frac{1}{2}\nu}K_\nu[a(x^2-b^2)^{\frac{1}{2}}]$ $\mathrm{Re}\,\nu<1, \quad a,\,b>0$	$2^{-3/2}\pi^{3/2}e^{-i\pi(\nu-1)}a^{-\nu}y(a^2+y^2)^{\nu-3/2}$ $\times b^{3/2-\nu}H^{(2)}_{\nu-3/2}[b(a^2+y^2)^{\frac{1}{2}}]$

Bessel functions of other arguments (cont'd)

	$f(x)$	$g(y) = \int_0^\infty f(x) \sin(xy)\,dx \qquad y > 0$
(58)	$J_0[2a \sinh(\tfrac{1}{2}x)] \qquad a > 0$	$2\pi^{-1} \sinh(\pi y)\,[K_{iy}(a)]^2$
(59)	$J_{2\nu}[2a \sinh(\tfrac{1}{2}x)]$ $\operatorname{Re} \nu > -1, \qquad a > 0$	$-i\,[I_{\nu-iy}(a) K_{\nu+iy}(a)$ $- I_{\nu+iy}(a)\, K_{\nu-iy}(a)]$
(60)	$\sinh(\tfrac{1}{2}\pi x)\, K_{ix}(a) \qquad a > 0$	$\tfrac{1}{2}\pi \sin(a \sinh y)$

2.14. Other higher transcendental functions

(1)	$e^{-\frac{1}{4}x^2} D_{2n+1}(x)$	$(-1)^n 2^{-\frac{1}{2}} \pi^{\frac{1}{2}} y^{2n+1} e^{-\frac{1}{2}y^2}$
(2)	$e^{-\frac{1}{4}x^2}[D_{2\nu-\frac{1}{2}}(x) - D_{2\nu-\frac{1}{2}}(-x)]$ $\operatorname{Re} \nu > \tfrac{1}{4}$	$2^{\frac{1}{2}} \pi^{\frac{1}{2}} \sin[(\nu - \tfrac{1}{4})\pi]\, y^{2\nu-\frac{1}{2}} e^{-\frac{1}{2}y^2}$
(3)	$[D_{-n-1}(ix)]^2 - [D_{-n-1}(-ix)]^2$	$(-1)^{n+1} i\,(\pi/n!)\,(2\pi)^{\frac{1}{2}} e^{-\frac{1}{2}y^2} L_n(y^2)$
(4)	$D_{\nu-\frac{1}{2}}[(2x)^{\frac{1}{2}}]\{D_{-\nu-\frac{1}{2}}[(2x)^{\frac{1}{2}}]$ $- D_{-\nu-\frac{1}{2}}[-(2x)^{\frac{1}{2}}]\}$	$-2^{\frac{1}{2}} \pi^{\frac{1}{2}} \sin[(\tfrac{1}{4} + \tfrac{1}{2}\nu)\pi]\, y^{-\nu-\frac{1}{2}}$ $\times (1+y^2)^{-\frac{1}{2}} [1 + (1+y^2)^{\frac{1}{2}}]^{\nu}$
(5)	$x^{4\nu} e^{-\frac{1}{2}x^2}$ $\times {}_1F_1(\tfrac{1}{2} - 2\nu; 2\nu + 1; \tfrac{1}{2}x^2)$ $\operatorname{Re} \nu > -\tfrac{1}{4}$	$2^{-\frac{1}{2}} \pi^{\frac{1}{2}} y^{4\nu} e^{-\frac{1}{2}y^2}$ $\times {}_1F_1(\tfrac{1}{2} - 2\nu; 1 + 2\nu; \tfrac{1}{2}y^2)$
(6)	$x\, {}_2F_1(\alpha; \beta; 3/2, -x^2 c^2)$ $\operatorname{Re} \alpha > \tfrac{1}{2}, \quad \operatorname{Re} \beta > \tfrac{1}{2}$	$2^{-\alpha-\beta+1} \pi c^{-\alpha-\beta} y^{\alpha+\beta-2}$ $\times K_{\alpha-\beta}(c^{-1}y)/[\Gamma(\alpha)\Gamma(\beta)]$

Higher functions (cont'd)

	$f(x)$	$g(y)=\int_0^\infty f(x)\sin(xy)\,dx \qquad y>0$
(7)	$x\;{}_1F_2(a;\beta,3/2;-\tfrac{1}{4}x^2)$ $\mathrm{Re}\,\beta>\mathrm{Re}\,a>\tfrac{1}{2}$	$\pi\dfrac{\Gamma(\beta)}{\Gamma(a)\Gamma(\beta-a)}y^{2a-2}(1-y^2)^{\beta-a-1}$ $\quad 0<y<1$ $0 \quad 1<y<\infty$
(8)	For other sine transforms of functions of the form $${}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;x^2)$$ see the table of Hankel transforms.	
(9)	$x^{-2\nu}e^{\frac{1}{4}x^2}W_{3\nu-1,\nu}(\tfrac{1}{2}x^2)$ $\mathrm{Re}\,\nu<\tfrac{1}{2}$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}y^{-2\nu}e^{\frac{1}{4}y^2}W_{3\nu-1,\nu}(\tfrac{1}{2}y^2)$
(10)	$x^{2\nu-1}e^{-\frac{1}{4}x^2}M_{3\nu,\nu}(\tfrac{1}{2}x^2)$ $\mathrm{Re}\,\nu>-\tfrac{1}{4}$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}y^{2\nu-1}e^{-\frac{1}{4}y^2}M_{3\nu,\nu}(\tfrac{1}{2}y^2)$
(11)	$x^{-3/2}W_{\mu+\rho,1/8+\lambda}(\tfrac{1}{2}x^2)$ $\times M_{\mu-\rho,1/8-\lambda}(\tfrac{1}{2}x^2)$ $\mathrm{Re}\,\rho<1/8,\quad \mathrm{Re}\,\lambda<5/8$	$(\tfrac{1}{2}\pi)^{1/2}y^{-3/2}\Gamma(5/4-2\lambda)[\Gamma(5/4-2\rho)]^{-1}$ $\times W_{\mu+\lambda,1/8+\rho}(\tfrac{1}{2}y^2)$ $\times M_{\mu-\lambda,1/8-\rho}(\tfrac{1}{2}y^2)$

CHAPTER III

EXPONENTIAL FOURIER TRANSFORMS

3.1. General formulas

	$f(x)$		$g(y)=\int_{-\infty}^{\infty} f(x)\, e^{-ixy}\, dx$
(1)	$g(x)$		$2\pi f(-y)$
(2)	$\overline{f(x)}$		$\overline{g(-y)}$
(3)	$f(x)=f(-x)$		$2\int_0^{\infty} f(x)\cos xy\, dx$
(4)	$f(x)=-f(-x)$		$-2i\int_0^{\infty} f(x)\sin xy\, dx$
(5)	$f(a^{-1}x+b)$	$a>0$	$ae^{iaby}\, g(ay)$
(6)	$f(-a^{-1}x+b)$	$a>0$	$ae^{-iaby}\, g(-ay)$
(7)	$f(ax)\, e^{ibx}$	$a>0$	$\frac{1}{a}\, g\left(\frac{y-b}{a}\right)$
(8)	$f(ax)\cos bx$	$a>0$	$\frac{1}{2a}\left[g\left(\frac{y-b}{a}\right)+g\left(\frac{y+b}{a}\right)\right]$
(9)	$f(ax)\sin bx$	$a>0$	$\frac{1}{2ai}\left[g\left(\frac{y-b}{a}\right)-g\left(\frac{y+b}{a}\right)\right]$
(10)	$x^n f(x)$		$i^n\, \frac{d^n g(y)}{dy^n}$

General formulas (cont'd)

	$f(x)$	$g(y)=\int_{-\infty}^{\infty} f(x)e^{-ixy}dx$
(11)	$f^{(n)}(x)$	$i^n y^n g(y)$

3.2. Elementary functions

(1)	$(1+x^2)^{-1}$	$\pi e^{-\lvert y\rvert}$
(2)	$(1+x^2)^{-1}(i-x)^n/(i+x)^n$ $n=1, 2, 3, \dots$	$(-1)^{n-1}2\pi y e^{-y} L_{n-1}^{1}(2y)$ $\quad y>0$ 0 $\quad y<0$
(3)	$(a-ix)^{-\nu}$ $\operatorname{Re} a>0, \quad \operatorname{Re}\nu>0$	$2\pi y^{\nu-1}e^{-ay}/\Gamma(\nu)$ $\quad y>0$ 0 $\quad y<0$
(4)	$(a+ix)^{-\nu}$ $\operatorname{Re}\nu>0, \quad \operatorname{Re} a>0$	0 $\quad y>0$ $-2\pi(-y)^{\nu-1}e^{ay}/\Gamma(\nu)$ $\quad y<0$
(5)	$(x^2+a^2)^{-1}(ix)^{-\nu}$ $\lvert\nu\rvert<1, \quad \operatorname{Re} a>0$ $\arg(ix)=\tfrac{1}{2}\pi \quad (x>0)$ $\arg(ix)=-\tfrac{1}{2}\pi \quad (x<0)$	$\pi a^{-\nu-1}e^{-\lvert y\rvert a}$
(6)	$(x^2+a^2)^{-1}(\beta+ix)^{-\nu} \quad \operatorname{Re}\nu>-1$ $\operatorname{Re} a>0, \quad \operatorname{Re}\beta>0$	$\pi a^{-1}(a+\beta)^{-\nu}e^{-ay}$ $\quad y>0$
(7)	$(x^2+a^2)^{-1}(\beta-ix)^{-\nu}$ $\operatorname{Re}\nu>-1, \quad \operatorname{Re} a>0$ $\operatorname{Re}\beta>0, \quad a\neq\beta$	$\pi a^{-1}(\beta-a)^{\nu}e^{ay}$ $\quad y>0$
(8)	$(a_0-ix)^{-1}(ix)^{\nu_0}$ $\times(a_1+ix)^{\nu_1}\cdots(a_n+ix)^{\nu_n}$ $\sum_0^n \operatorname{Re}\nu_i<1, \quad \operatorname{Re}\nu_0>-1, \quad \operatorname{Re} a_k>0$ $\arg(ix)=\tfrac{1}{2}\pi \quad (x>0)$ $\arg(ix)=-\tfrac{1}{2}\pi \quad (x<0)$	$2\pi e^{-a_0 y}a_0^{\nu_0}(a_0+a_1)^{\nu_1}\cdots(a_0+a_n)^{\nu_n}$ $y>0$

Elementary functions (cont'd)

	$f(x)$	$g(y)=\int_{-\infty}^{\infty} f(x)e^{-ixy}\,dx$
(9)	$(a_0+ix)^{-1}(ix)^{\nu_0}$ $\times(a_1+ix)^{\nu_1}\cdots(a_n+ix)^{\nu_n}$ $\sum_0^n \operatorname{Re}\nu_i<1, \quad \operatorname{Re}\nu_0>-1$ $\operatorname{Re} a_k>0$ $\arg(ix)=\tfrac12\pi \quad (x>0)$ $\arg(ix)=-\tfrac12\pi \quad (x<0)$	$0 \qquad y>0$
(10)	$(a-ix)^{-\mu}(\beta-ix)^{-\nu}$ $\operatorname{Re}(\mu+\nu)>1$ $\operatorname{Re} a>0, \quad \operatorname{Re}\beta>0$	$\dfrac{2\pi e^{-ay}y^{\mu+\nu-1}}{\Gamma(\mu+\nu)}$ $\times {}_1F_1[\nu;\mu+\nu;(a-\beta)y] \qquad y>0$ $0 \qquad y<0$
(11)	$(a+ix)^{-\mu}(\beta+ix)^{-\nu}$ $\operatorname{Re} a>0, \quad \operatorname{Re}\beta>0$ $\operatorname{Re}(\mu+\nu)>1$	$0 \qquad y>0$ $-\dfrac{2\pi e^{ay}(-y)^{\mu+\nu-1}}{\Gamma(\mu+\nu)}$ $\times {}_1F_1[\nu;\mu+\nu;(\beta-a)y] \qquad y<0$
(12)	$(a+ix)^{-2\mu}(\beta-ix)^{-2\nu}$ $\operatorname{Re}(\mu+\nu)>\tfrac12$ $\operatorname{Re} a>0, \quad \operatorname{Re}\beta>0$	$-2\pi(a+\beta)^{-\nu-\mu}[\Gamma(2\nu)]^{-1}$ $\times e^{\frac12(\beta-a)y}y^{\nu+\mu-1}$ $\times W_{\nu-\mu,\frac12-\nu-\mu}[(a+\beta)y] \qquad y>0$ $2\pi(a+\beta)^{-\mu-\nu}[\Gamma(2\mu)]^{-1}e^{\frac12(a-\beta)y}$ $\times(-y)^{\nu+\mu-1}W_{\mu-\nu,\frac12-\nu-\mu}[-(a+\beta)y]$ $y<0$
(13)	$0 \qquad -\infty<x<-1$ $(1-x)^{\nu-1}(1+x)^{\mu-1} \qquad -1<x<1$ $0 \qquad 1<x<\infty$ $\operatorname{Re}\nu>0, \quad \operatorname{Re}\mu>0$	$2^{\nu+\mu-1}\mathrm{B}(\mu,\nu)e^{iy}$ $\times {}_1F_1(\mu;\nu+\mu;-2iy)$

Elementary functions (cont'd)

	$f(x)$	$g(y)=\int_{-\infty}^{\infty} f(x)e^{-ixy}dx$
(14)	$(a-e^{-x})^{-1}e^{-\lambda x}$ $0<\operatorname{Re}\lambda<1,\quad a>0$	$\pi a^{\lambda-1+iy}\operatorname{ctn}[\pi\lambda+i\pi y)$ The integral is a Cauchy Principal value
(15)	$(a+e^{-x})^{-1}e^{-\lambda x}$ $0<\operatorname{Re}\lambda<1,\quad -\pi<\arg a<\pi$	$\pi a^{\lambda-1+iy}\csc(\pi\lambda+i\pi y)$
(16)	$x(a+e^{-x})^{-1}e^{-\lambda x}$ $0<\operatorname{Re}\lambda<1,\quad -\pi<\arg a<\pi$	$\pi a^{\lambda-1+iy}\csc(\pi\lambda+i\pi y)$ $\times[\log a-\pi\operatorname{ctn}(\pi\lambda+i\pi y)]$
(17)	$x^2(1+e^{-x})^{-1}e^{-\lambda x}$ $0<\operatorname{Re}\lambda<1$	$\pi^3\csc^3(\pi\lambda+iy\pi)[2-\sin^2(\pi\lambda+iy\pi)]$
(18)	$(a+e^{-x})^{-1}(\beta+e^{-x})^{-1}e^{-\lambda x}$ $0<\operatorname{Re}\lambda<2,\quad \beta\neq a$ $\lvert\arg a\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$	$\pi(\beta-a)^{-1}(a^{\lambda-1+iy}-\beta^{\lambda-1+iy})$ $\times\csc(\pi\lambda+iy\pi)$
(19)	$x(a+e^{-x})^{-1}(\beta+e^{-x})^{-1}e^{-\lambda x}$ $0<\operatorname{Re}\lambda<2,\quad a\neq\beta$ $\lvert\arg a\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$	$\dfrac{\pi(a^{\lambda-1+iy}\log a-\beta^{\lambda-1+iy}\log\beta)}{(a-\beta)\sin(\lambda\pi+iy\pi)}$ $+\dfrac{\pi^2(a^{\lambda-1+iy}-\beta^{\lambda-1+iy})\cos(\lambda\pi+iy\pi)]}{(\beta-a)\sin^2(\lambda\pi+iy\pi)}$
(20)	$(1+e^{-x})^{-n}e^{-\lambda x}$ $n=1,2,3,\ldots,\quad 0<\operatorname{Re}a<n$	$\pi\csc(\pi\lambda+iy\pi)\prod_{j=1}^{n-1}(j-\lambda-iy)/(n-1)!$
(21)	$\dfrac{e^{-ax}}{(e^{\beta/\gamma}+e^{-x/\gamma})^{\nu}}$ $\operatorname{Re}(\nu/\gamma)>\operatorname{Re}a>0$ $\lvert\operatorname{Im}\beta\rvert<\pi\operatorname{Re}\gamma$	$\gamma e^{\beta(a+iy-\nu/\gamma)}\mathrm{B}[\gamma(a+iy),\nu-\gamma(a+iy)]$

Elementary functions (cont'd)

	$f(x)$	$g(y)=\int_{-\infty}^{\infty} f(x)e^{-ixy}dx$
(22)	$\dfrac{e^{-ax}}{(e^{\beta}+e^{-x})^{\nu}(e^{\gamma}+e^{-x})^{\mu}}$ $0<\operatorname{Re}a<\operatorname{Re}(\mu+\nu)$ $\|\operatorname{Im}\beta\|<\pi, \quad \|\operatorname{Im}\gamma\|<\pi$	$e^{\gamma(a+iy-\mu)-\beta\nu}\dfrac{\Gamma(a+iy)\Gamma(\mu+\nu-a-iy)}{\Gamma(\mu+\nu)}$ $\times {}_2F_1(\nu, a+iy; \mu+\nu; 1-e^{\gamma-\beta})$
(23)	$(ix)^{\nu}e^{-a^2x^2}$ $\operatorname{Re}a>0, \quad \operatorname{Re}\nu>-1$ $\arg(ix)=\tfrac{1}{2}\pi \quad (x>0)$ $\arg(ix)=-\tfrac{1}{2}\pi \quad (x<0)$	$\pi^{1/2}2^{-1/2\nu}a^{-\nu-1}e^{-y^2a^{-2}/8}$ $\times D_{\nu}(2^{-1/2}a^{-1}y)$
(24)	$[\exp(e^{-x})-1]^{-1}e^{-\lambda x}$ $\operatorname{Re}(\lambda)>1$	$\zeta(iy+\lambda)\Gamma(iy+\lambda)$
(25)	$[\exp(e^{-x})+1]^{-1}e^{-\lambda x}$ $\operatorname{Re}\lambda>0$	$(1-2^{1-\lambda-iy})\Gamma(iy+\lambda)\zeta(iy+\lambda)$
(26)	$e^{-\lambda x}\log\|1-e^{-x}\|$ $-1<\operatorname{Re}\lambda<0$	$\pi(\lambda+iy)^{-1}\operatorname{ctn}(\pi\lambda+iy\pi)$
(27)	$e^{-\lambda x}\log(1+e^{-x})$ $-1<\operatorname{Re}\lambda<0$	$\pi(\lambda+iy)^{-1}\csc(\pi\lambda+iy\pi)$
(28)	$e^{-\lambda x}\log(\|1+e^{-x}\|/\|1-e^{-x}\|)$ $\|\operatorname{Re}\lambda\|<1$	$\pi(\lambda+iy)^{-1}\tan(\tfrac{1}{2}\pi\lambda+\tfrac{1}{2}iy\pi)$
(29)	$e^{-\lambda x}(a+e^{-x})^{-\nu}\log(a+e^{-x})$ $a>0, \quad \operatorname{Re}\nu>\operatorname{Re}\lambda>0$	$a^{\lambda+iy-\nu}B(\lambda+iy, \nu-\lambda-iy)$ $\times[\psi(\nu)-\psi(\nu-\lambda-iy)+\log a]$
(30)	$(\sinh x+\sinh a)^{-1} \quad a>0$	$-\pi ie^{iay}\operatorname{sech}a\operatorname{csch}(\pi y)$ $\times[\cosh(\pi y)-e^{-2iay}]$ The integral is a Cauchy Principal value

Elementary functions (cont'd)

	$f(x)$	$g(y)=\int_{-\infty}^{\infty} f(x)e^{-ixy}dx$
(31)	$0 \qquad -\infty < x < -\tfrac{1}{2}\pi$ $(\cos x)^{\mu}(a^2 e^{ix}+b^2 e^{-ix})^{\nu}$ $-\tfrac{1}{2}\pi < x < \tfrac{1}{2}\pi$ $0 \qquad \tfrac{1}{2}\pi < x < \infty$ $\operatorname{Re}\mu > -1$	$\dfrac{\pi b^{2\nu}2^{-\mu}\Gamma(1+\mu)}{\Gamma(1-\tfrac{1}{2}y-\tfrac{1}{2}\nu+\tfrac{1}{2}\mu)\Gamma(1+\tfrac{1}{2}y+\tfrac{1}{2}\nu+\tfrac{1}{2}\mu)}$ $\times {}_2F_1\left(-\nu, \dfrac{y+\nu+\mu}{-2}; 1+\dfrac{\mu-\nu-y}{2}; \dfrac{a^2}{b^2}\right)$ $a^2 < b^2$ $\dfrac{\pi a^{2\nu}2^{-\mu}\Gamma(1+\mu)}{\Gamma(1+\tfrac{1}{2}y-\tfrac{1}{2}\nu+\tfrac{1}{2}\mu)\Gamma(1+\tfrac{1}{2}\nu-\tfrac{1}{2}y+\tfrac{1}{2}\mu)}$ $\times {}_2F_1\left(-\nu, \dfrac{y-\nu-\mu}{2}; 1+\dfrac{\mu+y-\nu}{2}; \dfrac{b^2}{a^2}\right)$ $a^2 > b^2$
(32)	$\dfrac{e^{\nu \sinh^{-1} x}}{(1+x^2)^{1/2}} \qquad \lvert\operatorname{Re}\nu\rvert < 1$	$-2e^{-\frac{1}{2}\nu\pi i}K_{\nu}(y) \qquad y>0$ $-2e^{\frac{1}{2}\nu\pi i}K_{\nu}(-y) \qquad y<0$

3.3. Higher transcendental functions

(1)	$0 \qquad -\infty < x < -1$ $P_n(x) \qquad --1 < x < 1$ $0 \qquad 1 < x < \infty$	$(-1)^n i^n (2\pi)^{1/2} y^{-1/2} J_{n+1/2}(y)$
(2)	$0 \qquad -\infty < x < -1$ $(1-x^2)^{-1/2} T_n(x) \qquad -1 < x < 1$ $0 \qquad 1 < x < \infty$	$(-1)^n i^n \pi J_n(y)$
(3)	$0 \qquad -\infty < x < -1$ $(1-x^2)^{\nu} P_n^{(\nu,\nu)}(x) \qquad -1 < x < 1$ $0 \qquad 1 < x < \infty$ $\operatorname{Re}\nu > -1$	$(-1)^n i^n 2^{\nu+1/2} \pi^{1/2} y^{-\nu-1/2}(n!)^{-1}$ $\times\Gamma(n+\nu+1) J_{n+\nu+1/2}(y)$

Higher transcendental functions (cont'd)

	$f(x)$	$g(y) = \int_{-\infty}^{\infty} f(x)\, e^{-ixy}\, dx$
(4)	$0 \quad -\infty < x < -1$ $(1-x)^{\nu}(1+x)^{\mu} P_n^{(\nu,\nu)}(x) \quad -1 < x < 1$ $0 \quad 1 < x < \infty$ $\mathrm{Re}\,\nu > -1, \quad \mathrm{Re}\,\mu > -1$	$(-1)^n i^n 2^{n+\nu+\mu+1} y^n (n!)^{-1}$ $\times \mathrm{B}(n+\nu+1, n+\mu+1)\, e^{iy}$ $\times {}_1F_1(n+\nu+1; 2n+\mu+\nu+2; -2iy)$
(5)	$[\Gamma(\nu-x)\Gamma(\mu+x)]^{-1}$	$[2\cos(\tfrac{1}{2}y)]^{\mu+\nu-2} e^{\frac{1}{2}iy(\mu-\nu)}$ $\times [\Gamma(\mu+\nu-1)]^{-1} \quad \lvert y\rvert < \pi$ $0 \quad \lvert y\rvert > \pi$
(6)	$[\Gamma(a+x)]^{\pm 1}[\Gamma(\beta+x)]^{\pm 1}$	see Titchmarsh, E. C., 1937: *Introduction to the theory of Fourier integrals*, p. 185 and the following pages.
(7)	$0 \quad -\infty < x < -1$ $P_\nu(x) \quad -1 < x < 1$ $0 \quad 1 < x < \infty$	$2\pi(\nu+\nu^2)^{-1} \sin(\nu\pi)\, e^{iy}$ $\times {}_2F_2(1, 1; -\nu, 2+\nu; -2iy)$
(8)	$x^{-\frac{1}{2}} J_{n+\frac{1}{2}}(x)$	$(-1)^n i^n (2\pi)^{\frac{1}{2}} P_n(y) \quad \lvert y\rvert < 1$ $0 \quad \lvert y\rvert > 1$
(9)	$x^{-\nu-\frac{1}{2}} J_{n+\nu+\frac{1}{2}}(x)$ $\mathrm{Re}\,\nu > -1$	$2^{-\nu+\frac{1}{2}} \pi^{\frac{1}{2}} (-1)^n i^n n!\, [\Gamma(n+\nu+1)]^{-1}$ $\times (1-y^2)^{\nu} P_n^{(\nu,\nu)}(y) \quad \lvert y\rvert < 1$ $0 \quad \lvert y\rvert > 1$
(10)	$J_{\mu+x}(a)\, J_{\nu-x}(a)$ $\mathrm{Re}(\mu+\nu) > 1$	$e^{\frac{1}{2}iy(\mu-\nu)} J_{\mu+\nu}[2a\cos(\tfrac{1}{2}y)]$ $\lvert y\rvert < \pi$ $0 \quad \lvert y\rvert > \pi$
(11)	$\dfrac{J_{\mu+x}(a)\, J_{\nu-x}(\beta)}{a^{\mu+x}\beta^{\nu-x}}$	see Gröbner, W. and N. Hofreiter, 1950: *Integerltafel*, Part II p. 203.

Higher transcendental functions (cont'd)

	$f(x)$	$g(y) = \int_{-\infty}^{\infty} f(x) e^{-ixy} dx$
(12)	$[(x+c)^2+b^2]^{-\nu}$ $\times J_\nu\{a[(x+c)^2+b^2]\}$ $\mathrm{Re}\,\nu > -\tfrac{1}{2}, \quad a, b, c > 0$	$(2\pi)^{1/2} e^{icy} a^{-\nu} b^{-\nu+1/2} (a^2-y^2)^{1/2\nu-1/4}$ $\times J_{\nu-1/2}[b(a^2-y^2)^{1/2}] \qquad \lvert y\rvert < a$ $0 \qquad \lvert y\rvert > a$
(13)	$0 \qquad \lvert x\rvert > \tfrac{1}{2}\pi$ $(\cos x)^\nu (a^2 e^{ix} + b^2 e^{-ix})^{-\nu}$ $\times J_{2\nu}\{c[2(a^2e^{ix}+b^2e^{-ix})\cos x]^{1/2}\}$ $\lvert x\rvert < \tfrac{1}{2}\pi$ $\mathrm{Re}(\mu+\nu) > -1$	$\pi 2^{-\nu} a^{1/2y-\nu} b^{-1/2y-\nu} J_{\nu-1/2y}(ac)$ $\times J_{\nu+1/2y}(bc)$
(14)	$a^{-\mu-x} b^{-\nu+x} J_{\mu+x}(a) J_{\nu-x}(b)$ $\mathrm{Re}(\mu+\nu) > 1, \quad a, b > 0$	$(2\cos\tfrac{1}{2}y)^{1/2(\mu+\nu)}(a^2 e^{1/2iy}$ $+ b^2 e^{-1/2iy})^{-1/2(\mu+\nu)} e^{1/2iy(\mu-\nu)}$ $\times J_{\mu+\nu}\{[2(a^2e^{1/2iy}+b^2e^{-1/2iy})\cos\tfrac{1}{2}y]^{1/2}\}$ $\lvert y\rvert < \pi$ $0 \qquad \lvert y\rvert > \pi$
(15)	$e^{-1/4(1+\lambda)x^2} D_\nu[(1-\lambda)^{1/2}x]$ $\mathrm{Re}\,\lambda > 0$	$(2\pi)^{1/2}\lambda^{1/2\nu} e^{-1/4(1+\lambda)y^2/\lambda}$ $\times D_\nu[-iy(\lambda^{-1}-1)^{1/2}]$
(16)	$x^n e^{-ix} {}_1F_1(a;\beta;2ix)$ $\mathrm{Re}\,a > n, \quad \mathrm{Re}(\beta-a) > n$	$(-1)^n i^n \pi 2^{n+2-b} n!\,[\mathrm{B}(a, b-a)]^{-1}$ $\times(1-y)^{a-n-1}(1+y)^{b-a-n-1}$ $\times P_n^{(a-n-1,\, b-a-n-1)}(y) \qquad \lvert y\rvert < 1$ $0 \qquad \lvert y\rvert > 1$

LAPLACE TRANSFORMS

We call

$$g(p) = \mathfrak{L}\{f(t);\, p\} = \int_0^\infty e^{-pt} f(t)\, dt$$

the *Laplace transform* of $f(t)$, and regard p as a complex variable. The function $f(t)$ is called the *inverse Laplace transform* of $g(p)$. We give tables of both Laplace transforms and inverse Laplace transforms. In chapter IV transform pairs are classified according to $f(t)$, in chapter V according to $g(p)$. It should be noted that many authors use $p\,g(p)$ as the "operational image" or "operational representation" (sometimes also called the Laplace transform) of $f(t)$. The symbols $\doteq$, $\subset$, and similar notations usually indicate the relationship between a function $f(t)$ and its operational image $p\,g(p)$.

There is a very large number of books on the theory and application of Laplace transforms. The most important books and tables are listed on p. 128. Since Laplace transforms are virtually Fourier transforms in the complex domain, references given for Fourier transforms should also be consulted. In addition, many textbooks or works of reference contain material on Laplace transforms. Many books on Laplace transforms contain more or less elaborate tables of Laplace transforms or (more often) of inverse Laplace transforms. The most extensive liste are: Cossar and Erdélyi (1944-46, both Laplace and inverse Laplace transforms), Ditkin and Kuznecov (1951, inverses of operational images), Doetsch, Kniess, and Voelker (1947, inverse Laplace transforms), McLachlan and Humbert (1950, operational images), McLachlan, Humbert, and Poli (1950, operational images).

From the transform pairs given in chapters IV and V further pairs may be derived by means of the methods indicated in the introduction to this volume, and also by means of the general formulas given in sections 4.1 and 5.1. We do not list formulas for "chains" or "sequences" of transformations; for these see McLachlan, Humbert, and Poli (1950, p. 18-19).

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CHAPTER IV

LAPLACE TRANSFORMS

4.1. General formulas

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(1)	$\dfrac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{zt}g(z)\,dz$	$g(p)$
(2)	$f(t+a)=f(t)$	$(1-e^{ap})^{-1}\int_0^a e^{-pt}f(t)\,dt$
(3)	$f(t+a)=-f(t)$	$(1+e^{-ap})^{-1}\int_0^a e^{-pt}f(t)\,dt$
(4)	$0 \qquad t<ba^{-1}$ $f(at-b) \qquad t>ba^{-1}$ $a,\ b>0$	$a^{-1}e^{-ba^{-1}p}g(a^{-1}p)$
(5)	$e^{-at}f(t)$	$g(p+a)$
(6)	$t^n f(t)$	$(-1)^n \dfrac{d^n g(p)}{dp^n}$
(7)	$t^{-n}f(t)$	$\int_p^\infty \cdots \int_p^\infty g(p)(dp)^n$ n-th repeated integral
(8)	$f^{(n)}(t)$	$p^n g(p)-p^{n-1}f(0)$ $-p^{n-2}f'(0)-\cdots-f^{(n-1)}(0)$

General formulas (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(9)	$\int_0^t\cdots\int_0^t f(t)(dt)^n$	$p^{-n}g(p)$
(10)	$\left(t\frac{d}{dt}\right)^n f(t)$ where e.g. $\left(t\frac{d}{dt}\right)^2 f(t)=t\frac{d}{dt}\left\{t\frac{d}{dt}[f(t)]\right\}$	$\left(-\frac{d}{dp}p\right)^n g(p)$ $\left(\frac{d}{dp}p\right)^2 g(p)=\frac{d}{dp}\left\{p\frac{d}{dp}[pg(p)]\right\}$
(11)	$\left(\frac{d}{dt}t\right)^n f(t)$	$\left(-p\frac{d}{dp}\right)^n g(p)$
(12)	$\left(t^{-1}\frac{d}{dt}\right)^n f(t)$ if $\left(\frac{1}{t}\frac{d}{dt}\right)^k f(t)=0$ for $t=0,\quad k=0,\ldots,n-1$	$\int_p^\infty p\int_p^\infty\cdots p\int_p^\infty pg(p)(dp)^n$
(13)	$t^m f^{(n)}(t)$ $\qquad m\ge n$	$\left(-\frac{d}{dp}\right)^m[p^n g(p)]$
(14)	$t^m f^{(n)}(t)$ $\qquad m<n$	$\left(-\frac{d}{dp}\right)^m[p^n g(p)]+(-1)^{m-1}$ $\times\left[\frac{(n-1)!}{(n-m-1)!}p^{n-m-1}f(0)+\frac{(n-2)!}{(n-m-2)!}p^{n-m-2}f'(0)+\cdots+m!\,f^{(n-m-1)}(0)\right]$

General formulas (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(15)	$\dfrac{d^n}{dt^n}[t^m f(t)] \qquad m \geq n$	$(-1)^m p^n g^{(m)}(p)$
(16)	$\dfrac{d^n}{dt^n}[t^m f(t)] \qquad m < n$	$(-1)^m p^n g^{(m)}(p) - m!\,p^{n-m-1} f(0) - \dfrac{(m+1)!}{1!} p^{n-m-2} f'(0) - \cdots - \dfrac{(n-1)!}{(n-m-1)!} f^{(n-m-1)}(0)$
(17)	$\left(e^t \dfrac{d}{dt}\right)^n f(t)$ provided that $f^{(k)}(0)=0$ for $k=0, 1, \ldots, n-1$	$(p-1)\cdots(p-n)\,g(p-n)$
(18)	$\int_0^t t^{-1} f(t)\,dt$	$p^{-1}\int_p^\infty g(p)\,dp$
(19)	$\int_t^\infty t^{-1} f(t)\,dt$	$p^{-1}\int_0^p g(p)\,dp$
(20)	$\int_0^t f_1(u) f_2(t-u)\,du$	$g_1(p)\,g_2(p)$
(21)	$f_1(t)\,f_2(t)$	$\dfrac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} g_1(z)\,g_2(p-z)\,dz$
(22)	$f(t^2)$	$\pi^{-1/2}\int_0^\infty e^{-\frac14 p^2 u^{-2}}\,g(u^2)\,du$
(23)	$t^n f(t^2)$	$2^{-\frac12 n}\pi^{-1/2}\int_0^\infty u^{n-2} e^{-\frac14 p^2 u^2} \times \mathrm{He}_n(2^{-1/2} pu)\,g(u^{-2})\,du$

General formulas (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(24)	$t^{\nu}f(t^2)$	$2^{-1/2}\pi^{-1/2}\int_0^\infty u^{\nu-2}e^{-1/4 p^2u^2}$ $\times D_{\nu}(pu)g(\tfrac{1}{2}u^{-2})\,du$
(25)	$t^{\nu-1}f(t^{-1})$ $\quad \mathrm{Re}\,\nu>-1$	$p^{-1/2\nu}\int_0^\infty u^{1/2\nu}J_{\nu}(2u^{1/2}p^{1/2})g(u)\,du$
(26)	$f(ae^t-a)$ $\quad a>0$	$[a\Gamma(p+1)]^{-1}\int_0^\infty e^{-u}u^pg(u/a)\,du$
(27)	$f(a\sinh t)$ $\quad a>0$	$\int_0^\infty J_p(au)g(u)\,du$
(28)	$\sum_{n=1}^{\infty} n^{-1}f(n^{-1}t)$	$\int_0^\infty (e^{pu}-1)^{-1}f(u)\,du$
(29)	$\int_0^\infty \frac{t^{u-1}}{\Gamma(u)}f(u)\,du$	$g(\log p)$
(30)	$\int_0^\infty u^{-1/2}\sin(2u^{1/2}t^{1/2})f(u)\,du$	$\pi^{1/2}p^{-3/2}g(p^{-1})$
(31)	$t^{-1/2}\int_0^\infty \cos(2u^{1/2}t^{1/2})f(u)\,du$	$\pi^{1/2}p^{-1/2}g(p^{-1})$
(32)	$t^{\nu}\int_0^\infty J_{2\nu}(2u^{1/2}t^{1/2})u^{-\nu}f(u)\,du$	$p^{-2\nu-1}g(p^{-1})$
(33)	$t^{-1/2}\int_0^\infty e^{-1/4u^2/t}f(u)\,du$	$\pi^{1/2}p^{-1/2}g(p^{1/2})$
(34)	$t^{-1/2n-1/2}\int_0^\infty e^{-1/4u^2/t}$ $\times \mathrm{He}_n(2^{-1/2}ut^{-1/2})f(u)\,du$	$2^{1/2n}\pi^{1/2}p^{1/2n-1/2}g(p^{1/2})$

General formulas (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(35)	$t^{-\nu}\int_0^\infty e^{-u^2/(8t)}$ $\times D_{2\nu-1}(2^{-1/2}ut^{-1/2})f(u)\,du$	$2^{\nu-1/2}\pi^{1/2}p^{\nu-1}g(p^{1/2})$
(36)	$\int_0^t \left(\frac{t-u}{au}\right)^{\nu}$ $\times J_{2\nu}[2(aut-au^2)^{1/2}]f(u)\,du$	$p^{-2\nu-1}g(p+ap^{-1})$
(37)	$\int_0^t J_0[(t^2-u^2)^{1/2}]f(u)\,du$	$(p^2+1)^{-1/2}g[(p^2+1)^{1/2}]$
(38)	$f(t)-\int_0^t J_1(u)f[(t^2-u^2)^{1/2}]\,du$	$g[(p^2+1)^{1/2}]$
(39)	$\int_0^t \left(\frac{t-u}{t+u}\right)^{\nu}$ $\times J_{2\nu}[(t^2-u^2)^{1/2}]f(u)\,du$	$(p^2+1)^{-1/2}[(p^2+1)^{1/2}+p]^{-2\nu}$ $\times g[(p^2+1)^{1/2}]$

4.2. Algebraic functions

(1)	1		p^{-1}	$\operatorname{Re} p>0$
(2)	0 1 0	$0<t<a$ $a<t<b$ $t>b$	$p^{-1}(e^{-ap}-e^{-bp})$	
(3)	t^n		$n!\,p^{-n-1}$	$\operatorname{Re} p>0$
(4)	0 t^n	$0<t<b$ $t>b$	$e^{-bp}\sum_{m=0}^{n}\frac{n!}{m!}\frac{b^m}{p^{n-m+1}}$	$\operatorname{Re} p>0$

Algebraic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(5)	$t^n \quad 0<t<b$ $0 \quad t>b$	$\frac{n!}{p^{n+1}} - e^{-bp}\sum_{m=0}^{n}\frac{n!}{m!}\frac{b^m}{p^{n-m+1}}$
(6)	$0 \quad 0<t<b$ $(t+a)^{-1} \quad t>b$ $\lvert\arg(a+b)\rvert<\pi$	$-e^{ap}\,\mathrm{Ei}[-(a+b)p] \qquad \mathrm{Re}\,p>0$
(7)	$0 \quad 0<t<b$ $(t+a)^{-1} \quad b<t<c$ $0 \quad t>c$ $-a$ not between b and c	$e^{ap}\{\mathrm{Ei}[-(a+c)p]-\mathrm{Ei}[-(a+b)p]\}$
(8)	$(t-a)^{-1} \quad a\geq 0$	$-e^{-ap}\,\overline{\mathrm{Ei}}(ap) \qquad \mathrm{Re}\,p>0$ The integral is a Cauchy Principal value
(9)	$\frac{1}{(t+a)^n} \quad n\geq 2, \quad \lvert\arg a\rvert<\pi$	$\sum_{m=1}^{n-1}\frac{(m-1)!}{(n-1)!}\frac{(-p)^{n-m-1}}{a^m}$ $-\frac{(-p)^{n-1}}{(n-1)!}e^{ap}\,\mathrm{Ei}(-ap)$ $\mathrm{Re}\,p\geq 0$
(10)	$0 \quad 0<t<b$ $(t+a)^{-n} \quad t>b$ $\lvert\arg(a+b)\rvert<\pi, \quad n\geq 2$	$e^{-bp}\sum_{m=1}^{n-1}\frac{(m-1)!}{(n-1)!}\frac{(-p)^{n-m-1}}{(a+b)^m}$ $-\frac{(-p)^{n-1}}{(n-1)!}e^{ap}\,\mathrm{Ei}[-(a+b)p]$ $\mathrm{Re}\,p>0$

For further formulas of a similar type see Bierens de Haan, D., 1867: *Nouvelles tables d'intégrales définies*, 727 p.

Algebraic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(11)	$t^n(t+a)^{-1} \quad n \geq 1, \qquad \lvert\arg a\rvert < \pi$	$(-)^{n-1} a^n e^{ap}\,\mathrm{Ei}(-ap) + \sum_{m=1}^{n} (m-1)!(-a)^{n-m} p^{-m}$ $\qquad \mathrm{Re}\, p > 0$
(12)	$(At+Ba)(t^2-a^2)^{-1}$ $\qquad \lvert\arg(\pm a)\rvert < \pi$	$-\tfrac{1}{2}(A-B)e^{ap}\,\mathrm{Ei}(-ap)$ $-\tfrac{1}{2}(A+B)e^{-ap}\,\mathrm{Ei}(ap) \qquad \mathrm{Re}\, p > 0$
(13)	$(At+Ba)(t^2-a^2)^{-1} \qquad a > 0$ Cauchy Principal value	$-\tfrac{1}{2}(A-B)e^{-ap}\,\mathrm{Ei}(-ap)$ $-\tfrac{1}{2}(A+B)e^{-ap}\,\overline{\mathrm{Ei}}(ap) \qquad \mathrm{Re}\, p > 0$
(14)	$(At+Ba)(t^2+a^2)^{-1}$ $\qquad \lvert\arg(\pm ia)\rvert < \pi$	$(A\cos ap - B\sin ap)\,\mathrm{ci}(ap)$ $-(A\sin ap + B\cos ap)\,\mathrm{si}(ap)$ $\qquad \mathrm{Re}\, p > 0$
(15)	$0 \qquad 0 < t < b$ $t^{-1/2} \qquad t > b$	$\pi^{1/2} p^{-1/2}\,\mathrm{Erfc}(b^{1/2} p^{1/2}) \qquad \mathrm{Re}\, p > 0$
(16)	$t^{-1/2} \qquad 0 < t < b$ $0 \qquad t > b$	$\pi^{1/2} p^{-1/2}\,\mathrm{Erf}(b^{1/2} p^{1/2})$
(17)	$t^{n-1/2}$	$\pi^{1/2}\, 1/2(3/2)\cdots(n-1/2)\,p^{-n-1/2}$ $\qquad \mathrm{Re}\, p > 0$
(18)	$(t+a)^{-1/2} \qquad \lvert\arg a\rvert < \pi$	$\pi^{1/2} p^{-1/2} e^{ap}\,\mathrm{Erfc}(a^{1/2} p^{1/2}) \qquad \mathrm{Re}\, p > 0$
(19)	$0 \qquad 0 < t < b$ $t^{-3/2} \qquad t > b$	$2b^{-1/2} e^{-bp} - 2\pi^{1/2} p^{1/2}\,\mathrm{Erfc}(b^{1/2} p^{1/2})$ $\qquad \mathrm{Re}\, p \geq 0$
(20)	$(t+a)^{-3/2} \qquad \lvert\arg a\rvert < \pi$	$2a^{-1/2} - 2\pi^{1/2} p^{1/2} e^{ap}\,\mathrm{Erfc}(a^{1/2} p^{1/2})$ $\qquad \mathrm{Re}\, p \geq 0$

Algebraic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(22)	$t^{1/2}(t+a)^{-1}$ $\quad\lvert\arg a\rvert<\pi$	$(\pi/p)^{1/2}-\pi a^{1/2}e^{ap}\operatorname{Erfc}(a^{1/2}p^{1/2})$ $\quad\mathrm{Re}\,p>0$
(23)	$0 \quad 0<t<b$ $t^{-1}(t-b)^{1/2} \quad t>b$	$(\pi/p)^{1/2}e^{-bp}-\pi b^{1/2}\operatorname{Erfc}(b^{1/2}p^{1/2})$ $\quad\mathrm{Re}\,p>0$
(24)	$t^{-1/2}(1+2at)$	$\pi^{1/2}p^{-3/2}(p+a)$ $\quad\mathrm{Re}\,p>0$
(25)	$t^{-1/2}(t+a)^{-1}$ $\quad\lvert\arg a\rvert<\pi$	$\pi a^{-1/2}e^{ap}\operatorname{Erfc}(a^{1/2}p^{1/2})$ $\quad\mathrm{Re}\,p\geq 0$
(26)	$0 \quad 0<t<b$ $t^{-1}(t-b)^{-1/2} \quad t>b$	$\pi b^{-1/2}\operatorname{Erfc}(b^{1/2}p^{1/2})$ $\quad\mathrm{Re}\,p\geq 0$
(27)	$t(t^2+a^2)^{-1/2}$ $\quad\lvert\arg a\rvert<\pi/2$	$\tfrac{1}{2}\pi a[\mathbf{H}_1(ap)-Y_1(ap)]-a$ $\quad\mathrm{Re}\,p>0$
(28)	$t(b^2-t^2)^{-1/2} \quad 0<t<b$ $0 \quad t>b$	$\tfrac{1}{2}\pi b[\mathbf{L}_1(bp)-I_1(bp)]+b$ $\quad\mathrm{Re}\,p>0$
(29)	$0 \quad 0<t<b$ $t(t^2-b^2)^{-1/2} \quad t>b$	$bK_1(bp)$ $\quad\mathrm{Re}\,p>0$
(30)	$(t^2+2at)^{-1/2}(t+a)$ $\quad\lvert\arg a\rvert<\pi$	$ae^{ap}K_1(ap)$ $\quad\mathrm{Re}\,p>0$
(31)	$(2bt-t^2)^{-1/2}(b-t) \quad 0<t<2b$ $0 \quad t>2b$	$\pi b e^{-bp}I_1(bp)$ $\quad\mathrm{Re}\,p>0$
(32)	$[t+(t^2+a^2)^{1/2}]^{-1}$ $\quad\lvert\arg a\rvert<\pi/2$	$\tfrac{1}{2}\pi a^{-1}p^{-1}[\mathbf{H}_1(ap)-Y_1(ap)]-a^{-2}p^{-2}$ $\quad\mathrm{Re}\,p>0$
(33)	$\sin\theta(1+t+\cos\theta)^{-1}(t^2+2t)^{-1/2}$	$\exp[2p\cos^2(\tfrac{1}{2}\theta)]$ $\times[\theta-\sin\theta\int_0^p K_0(v)e^{-v\cos\theta}dv]$ $\quad\mathrm{Re}\,p>0$

Algebraic functions (cont'd)

	$f(t)$	$g(y)=\int_0^\infty e^{-pt} f(t)\,dt$
(34)	$[t+(1+t^2)^{1/2}]^n+[t-(1+t^2)^{1/2}]^n$	$2O_n(p)$ $\quad \mathrm{Re}\, p>0$
(35)	$[t+(1+t^2)^{1/2}]^n(1+t^2)^{-1/2}$	$\frac{1}{2}[S_n(p)-\pi \mathbf{E}_n(p)-\pi Y_n(p)]$ $\quad \mathrm{Re}\, p>0$
(36)	$[t-(1+t^2)^{1/2}]^n(1+t^2)^{-1/2}$	$-\frac{1}{2}[S_n(p)+\pi \mathbf{E}_n(p)+\pi Y_n(p)]$ $\quad \mathrm{Re}\, p>0$

4.3. Powers with an arbitrary index

(1)	t^ν $\quad \mathrm{Re}\, \nu>-1$	$\Gamma(\nu+1)p^{-\nu-1}$ $\quad \mathrm{Re}\, p>0$
(2)	0 $\quad 0<t<b$ t^ν $\quad t>b$	$p^{-\nu-1}\Gamma(\nu+1, bp)$ $\quad \mathrm{Re}\, p>0$
(3)	t^ν $\quad 0<t<b$ 0 $\quad t>b$ $\mathrm{Re}\, \nu>-1$	$p^{-\nu-1}\gamma(\nu+1, bp)$
(4)	$(t+a)^\nu$ $\quad \lvert\arg a\rvert<\pi$	$p^{-\nu-1}e^{ap}\Gamma(\nu+1, ap)$ $\quad \mathrm{Re}\, p>0$
(5)	0 $\quad 0<t<b$ $(t-b)^\nu$ $\quad t>b$ $\mathrm{Re}\, \nu>-1$	$\Gamma(\nu+1)p^{-\nu-1}e^{-bp}$ $\quad \mathrm{Re}\, p>0$
(6)	$(b-t)^\nu$ $\quad 0<t<b$ 0 $\quad t>b$ $\mathrm{Re}\, \nu>-1$	$p^{-\nu-1}e^{-bp}\gamma(\nu+1, -bp)$
(7)	$t^\nu(t+a)^{-1}$ $\lvert\arg a\rvert<\pi, \quad \mathrm{Re}\, \nu>-1$	$\Gamma(\nu+1)\, a^\nu e^{ap}\Gamma(-\nu, ap)$ $\quad \mathrm{Re}\, p>0$

Arbitrary powers (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(8)	$0 \qquad 0<t<b$ $t^{-1}(t-b)^{\nu} \qquad t>b$ $\operatorname{Re}\nu>-1$	$\Gamma(\nu+1)b^{\nu}\Gamma(-\nu, bp) \qquad \operatorname{Re} p>0$
(9)	$t^{\nu-1}(1+t^2)^{-1} \qquad \operatorname{Re}\nu>0$	$\pi\csc(\nu\pi)V_{\nu}(2p, 0) \qquad \operatorname{Re} p>0$
(10)	$(1+t^2)^{\nu-\frac{1}{2}}$	$2^{\nu-1}\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}[\mathbf{H}_{\nu}(p)-Y_{\nu}(p)]$ $\operatorname{Re} p>0$
(11)	$0 \qquad 0<t<b$ $(t^2-b^2)^{\nu-\frac{1}{2}} \qquad t>b$ $\operatorname{Re}\nu>-\frac{1}{2}$	$\pi^{-\frac{1}{2}}\Gamma(\nu+\frac{1}{2})(2b/p)^{\nu}K_{\nu}(bp)$ $\operatorname{Re} p>0$
(12)	$(b^2-t^2)^{\nu-\frac{1}{2}} \qquad 0<t<b$ $0 \qquad t>b$ $\operatorname{Re}\nu>-\frac{1}{2}$	$\frac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})(2b/p)^{\nu}[I_{\nu}(bp)$ $-\mathbf{L}_{\nu}(bp)]$
(13)	$(t^2+2at)^{\nu-\frac{1}{2}}$ $\lvert\arg a\rvert<\pi, \qquad \operatorname{Re}\nu>-\frac{1}{2}$	$\pi^{-\frac{1}{2}}\Gamma(\nu+\frac{1}{2})(2a/p)^{\nu}e^{ap}K_{\nu}(ap)$ $\operatorname{Re} p>0$
(14)	$(2bt-t^2)^{\nu-\frac{1}{2}} \qquad 0<t<2b$ $0 \qquad t>2b$ $\operatorname{Re}\nu>-\frac{1}{2}$	$\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})(2b/p)^{\nu}e^{-bp}I_{\nu}(bp)$
(15)	$(t^2+it)^{\nu-\frac{1}{2}} \qquad \operatorname{Re}\nu>-\frac{1}{2}$	$-\frac{1}{2}i\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}e^{\frac{1}{2}ip}H_{\nu}^{(2)}(\frac{1}{2}p)$ $\operatorname{Re} p>0$
(16)	$(t^2-it)^{\nu-\frac{1}{2}} \qquad \operatorname{Re}\nu>-\frac{1}{2}$	$\frac{1}{2}i\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})p^{-\nu}e^{-\frac{1}{2}ip}H_{\nu}^{(1)}(\frac{1}{2}p)$ $\operatorname{Re} p>0$

Arbitrary powers (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(17)	$0 \qquad 0<t<2b$ $(t+2a)^{\nu}(t-2b)^{-\nu} \qquad t>2b$ $\lvert\arg(a+b)\rvert<\pi, \quad \operatorname{Re}\nu<1$	$\nu\pi\csc(\nu\pi)p^{-1}e^{-(a+b)p}k_{2\nu}[(a+b)p]$ $\operatorname{Re}p>0$
(18)	$0 \qquad 0<t<b$ $(t-b)^{\nu-1}(t+b)^{-\nu+1/2} \qquad t>b$ $\operatorname{Re}\nu>0$	$2^{\nu-1/2}\Gamma(\nu)p^{-1/2}D_{1-2\nu}(2b^{1/2}p^{1/2})$ $\operatorname{Re}p>0$
(19)	$0 \qquad 0<t<b$ $(t-b)^{\nu-1}(t+b)^{-\nu-1/2} \qquad t>b$ $\operatorname{Re}\nu>0$	$2^{\nu-1/2}\Gamma(\nu)b^{-1/2}D_{-2\nu}(2b^{1/2}p^{1/2})$ $\operatorname{Re}p\geq 0$
(20)	$t^{\nu-1}(t+a)^{-\nu+1/2}$ $\operatorname{Re}\nu>0, \quad \lvert\arg a\rvert<\pi$	$2^{\nu-1/2}\Gamma(\nu)p^{-1/2}e^{1/2\,ap}D_{1-2\nu}(2^{1/2}a^{1/2}p^{1/2})$ $\operatorname{Re}p>0$
(21)	$t^{\nu-1}(t+a)^{-\nu-1/2}$ $\operatorname{Re}\nu>0, \quad \lvert\arg a\rvert<\pi$	$2^{\nu}\Gamma(\nu)a^{-1/2}e^{1/2\,ap}D_{-2\nu}(2^{1/2}a^{1/2}p^{1/2})$ $\operatorname{Re}p\geq 0$
(22)	$0 \qquad 0<t<b$ $(t+a)^{2\mu-1}(t-b)^{2\nu-1} \qquad t>b$ $\operatorname{Re}\nu>0, \quad \lvert\arg(a+b)\rvert<\pi$	$\Gamma(2\nu)(a+b)^{\mu+\nu-1}p^{-\mu-\nu}e^{1/2\,p(a-b)}$ $\times W_{\mu-\nu,\,\mu+\nu-1/2}(ap+bp) \qquad \operatorname{Re}p>0$
(23)	$0 \qquad 0<t<a$ $(t-a)^{2\mu-1}(b-t)^{2\nu-1} \qquad a<t<b$ $0 \qquad t>b$ $\operatorname{Re}\mu>0, \quad \operatorname{Re}\nu>0$	$\mathrm{B}(2\mu,2\nu)(b-a)^{\mu+\nu-1}p^{-\mu-\nu}e^{-1/2\,p(a+b)}$ $\times M_{\mu-\nu,\,\mu+\nu-1/2}(bp-ap)$
(24)	$t^{\alpha-1}(1-t)^{\beta-1}(1-\sigma t)^{-\gamma}$ $0<t<1$ $0 \qquad t>1$ $\operatorname{Re}\alpha>0, \quad \operatorname{Re}\beta>0$ $\lvert\arg(1-\sigma)\rvert<\pi$	$\mathrm{B}(\alpha,\beta)\,\Phi_1(\alpha,\gamma,\alpha+\beta;\sigma,-p)$

Arbitrary powers (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(25)	$[(t^2+1)^{1/2}+t]^\nu$	$p^{-1} S_{1,\nu}(p)+\nu p^{-1} S_{0,\nu}(p)$ $\quad \operatorname{Re} p>0$
(26)	$[(t^2+1)^{1/2}-t]^\nu$	$p^{-1} S_{1,\nu}(p)-\nu p^{-1} S_{0,\nu}(p)$ $\quad \operatorname{Re} p>0$
(27)	$(t^2+1)^{-1/2}[(t^2+1)^{1/2}+t]^\nu$	$\pi \csc(\nu\pi)[\mathbf{J}_{-\nu}(p)-J_{-\nu}(p)]$ $\quad \operatorname{Re} p>0$
(28)	$(t^2+1)^{-1/2}[(t^2+1)^{1/2}-t]^\nu$	$S_{0,\nu}(p)-\nu S_{-1,\nu}(p)$ $\quad \operatorname{Re} p>0$
(29)	$0 \quad 0<t<1$ $\dfrac{[(t^2-1)^{1/2}+t]^\nu+[(t^2-1)^{1/2}+t]^{-\nu}}{(t^2-1)^{1/2}} \quad t>1$	$2K_\nu(p) \quad \operatorname{Re} p>0$
(30)	$[(t+2a)^{1/2}+t^{1/2}]^{2\nu}$ $-[(t+2a)^{1/2}-t^{1/2}]^{2\nu} \quad \lvert\arg a\rvert<\pi$	$2^{\nu+1}\nu a^\nu p^{-1} e^{ap} K_\nu(ap) \quad \operatorname{Re} p>0$
(31)	$0 \quad 0<t<b$ $[(t+b)^{1/2}+(t-b)^{1/2}]^{2\nu}$ $-[(t+b)^{1/2}-(t-b)^{1/2}]^{2\nu} \quad t>b$	$2^{\nu+1}\nu b^\nu p^{-1} K_\nu(bp) \quad \operatorname{Re} p>0$
(32)	$t^{-\nu-1}(t^2+1)^{-1/2}$ $\times[1+(t^2+1)^{1/2}]^{\nu+1/2} \quad \operatorname{Re}\nu<0$	$2^{1/2}\Gamma(-\nu) D_\nu[(2ip)^{1/2}] D_\nu[(-2ip)^{1/2}]$ $\quad \operatorname{Re} p\geq 0$
(33)	$(2a)^{2\nu}[t+(t^2+4a^2)^{1/2}]^{2\nu}$ $\times(t^3+4a^2t)^{-1/2} \quad \operatorname{Re} a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2}[J_{\nu+1/4}(ap) Y_{\nu-1/4}(ap)$ $- J_{\nu-1/4}(ap) Y_{\nu+1/4}(ap)] \quad \operatorname{Re} p>0$
(34)	$0 \quad 0<t<1$ $t^{-1/2}(t^2-1)^{-1/2}\{[t+(t^2-1)^{1/2}]^{2\nu}$ $+[t-(t^2-1)^{1/2}]^{2\nu}\} \quad t>1$	$(2p/\pi)^{1/2} K_{\nu+1/4}(\tfrac{1}{2}p) K_{\nu-1/4}(\tfrac{1}{2}p)$ $\quad \operatorname{Re} p>0$

4.4. Step-, jump-, and other sectionally rational functions

$n = 0, 1, 2, \ldots$

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$0 \qquad 2nb < t < (2n+1)b$ $1 \qquad (2n-1)b < t < 2nb$	$p^{-1}(e^{bp}+1)^{-1} \qquad \mathrm{Re}\, p > 0$
(2)	$0 \qquad (4n-1)b < t < (4n+1)b$ $2 \qquad (4n+1)b < t < (4n+3)b$	$p^{-1}\,\mathrm{sech}(bp) \qquad \mathrm{Re}\, p > 0$
(3)	$\tfrac{1}{2} \qquad 2nb < t < (2n+1)b$ $-\tfrac{1}{2} \qquad (2n-1)b < t < 2nb$	$\tfrac{1}{2}p^{-1}\tanh(\tfrac{1}{2}bp) \qquad \mathrm{Re}\, p > 0$
(4)	$n \qquad nb < t < (n+1)b$	$p^{-1}(e^{bp}-1)^{-1} \qquad \mathrm{Re}\, p > 0$
(5)	$n+1 \qquad nb < t < (n+1)b$	$p^{-1}(1-e^{-bp})^{-1} \qquad \mathrm{Re}\, p > 0$
(6)	$2n+1 \qquad 2nb < t < 2(n+1)b$	$p^{-1}\,\mathrm{ctnh}(bp) \qquad \mathrm{Re}\, p > 0$
(7)	$0 \qquad 0 < t < b$ $2n \qquad (2n-1)b < t < (2n+1)b$	$p^{-1}\,\mathrm{csch}(bp) \qquad \mathrm{Re}\, p > 0$
(8)	$n \qquad b\pi^2 n^2 < t < b\pi^2(n+1)^2$	$\tfrac{1}{2}p^{-1}[\theta_3(0\|i\pi bp)-1] \qquad \mathrm{Re}\, p > 0$
(9)	$n \qquad \log n < t < \log(n+1)$	$p^{-1}\zeta(p) \qquad \mathrm{Re}\, p > 0$
(10)	$\sum_{0 \le \log n \le t} (t-\log n)^{a-1}$ $\mathrm{Re}\, a > 0$	$\Gamma(a)\,p^{-a}\zeta(p) \qquad \mathrm{Re}\, p > 0$
(11)	$(1-a)^{-1}(1-a^n)$ $nb < t < (n+1)b$	$p^{-1}(e^{bp}-a)^{-1}$ $\mathrm{Re}\, p > 0, \quad b\,\mathrm{Re}\, p > \mathrm{Re}(\log a)$
(12)	$\binom{n}{m} \qquad nb < t < (n+1)b$	$\dfrac{e^{-bp}}{p(e^{bp}-1)^m} \qquad \mathrm{Re}\, p > 0$

Sectionally rational functions (cont'd)

	$f(t)$	$g(y)=\int_0^\infty e^{-pt} f(t)\,dt$
(13)	$n^m \qquad nb<t<(n+1)b$	$\dfrac{1-e^{-bp}}{(-b)^m p}\,\dfrac{d^m}{dp^m}\,(1-e^{-bp})^{-1}$ $\mathrm{Re}\, p>0$
(14)	$t \qquad 0<t<1$ $1 \qquad t>1$	$p^{-2}(1-e^{-p}) \qquad \mathrm{Re}\, p>0$
(15)	$t \qquad 0<t<1$ $2-t \qquad 1<t<2$ $0 \qquad t>2$	$p^{-2}(1-e^{-p})^2$
(16)	$a(t-nb) \qquad nb<t<(n+1)b$	$ap^{-2}-\tfrac{1}{2}abp^{-1}[\mathrm{ctnh}(\tfrac{1}{2}bp)-1]$ $=ap^{-2}(e^{bp}-1)^{-1}(e^{bp}-bp-1)$ $\mathrm{Re}\, p>0$
(17)	$\dfrac{1-a^n}{1-a}\,t-b\,\dfrac{1-(n+1)a^n+na^{n+1}}{(1-a)^2}$ $nb<t<(n+1)b$	$\dfrac{1}{p^2(e^{bp}-a)}$ $\mathrm{Re}\, p>0, \quad b\,\mathrm{Re}\, p>\mathrm{Re}\log a$
(18)	$(2n+1)t-2bn(n+1)$ $2nb<t<2(n+1)b$	$p^{-2}\,\mathrm{ctnh}\,(bp) \qquad \mathrm{Re}\, p>0$
(19)	$b-(-1)^n(2bn+b-t)$ $2nb<t<2(n+1)b$	$p^{-2}\tanh(bp) \qquad \mathrm{Re}\, p>0$
(20)	$0 \qquad 0<t<b$ $t-(-1)^n(t-2nb)$ $(2n-1)b<t<(2n+1)b$ $n\geq 1$	$p^{-2}\,\mathrm{sech}\,(bp) \qquad \mathrm{Re}\, p>0$

Sectionally rational functions (cont'd)

	$f(t)$	$g(y)=\int_0^\infty e^{-pt} f(t)\,dt$
(21)	$0 \qquad 0<t<b$ $2n(t-bn)$ $(2n-1)b<t<(2n+1)b$ $n \geq 1$	$p^{-2}\operatorname{csch}(bp) \qquad \operatorname{Re} p>0$
(22)	$\tfrac{1}{4}[1-(-1)^n](2t-b)+\tfrac{1}{2}(-1)^n bn$ $nb<t<(n+1)b$	$p^{-2}(e^{bp}+1)^{-1} \qquad \operatorname{Re} p>0$
(23)	$0 \qquad 0<t<b$ $nt-\tfrac{1}{2}bn(n+1)$ $nb<t<(n+1)b$ $n \geq 1$	$p^{-2}(e^{bp}-1)^{-1} \qquad \operatorname{Re} p>0$
(24)	$\tfrac{1}{2}t^2 \qquad 0<t<1$ $1-\tfrac{1}{2}(t-2)^2 \qquad 1<t<2$ $1 \qquad t>2$	$p^{-3}(1-e^{-p})^2 \qquad \operatorname{Re} p>0$
(25)	$\tfrac{1}{2}t^2 \qquad 0<t<1$ $3/4-(t-3/2)^2 \qquad 1<t<2$ $\tfrac{1}{2}(t-3)^2 \qquad 2<t<3$ $0 \qquad t>3$	$p^{-3}(1-e^{-p})^3$
(26)	$(t-nb)^2 \qquad nb<t<(n+1)b$	$2p^{-3}-p^{-2}(b^2+2bp)(e^{bp}-1)^{-1}$ $\operatorname{Re} p>0$
	For further similar integrals see Gardner, M. F. and J. L. Barnes, 1942: *Transients in linear systems*, I, Wiley.	

4.5. Exponential functions

(1)	$e^{-\alpha t}$	$(p+\alpha)^{-1} \qquad \operatorname{Re} p>-\operatorname{Re} \alpha$

Exponential functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(2)	$te^{-\alpha t}$	$(p+\alpha)^{-2}$ $\qquad \operatorname{Re} p > -\operatorname{Re} \alpha$
(3)	$t^{\nu-1}e^{-\alpha t}$ $\qquad \operatorname{Re}\nu>0$	$\Gamma(\nu)(p+\alpha)^{-\nu}$ $\qquad \operatorname{Re} p > -\operatorname{Re}\alpha$
(4)	$t^{-1}(e^{-\alpha t}-e^{-\beta t})$	$\log(p+\beta)-\log(p+\alpha)$ $\qquad \operatorname{Re} p > -\operatorname{Re}\alpha,\ -\operatorname{Re}\beta$
(5)	$t^{-2}(1-e^{-\alpha t})^2$	$(p+2\alpha)\log(p+2\alpha)+p\log p - 2(p+\alpha)\log(p+\alpha)$ $\qquad \operatorname{Re} p \geq 0,\ -\operatorname{Re} 2\alpha$
(6)	$t^{-1}-\frac{1}{2}t^{-2}(t+2)(1-e^{-t})$	$-1+(p+\frac{1}{2})\log(1+1/p)$ $\qquad \operatorname{Re} p>0$
(7)	$(1+e^{-t})^{-1}$	$\frac{1}{2}\psi(\frac{1}{2}p+\frac{1}{2})-\frac{1}{2}\psi(\frac{1}{2}p)$ $\qquad \operatorname{Re} p>0$
(8)	$(1-e^{-t/\alpha})^{\nu-1}$ $\qquad \operatorname{Re}\alpha>0,\quad \operatorname{Re}\nu>0$	$\alpha\,\mathrm{B}(\alpha p,\nu)$ $\qquad \operatorname{Re} p>0$
(9)	$t^n(1-e^{-t/\alpha})^{-1}$ $\qquad \operatorname{Re}\alpha>0$	$(-\alpha)^{n+1}\psi^{(n)}(\alpha p)$ $\qquad \operatorname{Re} p>0$ $\psi^{(n)}(z)=\dfrac{d^n}{dz^n}\psi(z)$
(10)	$t^{\nu-1}(1-e^{-t/\alpha})^{-1}$ $\qquad \operatorname{Re}\nu>1$	$\alpha^\nu\Gamma(\nu)\zeta(\nu,\alpha p)$ $\qquad \operatorname{Re} p>0$
(11)	$t^{-1}(1-e^{-t})^{-1}-t^{-2}-\frac{1}{2}t^{-1}$	$p+\log\Gamma(p)-p\log p+\frac{1}{2}\log(\frac{1}{2}p/\pi)$ $\qquad \operatorname{Re} p>0$
(12)	$(1-e^{-\alpha t})(1-e^{-t})^{-1}$	$\psi(p+\alpha)-\psi(p)$ $\qquad \operatorname{Re} p>0,\ -\operatorname{Re}\alpha$
(13)	$\dfrac{1-e^{-\alpha t}}{t(1+e^{-t})}$	$\log\dfrac{\Gamma(\frac{1}{2}p)\Gamma(\frac{1}{2}\alpha+\frac{1}{2}p+\frac{1}{2})}{\Gamma(\frac{1}{2}p+\frac{1}{2})\Gamma(\frac{1}{2}\alpha+\frac{1}{2}p)}$ $\qquad \operatorname{Re} p>0,\ -\operatorname{Re}\alpha$

Exponential functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(14)	$(1-e^{-t})^{\nu-1}(1-ze^{-t})^{-\mu}$ $\mathrm{Re}\,\nu>0,\quad \lvert\arg(1-z)\rvert<\pi$	$B(p,\nu)\ {}_2F_1(\mu,p;p+\nu;z)$ $\mathrm{Re}\,p>0$
(15)	$(1-e^{-t})^{-1}(1-e^{-\alpha t})(1-e^{-\beta t})$	$\psi(p+\alpha)+\psi(p+\beta)$ $-\psi(p+\alpha+\beta)-\psi(p)$ $\mathrm{Re}\,p>0,\quad -\mathrm{Re}\,\alpha$ $\mathrm{Re}\,p>-\mathrm{Re}\,\beta,\quad -\mathrm{Re}(\alpha+\beta)$
(16)	$\dfrac{(1-e^{-\alpha t})(1-e^{-\beta t})}{t(1-e^{-t})}$	$\log\dfrac{\Gamma(p)\,\Gamma(p+\alpha+\beta)}{\Gamma(p+\alpha)\Gamma(p+\beta)}$ $\mathrm{Re}\,p>0,\quad -\mathrm{Re}\,\alpha$ $\mathrm{Re}\,p>-\mathrm{Re}\,\beta,\quad -\mathrm{Re}(\alpha+\beta)$
(17)	$\dfrac{(1-e^{-\alpha t})(1-e^{-\beta t})(1-e^{-\gamma t})}{t(1-e^{-t})}$	$\log\dfrac{\Gamma(p)\Gamma(p+\beta+\gamma)\Gamma(p+\alpha+\gamma)\Gamma(p+\alpha+\beta)}{\Gamma(p+\alpha)\Gamma(p+\beta)\Gamma(p+\gamma)\Gamma(p+\alpha+\beta+\gamma)}$ $2\,\mathrm{Re}\,p>\lvert\mathrm{Re}\,\alpha\rvert+\lvert\mathrm{Re}\,\beta\rvert+\lvert\mathrm{Re}\,\gamma\rvert$
(18)	$\dfrac{[a+(1-e^{-t})^{1/2}]^{-\nu}+[a-(1-e^{-t})^{1/2}]^{-\nu}}{(1-e^{-t})^{1/2}}$	$2^{p+1}e^{(p-\nu)\pi i}\dfrac{\Gamma(p)}{\Gamma(\nu)}(a^2-1)^{\frac{1}{2}p-\frac{1}{2}\nu}$ $\times Q_{p-1}^{\nu-p}(a)\qquad \mathrm{Re}\,p>0$
(19)	$0\qquad 0<t<b$ $(1-e^{-2t})^{-1/2}[e^{-b}(1-e^{-2t})^{1/2}$ $-e^{-t}(1-e^{-2b})^{1/2}]^{\nu}\qquad t>b$ $\mathrm{Re}\,\nu>-1$	$\dfrac{(\pi)^{1/2}\Gamma(p)\Gamma(\nu+1)}{2^{\frac{1}{2}p+\frac{1}{2}\nu}\Gamma(\frac{1}{2}p+\frac{1}{2}\nu+\frac{1}{2})}e^{-\frac{1}{2}b(p+\nu)}$ $\times P_{-\frac{1}{2}p+\frac{1}{2}\nu}^{-\frac{1}{2}p-\frac{1}{2}\nu}[(1-e^{-2b})^{1/2}]$ $\mathrm{Re}\,p>0$
(20)	$e^{(\mu-1)t}(1-e^{-t})^{\mu-\frac{1}{2}}[(1-e^{-t})\sin\theta$ $-i(1-e^{-t})\cos\theta]^{\mu-\frac{1}{2}}$ $\mathrm{Re}\,\mu>-\frac{1}{2}$	$\dfrac{2^{\mu-1}\Gamma(\mu+\frac{1}{2})\Gamma(p-\mu+1)}{\pi^{1/2}\,\Gamma(p+\mu+1)}\sin^{\mu}\theta$ $\times e^{(p+\frac{1}{2})i\theta+(\frac{1}{2}\mu-\frac{1}{4})\pi i}$ $\times[\pi P_{\nu}^{\mu}(\cos\theta)+2i\,Q_{\nu}^{\mu}(\cos\theta)]$ $\mathrm{Re}\,p>\mathrm{Re}\,\mu-1$
	Another formula may be derived from this by changing i into $-i$ throughout.	

Exponential functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(21)	$0 \quad 0<t<b$ $e^{-\frac{1}{4}t^2/a} \quad t>b$ $\mathrm{Re}\,a>0$	$\pi^{\frac{1}{2}}a^{\frac{1}{2}}e^{ap^2}\,\mathrm{Erfc}(a^{\frac{1}{2}}p+\frac{1}{2}a^{-\frac{1}{2}}b)$
(22)	$te^{-\frac{1}{4}t^2/a} \quad \mathrm{Re}\,a>0$	$2a-2\pi^{1/2}a^{3/2}pe^{ap^2}\,\mathrm{Erfc}(a^{1/2}p)$
(23)	$t^{-\frac{1}{2}}e^{-\frac{1}{4}t^2/a} \quad \mathrm{Re}\,a>0$	$a^{\frac{1}{4}}p^{\frac{1}{2}}e^{\frac{1}{2}ap^2}K_{\frac{1}{4}}(\frac{1}{2}ap^2)$
(24)	$t^{\nu-1}e^{-t^2/(8a)}$ $\mathrm{Re}\,a>0, \quad \mathrm{Re}\,\nu>0$	$\Gamma(\nu)2^{\nu}a^{\frac{1}{2}\nu}e^{ap^2}D_{-\nu}(2pa^{\frac{1}{2}})$
(25)	$e^{-\frac{1}{4}a/t} \quad \mathrm{Re}\,a\geq 0$	$a^{\frac{1}{2}}p^{-\frac{1}{2}}K_1(a^{\frac{1}{2}}p^{\frac{1}{2}}) \quad \mathrm{Re}\,p>0$
(26)	$t^{\frac{1}{2}}e^{-\frac{1}{4}a/t} \quad \mathrm{Re}\,a\geq 0$	$\frac{1}{2}\pi^{1/2}p^{-3/2}(1+a^{1/2}p^{1/2})e^{-a^{\frac{1}{2}}p^{\frac{1}{2}}}$ $\mathrm{Re}\,p>0$
(27)	$t^{-\frac{1}{2}}e^{-\frac{1}{4}a/t} \quad \mathrm{Re}\,a\geq 0$	$\pi^{\frac{1}{2}}p^{-\frac{1}{2}}e^{-a^{\frac{1}{2}}p^{\frac{1}{2}}} \quad \mathrm{Re}\,p>0$
(28)	$t^{-3/2}e^{-\frac{1}{4}a/t} \quad \mathrm{Re}\,a>0$	$2\pi^{\frac{1}{2}}a^{-\frac{1}{2}}e^{-a^{\frac{1}{2}}p^{\frac{1}{2}}} \quad \mathrm{Re}\,p\geq 0$
(29)	$t^{\nu-1}e^{-\frac{1}{4}a/t} \quad \mathrm{Re}\,a>0$	$2(\frac{1}{4}a/p)^{\frac{1}{2}\nu}K_{\nu}(a^{\frac{1}{2}}p^{\frac{1}{2}}) \quad \mathrm{Re}\,p>0$
(30)	$t^{-\frac{1}{2}}(e^{-\frac{1}{4}a/t}-1) \quad \mathrm{Re}\,a\geq 0$	$\pi^{\frac{1}{2}}p^{-\frac{1}{2}}(e^{-a^{\frac{1}{2}}p^{\frac{1}{2}}}-1) \quad \mathrm{Re}\,p\geq 0$
(31)	$e^{-2a^{\frac{1}{2}}t^{\frac{1}{2}}} \quad \lvert\arg a\rvert<\pi$	$p^{-1}-\pi^{1/2}a^{1/2}p^{-3/2}e^{a/p}$ $\times\,\mathrm{Erfc}(a^{1/2}p^{-1/2}) \quad \mathrm{Re}\,p>0$
(32)	$t^{\frac{1}{2}}e^{-2a^{\frac{1}{2}}t^{\frac{1}{2}}} \quad \lvert\arg a\rvert<\pi$	$-a^{1/2}p^{-2}+\pi^{1/2}p^{-5/2}(a+\frac{1}{2}p)e^{a/p}$ $\times\,\mathrm{Erfc}(a^{1/2}p^{-1/2}) \quad \mathrm{Re}\,p>0$

Exponential functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(33)	$t^{-1/2}e^{-2a^{1/2}t^{1/2}}$ $\qquad\lvert\arg a\rvert<\pi$	$\pi^{1/2}p^{-1/2}e^{a/p}\,\mathrm{Erfc}(a^{1/2}p^{-1/2})$ $\qquad\mathrm{Re}\,p>0$
(34)	$(2t)^{-3/4}e^{-2a^{1/2}t^{1/2}}$ $\qquad\lvert\arg a\rvert<\pi$	$(\tfrac12 a/p)^{1/4}e^{\frac12 a/p}K_{1/4}(\tfrac12 a/p)$ $\qquad\mathrm{Re}\,p>0$
(35)	$(2t)^{\nu-1}e^{-2a^{1/2}t^{1/2}}$ $\qquad\mathrm{Re}\,\nu>0$	$\Gamma(2\nu)\,p^{-\nu}e^{\frac12 a/p}D_{-2\nu}[(2a/p)^{1/2}]$ $\qquad\mathrm{Re}\,p>0$
(36)	$\exp(-ae^{-t})$	$a^{-p}\,\gamma(p,a)$ $\qquad\mathrm{Re}\,p>0$
(37)	$\exp(-ae^{t})$ $\qquad\mathrm{Re}\,a>0$	$a^{p}\,\Gamma(-p,a)$
(38)	$(1-e^{-t})^{\nu-1}\exp(ae^{-t})$ $\qquad\mathrm{Re}\,\nu>0$	$\dfrac{\Gamma(\nu)\Gamma(p)}{\Gamma(\nu+p)}\,a^{-\frac12\nu-\frac12 p}\,e^{\frac12 a}$ $\times M_{\frac12\nu-\frac12 p,\,\frac12\nu+\frac12 p-\frac12}(a)$ $\qquad\mathrm{Re}\,p>0$
(39)	$(1-e^{-t})^{\nu-1}\exp(-ae^{t})$ $\qquad\mathrm{Re}\,a>0,\quad\mathrm{Re}\,\nu>0$	$\Gamma(\nu)\,a^{\frac12 p-\frac12}e^{-\frac12 a}W_{\frac12-\frac12 p-\nu,\,-\frac12 p}(a)$
(40)	$(1-e^{-t})^{\nu-1}(1-\lambda e^{-t})^{-\mu}$ $\times\exp(ae^{-t})$ $\qquad\mathrm{Re}\,\nu>0,\quad\lvert\arg(1-\lambda)\rvert<\pi$	$\dfrac{\Gamma(\nu)\Gamma(p)}{\Gamma(\nu+p)}\,\Phi_1(p,\mu,\nu;\lambda,a)$ $\qquad\mathrm{Re}\,p>0$
(41)	$(e^{t}-1)^{\nu-1}\exp[-a/(e^{t}-1)]$ $\qquad\mathrm{Re}\,a>0$	$\Gamma(p-\nu+1)\,e^{\frac12 a}a^{\frac12\nu-\frac12}$ $\times W_{\frac12\nu-\frac12-p,\,\frac12\nu}(a)$ $\qquad\mathrm{Re}\,p>\mathrm{Re}\,\nu-1$

4.6. Logarithmic functions

	$f(t)$		$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	
(1)	$\log t$		$-p^{-1}\log(\gamma p)$	$\operatorname{Re} p>0$
(2)	0 $\log t$	$0<t<b$ $t>b$	$p^{-1}[e^{-bp}\log b-\operatorname{Ei}(-bp)]$	$\operatorname{Re} p>0$
(3)	0 $\log(t/b)$	$0<t<b$ $t>b$	$-p^{-1}\operatorname{Ei}(-bp)$	$\operatorname{Re} p>0$
(4)	$\log(1+at)$	$\lvert\arg a\rvert<\pi$	$-p^{-1}e^{p/a}\operatorname{Ei}(-p/a)$	$\operatorname{Re} p>0$
(5)	$\log(t+a)$	$\lvert\arg a\rvert<\pi$	$p^{-1}[\log a-e^{ap}\operatorname{Ei}(-ap)]$	$\operatorname{Re} p>0$
(6)	$\log\lvert b-t\rvert$	$b>0$	$p^{-1}[\log b-e^{-bp}\overline{\operatorname{Ei}}(bp)]$	$\operatorname{Re} p>0$
(7)	$t^n\log t$		$\dfrac{n!}{p^{n+1}}\left[1+\dfrac{1}{2}+\dfrac{1}{3}+\cdots+\dfrac{1}{n}-\log(\gamma p)\right]$	$\operatorname{Re} p>0$
(8)	0 $t^{-1}\log(2t-1)$	$0<t<1$ $t>1$	$\frac{1}{2}[\operatorname{Ei}(-\frac{1}{2}p)]^2$	$\operatorname{Re} p>0$
(9)	$t^{-\frac{1}{2}}\log t$		$-\pi^{\frac{1}{2}}p^{-\frac{1}{2}}\log(4\gamma p)$	$\operatorname{Re} p>0$
(10)	$t^{n-\frac{1}{2}}\log t$	$n\geq 1$	$\pi^{\frac{1}{2}}\dfrac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{p^{n+\frac{1}{2}}\,2^n}\left[2\left(1+\dfrac{1}{3}+\dfrac{1}{5}+\cdots+\dfrac{1}{2n-1}\right)-\log(4\gamma p)\right]$	$\operatorname{Re} p>0$
(11)	$t^{\nu-1}\log t$	$\operatorname{Re}\nu>0$	$\Gamma(\nu)\,p^{-\nu}[\psi(\nu)-\log p]$	$\operatorname{Re} p>0$

Logarithmic functions (cont'd)

	$f(t)$		$g(p) = \int_0^\infty e^{-pt} f(t)\,dt$	
(12)	$t^{\nu-1}[\psi(\nu) - \log t]$	$\operatorname{Re}\nu > 0$	$\Gamma(\nu)p^{-\nu}\log p$	$\operatorname{Re} p > 0$
(13)	$(\log t)^2$		$p^{-1}\{\pi^2/6 + [\log(\gamma p)]^2\}$	$\operatorname{Re} p > 0$
(14)	$\log\lvert t^2 - a^2\rvert$	$a > 0$	$p^{-1}[\log a^2 - e^{ap}\operatorname{Ei}(-ap) - e^{-ap}\overline{\operatorname{Ei}}(ap)]$	$\operatorname{Re} p > 0$
(15)	$\log(t^2 - a^2)$	$\lvert\operatorname{Im} a\rvert > 0$	$p^{-1}[\log a^2 - e^{ap}\operatorname{Ei}(-ap) - e^{-ap}\operatorname{Ei}(ap)]$	$\operatorname{Re} p > 0$
(16)	$\log(t^2 + a^2)$		$2p^{-1}[\log a - \operatorname{ci}(ap)\cos(ap) - \operatorname{si}(ap)\sin(ap)]$	$\operatorname{Re} p > 0$
(17)	$t^{-1}[\log(t^2 + a^2) - \log a^2]$		$[\operatorname{ci}(ap)]^2 + [\operatorname{si}(ap)]^2$	$\operatorname{Re} p > 0$
(18)	0	$0 < t < b$	$\frac{1}{2}p^{-1}K_0(bp)$	$\operatorname{Re} p > 0$
	$\log\dfrac{(t+b)^{1/2} + (t-b)^{1/2}}{2^{1/2}b^{1/2}}$	$t > b$		
(19)	$\log\dfrac{t^{1/2} + (t+2a)^{1/2}}{2^{1/2}a^{1/2}}$	$\lvert\arg a\rvert < \pi$	$\frac{1}{2}p^{-1}e^{ap}K_0(ap)$	$\operatorname{Re} p > 0$
(20)	$\log\dfrac{(t+ib)^{1/2} + (t-ib)^{1/2}}{2^{1/2}b^{1/2}}$	$b > 0$	$\frac{1}{4}\pi p^{-1}[\mathbf{H}_0(bp) - Y_0(bp)]$	$\operatorname{Re} p > 0$
(21)	$\dfrac{\log[4t(2b-t)/b^2]}{t^{1/2}(2b-t)^{1/2}}$	$0 < t < 2b$	$\pi e^{-bp}[\frac{1}{2}\pi Y_0(ibp) - \log(\frac{1}{2}\gamma)J_0(ibp)]$	
	0	$t > 2b$		

4.7. Trigonometric functions

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$\sin(at)$	$a(p^2+a^2)^{-1}$ $\qquad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a \rvert$
(2)	$\lvert\sin(at)\rvert$ $\qquad a>0$	$a(p^2+a^2)^{-1}\operatorname{ctnh}(\tfrac{1}{2}\pi a^{-1}p)$ $\mathrm{Re}\, p > 0$
(3)	$\sin^{2n}(at)$	$\dfrac{(2n)!\,a^{2n}}{p[p^2+(2a)^2][p^2+(4a)^2]\cdots[p^2+(2na)^2]}$ $\mathrm{Re}\, p > 2n\,\lvert \mathrm{Im}\, a \rvert$
(4)	$0 \qquad 0<t<\pi/2$ $\sin^{2n} t \qquad t>\pi/2$	$\dfrac{(2n)!\,e^{-\frac{1}{2}\pi p}}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$ $\times\left\{1+\dfrac{p^2}{2!}+\dfrac{p^2(2^2+p^2)}{4!}+\cdots+\dfrac{p^2(2^2+p^2)\cdots[4(n-1)^2+p^2]}{(2n)!}\right\}$ $\mathrm{Re}\, p > 0$
(5)	$\sin^{2n} t \qquad 0<t<\pi/2$ $0 \qquad t>\pi/2$	$\dfrac{(2n)!\,e^{-\frac{1}{2}\pi p}}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$ $\times\left\{e^{\frac{1}{2}\pi p}-1-\dfrac{p^2}{2!}-\cdots-\dfrac{p^2(2^2+p^2)\cdots[4(n-1)^2+p^2]}{(2n)!}\right\}$
(6)	$\sin^{2n} t \qquad 0<t<m\pi$ $0 \qquad t>m\pi$ $m=1, 2, 3, \ldots$	$\dfrac{(2n)!\,(1-e^{-mp\pi})}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$
(7)	$\sin^{2n+1}(at)$	$\dfrac{(2n+1)!\,a^{2n+1}}{(p^2+a^2)[p^2+(3a)^2]\cdots\{p^2+[(2n+1)a]^2\}}$ $\mathrm{Re}\, p > (2n+1)\,\lvert \mathrm{Im}\, a \rvert$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(8)	$0 \quad 0<t<\pi/2$ $\sin^{2n+1} \quad t>\pi/2$	$\dfrac{(2n+1)!\,pe^{-\frac{1}{2}\pi p}}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}$ $\times\left\{1+\dfrac{1^2+p^2}{3!}+\dfrac{(1^2+p^2)(3^2+p^2)}{5!}\right.$ $\left.+\cdots+\dfrac{(1^2+p^2)(3^2+p^2)\cdots[(2n-1)^2+p^2]}{(2n+1)!}\right\}$ $\operatorname{Re} p>0$
(9)	$\sin^{2n+1} t \quad 0<t<\pi/2$ $0 \quad t>\pi/2$	$\dfrac{(2n+1)!\,pe^{-\frac{1}{2}\pi p}}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}$ $\times\left\{\dfrac{e^{\frac{1}{2}\pi p}}{p}-1-\dfrac{1^2+p^2}{3!}\right.$ $\left.-\cdots-\dfrac{(1^2+p^2)(3^2+p^2)\cdots[(2n-1)^2+p^2]}{(2n+1)!}\right\}$
(10)	$\sin^{2n+1} t \quad 0<t<m\pi$ $0 \quad t>m\pi$ $m=1,2,3,\ldots$	$\dfrac{(2n+1)!\,[1-(-1)^m e^{-mp\pi}]}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}$
(11)	$\lvert\sin(at)\rvert^{2\nu}$ $a>0, \quad \operatorname{Re}\nu>-\frac{1}{2}$	$\dfrac{\mathrm{B}(1+\frac{1}{2}ip/a,\ 1-\frac{1}{2}ip/a)}{(2\nu+1)2^{2\nu}p\,\mathrm{B}(\nu+1+\frac{1}{2}ip/a,\ \nu+1-\frac{1}{2}ip/a)}$ $\operatorname{Re} p>0$
(12)	$0 \quad 0<t<\pi/2$ $t\sin t \quad t>\pi/2$	$\dfrac{e^{-\frac{1}{2}\pi p}}{(1+p^2)^2}[\tfrac{1}{2}p\pi(1+p^2)+p^2-1]$ $\operatorname{Re} p>0$
(13)	$t\sin t \quad 0<t<\pi/2$ $0 \quad t>\pi/2$	$(1+p^2)^{-2}\{2p-e^{-\frac{1}{2}\pi p}$ $\times[\tfrac{1}{2}p\pi(1+p^2)+p^2-1]\}$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(14)	$t^n \sin(at)$	$n!\,\dfrac{p^{n+1}}{(p^2+a^2)^{n+1}} \times \sum\limits_{0\le 2m\le n} (-1)^m \binom{n+1}{2m+1}\left(\dfrac{a}{p}\right)^{2m+1}$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$
(15)	$t^{\nu-1}\sin(at)$ $\quad \mathrm{Re}\,\nu > -1$	$\tfrac{1}{2} i\,\Gamma(\nu)\,[(p+ia)^{-\nu}-(p-ia)^{-\nu}]$ $= \Gamma(\nu)\,(p^2+a^2)^{-\frac{1}{2}\nu}\sin[\nu\tan^{-1}(a/p)]$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$
(16)	$t^{-1}\sin(at)$	$\tan^{-1}(a/p)$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$
(17)	$t^{-1}\sin^2(at)$	$\tfrac{1}{4}\log(1+4a^2p^{-2})$ $\quad \mathrm{Re}\, p > 2\lvert \mathrm{Im}\, a\rvert$
(18)	$t^{-1}\sin^3(at)$	$\tfrac{1}{2}\tan^{-1}(a/p)-\tfrac{1}{4}\tan^{-1}[2ap/(p^2+3a^2)]$ $\quad \mathrm{Re}\, p > 3\lvert \mathrm{Im}\, a\rvert$
(19)	$t^{-1}\sin^4(at)$	$\dfrac{1}{8}\log\dfrac{(p^2+4a^2)^2}{p^3}-\dfrac{1}{16}\log(p^2+16a^2)$ $\quad \mathrm{Re}\, p > 4\lvert \mathrm{Im}\, a\rvert$
(20)	$t^{-2}\sin^2(at)$	$a\tan^{-1}(2a/p)-\tfrac{1}{4}p\log(1+4a^2p^{-2})$ $\quad \mathrm{Re}\, p \geq 2\lvert \mathrm{Im}\, a\rvert$
(21)	$t^{-2}\sin^3(at)$	$\tfrac{1}{4}p\tan^{-1}(3a/p)-\tfrac{3}{4}p\tan^{-1}(a/p)$ $+(3a/8)\log[(p^2+3a^2)/(p^2+a^2)]$ $\quad \mathrm{Re}\, p \geq 3\,\mathrm{Im}\, a\rvert$
(22)	$(e^t-1)^{-1}\sin(at)$	$\tfrac{1}{2} i\,\psi(p-ia+1)-\tfrac{1}{2} i\,\psi(p+ia+1)$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert-1$
(23)	$(1-e^{-t})^{-1}\sin(at)$	$\tfrac{1}{2} i\,\psi(p-ia)-\tfrac{1}{2} i\,\psi(p+ia)$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(24)	$(1-e^{-t})^{\nu-1}\sin(at)$ $\operatorname{Re}\nu>-1$	$\tfrac{1}{2}i\,\mathrm{B}(\nu,p+ia)-\tfrac{1}{2}i\,\mathrm{B}(\nu,p-ia)$ $\operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(25)	$t^{\nu-1}e^{-\frac{1}{2}t^2/\alpha}\sin(\beta t)$ $\operatorname{Re}\alpha>0,\quad\operatorname{Re}\nu>-1$	$\tfrac{1}{2}i\,\Gamma(\nu)\ \alpha^{\frac{1}{2}\nu}e^{\frac{1}{4}\alpha(p^2-\beta^2)}$ $\times\{e^{\frac{1}{2}ip\alpha\beta}D_{-\nu}[\alpha^{\frac{1}{2}}(p+i\beta)]$ $-e^{-\frac{1}{2}ip\alpha\beta}D_{-\nu}[\alpha^{\frac{1}{2}}(p-i\beta)]\}$
(26)	$\log t\,\sin(at)$	$\dfrac{p\tan^{-1}(a/p)-a\log[\gamma(p^2+a^2)^{\frac{1}{2}}]}{p^2+a^2}$ $\operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(27)	$\log t\,\sin^2(\tfrac{1}{2}at)$	$p^{-1}(p^2+a^2)^{-1}[ap\tan^{-1}(a/p)$ $+p^2\log(p^2+a^2)^{\frac{1}{2}}$ $-(p^2+a^2)\log p-a^2\log\gamma]$ $\operatorname{Re}p>2\lvert\operatorname{Im}a\rvert$
(28)	$t^{-1}\log t\,\sin(at)$	$-\log[\gamma(p^2+a^2)^{\frac{1}{2}}]\tan^{-1}(a/p)$ $\operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(29)	$t^{\nu-1}\log t\,\sin(at)$ $\operatorname{Re}\nu>-1$	$\Gamma(\nu)(p^2+a^2)^{-\frac{1}{2}\nu}\sin[\nu\tan^{-1}(a/p)]$ $\times\{\psi(\nu)-\log(p^2+a^2)^{\frac{1}{2}}$ $+\tan^{-1}(a/p)\operatorname{ctn}[\nu\tan^{-1}(a/p)]\}$ $\operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(30)	$\sin(t^2)$	$(\tfrac{1}{2}\pi)^{\frac{1}{2}}[\tfrac{1}{2}-\cos(\tfrac{1}{4}p^2)C(\tfrac{1}{4}p^2)$ $-\sin(\tfrac{1}{4}p^2)S(\tfrac{1}{4}p^2)]\qquad\operatorname{Re}p>0$
(31)	$t^{-1}\sin(t^2)$	$\tfrac{1}{2}\pi[\tfrac{1}{2}-C(\tfrac{1}{4}p^2)]^2+\tfrac{1}{2}\pi[\tfrac{1}{2}-S(\tfrac{1}{4}p^2)]^2$ $\operatorname{Re}p>0$
(32)	$\sin(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$\pi^{1/2}a^{1/2}p^{-3/2}e^{-a/p}\qquad\operatorname{Re}p>0$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(33)	$t^n \sin(2a^{1/2}t^{1/2})$	$(-1)^n 2^{-n-1/2} \pi^{1/2} p^{-n-1} e^{-a/p}$ $\times \mathrm{He}_{2n+1}(2^{1/2} a^{1/2} p^{-1/2})$ Re $p>0$
(34)	$t^{-1} \sin(2a^{1/2}t^{1/2})$	$\pi \,\mathrm{Erf}(a^{1/2} p^{-1/2})$ Re $p>0$
(35)	$t^{1/2} \sin(2a^{1/2}t^{1/2})$	$a^{1/2}p^{-2} - i\pi^{1/2} p^{-5/2}(\tfrac{1}{2}p-a)e^{-a/p}$ $\times \mathrm{Erf}(i a^{1/2} p^{-1/2})$ Re $p>0$
(36)	$t^{-1/2} \sin(2a^{1/2}t^{1/2})$	$-i\pi^{1/2} p^{-1/2} e^{-a/p} \mathrm{Erf}(i a^{1/2} p^{-1/2})$ Re $p>0$
(37)	$t^{\nu-1} \sin(2^{1/2} a^{1/2} t^{1/2})$ Re $\nu>-\tfrac{1}{2}$	$2^{-\nu-1/2}\pi^{1/2} \sec(\nu\pi) p^{-\nu} e^{-1/4\, a/p}$ $\times [D_{2\nu-1}(-a^{1/2}p^{-1/2}) - D_{2\nu-1}(a^{1/2}p^{-1/2})]$ Re $p>0$
(38)	0 $\quad 0<t<b$ $\sin[a(t^2-b^2)^{1/2}]$ $\quad t>b$	$abp^{-1} K_1(bp)$ Re $p>\lvert\mathrm{Im}\, a\rvert$
(39)	$\sin(ae^{-t})$	$a^{-p}\Gamma(p)[U_p(2a,0)\sin a$ $- U_{p+1}(2a,0)\cos a]$ Re $p>0$
(40)	$\sin[a(1-e^{-t})]$	$a^{-p}\Gamma(p)\, U_{p+1}(2a,0)$ Re $p>0$
(41)	$(e^t-1)^{-1/2} \sin[a(1-e^{-t})^{1/2}]$	$\pi^{1/2}\Gamma(p+\tfrac{1}{2})(2/a)^p\, \mathbf{H}_p(a)$ Re $p>-\tfrac{1}{2}$
(42)	$(1-e^{-t})^{-1/2} \sin[a(e^t-1)^{1/2}]$ $a>0$	$\pi^{1/2}\Gamma(\tfrac{1}{2}-p)(\tfrac{1}{2}a)^p [I_p(a) - \mathbf{L}_{-p}(a)]$ Re $p>-\tfrac{1}{2}$
(43)	$\cos(at)$	$p(p^2+a^2)^{-1}$ Re $p>\lvert\mathrm{Im}\, a\rvert$

Trigonometric functions (cont'd)

	$f(t)$		$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(44)	$\lvert\cos(at)\rvert$	$a>0$	$(p^2+a^2)^{-1}[p+a\,\mathrm{csch}(\tfrac{1}{2}\pi a^{-1}p)]$ $\mathrm{Re}\,p>0$
(45)	$\cos^2(at)$		$(p^2+2a^2)(p^3+4a^2p)^{-1}$ $\mathrm{Re}\,p>2\lvert\mathrm{Im}\,a\rvert$
(46)	$\cos^3(at)$		$p(p^2+7a^2)(p^2+a^2)^{-1}(p^2+9a^2)^{-1}$ $\mathrm{Re}\,p>3\lvert\mathrm{Im}\,a\rvert$
(47)	$\cos^{2n}(at)$		$\dfrac{(2n)!\,a^{2n}}{p[p^2+(2a)^2][p^2+(4a)^2]\cdots[p^2+(2na)^2]}$ $\times\left\{1+\dfrac{p^2}{2!a^2}+\dfrac{p^2[p^2+(2a)^2]}{4!\,a^4}+\cdots+\dfrac{p^2(p^2+4a^2)\cdots[p^2+4(na-a)^2]}{(2n)!\,a^{2n}}\right\}$ $\mathrm{Re}\,p>2n\lvert\mathrm{Im}\,a\rvert$
(48)	0 $\cos^{2n} t$	$0<t<\pi/2$ $t>\pi/2$	$\dfrac{(2n)!\,e^{-\frac{1}{2}p\pi}}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$ $\mathrm{Re}\,p>0$
(49)	$\cos^{2n} t$ 0	$0<t<\pi/2$ $t>\pi/2$	$\dfrac{(2n)!}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$ $\times\left\{-e^{-\frac{1}{2}p\pi}+1+\dfrac{p^2}{2!}+\cdots+\dfrac{p^2(2^2+p^2)\cdots[4(n-1)^2+p^2]}{(2n)!}\right\}$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(50)	$0 \qquad 0<t<\pi/2$ $\cos^{2n} t \qquad \pi/2<t<(m+\tfrac{1}{2})\pi$ $0 \qquad t>(m+\tfrac{1}{2})\pi$ $m=1,2,3,\ldots$	$\dfrac{(2n)!\,e^{-\frac{1}{2}p\pi}(1-e^{-mp\pi})}{p(2^2+p^2)(4^2+p^2)\cdots(4n^2+p^2)}$
(51)	$\cos^{2n+1}(at)$	$\dfrac{(2n+1)!\,a^{2n}p}{(p^2+a^2)[p^2+(3a)^2]\cdots[p^2+(2na+a)^2]}\left\{1+\dfrac{p^2+a^2}{3!\,a^2}+\dfrac{(p^2+a^2)(p^2+9a^2)}{5!\,a^4}+\cdots+\dfrac{[p^2+a^2][p^2+(3a)^2]\cdots[p^2+(2na-a)^2]}{(2n+1)!\,a^{2n}}\right\}$ $\operatorname{Re} p>(2n+1)\,\lvert\operatorname{Im} a\rvert$
(52)	$0 \qquad 0<t<\pi/2$ $\cos^{2n+1} t \qquad t>\pi/2$	$\dfrac{-(2n+1)!\,e^{-\frac{1}{2}p\pi}}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}$ $\operatorname{Re} p>0$
(53)	$\cos^{2n+1} t \qquad 0<t<\pi/2$ $0 \qquad t>\pi/2$	$\dfrac{(2n+1)!\,p}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}\times\left\{\dfrac{e^{-\frac{1}{2}p\pi}}{p}+1+\dfrac{1^2+p^2}{3!}+\cdots+\dfrac{(1^2+p^2)(3^2+p^2)\cdots[(2n-1)^2+p^2]}{(2n+1)!}\right\}$
(54)	$0 \qquad 0<t<\pi/2$ $\cos^{2n+1} t \qquad \pi/2<t<(m+\tfrac{1}{2})\pi$ $0 \qquad t>(m+\tfrac{1}{2})\pi$ $m=1,2,3,\ldots$	$\dfrac{(2n+1)!\,e^{-\frac{1}{2}p\pi}(e^{-m\pi(p+i)}-1)}{(1^2+p^2)(3^2+p^2)\cdots[(2n+1)^2+p^2]}$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(55)	$0 \quad 0<t<\pi/2$ $t\cos t \quad t>\pi/2$	$-(1+p^2)^{-2} e^{-\frac{1}{2}p\pi}[\tfrac{1}{2}\pi(1+p^2)+2p]$ $\operatorname{Re} p>0$
(56)	$t\cos t \quad 0<t<\pi/2$ $0 \quad t>\pi/2$	$\dfrac{p^2-1+e^{-\frac{1}{2}p\pi}[\frac{1}{2}\pi(1+p^2)+2p]}{(1+p^2)^2}$
(57)	$t^n\cos(\alpha t)$	$\dfrac{n!p^{n+1}}{(p^2+\alpha^2)^{n+1}}$ $\times \sum_{0\le 2m\le n+1} (-)^m \binom{n+1}{2m}\left(\dfrac{\alpha}{p}\right)^{2m}$ $\operatorname{Re} p>\lvert\operatorname{Im}\alpha\rvert$
(58)	$t^{\nu-1}\cos(\alpha t) \quad \operatorname{Re}\nu>0$	$\tfrac{1}{2}\Gamma(\nu)[(p-i\alpha)^{-\nu}+(p+i\alpha)^{-\nu}]$ $=\Gamma(\nu)(p^2+\alpha^2)^{-\frac{1}{2}\nu}$ $\times\cos[\nu\tan^{-1}(\alpha/p)]$ $\operatorname{Re} p>\lvert\operatorname{Im}\alpha\rvert$
(59)	$t^{-1}(1-\cos\alpha t)$	$\tfrac{1}{2}\log(1+\alpha^2/p^2) \quad \operatorname{Re} p>\lvert\operatorname{Im}\alpha\rvert$
(60)	$(1-e^{-t})^{\nu-1}\cos(\alpha t) \quad \operatorname{Re}\nu>0$	$\tfrac{1}{2}B(\nu,p-i\alpha)+\tfrac{1}{2}B(\nu,p+i\alpha)$ $\operatorname{Re} p>\lvert\operatorname{Im}\alpha\rvert$
(61)	$t^{\nu-1}e^{-\frac{1}{2}t^2/\alpha}\cos(\beta t)$ $\operatorname{Re}\alpha>0, \quad \operatorname{Re}\nu>0$	$\tfrac{1}{2}\Gamma(\nu)\,\alpha^{\frac{1}{2}\nu}e^{\frac{1}{4}\alpha(p^2-\beta^2)}$ $\times\{e^{-\frac{1}{2}i\alpha\beta p}D_{-\nu}[\alpha^{\frac{1}{2}}(p-i\beta)]$ $+e^{\frac{1}{2}i\alpha\beta p}D_{-\nu}[\alpha^{\frac{1}{2}}(p+i\beta)]\}$
(62)	$\log t\cos(\alpha t)$	$-\dfrac{\alpha\tan^{-1}(\alpha/p)+p\log[\gamma(p^2+\alpha^2)^{\frac{1}{2}}]}{p^2+\alpha^2}$ $\operatorname{Re} p>\lvert\operatorname{Im}\alpha\rvert$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(63)	$t^{\nu-1}\log t\cos(at)$ $\quad \operatorname{Re}\nu>0$	$\dfrac{\Gamma(\nu)}{(p^2+a^2)^{\frac12\nu}}\cos[\nu\tan^{-1}(a/p)]$ $\times\{\psi(\nu)-\log(p^2+a^2)^{\frac12}$ $-\tan^{-1}(a/p)\tan[\nu\tan^{-1}(a/p)]\}$ $\quad \operatorname{Re}p>\lvert\operatorname{Im}a\rvert$
(64)	$\cos(t^2)$	$(\tfrac12\pi)^{\frac12}[\tfrac12-\cos(\tfrac14p^2)S(\tfrac14p^2)$ $+\sin(\tfrac14p^2)\,C(\tfrac14p^2)]$ $\quad \operatorname{Re}p>0$
(65)	$\cos(2a^{\frac12}t^{\frac12})$	$p^{-1}+i\pi^{1/2}a^{1/2}p^{-3/2}e^{-a/p}$ $\times\operatorname{Erf}(ia^{1/2}p^{-1/2})$ $\quad \operatorname{Re}p>0$
(66)	$t^{\frac12}\cos(2a^{\frac12}t^{\frac12})$	$\pi^{1/2}p^{-5/2}(\tfrac12p-a)e^{-a/p}$ $\quad \operatorname{Re}p>0$
(67)	$t^{-\frac12}\cos(2a^{\frac12}t^{\frac12})$	$\pi^{\frac12}p^{-\frac12}e^{-a/p}$ $\quad \operatorname{Re}p>0$
(68)	$t^{n-\frac12}\cos(2a^{\frac12}t^{\frac12})$	$(-2)^{-n}\pi^{\frac12}p^{-n-\frac12}e^{-a/p}$ $\times\operatorname{He}_{2n}(2^{\frac12}a^{\frac12}p^{-\frac12})$ $\quad \operatorname{Re}p>0$
(69)	$t^{\nu-1}\cos(2^{\frac12}a^{\frac12}t^{\frac12})$ $\quad \operatorname{Re}\nu>0$	$2^{-\frac12-\nu}\pi^{\frac12}\csc(\nu\pi)p^{-\nu}e^{-\frac14a/p}$ $\times[D_{2\nu-1}(a^{\frac12}p^{-\frac12})+D_{2\nu-1}(-a^{\frac12}/p^{\frac12})]$ $\quad \operatorname{Re}p>0$
(70)	$(2t-t^2)^{-\frac12}\cos[a(2t-t^2)^{\frac12}]$ $\quad 0<t<2$; 0 $\quad t>2$	$\pi e^{-p}J_0[(a^2-p^2)^{\frac12}]$
(71)	$t^{-\frac12}\sinh a[\cosh a-\cos(t^{\frac12})]^{-1}$ $\quad \operatorname{Re}a>0$	$2\pi e^{a^2p}[\theta_3(2ap,4p)+\hat\theta_3(2ap,4p)]$ $-\pi^{\frac12}p^{-\frac12}$ $\quad \operatorname{Re}p>0$
(72)	$\cos(ae^{-t})$	$a^{-p}\Gamma(p)[U_p(2a,0)\cos a$ $+U_{p+1}(2a,0)\sin a]$ $\quad \operatorname{Re}p>0$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(73)	$\cos[\alpha(1-e^{-t})]$	$\alpha^{-p}\,\Gamma(p)\,U_p(2\alpha,0)$ $\quad$ Re $p>0$
(74)	$(e^{t}-1)^{-1/2}\cos[\alpha(1-e^{-t})^{1/2}]$	$\pi^{1/2}\Gamma(p+\tfrac12)(2/\alpha)^p\,J_p(\alpha)$ $\quad$ Re $p>-\tfrac12$
(75)	$(1-e^{-t})^{-1/2}\cos[a(e^{t}-1)^{1/2}]$ $\quad$ $a>0$	$2\pi^{1/2}[\Gamma(p+\tfrac12)]^{-1}(\tfrac12 a)^p K_p(a)$ $\quad$ Re $p>-\tfrac12$
(76)	$t^{-1}[\cos(\alpha t)-\cos(\beta t)]$	$\tfrac12\log[(p^2+\beta^2)(p^2+\alpha^2)^{-1}]$ $\quad$ Re $p>\lvert\mathrm{Im}\,\alpha\rvert,\ \lvert\mathrm{Im}\,\beta\rvert$
(77)	$t^{-2}(\cos\alpha t-\cos\beta t)$	$\tfrac12 p\log[(p^2+\alpha^2)(p^2+\beta^2)^{-1}]$ $+\beta\tan^{-1}(\beta/p)-\alpha\tan^{-1}(\alpha/p)$ $\quad$ Re $p\geq\lvert\mathrm{Im}\,\alpha\rvert,\ \lvert\mathrm{Im}\,\beta\rvert$
(78)	$\sin(\alpha t)\sin(\beta t)$	$\dfrac{2\alpha\beta p}{[p^2+(\alpha+\beta)^2][p^2+(\alpha-\beta)^2]}$ $\quad$ Re $p\geq\lvert\mathrm{Im}(\pm\alpha\pm\beta)\rvert$
(79)	$\cos(\alpha t)\sin(\beta t)$	$\dfrac{\beta(p^2-\alpha^2+\beta^2)}{[p^2+(\alpha+\beta)^2][p^2+(\alpha-\beta)^2]}$ $\quad$ Re $p>\lvert\mathrm{Im}(\pm\alpha\pm\beta)\rvert$
(80)	$\cos(\alpha t)\cos(\beta t)$	$\dfrac{p(p^2+\alpha^2+\beta^2)}{[p^2+(\alpha+\beta)^2][p^2+(\alpha-\beta)^2]}$ $\quad$ Re $p>\lvert\mathrm{Im}(\pm\alpha\pm\beta)\rvert$
(81)	$t^{-1}\sin(\alpha t)\sin(\beta t)$	$\tfrac14\log\dfrac{p^2+(\alpha+\beta)^2}{p^2+(\alpha-\beta)^2}$ $\quad$ Re $p>\lvert\mathrm{Im}(\pm\alpha\pm\beta)\rvert$

Trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(82)	$t^{-1}\sin(at)\cos(\beta t)$	$\tfrac{1}{2}\tan^{-1}\dfrac{2ap}{p^2-a^2+\beta^2}$ $\quad \operatorname{Re} p > \lvert\operatorname{Im}(\pm a \pm\beta)\rvert$
(83)	$t^{-2}\sin(at)\sin(\beta t)$	$\tfrac{1}{2}a\tan^{-1}\dfrac{2\beta p}{p^2+a^2-\beta^2}+\tfrac{1}{2}\beta\tan^{-1}\dfrac{2ap}{p^2-a^2+\beta^2}+\tfrac{1}{4}p\log\dfrac{p^2+(a-\beta)^2}{p^2+(a+\beta)^2}$ $\quad \operatorname{Re} p \geq \lvert\operatorname{Im}(\pm a \pm\beta)\rvert$
(84)	$\csc t\,\sin[(2n+1)t]$	$\dfrac{1}{p}+\sum_{m=1}^{n}\dfrac{2p}{p^2+4m^2}$ $\quad \operatorname{Re} p > 0$
(85)	$\tan t\,\cos[(2n+1)t]$	$\dfrac{2n+1}{p^2+(2n+1)^2}+2\sum_{m=0}^{n-1}\dfrac{(-1)^m(2m+1)}{p^2+(2m+1)^2}$ $\quad \operatorname{Re} p > 0$
(86)	$\dfrac{(2at\cos at-\sin at)\sin at}{t^2}$	$\tfrac{1}{4}p\log\left(1+\dfrac{4a^2}{p^2}\right)$ $\quad \operatorname{Re} p > 2\lvert\operatorname{Im} a\rvert$
(87)	$\dfrac{at\cos at-\sin at}{t^2}$	$p\tan^{-1}\dfrac{a}{p}-a$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$

4.8. Inverse trigonometric functions

(1)	$\sin^{-1} t \quad 0<t<1$; $0 \quad t>1$	$\tfrac{1}{2}\pi p^{-1}[I_0(p)-\mathbf{L}_0(p)]$

Inverse trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(2)	$t \sin^{-1} t \qquad 0<t<1$ $0 \qquad t>1$	$\tfrac{1}{2}\pi p^{-2}[\mathbf{L}_0(p)-I_0(p)$ $+p\,\mathbf{L}_1(p)-p\,I_1(p)]+p^{-1}$
(3)	$\tan^{-1}(t/a)$	$p^{-1}[-\mathrm{ci}(ap)\sin(ap)$ $-\mathrm{si}(ap)\cos(ap)] \qquad \mathrm{Re}\,p>0$
(4)	$\mathrm{ctn}^{-1}(t/a)$	$p^{-1}[\tfrac{1}{2}\pi+\mathrm{ci}(ap)\sin(ap)$ $+\mathrm{si}(ap)\cos(ap)] \qquad \mathrm{Re}\,p>0$
(5)	$t\tan^{-1}(t/a)$	$p^{-2}[-\mathrm{ci}(ap)\sin(ap)$ $-\mathrm{si}(ap)\cos(ap)]$ $+ap^{-1}[\mathrm{ci}(ap)\cos(ap)$ $-\mathrm{si}(ap)\sin(ap)] \qquad \mathrm{Re}\,p>0$
(6)	$t\,\mathrm{ctn}^{-1}(t/a)$	$p^{-2}[\tfrac{1}{2}\pi+\mathrm{ci}(ap)\sin(ap)$ $+\mathrm{si}(ap)\cos(ap)]$ $+ap^{-1}[\mathrm{si}(ap)\sin(ap)$ $-\mathrm{ci}(ap)\cos(ap)] \qquad \mathrm{Re}\,p>0$
(7)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times e^{-i(\nu-\frac{1}{2})\,\mathrm{ctn}^{-1}t}$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}i\pi^{\frac{1}{2}}\Gamma(\nu+\tfrac{1}{2})p^{-\nu}e^{-\frac{1}{2}ip}H_\nu^{(1)}(\tfrac{1}{2}p)$ $\mathrm{Re}\,p>0$
(8)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times\sin[(\nu-\tfrac{1}{2})\,\mathrm{ctn}^{-1}t]$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$-\tfrac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\tfrac{1}{2})p^{-\nu}[J_\nu(\tfrac{1}{2}p)\cos(\tfrac{1}{2}p)$ $+Y_\nu(\tfrac{1}{2}p)\sin(\tfrac{1}{2}p)] \qquad \mathrm{Re}\,p>0$
(9)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times\cos[(\nu-\tfrac{1}{2})\,\mathrm{ctn}^{-1}t]$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\tfrac{1}{2})p^{-\nu}[J_\nu(\tfrac{1}{2}p)\sin(\tfrac{1}{2}p)$ $-Y_\nu(\tfrac{1}{2}p)\cos(\tfrac{1}{2}p)] \qquad \mathrm{Re}\,p>0$
(10)	$t^{\nu-\frac{1}{2}}(1+t^2)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times\sin[\beta-(\nu-\tfrac{1}{2})\,\mathrm{ctn}^{-1}t]$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}\pi^{\frac{1}{2}}\Gamma(\nu+\tfrac{1}{2})p^{-\nu}[J_\nu(\tfrac{1}{2}p)\cos(\tfrac{1}{2}p-\beta)$ $+Y_\nu(\tfrac{1}{2}p)\sin(\tfrac{1}{2}p-\beta)] \qquad \mathrm{Re}\,p>0$

Inverse trigonometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(11)	$\dfrac{\cos[(2n+\frac{1}{2})\cos^{-1}(\frac{1}{2}t/b)]}{(4b^2t-t^3)^{\frac{1}{2}}}$ $\quad 0<t<2b$ $0 \quad t>2b$	$(-1)^n(\frac{1}{2}\pi p)^{\frac{1}{2}} I_n(bp) K_{n+\frac{1}{2}}(bp)$
(12)	$\dfrac{\cos[\nu\cos^{-1}(\frac{1}{2}t/b)]}{(4b^2t-t^3)^{\frac{1}{2}}}$ $\quad 0<t<2b$ $0 \quad t>2b$	$(\frac{1}{2}\pi)^{3/2} p^{1/2}[I_{\frac{1}{2}\nu-\frac{1}{4}}(bp) I_{-\frac{1}{2}\nu-\frac{1}{4}}(bp)$ $- I_{\frac{1}{2}\nu+\frac{1}{4}}(bp) I_{-\frac{1}{2}\nu+\frac{1}{4}}(bp)]$
(13)	$0 \quad 0<t<a$ $\dfrac{\cos\{n\cos^{-1}[(2t-a-b)/(b-a)]\}}{(t-a)^{\frac{1}{2}}(b-t)^{\frac{1}{2}}}$ $\quad a<t<b$ $0 \quad t>b$	$\pi e^{-\frac{1}{2}(a+b)p} I_n[\frac{1}{2}(b-a)p]$
(14)	$[t(t+1)(t+2)]^{-\frac{1}{2}}$ $\times\cos[\nu\cos^{-1}(1+t)^{-1}]$	$\pi^{\frac{1}{2}} e^p D_{\nu-\frac{1}{2}}(2^{\frac{1}{2}}p^{\frac{1}{2}}) D_{-\nu-\frac{1}{2}}(2^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\, p>0$
(15)	$\dfrac{\cos[\nu\cos^{-1} e^{-t})}{(1-e^{-2t})^{\frac{1}{2}}}$	$\dfrac{\pi 2^{-p}}{p\,\mathrm{B}(\frac{1}{2}p+\frac{1}{2}\nu+\frac{1}{2}, \frac{1}{2}p-\frac{1}{2}\nu+\frac{1}{2})}$ $\mathrm{Re}\, p>0$

4.9. Hyperbolic functions

(1)	$\sinh(at)$	$a(p^2-a^2)^{-1}$ $\quad \mathrm{Re}\, p>\lvert\mathrm{Re}\, a\rvert$
(2)	$\cosh(at)$	$p(p^2-a^2)^{-1}$ $\quad \mathrm{Re}\, p>\lvert\mathrm{Re}\, a\rvert$
(3)	$\sinh^2(at)$	$2a^2(p^3-4a^2p)^{-1}$ $\mathrm{Re}\, p>2\lvert\mathrm{Re}\, a\rvert$

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(4)	$\cosh^2(at)$	$(p^2-2a^2)(p^3-4a^2p)^{-1}$ $\quad \operatorname{Re} p > 2\lvert\operatorname{Re} a\rvert$
(5)	$[\sinh(at)]^\nu$ $\quad \operatorname{Re} a>0,\ \operatorname{Re}\nu>-1$	$2^{-\nu-1}a^{-1}\,\mathrm{B}(\tfrac{1}{2}p/a-\tfrac{1}{2}\nu,\ \nu+1)$ $\quad \operatorname{Re} p > \operatorname{Re}\nu a$
(6)	$[\cosh(at)-1]^\nu$ $\quad \operatorname{Re} a>0,\ \operatorname{Re}\nu>-\tfrac{1}{2}$	$2^{-\nu}a^{-1}\,\mathrm{B}(p/a-\nu,\ 2\nu+1)$ $\quad \operatorname{Re} p > \operatorname{Re}\nu a$
(7)	$\operatorname{sech} t$	$\tfrac{1}{2}\psi(\tfrac{1}{4}p+\tfrac{3}{4})-\tfrac{1}{2}\psi(\tfrac{1}{4}p+\tfrac{1}{4})$ $\quad \operatorname{Re} p>-1$
(8)	$\operatorname{sech}^2 t$	$\tfrac{1}{2}p[\psi(\tfrac{1}{4}p+\tfrac{1}{2})-\psi(\tfrac{1}{4}p)]-1$ $\quad \operatorname{Re} p>-2$
(9)	$\tanh t$	$\tfrac{1}{2}\psi(\tfrac{1}{4}p+\tfrac{1}{2})-\tfrac{1}{2}\psi(\tfrac{1}{4}p)-p^{-1}$ $\quad \operatorname{Re} p>0$
(10)	$t^{-1}-\operatorname{csch} t$	$\psi(\tfrac{1}{2}p+\tfrac{1}{2})-\log(\tfrac{1}{2}p)$ $\quad \operatorname{Re} p>0$
(11)	$t^{-1}-\operatorname{ctnh} t$	$\psi(\tfrac{1}{2}p)+p^{-1}-\log(\tfrac{1}{2}p)$ $\quad \operatorname{Re} p>0$
(12)	$\dfrac{2}{t}\sinh(at)$	$\log\dfrac{p+a}{p-a}$ $\quad \operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(13)	$0 \quad 0<t<1$ $\dfrac{2}{t}\sinh(at) \quad t>1$	$-\mathrm{Ei}(a-p)+\mathrm{Ei}(-a-p)$ $\quad \operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(14)	$\dfrac{2}{t}\sinh(at) \quad 0<t<1$ $0 \quad t>1$	$\log\dfrac{p+a}{p-a}+\mathrm{Ei}(a-p)-\mathrm{Ei}(-a-p)$

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(15)	$0 \quad 0<t<1$ $\frac{2}{t}\cosh(at) \quad t>1$	$-\mathrm{Ei}(a-p)-\mathrm{Ei}(-a-p)$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(16)	$\frac{1}{t}\tanh t$	$\log(\tfrac14 p)+2\log\frac{\Gamma(\frac14 p)}{\Gamma(\frac14 p+\frac12)}$ $\mathrm{Re}\,p>0$
(17)	$\frac{1}{t}(1-\operatorname{sech} t)$	$2\log\frac{\Gamma(\frac14 p+\frac34)}{\Gamma(\frac14 p+\frac14)}-\log(\tfrac14 p)$ $\mathrm{Re}\,p>0$
(18)	$t^{\nu-1}\sinh(at) \quad \mathrm{Re}\,\nu>-1$	$\tfrac12\Gamma(\nu)[(p-a)^{-\nu}-(p+a)^{-\nu}]$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(19)	$t^{\nu-1}\cosh(at) \quad \mathrm{Re}\,\nu>0$	$\tfrac12\Gamma(\nu)[(p-a)^{-\nu}+(p+a)^{-\nu}]$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(20)	$t^{\nu-1}\operatorname{csch} t \quad \mathrm{Re}\,\nu>1$	$2^{1-\nu}\Gamma(\nu)\,\zeta(\nu,\tfrac12 p+\tfrac12)$ $\mathrm{Re}\,p>-1$
(21)	$t^{\nu-1}\operatorname{ctnh} t \quad \mathrm{Re}\,\nu>1$	$\Gamma(\nu)[2^{1-\nu}\zeta(\nu,\tfrac12 p)-p^{-\nu}]$ $\mathrm{Re}\,p>0$
(22)	$t^{\nu-1}(\operatorname{ctnh} t-1) \quad \mathrm{Re}\,\nu>1$	$2^{1-\nu}\Gamma(\nu)\,\zeta(\nu,\tfrac12 p+1) \quad \mathrm{Re}\,p>-2$
(23)	$0 \quad 0<t<b$ $(\cosh t-\cosh b)^{\nu-1} \quad t>b$ $\mathrm{Re}\,\nu>0$	$-i\,2^{\frac12}\pi^{-\frac12}e^{\nu\pi i}\Gamma(\nu)(\sinh b)^{\nu-\frac12}$ $\times Q^{\frac12-\nu}_{p-\frac12}(\cosh b)$ $\mathrm{Re}\,p>\mathrm{Re}\,\nu-1$
(24)	$\sin(at)\sinh(at)$	$2a^2p(p^4+4a^4)^{-1}$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert+\lvert\mathrm{Im}\,a\rvert$

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(25)	$\cos(at)\sinh(at)$	$(ap^2-2a^3)(p^4+4a^4)^{-1}$ $\operatorname{Re} p > \lvert\operatorname{Re} a\rvert + \lvert\operatorname{Im} a\rvert$
(26)	$\sin(at)\cosh(at)$	$(ap^2+2a^3)(p^4+4a^4)^{-1}$ $\operatorname{Re} p > \operatorname{Re}(\pm a \pm i a)$
(27)	$\cos(at)\cosh(at)$	$p^3(p^4+4a^4)^{-1}$ $\operatorname{Re} p > \operatorname{Re}(\pm a \pm i a)$
(28)	$e^{-a\sinh t}$ $\quad \operatorname{Re} a>0$	$\pi\csc(\pi p)\,[\mathbf{J}_p(a)-J_p(a)]$
(29)	$e^{-a\sinh(t+i\psi)}$ $\quad -\pi/2<\psi<\pi/2$ $\lvert\arg a\rvert<\pi/2-\psi$	$\csc(\pi p)\,[\int_0^\pi e^{ia\sin\psi\cos\theta}$ $\times\cos(p\theta-a\cos\psi\sin\theta)\,d\theta$ $-\pi e^{ip\psi}J_p(a)]$
(30)	$e^{-a\cosh t}$ $\quad \operatorname{Re} a>0$	$\csc(\pi p)\,[\int_0^\pi e^{a\cos\theta}\cos(p\theta)\,d\theta$ $-\pi I_p(a)]$
(31)	$(\sinh \tfrac{1}{2}t)^{2\beta}e^{-2a\operatorname{ctnh}\frac{1}{2}t}$ $\operatorname{Re} a>0$	$\tfrac{1}{2}a^{\frac{1}{2}\beta-\frac{1}{2}}\Gamma(p-\beta)\,[W_{-p+\frac{1}{2},\beta}(4a)$ $-(p-\beta)W_{-p-\frac{1}{2},\beta}(4a)]$ $\operatorname{Re} p>\operatorname{Re}\beta$
(32)	$\log\cosh t$	$\tfrac{1}{2}p^{-1}[\psi(\tfrac{1}{4}p+\tfrac{1}{2})-\psi(\tfrac{1}{4}p)]-p^{-2}$ $\operatorname{Re} p>0$
(33)	$\log(\sinh t)-\log t$	$p^{-1}[\log(\tfrac{1}{2}p)-\tfrac{1}{2}p^{-1}-\psi(\tfrac{1}{2}p)]$ $\operatorname{Re} p>0$
(34)	$\sinh(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$\pi^{1/2}a^{1/2}p^{-3/2}e^{a/p}$ $\quad \operatorname{Re} p>0$

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(35)	$\cosh(2a^{1/2}t^{1/2})$	$\pi^{1/2}a^{1/2}p^{-3/2}e^{a/p}\operatorname{Erf}(a^{1/2}p^{-1/2})+p^{-1}$ $\quad\operatorname{Re} p>0$
(36)	$t^{1/2}\sinh(2a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-5/2}(\tfrac{1}{2}p+a)e^{a/p}$ $\times\operatorname{Erf}(a^{1/2}p^{-1/2})-a^{1/2}p^{-2}$ $\quad\operatorname{Re} p>0$
(37)	$t^{1/2}\cosh(2a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-5/2}(\tfrac{1}{2}p+a)e^{a/p}$ $\quad\operatorname{Re} p>0$
(38)	$t^{-1/2}\sinh(2a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-1/2}e^{a/p}\operatorname{Erf}(a^{1/2}p^{-1/2})$ $\quad\operatorname{Re} p>0$
(39)	$t^{-1/2}\cosh(2a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-1/2}e^{a/p}$ $\quad\operatorname{Re} p>0$
(40)	$t^{-1/2}\sinh^2(a^{1/2}t^{1/2})$	$\tfrac{1}{2}\pi^{1/2}p^{-1/2}(e^{a/p}-1)$ $\quad\operatorname{Re} p>0$
(41)	$t^{-1/2}\cosh^2(a^{1/2}t^{1/2})$	$\tfrac{1}{2}\pi^{1/2}p^{-1/2}(e^{a/p}+1)$ $\quad\operatorname{Re} p>0$
(42)	$t^{-3/4}\sinh(2^{3/2}a^{1/2}t^{1/2})$	$\pi(2a)^{1/4}p^{-1/2}e^{a/p}I_{1/4}(a/p)$ $\quad\operatorname{Re} p>0$
(43)	$t^{-3/4}\cosh(2^{3/2}a^{1/2}t^{1/2})$	$\pi(2a)^{1/4}p^{-1/2}e^{a/p}I_{-1/4}(a/p)$ $\quad\operatorname{Re} p>0$
(44)	$t^{\nu-1}\sinh(2^{1/2}a^{1/2}t^{1/2})$ $\quad\operatorname{Re}\nu>-\tfrac{1}{2}$	$\Gamma(2\nu)(2p)^{-\nu}e^{\frac{1}{4}a/p}[D_{-2\nu}(-a^{1/2}p^{-1/2})$ $-D_{-2\nu}(a^{1/2}p^{-1/2})]$ $\quad\operatorname{Re} p>0$
(45)	$t^{\nu-1}\cosh(2^{1/2}a^{1/2}t^{1/2})$ $\quad\operatorname{Re}\nu>0$	$\Gamma(2\nu)(2p)^{-\nu}e^{\frac{1}{4}a/p}[D_{-2\nu}(-a^{1/2}p^{-1/2})$ $+D_{-2\nu}(a^{1/2}p^{-1/2})]$ $\quad\operatorname{Re} p>0$
(46)	$\begin{matrix}\sin\\ \cos\end{matrix}(bt)\begin{matrix}\tanh\\ \operatorname{ctnh}\end{matrix}(at^{1/2})$	see Mordell, L. J., 1920: *Mess. of Math.* 49, 65-72

Hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(47)	$\dfrac{\sinh[(2\nu+1)\pi^{1/2}t^{1/2}]}{\pi^{1/2}t^{1/2}\sinh(\pi^{1/2}t^{1/2})}$	For this and more generally for the Laplace transform of $\dfrac{a\cosh(At^{1/2})+b\cosh(Bt^{1/2})}{t^{1/2}\{a^2+2ab\cosh[(A+B)t^{1/2}]+b^2\}}$ see Mordell, L. J., 1933: *Acta Math.* 61, 323-360 and *Quart. J. Math.* 1920, 48, 329-342.
(48)	$(e^t-1)^{-1/2}\sinh[a(1-e^{-t})^{1/2}]$	$\pi^{1/2}\Gamma(p+\tfrac{1}{2})\, 2^p a^{-p}\mathbf{L}_p(a)$ $\quad \operatorname{Re} p>-\tfrac{1}{2}$
(49)	$(e^t-1)^{-1/2}\cosh[a(1-e^{-t})^{1/2}]$	$\pi^{1/2}\Gamma(p+\tfrac{1}{2})\, 2^p a^{-p} I_p(a)$ $\quad \operatorname{Re} p>-\tfrac{1}{2}$
(50)	$\tanh[\tfrac{1}{2}\pi(e^{2t}-1)^{1/2}]$	$2^{-p}\zeta(p-1)$ $\quad \operatorname{Re} p>0$

4.10. Inverse hyperbolic functions

(1)	$\sinh^{-1} t$	$\tfrac{1}{2}\pi p^{-1}[\mathbf{H}_0(p)-Y_0(p)]$ $\quad \operatorname{Re} p>0$
(2)	$0 \quad 0<t<b$ $\cosh^{-1}(t/b) \quad t>b$	$p^{-1}K_0(bp)$ $\quad \operatorname{Re} p>0$
(3)	$\cosh^{-1}(1+t/a) \quad \lvert\arg a\rvert<\pi$	$p^{-1}e^{ap}K_0(ap)$ $\quad \operatorname{Re} p>0$
(4)	$t\sinh^{-1} t$	$\pi\dfrac{\mathbf{H}_0(p)-Y_0(p)}{2p^2}+\pi\dfrac{\mathbf{H}_1(p)-Y_1(p)}{2p}-\dfrac{1}{p}$ $\quad \operatorname{Re} p>0$
(5)	$\sinh[(2n+1)\sinh^{-1} t]$	$O_{2n+1}(p)$ $\quad \operatorname{Re} p>0$

Inverse hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(6)	$\cosh(2n \sinh^{-1} t)$	$O_{2n}(p)$ $\quad$ $\operatorname{Re} p > 0$
(7)	$\sinh(\nu \sinh^{-1} t)$	$\nu p^{-1} S_{0,\nu}(p)$ $\quad$ $\operatorname{Re} p > 0$
(8)	$\cosh(\nu \sinh^{-1} t)$	$p^{-1} S_{1,\nu}(p)$ $\quad$ $\operatorname{Re} p > 0$
(9)	0 $\quad$ $0 < t < b$ $\sinh[\nu \cosh^{-1}(t/b)]$ $\quad$ $t > b$	$\nu p^{-1} K_\nu(bp)$ $\quad$ $\operatorname{Re} p > 0$
(10)	$\sinh[\nu \cosh^{-1}(1+t/a)]$ $\lvert\arg a\rvert < \pi$	$\nu p^{-1} e^{ap} K_\nu(ap)$ $\quad$ $\operatorname{Re} p > 0$
(11)	$(1+t^2)^{-1/2} \exp(n \sinh^{-1} t)$	$\frac{1}{2}[S_n(p) - \pi \mathbf{E}_n(p) - \pi Y_n(p)]$ $\operatorname{Re} p > 0$
(12)	$(1+t^2)^{-1/2} \exp(-n \sinh^{-1} t)$	$\frac{1}{2}(-1)^{n+1}[S_n(p) + \pi \mathbf{E}_n(p) + \pi Y_n(p)]$ $\operatorname{Re} p > 0$
(13)	$(1+t^2)^{-1/2} \exp(-\nu \sinh^{-1} t)$	$\pi \csc(\nu\pi)[\mathbf{J}_\nu(p) - J_\nu(p)]$ $\operatorname{Re} p > 0$
(14)	$\dfrac{\sinh(\nu \sinh^{-1} t)}{(t^2+1)^{1/2}}$	$\nu S_{-1,\nu}(p)$ $\quad$ $\operatorname{Re} p > 0$
(15)	0 $\quad$ $0 < t < b$ $\dfrac{\cosh(n \sinh^{-1} t)}{(t^2+1)^{1/2}}$ $\quad$ $t > b$	$S_n(\sinh^{-1} b, p)$ $\quad$ $\operatorname{Re} p > 0$
(16)	$\dfrac{\cosh(\nu \sinh^{-1} t)}{(t^2+1)^{1/2}}$	$S_{0,\nu}(p)$ $\quad$ $\operatorname{Re} p > 0$

Inverse hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(17)	0 $\quad 0<t<b$ $\dfrac{\cosh(n\cosh^{-1} t)}{(t^2-1)^{1/2}}$ $\quad t>b>1$	$C_n(\cosh^{-1} b,\ p)$ $\quad \operatorname{Re} p>0$
(18)	0 $\quad 0<t<b$ $\dfrac{\cosh[\nu\cosh^{-1}(t/b)]}{(t^2-b^2)^{1/2}}$ $\quad t>b$	$K_\nu(bp)$ $\quad \operatorname{Re} p>0$
(19)	$\dfrac{\cosh[\nu\cosh^{-1}(1+t/a)]}{(t^2+at)^{1/2}}$ $\lvert\arg a\rvert<\pi$	$e^{ap} K_\nu(ap)$ $\quad \operatorname{Re} p>0$
(20)	$(t^3+4a^2t)^{-1/2}$ $\times\exp(2\nu\sinh^{-1}(\tfrac{1}{2}t/a)]$ $\operatorname{Re} a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2}[J_{\nu+1/4}(ap)J_{\nu-1/4}(ap)$ $+Y_{\nu+1/4}(ap)\,Y_{\nu-1/4}(ap)]$ $\operatorname{Re} p>0$
(21)	$(t^3+4a^2t)^{-1/2}$ $\times\exp[-2\nu\sinh^{-1}(\tfrac{1}{2}t/a)]$ $\operatorname{Re} a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2}[J_{\nu+1/4}(ap)\,Y_{\nu-1/4}(ap)$ $-J_{\nu-1/4}(ap)\,Y_{\nu+1/4}(ap)]$ $\operatorname{Re} p>0$
(22)	$(t^3+4a^2t)^{-1/2}\{\cos[(\nu+\tfrac{1}{4})\pi]$ $\times\exp[-2\nu\sinh^{-1}(\tfrac{1}{2}t/a)]$ $+\sin[(\nu+\tfrac{1}{4})\pi]$ $\times\exp[2\nu\sinh^{-1}(\tfrac{1}{2}t/a)]\}$ $\operatorname{Re} a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2}[J_{1/4+\nu}(ap)J_{1/4-\nu}(ap)$ $+Y_{1/4+\nu}(ap)\,Y_{1/4-\nu}(ap)]$ $\operatorname{Re} p>0$
(23)	$(t^3+4a^2t)^{-1/2}\{\sin[(\nu+\tfrac{1}{4})\pi]$ $\times\exp[-2\nu\sinh^{-1}(\tfrac{1}{2}t/a)]$ $-\cos[(\nu+\tfrac{1}{4})\pi]$ $\times\exp[2\nu\sinh^{-1}(\tfrac{1}{2}t/a)]\}$ $\operatorname{Re} a>0$	$(\tfrac{1}{2}\pi)^{3/2} p^{1/2}[J_{1/4+\nu}(ap)\,Y_{1/4-\nu}(ap)$ $-J_{1/4-\nu}(ap)\,Y_{1/4+\nu}(ap)]$ $\quad \operatorname{Re} p>0$

Inverse hyperbolic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(24)	$\dfrac{\sinh[2\nu \sinh^{-1}(\frac{1}{2}t/a)]}{(t^3+4a^2t)^{\frac{1}{2}}}$ $\quad\lvert\arg a\rvert<\pi$	$\dfrac{\pi^{3/2}p^{1/2}}{8i}[e^{\nu\pi i}H^{(1)}_{\frac{1}{2}+\nu}(ap)H^{(2)}_{\frac{1}{2}-\nu}(ap)$ $-e^{-\nu\pi i}H^{(1)}_{\frac{1}{2}-\nu}(ap)H^{(2)}_{\frac{1}{2}+\nu}(ap)]$ $\quad\mathrm{Re}\,p>0$
(25)	$\dfrac{\cosh[2\nu \sinh^{-1}(\frac{1}{2}t/a)]}{(t^3+4a^2t)^{\frac{1}{2}}}$ $\quad\lvert\arg a\rvert<\pi$	$\dfrac{\pi^{3/2}p^{1/2}}{8}[e^{\nu\pi i}H^{(1)}_{\frac{1}{2}+\nu}(ap)H^{(2)}_{\frac{1}{2}-\nu}(ap)$ $+e^{-\nu\pi i}H^{(1)}_{\frac{1}{2}-\nu}(ap)H^{(2)}_{\frac{1}{2}+\nu}(ap)]$ $\quad\mathrm{Re}\,p>0$
(26)	$0 \qquad 0<t<2b$ $\dfrac{\cosh[2\nu\cosh^{-1}(\frac{1}{2}t/b)]}{(t^3-4b^2t)^{\frac{1}{2}}} \qquad t>2b$	$\dfrac{p^{\frac{1}{2}}}{(2\pi)^{\frac{1}{2}}}K_{\nu+\frac{1}{4}}(bp)\,K_{\nu-\frac{1}{4}}(bp)$ $\quad\mathrm{Re}\,p>0$
(27)	$\dfrac{\cosh[2\nu\cosh^{-1}(1+\frac{1}{2}t/a)]}{[t(t+2a)(t+4a)]^{\frac{1}{2}}}$ $\quad\lvert\arg a\rvert<\pi$	$\dfrac{p^{\frac{1}{2}}}{(2\pi)^{\frac{1}{2}}}e^{2ap}K_{\nu+\frac{1}{4}}(ap)\,K_{\nu-\frac{1}{4}}(ap)$ $\quad\mathrm{Re}\,p>0$

4.11. Orthogonal polynomials

(1)	$P_n(t)$	Sum of powers with negative indices in the expansion, in ascending powers of p, of $(-1)^n(\frac{1}{2}\pi)^{\frac{1}{2}}p^{-\frac{1}{2}}I_{-n-\frac{1}{2}}(p)$ $\quad\mathrm{Re}\,p>0$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(2)	$P_n(1-t)$	$e^{-p}p^n\left(\frac{1}{p}\frac{d}{dp}\right)^n\left(\frac{e^p}{p}\right)$ $=p^n\left(1+\frac{1}{2}\frac{d}{dp}\right)^n\left(\frac{1}{p^{n+1}}\right)$ $\operatorname{Re} p>0$
(3)	$P_n(e^{-t})$ $n\geq 2$	$\frac{(p-1)(p-2)(p-3)\cdots(p-n+1)}{(p+n)(p+n-2)\cdots(p-n+2)}$ $\operatorname{Re} p>0$
(4)	$P_{2n}(\cos t)$	$\frac{(p^2+1^2)(p^2+3^2)\cdots[p^2+(2n-1)^2]}{p(p^2+2^2)(p^2+4^2)\cdots[p^2+(2n)^2]}$ $\operatorname{Re} p>0$
(5)	$P_{2n+1}(\cos t)$	$\frac{p(p^2+2^2)(p^2+4^2)\cdots[p^2+(2n)^2]}{(p^2+1^2)(p^2+3^2)\cdots[p^2+(2n+1)^2]}$ $\operatorname{Re} p>0$
(6)	$P_{2n}(\cosh t)$	$\frac{(p^2-1^2)(p^2-3^2)\cdots[p^2-(2n-1)^2]}{p(p^2-2^2)(p^2-4^2)\cdots[p^2-(2n)^2]}$ $\operatorname{Re} p>2n$
(7)	$P_{2n+1}(\cosh t)$	$\frac{p(p^2-2^2)(p^2-4^2)\cdots[p^2-(2n)^2]}{(p^2-1^2)(p^2-3^2)\cdots[p^2-(2n+1)^2]}$ $\operatorname{Re} p>2n+1$
(8)	$2^\nu i^n(n+\nu)\Gamma(\nu)\,C_n^\nu(-it)$	$A_{n,\nu}(p)$ $\operatorname{Re} p>0$
(9)	$[t(2a-t)]^{\nu-\frac{1}{2}}\,C_n^\nu(t/a-1)$ $0<t<2a$ 0 $t>2a$ $\operatorname{Re}\nu>-\frac{1}{2}$	$(-1)^n\frac{\pi\Gamma(2\nu+n)}{n!\,\Gamma(\nu)}\left(\frac{a}{2p}\right)^\nu e^{-ap}$ $\times I_{\nu+n}(ap)$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(10)	$P_n^{(\alpha,\beta)}(t)$	$2(1-2\mu)_n A_{\kappa,\mu,n}(2p) \quad \text{Re}\,p>0$ $\kappa=\frac{1}{2}\alpha-\frac{1}{2}\beta$ $\mu=\frac{1}{2}\alpha+\frac{1}{2}\beta+\frac{1}{2}+n$
(11)	$t^{\alpha-1}\,\text{He}_n(t)$ $\text{Re}\,\alpha>\begin{cases}0 & \text{if } n \text{ is even}\\ -1 & \text{if } n \text{ is odd}\end{cases}$	$\sum_{m=0}^{[n/2]} \frac{n!\,\Gamma(\alpha+n-2m)}{m!\,(n-2m)!}(-\frac{1}{2})^m p^{2m-\alpha-n}$ $\text{Re}\,p>0$ $[n/2]=\begin{cases}\frac{1}{2}n & \text{if } n \text{ is even}\\ \frac{1}{2}n-\frac{1}{2} & \text{if } n \text{ is odd}\end{cases}$
(12)	$\text{He}_{2n+1}(t^{1/2})$	$\pi^{1/2}\,2^{-n-1}\frac{(2n+1)!}{n!}\frac{(\frac{1}{2}-p)^n}{p^{n+3/2}}$ $\text{Re}\,p>0$
(13)	$t^{-1/2}\,\text{He}_{2n}(t^{1/2})$	$\pi^{1/2}\,2^{-n}\frac{(2n)!}{n!}\frac{(\frac{1}{2}-p)^n}{p^{n+1/2}} \quad \text{Re}\,p>0$
(14)	$t^{\alpha-\frac{1}{2}n-1}\,\text{He}_n(t^{1/2})$ $\text{Re}\,\alpha>\begin{cases}\frac{1}{2}n & \text{if } n \text{ is even}\\ \frac{1}{2}n-\frac{1}{2} & \text{if } n \text{ is odd}\end{cases}$	$\Gamma(\alpha)p^{-\alpha}\,{}_2F_1(-\frac{1}{2}n,\frac{1}{2}-\frac{1}{2}n;1-\alpha;2p)$ If α is an integer, take the first $1+[n/2]$ terms of the series. $\text{Re}\,p>0$
(15)	$e^{\beta t}\,\text{He}_{2n+1}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$	$(-2)^{-n}(\frac{1}{2}\pi)^{1/2}(\alpha-\beta)^{1/2}$ $\times\frac{(2n+1)!}{n!}\frac{(p-\alpha)^n}{(p-\beta)^{n+3/2}}$ $\text{Re}\,p>\text{Re}\,\beta$
(16)	$e^{\beta t}t^{-1/2}\,\text{He}_{2n}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$	$(-2)^{-n}\pi^{1/2}\frac{(2n)!}{n!}\frac{(p-\alpha)^n}{(p-\beta)^{n+1/2}}$ $\text{Re}\,p>\text{Re}\,\beta$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(17)	$t^{-1/2}\left[\mathrm{He}_n\left(\dfrac{x+t^{1/2}}{\lambda}\right)+\mathrm{He}_n\left(\dfrac{x-t^{1/2}}{\lambda}\right)\right]$	$(2\pi/p)^{1/2}(1-\tfrac{1}{2}\lambda^{-2}p^{-1})^{\frac{1}{2}n}\times\mathrm{He}_n\left[\dfrac{x}{(\lambda^2-\frac{1}{2}p^{-1})^{1/2}}\right]$ Re $p>0$
(18)	$t^{-\frac{1}{2}(n+1)}e^{-\frac{1}{2}a/t}\mathrm{He}_n(a^{1/2}t^{-1/2})$ Re $a>0$	$2^{\frac{1}{2}n}\pi^{1/2}p^{\frac{1}{2}n-\frac{1}{2}}e^{-(2ap)^{1/2}}$ Re $p>0$
(19)	$t^{-1/2}\mathrm{He}_{2n}(2^{1/2}a^{1/2}t^{1/2})\times\mathrm{He}_{2m}(2^{1/2}\beta^{1/2}t^{1/2})$	$\dfrac{\pi^{1/2}(2m+2n)!}{(-2)^{m+n}(m+n)!}\dfrac{(p-a)^n(p-\beta)^m}{p^{m+n+\frac{1}{2}}}\times{}_2F_1\left[-m,-n;-m-n+\tfrac{1}{2};\dfrac{p(p-a-\beta)}{(p-a)(p-\beta)}\right]$ Re $p>0$
(20)	$(a\beta t)^{-1/2}\mathrm{He}_{2n+1}(2^{1/2}a^{1/2}t^{1/2})\times\mathrm{He}_{2m+1}(2^{1/2}\beta^{1/2}t^{1/2})$	$\dfrac{-\pi^{1/2}(2m+2n+2)!}{(-2)^{m+n+1}(m+n+1)!}\dfrac{(p-a)^n(p-\beta)^m}{p^{m+n+3/2}}\times{}_2F_1\left[-m,-n;-m-n-\tfrac{1}{2};\dfrac{p(p-a-\beta)}{(p-a)(p-\beta)}\right]$ Re $p>0$
(21)	$t^{-1/2}e^{-(a+\beta)t}\mathrm{He}_n(2a^{1/2}t^{1/2})\times\mathrm{He}_n(2\beta^{1/2}t^{1/2})$	$\pi^{1/2}n!\,\dfrac{(a+\beta-p)^{\frac{1}{2}n}}{(a+\beta+p)^{\frac{1}{2}n+\frac{1}{2}}}\times P_n\left\{\dfrac{2a^{1/2}\beta^{1/2}}{[(a+\beta)^2-p^2]^{1/2}}\right\}$ Re $(a+\beta+p)>0$
(22)	$t^{-1/2}\left[\mathrm{He}_m\left(\dfrac{x+t^{1/2}}{\lambda}\right)\mathrm{He}_n\left(\dfrac{y+t^{1/2}}{\mu}\right)+\mathrm{He}_m\left(\dfrac{x-t^{1/2}}{\lambda}\right)\mathrm{He}_n\left(\dfrac{y-t^{1/2}}{\mu}\right)\right]$	$\dfrac{2\pi^{1/2}\lambda^{-m}\mu^{-n}}{(2p)^{\frac{1}{2}m+\frac{1}{2}n+\frac{1}{2}}}\sum_{k=0}^{\min(m,n)}\left\{\binom{m}{k}\binom{n}{k}k!\times(2\lambda^2p-1)^{\frac{1}{2}m+\frac{1}{2}k}(2\mu^2p-1)^{\frac{1}{2}n+\frac{1}{2}k}\times\mathrm{He}_{m-k}\left[\dfrac{x}{(\lambda^2-\frac{1}{2}p^{-1})^{1/2}}\right]\times\mathrm{He}_{n-k}\left[\dfrac{y}{(\mu^2-\frac{1}{2}p^{-1})^{1/2}}\right]\right\}$ Re $p>0$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(23)	$(e^{t}-1)^{-\frac{1}{2}}\mathrm{He}_{2n}[x^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]$	$\dfrac{(-2)^n\pi^{\frac{1}{2}}(2n)!\,\Gamma(p+\frac{1}{2})}{\Gamma(n+p+1)}\,L_n^p(\tfrac{1}{2}x)$ Re $p>-\frac{1}{2}$
(24)	$\mathrm{He}_{2n+1}[x^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]$	$\dfrac{(-2)^n\pi^{\frac{1}{2}}(2n+1)!\,\Gamma(p)\,x^{\frac{1}{2}}}{\Gamma(n+p+3/2)}\,L_n^p(\tfrac{1}{2}x)$ Re $p>0$
(25)	$L_n(t)$	$(p-1)^n p^{-n-1}$ Re $p>0$
(26)	$t^n L_n(t)$	$n!\,p^{-n-1}P_n(1-2p^{-1})$ Re $p>0$
(27)	$L_n^{\alpha}(t)$	$\sum\limits_{m=0}^{n}\dbinom{\alpha+m-1}{m}\dfrac{(p-1)^{n-m}}{p^{n-m+1}}$ Re $p>0$
(28)	$t^{\alpha}L_n^{\alpha}(t)$ Re $\alpha>-1$	$\dfrac{\Gamma(\alpha+n+1)}{n!}\dfrac{(p-1)^n}{p^{\alpha+n+1}}$ Re $p>0$
(29)	$t^{\beta}L_n^{\alpha}(t)$ Re $\beta>-1$	$\dfrac{\Gamma(\beta+n+1)}{n!}\dfrac{(p-1)^n}{p^{\beta+n+1}}$ $\times\,{}_2F_1[-n,\alpha-\beta;-\beta-n;p/(p-1)]$ Re $p>0$
(30)	$t^{2\alpha}[L_n^{\alpha}(t)]^2$ Re $\alpha>-\frac{1}{2}$	$\dfrac{2^{2\alpha}\Gamma(\alpha+\frac{1}{2})\Gamma(n+\frac{1}{2})}{\pi(n!)^2p^{2\alpha+1}}$ $\times\,{}_2F_1[-n,\alpha+\frac{1}{2};\frac{1}{2}-n;(1-2/p)^2]$ Re $p>0$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(31)	$t^{\alpha} e^{\lambda t} L_n^{\alpha}(\kappa t)$ Re $\alpha > -1$	$\dfrac{\Gamma(\alpha+n+1)}{n!} \dfrac{(p-\kappa-\lambda)^n}{(p-\lambda)^{\alpha+n+1}}$ Re$(p-\lambda)>0$
(32)	$e^{-t} \sum_{m=0}^{n} a_{mn} L_m(2t)$ where a_{mn} is given by $P_n(z)=\sum_{m=0}^{n} a_{mn} z^m$	$\dfrac{1}{p+1} P_n\left(\dfrac{p-1}{p+1}\right)$ Re $p>-1$
(33)	$t^{-n} e^{-\lambda/t} L_n^{\alpha}(\lambda/t)$ Re $\lambda>0$	$(-1)^n (2/n!)\, \lambda^{-\frac{1}{2}\alpha} p^{\frac{1}{2}\alpha+n} K_{\alpha}(2\lambda^{\frac{1}{2}} p^{\frac{1}{2}})$ Re $p>0$
(34)	$L_n(\lambda t) L_n(\kappa t)$	$\dfrac{(p-\lambda-\kappa)^n}{p^{n+1}} P_n\left[\dfrac{p^2-(\lambda+\kappa)p+2\lambda\kappa}{p(p-\lambda-\kappa)}\right]$ Re $p>0$
(35)	$t^{\alpha} L_n^{\alpha}(\lambda t)\, L_m^{\alpha}(\kappa t)$ Re $\alpha>-1$	$\dfrac{\Gamma(m+n+\alpha+1)}{m!\,n!} \dfrac{(p-\lambda)^n (p-\kappa)^m}{p^{m+n+\alpha+1}} \times {}_2F_1\left[-m,-n;-m-n-\alpha; \dfrac{p(p-\lambda-\kappa)}{(p-\lambda)(p-\kappa)}\right]$ Re $p>0$
(36)	$t^{2\alpha} L_n^{\alpha}(\lambda t)\, L_n^{\alpha}(\kappa t)$ Re $\alpha > -\frac{1}{2}$	$\dfrac{\Gamma(2\alpha+1)\Gamma(n+\alpha+1)}{n!\,p^{2\alpha+1}} \times \sum_{r=0}^{\infty} \left\{ \dfrac{(-1)^r [1-(\lambda+\kappa)/(2p)]^{n-r}}{r!\,\Gamma(\alpha-r+1)} \times C_{n+r}^{\alpha+\frac{1}{2}} \left[\dfrac{p^2-(\lambda+\kappa)p+2\lambda\kappa}{p(p-\lambda-\kappa)}\right]\right\}$ Re $p>0$

Orthogonal polynomials (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(37)	$t^m\, p_n(m, t)$	$m!\,(p-1)^n\, p^{-m-1}$ $\quad$ Re $p>0$
(38)	$t^{\alpha-1}\, p_n(m, t)$ $\quad$ Re $\alpha > \min(n, m)$	$\dfrac{m!\,\Gamma(\alpha-n)}{(m-n)!\,p^{\alpha-n}}$ $\times {}_2F_1(-n, \alpha-n; m-n+1; r/p)$ $\quad$ Re $p>0$

4.12. Gamma function, error function, exponential integral and related functions

(1)	$\binom{t}{n} t^{\nu-1}$ $\quad$ Re $\nu>0$	$\dfrac{\Gamma(\nu)}{p^\nu}\,\Phi_n\left(\nu, \dfrac{1}{p}\right)$ $\quad$ Re $p>0$ where $\sum_0^\infty h^n \Phi_n(\nu, z) = [1 - z\log(h+1)]^{-\nu}$
(2)	$\mathrm{Erf}(\tfrac{1}{2} t/a)$ $\quad$ $\lvert\arg a\rvert < \tfrac{1}{4}\pi$	$p^{-1} e^{a^2 p^2}\,\mathrm{Erfc}(ap)$ $\quad$ Re $p>0$
(3)	$e^{-a^2 t^2}\,\mathrm{Erf}(iat)$ $\quad$ $\lvert\arg a\rvert < \tfrac{1}{4}\pi$	$(2ai\pi^{1/2})^{-1}\, e^{\frac{1}{4}a^{-2}p^2}\,\mathrm{Ei}(-\tfrac{1}{4}a^{-2}p^2)$ $\quad$ Re $p>0$
(4)	$\mathrm{Erf}(a^{1/2} t^{1/2})$	$a^{1/2} p^{-1} (p+a)^{-1/2}$ $\quad$ Re $p>0$, $\quad -$Re a
(5)	$e^{at}\,\mathrm{Erf}(a^{1/2} t^{1/2})$	$a^{1/2} p^{-1/2} (p-a)^{-1}$ $\quad$ Re $p>0$, $\quad$ Re a
(6)	$\mathrm{Erf}(\tfrac{1}{2} a^{1/2} t^{-1/2})$ $\quad$ Re $a>0$	$p^{-1}(1-e^{-a^{1/2} p^{1/2}})$ $\quad$ Re $p>0$

Gamma function etc. (cont'd)

	$f(t)$		$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	
(7)	$\mathrm{Erfc}\,(\tfrac{1}{2}t/\alpha)$	$\lvert\arg\alpha\rvert<\tfrac{1}{4}\pi$	$p^{-1}[1-e^{\alpha^2p^2}\,\mathrm{Erfc}\,(\alpha p)]$	
(8)	$e^{-\alpha^2t^2}\,\mathrm{Erfc}\,(i\alpha t)$		$\tfrac{1}{2}\pi^{\frac{1}{2}}\alpha^{-1}e^{\frac{1}{4}\alpha^{-2}p^2}[\mathrm{Erfc}\,(\tfrac{1}{2}\alpha^{-1}p)$ $+i\pi^{-1}\,\mathrm{Ei}\,(-\tfrac{1}{4}\alpha^{-2}p^2)]$	$\mathrm{Re}\,p>0$
(9)	$\mathrm{Erfc}\,(\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$		$p^{-1}(p+\alpha)^{-\frac{1}{2}}[(p+\alpha)^{\frac{1}{2}}-\alpha^{\frac{1}{2}}]$	$\mathrm{Re}\,p>-\mathrm{Re}\,\alpha$
(10)	$e^{\alpha t}\,\mathrm{Erfc}\,(\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$		$p^{-\frac{1}{2}}(p^{\frac{1}{2}}+\alpha^{\frac{1}{2}})^{-1}$	$\mathrm{Re}\,p>0$
(11)	$\mathrm{Erfc}\,(\tfrac{1}{2}\alpha^{\frac{1}{2}}t^{-\frac{1}{2}})$	$\mathrm{Re}\,\alpha>0$	$p^{-1}e^{-\alpha^{\frac{1}{2}}p^{\frac{1}{2}}}$	$\mathrm{Re}\,p>0$
(12)	$e^{\alpha t}\,\mathrm{Erfc}\,(\alpha^{\frac{1}{2}}t^{\frac{1}{2}}+\tfrac{1}{2}\beta^{\frac{1}{2}}t^{-\frac{1}{2}})$	$\mathrm{Re}\,\beta>0$	$p^{-\frac{1}{2}}(p^{\frac{1}{2}}+\alpha^{\frac{1}{2}})^{-1}$ $\times\exp(-\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}-\beta^{\frac{1}{2}}p^{\frac{1}{2}})$	$\mathrm{Re}\,p>0$
(13)	$S(t)$		$\dfrac{[(p^2+1)^{\frac{1}{2}}-p]^{\frac{1}{2}}}{2p(p^2+1)^{\frac{1}{2}}}$	$\mathrm{Re}\,p>0$
(14)	$C(t)$		$\dfrac{[(p^2+1)^{\frac{1}{2}}-p]^{-\frac{1}{2}}}{2p(p^2+1)^{\frac{1}{2}}}$	$\mathrm{Re}\,p>0$
(15)	$S(t^{\frac{1}{2}})$		$p^{-1}[\tfrac{1}{2}-\cos(\tfrac{1}{4}p^2)\,C(\tfrac{1}{4}p^2)$ $-\sin(\tfrac{1}{4}p^2)\,S(\tfrac{1}{4}p^2)]$	
(16)	$C(t^{\frac{1}{2}})$		$p^{-1}[\tfrac{1}{2}\cos(\tfrac{1}{2}p^2)-\cos(\tfrac{1}{4}p^2)S(\tfrac{1}{4}p^2)$ $+\sin(\tfrac{1}{4}p^2)\,C(\tfrac{1}{4}p^2)]$	
(17)	$\mathrm{Si}\,(t)$		$p^{-1}\,\mathrm{ctn}^{-1}\,p$	$\mathrm{Re}\,p>0$
(18)	$\mathrm{si}\,(t)$		$-p^{-1}\tan^{-1}p$	$\mathrm{Re}\,p>0$

Gamma function etc. (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(19)	$\mathrm{Ci}(t)=-\mathrm{ci}(t)$	$\tfrac{1}{2}p^{-1}\log(p^2+1)$ $\quad \mathrm{Re}\, p>0$
(20)	$\cos t\, \mathrm{Si}(t)-\sin t\, \mathrm{Ci}(t)$	$(p^2+1)^{-1}\log p$ $\quad \mathrm{Re}\, p>0$
(21)	$\sin t\, \mathrm{Si}(t)+\cos t\, \mathrm{Ci}(t)$	$-(p^2+1)^{-1}\, p\log p$ $\quad \mathrm{Re}\, p>0$
(22)	$\mathrm{Si}(t^2)$	$\pi[\tfrac{1}{2}-C(\tfrac{1}{4}p^2)]^2+\pi[\tfrac{1}{2}-S(\tfrac{1}{4}p^2)]^2$ $\quad \mathrm{Re}\, p>0$
(23)	$\overline{\mathrm{Ei}}(t)$	$-p^{-1}\log(p-1)$ $\quad \mathrm{Re}\, p>1$
(24)	$\mathrm{Ei}(-t)$	$-p^{-1}\log(p+1)$ $\quad \mathrm{Re}\, p>0$
(25)	$t^{-1/2}\,\mathrm{Ei}(-t)$	$-2\pi^{1/2}p^{-1/2}\log[p^{1/2}+(p+1)^{1/2}]$ $\quad \mathrm{Re}\, p>0$
(26)	$\sin(at)\,\mathrm{Ei}(-t)$	$-(p^2+a^2)^{-1}\{\tfrac{1}{2}a\log[(p+1)^2+a^2]$ $-p\tan^{-1}[a/(p+1)]\}$ $\quad \mathrm{Re}\, p>\lvert\mathrm{Im}\, a\rvert$
(27)	$\cos(at)\,\mathrm{Ei}(-t)$	$-(p^2+a^2)^{-1}\{\tfrac{1}{2}p\log[(p+1)^2+a^2]$ $+a\tan^{-1}[a/(p+1)]\}$ $\quad \mathrm{Re}\, p>\lvert\mathrm{Im}\, a\rvert$
(28)	$\mathrm{li}(e^t)$	$-p^{-1}\log(p-1)$ $\quad \mathrm{Re}\, p>1$
(29)	$\mathrm{li}(e^{-t})$	$-p^{-1}\log(p+1)$ $\quad \mathrm{Re}\, p>0$
(30)	$\Gamma(\nu, at)$ $\quad \mathrm{Re}\,\nu>-1$	$\Gamma(\nu)p^{-1}[1-(1+p/a)^{-\nu}]$ $\quad \mathrm{Re}\, p>-\mathrm{Re}\, a$
(31)	$e^{at}\Gamma(\nu, at)$ $\quad \mathrm{Re}\,\nu>-1$	$\Gamma(\nu)(p-a)^{-1}(1-a^{\nu}p^{-\nu})$ $\quad \mathrm{Re}\, p>0$

Gamma function etc. (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(32)	$\Gamma(\nu, a/t)$ $\qquad \lvert\arg a\rvert < \frac{1}{2}\pi$	$2a^{\frac{1}{2}\nu} p^{\frac{1}{2}\nu-1} K_\nu(2a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\qquad \mathrm{Re}\, p > 0$
(33)	$t^{\mu-1} e^{a/t}\Gamma(\nu, a/t)$ $\mathrm{Re}(\nu-\mu) < 1, \quad \lvert\arg a\rvert < \pi$	$2^{2+\mu-2\nu}\Gamma(1+\mu-\nu)(a/p)^{\frac{1}{2}\mu}$ $\times S_{2\nu-\mu-1,\,\mu}(2a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\qquad \mathrm{Re}\, p > 0$
(34)	$e^{\beta t}\gamma(\nu, at)$ $\qquad \mathrm{Re}\,\nu > -1$	$\Gamma(\nu)\, a^\nu (p-\beta)^{-1}(p+a-\beta)^{-\nu}$ $\mathrm{Re}\, p > \mathrm{Re}\,\beta, \quad \mathrm{Re}(\beta - a)$
(35)	$\gamma(\frac{1}{4}, 2^{-3}a^{-2}t^2)$ $\qquad \lvert\arg a\rvert < \frac{1}{4}\pi$	$2^{\frac{3}{4}} a^{\frac{1}{2}} p^{-\frac{1}{2}} e^{a^2p^2} K_{\frac{1}{4}}(a^2p^2)$ $\mathrm{Re}\, p > 0$
(36)	$\gamma(\nu, 2^{-3}a^{-2}t^2)$ $\lvert\arg a\rvert < \frac{1}{4}\pi, \quad \mathrm{Re}\,\nu > -\frac{1}{2}$	$2^{-\nu-1}\Gamma(2\nu)\, p^{-1} e^{a^2p^2} D_{-2\nu}(2ap)$ $\mathrm{Re}\, p > 0$
(37)	$e^{-\frac{1}{4}t^2/a}\gamma(\nu, \frac{1}{4}e^{i\pi}t^2/a)$ $\lvert\arg a\rvert < \frac{1}{2}\pi, \quad \mathrm{Re}\,\nu > -\frac{1}{2}$	$2^{1-2\nu}\Gamma(2\nu)\, a^{\frac{1}{2}} e^{ap^2+\nu\pi i}$ $\times\Gamma(\frac{1}{2}-\nu, ap^2)$ $\qquad \mathrm{Re}\, p > 0$

4.13. Legendre functions

(1)	$[t(1+t)]^{-\frac{1}{2}\mu} P_\nu^\mu(1+2t)$ $\mathrm{Re}\,\mu < 1$	$\pi^{-\frac{1}{2}} p^{\mu-\frac{1}{2}} e^{\frac{1}{2}p} K_{\nu+\frac{1}{2}}(\frac{1}{2}p)$ $\mathrm{Re}\, p > 0$
(2)	$(1+t^{-1})^{\frac{1}{2}\mu} P_\nu^\mu(1+2t)$ $\mathrm{Re}\,\mu < 1$	$p^{-1} e^{\frac{1}{2}p} W_{\mu,\,\nu+\frac{1}{2}}(p)$ $\qquad \mathrm{Re}\, p > 0$
(3)	$t^{\lambda+\frac{1}{2}\mu-1}(t+2)^{\frac{1}{2}\mu} P_\nu^{-\mu}(1+t)$ $\mathrm{Re}(\lambda+\mu) > 0$	$-\pi^{-1}\sin(\nu\pi)\, p^{-\lambda-\mu}$ $\times E(-\nu, \nu+1, \lambda+\mu{:}\ \mu+1{:}\ 2p)$ $\mathrm{Re}\, p > 0$
(4)	$t^{\lambda-\frac{1}{2}\mu-1}(t+2)^{-\frac{1}{2}\mu} P_\nu^{-\mu}(1+t)$ $\mathrm{Re}\,\lambda > 0$	$\dfrac{E(\mu+\nu+1, \mu-\nu, \lambda{:}\ \mu+1{:}\ 2p)}{2^\mu p^\lambda \Gamma(\mu+\nu+1)\Gamma(\mu-\nu)}$ $\mathrm{Re}\, p > 0$

Legendre functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(5)	$(a+t)^{\frac{1}{2}\nu}(\beta+t)^{\frac{1}{2}\nu}$ $\times P_\nu[2a^{-1}\beta^{-1}(a+t)(\beta+t)-1]$ $\lvert\arg a\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$	$\pi^{-1}(a\beta)^{\frac{1}{2}\nu+\frac{1}{2}} e^{\frac{1}{2}(a+\beta)p}$ $\times K_{\nu+\frac{1}{2}}(\tfrac{1}{2}ap)\, K_{\nu+\frac{1}{2}}(\tfrac{1}{2}\beta p)$ $\lvert\arg(ap)\rvert<\pi,\quad \lvert\arg(\beta p)\rvert<\pi$
(6)	$(1+t)^{-1} P_\nu[2(1+t)^{-2}-1]$	$p^{-1} e^{p} W_{\nu+\frac{1}{2},0}(p)\, W_{-\nu-\frac{1}{2},0}(p)$ $\mathrm{Re}\, p>0$
(7)	$t^{-\frac{1}{2}\mu} P_\nu^\mu[(1+t)^{\frac{1}{2}}]$ $\mathrm{Re}\,\mu<1$	$2^\mu p^{\mu/2-5/4} e^{p/2} W_{\frac{1}{2}\mu+\frac{1}{4},\,\frac{1}{2}\nu+\frac{1}{4}}(p)$ $\mathrm{Re}\, p>0$
(8)	$t^{-\frac{1}{2}\mu}(1+t)^{-\frac{1}{2}} P_\nu^\mu[(1+t)^{\frac{1}{2}}]$ $\mathrm{Re}\,\mu<1$	$2^\mu p^{\frac{1}{2}\mu-\frac{3}{4}} p^{\frac{1}{2}p} W_{\frac{1}{2}\mu-\frac{1}{4},\,\frac{1}{2}\nu+\frac{1}{4}}(p)$ $\mathrm{Re}\, p>0$
(9)	$t^{\frac{1}{2}} P_\nu^{\frac{1}{4}}[(1+t^2)^{\frac{1}{2}}] P_\nu^{-\frac{1}{4}}[(1+t^2)^{\frac{1}{2}}]$	$\tfrac{1}{2}(\tfrac{1}{2}\pi/p)^{\frac{1}{2}} H_{\nu+\frac{1}{2}}^{(1)}(\tfrac{1}{2}p)\, H_{\nu+\frac{1}{2}}^{(2)}(\tfrac{1}{2}p)$ $\mathrm{Re}\, p>0$
(10)	$(a+t)^{-\frac{1}{2}\nu-\frac{1}{2}}(\beta+t)^{\frac{1}{2}\nu}$ $\times[-1-(a+\beta)/t]^{\frac{1}{2}\mu}$ $\times P_\nu^\mu[a^{\frac{1}{2}}\beta^{\frac{1}{2}}(a+t)^{-\frac{1}{2}}(\beta+t)^{-\frac{1}{2}}]$ $\lvert\arg a\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$ $\mathrm{Re}\,\mu<1$	$2^{\frac{1}{2}} p^{-\frac{1}{2}} e^{\frac{1}{2}(a+\beta)p} D_{\mu-\nu-1}(2^{\frac{1}{2}} a^{\frac{1}{2}} p^{\frac{1}{2}})$ $\times D_{\mu+\nu}(2^{\frac{1}{2}}\beta^{\frac{1}{2}} p^{\frac{1}{2}})$ $\mathrm{Re}\, p>0$ $\lvert\arg(ap)\rvert<\pi,\quad \lvert\arg(\beta p)\rvert<\pi$
(11)	$(1-e^{-2t})^{\frac{1}{2}\mu} P_\nu^{-\mu}(e^t)$ $\mathrm{Re}\,\mu>-1$	$\dfrac{2^{p-1}\Gamma(\frac{1}{2}p+\frac{1}{2}\nu+\frac{1}{2})\Gamma(\frac{1}{2}p-\frac{1}{2}\nu)}{\pi^{\frac{1}{2}}\Gamma(p+\mu+1)}$ $\mathrm{Re}\, p>\mathrm{Re}\,\nu,\quad -1-\mathrm{Re}\,\nu$
(12)	$\left[(e^t-1)\left(\dfrac{ae^t}{a-2}-1\right)\right]^{\frac{1}{2}\mu}$ $\times P_\nu^{-\mu}(ae^t-a+1)$ $\mathrm{Re}\, a>0,\quad \mathrm{Re}\,\mu>-1$	$\dfrac{\Gamma(p-\mu+\nu+1)\Gamma(p-\nu-\mu)}{\Gamma(p+1)}\left(\dfrac{a}{a-2}\right)^{\frac{1}{2}p}$ $\times P_\nu^{\mu-p}(a-1)$ $\mathrm{Re}\, p>\mathrm{Re}(\mu-\nu)-1,\ \mathrm{Re}(\mu+\nu)$

Legendre functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(13)	$(1-z^2+z^2e^{-t})^{\mu}$ $\times\{P_{2\nu}^{2\mu}[z(1-e^{-t})^{1/2}]$ $-P_{2\nu}^{2\mu}[-z(1-e^{-t})^{1/2}]\}$ $\quad\vert z\vert<1$	$\dfrac{-2^{2\mu+1}\pi z\,\Gamma(p)}{\Gamma(-\mu-\nu)\,\Gamma(\tfrac{1}{2}-\mu+\nu)\,\Gamma(p+3/2)}$ $\times{}_2F_1(1/2-\mu-\nu,\nu-\mu+1;p+3/2;z^2)$ $\quad\mathrm{Re}\,p>0$
(14)	$(1-e^{-t})^{-1/2}(1-z^2+z^2e^{-t})^{\mu}$ $\times\{P_{2\nu}^{2\mu}[z(1-e^{-t})^{1/2}]$ $+P_{2\nu}^{2\mu}[-z(1-e^{-t})^{1/2}]\}$ $\vert z\vert<1$	$\dfrac{2^{2\mu+1}\pi\,\Gamma(p)}{\Gamma(\tfrac{1}{2}-\mu-\nu)\,\Gamma(1-\mu+\nu)\,\Gamma(p+\tfrac{1}{2})}$ $\times{}_2F_1(-\mu-\nu,\tfrac{1}{2}-\mu+\nu;p+\tfrac{1}{2};z^2)$ $\quad\mathrm{Re}\,p>0$
(15)	$\sinh^{2\mu}(\tfrac{1}{2}t)\,P_{2n}^{-2\mu}[\cosh(\tfrac{1}{2}t)]$ $\quad\mathrm{Re}\,\mu>-\tfrac{1}{4}$	$\dfrac{\Gamma(2\mu+\tfrac{1}{2})\,\Gamma(p-n-\mu)\,\Gamma(p+n-\mu+\tfrac{1}{2})}{4^{\mu}\pi^{1/2}\,\Gamma(p+n+\mu+1)\,\Gamma(p-n+\mu+\tfrac{1}{2})}$ $\quad\mathrm{Re}\,p>n+\mathrm{Re}\,\mu$
(16)	$t^{\lambda+\frac{1}{2}\mu-1}(t+2)^{\frac{1}{2}\mu}Q_{\nu}^{\mu}(1+t)$ $\quad\mathrm{Re}\,\lambda>0,\quad\mathrm{Re}(\lambda+\mu)>0$	$\dfrac{\Gamma(\nu+\mu+1)}{\Gamma(\nu-\mu+1)}\Bigg\{\dfrac{\sin(\nu\pi)}{2p^{\lambda+\mu}\sin(\mu\pi)}$ $\times E(-\nu,\nu+1,\lambda+\mu:\mu+1:2p)$ $-\dfrac{\sin[(\mu+\nu)\pi)}{2^{1-\mu}p^{\lambda}\sin(\mu\pi)}$ $\times E(\nu-\mu+1,-\nu-\mu,\lambda:1-\mu:2p)\Bigg\}$ $\quad\mathrm{Re}\,p>0$
(17)	$t^{\lambda-\frac{1}{2}\mu-1}(t+2)^{\frac{1}{2}\mu}Q_{\nu}^{\mu}(1+t)$ $\quad\mathrm{Re}\,\lambda>0,\quad\mathrm{Re}(\lambda-\mu)>0$	$-\dfrac{\sin(\nu\pi)}{2p^{\lambda-\mu}\sin(\mu\pi)}$ $\times E(-\nu,\nu+1,\lambda-\mu:1-\mu:2p)$ $-\dfrac{\sin[(\mu-\nu)\pi]}{2^{1+\mu}p^{\lambda}\sin(\mu\pi)}$ $\times E(\mu+\nu+1,\mu-\nu,\lambda:1+\mu:2p)$ $\quad\mathrm{Re}\,p>0$

4.14. Bessel functions of arguments kt and $kt^{1/2}$

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$J_\nu(at)$ Re $\nu>-1$	$r^{-1}(a/R)^\nu = r^{-1}e^{-\nu\sinh^{-1}(p/a)}$ Re $p>\lvert\text{Im}\,a\rvert$
(2)	$tJ_\nu(at)$ Re $\nu>-2$	$r^{-3}(p+\nu r)(a/R)^\nu$ Re $p>\lvert\text{Im}\,a\rvert$
(3)	$t^2J_\nu(at)$ Re $\nu>-3$	$\left(\dfrac{\nu^2-1}{r^3}+3p\,\dfrac{p+\nu r}{r^5}\right)\left(\dfrac{a}{R}\right)^\nu$ Re $p>\lvert\text{Im}\,a\rvert$
(4)	$t^n J_n(at)$	$1\cdot3\cdot5\cdot\cdots\cdot(2n-1)\,a^n r^{-2n-1}$ Re $p>\lvert\text{Im}\,a\rvert$
(5)	$t^{-1}J_\nu(at)$ Re $\nu>0$	$\nu^{-1}(a/R)^\nu$ Re $p\geq\lvert\text{Im}\,a\rvert$
(6)	$t^{-2}J_\nu(at)$ Re $\nu>1$	$\dfrac{a}{2\nu}\left[\dfrac{1}{\nu-1}\left(\dfrac{a}{R}\right)^{\nu-1}+\dfrac{1}{\nu+1}\left(\dfrac{a}{R}\right)^{\nu+1}\right]$ Re $p\geq\lvert\text{Im}\,a\rvert$
(7)	$t^\nu J_\nu(at)$ Re $\nu>-\tfrac{1}{2}$	$2^\nu\pi^{-1/2}\Gamma(\nu+\tfrac{1}{2})\,a^\nu r^{-2\nu-1}$ Re $p>\lvert\text{Im}\,a\rvert$
(8)	$t^{\nu+1}J_\nu(at)$ Re $\nu>-1$	$2^{\nu+1}\pi^{-1/2}\Gamma(\nu+3/2)\,a^\nu r^{-2\nu-3}p$ Re $p>\lvert\text{Im}\,a\rvert$
(9)	$t^\mu J_\nu(at)$ Re $(\mu+\nu)>-1$	$\Gamma(\mu+\nu+1)\,r^{-\mu-1}P_\mu^{-\nu}(p/r)$ Re $p>\lvert\text{Im}\,a\rvert$
(10)	$t^\mu\sin(at)\,J_\mu(at)$ $a>0$, Re $\mu>-1$	$\dfrac{\Gamma(\mu+1)\,a^{\mu+1}}{2^{1/2}\pi}\times\displaystyle\int_0^{\frac{1}{2}\pi}\frac{(\cos\theta)^{\mu+1/2}\cos[(\mu-\tfrac{1}{2})\theta]}{(\tfrac{1}{4}p^2+a^2\cos^2\theta)^{\mu+1}}\,d\theta$ Re $p>0$

$r=(p^2+a^2)^{1/2},\quad R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(11)	$t^{\mu-1}\cos(at)\,J_\mu(at)$ $\quad a>0,\quad \mathrm{Re}\,\mu>0$	$\dfrac{\Gamma(\mu)\,a^\mu}{2^{1/2}\,\pi} \times \displaystyle\int_0^{\frac{1}{2}\pi} \frac{(\cos\theta)^{\mu-1/2}\cos[(\mu+\frac{1}{2})\theta]}{(\frac{1}{4}p^2+a^2\cos^2\theta)^\mu}\,d\theta$ $\quad \mathrm{Re}\,p>0$
(12)	$[J_0^2(\frac{1}{2}at)]^2$	$2\pi^{-1}r^{-1}K(a/r) \qquad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(13)	$[J_1^2(\frac{1}{2}at)]^2$	$2\pi^{-1}a^{-2}r^{-2}[(2p^2+a^2)K(a/r) - 2(p^2+a^2)E(a/r)]$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(14)	$J_\nu(at)\,J_\nu(\beta t) \qquad \mathrm{Re}\,\nu>-\frac{1}{2}$	$\dfrac{1}{\pi a^{1/2}\beta^{1/2}}\,Q_{\nu-1/2}\left(\dfrac{p^2+a^2+\beta^2}{2a\beta}\right)$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert+\lvert\mathrm{Im}\,\beta\rvert$
(15)	$t\,J_0(\frac{1}{2}at)\,J_1(\frac{1}{2}at)$	$2\pi^{-1}a^{-1}r^{-1}[K(a/r)-E(a/r)]$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(16)	$t\,J_\nu^2(at) \qquad \mathrm{Re}\,\nu>-1$	$2^{2\nu+1}(\nu+\frac{1}{2})\pi^{-1}a^{2\nu}p^{-2\nu-2} \times \mathrm{B}(\nu+\frac{1}{2},\ \nu+\frac{1}{2}) \times {}_2F_1(\nu+1/2,\nu+3/2;2\nu+1;-4a^2/p^2)$ $\quad \mathrm{Re}\,p>2\lvert\mathrm{Im}\,a\rvert$
(17)	$t^{-2}J_1^2(t)$	$\frac{1}{2}\pi^{-1}\int_0^\pi[(p^2+2-2\cos\phi)^{1/2}-p] \times(1+\cos\phi)\,d\phi \qquad \mathrm{Re}\,p>0$
(18)	$t^{1/2}J_\nu^2(\frac{1}{2}at) \qquad \mathrm{Re}\,\nu>-\frac{3}{4}$	$\dfrac{a\,\Gamma(2\nu+3/2)}{2^{\nu+3/2}p^{1/2}r}\,P_{1/4}^{-\nu}\left(\dfrac{r}{p}\right)P_{-1/4}^{-\nu}\left(\dfrac{r}{p}\right)$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$

$r=(p^2+a^2)^{1/2}, \qquad R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(19)	$t^{-1/2} J_\nu^2(\tfrac{1}{2}at)$ Re $\nu > -\tfrac{1}{4}$	$2^{-\nu-1/2}\Gamma(2\nu+\tfrac{1}{2})\,p^{-1/2}\,[P_{-1/4}^{-\nu}(r/p)]^2$ Re $p > \lvert\mathrm{Im}\,a\rvert$
(20)	$t^{1/2} J_\nu(\tfrac{1}{2}at)\,J_{-\nu}(\tfrac{1}{2}at)$	$\dfrac{a\pi^{1/2}}{2p^{1/2}r}[(\nu+\tfrac{1}{4})P_{-1/4}^{\nu}(r/p)\,P_{1/4}^{-\nu}(r/p)$ $-(\nu-\tfrac{1}{4})P_{1/4}^{\nu}(r/p)\,P_{-1/4}^{-\nu}(r/p)]$ Re $p > \lvert\mathrm{Im}\,a\rvert$
(21)	$t^{1/2} J_\nu(\tfrac{1}{2}at)\,J_{\nu+1}(\tfrac{1}{2}at)$ Re $\nu > -5/4$	$\dfrac{a\,\Gamma(2\nu+5/2)}{2^{\nu+5/2}p^{1/2}r} P_{-1/4}^{-\nu}\left(\dfrac{r}{p}\right)P_{-1/4}^{-\nu-1}\left(\dfrac{r}{p}\right)$ Re $p > \lvert\mathrm{Im}\,a\rvert$
(22)	$t^{-1/2} J_\nu(\tfrac{1}{2}at)\,J_{-\nu}(\tfrac{1}{2}at)$	$2^{-1/2}\pi^{1/2}p^{-1/2}\,P_{-1/4}^{\nu}(r/p)\,P_{-1/4}^{-\nu}(r/p)$ Re $p > \lvert\mathrm{Im}\,a\rvert$
(23)	$t^{2\nu} J_\nu^2(at)$ Re $\nu > -\tfrac{1}{4}$	$\dfrac{2^{4\nu}a^{2\nu}\Gamma(\nu+\tfrac{1}{2})\Gamma(2\nu+\tfrac{1}{2})}{\pi\Gamma(\nu+1)\,p^{4\nu+1}}$ $\times\,{}_2F_1(\nu+\tfrac{1}{2},2\nu+\tfrac{1}{2};\nu+1;-4a^2/p^2)$ Re $p > 2\lvert\mathrm{Im}\,a\rvert$
(24)	$t^{\mu-1} J_{\nu_1}(a_1t)\cdots J_{\nu_n}(a_nt)$ Re$(\mu+N)>0$ $N=\nu_1+\cdots+\nu_n$ $a=a_1+\cdots+a_n$	$\dfrac{2^{-N}\Gamma(\mu+N)}{\Gamma(\nu_1+1)\cdots\Gamma(\nu_n+1)}\,a_1^{\nu_1}\cdots a_n^{\nu_n}$ $\times(p+ia)^{-\mu-N}F_A\left(\mu+N;\nu_1+\tfrac{1}{2},\dots,\nu_n+\tfrac{1}{2};\ 2\nu_1+1,\dots,2\nu_n+1;\dfrac{2a_1i}{p+ia},\dots,\dfrac{2a_ni}{p+ia}\right)$ Re$(p\pm ia_1\pm\cdots\pm ia_n)>0$ If $n=2$, F_A is to be replaced by F_2

$r=(p^2+a^2)$, $\quad R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(25)	$J_0(2a^{1/2}t^{1/2})$	$p^{-1}e^{-a/p}$ $\mathrm{Re}\,p>0$
(26)	$J_\nu(2a^{1/2}t^{1/2})$ $\mathrm{Re}\,\nu>-2$	$\tfrac{1}{2}a^{1/2}\pi^{1/2}p^{-3/2}e^{-\frac{1}{2}a/p}[I_{\frac{1}{2}\nu-\frac{1}{2}}(\tfrac{1}{2}a/p)$ $-I_{\frac{1}{2}\nu+\frac{1}{2}}(\tfrac{1}{2}a/p)]$ $\mathrm{Re}\,p>0$
(27)	$t^{1/2}J_1(2a^{1/2}t^{1/2})$	$a^{1/2}p^{-2}e^{-a/p}$ $\mathrm{Re}\,p>0$
(28)	$t^{n-1/2}J_1(2a^{1/2}t^{1/2})$	$(-1)^{n-1}a^{-1/2}n!\,p^{-n}e^{-\frac{1}{2}a/p}k_{2n}(\tfrac{1}{2}a/p)$ $\mathrm{Re}\,p>0$
(29)	$t^{-1/2}J_\nu(2a^{1/2}t^{1/2})$ $\mathrm{Re}\,\nu>-1$	$\pi^{1/2}p^{-1/2}e^{-\frac{1}{2}a/p}I_{\frac{1}{2}\nu}(\tfrac{1}{2}a/p)$ $\mathrm{Re}\,p>0$
(30)	$t^{\frac{1}{2}\nu}J_\nu(2a^{1/2}t^{1/2})$ $\mathrm{Re}\,\nu>-1$	$a^{\frac{1}{2}\nu}p^{-\nu-1}e^{-a/p}$ $\mathrm{Re}\,p>0$
(31)	$t^{-\frac{1}{2}\nu}J_\nu(2a^{1/2}t^{1/2})$	$\dfrac{e^{i\nu\pi}p^{\nu-1}}{a^{\frac{1}{2}\nu}\Gamma(\nu)}e^{-a/p}\,\gamma\left(\nu,\dfrac{a}{p}e^{-i\pi}\right)$ $\mathrm{Re}\,p>0$
(32)	$t^{\frac{1}{2}\nu-1}J_\nu(2a^{1/2}t^{1/2})$ $\mathrm{Re}\,\nu>0$	$a^{-\frac{1}{2}\nu}\gamma(\nu,\,a/p)$ $\mathrm{Re}\,p>0$
(33)	$t^{\frac{1}{2}\nu+n}J_\nu(2a^{1/2}t^{1/2})$ $\mathrm{Re}\,\nu+n>-1$	$n!\,a^{\frac{1}{2}\nu}p^{-n-\nu-1}e^{-a/p}L_n^\nu(a/p)$ $\mathrm{Re}\,p>0$
(34)	$t^{\mu-1/2}J_{2\nu}(2a^{1/2}t^{1/2})$ $\mathrm{Re}(\mu+\nu)>-\tfrac{1}{2}$	$\dfrac{\Gamma(\mu+\nu+\frac{1}{2})}{a^{1/2}\Gamma(2\nu+1)}p^{-\mu}e^{-\frac{1}{2}a/p}M_{\mu,\nu}(a/p)$ $\mathrm{Re}\,p>0$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(35)	$t^{\mu-1} J_{2\nu}(2a^{1/2} t^{1/2})$ $\quad \operatorname{Re}(\mu+\nu)>0$	$\dfrac{\Gamma(\mu+\nu)\,a^{\nu}}{\Gamma(2\nu+1)\,p^{\mu+\nu}} \times {}_1F_1(\mu+\nu; 2\nu+1; -a/p)$ $\quad \operatorname{Re} p>0$
(36)	$t^{\nu-1/2}\{J_{2\mu}(2t^{1/2})\cos[(\nu+\mu)\pi] - J_{-2\mu}(2t^{1/2})\cos[(\nu-\mu)\pi]\}$ $\quad \operatorname{Re}(\nu\pm\mu)>-1/2$	$-\sin(2\mu\pi)\,p^{-\nu} e^{-1/2 p^{-1}} W_{\nu,\mu}(p^{-1})$ $\quad \operatorname{Re} p>0$
(37)	$t^{1/2\nu} L_n^{\nu}(t)\, J_{\nu}(2a^{1/2} t^{1/2})$	$a^{1/2\nu} e^{-a/p} \dfrac{(p-1)^n}{p^{\nu+n+1}} L_n^{\nu}\left[\dfrac{a}{p(1-p)}\right]$ $\quad \operatorname{Re} p>0$
(38)	$J_{\nu}(t)\, J_{2\nu}(2a^{1/2} t^{1/2})$ $\quad \operatorname{Re}\nu>-1/2$	$\dfrac{e^{-ap/(p^2+1)}}{(p^2+1)^{1/2}} J_{\nu}\left(\dfrac{a}{p^2+1}\right)$ $\quad \operatorname{Re} p>0$
(39)	$J_{\nu}(at^{1/2})\, J_{\nu}(\beta t^{1/2})$ $\quad \operatorname{Re}\nu>-1/2$	$p^{-1} e^{-1/4(a^2+\beta^2)/p} I_{\nu}(1/2\, a\beta/p)$ $\quad \operatorname{Re} p>0$
(40)	$t^{-1} J_{\nu}^2(2t^{1/2})$ $\quad \operatorname{Re}\nu>0$	$\nu^{-1} e^{-2/p}[I_{\nu}(2/p)+2\sum_{n=1}^{\infty} I_{\nu+n}(2/p)]$ $\quad \operatorname{Re} p>0$
(41)	$t^{-1/2} J_{\nu}(ae^{1/4\pi i} t^{1/2}) \times J_{\nu}(ae^{-1/4\pi i} t^{1/2})$ $\quad \operatorname{Re}\nu>-1/2$	$\dfrac{p^{1/2} 2^{1-\nu}\Gamma(\nu+1/2)}{a^2[\Gamma(\nu+1)]^2} M_{1/4,1/2\nu}\left(\dfrac{a^2}{2p}\right) \times M_{-1/4,1/2\nu}\left(\dfrac{a^2}{2p}\right)$ $\quad \operatorname{Re} p>0$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(42)	$t^{\lambda-1}J_{2\mu}(2a^{1/2}t^{1/2})J_{2\nu}(2a^{1/2}t^{1/2})$ $\mathrm{Re}(\lambda+\mu+\nu)>0$	$\dfrac{2\Gamma(\lambda+\mu+\nu)\,a^{\mu+\nu}}{\Gamma(2\mu+1)\Gamma(2\nu+1)\,p^{\lambda+\mu+\nu}}$ $\times\,{}_3F_3(\mu+\nu+\tfrac12,\ \mu+\nu+1,\ \lambda+\mu+\nu;$ $2\mu+1, 2\nu+1, 2\mu+2\nu+1; -4a/p)$ $\mathrm{Re}\,p>0$
(43)	$t^{\nu-1}J_{2\mu_1}(2a_1^{1/2}t^{1/2})\cdots J_{2\mu_n}(2a_n^{1/2}t^{1/2})$ $M=\mu_1+\cdots+\mu_n,\quad \mathrm{Re}(\nu+M)>0$	$\dfrac{\Gamma(\nu+M)\,p^{-\nu-M}\,a_1^{\mu_1}\cdots a_n^{\mu_n}}{\Gamma(2\mu_1+1)\cdots\Gamma(2\mu_n+1)}$ $\times\Psi_2\left(\nu+M; 2\mu_1+1,\ldots,2\mu_n+1; \dfrac{a_1}{p},\ldots,\dfrac{a_n}{p}\right)$ $\mathrm{Re}\,p>0$
(44)	$Y_0(at)$	$-2\pi^{-1}r^{-1}\sinh^{-1}(p/a)$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(45)	$Y_\nu(at)$ $\quad\lvert\mathrm{Re}\,\nu\rvert<1$	$a^\nu\,\mathrm{ctn}(\nu\pi)r^{-1}R^{-\nu}-a^{-\nu}\csc(\nu\pi)r^{-1}R^{\nu}$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(46)	$t\,Y_0(at)$	$2\pi^{-1}r^{-2}[1-pr^{-1}\log(R/a)]$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(47)	$t\,Y_1(at)$	$-2\pi^{-1}r^{-2}[p\,a^{-1}+ar^{-1}\log(R/a)]$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(48)	$t^\mu Y_\nu(at)$ $\quad\mathrm{Re}(\mu\pm\nu)>-1$	$r^{-\mu-1}[\Gamma(\mu+\nu+1)\,\mathrm{ctn}(\nu\pi)P_\mu^{-\nu}(p/r)$ $-\Gamma(\mu-\nu+1)\csc(\nu\pi)P_\mu^{\nu}(p/r)]$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(49)	$t^{-1/2}Y_{2\nu}(2a^{1/2}t^{1/2})$ $\quad\lvert\mathrm{Re}\,\nu\rvert<\tfrac12$	$-\pi^{1/2}p^{-1/2}e^{-\frac12 a/p}[\tan(\nu\pi)\,I_\nu(\tfrac12 a/p)$ $+\pi^{-1}\sec(\nu\pi)\,K_\nu(\tfrac12 a/p)]$ $\mathrm{Re}\,p>0$

$r=(p^2+a^2)^{1/2},\qquad R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(50)	$t^{\mu-\frac{1}{2}}\, Y_{2\nu}(2a^{\frac{1}{2}} t^{\frac{1}{2}})$ $\quad \mathrm{Re}(\mu \pm \nu) > -\frac{1}{2}$	$a^{-\frac{1}{2}}\, p^{-\mu} e^{-\frac{1}{2}a/p} \times \left\{ \dfrac{\tan[(\mu-\nu)\pi]\,\Gamma(\mu+\nu+\frac{1}{2})}{\Gamma(2\nu+1)} M_{\mu,\nu}(a/p) - \sec[(\mu-\nu)\pi]\, W_{\mu,\nu}(a/p) \right\}$ $\quad \mathrm{Re}\, p > 0$
(51)	$t^{-1}\{[at^{\frac{1}{2}} J_1(t^{\frac{1}{2}}) + b\, J_0(t^{\frac{1}{2}})]^2 + [at^{\frac{1}{2}} Y_1(t^{\frac{1}{2}}) + b\, Y_0(t^{\frac{1}{2}})]^2\}^{-1}$	$2\, I(a, b; p)$ $\quad \mathrm{Re}\, p > 0$ For expansions of $I(a, b; p)$ and a short numerical table of $I(0,1;p)$ cf. Jaeger, J. C., and Martha Clarke 1942: *Proc. Roy. Soc. Edinburgh Sect. A* 61, 229-230.
(52)	$H_0^{(1)}(at)$	$r^{-1} - 2i\pi^{-1} r^{-1} \sinh^{-1}(p/a)$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$
(53)	$H_0^{(2)}(at)$	$r^{-1} + 2i\pi^{-1} r^{-1} \sinh^{-1}(p/a)$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$
(54)	$H_\nu^{(1)}(at)$ $\quad \lvert \mathrm{Re}\,\nu\rvert < 1$	$ir^{-1} \csc(\nu\pi)(e^{-i\nu\pi} a^\nu R^{-\nu} - a^{-\nu} R^\nu)$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$
(55)	$H_\nu^{(2)}(at)$ $\quad \lvert \mathrm{Re}\,\nu\rvert < 1$	$ir^{-1} \csc(\nu\pi)(a^{-\nu} R^\nu - e^{i\nu\pi} a^\nu R^{-\nu})$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$
(56)	$t\, H_0^{(1)}(at)$	$\dfrac{p}{r^3}\left(1 - \dfrac{2i}{\pi} \log \dfrac{R}{a}\right) + \dfrac{2i}{\pi r^2}$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$
(57)	$t\, H_0^{(2)}(at)$	$\dfrac{p}{r^3}\left(1 + \dfrac{2i}{\pi} \log \dfrac{R}{a}\right) - \dfrac{2i}{\pi r^2}$ $\quad \mathrm{Re}\, p > \lvert \mathrm{Im}\, a\rvert$

$r = (p^2 + a^2)^{\frac{1}{2}}, \quad R = p + r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(58)	$t\,H_1^{(1)}(at)$	$\frac{a}{r^3}\left(1-\frac{2i}{\pi}\log\frac{R}{a}\right)-\frac{2ip}{\pi a r^2}$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(59)	$t\,H_1^{(2)}(at)$	$\frac{a}{r^3}\left(1+\frac{2i}{\pi}\log\frac{R}{a}\right)+\frac{2ip}{\pi a r^2}$ $\quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(60)	$t^{-1/2}\,H_{2\nu}^{(1)}(2a^{1/2}t^{1/2})\quad \lvert\mathrm{Re}\,\nu\rvert<1/2$	$\pi^{1/2}p^{-1/2}\sec(\nu\pi)\,e^{-1/2\,a/p}$ $\times[e^{-i\nu\pi}I_\nu(1/2\,a/p)$ $-i\pi^{-1}K_\nu(1/2\,a/p)]\quad \mathrm{Re}\,p>0$
(61)	$t^{-1/2}\,H_{2\nu}^{(2)}(2a^{1/2}t^{1/2})\quad \lvert\mathrm{Re}\,\nu\rvert<1/2$	$\pi^{1/2}p^{-1/2}e^{-1/2\,a/p}\sec(\nu\pi)$ $\times[e^{i\nu\pi}I_\nu(1/2\,a/p)$ $+i\pi^{-1}K_\nu(1/2\,a/p)]\quad \mathrm{Re}\,p>0$
(62)	$t^{-1/2\nu}\,H_\nu^{(1)}(2a^{1/2}t^{1/2})\quad \mathrm{Re}\,\nu<1$	$\frac{p^{\nu-1}e^{-a/p}}{\pi i\,a^{1/2\nu}}\Gamma(1-\nu)\Gamma(\nu,e^{-i\pi}a/p)$ $\quad \mathrm{Re}\,p>0$
(63)	$t^{-1/2\nu}\,H_\nu^{(2)}(2a^{1/2}t^{1/2})\quad \mathrm{Re}\,\nu<1$	$-\frac{p^{\nu-1}e^{-a/p}}{\pi i\,a^{1/2\nu}}\Gamma(1-\nu)\Gamma(\nu,e^{i\pi}a/p)$ $\quad \mathrm{Re}\,p>0$
(64)	$t^{\nu-1/2}\,H_1^{(1)}(2a^{1/2}t^{1/2})\quad \mathrm{Re}\,\nu>0$	$\frac{\Gamma(\nu+1)}{ip^\nu\sin(\nu\pi)}\;e^{-1/2\,a/p}\,k_{-2\nu}\left(\frac{ae^{-\pi i}}{2p}\right)$ $\quad \mathrm{Re}\,p>0$
(65)	$t^{\nu-1/2}\,H_1^{(2)}(2a^{1/2}t^{1/2})\quad \mathrm{Re}\,\nu>0$	$\frac{i\,\Gamma(\nu+1)}{p^\nu\sin(\nu\pi)}\;e^{-1/2\,a/p}\,k_{-2\nu}\left(\frac{ae^{\pi i}}{2p}\right)$ $\quad \mathrm{Re}\,p>0$

$r=(p^2+a^2)^{1/2},\quad R=p+r$

Bessel functions of kt and $kt^{1/2}$ (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(66)	$t^{\mu-1/2} H^{(1)}_{2\nu}(2a^{1/2} t^{1/2})$ $\operatorname{Re}(\mu \pm \nu) > -1/2$	$\dfrac{\Gamma(\mu+\nu+1/2)\Gamma(\mu-\nu+1/2)}{\pi a^{1/2} e^{\nu\pi i+1/2 a/p} p^{\mu}} \times W_{-\mu,\nu}(e^{-i\pi} a/p)$ $\operatorname{Re} p > 0$
(67)	$t^{\mu-1/2} H^{(2)}_{2\nu}(2a^{1/2} t^{1/2})$ $\operatorname{Re}(\mu \pm \nu) > -1/2$	$\dfrac{\Gamma(\mu+\nu+1/2)\Gamma(\mu-\nu+1/2)}{\pi a^{1/2} e^{-\nu\pi i+1/2 a/p} p^{\mu}} \times W_{-\mu,\nu}(e^{i\pi} a/p)$ $\operatorname{Re} p > 0$

4.15. Bessel functions of other arguments

(1)	$J_{\nu+1/2}(1/2 t^2)$ $\operatorname{Re}\nu > -1$	$\pi^{-1/2}\Gamma(\nu+1) D_{-\nu-1}(pe^{1/4\pi i}) \times D_{-\nu-1}(pe^{-1/4\pi i})$ $\operatorname{Re} p > 0$
(2)	$t^{1/2} J_{1/4}(at^2)$ $a > 0$	$\dfrac{\pi^{1/2} p^{1/2}}{4a}\left[\mathbf{H}_{-1/4}\left(\dfrac{p^2}{4a}\right) - Y_{-1/4}\left(\dfrac{p^2}{4a}\right)\right]$ $\operatorname{Re} p > 0$
(3)	$t^{1/2} J_{-1/4}(at^2)$ $a > 0$	$\dfrac{\pi^{1/2} p^{1/2}}{4a}\left[\mathbf{H}_{1/4}\left(\dfrac{p^2}{4a}\right) - Y_{1/4}\left(\dfrac{p^2}{4a}\right)\right]$ $\operatorname{Re} p > 0$
(4)	$t^{3/2} J_{-1/4}(at^2)$ $a > 0$	$-\dfrac{\pi^{1/2} p^{3/2}}{8a^2}\left[\mathbf{H}_{-1/4}\left(\dfrac{p^2}{4a}\right) - Y_{-1/4}\left(\dfrac{p^2}{4a}\right)\right]$ $\operatorname{Re} p > 0$
(5)	$t^{3/2} J_{-1/4}(at^2)$ $a > 0$	$\dfrac{\pi^{1/2} p^{3/2}}{8a^2}\left[\mathbf{H}_{-1/4}\left(\dfrac{p^2}{4a}\right) - Y_{-1/4}\left(\dfrac{p^2}{4a}\right)\right]$ $\operatorname{Re} p > 0$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(6)	$t^{1/2} J_{1/8}(t^2/16)\, J_{-1/8}(t^2/16)$	$2^{-1/2} \pi^{1/2} p^{1/2} \sec(\pi/8)$ $\times H^{(1)}_{1/8}(p^2)\, H^{(2)}_{1/8}(p^2)$ Re $p>0$
(7)	$t^{1/2} J_{\nu+1/8}(t^2/16) J_{\nu-1/8}(t^2/16)$ Re $\nu > -3/8$	$2^{1/2}(\pi p)^{-3/2}\Gamma(\nu+3/8)\Gamma(\nu+5/8)$ $\times W_{-\nu,\,1/8}(2e^{\pi i/2}p^2)$ $\times W_{-\nu,\,1/8}(2e^{-\pi i/2}p^2)$ Re $p>0$
(8)	$t^{-1} J_\nu(t^{-1})$	$2 J_\nu(2^{1/2} p^{1/2})\, K_\nu(2^{1/2} p^{1/2})$ Re $p>0$
(9)	0 $\quad 0<t<b$ $J_0(a\gamma)$ $\quad t>b$	$r^{-1} e^{-br}$ Re $p > \lvert \mathrm{Im}\, a \rvert$
(10)	0 $\quad 0<t<b$ $t\, J_0(a\gamma)$ $\quad t>b$	$pr^{-3}(br+1)e^{-br}$ Re $p > \lvert \mathrm{Im}\, a \rvert$
(11)	0 $\quad 0<t<b$ $\dfrac{J_0(a\gamma)}{t-\lambda}$ $\quad t>b$ $\lvert \arg(b-\lambda)\rvert < \pi$	$-e^{-br}\int_0^\infty e^{-u}\,[u^2-2(\lambda p-br)u$ $+\lambda^2(r-p)^2]^{-1/2}\,du$ Re $p > \lvert \mathrm{Im}\, a \rvert$
(12)	0 $\quad 0<t<b$ $\gamma\, J_1(a\gamma)$ $\quad t>b$	$ar^{-3}(br+1)e^{-br}$ Re $p > \lvert \mathrm{Im}\, a \rvert$
(13)	0 $\quad 0<t<b$ $\gamma^{-1} J_1(a\gamma)$ $\quad t>b$	$a^{-1}b^{-1}(e^{-bp}-e^{-br})$ Re $p > \lvert \mathrm{Im}\, a \rvert$
(14)	0 $\quad 0<t<b$ $\gamma^{-1} J_\nu(a\gamma)$ $\quad t>b$ Re $\nu > -1$	$I_{\frac{1}{2}\nu}[\tfrac{1}{2}b(r-p)]\, K_{\frac{1}{2}\nu}[\tfrac{1}{2}b(r+p)]$ Re $p > \lvert \mathrm{Im}\, a \rvert$

$\gamma = (t^2-b^2)^{1/2}, \quad r = (p^2+a^2)^{1/2}, \quad R = p + r$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(15)	$0 \qquad 0<t<b$ $ty^{-1} J_1(ay) \qquad t>b$	$a^{-1} e^{-bp} - a^{-1} pr^{-1} e^{-br}$ $\operatorname{Re} p > \lvert \operatorname{Im} a \rvert$
(16)	$0 \qquad 0<t<b$ $(t-b)^{\frac{1}{2}\nu} (t+b)^{-\frac{1}{2}\nu} J_\nu(ay)$ $t>b$ $\operatorname{Re} \nu > -1$	$a^\nu r^{-1} R^{-\nu} e^{-br} \qquad \operatorname{Re} p > \lvert \operatorname{Im} a \rvert$
(17)	$0 \qquad 0<t<b$ $y^\nu J_\nu(ay) \qquad t>b$ $\operatorname{Re} \nu > -1$	$2^{\frac{1}{2}} \pi^{-\frac{1}{2}} a^\nu b^{\nu+\frac{1}{2}} r^{-\nu-\frac{1}{2}} K_{\nu+\frac{1}{2}}(br)$ $\operatorname{Re} p > \lvert \operatorname{Im} a \rvert$
(18)	$0 \qquad 0<t<b$ $y^{2\mu} J_{2\nu}(ay) \qquad t>b$ $\operatorname{Re}(\mu+\nu) > -1$	$\sum_{n=0}^{\infty} \frac{(-1)^n (ab)^{2\nu+2n} (2b)^{2\mu+1} \Gamma(\mu+\nu+n+1)}{\pi^{\frac{1}{2}} n! \Gamma(2\nu+n+1) (2p)^{\mu+\nu+n+\frac{1}{2}}}$ $\times K_{\mu+\nu+n+\frac{1}{2}}(bp) \qquad \operatorname{Re} p > \lvert \operatorname{Im} a \rvert$
(19)	$J_0[a(t^2+\beta t)^{\frac{1}{2}}] \qquad \lvert \arg\beta \rvert < \pi$	$r^{-1} e^{\frac{1}{2}\beta(p-r)} \qquad \operatorname{Re} p > \lvert \operatorname{Im} a \rvert$
(20)	$(t^2+\beta t)^{\frac{1}{2}\nu} J_\nu[a(t^2+\beta t)^{\frac{1}{2}}]$ $\operatorname{Re} \nu > -1, \quad \lvert \arg\beta \rvert < \pi$	$\pi^{-\frac{1}{2}} (\frac{1}{2}a)^\nu (\beta/r)^{\nu+\frac{1}{2}} e^{\frac{1}{2}\beta p} K_{\nu+\frac{1}{2}}(\frac{1}{2}\beta r)$ $\operatorname{Re} p > \lvert \operatorname{Im} a \rvert$
(21)	$t^{\frac{1}{2}\nu} (t+\beta)^{-\frac{1}{2}\nu} J_\nu[a(t^2+\beta t)^{\frac{1}{2}}]$ $\operatorname{Re} \nu > -1, \quad \lvert \arg\beta \rvert < \pi$	$a^\nu r^{-1} R^{-\nu} e^{\frac{1}{2}\beta(p-r)} \qquad \operatorname{Re} p > \lvert \operatorname{Im} a \rvert$
(22)	$t^{\frac{1}{2}\nu-1} (t+1)^{-\frac{1}{2}\nu} J_\nu[a(t^2+t)^{\frac{1}{2}}]$ $\operatorname{Re} \nu > 0$	$2^\nu a^{-\nu} \gamma(\nu, \frac{1}{2}r - \frac{1}{2}p) \qquad \operatorname{Re} p > \lvert \operatorname{Im} a \rvert$

$y = (t^2 - b^2)^{\frac{1}{2}}, \quad r = (p^2 + a^2)^{\frac{1}{2}}, \quad R = p + r$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(22)	$t^{\lambda-\frac{1}{2}\nu-1}(t+1)^{-\frac{1}{2}\nu} J_\nu[a(t^2+t)^{\frac{1}{2}}]$ $\mathrm{Re}\,\nu+1>\mathrm{Re}\,\lambda>0$	$\dfrac{2^\nu a^{-\nu}}{\Gamma(\nu-\lambda+1)}$ $\times\int_0^{\frac{1}{2}r-\frac{1}{2}p} e^{-u} u^{\lambda-1}(\tfrac{1}{4}a^2-pu-u^2)^{\nu-\lambda}\,du$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(23)	$(t^2+2it)^{\frac{1}{2}\nu} J_\nu[a(t^2+2it)^{\frac{1}{2}}]$ $\mathrm{Re}\,\nu>-1$	$-i\,2^{-\frac{1}{2}}\pi^{\frac{1}{2}} a^\nu r^{-\nu-\frac{1}{2}} e^{ip} H^{(2)}_{\nu+\frac{1}{2}}(r)$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(24)	$(t^2+2it)^{\lambda-\frac{1}{2}\nu} J_\nu[a(t^2+2it)^{\frac{1}{2}}]$ $\mathrm{Re}\,\lambda>-1$	$\dfrac{2^{\lambda-\nu-\frac{1}{2}}\pi^{\frac{1}{2}} e^{ip}\Gamma(\lambda+1)}{ir^{\lambda+\frac{1}{2}}\Gamma(\nu-\lambda)}$ $\times\sum_{n=0}^{\infty}\dfrac{\Gamma(\nu-\lambda+n)}{2^n n!\,\Gamma(\nu+n+1)\,r^n} H^{(2)}_{\lambda+n+\frac{1}{2}}(r)$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(25)	$\exp[ia(1-e^{-t})]\,J_\nu(ae^{-t})$	$\dfrac{J_\nu(a)}{\nu+p}+2\sum_{n=1}^{\infty} i^n \dfrac{(\nu-p+1)_{n-1}}{(\nu+p)_{n+1}}(\nu+n)$ $\times J_{\nu+n}(a)$ $\quad\mathrm{Re}\,p>-\mathrm{Re}\,\nu$
(26)	$\sin[a(1-e^{-t})]\,J_\nu(ae^{-t})$	$2\sum_{n=0}^{\infty}\dfrac{(-1)^n(\nu-p+1)_{2n}}{(\nu+p)_{2n+2}}(\nu+2n-1)$ $\times J_{\nu+2n+1}(a)$ $\quad\mathrm{Re}\,p>-\mathrm{Re}\,\nu$
(27)	$\cos[a(1-e^{-t})]J_\nu(ae^{-t})$	$\dfrac{J_\nu(a)}{\nu+p}+\sum_{n=0}^{\infty} 2(-1)^n\dfrac{(\nu-p+1)_{2n-1}}{(\nu+p)_{2n+1}}$ $\times(\nu+2n)\,J_{\nu+2n}(a)$ $\mathrm{Re}\,p>-\mathrm{Re}\,\nu$

$y=(t^2-b^2)^{\frac{1}{2}}$, $\quad r=(p^2+a^2)^{\frac{1}{2}}$, $\quad R=p+r$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(28)	$J_\mu(ae^{-t})\, J_\nu[a(1-e^{-t})]$ $\quad \mathrm{Re}\,\nu>-1$	$\left(\frac{2}{a}\right)^p \sum_{n=0}^{\infty} \frac{(-1)^n\,\Gamma(p+n)}{(\mu+n)\,n!\,\mathrm{B}(p,\mu+n)}$ $\times J_{\mu+\nu+p+2n}(a) \quad \mathrm{Re}\,p>-\mathrm{Re}\,\mu$
(29)	$(1-e^{-t})^{\frac{1}{2}\nu}\, J_\nu[a(1-e^{-t})^{\frac{1}{2}}]$ $\quad \mathrm{Re}\,\nu>-1$	$\Gamma(p)\,(2/a)^p\, J_{\nu+p}(a) \quad \mathrm{Re}\,p>0$
(30)	$(1-e^{-t})^{-\frac{1}{2}\nu}\, J_\nu[a(1-e^{-t})^{\frac{1}{2}}]$	$\frac{s_{\nu+p-1,\,p-\nu}(a)}{2^\nu\, a^p\, \Gamma(\nu)} \quad \mathrm{Re}\,p>0$
(31)	$(e^t-1)^{\frac{1}{2}\nu}\, J_\nu[2a(e^t-1)^{\frac{1}{2}}]$ $\quad a>0,\quad \mathrm{Re}\,\nu>-1$	$\frac{2a^p}{\Gamma(p+1)} K_{\nu-p}(2a)$ $\quad \mathrm{Re}\,p>\frac{1}{2}\mathrm{Re}\,\nu-\frac{3}{4}$
(32)	$(e^t-1)^\mu\, J_{2\nu}[2a(e^t-1)^{\frac{1}{2}}]$ $\quad a>0,\quad \mathrm{Re}(\mu+\nu)>-1$	$\frac{a^{2\nu}\,\mathrm{B}(\mu+\nu+1,\,p-\mu-\nu)}{\Gamma(2\nu+1)}$ $\times\, {}_1F_2(\mu+\nu+1;\,\mu+\nu+1-p;\,2\nu+1;\,a^2)$ $+\frac{a^{2p-2\mu}\,\Gamma(\mu+\nu-p)}{\Gamma(\nu-\mu+p+1)}$ $\times\, {}_1F_2(p+1;\,p+1+\nu-\mu,\,p+1-\mu-\nu;\,a^2)$ $\quad \mathrm{Re}\,p>\mathrm{Re}\,\mu-7/4$
(33)	$J_\nu(2a\sinh t)$ $\quad \mathrm{Re}\,\nu>-1,\quad a>0$	$I_{\frac{1}{2}\nu+\frac{1}{2}p}(a)\, K_{\frac{1}{2}\nu-\frac{1}{2}p}(a) \quad \mathrm{Re}\,p>-\frac{1}{2}$
(34)	$\mathrm{csch}(t)\, J_\nu(a\,\mathrm{csch}\,t) \quad a>0$	$\frac{\Gamma(\frac{1}{2}p+\frac{1}{2}\nu+\frac{1}{2})}{a\,\Gamma(\nu+1)} W_{-\frac{1}{2}p,\,\frac{1}{2}\nu}(a)$ $\times M_{\frac{1}{2}p,\,\frac{1}{2}\nu}(a) \quad \mathrm{Re}\,p>-\mathrm{Re}\,\nu-1$

Bessel functions of other arguments (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(35)	$\csc(\tfrac{1}{2}t) \exp\left(\frac{\alpha-\beta e^{t}}{e^{t}-1}\right) \times J_{2\nu}\left[\frac{\alpha^{1/2}\beta^{1/2}}{\sinh(\tfrac{1}{2}t)}\right]$ $\mathrm{Re}\, \alpha>0, \quad \mathrm{Re}\, \beta>0$	$\frac{2\Gamma(p+\nu+\tfrac{1}{2})}{\alpha^{1/2}\beta^{1/2}\Gamma(2\nu+1)} e^{-\frac{1}{2}(\alpha+\beta)} \times W_{-p,\nu}(\beta) M_{p,\nu}(\alpha)$ $\mathrm{Re}\, p>-\mathrm{Re}\, \nu-\tfrac{1}{2}$
(36)	$t^{-1} Y_{\nu}(t^{-1})$	$2Y_{\nu}(2^{1/2}p^{1/2}) K_{\nu}(2^{1/2}p^{1/2}) \qquad \mathrm{Re}\, p>0$
(37)	$t^{-1} H_{\nu}^{(1)}(t^{-1})$	$2H_{\nu}^{(1)}(2^{1/2}p^{1/2}) K_{\nu}(2^{1/2}p^{1/2}) \qquad \mathrm{Re}\, p>0$
(38)	$t^{-1} H_{\nu}^{(2)}(t^{-1})$	$2H_{\nu}^{(2)}(2^{1/2}p^{1/2}) K_{\nu}(2^{1/2}p^{1/2}) \qquad \mathrm{Re}\, p>0$

4.16. Modified Bessel functions of arguments kt and $kt^{1/2}$

(1)	$I_{\nu}(at) \qquad \mathrm{Re}\, \nu>-1$	$a^{\nu} s^{-1} S^{-\nu} \qquad \mathrm{Re}\, p>\lvert\mathrm{Re}\, a\rvert$
(2)	$t\, I_{\nu}(at) \qquad \mathrm{Re}\, \nu>-2$	$a^{\nu}(p+\nu s) s^{-3} S^{-\nu} \qquad \mathrm{Re}\, p>\lvert\mathrm{Re}\, a\rvert$
(3)	$t^{-1} I_{1}(at)$	$\frac{(p+a)^{1/2}-(p-a)^{1/2}}{(p+a)^{1/2}+(p-a)^{1/2}} \qquad \mathrm{Re}\, p>\lvert\mathrm{Re}\, a\rvert$
(4)	$t^{-1} I_{\nu}(at) \qquad \mathrm{Re}\, \nu>0$	$\nu^{-1} a^{\nu} S^{-\nu} \qquad \mathrm{Re}\, p>\lvert\mathrm{Re}\, a\rvert$
(5)	$t^{-1/2} I_{\nu}(t) \qquad \mathrm{Re}\, \nu>-\tfrac{1}{2}$	$2^{1/2}\pi^{-1/2} Q_{\nu-1/2}(p) \qquad \mathrm{Re}\, p>1$
(6)	$t^{\nu} I_{\nu}(at) \qquad \mathrm{Re}\, \nu>-\tfrac{1}{2}$	$2^{\nu}\pi^{-1/2}\Gamma(\nu+\tfrac{1}{2}) a^{\nu} s^{-2\nu-1} \qquad \mathrm{Re}\, p>\lvert\mathrm{Re}\, a\rvert$

$s=(p^2-a^2)^{1/2}, \qquad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(7)	$t^{\nu+1} I_\nu(at)$ $\quad$ $\operatorname{Re}\nu>-1$	$2^{\nu+1}\pi^{-1/2}\Gamma(\nu+3/2)\,a^\nu p s^{-2\nu-3}$ $\quad$ $\operatorname{Re}p>\lvert\operatorname{Re}a\rvert$
(8)	$t^\mu I_\nu(at)$ $\quad$ $\operatorname{Re}(\mu+\nu)>-1$	$\Gamma(\mu+\nu+1)\,s^{-\mu-1}P_\mu^{-\nu}(p/s)$ $\quad$ $\operatorname{Re}p>\lvert\operatorname{Re}a\rvert$
(9)	$t^{\mu-1/2} I_{\nu+1/2}(at)$ $\quad$ $\operatorname{Re}(\mu+\nu)>-1$	$\dfrac{2^{1/2}\sin(\nu\pi)\,s^{-\mu}}{\pi^{1/2}a^{1/2}\sin[(\mu+\nu)\pi]}\,Q_\nu^\mu\left(\dfrac{p}{a}\right)$ $\quad$ $\operatorname{Re}p>\lvert\operatorname{Re}a\rvert$
(10)	$I_0^2(\tfrac{1}{2}at)$	$2\pi^{-1}p^{-1}E(a/p)$ $\quad$ $\operatorname{Re}p>\lvert\operatorname{Re}a\rvert$
(11)	$t\,I_0(\tfrac{1}{2}at)\,I_1(\tfrac{1}{2}at)$	$\dfrac{2p\,E(a/p)}{\pi a(p^2-a^2)}-\dfrac{2K(a/p)}{\pi a p}$ $\quad$ $\operatorname{Re}p>\lvert\operatorname{Re}a\rvert$
(12)	$t^{-1/2} I_\mu(at)\,I_\nu(bt)$ $\quad$ $\operatorname{Re}(\mu+\nu)>-\tfrac{1}{2}$	$c^{1/2}\Gamma(\mu+\nu+\tfrac{1}{2})\,P_{\nu-1/2}^{-\mu}(\cosh\alpha)$ $\times P_{\mu-1/2}^{-\nu}(\cosh\beta)$ $\quad$ $\operatorname{Re}(p\pm a\pm b)>0$ where $\sinh\alpha=ac,\quad \sinh\beta=bc,$ $\cosh\alpha\cosh\beta=pc,$ $\lvert\operatorname{Im}\alpha\rvert,\quad \lvert\operatorname{Im}\beta\rvert<\tfrac{1}{2}\pi$
(13)	$t^{2\lambda-1} I_{2\mu}(\alpha t)\,I_{2\nu}(\beta t)$ $\quad$ $\operatorname{Re}(\lambda+\mu+\nu)>0$	$\dfrac{2^{2\lambda-1}\alpha^{2\mu}\beta^{2\nu}\Gamma(\lambda+\mu+\nu)\Gamma(\lambda+\mu+\nu+\tfrac{1}{2})}{\pi^{1/2}p^{2\lambda+2\mu+2\nu}\Gamma(2\mu+1)\Gamma(2\nu+1)}$ $\times F_4(\lambda+\mu+\nu,\lambda+\mu+\nu+\tfrac{1}{2};2\mu+1,$ $2\nu+1;\alpha^2/p^2,\beta^2/p^2)$ $\quad$ $\operatorname{Re}p>\lvert\operatorname{Re}\alpha\rvert+\lvert\operatorname{Re}\beta\rvert$

$s=(p^2-a^2)^{1/2},\quad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(14)	$I_0(2a^{1/2}t^{1/2})$	$p^{-1}e^{a/p}$ $\quad$ $\operatorname{Re} p>0$
(15)	$t^{-1/2}I_0(2^{3/2}a^{1/2}t^{1/2})$	$\pi^{1/2}p^{-1/2}e^{a/p}I_0(a/p)$ $\quad$ $\operatorname{Re} p>0$
(16)	$t^{-1/2}I_1(2a^{1/2}t^{1/2})$	$a^{-1/2}(e^{a/p}-1)$ $\quad$ $\operatorname{Re} p>0$
(17)	$t^{-1/2}I_\nu(2^{3/2}a^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}\nu>-1$	$\pi^{1/2}p^{-1/2}e^{a/p}I_{\frac{1}{2}\nu}(a/p)$ $\quad$ $\operatorname{Re} p>0$
(18)	$t^{\frac{1}{2}\nu}I_\nu(2a^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}\nu>-1$	$a^{\frac{1}{2}\nu}p^{-\nu-1}e^{a/p}$ $\quad$ $\operatorname{Re} p>0$
(19)	$t^{-\frac{1}{2}\nu}I_\nu(2a^{1/2}t^{1/2})$	$a^{-\frac{1}{2}\nu}[\Gamma(\nu)]^{-1}p^{\nu-1}e^{a/p}\gamma(\nu,a/p)$ $\quad$ $\operatorname{Re} p>0$
(20)	$t^{\mu-\frac{1}{2}}I_{2\nu}(2a^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}(\mu+\nu)>-\frac{1}{2}$	$\dfrac{\Gamma(\mu+\nu+\frac{1}{2})\,e^{\frac{1}{2}a/p}}{a^{1/2}\Gamma(2\nu+1)\,p^{\mu}}M_{-\mu,\nu}\left(\dfrac{a}{p}\right)$ $\quad$ $\operatorname{Re} p>0$
(21)	$I_\nu^2(2^{1/2}a^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}\nu>-1$	$p^{-1}e^{a/p}I_\nu(a/p)$ $\quad$ $\operatorname{Re} p>0$
(22)	$I_\nu(2^{1/2}a^{1/2}t^{1/2})\,I_\nu(2^{1/2}\beta^{1/2}t^{1/2})$ $\quad$ $\operatorname{Re}\nu>-1$	$p^{-1}\exp[\frac{1}{2}(a+\beta)/p]\,I_\nu(a^{1/2}\beta^{1/2}/p)$ $\quad$ $\operatorname{Re} p>0$
(23)	$K_0(at)$	$s^{-1}\log(S/a)=s^{-1}\sinh^{-1}(s/a)$ $\quad$ $\operatorname{Re} p>-\operatorname{Re} a$
(24)	$K_\nu(at)$ $\quad$ $\lvert\operatorname{Re}\nu\rvert<1$	$\frac{1}{2}\pi\csc(\nu\pi)s^{-1}[a^{-\nu}S^{\nu}-a^{\nu}S^{-\nu}]$ $\quad$ $\operatorname{Re} p>-\operatorname{Re} a$

$s=(p^2-a^2)^{1/2}, \quad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(25)	$tK_0(at)$	$ps^{-3}\log(S/a)-s^{-2}$ $\operatorname{Re}p>-\operatorname{Re}a$
(26)	$tK_1(at)$	$a^{-1}ps^{-2}-as^{-3}\log(S/a)$ $\operatorname{Re}p>-\operatorname{Re}a$
(27)	$t^{\mu-\frac{1}{2}}K_{\nu+\frac{1}{2}}(at)$ $\operatorname{Re}(\mu+\nu)>-1$ $\operatorname{Re}(\mu-\nu)>0$	$2^{-\frac{1}{2}}a^{-\frac{1}{2}}\pi^{\frac{1}{2}}\Gamma(\mu-\nu)\Gamma(\mu+\nu+1)$ $\times s^{-\mu}P_\nu^{-\mu}(p/a)$ $\operatorname{Re}p>-\operatorname{Re}a$
(28)	$t^\mu K_\nu(at)$ $\operatorname{Re}(\mu\pm\nu)>-1$	$\dfrac{\sin(\mu\pi)\,\Gamma(\mu-\nu+1)}{\sin[(\mu+\nu)\pi]\,s^{\mu+1}}\,Q_\mu^\nu\left(\dfrac{p}{s}\right)$ $\operatorname{Re}p>-\operatorname{Re}a$
(29)	$\dfrac{1}{2t}\exp\left(-\dfrac{\lambda}{2at}\right)K_\nu(a\lambda t)$ $\operatorname{Re}(\lambda/a)>0$	$K_\nu(a^{-\frac{1}{2}}\lambda^{\frac{1}{2}}S^{\frac{1}{2}})\,K_\nu(a^{\frac{1}{2}}\lambda^{\frac{1}{2}}S^{-\frac{1}{2}})$ $\operatorname{Re}p>-\operatorname{Re}(a\lambda)$
(30)	$t^{-\frac{1}{2}}I_\mu(at)\,K_\nu(bt)$ $\operatorname{Re}(\mu\pm\nu)>-\frac{1}{2}$	$\dfrac{c^{\frac{1}{2}}\Gamma(\mu-\nu+\frac{1}{2})\cos(\mu\pi)}{\cos(\mu+\nu)\pi}\,P_{\nu-\frac{1}{2}}^{-\mu}(\cosh\alpha)$ $\times Q_{\mu-\frac{1}{2}}^{-\nu}(\cosh\beta)$ $\operatorname{Re}(p\pm a+b)>0$ for definition of α, β, and c see (12) of this section.
(31)	$t^{-\frac{1}{2}}K_\mu(at)\,K_\nu(bt)$ $\lvert\operatorname{Re}\mu\rvert+\lvert\operatorname{Re}\nu\rvert<\frac{1}{2}$	$\dfrac{c^{\frac{1}{2}}\Gamma(\frac{1}{2}-\mu-\nu)\cos(\mu\pi)\cos(\nu\pi)}{\cos(\mu+\nu)\pi\,\cos(\mu-\nu)\,\pi}$ $\times Q_{\nu+\frac{1}{2}}^{-\mu}(\cosh\alpha)\,Q_{\mu-\frac{1}{2}}^{-\nu}(\cosh\beta)$ $\operatorname{Re}(p+a+b)>0$ α, β, c defined in (12).

$s=(p^2-a^2)^{\frac{1}{2}},\quad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(32)	$K_0(2a^{1/2} t^{1/2})$	$-\tfrac{1}{2}p^{-1} e^{a/p}\,\mathrm{Ei}(-a/p)$ $\quad \mathrm{Re}\, p>0$
(33)	$K_1(2^{3/2} a^{1/2} t^{1/2})$	$2^{-3/2} a^{1/2} \pi^{1/2} p^{-3/2} e^{a/p}$ $\times[K_1(a/p)-K_0(a/p)]$ $\quad \mathrm{Re}\, p>0$
(34)	$t^{-1/2} K_0(2^{3/2} a^{1/2} t^{1/2})$	$\tfrac{1}{2}\pi^{1/2} p^{-1/2} e^{a/p} K_0(a/p)$ $\quad \mathrm{Re}\, p>0$
(35)	$t^{-1/2} K_\nu(2^{3/2} a^{1/2} t^{1/2})$ $\quad \lvert\mathrm{Re}\,\nu\rvert<1$	$\tfrac{1}{2}\pi^{1/2} p^{-1/2} \sec(\tfrac{1}{2}\nu\pi)\, e^{a/p} K_{\frac{1}{2}\nu}(a/p)$ $\quad \mathrm{Re}\, p>0$
(36)	$t^{\frac{1}{2}\nu} K_\nu(2a^{1/2} t^{1/2})$ $\quad \mathrm{Re}\,\nu>-1$	$\tfrac{1}{2}a^{\frac{1}{2}\nu}\Gamma(\nu+1) p^{-\nu-1} e^{a/p}$ $\times\Gamma(-\nu, a/p)$ $\quad \mathrm{Re}\, p>0$
(37)	$t^{\mu-\frac{1}{2}} K_{2\nu}(2a^{1/2} t^{1/2})$ $\quad \mathrm{Re}(\mu\pm\nu)>-\tfrac{1}{2}$	$\dfrac{\Gamma(\mu+\nu+\frac{1}{2})\Gamma(\mu-\nu+\frac{1}{2})}{2a^{1/2} p^{\mu}} e^{\frac{1}{2}a/p}$ $\times W_{-\mu,\nu}(a/p)$ $\quad \mathrm{Re}\, p>0$
(38)	$t^{-1/2} K_{2\nu}(2^{1/2} a^{1/2} t^{1/2})$ $\times\{\sin[(\nu-\tfrac{1}{4})\pi] J_{2\nu}(2^{1/2} a^{1/2} t^{1/2})$ $+\cos[(\nu-\tfrac{1}{4})\pi] Y_{2\nu}(2^{1/2} a^{1/2} t^{1/2})\}$ $\quad \lvert\mathrm{Re}\,\nu\rvert<\tfrac{1}{4}$	$-2^{-3/2}\pi^{-1/2} p^{1/2} a^{-1}\Gamma(\tfrac{1}{4}+\nu)$ $\times\Gamma(\tfrac{1}{4}-\nu)\, W_{\frac{1}{4},\nu}(e^{\frac{1}{2}\pi i} a/p)$ $\times W_{\frac{1}{4},\nu}(e^{-\frac{1}{2}\pi i} a/p)$ $\quad \mathrm{Re}\, p>0$
(39)	$t^{2\nu} K_{2\nu}(t^{1/2})\, I_{2\nu}(t^{1/2})$ $\quad \mathrm{Re}\,\nu>-\tfrac{1}{4}$	$\tfrac{1}{2}\Gamma(2\nu+\tfrac{1}{2}) p^{-3\nu-\frac{1}{2}} e^{\frac{1}{2}p^{-1}}$ $\times W_{-\nu,\nu}(p^{-1})$ $\quad \mathrm{Re}\, p>0$

4.17. Modified Bessel functions of other arguments

(1)	$\exp\left(-\dfrac{t^2}{16a}\right) I_0\left(\dfrac{t^2}{16a}\right)$ $\quad \mathrm{Re}\, a\geq 0$	$\dfrac{2^{1/2} a^{1/2}}{\pi^{1/2}} e^{ap^2} K_0(ap^2)$ $\quad \mathrm{Re}\, p>0$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(2)	$t^{\frac{1}{2}}\exp\left(-\frac{t^2}{8a}\right)I_{\frac{1}{4}}\left(\frac{t^2}{8a}\right)$ $\quad$ $\operatorname{Re} a\geq 0$	$\frac{2}{\Gamma(\frac{1}{4})}\,\frac{a^{\frac{1}{2}}}{p^{\frac{1}{2}}}\,e^{ap^2}\,\Gamma(\tfrac{1}{4},ap^2)$ $\quad$ $\operatorname{Re} p>0$
(3)	$t^{2\nu}\exp\left(-\frac{t^2}{8a}\right)I_{\nu}\left(\frac{t^2}{8a}\right)$ $\quad$ $\operatorname{Re} a\geq 0,\quad \operatorname{Re}\nu>-\frac{1}{4}$	$\frac{a^{\frac{1}{2}\nu}\Gamma(4\nu+1)}{2^{4\nu}\Gamma(\nu+1)}\,p^{-\nu-1}\,e^{\frac{1}{2}ap^2}$ $\times W_{-3\nu/2,\,\nu/2}(ap^2)$ $\quad$ $\operatorname{Re} p>0$
(4)	$\frac{1}{t}\exp\left(-\frac{\alpha+\beta}{2t}\right)I_{\nu}\left(\frac{\alpha-\beta}{2t}\right)$ $\quad$ $\operatorname{Re}\alpha\geq\operatorname{Re}\beta>0$	$2K_{\nu}[(\alpha^{\frac{1}{2}}+\beta^{\frac{1}{2}})p^{\frac{1}{2}}]\,I_{\nu}[(\alpha^{\frac{1}{2}}-\beta^{\frac{1}{2}})p^{\frac{1}{2}}]$ $\quad$ $\operatorname{Re} p>0$
(5)	$0 \quad 0<t<b$ $I_0(a\gamma) \quad t>b$	$s^{-1}e^{-bs}$ $\quad$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(6)	$0 \quad 0<t<b$ $t\,I_0(a\gamma) \quad t>b$	$p(bs^{-2}+s^{-3})e^{-bs}$ $\quad$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(7)	$0 \quad 0<t<b$ $\gamma\,I_1(a\gamma) \quad t>b$	$a(bs^{-2}-s^{-3})e^{-bs}$ $\quad$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(8)	$0 \quad 0<t<b$ $\gamma^{-1}I_1(a\gamma) \quad t>b$	$a^{-1}b^{-1}(e^{-bs}-e^{-bp})$ $\quad$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(9)	$0 \quad 0<t<b$ $\gamma^{-1}t\,I_1(a\gamma) \quad t>b$	$a^{-1}ps^{-1}e^{-bs}-a^{-1}e^{-bp}$ $\quad$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$

$\gamma=(t^2-b^2)^{\frac{1}{2}} \qquad s=(p^2-a^2)^{\frac{1}{2}}, \qquad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(10)	$0 \qquad 0<t<b$ $y^{\nu} I_{\nu}(ay) \qquad t>b$ $\mathrm{Re}\,\nu>-1$	$2^{1/2}\pi^{-1/2} a^{\nu} b^{\nu+1/2} s^{-\nu-1/2} K_{\nu+1/2}(bs)$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(11)	$0 \qquad 0<t<b$ $(t-b)^{1/2\nu}(t+b)^{-1/2\nu} I_{\nu}(ay)$ $t>b$ $\mathrm{Re}\,\nu>-1$	$a^{\nu} s^{-1} S^{-\nu} e^{-bs} \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(12)	$I_0[a(t^2+\beta t)^{1/2}] \qquad \lvert\arg\beta\rvert<\pi$	$s^{-1} e^{1/2\beta(p-s)} \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(13)	$(t^2+\beta t)^{1/2\nu} I_{\nu}[a(t^2+\beta t)^{1/2}]$ $\mathrm{Re}\,\nu>-1, \quad \lvert\arg\beta\rvert<\pi$	$\pi^{-1/2}(\tfrac{1}{2}a)^{\nu}(\beta/s)^{\nu+1/2} e^{1/2\beta p}$ $\times K_{\nu+1/2}(\tfrac{1}{2}\beta s) \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(14)	$t^{1/2\nu}(t+\beta)^{-1/2\nu} I_{\nu}[a(t^2+\beta t)^{1/2}]$ $\mathrm{Re}\,\nu>-1, \quad \lvert\arg\beta\rvert<\pi$	$a^{\nu} s^{-1} S^{-\nu} e^{1/2\beta(p-s)} \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(15)	$t^{\mu-1}(t+\beta)^{-\mu} I_{2\nu}[a(t^2+\beta t)^{1/2}]$ $\mathrm{Re}(\mu+\nu)>0, \quad \lvert\arg\beta\rvert<\pi$	$\dfrac{2\Gamma(\mu+\nu)e^{1/2\beta p}}{a\beta\Gamma(2\nu+1)} M_{1/2-\mu,\nu}\left(\dfrac{a^2\beta}{2S}\right)$ $\times W_{1/2-\mu,\nu}\left(\dfrac{\beta S}{2}\right) \qquad \mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
	For more general formulas see MacRobert, T. M., 1948: *Philos. Mag.* (7) 39, pp. 466-471.	
(16)	$(2t-t^2)^{1/2\nu-1/4} C_n^{\nu}(t-1)$ $\times I_{\nu-1/2}[a(2t-t^2)^{1/2}]$ $0<t<2$ $0 \qquad t>2$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$(-1)^n \dfrac{2^{1/2}\pi^{1/2}a^{\nu-1/2}}{r^{\nu}e^{p}} C_n^{\nu}\left(\dfrac{p}{r}\right) I_{\nu+n}(r)$ $r=(p^2+a^2)^{1/2}$

$y=(t^2-b^2)^{1/2}, \quad s=(p^2-a^2)^{1/2}, \quad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(17)	$\exp[a(1-e^{-t})]\,I_\nu(ae^{-t})$	$\dfrac{I_\nu(a)}{\nu+p}+\sum_{n=1}^{\infty}\dfrac{(\nu-p+1)_{n-1}}{(\nu+p)_{n+1}}(\nu+n)$ $\times I_{\nu+p}(a)$ Re $p>-$ Re ν
(18)	$t^{-1/2}e^{-a/t}K_\nu(a/t)$ Re $a>0$	$2\pi^{1/2}p^{-1/2}K_{2\nu}(2^{3/2}a^{1/2}p^{1/2})$ Re $p>0$
(19)	$\dfrac{1}{t}\exp\left(-\dfrac{a+\beta}{2t}\right)K_\nu\left(\dfrac{a-\beta}{2t}\right)$ Re $a>$ Re $\beta>0$	$2K_\nu[(a^{1/2}+\beta^{1/2})p^{1/2}]K_\nu[(a^{1/2}-\beta^{1/2})p^{1/2}]$ Re $p>0$
(20)	$t^{\mu-1}(t+\beta)^{-\mu}K_{2\nu}[a(t^2+\beta t)^{1/2}]$ Re$(\mu\pm\nu)>0$, $\lvert\arg\beta\rvert<\pi$	$a^{-1}\beta^{-1}\Gamma(\mu+\nu)\Gamma(\mu-\nu)e^{\frac{1}{2}\beta p}$ $\times W_{\frac{1}{2}-\mu,\nu}(\tfrac{1}{2}a^2\beta S^{-1})$ $\times W_{\frac{1}{2}-\mu,\nu}(\tfrac{1}{2}\beta S)$ Re $p>\lvert$Re $a\rvert$
(21)	$-2\pi^{-1}K_0[2a\sinh(\tfrac{1}{2}t)]$ Re $a>0$	$J_p(a)\dfrac{\partial Y_p(a)}{\partial p}-Y_p(a)\dfrac{\partial J_p(a)}{\partial p}$
(22)	$-2\pi^{-1}\cosh t\,K_0[2a\sinh(\tfrac{1}{2}t)]$ Re $a>0$	$J'_p(a)\dfrac{\partial Y'_p(a)}{\partial p}-Y'_p(a)\dfrac{\partial J'_p(a)}{\partial p}$ $+\dfrac{p^2}{a^2}\left[J_p(a)\dfrac{\partial Y_p(a)}{\partial p}\right.$ $\left.-Y_p(a)\dfrac{\partial J_p(a)}{\partial p}\right]$ $\left[J'_p=\dfrac{dJ_p}{da}\right]$
(23)	$2\pi^{-2}\sin(2\nu\pi)$ $\times K_{2\nu}[2a\sinh(\tfrac{1}{2}t)]$ Re $a>0$	$J_{\nu-p}(a)Y_{-\nu-p}(a)-J_{-\nu-p}(a)Y_{\nu-p}(a)$

$\gamma=(t^2-b^2)^{1/2},\quad s=(p^2-a^2)^{1/2},\quad S=p+s$

Modified Bessel functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(24)	$\operatorname{csch}(\tfrac{1}{2}t)\,K_{2\nu}[a\operatorname{csch}(\tfrac{1}{2}t)]$ $\quad \operatorname{Re} a>0$	$a^{-1}\,\Gamma(p+\nu+\tfrac{1}{2})\Gamma(p-\nu+\tfrac{1}{2})$ $\times W_{-p,\nu}(ia)\,W_{-p,\nu}(-ia)$ $\quad \operatorname{Re}(p\pm\nu)>-1$
(25)	$\dfrac{1}{\sinh(\tfrac{1}{2}t)}\exp\left(-\dfrac{ae^t+\beta}{e^t-1}\right)$ $\times K_{2\nu}\left[\dfrac{a^{1/2}\beta^{1/2}}{\sinh(\tfrac{1}{2}t)}\right]$ $\quad \operatorname{Re} a>0,\quad \operatorname{Re}\beta>0$	$\dfrac{1}{a^{1/2}\beta^{1/2}}\,\Gamma(p+\nu+\tfrac{1}{2})\Gamma(p-\nu+\tfrac{1}{2})$ $\times e^{-\frac{1}{2}a+\frac{1}{2}\beta}\,W_{-p,\nu}(a)\,W_{-p,\nu}(\beta)$ $\quad \operatorname{Re}(p\pm\nu)>-\tfrac{1}{2}$

4.18. Kelvin's functions and related functions

(1)	$\operatorname{ber} t$	$[\tfrac{1}{2}(p^4+1)^{-1/2}+\tfrac{1}{2}p^2(p^4+1)^{-1}]^{1/2}$ $\quad \operatorname{Re} p>2^{-1/2}$
(2)	$\operatorname{bei} t$	$[\tfrac{1}{2}(p^4+1)^{-1/2}-\tfrac{1}{2}p^2(p^4+1)^{-1}]^{1/2}$ $\quad \operatorname{Re} p>2^{-1/2}$
(3)	$\operatorname{ber}(2t^{1/2})$	$p^{-1}\cos p^{-1}$ $\quad \operatorname{Re} p>0$
(4)	$\operatorname{bei}(2t^{1/2})$	$p^{-1}\sin p^{-1}$ $\quad \operatorname{Re} p>0$
(5)	$t^{\frac{1}{2}\nu}\operatorname{ber}_\nu(t^{1/2})$ $\quad \operatorname{Re}\nu>-1$	$2^{-\nu}p^{-\nu-1}\cos[\tfrac{1}{4}(1+3\nu\pi p)/p]$ $\quad \operatorname{Re} p>0$
(6)	$t^{\frac{1}{2}\nu}\operatorname{bei}_\nu(t^{1/2})$ $\quad \operatorname{Re}\nu>-1$	$2^{-\nu}p^{-\nu-1}\sin[\tfrac{1}{4}(1+3\nu\pi p)/p]$ $\quad \operatorname{Re} p>0$
(7)	$0 \quad 0<t<b$ $\operatorname{ber}(a\gamma)+i\operatorname{bei}(a\gamma) \quad t>b$	$v^{-1}e^{-bv}$ $\quad \operatorname{Re}(p\pm ai^{1/2})>0$

$\gamma=(t^2-b^2)^{1/2},\quad v=(p^2-ia^2)^{1/2},\quad V=p+v$

Kelvin's functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(8)	$0 \qquad 0<t<b$ $t[\operatorname{ber}(ay)+i\operatorname{bei}(ay)] \qquad t>b$	$pv^{-3}(bv+1)e^{-bv}$ $\operatorname{Re}(p \pm ai^{1/2})>0$
(9)	$0 \qquad 0<t<b$ $y[\operatorname{ber}_1(ay)+i\operatorname{bei}_1(ay)] \qquad t>b$	$av^{-3}(bv+1)e^{-bv+\frac{3}{4}\pi i}$ $\operatorname{Re}(p \pm ai^{1/2})>0$
(10)	$0 \qquad 0<t<b$ $y^{-1}[\operatorname{ber}_1(ay)+i\operatorname{bei}_1(ay)]$ $t>b$	$a^{-1}b^{-1}e^{-\frac{3}{4}\pi i}(e^{-bp}-e^{-bv})$ $\operatorname{Re}(p \pm ai^{1/2})>0$
(11)	$0 \qquad 0<t<b$ $ty^{-1}[\operatorname{ber}_1(ay)+i\operatorname{bei}_1(ay)]$ $t>b$	$a^{-1}e^{-\frac{3}{4}\pi i}(e^{-bp}-pv^{-1}e^{-bv})$ $\operatorname{Re}(p \pm ai^{1/2})>0$
(12)	$0 \qquad 0<t<b$ $\left(\frac{t-b}{t+b}\right)^{\frac{1}{2}\nu}[\operatorname{ber}_\nu(ay)$ $+i\operatorname{bei}_\nu(ay)] \qquad t>b$ $\operatorname{Re}\nu>-1$	$a^{\nu}v^{-1}V^{-\nu}e^{\frac{3}{4}\nu\pi i-bv}$ $\operatorname{Re}(p \pm ai^{1/2})>0$
(13)	$t[\operatorname{ker}(at)+i\operatorname{kei}(at)]$	$pv^{-3}\log(i^{-1/2}V/a)-v^{-2}$ $\operatorname{Re}(p \pm ai^{1/2})>0$
(14)	$t[\operatorname{ker}_1(at)+i\operatorname{kei}_1(at)]$	$i^{1/2}p\,a^{-1}v^{-2}+ai^{3/2}v^{-3}\log(i^{-1/2}V/a)$ $\operatorname{Re}(p \pm ai^{1/2})>0$
(15)	$0 \qquad 0<t<b$ $\operatorname{ker}(ay)+i\operatorname{kei}(ay) \qquad t>b$	$v^{-1}e^{-bv}\log(i^{-1/2}V/a)$ $\operatorname{Re}(p+ai^{1/2})>0$

$y=(t^2-b^2)^{1/2}, \quad v=(p^2-ia^2)^{1/2}, \quad V=p+v$

Kelvin's functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(16)	$0 \quad 0<t<b$ $t[\ker(ay)+i\,\mathrm{kei}(ay)] \quad t>b$	$-v^{-2}e^{-bv}[1+(bp-pv^{-1})\log(i^{-\frac{1}{2}}V/a)]$ $\mathrm{Re}\,(p+ai^{\frac{1}{2}})>0$
(17)	$0 \quad 0<t<b$ $y[\ker_1(ay)+i\,\mathrm{kei}_1(ay)] \quad t>b$	$v^{-1}e^{-bv}[i^{1/2}p/a+i^{3/2}a(b+1/v)$ $\times\log(i^{-1/2}V/a)] \quad \mathrm{Re}\,(p+ai^{\frac{1}{2}})>0$
(18)	$0 \quad 0<t<b$ $\left(\dfrac{t-b}{t+b}\right)^{\frac{1}{2}\nu}[\ker_\nu(ay)$ $+i\,\mathrm{kei}_\nu(ay)] \quad t>b$ $\lvert\mathrm{Re}\,\nu\rvert<1$	$\dfrac{\pi e^{-bp-\frac{1}{2}i\nu\pi}}{2v\sin(\nu\pi)}\left[\left(\dfrac{V}{ai^{\frac{1}{2}}}\right)^{\nu}-\left(\dfrac{ai^{\frac{1}{2}}}{V}\right)^{\nu}\right]$ $\mathrm{Re}\,(p+ai^{\frac{1}{2}})>0$
(19)	$V_\nu^{(b)}(2t^{\frac{1}{2}}) \quad \mathrm{Re}\,\nu>0$	$p\,I_\nu(2p^{-1}) \quad \mathrm{Re}\,p>0$
(20)	$t^{\frac{1}{2}}W_\nu^{(b)}(2t^{\frac{1}{2}}) \quad \mathrm{Re}\,\nu>-2$	$p^{-2}I_\nu(2p^{-1}) \quad \mathrm{Re}\,p>0$
(21)	$X_\nu^{(b)}(2t^{\frac{1}{2}}) \quad \mathrm{Re}\,\nu>-1$	$p^{-1}I_\nu(2p^{-1}) \quad \mathrm{Re}\,p>0$
(22)	$t^{-\frac{1}{2}}Z_\nu^{(b)}(2t^{\frac{1}{2}}) \quad \mathrm{Re}\,\nu>0$	$I_\nu(2p^{-1}) \quad \mathrm{Re}\,p>0$

4.19. Functions related to Bessel functions, Struve, Lommel, and Bessel integral functions

(1)	$\mathbf{H}_0(at)$	$2\pi^{-1}r^{-1}\log(r/p+a/p)$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(2)	$\mathbf{H}_1(at)$	$\dfrac{2}{\pi p}-\dfrac{2p}{\pi ar}\log\dfrac{r+a}{p} \quad \mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$

$y=(t^2-b^2)^{\frac{1}{2}}, \quad v=(p^2-i\,a^2)^{\frac{1}{2}}, \quad V=p+v, \quad r=(p^2+a^2)^{\frac{1}{2}}$

Related functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(3)	$\mathbf{H}_2(at)$	$\frac{2}{\pi}\left(-\frac{2}{a}+\frac{a}{3p^2}+\frac{a^2+2p^2}{a^2 r}\log\frac{r+a}{p}\right)$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(4)	$\mathbf{H}_3(at)$	$2\pi^{-1}p^{-1}(1/3+4a^{-2}p^2+2a^2p^{-2}/15)$ $-\frac{6a^2p+8p^3}{\pi a^3 r}\log\frac{r+a}{p}$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(5)	$\mathbf{H}_{1/2}(at)$ $\quad \operatorname{Re} a > 0$	$(\tfrac{1}{2}ap)^{-1/2}-a^{-1/2}R^{1/2}/r$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(6)	$\mathbf{H}_{-n-1/2}(at)$	$(-1)^n a^{n+1/2}R^{-n-1/2}/r$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(7)	$t^{-1}\mathbf{H}_1(at)$	$\frac{2}{\pi}\left(-1+\frac{r}{a}\log\frac{r+a}{p}\right)$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(8)	$t^{-1}\mathbf{H}_2(at)$	$\frac{2}{\pi}\left(\frac{p}{a}+\frac{a}{3p}-\frac{r}{a}\log\frac{a+r}{p}\right)$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(9)	$t^{-1}\mathbf{H}_3(at)$	$\frac{2}{\pi}\left(\frac{a^2}{15p^2}-\frac{4p^2}{3a^2}-\frac{7}{9}+\frac{4p^2r+a^2r}{3a^3}\log\frac{r+a}{p}\right)$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(10)	$t^{1/2}\mathbf{H}_{1/2}(at)$	$2^{1/2}\pi^{-1/2}a^{3/2}p^{-1}r^{-2}$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$
(11)	$t^{1/2}\mathbf{H}_{-1/2}(at)$	$2^{1/2}\pi^{-1/2}a^{1/2}r^{-2}$ $\quad \operatorname{Re} p > \lvert\operatorname{Im} a\rvert$

$r=(p^2+a^2)^{1/2}, \quad R=p+r$

Related functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(12)	$t^{1/2}\,\mathbf{H}_{3/2}(at)$	$2^{1/2}\pi^{-1/2}a^{1/2}[\tfrac{1}{2}p^{-2}-r^{-2}+a^{-2}\log(r/p)]$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(13)	$t^{1/2}\,\mathbf{H}_{-3/2}(at)$	$2^{1/2}\pi^{-1/2}[a^{-1/2}pr^{-2}$ $-a^{-3/2}\tan^{-1}(a/p)]$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(14)	$t^{-1/2}\,\mathbf{H}_{1/2}(at)$	$(\tfrac{1}{2}\pi a)^{-1/2}\log(r/p)$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(15)	$t^{-1/2}\,\mathbf{H}_{-1/2}(at)$	$(\tfrac{1}{2}\pi a)^{-1/2}\tan^{-1}(a/p)$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(16)	$t^{-1/2}\,\mathbf{H}_{3/2}(at)$	$(\tfrac{1}{2}\pi a)^{-1/2}[\tfrac{1}{2}ap^{-1}-a^{-1}p\log(r/p)]$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(17)	$t^{3/2}\,\mathbf{H}_{3/2}(at)$	$2^{1/2}\pi^{-1/2}a^{5/2}(3p^2+a^2)p^{-3}r^{-4}$ $\mathrm{Re}\,p>\lvert\mathrm{Im}\,a\rvert$
(18)	$\mathbf{L}_0(at)$	$2\pi^{-1}s^{-1}\sin^{-1}(a/p)$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(19)	$\mathbf{L}_1(at)$	$2\pi^{-1}p^{-1}[-1+a^{-1}p^2s^{-1}\sin^{-1}(a/p)]$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(20)	$\mathbf{L}_2(at)$	$\dfrac{2}{a\pi}\left(-2-\dfrac{a^2}{3p^2}+\dfrac{2p^2-a^2}{as}\sin^{-1}\dfrac{a}{p}\right)$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$
(21)	$\mathbf{L}_3(at)$	$\dfrac{2}{\pi p}\left(\dfrac{1}{3}-\dfrac{4p^2}{a^2}-\dfrac{2a^2}{15p^2}+\dfrac{4p^4-3ap^2}{a^3s}\sin^{-1}\dfrac{a}{p}\right)$ $\mathrm{Re}\,p>\lvert\mathrm{Re}\,a\rvert$

$r=(p^2+a^2)^{1/2}$, $R=p+r$, $s=(p^2-a^2)^{1/2}$, $S=p+s$

Related functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(22)	$t^{-1}\,\mathbf{L}_1(at)$	$2\pi^{-1}[1-a^{-1}s\sin^{-1}(a/p)]$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(23)	$t^{-2}\,\mathbf{L}_2(at)$	$2\pi^{-1}[p/a-ap^{-1}/3$ $-a^{-2}ps\sin^{-1}(a/p)]$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(24)	$t^{-1}\,\mathbf{L}_3(at)$	$\frac{2}{\pi}\left(\frac{4p^2}{3a^2}-\frac{7}{9}-\frac{a^2}{15p^2}-\frac{4p^2s-a^2s}{3a^3}\sin^{-1}\frac{a}{p}\right)$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(25)	$\mathbf{L}_{1/2}(at)$	$a^{-1/2}S^{1/2}/s-(\tfrac{1}{2}ap)^{-1/2}$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(26)	$\mathbf{L}_{-n-1/2}(at)$	$a^{n+1/2}s^{-1}S^{-n-1/2}$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(27)	$t^{1/2}\,\mathbf{L}_{1/2}(at)$	$2^{1/2}\pi^{-1/2}a^{3/2}p^{-1}s^{-2}$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(28)	$t^{1/2}\,\mathbf{L}_{-1/2}(at)$	$2^{1/2}\pi^{-1/2}a^{1/2}s^{-2}$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(29)	$t^{1/2}\,\mathbf{L}_{3/2}(at)$	$2^{1/2}a^{1/2}\pi^{-1/2}[s^{-2}-\tfrac{1}{2}p^{-2}$ $-a^{-2}\log(s/p)]$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(30)	$t^{1/2}\,\mathbf{L}_{-3/2}(at)$	$(\tfrac{1}{2}\pi a)^{-1/2}[ps^{-2}-a^{-1}\operatorname{ctnh}^{-1}(p/a)]$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$
(31)	$t^{-1/2}\,\mathbf{L}_{1/2}(at)$	$-(\tfrac{1}{2}\pi a)^{-1/2}\log(s/p)$ $\operatorname{Re} p>\lvert\operatorname{Re} a\rvert$

$s=(p^2-a^2)^{1/2}$, $\quad S=p+s$

Related functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(32)	$t^{-1/2}\,\mathbf{L}_{-1/2}(at)$	$(\tfrac12 a\pi)^{-1/2}\,\mathrm{ctnh}^{-1}(p/a)$ $\mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(33)	$t^{-1/2}\,\mathbf{L}_{3/2}(at)$	$(\tfrac12 a\pi)^{-1/2}\,[a^{-1}p\,\log(s/p)-\tfrac12 ap^{-1}]$ $\mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(34)	$t^{3/2}\,\mathbf{L}_{3/2}(at)$	$2^{1/2}\pi^{-1/2}a^{5/2}(3p^2-a^2)p^{-3}s^{-4}$ $\mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(35)	$t^{\nu}\,\mathbf{L}_{\nu}(at)$ $\mathrm{Re}\,\nu > -\tfrac12$	$\dfrac{(2a)^{\nu}\,\Gamma(\nu+\tfrac12)}{\pi^{1/2}\,s^{2\nu+1}} - \dfrac{\Gamma(2\nu+1)\,(a/p)^{\nu}}{(\tfrac12\pi p)^{1/2}(a^2-p^2)^{-\frac12\nu-\frac14}}\,P^{-\nu-\frac12}_{-\nu-\frac12}\left(\dfrac{a}{p}\right)$ $\mathrm{Re}\,p > \lvert\mathrm{Re}\,a\rvert$
(36)	$t^{\frac12\nu}\,\mathbf{L}_{\nu}(t^{1/2})$ $\mathrm{Re}\,\nu > -3/2$	$2^{-\nu}p^{-\nu-1}e^{\frac14 p^{-1}}\,\mathrm{Erf}(\tfrac12 p^{-1/2})$ $\mathrm{Re}\,p > 0$
(37)	$t^{\frac12\nu}\,\mathbf{L}_{-\nu}(t^{1/2})$	$\dfrac{2^{-\nu}p^{-\nu-1}}{\Gamma(\tfrac12-\nu)}\,e^{\frac14 p^{-1}}\,\gamma(\tfrac12-\nu,\tfrac14 p^{-1})$ $\mathrm{Re}\,p > 0$
(38)	$t^{1/2}\,s_{\mu,\frac14}(\tfrac12 t^2)$ $\mathrm{Re}\,\mu > -\tfrac34$	$2^{-2\mu-1}p^{1/2}\,\Gamma(2\mu+3/2) \times S_{-\mu-1,\frac14}(\tfrac12 p^2)$ $\mathrm{Re}\,p > 0$
	For further similar formulas see Meijer, C. S., 1935: *Nederl. Akad. Wetensch., Proc.* 38, 628-634.	
(39)	$Ji_0(t)$	$-p^{-1}\sinh^{-1}p$ $\mathrm{Re}\,p > 0$
(40)	$Ji_{\nu}(t)$ $\mathrm{Re}\,\nu > 0$	$\nu^{-1}p^{-1}[(p^2+1)^{1/2}-p]^{\nu}-\nu^{-1}p^{-1}$ $\mathrm{Re}\,p > 0$
(41)	$Ji_0(2t^{1/2})$	$\tfrac12 p^{-1}\,\mathrm{Ei}(-p^{-1})$ $\mathrm{Re}\,p > 0$

$s=(p^2-a^2)^{1/2}, \quad S=p+s$

4.20. Parabolic cylinder functions

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(1)	$t^{\nu}e^{\frac{1}{4}t^2}D_{-\mu}(t)$ $\quad$ Re $\nu>-1$	$\frac{\Gamma(\nu+1)}{\Gamma(\mu)}\int_0^\infty x^{\mu-1}(p+x)^{-\nu-1}$ $\times e^{-\frac{1}{2}x^2}\,dx$ $=p^{\mu-\nu-1}\sum_{r=0}^{\infty}(r!)^{-1}\,(\mu)_{2r}$ $\times\Gamma(\nu-2r-\mu+1)(-\frac{1}{2}p^2)^r$ $\quad$ Re $p>0$
(2)	$\exp\left(-\frac{t^2}{4a}\right)\left[D_{-2\nu}\left(-\frac{t}{a}\right)\right.$ $\left.-D_{-2\nu}\left(\frac{t}{a}\right)\right]$	$2^{\frac{1}{2}}\pi^{\frac{1}{2}}a^{1-2\nu}p^{-2\nu}e^{\frac{1}{2}a^2p^2}$ $\times\frac{\Gamma(\nu,\frac{1}{2}a^2p^2)}{\Gamma(\nu)}$ $\quad$ Re $p>0$
(3)	$D_{2n+1}(2^{\frac{1}{2}}t^{\frac{1}{2}})$	$(-2)^n\Gamma(n+3/2)(p-1/2)^n$ $\times(p+1/2)^{-n-3/2}$ $\quad$ Re $p>-\frac{1}{2}$
(4)	$D_{2\nu}(-2a^{\frac{1}{2}}t^{\frac{1}{2}})-D_{2\nu}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$	$\frac{2^{\nu+3/2}\pi a^{1/2}(p-a)^{\nu-1/2}}{\Gamma(-\nu)(p+a)^{\nu+1}}$ $\quad$ Re $p>\lvert\text{Re } a\rvert$
(5)	$t^{-\frac{1}{2}}D_{2n}(2^{\frac{1}{2}}t^{\frac{1}{2}})$	$(-2)^n\Gamma(n+\frac{1}{2})(p-\frac{1}{2})^n(p+\frac{1}{2})^{-n-\frac{1}{2}}$ $\quad$ Re $p>-\frac{1}{2}$
(6)	$t^{-\frac{1}{2}}[D_{2\nu}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$ $+D_{2\nu}(-2a^{\frac{1}{2}}t^{\frac{1}{2}})]$	$2^{\nu+1}\pi(p-a)^{\nu}(p+a)^{-\nu-\frac{1}{2}}/\Gamma(\frac{1}{2}-\nu)$ $\quad$ Re $p>\lvert\text{Re } a\rvert$
(7)	$t^{-\frac{1}{2}\nu-\frac{1}{2}}e^{\frac{1}{4}t}D_{\nu}(t^{\frac{1}{2}})$ $\quad$ Re $\nu<1$	$\pi^{\frac{1}{2}}p^{-\frac{1}{2}}(1+2^{\frac{1}{2}}p^{\frac{1}{2}})^{\nu}$ $\quad$ Re $p>0$
(8)	$t^{-\nu/2-3/2}e^{t/4}D_{\nu}(t^{1/2})$ $\quad$ Re $\nu<-1$	$-2^{\frac{1}{2}}\pi^{\frac{1}{2}}(\nu+1)^{-1}(1+2^{\frac{1}{2}}p^{\frac{1}{2}})^{\nu+1}$ $\quad$ Re $p>0$

Parabolic cylinder functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(9)	$t^{\nu-1}e^{\frac{1}{4}t}D_{2\nu+2n-1}(t^{\frac{1}{2}})$ $\operatorname{Re}\nu>0$	$\dfrac{\pi^{\frac{1}{2}}\Gamma(2n+2\nu)(1-2p)^n}{2^{2n-\frac{1}{2}+\nu}n!\,p^{n+\nu}}$ $\times{}_2F_1(n+\nu,\tfrac{1}{2}-\nu;n+1;1-\tfrac{1}{2}p^{-1})$ $\operatorname{Re}p>0$
(10)	$t^{\nu-1}e^{\frac{1}{4}t}D_{2\mu-1}(t^{\frac{1}{2}})$ $\operatorname{Re}\nu>0,\quad \operatorname{Re}(\nu-\mu)>-1$	$2^{\frac{1}{2}}\pi^{\frac{1}{2}}\Gamma(2\nu)(2p)^{-\frac{1}{2}\mu-\frac{1}{2}\nu}$ $\times(2p-1)^{\frac{1}{2}\mu-\frac{1}{2}\nu}P^{\mu-\nu}_{\mu+\nu-1}(2^{-\frac{1}{2}}p^{-\frac{1}{2}})$ $\operatorname{Re}p>0$
(11)	$[D_{-n-1}(-i2^{\frac{1}{2}}t^{\frac{1}{2}})]^2$ $-[D_{-n-1}(i2^{\frac{1}{2}}t^{\frac{1}{2}})]^2$	$\dfrac{2\pi i}{n!\,p^{\frac{1}{2}}}\dfrac{(p-1)^n}{(p+1)^{n+1}}$ $\quad\operatorname{Re}p>0$
(12)	$t^{-\nu}e^{-a/(8t)}$ $\times D_{2\nu-1}(2^{-\frac{1}{2}}a^{\frac{1}{2}}t^{-\frac{1}{2}})$ $\operatorname{Re}a>0$	$2^{\nu-\frac{1}{2}}\pi^{\frac{1}{2}}p^{\nu-1}e^{-a^{\frac{1}{2}}p^{\frac{1}{2}}}$ $\operatorname{Re}p>0$
(13)	$\dfrac{e^{\frac{1}{2}t}}{(e^t-1)^{\mu+\frac{1}{2}}}\exp\left(-\dfrac{a}{1-e^{-t}}\right)$ $\times D_{2\mu}\left[\dfrac{2a^{\frac{1}{2}}}{(1-e^{-t})^{\frac{1}{2}}}\right]$ $\operatorname{Re}a>0$	$e^{-a}2^{p+\mu}\Gamma(p+\mu)D_{-2p}(2a^{\frac{1}{2}})$ $\operatorname{Re}p>-\operatorname{Re}\mu$

4.21. Gauss' hypergeometric function

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$t^{a-1}F(\tfrac{1}{2}+\nu, \tfrac{1}{2}-\nu;\ a; -\tfrac{1}{2}t)$ $\operatorname{Re} a>0$	$\pi^{-1/2}\Gamma(a)(2p)^{1/2-a}K_\nu(p)$ $\operatorname{Re} p>0$
(2)	$t^{\gamma-1}F(\alpha,\beta;\delta;-t)$ $\quad \operatorname{Re}\gamma>0$	$\dfrac{\Gamma(\delta)\,p^{-\gamma}}{\Gamma(\alpha)\,\Gamma(\beta)}E(\alpha,\beta,\gamma:\delta:p)$ $\operatorname{Re} p>0$
(3)	$t^{\gamma-1}(1+t)^{\alpha+\beta-\delta}F(\alpha,\beta;\delta;-t)$ $\operatorname{Re}\gamma>0$	$\dfrac{\Gamma(\delta)\,p^{-\gamma}}{\Gamma(\delta-\alpha)\Gamma(\delta-\beta)}$ $\times E(\delta-\alpha,\delta-\beta,\gamma:\delta:p)$ $\quad \operatorname{Re} p>0$
(4)	$t^{\gamma-1}F(2\alpha,2\beta;\gamma;-\lambda t)$ $\operatorname{Re}\gamma>0,\quad \lvert\arg\lambda\rvert<\pi$	$\Gamma(\gamma)\,p^{-\gamma}(p/\lambda)^{\alpha+\beta-1/2}e^{1/2 p/\lambda}$ $\times W_{1/2-\alpha-\beta,\,\alpha-\beta}(\tfrac{1}{2}p/\lambda)$ $\quad \operatorname{Re} p>0$
(5)	$0 \qquad 0<t<1$ $(t^2-1)^{2a-1/2}$ $\times F(a-\tfrac{1}{2}\nu, a+\tfrac{1}{2}\nu; 2a+\tfrac{1}{2}; 1-t^2)$ $t>1$ $\operatorname{Re} a>-\tfrac{1}{4}$	$2^{2a}\pi^{-1/2}p^{-2a}\Gamma(2a+\tfrac{1}{2})K_\nu(p)$ $\operatorname{Re} p>0$
(6)	$[(\alpha+t)(\beta+t)]^{-1/2-\nu}$ $\times F\left[\dfrac{1}{2}+\nu,\dfrac{1}{2}+\nu;1;\dfrac{t(\alpha+\beta+t)}{(\alpha+t)(\beta+t)}\right]$ $\lvert\arg\alpha\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$	$\pi^{-1}(\alpha\beta)^{-\nu}e^{1/2(\alpha+\beta)p}K_\nu(\tfrac{1}{2}\alpha p)K_\nu(\tfrac{1}{2}\beta p)$ $\lvert\arg\alpha p\rvert<\pi,\quad \lvert\arg\beta\pi\rvert<\pi$ $\operatorname{Re} p>0$
(7)	$t^{-1/2}(1+\alpha/t)^{\mu}(1+\beta/t)^{\nu}$ $\times F\left[-\mu,-\nu;\tfrac{1}{2}-\mu-\nu;\dfrac{t(\alpha+\beta+t)}{(\alpha+t)(\beta+t)}\right]$ $\lvert\arg\alpha\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$ $\operatorname{Re}(\mu+\nu)<1$	$2^{-\mu-\nu}\Gamma(\tfrac{1}{2}-\mu-\nu)\,p^{-1/2}e^{1/2(\alpha+\beta)p}$ $\times D_{2\mu}(2^{1/2}\alpha^{1/2}p^{1/2})D_{2\nu}(2^{1/2}\beta^{1/2}p^{1/2})$ $\lvert\arg\alpha p\rvert<\pi,\quad \lvert\arg\beta p\rvert<\pi$ $\operatorname{Re} p>0$

Gauss' hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(8)	$t^{-\kappa-\lambda}(a+t)^{\kappa-\mu-\frac{1}{2}}(\beta+t)^{\lambda-\mu-\frac{1}{2}}$ $\times F\left[\frac{1}{2}-\kappa+\mu, \frac{1}{2}-\lambda+\mu; 1-\kappa-\lambda; \frac{t(a+\beta+t)}{(a+t)(\beta+t)}\right]$ $\lvert\arg a\rvert<\pi, \quad \lvert\arg\beta\rvert<\pi$ $\mathrm{Re}(\kappa+\lambda)<1$	$\Gamma(1-\kappa-\lambda)(a\beta)^{-\mu-\frac{1}{2}}p^{-1}e^{\frac{1}{2}(a+\beta)p}$ $\times W_{\kappa,\mu}(ap)W_{\lambda,\mu}(\beta p) \quad \mathrm{Re}\,p>0$ $\lvert\arg ap\rvert<\pi, \quad \lvert\arg\beta p\rvert<\pi$
(9)	$(1-e^{-t})^{\lambda-1}F(a,\beta;\gamma;\delta e^{-t})$ $\mathrm{Re}\,\lambda>0, \quad \lvert\arg(1-\delta)\rvert<\pi$	$\mathrm{B}(p,\lambda)\,{}_3F_2(a,\beta,p;\gamma,p+\lambda;\delta)$ $\mathrm{Re}\,p>0$
(10)	$(1-e^{-t})^{\mu}$ $\times F(-n,\mu+\beta+n;\beta;e^{-t})$ $\mathrm{Re}\,\mu>-1$	$\mathrm{B}(p,\mu+n+1)\,\mathrm{B}(p,\beta+n-p)/\mathrm{B}(p,\beta-p)$ $\mathrm{Re}\,p>0$
(11)	$(1-e^{-t})^{\gamma-1}F(a,\beta;\gamma;1-e^{-t})$ $\mathrm{Re}\,\gamma>0$	$\dfrac{\Gamma(p)\Gamma(\gamma-a-\beta+p)\Gamma(\gamma)}{\Gamma(\gamma-a+p)\Gamma(\gamma-\beta+p)}$ $\mathrm{Re}\,p>0, \quad \mathrm{Re}\,p>\mathrm{Re}(a+\beta-\gamma)$
(12)	$(1-e^{-t})^{\gamma-1}$ $\times F[a,\beta;\gamma;\delta(1-e^{-t})]$ $\mathrm{Re}\,\gamma>0, \quad \lvert\arg(1-\delta)\rvert<\pi$	$\mathrm{B}(p,\gamma)F(a,\beta;p+\gamma;\delta) \quad \mathrm{Re}\,p>0$
(13)	$(1-e^{-t})^{\lambda-1}$ $\times F[a,\beta;\gamma;\delta(1-e^{-t})]$ $\mathrm{Re}\,\lambda>0, \quad \lvert\arg(1-\delta)\rvert<\pi$	$\mathrm{B}(p,\lambda)\,{}_3F_2(a,\beta,\lambda;\gamma,p+\lambda;\delta)$ $\mathrm{Re}\,p>0$

4.22. Confluent hypergeometric functions

Particular confluent hypergeometric functions occur in sections 4.11, 4.12, 4.14 - 4.18, 4.20

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(1)	$k_0(t)$	$(p+1)^{-1}$ Re $p>-1$
(2)	$k_{2n+2}(t)$	$2(1-p)^n(1+p)^{-n-2}$ Re $p>-1$
(3)	$k_{2\nu}(t)$	$[2\pi\nu(1-\nu)]^{-1}\sin(\nu\pi)$ $\times {}_2F_1(1,2;2-\nu;\tfrac{1}{2}-\tfrac{1}{2}p)$ Re $p>0$
(4)	$t^{n-\frac{1}{2}} k_{2n+2}(t)$	$(-1)^{n-1}\dfrac{(2n)!\,\pi^{\frac{1}{2}}}{(n+1)!\,2^{2n+\frac{1}{2}}}(p+1)^{-n-1}$ $\times P^1_{2n+1}(p-1)^{\frac{1}{2}}(p+1)^{-\frac{1}{2}}$ Re $p>0$
(5)	$e^{-t^2} k_{2n}(t^2)$	$(-1)^{n-1} 2^{-1/4-3n/2} p^{n-3/2} e^{p^2/16}$ $\times W_{-\frac{3}{4}-\frac{1}{2}n,\frac{1}{4}-\frac{1}{2}n}(p^2/8)$
(6)	$t^{-\frac{1}{2}} e^{-t^{\frac{1}{2}}} k_{2n}(t^{\frac{1}{2}})$	$\sum_{m=0}^{n-1}(-1)^r\binom{n-1}{r}\left(\dfrac{2}{p}\right)^{\frac{1}{2}(n+1-r)}$ $\times e^{1/(2p)} D_{-n+r-1}(2^{\frac{1}{2}}p^{-\frac{1}{2}})$ Re $p>0$
(7)	$t^{-1} k_{2m+2}(\tfrac{1}{2}t) k_{2n+2}(\tfrac{1}{2}t)$	$(-p)^{m+n}(p+1)^{-m-n-2}$ $\times {}_2F_1(-m,-n;2;p^{-2})$ Re $p>-1$

Confluent hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(8)	$\dfrac{e^{\frac{1}{2}(\alpha+\beta)t}}{\alpha\beta t}\,k_{2m+2}(\frac{1}{2}\alpha t)$ $\times k_{2n+2}(\frac{1}{2}\beta t)$	$\dfrac{(-1)^{m+n}(m+n+1)!}{(m+1)!(n+1)!}\,\dfrac{(p-\alpha)^m(p-\beta)^n}{p^{m+n+2}}$ $\times {}_2F_1\left[-m,-n;-m-n-1;\dfrac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$ Re $p>0$
(9)	$t^{\lambda-1}k_{2m_1+2}(a_1t)\cdots k_{2m_n+2}(a_nt)$ Re $\lambda+n>0$	$(-1)^M 2^n a_1\cdots a_n(p+A)^{-\lambda-n}\Gamma(\lambda+n)$ $\times F_A\left(\lambda+n;-m_1,\ldots,-m_n;2,\ldots,2;\dfrac{2a_1}{p+A},\ldots,\dfrac{2a_n}{p+A}\right)$ Re $p>0$ $M=m_1+\cdots+m_n$ $A=a_1+\cdots+a_n$
(10)	$t^{\mu-\frac{1}{2}}M_{\kappa,\mu}(a t)$ Re $\mu>-\frac{1}{2}$	$a^{\mu+\frac{1}{2}}\Gamma(2\mu+1)\,\dfrac{(p-\frac{1}{2}a)^{\kappa-\mu-\frac{1}{2}}}{(p+\frac{1}{2}a)^{\kappa+\mu+\frac{1}{2}}}$ Re $p>\frac{1}{2}\lvert\mathrm{Re}\ a\rvert$
(11)	$t^{\nu-1}M_{\kappa,\mu}(at)$ Re$(\mu+\nu)>-\frac{1}{2}$	$a^{\mu+\frac{1}{2}}\Gamma(\mu+\nu+\frac{1}{2})(p+\frac{1}{2}a)^{-\mu-\nu-\frac{1}{2}}$ $\times {}_2F_1[\mu+\nu+\frac{1}{2},\ \mu-\kappa+\frac{1}{2};\ 2\mu+1;\ a/(p+\frac{1}{2}a)]$ Re $p>\frac{1}{2}\lvert\mathrm{Re}\ a\rvert$
(12)	$t^{2\nu-1}e^{-\frac{1}{2}t^2/a}M_{-3\nu,\nu}(t^2/a)$ Re $a>0$, Re $\nu>-\frac{1}{4}$	$\frac{1}{2}\pi^{-\frac{1}{2}}\Gamma(4\nu+1)\,a^{-\nu}p^{-4\nu}e^{ap^2/8}$ $\times K_{2\nu}(ap^2/8)$ Re $p>0$
(13)	$t^{2\mu-1}e^{-\frac{1}{2}t^2/a}M_{\kappa,\mu}(t^2/a)$ Re $a>0$, Re $\mu>-\frac{1}{4}$	$2^{-3\mu-\kappa}\Gamma(4\mu+1)\,a^{\frac{1}{2}(\kappa+\mu-1)}p^{\kappa-\mu-1}$ $\times e^{ap^2/8}W_{-\frac{1}{2}(\kappa+3\mu),\frac{1}{2}(\kappa-\mu)}(\frac{1}{4}ap^2)$ Re $p>0$

Confluent hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(14)	$t^{\nu-1}M_{\kappa_1,\mu_1-\frac{1}{2}}(a_1t)$ $\cdots M_{\kappa_n,\mu_n-\frac{1}{2}}(a_nt)$ $M=\mu_1+\cdots+\mu_n$ $\mathrm{Re}(\nu+M)>0$	$a_1^{\mu_1}\cdots a_n^{\mu_n}(p+A)^{-\nu-M}\Gamma(\nu+M)$ $\times F_A\left(\nu+M;\mu_1-\kappa_1,\ldots,\mu_n-\kappa_n;\ 2\mu_1,\ldots,2\mu_n;\frac{a_1}{p+A},\ldots,\frac{a_n}{p+A}\right)$ $A=\frac{1}{2}(a_1+\cdots+a_n)$ $\mathrm{Re}(p\pm\frac{1}{2}a_1\pm\cdots\pm\frac{1}{2}a_n)>0$
(15)	$(e^t-1)^{\mu-\frac{1}{2}}\exp(-\frac{1}{2}\lambda e^t)$ $\times M_{\kappa,\mu}(\lambda e^t-\lambda)$ $\mathrm{Re}\,\mu>-\frac{1}{2}$	$\dfrac{\Gamma(2\mu+1)\Gamma(\frac{1}{2}+\kappa-\mu+p)}{\Gamma(p+1)}$ $\times W_{-\kappa-\frac{1}{2}p,\,\mu-\frac{1}{2}p}(\lambda)$ $\mathrm{Re}\,p>\mathrm{Re}(\mu-\kappa)-\frac{1}{2}$
(16)	$t^{\nu-1}W_{\kappa,\mu}(at)$ $\mathrm{Re}(\nu\pm\mu)>-\frac{1}{2}$	$\dfrac{\Gamma(\mu+\nu+\frac{1}{2})\Gamma(\nu-\mu+\frac{1}{2})a^{\mu+\frac{1}{2}}}{\Gamma(\nu-\kappa+1)(p+\frac{1}{2}a)^{\mu+\nu+\frac{1}{2}}}$ $\times{}_2F_1\left(\mu+\nu+\frac{1}{2},\mu-\kappa+\frac{1}{2};\nu-\kappa+1;\frac{p-\frac{1}{2}a}{p+\frac{1}{2}a}\right)$ $\mathrm{Re}(p+\frac{1}{2}a)>0$
(17)	$t^{-1}\exp(-\frac{1}{2}a/t)W_{\frac{1}{2},\mu}(a/t)$ $\mathrm{Re}\,a>0$	$2\pi^{-\frac{1}{2}}(2ap)^{\frac{1}{2}}K_{\mu+\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times K_{\mu-\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,p>0$
(18)	$t^{-1}\exp(\frac{1}{2}a/t)W_{-\frac{1}{2},\mu}(a/t)$ $\lvert\arg a\rvert<\pi$	$\frac{1}{4}\mu^{-1}(a\pi^3p)^{\frac{1}{2}}[H^{(1)}_{\mu+\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times H^{(2)}_{\mu-\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $+H^{(1)}_{\mu-\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})H^{(2)}_{\mu+\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})]$ $\mathrm{Re}\,p>0$
(19)	$t^{3\nu-\frac{1}{2}}\exp(\frac{1}{2}a/t)W_{\nu,\nu}(a/t)$ $\lvert\arg a\rvert<\pi,\quad \mathrm{Re}\,\nu>-\frac{1}{4}$	$\frac{1}{2}\Gamma(2\nu+\frac{1}{2})a^{\nu+\frac{1}{2}}p^{-2\nu}H^{(1)}_{2\nu}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times H^{(2)}_{2\nu}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,p>0$

Confluent hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(20)	$t^{-3\nu-\frac{1}{2}}\exp(-\frac{1}{2}a/t)\,W_{\nu,\nu}(a/t)$ $\quad \operatorname{Re} a>0$	$2\pi^{-\frac{1}{2}}a^{\frac{1}{2}-\nu}p^{2\nu}\,[K_{2\nu}(a^{\frac{1}{2}}p^{\frac{1}{2}})]^2$ $\quad \operatorname{Re} p>0$
(21)	$t^{\kappa}\exp(\frac{1}{2}a/t)\,W_{\kappa,\mu}(a/t)$ $\lvert\arg a\rvert<\pi, \quad \operatorname{Re}(\kappa\pm\mu)>-\frac{1}{2}$	$2^{1-2\kappa}a^{\frac{1}{2}}p^{-\kappa-\frac{1}{2}}S_{2\kappa,2\mu}(2a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\quad \operatorname{Re} p>0$
(22)	$t^{-\kappa}\exp(-\frac{1}{2}a/t)\,W_{\kappa,\mu}(a/t)$ $\quad \operatorname{Re} a>0$	$2a^{\frac{1}{2}}p^{\kappa-\frac{1}{2}}K_{2\mu}(2a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\quad \operatorname{Re} p>0$
(23)	$(1-e^{-t})^{-\kappa}\exp\left[-\dfrac{\lambda}{2(e^t-1)}\right]$ $\times W_{\kappa,\mu}\left[\dfrac{\lambda}{(e^t-1)}\right]$ $\quad \operatorname{Re}\lambda>0$	$\dfrac{\Gamma(\frac{1}{2}+\mu+p)\,\Gamma(\frac{1}{2}-\mu+p)}{\Gamma(1-\kappa+p)}\,e^{\frac{1}{2}\lambda}W_{-p,\mu}(\lambda)$ $\operatorname{Re}(\frac{1}{2}\pm\mu+p)>0$
(24)	$\lambda e^t(e^t-1)^{-\kappa-1}\exp\left[-\dfrac{\lambda}{2(e^t-1)}\right]$ $\times W_{\kappa,\mu}\left(\dfrac{\lambda}{1-e^{-t}}\right)$ $\quad \operatorname{Re}\lambda>0$	$\Gamma(\kappa+p)\,W_{-p,\mu}(\lambda)$ $\quad \operatorname{Re} p>-\operatorname{Re}\kappa$

4.23. Generalized hypergeometric series

(1)	$t^{\gamma-1}\,{}_1F_1(a;\gamma;\lambda t)$ $\quad \operatorname{Re}\gamma>0$	$\Gamma(\gamma)\,p^{a-\gamma}(p-\lambda)^{-a}$ $\operatorname{Re} p>0, \quad \operatorname{Re}\lambda>0$
(2)	$\dfrac{t^{\gamma-1}e^{-t}}{(1-\lambda)^{2a}}\,{}_1F_1\left[a;\gamma;\dfrac{-4\lambda t}{(1-\lambda)^2}\right]$ $\operatorname{Re}\gamma>0$	$\dfrac{\Gamma(\gamma)}{(p+1)^{\gamma}}\left(1-2\,\dfrac{p-1}{p+1}\,\lambda+\lambda^2\right)^{-a}$ $\operatorname{Re} p>-1$ $\operatorname{Re} p>-\operatorname{Re}[(1+\lambda)^2(1-\lambda)^{-2}]>0$
(3)	$t^{a+\nu-\frac{1}{2}}$ $\times{}_1F_2(\frac{1}{2}+\nu;1+2\nu,\frac{1}{2}+\nu+a;-2t)$ $\operatorname{Re}(a+\nu+\frac{1}{2})>0$	$2^{\nu}\Gamma(\nu+1)\,\Gamma(a+\nu+\frac{1}{2})\,p^{-a-\frac{1}{2}}$ $\times e^{-1/p}\,I_{\nu}(p^{-1})$ $\quad \operatorname{Re} p>0$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\, dt$
(4)	$t^{\beta-1}\,{}_1F_2(-n; a+1, \beta; \lambda t)$ $\operatorname{Re}\beta>0$	$n![\Gamma(\beta)/(a+1)_n]\,p^{-\beta}L_n^{a}(\lambda/p)$ $\operatorname{Re} p>0$
(5)	${}_2F_2(-n, n+1; 1, 1; t)$	$p^{-1}P_n(1-2/p) \qquad \operatorname{Re} p>0$
(6)	$t^{\gamma-1}\,{}_2F_2(-n, n+1; 1, \gamma; t)$ $\operatorname{Re}\gamma>0$	$\Gamma(\gamma)\,p^{-\gamma}P_n(1-2/p) \qquad \operatorname{Re} p>0$
(7)	$t^{\gamma-1}\,{}_2F_2(-n, n; \gamma, ½; t)$ $\operatorname{Re}\gamma>0$	$\Gamma(\gamma)\,p^{-\gamma}\cos[2n\sin^{-1}(p^{-½})]$ $\operatorname{Re} p>0$
(8)	$t^{\gamma-1}\,{}_2F_2(-n, n+1; \gamma, 3/2; t)$ $\operatorname{Re}\gamma>0$	$\dfrac{\Gamma(\gamma)}{(2n+1)\,p^{\gamma}}\sin[(2n+1)\sin^{-1}(p^{-½})]$ $\operatorname{Re} p>0$
(9)	$t^{\gamma-1}\,{}_2F_2(-n, n+2\nu; \nu+½, \gamma; t)$ $\operatorname{Re}\gamma>0$	$n\,\mathrm{B}(n, 2\nu)\,\Gamma(\gamma)\,p^{-\gamma}\,C_n^{\nu}(1-2/p)$ $\operatorname{Re} p>0$
(10)	$t^{\gamma-1}\,{}_2F_2(-n, a+n; \beta, \gamma; t)$ $\operatorname{Re}\gamma>0$	$\Gamma(\gamma)\,p^{-\gamma}F(-n, a+n; \beta; p^{-1})$ $\operatorname{Re} p>0$
(11)	$t^{\mu+\nu-1}e^{-½t^2}$ $\times{}_2F_2\left(\mu, \nu; \dfrac{\mu+\nu}{2}, \dfrac{1+\mu+\nu}{2}; \dfrac{t^2}{4}\right)$ $\operatorname{Re}(\mu+\nu)>0$	$\Gamma(\mu+\nu)\,e^{¼p^2}D_{-\mu}(p)\,D_{-\nu}(p)$
(12)	t^{2a-1} $\times{}_3F_2(1, ½-\mu+\nu, ½-\mu-\nu;$ $a, a+½; -\lambda^2t^2)$ $\operatorname{Re}\lambda>0, \quad \operatorname{Re} a>0$	$\Gamma(2a)\,\lambda^{2\mu-1}\,p^{1-2a-2\mu}S_{2\mu,2\nu}(p/\lambda)$ $\operatorname{Re} p>0$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(13)	$t^{2\alpha-1}$ $\times {}_4F_3(\frac{1}{2}+\mu+\nu,\ \frac{1}{2}-\mu+\nu,\ \frac{1}{2}+\mu-\nu,$ $\frac{1}{2}-\mu-\nu; \frac{1}{2}, \alpha, \alpha+\frac{1}{2}; -\lambda^2t^2/4)$ $\operatorname{Re}\alpha>0, \quad \operatorname{Re}\lambda>0$	$\dfrac{\pi\Gamma(2\alpha)}{4\lambda p^{2\alpha-1}}[e^{(\mu-\nu)\pi i}H_{2\mu}^{(1)}(p/\lambda)H_{2\nu}^{(2)}(p/\lambda)$ $+e^{(\nu-\mu)\pi i}H_{2\mu}^{(2)}(p/\lambda)H_{2\nu}^{(1)}(p/\lambda)]$ $\lvert\arg p\rvert<\frac{1}{2}\pi$
(14)	$t^{2\alpha-1}$ $\times {}_4F_3(1+\mu+\nu, 1-\mu+\nu, 1+\mu-\nu, 1-\mu-\nu;$ $3/2, \alpha, \alpha+\frac{1}{2}; -\lambda^2t^2/4)$ $\operatorname{Re}\lambda>0, \quad \operatorname{Re}\alpha>0$	$\dfrac{\pi\Gamma(2\alpha)p^{2-2\alpha}}{8i\lambda^2(\mu^2-\nu^2)}$ $\times[e^{(\mu-\nu)\pi i}H_{2\mu}^{(1)}(p/\lambda)H_{2\nu}^{(2)}(p/\lambda)$ $-e^{(\nu-\mu)\pi i}H_{2\mu}^{(2)}(p/\lambda)H_{2\nu}^{(1)}(p/\lambda)]$ $\operatorname{Re}p>0$
(15)	$t^{2\alpha-1}$ $\times {}_4F_3(\frac{1}{2}+\mu-\kappa, \frac{1}{2}-\mu-\kappa, \frac{1}{2}-\kappa, 1-\kappa;$ $1-2\kappa, \alpha, \alpha+\frac{1}{2}; -\lambda^2t^2)$ $\operatorname{Re}\lambda>0, \quad \operatorname{Re}\alpha>0$	$\Gamma(2\alpha)\lambda^{2\kappa}p^{-2\alpha-2\kappa}W_{\kappa,\mu}(ip/\lambda)$ $\times W_{\kappa,\mu}(-ip/\lambda) \qquad \operatorname{Re}p>0$
(16)	t^{ρ_n-1} $\times {}_mF_n(\alpha_1,\dots,\alpha_m; \rho_1,\dots,\rho_n; \lambda t)$ $m\le n, \quad \operatorname{Re}\rho_n>0$	$\Gamma(\rho_n)\,p^{-\rho_n}$ $\times {}_mF_{n-1}(\alpha_1,\dots,\alpha_m; \rho_1,\dots,\rho_{n-1}; \lambda/p)$ $\operatorname{Re}p>0$ if $m<n$ $\operatorname{Re}p>\operatorname{Re}\lambda$ if $m=n$
(17)	$t^{\sigma-1}$ $\times {}_mF_n(\alpha_1,\dots,\alpha_m; \rho_1,\dots,\rho_n; \lambda t)$ $m\le n, \quad \operatorname{Re}\sigma>0$	$\Gamma(\sigma)p^{-\sigma}$ $\times {}_{m+1}F_n(\alpha_1,\dots,\alpha_m,\sigma; \rho_1,\dots,\rho_n; \lambda/p)$ $\operatorname{Re}p>0$ if $m<n$ $\operatorname{Re}p>\operatorname{Re}\lambda$ if $m=n$
(18)	$t^{2\sigma-1}$ $\times {}_mF_n(\alpha_1,\dots,\alpha_m; \rho_1,\dots,\rho_n; \lambda^2t^2)$ $m<n, \quad \operatorname{Re}\sigma>0$	$\Gamma(2\sigma)p^{-2\sigma}$ $\times {}_{m+2}F_n(\alpha_1,\dots,\alpha_m, \frac{1}{2}\sigma, \frac{1}{2}\sigma+\frac{1}{2};$ $\rho_1,\dots,\rho_n; 4\lambda^2p^{-2})$ $\operatorname{Re}p>0$ if $m<n-1$ $\operatorname{Re}p>\lvert\operatorname{Re}\lambda\rvert$ if $m=n-1$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(19)	$t^{\sigma-1}$ $\times {}_mF_n[\alpha_1,\dots,\alpha_m;\rho_1,\dots,\rho_n;(\lambda t)^k]$ $m+k\le n+1, \quad \mathrm{Re}\,\sigma>0$	$\Gamma(\sigma)p^{-\sigma}$ $\times {}_{m+k}F_n\left[\alpha_1,\dots\alpha_m,\frac{\sigma}{k},\frac{\sigma+1}{k},\dots,\frac{\sigma+k-1}{k};\ \rho_1,\dots,\rho_n;\left(\frac{k\lambda}{p}\right)^k\right]$ $\mathrm{Re}\,p>0$ if $m+k\le n$ $\mathrm{Re}\,(p+k\lambda e^{2\pi ir/k})>0$ for $r=0,1,\dots,k-1$ if $m+k=n+1$
(20)	$t^{-1/2}$ $\times {}_{2m}F_{2n}\left(\frac{\alpha_1}{2},\frac{\alpha_1+1}{2},\dots,\frac{\alpha_m}{2},\frac{\alpha_m+1}{2};\ \frac{\rho_1}{2},\frac{\rho_1+1}{2},\dots,\frac{\rho_n}{2},\frac{\rho_n+1}{2};\ -2^{m-n-2}\frac{k^2}{t}\right)$ $k>0, \quad m\le n$	$\pi^{1/2}p^{-1/2}\,{}_mF_n(\alpha_1,\dots,\alpha_m;\rho_1,\dots,\rho_n;-kp^{1/2})$ $\mathrm{Re}\,p>0$
(21)	$(1-e^{-t})^{\lambda-1}$ $\times {}_mF_n(\alpha_1,\dots,\alpha_m;\rho_1,\dots,\rho_n;\gamma e^{-t})$ $\mathrm{Re}\,\lambda>0, \quad m\le n$ Valid for $m=n+1$ if $\lvert\gamma\rvert<1$	$\mathrm{B}(\lambda,p)$ $\times {}_{m+1}F_{n+1}(\alpha_1,\dots,\alpha_m,p;\rho_1,\dots,\rho_n,p+\lambda;\gamma)$ $\mathrm{Re}\,p>0$
(22)	$(1-e^{-t})^{\lambda-1}$ $\times {}_mF_n[\alpha_1,\dots,\alpha_m;\rho_1,\dots,\rho_n;\ \gamma(1-e^{-t})]$ $\mathrm{Re}\,\lambda>0, \quad m\le n$ Valid for $m=n+1$ if $\lvert\gamma\rvert<1$	$\mathrm{B}(\lambda,p)$ $\times {}_{m+1}F_{n+1}(\alpha_1,\dots,\alpha_m,\lambda;\rho_1,\dots,\rho_n,p+\lambda;\gamma)$ $\mathrm{Re}\,p>0$
(23)	$t^{\alpha_{m+1}-1}E(m;\alpha_r:n;\beta_s:t^{-1})$ $\mathrm{Re}\,\alpha_{m+1}>0$	$p^{-\alpha_{m+1}}E(m+1;\alpha_r:n;\beta_s:p) \quad \mathrm{Re}\,p>0$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(24)	$(e^t-1)^{\alpha_{m+1}} \times E\left(m; \alpha_r : n; \beta_s : \frac{\lambda}{1-e^{-t}}\right)$ $\operatorname{Re} \alpha_{m+1} > -1$	$\Gamma(p-\alpha_{m+1}) E(m+1; \alpha_r : n; \beta_s, p : \lambda)$ $\operatorname{Re} p > \operatorname{Re} \alpha_{m+1}$
(25)	$t^{-2\nu} S_1(\nu, \nu-\frac{1}{2}, -\nu-\frac{1}{2}, \nu-\frac{1}{2}; at)$	$2^{-2\nu-\frac{1}{2}} \pi^{-\frac{1}{2}} p^{2\nu-1} \mathbf{H}_{2\nu}(4a/p)$ $\operatorname{Re} p > 0$
(26)	$t^{-2\nu-1}$ $\times S_1(\nu, \nu-\frac{1}{2}, -\nu-\frac{1}{2}, \nu+\frac{1}{2}; at)$	$2^{-2\nu-3/2} \pi^{-1/2} p^{2\nu} \mathbf{H}_{2\nu}(4a/p)$ $\operatorname{Re} p > 0$
(27)	$t^{-2\lambda-1}$ $\times S_1(\nu-\frac{1}{2}, -\nu-\frac{1}{2}, \lambda, \lambda+\frac{1}{2}; at)$ $\operatorname{Re}(\nu-\lambda) > 0$	$2^{-2\lambda-1} \pi^{-\frac{1}{2}} p^{2\lambda} J_{2\nu}(4a/p)$ $\operatorname{Re} p > 0$
(28)	$t^{-2\nu} S_2(\nu, \nu-\frac{1}{2}, -\nu-\frac{1}{2}, \nu-\frac{1}{2}; at)$	$2^{-2\nu} \pi^{\frac{1}{2}} p^{2\nu-1} [I_{2\nu}(4a/p)$ $-\mathbf{L}_{2\nu}(4a/p)]$ $\operatorname{Re} p > 0$
(29)	$t^{-2\nu-1}$ $\times S_2(\nu, -\nu-\frac{1}{2}, \nu-\frac{1}{2}, \nu+\frac{1}{2}; at)$ $\operatorname{Re} \nu < 0$	$2^{-2\nu-1} \pi^{\frac{1}{2}} \sec(2\nu\pi) p^{2\nu}$ $\times[I_{-2\nu}(4a/p) - \mathbf{L}_{2\nu}(4a/p)]$ $\operatorname{Re} p > 0$
(30)	$t^{-2\nu} S_2(\nu, -\nu-\frac{1}{2}, \nu-\frac{1}{2}, \nu-\frac{1}{2}; at)$ $\operatorname{Re} \nu < \frac{1}{2}$	$2^{-2\nu} \pi^{\frac{1}{2}} \sec(2\nu\pi) p^{2\nu-1}$ $\times[I_{-2\nu}(4a/p) - \mathbf{L}_{2\nu}(4a/p)]$ $\operatorname{Re} p > 0$
(31)	$t^{-2\lambda-1}$ $\times S_2(\nu-\frac{1}{2}, -\nu-\frac{1}{2}, \lambda+\frac{1}{2}, \lambda; at)$ $\operatorname{Re}(\lambda \pm \nu) < 0$	$2^{-2\lambda} \pi^{-\frac{1}{2}} p^{2\lambda} K_{2\nu}(4a/p)$ $\operatorname{Re} p > 0$
(32)	$t^{-2\nu} S_3(\nu, \nu-\frac{1}{2}, -\nu-\frac{1}{2}, \nu-\frac{1}{2}; at)$ $\operatorname{Re} \nu < \frac{1}{2}$	$2^{-2\nu} \pi^{3/2} \sec(2\nu\pi) p^{2\nu-1}$ $\times[\mathbf{H}_{2\nu}(4a/p) - Y_{2\nu}(4a/p)]$ $\operatorname{Re} p > 0$

Generalized hypergeometric series (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(33)	$t^{\alpha-1}$ $\times E(a_1,\dots,a_h:\rho_1,\dots,\rho_k:t^{-1})$ $\operatorname{Re}\alpha>0$	$p^{-\alpha}E(a_1,\dots,a_h,\alpha:\rho_1,\dots,\rho_k:p)$
(34)	$t^{-\alpha}G^{m,n}_{h,k}\left(t\,\middle\vert\,\begin{matrix}a_1,\dots,a_h\\ b_1,\dots,b_k\end{matrix}\right)$ $h+k<2(m+n)$ $\operatorname{Re}\alpha>\operatorname{Re}b_j+1,\quad j=1,\dots,m$	$p^{\alpha-1}\,G^{m,n+1}_{h+1,k}\left(p^{-1}\,\middle\vert\,\begin{matrix}\alpha,a_1,\dots,a_h\\ b_1,\dots,b_k\end{matrix}\right)$ $\vert\arg p\vert<(m+n-\tfrac12 h-\tfrac12 k)\pi$
	The same formula is valid if $h<k$ (or $h=k$ and $\operatorname{Re}p>1$) and $\operatorname{Re}\alpha<\operatorname{Re}b_j+1,\quad j=1,\dots,m.$	

4.24. Hypergeometric functions of several variables

(1)	$t^{\beta'-1}\Phi_1(\alpha,\beta,\gamma;x,yt)$ $\operatorname{Re}\beta'>0$	$\Gamma(\beta')p^{-\beta'}F_1(\alpha,\beta,\beta',\gamma;x,y/p)$ $\operatorname{Re}p>0,\quad\operatorname{Re}p>\operatorname{Re}y$
(2)	$t^{\beta-1}\Phi_2(\alpha,\alpha',\gamma;xt,y)$ $\operatorname{Re}\beta>0$	$\Gamma(\beta)p^{-\beta}\Xi_1(\alpha,\alpha',\beta,\gamma;x/p,y)$ $\operatorname{Re}p>0,\quad\operatorname{Re}p>\operatorname{Re}x$
(3)	$t^{\gamma-1}\Phi_2(\beta,\beta',\gamma;xt,yt)$ $\operatorname{Re}\gamma>0$	$\Gamma(\gamma)p^{-\gamma}(1-x/p)^{-\beta}(1-y/p)^{-\beta'}$ $\operatorname{Re}p>0,\quad\operatorname{Re}x,\quad\operatorname{Re}y$
(4)	$t^{\alpha-1}\Phi_2(\beta,\beta',\gamma;xt,yt)$ $\operatorname{Re}\alpha>0$	$\Gamma(\alpha)p^{-\alpha}F_1(\alpha,\beta,\beta',\gamma;x/p,y/p)$ $\operatorname{Re}p>0,\quad\operatorname{Re}x,\quad\operatorname{Re}y$
(5)	$t^{\gamma-1}\Phi_2(\beta_1,\dots,\beta_n;\gamma;\lambda_1t,\dots,\lambda_nt)$ $\operatorname{Re}\gamma>0$	$\dfrac{\Gamma(\gamma)}{p^\gamma}\left(1-\dfrac{\lambda_1}{p}\right)^{-\beta_1}\cdots\left(1-\dfrac{\lambda_n}{p}\right)^{-\beta_n}$ $\operatorname{Re}p>0,\quad\operatorname{Re}\lambda,\quad m=1,\dots,n$
(6)	$t^{\alpha-1}\Phi_3(\beta,\gamma;xt,y)\qquad\operatorname{Re}\alpha>0$	$\Gamma(\alpha)p^{-\alpha}\Xi_2(\alpha,\beta,\gamma;x/p,y)$ $\operatorname{Re}p>0,\quad\operatorname{Re}x$

Hypergeometric functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$
(7)	$t^{\beta'-1}\Phi_3(\beta,\gamma;x,yt)$ $\operatorname{Re}\beta'>0$	$\Gamma(\beta')p^{-\beta'}\Phi_2(\beta,\beta',\gamma;x,y/p)$ $\operatorname{Re}p>0,\quad \operatorname{Re}\gamma$
(8)	$t^{2\alpha-1}\Phi_3(\beta,\gamma;x,yt^2)$ $\operatorname{Re}\alpha>0$	$\Gamma(2\alpha)p^{-2\alpha}\Xi_1(\alpha,\beta,\alpha+\tfrac{1}{2},\gamma;4yp^{-2},x)$ $\operatorname{Re}p>2\lvert\operatorname{Re}\gamma^{1/2}\rvert$
(9)	$t^{\gamma-1}\Phi_3(\beta,\gamma;xt,yt)\quad \operatorname{Re}\gamma>0$	$\Gamma(\gamma)p^{-\gamma}(1-x/p)^{-\beta}e^{y/p}$ $\operatorname{Re}p>0,\quad \operatorname{Re}x$
(10)	$t^{\alpha-1}\Phi_3(\beta,\gamma;xt,yt)\quad \operatorname{Re}\alpha>0$	$\Gamma(\alpha)p^{-\alpha}\Phi_1(\alpha,\beta,\gamma;x/p,y/p)$ $\operatorname{Re}p>0,\quad \operatorname{Re}x$
(11)	$t^{\beta'-1}\Psi_1(\alpha,\beta,\gamma,\gamma';x,yt)$ $\operatorname{Re}\beta'>0$	$\Gamma(\beta')p^{-\beta'}F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y/p)$ $\operatorname{Re}p>0,\quad \operatorname{Re}\gamma$
(12)	$t^{\beta-1}\Psi_2(\alpha,\gamma,\gamma';xt,y)$ $\operatorname{Re}\beta>0$	$\Gamma(\beta)p^{-\beta}\Psi_1(\alpha,\beta,\gamma,\gamma';x/p,y)$ $\operatorname{Re}p>0,\quad \operatorname{Re}x$
(13)	$t^{\alpha-1}\Psi_2(\beta,\gamma,\gamma';xt,yt)$ $\operatorname{Re}\alpha>0$	$\Gamma(\alpha)p^{-\alpha}F_4(\alpha,\beta,\gamma,\gamma';x/p,y/p)$ $\operatorname{Re}p>0,\quad \operatorname{Re}x,\quad \operatorname{Re}\gamma$
(14)	$t^{\beta'-1}\Xi_1(\alpha,\alpha',\beta,\gamma;x,yt)$ $\operatorname{Re}\beta'>0$	$\Gamma(\beta')p^{-\beta'}F_3(\alpha,\alpha',\beta,\beta',\gamma;x,y/p)$ $\operatorname{Re}p>0,\quad \operatorname{Re}\gamma$
(15)	$t^{\alpha'-1}\Xi_2(\alpha,\beta,\gamma;x,yt)$ $\operatorname{Re}\alpha'>0$	$\Gamma(\alpha')p^{-\alpha'}\Xi_1(\alpha,\alpha',\beta,\gamma;x,y/p)$ $\operatorname{Re}p>0,\quad \operatorname{Re}\gamma$
(16)	$t^{2\alpha'-1}\Xi_2(\alpha,\beta,\gamma;x,yt^2)$ $\operatorname{Re}\alpha'>0$	$\Gamma(2\alpha')p^{-2\alpha'}$ $\times F_3(\alpha,\alpha',\beta,\alpha'+\tfrac{1}{2},\gamma;x,4yp^{-2})$ $\operatorname{Re}p>2\lvert\operatorname{Re}\gamma^{1/2}\rvert$

4.25. Elliptic functions

	$f(t)$	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$
(1)	$\theta_4(\tfrac{1}{2}x/l \mid i\pi t/l^2)$ $\quad -l \le x \le l$	$lp^{-1/2}\cosh(xp^{1/2})\operatorname{csch}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(2)	$\theta_1(\tfrac{1}{2}x/l \mid i\pi t/l^2)$ $\quad -l \le x \le l$	$-lp^{-1/2}\sinh(xp^{1/2})\operatorname{sech}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(3)	$\theta_2(\tfrac{1}{2}+\tfrac{1}{2}x/l \mid i\pi t/l^2)$ $\quad -l \le x \le l$	$-lp^{-1/2}\sinh(xp^{1/2})\operatorname{sech}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(4)	$\theta_3(\tfrac{1}{2}+\tfrac{1}{2}x/l \mid i\pi t/l^2)$ $\quad -l \le x \le l$	$lp^{-1/2}\cosh(xp^{1/2})\operatorname{csch}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(5)	$\hat{\theta}_4(\tfrac{1}{2}x/l \mid i\pi t/l^2)$ $\quad -l \le x \le l$	$-lp^{-1/2}\sinh(xp^{1/2})\operatorname{csch}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(6)	$\hat{\theta}_3(\tfrac{1}{2}+\tfrac{1}{2}x/l \mid i\pi t/l^2)$ $\quad -l \le x \le l$	$-lp^{-1/2}\sinh(xp^{1/2})\operatorname{csch}(lp^{1/2})$ $\quad \operatorname{Re} p > 0$
(7)	$e^{\alpha t}\ \theta_3(\alpha^{1/2} t \mid i\pi t)$	$\tfrac{1}{2}p^{-1/2}[\tanh(p^{1/2}+\alpha^{1/2}) + \tanh(p^{1/2}-\alpha^{1/2})]$ $\quad \operatorname{Re} p > 0$
(8)	$e^{\alpha t}\hat{\theta}_3(\alpha^{1/2} t \mid i\pi t)$	$\tfrac{1}{2}p^{-1/2}[\tanh(p^{1/2}+\alpha^{1/2}) - \tanh(p^{1/2}-\alpha^{1/2})+2]$ $\quad \operatorname{Re} p > 0$
(9)	$\dfrac{\partial}{\partial x}\ \theta_4\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$ $\quad -l < x < l$	$\dfrac{l\sinh(xp^{1/2})}{\sinh(lp^{1/2})}$ $\quad \operatorname{Re} p > 0$
(10)	$\dfrac{\partial}{\partial x}\ \theta_1\left(\dfrac{x}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$ $\quad -l < x < l$	$-\dfrac{l\cosh(xp^{1/2})}{\sinh(lp^{1/2})}$ $\quad \operatorname{Re} p > 0$
(11)	$\dfrac{\partial}{\partial x}\ \theta_2\left(\dfrac{x+l}{2l}\middle\vert\dfrac{i\pi t}{l^2}\right)$ $\quad -l < x < l$	$-\dfrac{l\cosh(xp^{1/2})}{\cosh(lp^{1/2})}$ $\quad \operatorname{Re} p > 0$

Elliptic functions (cont'd)

	$f(t)$	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	
(12)	$\frac{\partial}{\partial x}\theta_3\left(\frac{x+l}{2l}\middle\vert\frac{i\pi t}{l^2}\right)$	$\frac{l\sinh(xp^{1/2})}{\sinh(lp^{1/2})}$	$\mathrm{Re}\,p>0$

4.26. Miscellaneous functions

(1)	$\nu(t)$		$(p\log p)^{-1}$	$\mathrm{Re}\,p>0$
(2)	$(1-e^{-t})^{-1}\nu(t)$		$\int_0^\infty \zeta(u+1,p)\,du$	$\mathrm{Re}\,p>1$
(3)	$t^{-1/2}\nu(2t^{1/2})$		$2\pi^{1/2}p^{-1/2}\nu(p^{-1})$	$\mathrm{Re}\,p>0$
(4)	$\nu(e^{-t})$		$\int_0^\infty \frac{du}{(p+u)\Gamma(u+1)}$	$\mathrm{Re}\,p>0$
(5)	$\nu(1-e^{-t})$		$\Gamma(p)\,\nu(1,p)$	$\mathrm{Re}\,p>0$
(6)	$\nu(t,a)$	$\mathrm{Re}\,a>-1$	$p^{-a-1}/\log p$	$\mathrm{Re}\,p>1$
(7)	$\nu(2t^{1/2},2a)$	$\mathrm{Re}\,a>-1$	$\tfrac{1}{2}\pi^{1/2}p^{-3/2}\nu(p^{-1},a-\tfrac{1}{2})$	$\mathrm{Re}\,p>0$
(8)	$t^{-1/2}\nu(2t^{1/2},2a)$	$\mathrm{Re}\,a>-\tfrac{1}{2}$	$2\pi^{1/2}p^{-1/2}\nu(p^{-1},a)$	$\mathrm{Re}\,p>0$
(9)	$\mu(t,a-1)$	$\mathrm{Re}\,a>0$	$\Gamma(a)p^{-1}(\log p)^{-a}$	$\mathrm{Re}\,p>1$
(10)	$t^{-1/2}\mu(2t^{1/2},a)$		$2^{a+1}\pi^{1/2}p^{-1/2}\mu(p^{-1},a)$	$\mathrm{Re}\,p>0$

Miscellaneous functions (cont'd)

	$f(t)$	$g(p) = \int_0^\infty e^{-pt} f(t)\, dt$
(11)	$V_n(t)$ $V_n(t)$ is defined by the generating function $\frac{1}{1-z}\exp\left(-\frac{1+z}{1-z}\,t\right)$ $= \sum_{n=0}^{\infty} (n+\frac{1}{2})\, V_n(t) P_n(z)$	$\frac{2}{p-1}\, Q_n\left(\frac{p+1}{p-1}\right)$ $\qquad \mathrm{Re}\, p > 0$
(12)	$U^{m,n}(t)$ $U^{m,n}(t)$ is defined by the generating function $\frac{e^{-at} I_0(bt)}{(1-x)(1-y)} = \sum_{m,n=0}^{\infty} x^m y^n U^{m,n}(t)$ where $a+b = \left(\frac{1+x}{1-x}\right)^2, \quad a-b = \left(\frac{1+y}{1-y}\right)^2$	$\frac{1}{p+1} P_m\left(\frac{p-1}{p+1}\right) \; P_n\left(\frac{p-1}{p+1}\right)$ $\mathrm{Re}\, p > -1$

CHAPTER V

INVERSE LAPLACE TRANSFORMS

5.1. General formulas

Most general formulas are in section 4.1. The present section contains a few additions.

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(1)	$g(p)$	$f(t)=\dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{pt} g(p)\,dp$
(2)	$g(p+p^{1/2})$	$\frac{1}{2}\pi^{-1/2}\int_0^t u(t-u)^{-3/2} e^{-\frac{1}{4}u^2/(t-u)}$ $\times f(u)\,du$
(3)	$p^{-1/2} g(p+p^{1/2})$	$\pi^{-1/2}\int_0^t (t-u)^{-1/2} e^{-\frac{1}{4}u^2/(t-u)} f(u)\,du$
(4)	$(p+a)^{-\nu} g[cp+(p+a)^{1/2}]$ $\quad c>0$	$2^{\nu-1/2}\pi^{-1/2}\int_0^{t/c}(t-cu)^{\nu-1}\exp\left[-a(t-cu)\right.$ $\left.-\dfrac{u^2}{8(t-cu)}\right] D_{1-2\nu}\left[\dfrac{u}{2^{1/2}(t-cu)^{1/2}}\right]$ $\times f(u)\,du$
(5)	$g(r)$	$f(t)-a\int_0^t f[(t^2-u^2)^{1/2}]\,J_1(au)\,du$
(6)	$r^{-1} g(r)$	$\int_0^t J_0[a(t^2-u^2)^{1/2}]\,f(u)\,du$

$r=(p^2+a^2)^{1/2}, \quad R=p+r$

General formulas (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(7)	$pr^{-1}g(r)$	$f(t)-at\int_0^t (t^2-u^2)^{-1/2} \times J_1[a(t^2-u^2)^{1/2}]\,f(u)\,du$
(8)	$r^{-1}R^{-\nu}g(r)$ $\quad$ $\operatorname{Re}\nu>-1$	$a^{-\nu}\int_0^t \left(\frac{t-u}{t+u}\right)^{\frac{1}{2}\nu} \times J_\nu[a(t^2-u^2)^{1/2}]\,f(u)\,du$
(9)	$g(\beta+r-p)-g(\beta)$	$-at^{-1/2}\int_0^t t^{-1/2}(t+2u)^{-1/2} \times J_1[at^{1/2}(t+2u)^{1/2}]\,e^{-\beta u}\,uf(u)\,du$
(10)	$r^{-1}R^{-\nu}g(r-p)$ $\quad$ $\operatorname{Re}\nu>-1$	$a^{-\nu}t^{\frac{1}{2}\nu}\int_0^\infty (t+2u)^{-\frac{1}{2}\nu} \times I_\nu[at^{1/2}(t+2u)^{1/2}]\,f(u)\,du$
(11)	$r^{-1}R^{-\nu}g(p-r)$ $\quad$ $\operatorname{Re}\nu>-1$	$a^{-\nu}t^{\frac{1}{2}\nu}\int_0^\infty (t-2u)^{-\frac{1}{2}\nu} \times I_\nu[at^{1/2}(t-2u)^{1/2}]\,f(u)\,du$
(12)	$g(s)$	$f(t)+a\int_0^t f[(t^2-u^2)^{1/2}]\;I_1(u)\,du$
(13)	$s^{-1}g(s)$	$\int_0^t I_0[a(t^2-u^2)^{1/2}]f(u)\,du$
(14)	$ps^{-1}g(s)$	$f(t)+at\int_0^t (t^2-u^2)^{-1/2} \times I_1[a(t^2-u^2)^{1/2}]\,f(u)\,du$
(15)	$s^{-1}S^{-\nu}g(s)$ $\quad$ $\operatorname{Re}\nu>-1$	$a^{-\nu}\int_0^t \left(\frac{t-u}{t+u}\right)^{\frac{1}{2}\nu} \times I_\nu[a(t^2-u^2)^{1/2}]\,f(u)\,du$

$r=(p^2+a^2)^{1/2}, \quad R=p+r, \quad s=(p^2-a^2)^{1/2}, \quad S=p+s$

General formulas (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(16)	$g(\beta+s-p)-g(\beta)$	$at^{-1/2}\int_0^\infty e^{-\beta u}(t+2u)^{-1/2}$ $\times I_1[at^{1/2}(t+2u)^{1/2}]\,uf(u)\,du$
(17)	$s^{-1}S^{-\nu}g(s-p)$ $\quad \mathrm{Re}\,\nu>-1$	$a^{-\nu}t^{1/2\nu}\int_0^\infty(t+2u)^{-1/2\nu}$ $\times I_\nu[at^{1/2}(t+2u)^{1/2}]\,f(u)\,du$
(18)	$s^{-1}S^{-\nu}g(p-s)$ $\quad \mathrm{Re}\,\nu>-1$	$a^{-\nu}t^{1/2\nu}\int_0^\infty(t-2u)^{-1/2\nu}$ $\times I_\nu[at^{1/2}(t-2u)^{1/2}]\,f(u)\,du$

5.2. Rational functions

(1)	$(p+a)^{-1}$	e^{-at}
(2)	$(\lambda p+\mu)(p+a)^{-2}$	$[\lambda+(\mu-a\lambda)t]\,e^{-at}$
(3)	$(\lambda p+\mu)[(p+a)^2-\beta^2]^{-1}$	$\lambda e^{-at}\cosh(\beta t)+\beta^{-1}(\mu-a\lambda)$ $\times e^{-at}\sinh(\beta t)$
(4)	$(\lambda p+\mu)[(p+a)^2+\beta^2]^{-1}$	$\lambda e^{-at}\cos(\beta t)+\beta^{-1}(\mu-a\lambda)$ $\times e^{-at}\sin(\beta t)$
(5)	$\dfrac{\lambda p+\mu}{(p+a)(p+\beta)}$	$\dfrac{a\lambda-\mu}{a-\beta}\,e^{-at}+\dfrac{\beta\lambda-\mu}{\beta-a}\,e^{-\beta t}$
(6)	$(\lambda p^2+\mu p+\nu)(p+a)^{-3}$	$[\lambda+(\mu-2a\lambda)t+\tfrac{1}{2}(a^2\lambda-a\mu+\nu)t^2]$ $\times e^{-at}$

$s=(p^2-a^2)^{1/2},\qquad S=p+s$

Rational functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(7)	$3\alpha^2 \dfrac{\lambda p^2+\mu p+\nu}{p^3+\alpha^3}$	$(\lambda\alpha^2-\mu\alpha+\nu)\,e^{-\alpha t}$ $-(2\lambda\alpha^2-\mu\alpha-\nu)\,e^{\frac{1}{2}\alpha t}\cos(\tfrac{1}{2}3^{\frac{1}{2}}\alpha t)$ $+3^{\frac{1}{2}}(\mu\alpha-\nu)\,e^{\frac{1}{2}\alpha t}\sin(\tfrac{1}{2}3^{\frac{1}{2}}\alpha t)$
(8)	$\dfrac{\lambda p^2+\mu p+\nu}{(p+\alpha)^2(p+\beta)}$	$\left[\dfrac{\alpha(\alpha-2\beta)\lambda+\mu\beta-\nu}{(\alpha-\beta)^2} - \dfrac{\alpha^2\lambda-\alpha\mu+\nu}{\alpha-\beta}\,t\right]e^{-\alpha t}$ $+\dfrac{\beta^2\lambda-\beta\mu+\nu}{(\alpha-\beta)^2}\,e^{-\beta t}$
(9)	$\dfrac{\lambda p^2+\mu p+\nu}{[(p+\alpha)^2+\beta^2](p+\gamma)}$	$\left[\lambda-\dfrac{\lambda\gamma^2-\mu\gamma+\nu}{(\alpha-\gamma)^2+\beta^2}\right]e^{-\alpha t}\cos(\beta t)$ $+\dfrac{1}{\beta}\left[\mu-(\alpha+\gamma)\lambda - (\alpha-\gamma)\dfrac{\lambda\gamma^2-\mu\gamma+\nu}{(\alpha-\gamma)^2+\beta^2}\right]e^{-\alpha t}\sin(\beta t)$ $+\dfrac{\lambda\gamma^2-\mu\gamma+\nu}{(\alpha-\gamma)^2+\beta^2}\,e^{-\gamma t}$
(10)	$\dfrac{\lambda^2 p+\mu p+\nu}{(p+\alpha)(p+\beta)(p+\gamma)}$	$\dfrac{\lambda\alpha^2+\mu\alpha+\nu}{(\alpha-\beta)(\alpha-\gamma)}\,e^{-\alpha t}$ $+\dfrac{\lambda\beta^2-\mu\beta+\nu}{(\beta-\alpha)(\beta-\gamma)}\,e^{-\beta t}$ $+\dfrac{\lambda\gamma^2-\mu\gamma+\nu}{(\gamma-\alpha)(\gamma-\beta)}\,e^{-\gamma t}$
(11)	$4\alpha^3\dfrac{\lambda p^3+\mu p^2+\nu p+\rho}{p^4+4\alpha^4}$	$4\alpha^3\lambda\cos(\alpha t)\cosh(\alpha t)$ $+(2\alpha^2\mu-\rho)\cos(\alpha t)\sinh(\alpha t)$ $+(2\alpha^2\mu+\rho)\sin(\alpha t)\cosh(\alpha t)$ $+2\alpha\nu\sin(\alpha t)\sinh(\alpha t)$

Rational functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(12)	$2a^3 \dfrac{\lambda p^3+\mu p^2+\nu p+\rho}{p^4-a^4}$	$(a^3\lambda-a\nu)\cos(at)+(a^2\mu-\rho)\sin(at)$ $+(a^3\lambda+a\nu)\cosh(at)$ $+(a^2\mu+\rho)\sinh(at)$
(13)	$\dfrac{\lambda p^3+\mu p^2+\nu p+\rho}{(p^2+a^2)^2}$	$\lambda\cos(at)+\dfrac{\rho+a^2\mu}{2a^3}\sin(at)$ $+\dfrac{\nu-a^2\lambda}{2a}t\sin(at)-\dfrac{\rho-a^2\mu}{2a^2}t\cos(at)$
(14)	$(\beta^2-a^2)\dfrac{\lambda p^3+\mu p^2+\nu p+\rho}{(p^2+a^2)(p^2+\beta^2)}$	$(\nu-a^2\lambda)\cos(at)+(\rho/a-a\mu)\sin(at)$ $-(\nu-\beta^2\lambda)\cos(\beta t)-(\rho/\beta-\beta\mu)\sin(\beta t)$
	For further similar formulas see Gardner, M. F. and J. L. Barnes, 1942: *Transients in linear systems*, I, Wiley.	
(15)	$p^{-1}(p^{-1}-1)(p^{-1}-\frac{1}{2})\cdots(p^{-1}-1/n)$	$A_n(t)$
(16)	$2(1-p)^n(1+p)^{-n-2}$	$k_{2n+2}(t)$
(17)	$\dfrac{\lambda_1 p^{n-1}+\lambda_2 p^{n-2}+\cdots+\lambda_n}{(p+a)^n}$	$\Big\{\lambda_1+\left[\lambda_2-\binom{n-1}{1}\lambda_1 a\right]t$ $+\left[\lambda_3-\binom{n-2}{1}\lambda_2 a+\binom{n-1}{2}\lambda_1 a^2\right]\dfrac{t^2}{2!}$ $+\Big[\lambda_4-\binom{n-3}{1}\lambda_3 a+\binom{n-2}{2}\lambda_2 a^2$ $-\binom{n-1}{3}\lambda_1 a^3\Big]\dfrac{t^3}{3!}$ $+\cdots+[\lambda_n-\lambda_{n-1}a+\cdots+\lambda_1(-a)^{n-1}]$ $\times\dfrac{t^{n-1}}{(n-1)!}\Big\}e^{-at}$

Rational functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(18)	$p^{-1}(p+\alpha)^{-n}$	$\alpha^{-n}[1-e^{-\alpha t}e_{n-1}(\alpha t)]$ $e_n(z)=1+\dfrac{z}{1!}+\cdots+\dfrac{z^n}{n!}$
(19)	$\dfrac{\lambda_1 p^{n-1}+\lambda_2 p^{n-2}+\cdots+\lambda_n}{(p+\alpha_1)(p+\alpha_2)\cdots(p+\alpha_n)}$	$\dfrac{\lambda_1(-\alpha_1)^{n-1}+\cdots+\lambda_n}{(\alpha_2-\alpha_1)(\alpha_3-\alpha_1)\cdots(\alpha_n-\alpha_1)}\,e^{-\alpha_1 t}$ + $n-1$ similar terms obtained by cyclic permutation of $\alpha_1,\ldots,\alpha_n$
(20)	$\dfrac{Q(p)}{P(p)}$ $P(p)=(p-\alpha_1)(p-\alpha_2)\cdots(p-\alpha_n)$ $Q(p)$ = polynomial of degree $\leq n-1$ $\alpha_i \neq \alpha_k$, for $i \neq k$	$\sum_{m=1}^{n}\dfrac{Q(\alpha_m)}{P_m(\alpha_m)}\,e^{\alpha_m t}$ $P_m(p)=\dfrac{P(p)}{p-\alpha_m}$
(21)	$\dfrac{Q(p)}{P(p)}$ $P(p)=(p-\alpha_1)^{m_1}\cdots(p-\alpha_n)^{m_n}$ $Q(p)$ = polynomial of degree $< m_1+\cdots+m_n-1$ $\alpha_i \neq \alpha_k$, for $i \neq k$	$\sum_{k=1}^{n}\sum_{l=1}^{m_k}\dfrac{\Phi_{kl}(\alpha_k)}{(m_k-l)!(l-1)!}\,t^{m_k-l}e^{\alpha_k t}$ $\Phi_{kl}(p)=\dfrac{d^{l-1}}{dp^{l-1}}\left[\dfrac{Q(p)}{P_k(p)}\right]$ $P_k(p)=\dfrac{P(p)}{(p-\alpha_k)^{m_k}}$
(22)	$\dfrac{(2n+1)!\,\alpha^{2n+1}}{(p^2+\alpha^2)(p^2+3^2\alpha^2)\cdots[p^2+(2n+1)^2\alpha^2]}$	$\sin^{2n+1}(\alpha t)$
(23)	$\dfrac{(2n)!\,\alpha^{2n}}{p(p^2+2^2\alpha^2)(p^2+4^2\alpha^2)\cdots(p^2+4n^2\alpha^2)}$	$\sin^{2n}(\alpha t)$

Rational functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(24)	$\dfrac{p(p^2+2^2\alpha^2)(p^2+4^2\alpha^2)\cdots[p^2+(2n)^2\alpha^2]}{(p^2+\alpha^2)(p^2+3^2\alpha^2)\cdots[p^2+(2n+1)^2\alpha^2]}$	$P_{2n+1}[\cos(\alpha t)]$
(25)	$\dfrac{(p^2+\alpha^2)(p^2+\ {}^2\alpha^2)\cdots[p^2+(2n-1)^2\alpha^2]}{p(p^2+2^2\alpha^2)(p^2+4^2\alpha^2)\cdots[p^2+(2n)^2\alpha^2]}$	$P_{2n}[\cos(\alpha t)]$
(26)	$\dfrac{Q(p)+p\,\eta(p)}{P(p)}$ $P(p)=(p^2+\alpha_1^2)\cdots(p^2+\alpha_n^2)$ $Q(p)$, $\eta(p)$ even polynomials of degree $\leq 2n-2$ $\alpha_i \neq \alpha_k$, for $i \neq k$	$\sum_{m=1}^{n} \dfrac{1}{P_m(i\alpha_m)}[\eta(i\alpha_m)\cos(\alpha_m t)$ $+(\alpha_m)^{-1}Q(i\alpha_m)\sin(\alpha_m t)]$ $P_m(p)=\dfrac{P(p)}{p^2+\alpha_m^2}$

5.3. Irrational algebraic functions

(1)	$(p-\alpha)^{-1}p^{-1/2}$	$\alpha^{-1/2}e^{\alpha t}\operatorname{Erf}(\alpha^{1/2}t^{1/2})$
(2)	$p^{-1}(p-\alpha)^{-1}p^{-1/2}$	$\alpha^{-3/2}e^{\alpha t}\operatorname{Erf}(\alpha^{1/2}t^{1/2})$ $-2\alpha^{-1}\pi^{-1/2}t^{1/2}$
(3)	$2\pi i(p-1)^n(p+1)^{-n-1}p^{-1/2}$	$n![D^2_{-n-1}(-i\,2^{1/2}t^{1/2})-D^2_{-n-1}(i2^{1/2}t^{1/2})]$
(4)	$(p^{1/2}+\alpha)^{-1}$	$\pi^{-1/2}t^{-1/2}-\alpha e^{\alpha^2 t}\operatorname{Erfc}(\alpha t^{1/2})$
(5)	$\alpha p^{-1}(p^{1/2}+\alpha)^{-1}$	$1-e^{\alpha^2 t}\operatorname{Erfc}(\alpha t^{1/2})$
(6)	$(\alpha-\beta)p^{1/2}(p^{1/2}+\alpha^{1/2})^{-1}(p-\beta)^{-1}$	$\alpha e^{\alpha t}\operatorname{Erfc}(\alpha^{1/2}t^{1/2})$ $+\alpha^{1/2}\beta^{1/2}e^{\beta t}\operatorname{Erfc}(\beta^{1/2}t^{1/2})-\beta e^{\beta t}$

Irrational algebraic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(7)	$p^{-1/2}(p^{1/2}+\alpha)^{-1}$	$e^{\alpha^2 t}\,\mathrm{Erfc}(\alpha t^{1/2})$
(8)	$\alpha^2 p^{-3/2}(p^{1/2}+\alpha)$	$2\pi^{-1/2}\alpha t^{1/2}+e^{\alpha^2 t}\,\mathrm{Erfc}(\alpha t^{1/2})-1$
(9)	$(\alpha-\beta)\beta^{1/2}(p-\beta)^{-1}$ $\times p^{-1/2}(p^{1/2}+\alpha^{1/2})^{-1}$	$\beta^{1/2}e^{\alpha t}\,\mathrm{Erfc}(\alpha^{1/2}t^{1/2})$ $+\alpha^{1/2}e^{\beta t}\,\mathrm{Erf}(\beta^{1/2}t^{1/2})-\beta^{1/2}e^{\beta t}$
(10)	$(p^{1/2}+\alpha^{1/2})^{-2}$	$1-2\pi^{-1/2}\alpha^{1/2}t^{1/2}$ $+(1-2\alpha t)e^{\alpha t}[\mathrm{Erf}(\alpha^{1/2}t^{1/2})-1]$
(11)	$p^{-1}(p^{1/2}+\alpha^{1/2})^{-2}$	$\alpha^{-1}+(2t-\alpha^{-1})e^{\alpha t}\,\mathrm{Erfc}(\alpha^{1/2}t^{1/2})$ $-2\pi^{-1/2}\alpha^{-1/2}t^{1/2}$
(12)	$p^{-1/2}(p^{1/2}+\alpha)^{-2}$	$2\pi^{-1/2}t^{1/2}-2\alpha te^{\alpha^2 t}\,\mathrm{Erfc}(\alpha t^{1/2})$
(13)	$(p^{1/2}+\alpha)^{-3}$	$2\pi^{-1/2}(\alpha^2 t+1)t^{1/2}$ $-\alpha te^{\alpha^2 t}(2\alpha^2 t+3)\,\mathrm{Erfc}(\alpha t^{1/2})$
(14)	$p^{1/2}(p^{1/2}+\alpha)^{-3}$	$(2\alpha^4t^2+5\alpha^2t+1)e^{\alpha^2 t}\,\mathrm{Erfc}(\alpha t^{1/2})$ $-2\pi^{-1/2}\alpha(\alpha^2t+2)t^{1/2}$
(15)	$p^{-1/2}(p^{1/2}+\alpha)^{-3}$	$(2\alpha t^2+1)te^{\alpha^2 t}\,\mathrm{Erfc}(\alpha t^{1/2})-2\pi^{-1/2}\alpha t^{3/2}$
(16)	$3(p^{1/2}+\alpha)^{-4}$	$-2\pi^{-1/2}\alpha^3t^{5/2}(2\alpha^2t+5)$ $+t(4\alpha^4t^2+12\alpha^2t+3)e^{\alpha^2 t}\,\mathrm{Erfc}(\alpha t^{1/2})$
(17)	$p^{-1}(p^{1/2}-\alpha)(p^{1/2}+\alpha)^{-1}$	$2e^{\alpha^2 t}\,\mathrm{Erfc}(\alpha t^{1/2})-1$
(18)	$p^{-1}(p^{1/2}-\alpha)^2(p^{1/2}+\alpha)^{-2}$	$1+8\alpha^2te^{\alpha^2 t}\,\mathrm{Erfc}(\alpha t^{1/2})-8\pi^{-1/2}\alpha t^{1/2}$

Irrational algebraic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(19)	$p^{-1}(p^{1/2}-\alpha)^3(p^{1/2}+\alpha)^{-3}$	$2(8\alpha^4t^2+8\alpha^2t+1)e^{\alpha^2t}\,\mathrm{Erfc}(\alpha t^{1/2})$ $-8\pi^{-1/2}\alpha t^{1/2}(2\alpha^2t+1)-1$
(20)	$(p-\alpha)^{1/2}-(p-\beta)^{1/2}$	$\tfrac{1}{2}\pi^{-1/2}t^{-3/2}(e^{\beta t}-e^{\alpha t})$
(21)	$(p+\alpha)^{-1/2}$	$\pi^{-1/2}t^{-1/2}e^{-\alpha t}$
(22)	$(p+\beta)^{-1}(p+\alpha)^{1/2}$	$\pi^{-1/2}t^{-1/2}e^{-\alpha t}$ $+(\alpha-\beta)^{1/2}e^{-\beta t}\,\mathrm{Erf}[(\alpha-\beta)^{1/2}t^{1/2}]$
(23)	$(p+\alpha)^{-1}(p+\beta)^{-1/2}$	$(\beta-\alpha)^{-1/2}e^{-\alpha t}\,\mathrm{Erf}[(\beta-\alpha)^{1/2}t^{1/2}]$
(24)	$(p+\alpha)^{1/2}(p-\alpha)^{-1/2}-1$	$\alpha[I_0(\alpha t)+I_1(\alpha t)]$
(25)	$(p+\alpha+\beta)^{1/2}(p+\alpha-\beta)^{-3/2}$	$e^{-\alpha t}[(1+2\beta t)I_0(\beta t)+2\beta t\,I_1(\beta t)]$
(26)	$(p+\alpha+\beta)^{-1/2}(p+\alpha-\beta)^{-3/2}$	$te^{-\alpha t}[I_0(\beta t)+I_1(\beta t)]$
(27)	$p^{-1}(p+\alpha-\beta)^{1/2}(p+\alpha+\beta)^{-1/2}$	$e^{-\alpha t}I_0(\beta t)$ $+(\alpha-\beta)\int_0^t e^{-\alpha u}I_0(\beta u)\,du$
(28)	$\dfrac{(p+\alpha)^{1/2}-(p-\alpha)^{1/2}}{(p+\alpha)^{1/2}+(p-\alpha)^{1/2}}$	$t^{-1}I_1(\alpha t)$
	Campbell, G. A., and R. M. Foster, 1931: *Fourier integrals for practical applications*, Bell Telephone Laboratories, New York, contains other similar integrals.	
(29)	$(p+\alpha)^{-n-1/2}$	$\dfrac{\pi^{-1/2}t^{n-1/2}e^{-\alpha t}}{\frac{1}{2}\cdot\frac{3}{2}\cdots(n-\frac{1}{2})}$

Irrational algebraic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(30)	$(p-\alpha)^n(p-\beta)^{-n-1/2}$	$\dfrac{(-2)^n n!\,e^{\beta t}}{(2n)!\,\pi^{1/2}t^{1/2}}\,\mathrm{He}_{2n}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$
(31)	$(p-\alpha)^n(p-\beta)^{-n-3/2}$	$\dfrac{(-2)^n 2^{1/2} n!}{(2n+1)!\,\pi^{1/2}}e^{\beta t}\,\mathrm{He}_{2n+1}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$
(32)	$(p-\alpha)^n(p-\beta)^{-m-n-3/2}$	$\dfrac{(-1)^n e^{\beta t}2^{n+k+1/2}}{(\alpha-\beta)^m\,\pi^{1/2}}\times\sum_{k=1}^{m}\binom{m}{k}\dfrac{(n+k)!}{(2n+2k+1)!}\times\mathrm{He}_{2n+2k+1}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$
(33)	$(p-\alpha)^n(p-\beta)^{-m-n-1/2}$	$\dfrac{(-1)^n e^{\beta t}2^{n+k}}{(\alpha-\beta)^m\,\pi^{1/2}t^{1/2}}\sum_{k=1}^{m}\binom{m}{k}\dfrac{(n+k)!}{(2n+2k)!}\times\mathrm{He}_{2n+2k}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$
(34)	$(p^2+\alpha p+\beta)^{-1/2}$	$e^{-1/2\,\alpha t}J_0[(\beta-\tfrac14\alpha^2)^{1/2}t]$
(35)	r^{-1}	$J_0(at)$
(36)	$p^{-1}r^{-3}$	$\tfrac12\pi a^{-2}t[J_1(at)\mathbf{H}_0(at)-J_0(at)\mathbf{H}_1(at)]$
(37)	$r^{-1}R^{1/2}$	$2^{1/2}\pi^{-1/2}t^{-1/2}\cos(at)$
(38)	$(r-p)^{1/2}=aR^{-1/2}$	$2^{-1/2}\pi^{-1/2}t^{-3/2}\sin(at)$
(39)	$ar^{-1}R^{-1/2}$	$2^{1/2}\pi^{-1/2}t^{-1/2}\sin(at)$

$r=(p^2+a^2)^{1/2},\quad R=p+r$

Irrational algebraic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(40)	$p^{-1}r^{-1}R^{\frac{1}{2}}$	$2a^{-\frac{1}{2}}C(at)$
(41)	$p^{-1}r^{-1}R^{-\frac{1}{2}}$	$2a^{-3/2}S(at)$
(42)	r^{-2n-1}	$\dfrac{a^{-n}t^n J_n(at)}{1\cdot3\cdot5\cdots(2n-1)}$
(43)	R^{-n}	$n\,a^{-n}t^{-1}J_n(at)$
(44)	s^{-1}	$I_0(at)$
(45)	$p^{-1}s^{-3}$	$\frac{1}{2}\pi a^{-2}t[I_1(at)\mathbf{L}_0(at) - I_0(at)\mathbf{L}_1(at)]$
(46)	$s^{-1}S^{\frac{1}{2}}$	$2^{\frac{1}{2}}\pi^{-\frac{1}{2}}t^{-\frac{1}{2}}\cosh(at)$
(47)	$as^{-1}S^{-\frac{1}{2}}$	$2^{\frac{1}{2}}\pi^{-\frac{1}{2}}t^{-\frac{1}{2}}\sinh(at)$
(48)	s^{-2n-1}	$\dfrac{t^n I_n(at)}{1\cdot3\cdot5\cdots(2n-1)a^n}$
(49)	S^{-n}	$n\,a^{-n}t^{-1}I_n(at)$
(50)	$[(p^4+a^4)^{\frac{1}{2}}+p^2]^{\frac{1}{2}}(p^4+a^4)^{-\frac{1}{2}}$	$2^{\frac{1}{2}}\,\mathrm{ber}(at)$
(51)	$[(p^4+a^4)^{\frac{1}{2}}-p^2]^{\frac{1}{2}}(p^4+a^4)^{-\frac{1}{2}}$	$2^{\frac{1}{2}}\,\mathrm{bei}(at)$

$r=(p^2+a^2)^{\frac{1}{2}},\quad R=p+r,\qquad s=(p^2-a^2)^{\frac{1}{2}},\quad S=p+s$

5.4. Powers with an arbitrary index

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(1)	$\Gamma(\nu)(p+\alpha)^{-\nu}$ $\quad$ Re $\nu>0$	$t^{\nu-1}e^{-\alpha t}$
(2)	$p^{-m-1}(p-1)^n$	$t^m p_n(m,t)/m!$
(3)	$\Gamma(\nu)(p+\alpha)^{-\nu}(p-\beta)^{-1}$ Re $\nu>0$	$(\alpha+\beta)^{-\nu}e^{\beta t}\gamma[\nu,(\alpha+\beta)t]$
(4)	$\Gamma(\nu+1)(p-\lambda)^n(p-\mu)^{-\nu-1}$ Re $\nu>n-1$	$n!\,t^{\nu-n}e^{\mu t}L_n^{\nu-n}[(\mu-\lambda)t]$
(5)	$\Gamma(\nu)(p+\alpha)^{-\nu}(p+\beta)^{-\nu}$ Re $\nu>0$	$\pi^{1/2}(\alpha-\beta)^{1/2-\nu}t^{\nu-1/2}e^{-1/2(\alpha+\beta)t}$ $\times I_{\nu-1/2}[\tfrac{1}{2}(\alpha-\beta)t]$
(6)	$(p-\alpha)^{\lambda}(p-\beta)^{-\lambda-1/2}$	$2^{-\lambda-1}\pi^{-1}\Gamma(\tfrac{1}{2}-\lambda)t^{-1/2}e^{1/2(\alpha+\beta)t}$ $\times\{D_{2\lambda}[-2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$ $+D_{2\lambda}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]\}$
(7)	$(p-\alpha)^{\lambda}(p-\beta)^{-\lambda-3/2}$	$2^{-\lambda-3/2}\pi^{-1}(\alpha-\beta)^{-1/2}\Gamma(-\tfrac{1}{2}-\lambda)$ $\times e^{1/2(\alpha+\beta)t}\{D_{2\lambda+1}[-2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]$ $-D_{2\lambda+1}[2^{1/2}(\alpha-\beta)^{1/2}t^{1/2}]\}$
(8)	$\Gamma(2\nu-2\lambda)(p-\alpha)^{2\lambda}(p-\beta)^{-2\nu}$ Re$(\nu-\lambda)>0$	$t^{2\nu-2\lambda-1}e^{\alpha t}\,{}_1F_1[2\nu,2\nu-2\lambda;(\beta-\alpha)t]$ $=(\alpha-\beta)^{\lambda-\nu}t^{\nu-\lambda-1}e^{1/2(\alpha+\beta)t}$ $\times M_{\lambda+\nu,\,\nu-\lambda-1/2}[(\alpha-\beta)t]$
(9)	$\Gamma(\gamma)p^{-\gamma}$ $\times(1-\lambda_1/p)^{-\beta_1}\cdots(1-\lambda_n/p)^{-\beta_n}$ Re $\gamma>0$	$t^{\gamma-1}\Phi_2(\beta_1,\ldots,\beta_n;\gamma;\lambda_1 t,\ldots,\lambda_n t)$
(10)	$p^{-2\lambda}(p^2+\alpha^2)^{-\nu}$ Re$(\lambda+\nu)>0$	$[\Gamma(2\lambda+2\nu)]^{-1}t^{2\lambda+2\nu-1}$ $\times\,{}_1F_2(\nu;\lambda+\nu,\lambda+\nu+\tfrac{1}{2};-\tfrac{1}{4}\alpha^2t^2)$

Arbitrary powers (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(11)	$p^{-3\lambda}(p^3+a^3)^{-\nu}$ $\quad\mathrm{Re}(\lambda+\nu)>0$	$\dfrac{t^{3\lambda+3\nu-1}}{\Gamma(3\lambda+3\nu)}$ $\times{}_1F_3\left(\nu;\lambda+\nu,\lambda+\nu+\frac{1}{3},\lambda+\nu+\frac{2}{3};-\frac{a^3t^3}{27}\right)$
(12)	$\dfrac{2\Gamma(\nu)(\lambda p+\mu)}{(p^2-\beta^2)(p+a)^\nu}$ $\quad\mathrm{Re}\,\nu>0$	$\left(\lambda+\dfrac{\mu}{\beta}\right)e^{\beta t}\dfrac{\gamma[\nu,(a+\beta)t]}{(a+\beta)^\nu}$ $+\left(\lambda-\dfrac{\mu}{\beta}\right)e^{-\beta t}\dfrac{\gamma[\nu,(a-\beta)t]}{(a-\beta)^\nu}$
(13)	$[(p+a)^{1/2}+\beta^{1/2}]^\nu$ $\quad\mathrm{Re}\,\nu<0$	$-2^{1/2}\pi^{-1/2}\nu(2t)^{-1/2\nu-1}e^{(1/2\beta-a)t}$ $\times D_{\nu-1}(2^{1/2}\beta^{1/2}t^{1/2})$
(14)	$(p+a)^{-1/2}[(p+a)^{1/2}+\beta^{1/2}]^\nu$ $\quad\mathrm{Re}\,\nu<1$	$2^{1/2}\pi^{-1/2}(2t)^{-1/2\nu-1/2}e^{(1/2\beta-a)t}$ $\times D_\nu(2^{1/2}\beta^{1/2}t^{1/2})$
(15)	$[(p+a)^{1/2}+(p+\beta)^{1/2}]^{-2\nu}$ $\quad\mathrm{Re}\,\nu>0$	$\nu(a-\beta)^{-\nu}t^{-1}e^{-1/2(a+\beta)t}I_\nu[1/2(a-\beta)t]$
(16)	$[(p+a)^{1/2}+(p-a)^{1/2}]^{-2\nu}$ $\times(p+a)^{1/2}(p-a)^{-1/2}$ $\quad\mathrm{Re}\,\nu>0$	$1/4(2a)^{1-\nu}[I_{\nu-1}(at)$ $+2I_\nu(at)+I_{\nu+1}(at)]$
(17)	$[(p+a)^{1/2}+(p+\beta)^{1/2}]^{-2\nu}$ $\times(p+a)^{-1/2}(p+\beta)^{-1/2}$ $\quad\mathrm{Re}\,\nu>-1$	$(a-\beta)^{-\nu}e^{-1/2(a+\beta)t}I_\nu[1/2(a-\beta)t]$
(18)	$\Gamma(\nu+1/2)r^{-2\nu-1}$ $\quad\mathrm{Re}\,\nu>-1/2$	$\pi^{1/2}(2a)^{-\nu}t^\nu J_\nu(at)$
(19)	$\Gamma(\nu+1/2)s^{-2\nu-1}$ $\quad\mathrm{Re}\,\nu>-1/2$	$\pi^{1/2}(2a)^{-\nu}t^\nu I_\nu(at)$

$r=(p^2+a^2)^{1/2},\quad R=p+r,\qquad s=(p^2-a^2)^{1/2},\quad S=p+s$

Arbitrary powers (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$		$f(t)$
(20)	$\Gamma(\nu+\frac{1}{2})\,pr^{-2\nu-1}$	$\mathrm{Re}\,\nu>0$	$\pi^{\frac{1}{2}}a(2a)^{-\nu}t^{\nu}J_{\nu-1}(at)$
(21)	$a^{\nu}R^{-\nu}$	$\mathrm{Re}\,\nu>0$	$\nu t^{-1}J_{\nu}(at)$
(22)	$a^{\nu}pR^{-\nu}$	$\mathrm{Re}\,\nu>1$	$a\nu t^{-1}J_{\nu-1}(at)$ $-\nu(\nu+1)t^{-2}J_{\nu}(at)$
(23)	$r^{-1}R^{-\nu}$	$\mathrm{Re}\,\nu>-1$	$a^{-\nu}J_{\nu}(at)$
(24)	$a^{\nu-1}pr^{-1}R^{-\nu}$	$\mathrm{Re}\,\nu>0$	$\frac{1}{2}J_{\nu-1}(at)-\frac{1}{2}J_{\nu+1}(at)$
(25)	$p^{-1}[a^{-\nu}R^{\nu}+a^{\nu}R^{-\nu}\cos(\nu\pi)]$	$\lvert\mathrm{Re}\,\nu\rvert<1$	$1+\cos(\nu\pi)-\nu\sin(\nu\pi)$ $\times\int_t^\infty x^{-1}Y_{\nu}(ax)\,dx$
(26)	$(p+\nu r)r^{-3}R^{-\nu}$	$\mathrm{Re}\,\nu>-2$	$a^{-\nu}t\,J_{\nu}(at)$
(27)	$\Gamma(\nu+\frac{1}{2})\,ps^{-2\nu-1}$	$\mathrm{Re}\,\nu>0$	$\pi^{\frac{1}{2}}a(2a)^{-\nu}t^{\nu}I_{\nu-1}(at)$
(28)	$a^{\nu}S^{-\nu}$	$\mathrm{Re}\,\nu>0$	$\nu t^{-1}I_{\nu}(at)$
(29)	$a^{\nu}pS^{-\nu}$	$\mathrm{Re}\,\nu>1$	$a\nu t^{-1}I_{\nu-1}(at)$ $-\nu(\nu+1)t^{-2}I_{\nu}(at)$
(30)	$a^{\nu}s^{-1}S^{-\nu}$	$\mathrm{Re}\,\nu>-1$	$I_{\nu}(at)$
(31)	$a^{\nu-1}ps^{-1}S^{-\nu}$	$\mathrm{Re}\,\nu>0$	$\frac{1}{2}I_{\nu-1}(at)+\frac{1}{2}I_{\nu+1}(at)$
(32)	$s^{-1}(a^{-\nu}S^{\nu}-a^{\nu}S^{-\nu})$	$\lvert\mathrm{Re}\,\nu\rvert<1$	$2\pi^{-1}\sin(\nu\pi)K_{\nu}(at)$

$r=(p^2+a^2)^{\frac{1}{2}}$, $R=p+r$, $s=(p^2-a^2)^{\frac{1}{2}}$, $S=p+s$

Arbitrary powers (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(33)	$(p+\nu s)s^{-3}S^{-\nu}$ Re $\nu>-2$	$a^{-\nu}t\;I_\nu(at)$
(34)	$a^\nu v^{-1}\;V^{-\nu}$ Re $\nu>-1$	$e^{-\frac{1}{4}i\pi\nu}[\mathrm{ber}_\nu(at)+i\,\mathrm{bei}_\nu(at)]$
(35)	$v^{-1}(a^{-\nu}V^{\nu}-e^{i\pi\nu}a^\nu V^{-\nu})$ $\lvert\mathrm{Re}\,\nu\rvert<1$	$2\pi^{-1}e^{3/2\,i\pi\nu}\sin(\nu\pi)[\mathrm{ker}_\nu(at)$ $+i\,\mathrm{kei}_\nu(at)]$

5.5. Exponential functions of arguments p and $1/p$

(1)	$p^{-1}e^{-ap}$	$a>0$	0 1	$0<t<a$ $t>a$
(2)	$p^{-1}(1-e^{-ap})$	$a>0$	1 0	$0<t<a$ $t>a$
(3)	$p^{-1}(e^{-ap}-e^{-bp})$	$0\le a<b$	0 1 0	$0<t<a$ $a<t<b$ $t>b$
(4)	$p^{-2}(e^{-ap}-e^{-bp})$	$0\le a<b$	0 $t-a$ $b-a$	$0<t<a$ $a<t<b$ $t>b$
(5)	$p^{-3}(e^{-ap}-e^{-bp})$	$0\le a<b$	0 $\frac{1}{2}(t-a)^2$ $t(b-a)+\frac{1}{2}(a^2-b^2)$	$0<t<a$ $a<t<b$ $t>b$

$s=(p^2-a^2)^{\frac{1}{2}},\quad S=p+s,\qquad v=(p^2-ia^2)^{\frac{1}{2}},\quad V=p+v$

Exponential functions of p and 1/p (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(6)	$p^{-2}(e^{-ap}-e^{-bp})^2 \qquad 0\le a<b$	$0 \qquad t<2a$ $t-2a \qquad 2a<t<a+b$ $2b-t \qquad a+b<t<2b$ $0 \qquad t>2b$
(7)	$p^{-3}(e^{-ap}-e^{-bp})^2 \qquad 0\le a<b$	$0 \qquad t<2a$ $\tfrac{1}{2}(t-2a)^2 \qquad 2a<t<a+b$ $(b-a)^2-\tfrac{1}{2}(t-2b)^2$ $a+b<t<2b$ $(b-a)^2 \qquad t>2b$
(8)	$p^{-3}(e^{-ap}-e^{-bp})^3 \qquad 0\le a<b$	$0 \qquad t<3a$ $\tfrac{1}{2}(t-3a)^2 \qquad 3a<t<2a+b$ $[(3/4)(b-a)^2]-[t-(3/2)(a+b)]^2$ $2a+b<t<a+2b$ $\tfrac{1}{2}(3b-t)^2 \qquad a+2b<t<3b$ $0 \qquad t>3b$
(9)	$(p+\beta)^{-1}e^{-ap} \qquad a>0$	$0 \qquad 0<t<a$ $e^{-\beta(t-a)} \qquad t>a$
(10)	$(\lambda p+\mu)(p^2-\beta^2)^{-1}e^{-ap}$ $a>0$	$0 \qquad 0<t<a$ $\lambda\cosh[\beta(t-a)]+\mu\beta^{-1}\sinh[\beta(t-a)]$ $t>a$
(11)	$(\lambda p+\mu)(p^2+\beta^2)^{-1}e^{-ap}$ $a>0$	$0 \qquad 0<t<a$ $\lambda\cos[\beta(t-a)]+\mu\beta^{-1}\sin[\beta(t-a)]$ $t>a$
(12)	$(p^2+a^2)^{-1}(1-e^{-2m\pi p/a})$ $a>0, \quad m=1, 2, \ldots$	$a^{-1}\sin(at) \qquad 0<t<2m\pi/a$ $0 \qquad t>2m\pi/a$

Exponential functions of p and $1/p$ (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(13)	$p(p^2+a^2)^{-1}(1-e^{-2m\pi p/a})$ $a>0, \quad m=1,2,3,\ldots$	$\cos(at) \quad 0<t<2m\pi/a$ $0 \quad t>2m\pi/a$
(14)	$a^2p^{-1}(p^2+a^2)^{-1}$ $\times(1-e^{-2m\pi p/a})$	$2\sin^2(\tfrac{1}{2}at) \quad 0<t<2m\pi/a$ $0 \quad t>2m\pi/a$
(15)	$p^{-1}(e^{ap}-1)^{-1} \quad a>0$	$n \quad na<t<(n+1)a$
(16)	$p^{-2}(e^{ap}-1)^{-1} \quad a>0$	$nt-\tfrac{1}{2}an(n+1) \quad na<t<(n+1)a$
(17)	$p^{-1}(e^{ap}+1)^{-1} \quad a>0$	$0 \quad 2na<t<(2n+1)a$ $1 \quad (2n+1)a<t<(2n+2)a$
(18)	$p^{-2}(e^{ap}+1)^{-1} \quad a>0$	$\tfrac{1}{4}[1-(-1)^n](2t-a)+\tfrac{1}{2}(-1)^n na$ $na<t<(n+1)a$
(19)	$p^{-1}(e^{ap}-\beta)^{-1} \quad a>0$	$(1-\beta^n)/(1-\beta) \quad na<t<(n+1)a$
(20)	$p^{-2}(e^{ap}-\beta)^{-1} \quad a>0$	$(1-\beta^n)(1-\beta)^{-1}t-a(1-\beta)^{-2}$ $\times[1-(n+1)\beta^n+n\beta^{n+1}]$ $na<t<(n+1)a$
(21)	$\dfrac{1}{(p^2+c^2)(e^{-ap}+1)} \quad a>0$ $c>0, \quad ac\neq(2n+1)\pi$	$\dfrac{\sin(ct+\tfrac{1}{2}ac)}{2c\cos(\tfrac{1}{2}ac)}$ $+2a\sum_{n=0}^{\infty}\dfrac{\cos[(2n+1)\pi t/a]}{a^2c^2-(2n+1)^2\pi^2}$
(22)	$g(p)(e^{ap}+\beta)^{-c} \quad a,\ c>0$	$\sum_{0\le n<t/a-c}\binom{-c}{n}\beta^n f(t-ac-an)$

Exponential functions of p and $1/p$ (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(23)	$g(p)(1+\beta e^{-ap})^{\nu}$ $\quad a>0$	$\sum_{0\le n\le t/a}\binom{\nu}{n}\beta^n f(t-an)$
(24)	$a(p^2+a^2)^{-1}(1+e^{-2m\pi p/a})^{-1}$ $\quad a>0,\quad m=1,2,\dots$	$\sin(at)\quad 2n<at/(2m\pi)<2n+1$ $0\quad 2n+1<at/(2m\pi)<2n+2$ $n=0,1,2,\dots$
(25)	$p(p^2+a^2)^{-1}(1+e^{-2m\pi p/a})^{-1}$ $\quad a>0,\quad m=1,2,\dots$	$\cos(at)\quad 2n<at/(2m\pi)<2n+1$ $0\quad 2n+1<at/(2m\pi)<2n+2$ $n=0,1,2,\dots$
(26)	$a^2p^{-1}(p^2+a^2)^{-1}$ $\times(1+e^{-2m\pi p/a})^{-1}$ $a>0,\quad m=1,2,\dots$	$2\sin^2(\tfrac{1}{2}at)$ $2n<at/(2m\pi)<2n+1$ $0\quad 2n+1<at/(2m\pi)<2n+2$ $n=0,1,2,\dots$
(27)	$(p^2+a^2)^{-1}(1+e^{-\pi p/a})$ $\times(1-e^{-\pi p/a})^{-1}$ $\quad a>0$	$a^{-1}\lvert\sin(at)\rvert$
(28)	$p(p^2+a^2)^{-1}(1+e^{-\pi p/a})$ $\times(1-e^{-\pi p/a})^{-1}$	$\cos(at)\quad 2n\pi<at<(2n+1)\pi$ $-\cos(at)$ $(2n+1)\pi<at<(2n+2)\pi$ $n=0,1,2,\dots$
(29)	$g(p)(1+e^{-ap})(1-e^{-ap})^{-1}$ $a>0$	$f(t)+2\sum_{1\le n<t/a}f(t-an)$
(30)	$p^{-1}e^{-ap}(e^{ap}-1)^n$ $\quad a>0$	$\binom{n}{m}\quad na<t<(n+1)a$ $n=0,1,2,\dots$
(31)	$e^{a/p}-1$	$a^{1/2}t^{-1/2}I_1(2a^{1/2}t^{1/2})$

Exponential functions of p and $1/p$ (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$		$f(t)$
(32)	$p^{-1/2}e^{\alpha/p}$		$\pi^{-1/2}t^{-1/2}\cosh(2\alpha^{1/2}t^{1/2})$
(33)	$p^{-3/2}e^{\alpha/p}$		$\pi^{-1/2}\alpha^{-1/2}\sinh(2\alpha^{1/2}t^{1/2})$
(34)	$p^{-5/2}e^{\alpha/p}$		$\pi^{-1/2}\alpha^{-1}t^{1/2}\cosh(2\alpha^{1/2}t^{1/2})$ $-\frac{1}{2}\pi^{-1/2}\alpha^{-3/2}\sinh(2\alpha^{1/2}t^{1/2})$
(35)	$p^{-\nu-1}e^{\alpha/p}$	$\mathrm{Re}\,\nu>-1$	$\alpha^{-\frac{1}{2}\nu}t^{\frac{1}{2}\nu}I_\nu(2\alpha^{1/2}t^{1/2})$
(36)	$1-e^{-\alpha/p}$		$\alpha^{1/2}t^{-1/2}J_1(2\alpha^{1/2}t^{1/2})$
(37)	$p^{-1/2}e^{-\alpha/p}$		$\pi^{-1/2}t^{-1/2}\cos(2\alpha^{1/2}t^{1/2})$
(38)	$p^{-3/2}e^{-\alpha/p}$		$\pi^{-1/2}\alpha^{-1/2}\sin(2\alpha^{1/2}t^{1/2})$
(39)	$p^{-5/2}e^{-\alpha/p}$		$\frac{1}{2}\pi^{-1/2}\alpha^{-3/2}\sin(2\alpha^{1/2}t^{1/2})$ $-\alpha^{-1}\pi^{-1/2}t^{1/2}\cos(2\alpha^{1/2}t^{1/2})$
(40)	$p^{-\nu-1}e^{-\alpha/p}$	$\mathrm{Re}\,\nu>-1$	$\alpha^{-\frac{1}{2}\nu}t^{\frac{1}{2}\nu}J_\nu(2\alpha^{1/2}t^{1/2})$

5.6. Exponential functions of other arguments

(1)	$e^{-\alpha^{1/2}p^{1/2}}$	$\mathrm{Re}\,\alpha>0$	$\frac{1}{2}\pi^{-1/2}\alpha^{1/2}t^{-3/2}e^{-\frac{1}{4}\alpha/t}$
(2)	$pe^{-\alpha^{1/2}p^{1/2}}$	$\mathrm{Re}\,\alpha>0$	$\frac{1}{4}\pi^{-1/2}\alpha^{1/2}(\frac{1}{2}\alpha t^{-1}-3)t^{-5/2}e^{-\frac{1}{4}\alpha/t}$
(3)	$p^{-1}e^{-\alpha^{1/2}p^{1/2}}$	$\mathrm{Re}\,\alpha\geq 0$	$\mathrm{Erfc}(\frac{1}{2}\alpha^{1/2}t^{-1/2})$
(4)	$p^{-2}e^{-\alpha^{1/2}p^{1/2}}$	$\mathrm{Re}\,\alpha\geq 0$	$(t+\frac{1}{2}\alpha)\,\mathrm{Erfc}(\frac{1}{2}\alpha^{1/2}t^{-1/2})$ $-\pi^{-1/2}\alpha^{1/2}t^{1/2}e^{-\frac{1}{4}\alpha/t}$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(5)	$p^{1/2}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha>0$	$\pi^{-1/2}t^{-5/2}(\frac{1}{4}\alpha-\frac{1}{2}t)e^{-\frac{1}{4}\alpha/t}$
(6)	$p^{-1/2}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha\geq 0$	$\pi^{-1/2}t^{-1/2}e^{-\frac{1}{4}\alpha/t}$
(7)	$p^{-3/2}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha\geq 0$	$2\pi^{-1/2}t^{1/2}e^{-\frac{1}{4}\alpha/t}$ $-\alpha^{1/2}\,\mathrm{Erfc}(\frac{1}{2}\alpha^{1/2}t^{-1/2})$
(8)	$p^{\frac{1}{2}n-\frac{1}{2}}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha>0$	$2^{-\frac{1}{2}n}\pi^{-1/2}t^{-\frac{1}{2}n-\frac{1}{2}}e^{-\frac{1}{4}\alpha/t}$ $\times\,\mathrm{He}_n(2^{-1/2}\alpha^{1/2}t^{-1/2})$
(9)	$p^{\nu-1/2}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha>0$	$2^{-\nu}\pi^{-1/2}t^{-\nu-1/2}e^{-\alpha t^{-1}/8}$ $\times D_{2\nu}(2^{-1/2}\alpha^{1/2}t^{-1/2})$
(10)	$2(p+\beta)^{-1}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha\geq 0$	$e^{-\beta t}[e^{-i\alpha^{1/2}\beta^{1/2}}\,\mathrm{Erfc}(\frac{1}{2}\alpha^{1/2}t^{-1/2}-i\beta^{1/2}t^{1/2})$ $+e^{i\alpha^{1/2}\beta^{1/2}}\,\mathrm{Erfc}(\frac{1}{2}\alpha^{1/2}t^{-1/2}+i\beta^{1/2}t^{1/2})]$
(11)	$p(p^2+\beta^2)^{-1}e^{-\alpha^{1/2}p^{1/2}}$ Re $\alpha\geq 0$, Re $\beta\geq 0$	$\exp[-(\frac{1}{2}\alpha\beta)^{1/2}]\cos[\beta t-(\frac{1}{2}\alpha\beta)^{1/2}]$ $-\dfrac{1}{\pi}\displaystyle\int_0^\infty e^{-ut}\sin(\alpha^{1/2}u^{1/2})\frac{u}{u^2+\beta^2}\,du$
(12)	$(p^{1/2}+\beta)^{-1}e^{-\alpha p^{1/2}}$ Re $\alpha^2\geq 0$	$\pi^{-1/2}t^{-1/2}e^{-\frac{1}{4}\alpha^2/t}$ $-\beta e^{\alpha\beta+\beta^2t}\,\mathrm{Erfc}(\frac{1}{2}\alpha t^{-1/2}+\beta t^{1/2})$
(13)	$p(p^{1/2}+\beta)^{-1}e^{-\alpha p^{1/2}}$ Re $\alpha^2>0$	$\pi^{-1/2}t^{-3/2}(\frac{1}{4}\alpha^2t^{-1}-\frac{1}{2}$ $-\frac{1}{2}\alpha\beta+\beta^2t)e^{-\frac{1}{4}\alpha^2/t}$ $-\beta^3e^{\alpha\beta+\beta^2t}\,\mathrm{Erfc}(\frac{1}{2}\alpha t^{-1/2}+\beta t^{1/2})$
(14)	$\beta p^{-1}(p^{1/2}+\beta)^{-1}e^{-\alpha p^{1/2}}$ Re $\alpha^2\geq 0$	$\mathrm{Erfc}(\frac{1}{2}\alpha t^{-1/2})$ $-e^{\alpha\beta+\beta^2t}\,\mathrm{Erfc}(\frac{1}{2}\alpha t^{-1/2}+\beta t^{1/2})$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(15)	$p^{1/2}(p^{1/2}+\beta)^{-1}e^{-ap^{1/2}}$ $\mathrm{Re}\ a^2\ge 0$	$\pi^{-1/2}t^{-1}(\tfrac{1}{2}at^{-1/2}-\beta t^{1/2})e^{-\frac{1}{4}a^2/t}$ $+\beta^2 e^{a\beta+\beta^2 t}\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
(16)	$p^{-1/2}(p^{1/2}+\beta)^{-1}e^{-ap^{1/2}}$ $\mathrm{Re}\ a^2\ge 0$	$e^{a\beta+\beta^2 t}\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
(17)	$p^{-3/2}(p^{1/2}+\beta)^{-1}e^{-ap^{1/2}}$ $\mathrm{Re}\ a^2\ge 0$	$2\pi^{-1/2}\beta^{-1}t^{1/2}e^{-\frac{1}{4}a^2/t}$ $-(\beta^{-2}+a\beta^{-1})\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2})$ $+\beta^{-2}e^{a\beta+\beta^2 t}\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
(18)	$p^{-1}(p^{1/2}+\beta)^{-2}e^{-ap^{1/2}}$ $\mathrm{Re}\ a^2>0$	$\beta^{-2}\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2})$ $-2\pi^{-1/2}\beta^{-1}t^{1/2}e^{-\frac{1}{4}a^2/t}$ $+(2t+a\beta^{-1}-\beta^{-2})e^{a\beta+\beta^2 t}$ $\times\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
(19)	$p^{-1/2}(p^{1/2}+\beta)^{-2}e^{-ap^{1/2}}$ $\mathrm{Re}\ a^2>0$	$2\pi^{-1/2}t^{1/2}e^{-\frac{1}{4}a^2/t}-(2\beta t+a)e^{a\beta+\beta^2 t}$ $\times\,\mathrm{Erfc}(\tfrac{1}{2}at^{-1/2}+\beta t^{1/2})$
	See also Campbell, G. A. and R. M. Foster, 1931: *Fourier integrals for practical applications*, Bell Telephone Laboratories, New York	
(20)	$(p-1)^{-1/2}e^{-ap^{1/2}}$ $\mathrm{Re}\ a^2>0$	$\pi^{-1/2}t^{-1/2}e^{t}\{\exp(-\tfrac{1}{4}a^2/t)$ $-a\int_0^\infty e^{-\frac{1}{4}u^2/t}(u^2-a^2)^{-1/2}$ $\times J_1[(u^2-a^2)^{1/2}]\,du\}$
(21)	$p^{-\frac{1}{2}\nu-\frac{1}{2}}e^{-ap^{1/2}}$ $\mathrm{Re}\ a>0,\quad \mathrm{Re}\ \nu>-1$	$\tfrac{1}{2}\pi^{-1/2}a^{-\nu/2}t^{-3/2}$ $\times\int_0^\infty u^{1+\nu/2}e^{-\frac{1}{4}u^2/t}$ $\times J_\nu(2a^{1/2}u^{1/2})\,du$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(22)	$p^{-\nu-1}\exp[-a^{-1}p^{-1}(p^2+1)^{1/2}]$ $\quad \mathrm{Re}\,\nu>-1$	$a^{-\frac{1}{2}\nu}t^{\frac{1}{2}\nu}\left\{J_\nu(2a^{1/2}t^{1/2}) - a\int_0^\infty \frac{J_\nu(2a^{1/2}t^{1/2})J_1[(u^2-a^2)^{1/2}]}{(u^2-a^2)^{1/2}}du\right\}$
(23)	$e^{-bp}-e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $aby^{-1}J_1(ay) \quad t>b$
(24)	$r^{-1}e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $J_0(ay) \quad t>b$
(25)	$(1-pr^{-1})e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $a\left(\frac{t-b}{t+b}\right)^{1/2}J_1(ay) \quad t>b$
(26)	$r^{-2}(b+r^{-1})e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $a^{-1}y\,J_1(ay) \quad t>b$
(27)	$e^{-bp}-pr^{-1}e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $aty^{-1}J_1(ay) \quad t>b$
(28)	$r^{-1}R^{1/2}e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $2^{1/2}\pi^{-1/2}(t+b)^{-1/2}\cos(ay) \quad t>b$
(29)	$ar^{-1}R^{-1/2}e^{-br}$ $\quad b>0$	$0 \quad 0<t<b$ $2^{1/2}\pi^{-1/2}(t+b)^{-1/2}\sin(ay) \quad t>b$

$y=(t^2-b^2)^{1/2}, \quad r=(p^2+a^2)^{1/2}, \quad R=p+r$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(30)	$a^{\nu} r^{-1} R^{-\nu} e^{-br}$ $\quad b>0$, $\mathrm{Re}\,\nu>-1$	$0 \quad 0<t<b$ $(t-b)^{\frac{1}{2}\nu}(t+b)^{-\frac{1}{2}\nu} J_{\nu}(a\gamma) \quad t>b$
(31)	$1-e^{-\beta(r-p)}$	$a\beta(t^2+2\beta t)^{-\frac{1}{2}} J_1[a(t^2+2\beta t)^{\frac{1}{2}}]$
(32)	$r^{-1} R^{\frac{1}{2}} e^{\beta(p-r)}$	$2^{\frac{1}{2}}\pi^{-\frac{1}{2}}(t+2\beta)^{-\frac{1}{2}}\cos[a(t^2+2\beta t)^{\frac{1}{2}}]$
(33)	$ar^{-1} R^{-\frac{1}{2}} e^{\beta(p-r)}$	$2^{\frac{1}{2}}\pi^{-\frac{1}{2}}(t+2\beta)^{-\frac{1}{2}}\sin[a(t^2+2\beta t)^{\frac{1}{2}}]$
(34)	$a^{\nu} r^{-1} R^{-\nu} e^{-\beta(r-p)}$ $\quad \mathrm{Re}\,\nu>-1$	$t^{\frac{1}{2}\nu}(t+2\beta)^{-\frac{1}{2}\nu} J_{\nu}[a(t^2+2\beta t)^{\frac{1}{2}}]$
(35)	$e^{-bs}-e^{-bp}$ $\quad b>0$	$0 \quad 0<t<b$ $ab\gamma^{-1} I_1(a\gamma) \quad t>b$
(36)	$s^{-1} e^{-bs}$ $\quad b>0$	$0 \quad 0<t<b$ $I_0(a\gamma) \quad t>b$
(37)	$ps^{-1} e^{-bs}-e^{-bp}$ $\quad b>0$	$0 \quad 0<t<b$ $at\gamma^{-1} I_1(a\gamma) \quad t>b$
(38)	$(ps^{-1}-1)e^{-bs}$ $\quad b>0$	$0 \quad 0<t<b$ $a\left(\dfrac{t-b}{t+b}\right)^{\frac{1}{2}} I_1(a\gamma) \quad t>b$
(39)	$s^{-2}(b+s^{-1})e^{-bs}$ $\quad b>0$	$0 \quad 0<t<b$ $a^{-1}\gamma\, I_1(a\gamma) \quad t>b$

$\gamma=(t^2-b^2)^{\frac{1}{2}},\quad r=(p^2+a^2)^{\frac{1}{2}},\quad R=p+r,$
$s=(p^2-a^2)^{\frac{1}{2}},\quad S=p+s$

Exponential functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(40)	$a^\nu s^{-1}S^{-\nu}e^{-bs}$ $\quad b>0$, $\operatorname{Re}\nu>-1$	$0 \quad 0<t<b$; $(t-b)^{\frac12\nu}(t+b)^{-\frac12\nu}I_\nu(a\gamma) \quad t>b$
(41)	$e^{\beta(p-s)}-1$	$a\beta(t^2+2\beta t)^{-\frac12}I_1[a(t^2+2\beta t)^{\frac12}]$
(42)	$s^{-1}e^{\beta(p-s)}$	$I_0[a(t^2+2\beta t)]$
(43)	$1-ps^{-1}e^{\beta(p-s)}$	$-a(t+\beta)(t^2+2\beta t)^{-\frac12}I_1[a(t^2+2\beta t)^{\frac12}]$
(44)	$a^\nu s^{-1}S^{-\nu}e^{\beta(p-s)}$ $\quad \operatorname{Re}\nu>-1$	$t^{\frac12\nu}(t+2\beta)^{-\frac12\nu}I_\nu[a(t^2+2\beta t)^{\frac12}]$
(45)	$(p+a)^{-\frac12}(p+\beta)^{-\frac12}[p+\frac12(a+\beta)$ $+(p+a)^{\frac12}(p+\beta)^{\frac12}]^{-\nu}$ $\times\exp[-b(p+a)^{\frac12}(p+\beta)^{\frac12}]$ $\quad \operatorname{Re}\nu>-1,\quad b>0$	$0 \quad 0<t<b$; $[\frac12(a-\beta)]^{-\nu}(t-b)^{\frac12\nu}(t+b)^{-\frac12\nu}$ $\times e^{-\frac12(a+\beta)t}I_\nu[\frac12(a-\beta)\gamma] \quad t>b$
(46)	$(p+a)^{-\frac12}(p+\beta)^{-\frac12}[p+\frac12(a+\beta)$ $+(p+a)^{\frac12}(p+\beta)^{\frac12}]^{-\nu}$ $\times\exp[\gamma p-\gamma(p+a)^{\frac12}(p+\beta)^{\frac12}]$ $\quad \operatorname{Re}\nu>-1$	$[\frac12(a-\beta)]^{-\nu}t^{\frac12\nu}(t+2\gamma)^{-\frac12\nu}$ $\times\exp[-\frac12(a+\beta)(t+\gamma)]$ $\times I_\nu[\frac12(a-\beta)(t^2+2\gamma t)^{\frac12}]$

5.7. Logarithmic functions

(1)	$p^{-1}\log p$	$-\log(\gamma t)$
(2)	$p^{-n-1}\log p$	$[1+1/2+1/3+\cdots+1/n-\log(\gamma t)]t^n/n!$
(3)	$p^{-n-\frac12}\log p$	$\dfrac{2^n t^{n-\frac12}}{1\cdot3\cdot5\cdots(2n-1)\pi^{\frac12}}\left[2\left(1+\dfrac13+\cdots+\dfrac{1}{2n-1}\right)-\log(4\gamma t)\right]$

$\gamma=(t^2-b^2)^{\frac12}$; $\quad s=(p^2-a^2)^{\frac12}$, $\quad S=p+s$

Logarithmic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(4)	$\Gamma(\nu)p^{-\nu}\log p \qquad \mathrm{Re}\,\nu>0$	$t^{\nu-1}[\psi(\nu)-\log t]$
(5)	$\dfrac{\log(p+\beta)}{p+a}$	$e^{-at}\{\log(\beta-a)-\mathrm{Ei}[(a-\beta)t]\}$
(6)	$a(p^2+a^2)^{-1}\log p$	$\cos(at)\,\mathrm{Si}(at)$ $+\sin(at)[\log a-\mathrm{Ci}(at)]$
(7)	$p(p^2+a^2)^{-1}\log p$	$\cos(at)[\log a-\mathrm{Ci}(at)]$ $-\sin(at)\,\mathrm{Si}(at)$
(8)	$p^{-1}(\log p)^2$	$[\log(\gamma t)]^2-\pi^2/6$
(9)	$p^{-1}[\log(\gamma p)]^2$	$(\log t)^2-\pi^2/6$
(10)	$p^{-2}(\log p)^2$	$t\{[1-\log(\gamma t)]^2+1-\pi^2/6\}$
(11)	$p^{-a}(\log p)^{-1} \qquad a\geq 0$	$\nu(t,a-1)$
(12)	$\log\dfrac{p+\beta}{p+a}$	$\dfrac{e^{-at}-e^{-\beta t}}{t}$
(13)	$p\log\dfrac{p+a}{p+\beta}+\beta-a$	$(at^{-1}+t^{-2})e^{-at}-(\beta t^{-1}+t^{-2})e^{-\beta t}$
(14)	$p^{-1}\log(p^2+a^2)$	$2\,\mathrm{ci}(at)+2\log a$
(15)	$\log\dfrac{p^2+\beta^2}{p^2+a^2}$	$\dfrac{2}{t}[\cos(at)-\cos(\beta t)]$
(16)	$p\log\dfrac{p^2+\beta^2}{p^2+a^2}$	$\dfrac{2}{t^2}[\cos(\beta t)+\beta t\sin(\beta t)$ $-\cos(at)-at\sin(at)]$

Logarithmic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(17)	$\log\dfrac{(p+\alpha)^2+\lambda^2}{(p+\beta)^2+\lambda^2}$	$2t^{-1}\cos(\lambda t)(e^{-\beta t}-e^{-\alpha t})$
(18)	$\dfrac{\log[(p+\alpha)^{1/2}+(p+\beta)^{1/2}]}{(p+\beta)^{1/2}}$	$\dfrac{e^{-\beta t}}{2\pi^{1/2}t^{1/2}}\{\log(\alpha-\beta)-\mathrm{Ei}[(\beta-\alpha)t]\}$
(19)	$p^{-1}\log r$	$\log a+\mathrm{ci}(at)$
(20)	$p^{-2}\log r$ $\qquad$ $\mathrm{Re}\,a>0$	$t[\log a+a^{-1}t^{-1}\sin(at)+\mathrm{ci}(at)]$
(21)	$ar^{-2}\log r$	$\frac{1}{2}\sin(at)\left[\log\left(\dfrac{2a}{\gamma t}\right)-\mathrm{Ci}(2at)\right]$ $+\frac{1}{2}\cos(at)\,\mathrm{Si}(2at)$
(22)	$pr^{-2}\log r$	$\frac{1}{2}\cos(at)\left[\log\dfrac{2a}{\gamma t}-\mathrm{Ci}(2at)\right]$ $-\frac{1}{2}\sin(at)\,\mathrm{Si}(2at)$
(23)	$p\log(r/p)$	$t^{-2}[\cos(at)-1]+at^{-1}\sin(at)$
(24)	$r\log\dfrac{a+r}{p}-a$	$\frac{1}{2}\pi at^{-1}\mathbf{H}_1(at)$
(25)	$pr^{-1}\log\dfrac{a+r}{p}$	$a-\frac{1}{2}\pi a\,\mathbf{H}_1(at)$
(26)	$r^{-1}\log R$	$\log a\,J_0(at)-\frac{1}{2}\pi Y_0(at)$
(27)	$ar^{-3}\log R$	$t\,[J_1(at)\log a-\frac{1}{2}\pi Y_1(at)]$ $-a^{-1}\cos(at)$

Logarithmic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(28)	$pr^{-3}\log R$	$t\{J_0(at)\log a-\frac{1}{2}\pi Y_0(at)]$ $+a^{-1}\sin(at)$
(29)	$p\log(s/p)$	$t^{-2}[\cosh(at)-1]-at^{-1}\sinh(at)$
(30)	$s^{-1}\log S$	$I_0(at)\log a+K_0(at)$
(31)	$as^{-3}\log S$	$t[I_1(at)\log a-K_1(at)]$ $+a^{-1}\cosh(at)$
(32)	$ps^{-3}\log S$	$t[I_0(at)\log a+K_0(at)]$ $+a^{-1}\sinh(at)$
(33)	$2(A^2-1)^{-\frac{1}{2}}(B^2-1)^{-\frac{1}{2}}$ $\times\log\dfrac{(A+1)(B+1)+(A^2-1)^{\frac{1}{2}}(B^2-1)^{\frac{1}{2}}}{(A+1)(B+1)-(A^2-1)^{\frac{1}{2}}(B^2-1)^{\frac{1}{2}}}$ where $A^2=p+\beta,\quad B^2=p-\beta$	$-e^{t}I_0(\beta t)\operatorname{Ei}(-t)$

5.8. Trigonometric functions

(1)	$p^{-1}\sin(ap^{-1})$	$\operatorname{bei}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(2)	$p^{-1}\cos(ap^{-1})$	$\operatorname{ber}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(3)	$p^{-\frac{1}{2}}\sin(ap^{-1})$	$\pi^{-\frac{1}{2}}t^{-\frac{1}{2}}\sinh(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})\sin(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})$
(4)	$p^{-\frac{1}{2}}\cos(ap^{-1})$	$\pi^{-\frac{1}{2}}t^{-\frac{1}{2}}\cosh(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})\cos(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})$
(5)	$p^{-3/2}\sin(ap^{-1})$	$\pi^{-\frac{1}{2}}a^{-\frac{1}{2}}\cosh(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})\sin(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})$

Trigonometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(6)	$p^{-3/2}\cos(ap^{-1})$	$\pi^{-1/2} a^{-1/2}\sinh(2^{1/2}a^{1/2}t^{1/2})\cos(2^{1/2}a^{1/2}t^{1/2})$
(7)	$p^{-\nu-1}\sin(ap^{-1})$ Re $\nu>-2$	$(t/a)^{1/2\nu}[\cos(\tfrac{3}{4}\pi\nu)\,\mathrm{bei}_\nu(2a^{1/2}t^{1/2})$ $-\sin(\tfrac{3}{4}\pi\nu)\,\mathrm{ber}_\nu(2a^{1/2}t^{1/2})]$
(8)	$p^{-\nu-1}\cos(ap^{-1})$ Re $\nu>-1$	$(t/a)^{1/2\nu}[\cos(\tfrac{3}{4}\pi\nu)\,\mathrm{ber}_\nu(2a^{1/2}t^{1/2})$ $+\sin(\tfrac{3}{4}\pi\nu)\,\mathrm{bei}_\nu(2a^{1/2}t^{1/2})]$
(9)	$p^{-1/2}e^{-a^{1/2}p^{1/2}}\sin(a^{1/2}p^{1/2})$	$\pi^{-1/2}t^{-1/2}\sin(\tfrac{1}{2}at^{-1})$
(10)	$p^{-1/2}e^{-a^{1/2}p^{1/2}}\cos(a^{1/2}p^{1/2})$	$\pi^{-1/2}t^{-1/2}\cos(\tfrac{1}{2}at^{-1})$
(11)	$p^{-\mu-1/2}\sin(a^{-1/2}p^{-1/2})$ Re $\mu>-1$	$a^{-1/2}[\Gamma(\mu+1)]^{-1}t^{\mu}$ $\times\,{}_0F_2(\mu+1,3/2;-\tfrac{1}{4}t/a)$
(12)	$p^{-\mu-1}\cos(a^{-1/2}p^{-1/2})$ Re $\mu>-1$	$[\Gamma(\mu+1)]^{-1}t^{\mu}\,{}_0F_2(\mu+1,\tfrac{1}{2};-\tfrac{1}{4}t/a)$
(13)	$\Gamma(\nu+\tfrac{1}{2})p^{-\nu}\sin[(2n+1)\sin^{-1}(p^{-1/2})]$ Re $\nu>-\tfrac{1}{2}$	$t^{\nu-1/2}(2n+1)$ $\times\,{}_2F_2(-n,n;\nu+1/2,3/2;t)$
(14)	$\Gamma(\nu)p^{-\nu}\cos[2n\sin^{-1}(p^{-1/2})]$ Re $\nu>0$	$t^{\nu-1}\,{}_2F_2(-n,n;\nu,\tfrac{1}{2};t)$
(15)	$p^{-1}\tan^{-1}p$	$-\mathrm{si}(t)$
(16)	$p^{-1}\mathrm{ctn}^{-1}p$	$\mathrm{Si}(t)$
(17)	$\tan^{-1}(ap^{-1})$	$t^{-1}\sin(at)$

Trigonometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(18)	$p \tan^{-1}(ap^{-1})-a$	$t^{-2}[at\cos(at)-\sin(at)]$
(19)	$\log(p^2+a^2)^{1/2}\tan^{-1}(ap^{-1})$	$-t^{-1}\log(\gamma t)\sin(at)$
(20)	$\dfrac{\sin[\beta+\tan^{-1}(ap^{-1})]}{(p^2+a^2)^{1/2}}$	$\sin(at+\beta)$
(21)	$\dfrac{\cos[\beta+\tan^{-1}(ap^{-1})]}{(p^2+a^2)^{1/2}}$	$\cos(at+\beta)$
(22)	$\tan^{-1}[2ap(p^2+\beta^2)^{-1}]$	$2t^{-1}\sin(at)\cos[(a^2+\beta^2)^{1/2}t]$

5.9. Hyperbolic functions

(1)	$p^{-1}\operatorname{sech}(ap)$ $\quad a>0$	$0 \quad 4n-1<t/a<4n+1$ $2 \quad 4n+1<t/a<4n+3$
(2)	$p^{-2}\operatorname{sech}(ap)$ $\quad a>0$	$t-(-1)^n(t-2an)$ $2n-1<t/a<2n+1$
(3)	$p^{-1}\operatorname{csch}(ap)$ $\quad a>0$	$2n \quad 2n-1<t/a<2n+1$
(4)	$p^{-2}\operatorname{csch}(ap)$ $\quad a>0$	$2n(t-an) \quad 2n-1<t/a<2n+1$
(5)	$a(p^2+a^2)^{-1}\operatorname{csch}(\tfrac{1}{2}\pi p/a)$ $a>0$	$\lvert\cos(at)\rvert-\cos(at)$
(6)	$p^{-1}\tanh(pa)$ $\quad a>0$	$(-1)^{n-1} \quad n-1<\tfrac{1}{2}t/a<n$
(7)	$p^{-2}\tanh(ap)$ $\quad a>0$	$a+(-1)^n(2an-a-t)$ $n-1<\tfrac{1}{2}t/a<n$

Hyperbolic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(8)	$p^{-1}\,\mathrm{ctnh}\,(ap)$ $\qquad a>0$	$2n-1$ $\qquad n-1<\tfrac{1}{2}t/a<n$
(9)	$p^{-2}\,\mathrm{ctnh}\,(ap)$ $\qquad a>0$	$(2n-1)t-2an(n-1)$ $\qquad n-1<\tfrac{1}{2}t/a<n$
(10)	$a(p^2+a^2)^{-1}\,\mathrm{ctnh}\,(\tfrac{1}{2}\pi p/a)$ $\qquad a>0$	$\lvert\sin(at)\rvert$
(11)	$p^{-1/2}\sinh(ap^{-1})$	$\tfrac{1}{2}\pi^{-1/2}t^{-1/2}[\cosh(2a^{1/2}t^{1/2}) - \cos(2a^{1/2}t^{1/2})]$
(12)	$p^{-3/2}\sinh(ap^{-1})$	$\tfrac{1}{2}a^{-1/2}\pi^{-1/2}[\sinh(2a^{1/2}t^{1/2}) - \sin(2a^{1/2}t^{1/2})]$
(13)	$p^{-5/2}\sinh(ap^{-1})$	$\tfrac{1}{2}a^{-1}\pi^{-1/2}t^{1/2}[\cosh(2a^{1/2}t^{1/2}) + \cos(2a^{1/2}t^{1/2})]-\tfrac{1}{4}a^{-3/2}\pi^{-1/2} \times[\sinh(2a^{1/2}t^{1/2})+\sin(2a^{1/2}t^{1/2})]$
(14)	$p^{-\nu-1}\sinh(ap^{-1})$ $\qquad \mathrm{Re}\,\nu>-2$	$\tfrac{1}{2}a^{-\frac{1}{2}\nu}t^{\frac{1}{2}\nu}[I_\nu(2a^{1/2}t^{1/2})-J_\nu(2a^{1/2}t^{1/2})]$
(15)	$p^{-1/2}\cosh(ap^{-1})$	$\tfrac{1}{2}\pi^{-1/2}t^{-1/2}[\cos(2a^{1/2}t^{1/2}) + \cosh(2a^{1/2}t^{1/2})]$
(16)	$p^{-3/2}\cosh(ap^{-1})$	$\tfrac{1}{2}a^{-1/2}\pi^{-1/2}[\sinh(2a^{1/2}t^{1/2}) + \sin(2a^{1/2}t^{1/2})]$
(17)	$p^{-5/2}\cosh(ap^{-1})$	$\tfrac{1}{2}a^{-1}\pi^{-1/2}t^{1/2}[\cosh(2a^{1/2}t^{1/2}) - \cos(2a^{1/2}t^{1/2})]-\tfrac{1}{4}a^{-3/2}\pi^{-1/2} \times[\sinh(2a^{1/2}t^{1/2})-\sin(2a^{1/2}t^{1/2})]$
(18)	$p^{-\nu-1}\cosh(ap^{-1})$ $\qquad \mathrm{Re}\,\nu>-1$	$\tfrac{1}{2}a^{-\frac{1}{2}\nu}t^{\frac{1}{2}\nu}[I_\nu(2a^{1/2}t^{1/2})+J_\nu(2a^{1/2}t^{1/2})]$

Hyperbolic functions (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(19)	$\operatorname{sech}(p^{1/2})$	$-\left[\frac{\partial}{\partial v}\theta_1(\tfrac{1}{2}v\|i\pi t)\right]_{v=0}$
(20)	$p^{-1/2}\operatorname{sech}(p^{1/2})$	$\hat{\theta}_2(\tfrac{1}{2}\|i\pi t)$
(21)	$\operatorname{csch}(p^{1/2})$	$-\left[\frac{\partial}{\partial v}\hat{\theta}_4(\tfrac{1}{2}v\|i\pi t)\right]_{v=0}$
(22)	$p^{-1/2}\operatorname{csch}(p^{1/2})$	$\theta_4(0\|i\pi t)$
(23)	$p^{-1}\tanh(p^{1/2})$	$\int_0^1 \hat{\theta}_2(\tfrac{1}{2}v\|i\pi t)\,dv$
(24)	$p^{-1/2}\tanh(p^{1/2})$	$\theta_2(0\|i\pi t)$
(25)	$p^{-1/2}\tanh(p^{1/2}+a)$	$e^{a^2 t}[\theta_3(at\|i\pi t)+\hat{\theta}_3(at\|i\pi t)]-\pi^{-1/2}t^{-1/2}$
(26)	$p^{-1/2}\operatorname{ctnh}(p^{1/2})$	$\theta_3(0\|i\pi t)$
(27)	$\frac{\sinh(xp)}{p\cosh(ap)}$ $\quad 0 \le x \le a$	$\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n-\tfrac{1}{2}}\sin[(n-\tfrac{1}{2})\pi x/a]$ $\times\sin[(n-\tfrac{1}{2})\pi t/a]$
(28)	$\frac{\cosh(xp)}{p\cosh(ap)}$ $\quad -a \le x \le a$	$1+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n}}{n-\tfrac{1}{2}}\cos[(n-\tfrac{1}{2})\pi x/a]$ $\times\cos[(n-\tfrac{1}{2})\pi t/a]$

Hyperbolic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(29)	$\dfrac{\sinh(xp)}{p^2\cosh(ap)}$ $\quad 0\le x\le a$	$x+\dfrac{2a}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{(n-\frac12)^2}\sin[(n-\tfrac12)\pi x/a]$ $\times\cos[(n-\tfrac12)\pi t/a]$
(30)	$\dfrac{\cosh(xp)}{p^2\cosh(ap)}$ $\quad -a\le x\le a$	$t+\dfrac{2a}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{(n-\frac12)^2}\cos[(n-\tfrac12)\pi x/a]$ $\times\sin[(n-\tfrac12)\pi t/a]$
(31)	$\dfrac{\sinh(xp^{1/2})}{\sinh(lp^{1/2})}$ $\quad -l<x<l$	$\dfrac{1}{l}\dfrac{\partial}{\partial x}\;\theta_4\left(\dfrac{x}{2l}\middle\|\dfrac{i\pi t}{l^2}\right)$
(32)	$\dfrac{\sinh(xp^{1/2})}{p^{1/2}\sinh(lp^{1/2})}$ $\quad -l\le x\le l$	$-\dfrac{1}{l}\;\hat{\theta}_4\left(\dfrac{x}{2l}\middle\|\dfrac{i\pi t}{l^2}\right)$
(33)	$\dfrac{\sinh(xp^{1/2})}{\cosh(lp^{1/2})}$ $\quad -l<x<l$	$-\dfrac{1}{l}\dfrac{\partial}{\partial x}\;\hat{\theta}_1\left(\dfrac{x}{2l}\middle\|\dfrac{i\pi t}{l^2}\right)$
(34)	$\dfrac{\sinh(xp^{1/2})}{p^{1/2}\cosh(lp^{1/2})}$ $\quad -l\le x\le l$	$-\dfrac{1}{l}\;\theta_1\left(\dfrac{x}{2l}\middle\|\dfrac{i\pi t}{l^2}\right)$
(35)	$\dfrac{\cosh(xp^{1/2})}{\sinh(lp^{1/2})}$ $\quad -l\le x\le l$	$-\dfrac{1}{l}\dfrac{\partial}{\partial x}\;\hat{\theta}_4\left(\dfrac{x}{2l}\middle\|\dfrac{i\pi t}{l^2}\right)$
(36)	$\dfrac{\cosh(xp^{1/2})}{p^{1/2}\sinh(lp^{1/2})}$ $\quad -l\le x\le l$	$\dfrac{1}{l}\;\theta_4\left(\dfrac{x}{2l}\middle\|\dfrac{i\pi t}{l^2}\right)$
(37)	$\dfrac{\cosh(xp^{1/2})}{\cosh(lp^{1/2})}$ $\quad -l\le x\le l$	$-\dfrac{1}{l}\dfrac{\partial}{\partial x}\;\theta_1\left(\dfrac{x}{2l}\middle\|\dfrac{i\pi t}{l^2}\right)$

Hyperbolic functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(38)	$\dfrac{\cosh(xp^{1/2})}{p^{1/2}\cosh(lp^{1/2})}\qquad -l\le x\le l$	$-\dfrac{1}{l}\,\hat{\theta}_1\left(\dfrac{x}{2l},\dfrac{t}{l^2}\right)$
(39)	$\dfrac{1}{p-i\omega}\,\dfrac{\sinh(xp^{1/2})}{\sinh(lp^{1/2})}\qquad l\ge x>0$	$\dfrac{\sinh(xi^{1/2}\omega^{1/2})}{\sinh(li^{1/2}\omega^{1/2})}\,e^{i\omega t}$ $+2\pi\sum_{n=1}^{\infty}\dfrac{n(-1)^n\sin(n\pi x/l)}{n^2\pi^2+i\omega l^2}$ $\times e^{-n^2\pi^2 t/l^2}$
(40)	$\dfrac{1}{p-i\omega}\,\dfrac{\cosh(xp^{1/2})}{\cosh(lp^{1/2})}$	$\dfrac{\cosh(xi^{1/2}\omega^{1/2})}{\cosh(xi^{1/2}\omega^{1/2})}\,e^{i\omega t}$ $-2p\sum_{n=0}^{\infty}\dfrac{(n+\frac{1}{2})(-1)^n\cos[(n+\frac{1}{2})\pi x/l]}{(n+\frac{1}{2})^2\pi^2+i\omega l^2}$ $\times e^{-(n+\frac{1}{2})^2\pi^2 t/l^2}$
(41)	$p^{-1}\sinh^{-1}(p/a)$	$-Ji_0(at)$
(42)	$(p^2+a^2)^{-1/2}\sinh^{-1}(p/a)$	$-\frac{1}{2}\pi Y_0(at)$
(43)	$(p^2-a^2)^{-1/2}\cosh^{-1}(p/a)$	$K_0(at)$
(44)	$\operatorname{ctnh}^{-1}(p/a)$	$t^{-1}\sin(at)$
(45)	$p^{-1}(\sinh^{-1}p)^2$	$\int_t^\infty \tau^{-1}Y_0(\tau)\,d\tau$

5.10. Orthogonal polynomials

(1)	$(p+\beta)^{-n-1}P_n\left(\dfrac{p+a}{p+\beta}\right)$	$\dfrac{t^n}{n!}\,e^{-\beta t}L_n[\frac{1}{2}(\beta-a)t]$

Orthogonal polynomials (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(2)	$(p+\beta)^{-\nu}P_n\left(\frac{p+\alpha}{p+\beta}\right)$ Re $\nu>0$	$\frac{t^{\nu-1}}{\Gamma(\nu)}e^{-\beta t}\,{}_2F_2[-n,n+1;\,1,\nu;\tfrac{1}{2}(\beta-\alpha)t]$
(3)	$\frac{(p-\alpha-\beta)^n}{p^{n+1}} \times P_n\left[\frac{p^2-(\alpha+\beta)p+2\alpha\beta}{p(p-\alpha-\beta)}\right]$	$L_n(\alpha t)\,L_n(\beta t)$
(4)	$n!\,p^{-1/2}P_n(p^{-1})$	$i^{-n}\pi^{-1/2}t^{-1/2}\,\mathrm{He}_n(2^{1/2}t^{1/2})\,\mathrm{He}_n(i\,2^{1/2}t^{1/2})$
(5)	$\frac{(\alpha+\beta-p)^{1/2 n}}{(\alpha+\beta+p)^{1/2 n+1/2}} \times P_n\left\{\frac{2\alpha^{1/2}\beta^{1/2}}{[(\alpha+\beta)^2-p^2]^{1/2}}\right\}$	$\frac{e^{-2\alpha t}\mathrm{He}_n(2\alpha^{1/2}t^{1/2})\,\mathrm{He}_n(2\beta^{1/2}t^{1/2})}{n!\,\pi^{1/2}t^{1/2}}$
(6)	$(p+\beta)^{-\mu}C_n^{\nu}\left(\frac{p+\alpha}{p+\beta}\right)$ Re $\mu>0$, Re $\nu>0$	$\frac{t^{\mu-1}e^{-\beta t}}{n\,\mathrm{B}(n,2\nu)\,\Gamma(\mu)} \times {}_2F_2[-n,n+2\nu;\,\mu,\nu+\tfrac{1}{2};\,\tfrac{1}{2}(\beta-\alpha)t]$
(7)	$p^{-n-1/2}e^{-\alpha/p}\,\mathrm{He}_{2n}(2^{1/2}\alpha^{1/2}p^{-1/2})$	$(-2)^n\pi^{-1/2}t^{n-1/2}\cos(2\alpha^{1/2}t^{1/2})$
(8)	$p^{-n-1}e^{-\alpha/p}\,\mathrm{He}_{2n+1}(2^{1/2}\alpha^{1/2}p^{-1/2})$	$(-1)^n2^{n+1/2}\pi^{-1/2}t^n\sin(2\alpha^{1/2}t^{1/2})$
(9)	$p^{-\beta}L_n^{\alpha}\left(\frac{\lambda}{p}\right)$ Re $\beta>0$	$\frac{t^{\beta-1}\,{}_1F_2(-n;\,\alpha+1,\beta;\,\lambda t)}{n\,\Gamma(\beta)\,\mathrm{B}(n,\alpha+1)}$
(10)	$n!\,p^{-n-\alpha-1}e^{-\lambda/p}L_n^{\alpha}(\lambda p^{-1})$ Re $\alpha>-n-1$	$\lambda^{-1/2\alpha}t^{1/2\alpha+n}J_\alpha(2\lambda^{1/2}t^{1/2})$

Orthogonal polynomials (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(11)	$\dfrac{(p-1)^n e^{-\lambda/p}}{p^{n+\alpha+1}} L_n^\alpha\left[\dfrac{\lambda}{p(1-p)}\right]$ Re $\alpha>-1$	$\lambda^{-\frac{1}{2}\alpha} t^{\frac{1}{2}\alpha} L_n^\alpha(t)\, J_\alpha(2\lambda^{\frac{1}{2}} t^{\frac{1}{2}})$
(12)	$n!\,\mathrm{B}(n+\frac{1}{2}, p+\frac{1}{2})\, L_n^p(\lambda)$	$(-2)^{-n}(e^t-1)^{-\frac{1}{2}}\,\mathrm{He}_{2n}\{[2\lambda(1-e^{-t})]^{\frac{1}{2}}\}$
(13)	$n!\,\mathrm{B}(n+3/2, p)\, L_n^p(\lambda)$	$(-1)^n 2^{-n-\frac{1}{2}} \lambda^{-\frac{1}{2}} \pi^{-\frac{1}{2}}$ $\times \mathrm{He}_{2n+1}\{[2\lambda(1-e^{-t})]^{\frac{1}{2}}\}$

5.11. Gamma function, incomplete gamma functions, zeta function and related functions

(1)	$\Gamma(\nu)\,\Gamma(ap)/\Gamma(ap+\nu)$ Re $a>0$, Re $\nu>0$	$a^{-1}(1-e^{-t/a})^{\nu-1}$
(2)	$2^{1-2p}\,\Gamma(2p)\,[\Gamma(p+\lambda+\frac{1}{2})$ $\times\,\Gamma(p-\lambda+\frac{1}{2})]^{-1}$	$\pi^{-1}(1-e^{-t})^{-\frac{1}{2}}\cos[2\lambda\cos^{-1}(e^{-\frac{1}{2}t})]$
(3)	$\dfrac{2^{p-1}\Gamma(\frac{1}{2}p+\frac{1}{2}\nu+\frac{1}{2})\,\Gamma(\frac{1}{2}p-\frac{1}{2}\nu)}{\pi^{\frac{1}{2}}\,\Gamma(p+\mu+1)}$ Re $\mu>-\frac{1}{2}$	$(1-e^{-2t})^{\frac{1}{2}\mu} P_\nu^{-\mu}(e^t)$
(4)	$2^{-p}\,\Gamma(p)\,[\Gamma(\frac{1}{2}p+\frac{1}{2}n+\frac{1}{2})$ $\times\Gamma(\frac{1}{2}p-\frac{1}{2}n+\frac{1}{2})]^{-1}$	$\pi^{-1}(1-e^{-2t})^{-\frac{1}{2}}\, T_n(e^{-t})$
(5)	$\dfrac{\Gamma(p+a)}{\Gamma(p+\beta)}(\gamma+p)_n$ Re $(\beta-a)>n$	$\dfrac{e^{-at}}{\Gamma(\beta-a-n)}(1-e^{-t})^{\beta-a-n-1}$ $\times {}_2F_1(-n, \beta-\gamma-n; \beta-a-n; 1-e^{-t})$

Gamma functions etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(6)	$\dfrac{\Gamma[\frac{1}{2}(p-n-\mu)]\,\Gamma[\frac{1}{2}(p+n-\mu+1)]}{\Gamma[\frac{1}{2}(p+n+\mu)+1]\,\Gamma[\frac{1}{2}(p-n+\mu+1)]}$ Re $\mu>-\frac{1}{2}$	$\dfrac{2^{\mu+1}\pi^{\frac{1}{2}}}{\Gamma(\mu+\frac{1}{2})}\sinh^{\mu}t\,P_n^{-\mu}(\cosh t)$
(7)	$\dfrac{\Gamma(p+\alpha)\,\Gamma(p+\beta)}{\Gamma(p+\gamma)\,\Gamma(p+\delta)}$ Re$(\gamma+\delta-\alpha-\beta)>0$	$\dfrac{e^{-\alpha t}(1-e^{-t})^{\gamma+\delta-\alpha-\beta-1}}{\Gamma(\gamma+\delta-\alpha-\beta)}$ $\times{}_2F_1(\delta-\beta,\gamma-\beta;\gamma+\delta-\alpha-\beta;1-e^{-t})$
(8)	$\log\dfrac{e^{p}\Gamma(p)}{2^{\frac{1}{2}}\pi^{\frac{1}{2}}p^{p-\frac{1}{2}}}$	$\dfrac{1}{t}\left(\dfrac{1}{1-e^{-t}}-\dfrac{1}{t}-\dfrac{1}{2}\right)$
(9)	$\log\dfrac{(p+\alpha)^{\frac{1}{2}}\Gamma(p+\alpha)}{\Gamma(p+\alpha+\frac{1}{2})}$	$\frac{1}{2}t^{-1}e^{-\alpha t}\tanh(\frac{1}{4}t)$
(10)	$\log\dfrac{\Gamma(p+\alpha+\frac{3}{4})}{(p+\alpha)^{\frac{1}{2}}\Gamma(p+\alpha+\frac{1}{4})}$	$\frac{1}{2}t^{-1}e^{-\alpha t}[1-\operatorname{sech}(\frac{1}{4}t)]$
(11)	$\log\dfrac{\Gamma(p+\alpha)\,\Gamma(p+\beta+\frac{1}{2})}{\Gamma(p+\alpha+\frac{1}{2})\,\Gamma(p+\beta)}$	$\dfrac{e^{-\alpha t}-e^{-\beta t}}{t(1+e^{-\frac{1}{2}t})}$
(12)	$\log\dfrac{\Gamma(p+\alpha)\,\Gamma(p+\beta+\gamma)}{\Gamma(p+\alpha+\gamma)\,\Gamma(p+\beta)}$	$\dfrac{(e^{-\alpha t}-e^{-\beta t})(1-e^{-\gamma t})}{t(1-e^{-t})}$
(13)	$p^{-1}\psi(ap)$ Re $a>0$	$-\log[\gamma(e^{t/a}-1)]$
(14)	$\psi(\frac{1}{2}p+\frac{1}{2})-\psi(\frac{1}{2}p)$	$2(1+e^{-t})^{-1}$
(15)	$p^{-1}[\psi(\frac{1}{2}p+\frac{1}{2})-\psi(\frac{1}{2}p)]$	$2\log(\frac{1}{2}+\frac{1}{2}e^{t})$

Gamma functions etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(16)	$\psi(ap+\beta)-\psi(ap+\gamma)$ $\operatorname{Re} a>0$	$a^{-1}(e^{-\gamma t/a}-e^{-\beta t/a})(1-e^{-t/a})^{-1}$
(17)	$\psi(p+a)+\psi(p+\beta)-\psi(p)$ $-\psi(p+a+\beta)$	$(1-e^{-at})(1-e^{-\beta t})(1-e^{-t})^{-1}$
(18)	$p^{-1}[\psi(p)-\log p]$	$\log[t(e^{t}-1)^{-1}]$
(19)	$\psi(p)-\log p$	$t^{-1}-(1-e^{-t})^{-1}$
(20)	$\dfrac{\Gamma(p)\Gamma(a)}{\Gamma(p+a)}[\psi(p+a)-\psi(p)]$ $\operatorname{Re} a>0$	$t(1-e^{-t})^{a-1}$
(21)	$\log\dfrac{\Gamma(ap+\beta)}{\Gamma(ap+\lambda)}+(\lambda-\beta)\psi(ap+\delta)$ $\operatorname{Re} a>0$	$(1-e^{-t/a})^{-1}[t^{-1}(e^{-\beta t/a}-e^{-\lambda t/a})$ $+a^{-1}(\beta-\lambda)e^{-\delta t/a}]$
(22)	$\psi^{(n)}(ap)$	$(-a)^{-n-1}t^{n}(1-e^{-t/a})^{-1}$
(23)	$p^{-\nu}\gamma(\nu, bp)$ $\operatorname{Re}\nu>0, \quad b>0$	$t^{\nu-1} \quad 0<t<b$ $0 \quad t>b$
(24)	$p^{-\nu}e^{-bp}\gamma(\nu,-bp)$ $\operatorname{Re}\nu>0, \quad b>0$	$(b-t)^{\nu-1} \quad 0<t<b$ $0 \quad t>b$
(25)	$\gamma(\nu, a/p) \quad \operatorname{Re}\nu>0$	$a^{\frac12\nu}t^{\frac12\nu-1}J_\nu(2a^{\frac12}t^{\frac12})$
(26)	$p^{\nu-1}e^{a/p}\gamma(\nu, a/p) \quad \operatorname{Re}\nu>0$	$\Gamma(\nu)a^{\frac12\nu}t^{-\frac12\nu}I_\nu(2a^{\frac12}t^{\frac12})$
(27)	$p^{\nu-3/2}e^{a/p}\gamma(\nu, a/p)$	$\Gamma(\nu)(t/a)^{\frac14-\frac12\nu}\mathbf{L}_{\nu-\frac12}(2a^{\frac12}t^{\frac12})$

Gamma functions etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(28)	$p^{\mu}\gamma(\nu, a/p)$ $\operatorname{Re}\nu>0,\quad \operatorname{Re}(\nu-\mu)>0$	$t^{-\mu-1}\int_0^{at} u^{\frac12\nu+\frac12\mu-\frac12} J_{\nu-\mu-1}(2u^{\frac12})\,du$
(29)	$\nu\Gamma(\nu-\mu)\,p^{\mu}e^{a/p}\gamma(\nu, a/p)$ $\operatorname{Re}\nu>0,\quad \operatorname{Re}\mu>0$	$a^{\nu}t^{\nu-\mu-1}\,{}_1F_2(1;\ \nu+1, \nu-\mu; at)$
(30)	$\gamma[\nu, \frac12(p^2+a^2)^{\frac12}-\frac12 p]\quad \operatorname{Re}\nu>0$	$t^{\frac12\nu-1}(t+1)^{-\frac12\nu}J_{\nu}[at^{\frac12}(t+1)^{\frac12}]$
(31)	$a^{-p}\gamma(p, a)$	$\exp(-ae^{-t})$
(32)	$\Gamma(\nu, bp)\qquad b>0,\quad \operatorname{Re}\nu<1$	$0\qquad 0<t<b$ $\dfrac{b^{\nu}}{\Gamma(1-\nu)t(t-b)^{\nu}}\qquad t>b$
(33)	$\Gamma(1-\nu)e^{ap}\Gamma(\nu, ap)$ $\operatorname{Re}\nu<1$	$a^{\nu}(t+a)^{-1}t^{-\nu}$
(34)	$p^{-\nu}\Gamma(\nu, bp)\qquad b>0$	$0\qquad 0<t<b$ $t^{\nu-1}\qquad t>b$
(35)	$p^{-\nu}e^{ap}\Gamma(\nu, ap)$	$(t+a)^{\nu-1}$
(36)	$a^{\frac12}\Gamma(1-2\nu)e^{ap^2}\Gamma(\nu, ap^2)$ $\operatorname{Re}a>0,\quad \operatorname{Re}\nu<1$	$4^{\nu}e^{i\pi(\nu-\frac12)-\frac14 t^2/a}$ $\times\gamma(\frac12-\nu, \frac14 e^{i\pi}t^2/a)$
(37)	$p^{-\frac12}e^{\frac14 ap^2}\Gamma(\frac14, \frac14 ap^2)$ $\operatorname{Re}a>0$	$\Gamma(\frac14)a^{-\frac12}t^{\frac12}e^{-\frac12 t^2/a}I_{\frac14}(\frac12 t^2/a)$
(38)	$p^{-2\nu}e^{\frac12 a^2p^2}\Gamma(\nu, \frac12 a^2p^2)$	$2^{-\frac12}\pi^{-\frac12}\Gamma(\nu)a^{2\nu-1}e^{-\frac14 t^2/a^2}$ $\times[D_{-2\nu}(-t/a)-D_{-2\nu}(t/a)]$

Gamma functions etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(39)	$\Gamma(1-\nu)p^{\nu-1}e^{a/p}\Gamma(\nu, a/p)$ $\quad \mathrm{Re}\,\nu<1$	$2a^{\frac{1}{2}\nu}t^{-\frac{1}{2}\nu}K_\nu(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(40)	$p^{\nu-3/2}e^{a/p}\Gamma(\nu, a/p)$ $\quad \mathrm{Re}\,\nu<3/2$	$\Gamma(\nu)(a/t)^{\frac{1}{2}\nu-\frac{1}{4}}[I_{\frac{1}{2}-\nu}(2a^{\frac{1}{2}}t^{\frac{1}{2}}) - \mathbf{L}_{\nu-\frac{1}{2}}(2a^{\frac{1}{2}}t^{\frac{1}{2}})]$
(41)	$p^{\mu}\Gamma(\nu, a/p)$ $\quad \mathrm{Re}(\mu+\nu)<-\frac{1}{2},\quad \mathrm{Re}\,\mu>0$	$t^{-\mu-1}\int_{at}^\infty u^{\frac{1}{2}\mu+\frac{1}{2}\nu-\frac{1}{2}}J_{\nu-\mu-1}(2u^{\frac{1}{2}})\,du$
(42)	$a^p\Gamma(-p, a)$ $\quad \mathrm{Re}\,a>0$	e^{-ae^t}
(43)	$p^{-1}\zeta(p)$	$n \quad \log n<t\le\log(n+1)$
(44)	$p^{-1}\zeta(p+a)$	$\sum_{1\le n\le\exp t} n^{-a}$
(45)	$\Gamma(a)p^{-a}\zeta(p)$ $\quad \mathrm{Re}\,a>0$	$\sum_{1\le n\le\exp t}(t-\log n)^{a-1}$
(46)	$p^{-1}\zeta'(p)/\zeta(p)$	$-\psi(e^t)$
(47)	$\Gamma(a)\zeta(a,\beta p)$ $\quad \mathrm{Re}\,a>1,\quad \mathrm{Re}\,\beta>0$	$\beta^{-a}t^{a-1}(1-e^{-t/\beta})^{-1}$

5.12. Error function, exponential integral and related functions

(1)	$e^{ap^2}\,\mathrm{Erfc}(a^{\frac{1}{2}}p)$ $\quad \mathrm{Re}\,a>0$	$\pi^{-\frac{1}{2}}a^{-\frac{1}{2}}e^{-\frac{1}{4}t^2/a}$

Error function etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)dt$	$f(t)$
(2)	$e^{ap^2}\operatorname{Erfc}(a^{1/2}p+\tfrac{1}{2}a^{-1/2}b)$ $\operatorname{Re} a>0, \quad b>0$	$0 \qquad 0<t<b$ $\pi^{-1/2}a^{-1/2}e^{-1/4t^2/a} \qquad t>b$
(3)	$p^{-1}e^{ap^2}\operatorname{Erfc}(a^{1/2}p)$ $\operatorname{Re} a>0$	$\operatorname{Erf}(\tfrac{1}{2}a^{-1/2}t)$
(4)	$(p-a)^{-1}e^{p^2}\operatorname{Erfc}(p)$	$e^{a(t+a)}[\operatorname{Erf}(\tfrac{1}{2}t+a)-\operatorname{Erf}(a)]$
(5)	$1-a^{1/2}\pi^{1/2}pe^{ap^2}\operatorname{Erfc}(a^{1/2}p)$ $\operatorname{Re} a>0$	$\tfrac{1}{2}a^{-1}te^{-1/4t^2/a}$
(6)	$p^{-1}(p+1)^{-1}e^{1/4p^2}\operatorname{Erfc}(\tfrac{1}{2}p)$	$e^{t+1/4}[\operatorname{Erf}(t+\tfrac{1}{2})-\operatorname{Erf}(\tfrac{1}{2})]$
(7)	$p^{-1}e^{p^2}[\operatorname{Erf}(p)-\operatorname{Erf}(p+b)]$ $b>0$	$\operatorname{Erf}(\tfrac{1}{2}t) \qquad 0<t<2b$ $\operatorname{Erf}(b) \qquad t>2b$
(8)	$\operatorname{Erfc}(b^{1/2}p^{1/2}) \qquad b>0$	$0 \qquad 0<t<b$ $\pi^{-1}b^{1/2}t^{-1}(t-b)^{-1/2} \qquad t>b$
(9)	$e^{-bp}-\pi^{1/2}b^{1/2}p^{1/2}\operatorname{Erfc}(b^{1/2}p^{1/2})$ $b>0$	$0 \qquad 0<t<b$ $\tfrac{1}{2}b^{1/2}t^{-3/2} \qquad t>b$
(10)	$p^{-1/2}\operatorname{Erf}(b^{1/2}p^{1/2}) \qquad b>0$	$\pi^{-1/2}t^{-1/2} \qquad 0<t<b$ $0 \qquad t>b$
(11)	$p^{-1/2}\operatorname{Erfc}(b^{1/2}p^{1/2}) \qquad b>0$	$0 \qquad 0<t<b$ $\pi^{-1/2}t^{-1/2} \qquad t>b$
(12)	$e^{ap}\operatorname{Erfc}(a^{1/2}p^{1/2}) \qquad \lvert\arg a\rvert<\pi$	$\pi^{-1}a^{1/2}(t+a)^{-1}t^{-1/2}$

Error function etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(13)	$1-(\pi a p)^{1/2}e^{ap}\,\mathrm{Erfc}\,(a^{1/2}p^{1/2})$ $\lvert\arg a\rvert<\pi$	$\tfrac{1}{2}a^{1/2}(t+a)^{-3/2}$
(14)	$p^{-1/2}e^{ap}\,\mathrm{Erfc}\,(a^{1/2}p^{1/2})$ $\lvert\arg a\rvert<\pi$	$\pi^{-1/2}(t+a)^{-1/2}$
(15)	$\Gamma(\nu+\tfrac{1}{2})\,p^{-\nu}e^{ap}\,\mathrm{Erfc}\,(a^{1/2}p^{1/2})$ $\mathrm{Re}\,\nu>-\tfrac{1}{2},\quad \lvert\arg a\rvert<\pi$	$\pi^{-1/2}a^{-1/2}t^{\nu-1/2}\,{}_2F_1(1,\tfrac{1}{2};\nu+\tfrac{1}{2};-t/a)$
(16)	$\mathrm{Erf}\,(a^{1/2}p^{-1/2})$	$\pi^{-1}t^{-1}\sin(2a^{1/2}t^{1/2})$
(17)	$p^{-1/2}e^{a/p}\,\mathrm{Erf}\,(a^{1/2}p^{-1/2})$	$\pi^{-1/2}t^{-1/2}\sinh(2a^{1/2}t^{1/2})$
(18)	$p^{-3/2}e^{a/p}\,\mathrm{Erf}\,(a^{1/2}p^{-1/2})$	$\pi^{-1/2}a^{-1/2}[\cosh(2a^{1/2}t^{1/2})-1]$
(19)	$\pi^{1/2}p^{-5/2}e^{a/p}\,\mathrm{Erf}\,(a^{1/2}p^{-1/2})$	$a^{-1}t^{1/2}\sinh(2a^{1/2}t^{1/2})-a^{-1/2}t$ $-\tfrac{1}{2}a^{-3/2}[\cosh(2a^{1/2}t^{1/2})-1]$
(20)	$p^{-1/2}e^{a/p}\,\mathrm{Erfc}\,(a^{1/2}p^{-1/2})$	$\pi^{-1/2}t^{-1/2}e^{-2a^{1/2}t^{1/2}}$
(21)	$p^{-3/2}e^{a/p}\,\mathrm{Erfc}\,(a^{1/2}p^{-1/2})$	$a^{-1/2}\pi^{-1/2}(1-e^{-2a^{1/2}t^{1/2}})$
(22)	$p^{-\nu-1}e^{a/p}\,\mathrm{Erfc}\,(a^{1/2}p^{-1/2})$ $\mathrm{Re}\,\nu>-1$	$(t/a)^{\frac{1}{2}\nu}[I_\nu(2a^{1/2}t^{1/2})-\mathbf{L}_\nu(2a^{1/2}t^{1/2})]$
(23)	$p^{-\nu-1}e^{a/p}\,\mathrm{Erf}\,(a^{1/2}p^{-1/2})$ $\mathrm{Re}\,\nu>-1$	$(t/a)^{\frac{1}{2}\nu}\,\mathbf{L}_\nu(2a^{1/2}t^{1/2})$
(24)	$\mathrm{Ei}\,(-bp)$ $\quad b>0$	$0 \quad 0<t<b$ $-1/t \quad t>b$
(25)	$p^{-1}\,\mathrm{Ei}\,(-bp)$ $\quad b>0$	$0 \quad 0<t<b$ $\log(b/t) \quad t>0$

Error function etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(26)	$p^{-1}\operatorname{Ei}[-b(p+a)]$ $b>0, \quad a\neq 0$	$0 \qquad 0<t<b$ $\operatorname{Ei}(-ba)-\operatorname{Ei}(-at) \qquad t>b$
(27)	$p^{-1}[\operatorname{Ei}(-bp)-\log(\gamma p)] \qquad b>0$	$\log t \qquad 0<t<b$ $\log b \qquad t>b$
(28)	$e^{ap}\operatorname{Ei}(-ap) \qquad \lvert\arg a\rvert<\pi$	$-(t+a)^{-1}$
(29)	$p^{-1}e^{ap}\operatorname{Ei}(-ap) \qquad \lvert\arg a\rvert<\pi$	$-\log(1+t/a)$
(30)	$-p^{-1}e^{-bp}\,\overline{\operatorname{Ei}}(bp) \qquad b>0$	$\log\lvert 1-t/b\rvert$
(31)	$pe^{ap}\operatorname{Ei}(-ap)+a^{-1} \qquad \lvert\arg a\rvert<\pi$	$(t+a)^{-2}$
(32)	$e^{-bp}+bp\operatorname{Ei}(-bp) \qquad b>0$	$0 \qquad 0<t<b$ $bt^{-2} \qquad t>b$
(33)	$[\operatorname{Ei}(-\tfrac{1}{2}p)]^2$	$0 \qquad 0<t<1$ $2t^{-1}\log(2t-1) \qquad t>1$
(34)	$\operatorname{Ei}(-ap)\operatorname{Ei}(-bp)$ $a,\ b>0$	$0 \qquad 0<t<a+b$ $t^{-1}\log[a^{-1}b^{-1}(t-a)(t-b)]$ $t>a+b$
(35)	$\overline{\operatorname{Ei}}(p)\operatorname{Ei}(-p)$	$t^{-1}\log\lvert 1-t^2\rvert$
(36)	$e^{bp}[\operatorname{Ei}(-bp)]^2 \qquad b>0$	$0 \qquad 0<t<b$ $2(t+b)^{-1}\log(t/b) \qquad t>b$
(37)	$e^{bp}\{[\operatorname{Ei}(-bp)]^2$ $-2\log b\operatorname{Ei}(-2bp)\} \qquad b>0$	$0 \qquad 0<t<b$ $2(t+b)^{-1}\log t \qquad t>b$

Error function etc. (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(38)	$e^{(\alpha+\beta)p}\,\mathrm{Ei}(-\alpha p)\,\mathrm{Ei}(-\beta p)$ $\quad \lvert\arg(\alpha+\beta)\rvert < \pi$	$(t+\alpha+\beta)^{-1} \times \log[\alpha^{-1}\beta^{-1}(t+\alpha)(t+\beta)]$
(39)	$e^{(\alpha+\beta)p}[\mathrm{Ei}(-\alpha p)\,\mathrm{Ei}(-\beta p) - \log(\alpha\beta)\,\mathrm{Ei}(-\alpha p-\beta p)]$ $\quad \lvert\arg(\alpha+\beta)\rvert < \pi$	$(t+\alpha+\beta)^{-1}\log[(t+\alpha)(t+\beta)]$
(40)	$\exp(\tfrac{1}{4}\alpha^{-2}p^2)\,\mathrm{Ei}(-\tfrac{1}{4}\alpha^{-2}p^2)$ $\quad \lvert\arg\alpha\rvert < \tfrac{1}{4}\pi$	$2i\pi^{-1/2}\,\alpha e^{-\alpha^2 t^2}\,\mathrm{Erf}(i\alpha t)$
(41)	$p^{-1}\,\mathrm{Ei}(-p^{-1})$	$2\,\mathcal{J}i_0(2t^{1/2})$
(42)	$p^{-\nu-1}\,\mathrm{Ei}(-\alpha p^{-1})$ $\quad \mathrm{Re}\,\nu > -1, \quad \mathrm{Re}\,\alpha > 0$	$2t^\nu \int_\infty^{\alpha^{1/2}t^{1/2}} u^{-\nu-1}\,\mathcal{J}_\nu(2u)\,du$
(43)	$-p^{-1}e^{\alpha/p}\,\mathrm{Ei}(-\alpha p^{-1})$	$2K_0(2\alpha^{1/2}t^{1/2})$
(44)	$p^{-\nu-1}e^{\alpha/p}\,\mathrm{Ei}(-\alpha p^{-1})$ $\quad \mathrm{Re}\,\nu > -1$	$t^\nu \int_\infty^{\alpha t} u^{-\frac{1}{2}\nu-1}\,\mathcal{J}_\nu[2(u-\alpha t)^{1/2}]\,du$
(45)	$\mathrm{ci}(\alpha p)\cos(\alpha p) - \mathrm{si}(\alpha p)\sin(\alpha p)$ $\quad \mathrm{Re}\,\alpha > 0$	$t(t^2+\alpha^2)^{-1}$
(46)	$\mathrm{ci}(\alpha p)\sin(\alpha p) + \mathrm{si}(\alpha p)\cos(\alpha p)$ $\quad \mathrm{Re}\,\alpha > 0$	$-\alpha(t^2+\alpha^2)^{-1}$
(47)	$p^{-1}[\mathrm{ci}(\alpha p)\cos(\alpha p) - \mathrm{si}(\alpha p)\sin(\alpha p)]$ $\quad \mathrm{Re}\,\alpha > 0$	$\tfrac{1}{2}\log(1+t^2/\alpha^2)$
(48)	$p^{-1}[\mathrm{ci}(\alpha p)\sin(\alpha p) + \mathrm{si}(\alpha p)\cos(\alpha p)]$ $\quad \mathrm{Re}\,\alpha > 0$	$-\tan^{-1}(t/\alpha)$

Error function etc. (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(49)	$[\mathrm{ci}(ap)]^2+[\mathrm{si}(ap)]^2$	$t^{-1}\log(1+t^2/a^2)$
(50)	$\tfrac12-\cos(\tfrac14 p^2)C(\tfrac14 p^2)$ $-\sin(\tfrac14 p^2)S(\tfrac14 p^2)$	$2^{1/2}\pi^{-1/2}\sin(t^2)$
(51)	$\tfrac12\cos(\tfrac12 p^2)-\cos(\tfrac14 p^2)S(\tfrac14 p^2)$ $+\sin(\tfrac14 p^2)C(\tfrac14 p^2)$	$2^{1/2}\pi^{-1/2}\cos(t^2)$
(52)	$[\tfrac12-C(\tfrac14 p^2)]^2+[\tfrac12-S(\tfrac14 p^2)]^2$	$2\pi^{-1}t^{-1}\sin(t^2)$
(53)	$C_n(a,p)$ $\quad a>0$	$0 \quad 0<t<\cosh a$ $(t^2-1)^{-1/2}\cosh(n\cosh^{-1}t) \quad t>\cosh a$
(54)	$S_n(a,p)$ $\quad a>0$	$0 \quad 0<t<\sinh a$ $(t^2-1)^{-1/2}\cosh(n\sinh^{-1}t) \quad t>\sinh a$

5.13. Legendre functions

(1)	$\pi p^{-1}P_\nu(p)$ $\quad 0<\mathrm{Re}\,\nu<1$	$-t^{-1}\sin(\nu\pi)W_{0,\nu+1/2}(2t)$
(2)	$s^\mu P_\nu^\mu\left(\dfrac{p}{a}\right)$ $\quad \mathrm{Re}\,\mu-1<\mathrm{Re}\,\nu<-\mathrm{Re}\,\mu$	$\dfrac{2^{1/2}a^{1/2}t^{-\mu-1/2}K_{\nu+1/2}(at)}{\pi^{1/2}\Gamma(-\mu+\nu+1)\Gamma(-\mu-\nu)}$
(3)	$s^{-\mu}Q_\nu^\mu\left(\dfrac{p}{a}\right)$ $\quad \mathrm{Re}(\mu+\nu)>-1$	$\dfrac{\pi^{1/2}a^{1/2}\sin[(\mu+\nu)\pi]}{2^{1/2}\sin(\nu\pi)}\,t^{\mu-1/2}I_{\nu+1/2}(at)$
(4)	$Q_\nu\left(\dfrac{p^2+a^2+\beta^2}{2a\beta}\right)$ $\quad \mathrm{Re}\,\nu>-1$	$\pi a^{1/2}\beta^{1/2}J_{\nu+1/2}(at)J_{\nu+1/2}(\beta t)$

$s=(p^2-a^2)^{1/2}$

Legendre functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(5)	$\Gamma(-2\nu)p^{\nu}(a^2-p^2)^{\frac{1}{2}\nu}P_\nu^\nu(a/p)$ $\quad \mathrm{Re}\,\nu<0$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}(t/a)^{-\nu-\frac{1}{2}}[I_{-\nu-\frac{1}{2}}(at)$ $-\mathbf{L}_{-\nu-\frac{1}{2}}(at)]$
(6)	$\Gamma(-2\nu)p^{\nu+1}(a^2-p^2)^{\frac{1}{2}\nu}P_\nu^\nu(a/p)$ $\quad \mathrm{Re}\,\nu<-\frac{1}{2}$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}a(t/a)^{-\nu-\frac{1}{2}}[I_{-\nu-3/2}(at)$ $-\mathbf{L}_{-\nu-3/2}(at)]$
(7)	$p^{-\frac{1}{2}\nu-\frac{1}{2}}(p-a)^{\frac{1}{2}\mu}P_\nu^\mu(a^{\frac{1}{2}}p^{-\frac{1}{2}})$ $\quad \mathrm{Re}\,\mu<1,\quad \mathrm{Re}(\nu-\mu)>-1$	$\dfrac{t^{\frac{1}{2}(\nu-\mu-1)}e^{\frac{1}{2}at}D_{\mu+\nu}(2^{\frac{1}{2}}a^{\frac{1}{2}}t^{\frac{1}{2}})}{\pi^{\frac{1}{2}}2^{\frac{1}{2}(\mu-\nu-1)}\Gamma(\nu-\mu+1)}$
(8)	$s^{-\nu-1}P_\nu^\mu(p/s)\quad \mathrm{Re}(\nu-\mu)>-1$	$[\Gamma(\nu-\mu+1)]^{-1}t^\nu I_{-\mu}(at)$
(9)	$s^{-\nu-1}Q_\nu^\mu(p/s)$ $\quad \mathrm{Re}(\nu\pm\mu)>-1$	$\dfrac{\sin(\mu+\nu)\pi}{\sin(\nu\pi)}\;\dfrac{t^\nu K_\mu(at)}{\Gamma(\nu-\mu+1)}$
(10)	$p^{-\lambda}Q_{2\nu}(p^{\frac{1}{2}})\quad \mathrm{Re}(\lambda+\nu)>-\frac{1}{2}$	$\dfrac{\pi^{\frac{1}{2}}\Gamma(2\nu+1)t^{\lambda+\nu-\frac{1}{2}}}{2^{2\nu+1}\Gamma(2\nu+3/2)\Gamma(\lambda+\nu+1/2)}$ $\times{}_2F_2\left(\nu+\dfrac{1}{2},\nu+1;2\nu+\dfrac{3}{2},\lambda+\nu+\dfrac{1}{2};t\right)$
(11)	$p^{-\frac{1}{2}}[P_{-\frac{1}{4}}^\mu(r/p)]^2\quad \mathrm{Re}\,\mu<\frac{1}{4}$	$2^{\frac{1}{2}-\mu}[\Gamma(\frac{1}{2}-2\mu)]^{-1}t^{-\frac{1}{2}}$ $\times[J_{-\mu}(\frac{1}{2}at)]^2$
(12)	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}p^{-\frac{1}{2}}P_{-\frac{1}{4}}^\mu(r/p)P_{-\frac{1}{4}}^{-\mu}(r/p)$	$t^{-\frac{1}{2}}J_\mu(\frac{1}{2}at)J_{-\mu}(\frac{1}{2}at)$
(13)	$ar^{-1}p^{-\frac{1}{2}}P_{\frac{1}{4}}^\mu(r/p)P_{-\frac{1}{4}}^\mu(r/p)$ $\quad \mathrm{Re}\,\mu<\frac{3}{4}$	$2^{3/2-\mu}[\Gamma(3/2-2\mu)]^{-1}t^{1/2}$ $\times[J_{-\mu}(\frac{1}{2}at)]^2$

$s=(p^2-a^2)^{\frac{1}{2}},\quad r=(p^2+a^2)^{\frac{1}{2}}$

Legendre functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(14)	$\dfrac{\Gamma(p-\mu+\nu+1)\Gamma(p-\mu-\nu)}{\Gamma(p+1)}$ $\times\left(\dfrac{a}{a-2}\right)^{\frac{1}{2}p} P_{\nu}^{\mu-p}(a-1)$ $\mathrm{Re}\,a>0,\quad \mathrm{Re}\,\mu>-1$	$\left[(e^{t}-1)\left(\dfrac{ae^{t}}{a-2}-1\right)\right]^{\frac{1}{2}\mu}$ $\times P_{\nu}^{-\mu}(ae^{t}+1-a)$
(15)	$\Gamma(\tfrac{1}{2}-\mu)\,Q_{p-\frac{1}{2}}^{\mu}(\cosh a)$ $\mathrm{Re}\,\mu<\tfrac{1}{2},\quad a>0$	$0 \qquad 0<t<a$ $2^{-\frac{1}{2}}\pi^{\frac{1}{2}}e^{\mu\pi i}(\sinh a)^{\mu}$ $\times(\cosh t-\cosh a)^{-\mu-\frac{1}{2}} \qquad t>a$
(16)	$\Gamma(\tfrac{1}{2}-\mu)e^{ap}\,Q_{p-\frac{1}{2}}^{\mu}(\cosh a)$ $\mathrm{Re}\,\mu<\tfrac{1}{2},\quad \lvert\arg a\rvert<\pi$	$\pi^{\frac{1}{2}}2^{-\mu-1}e^{\mu\pi i}\sinh^{\mu}a$ $\times[\sinh(\tfrac{1}{2}t)\sinh(a+\tfrac{1}{2}t)]^{-\mu-\frac{1}{2}}$
(17)	$2^{p+1}e^{(p-a)\pi i}(\mu^2-1)^{\frac{1}{2}(p-a)}$ $\times\Gamma(p)\,Q_{p-1}^{a-p}(\mu)$	$\Gamma(a)(1-e^{-t})^{-\frac{1}{2}}\{[\mu+(1-e^{-t})^{\frac{1}{2}}]^{-a}$ $+[\mu-(1-e^{-t})^{\frac{1}{2}}]^{-a}\}$
(18)	$\pi^{\frac{1}{2}}2^{p+\frac{1}{2}}\Gamma(p)(\mu^2-1)^{\frac{1}{4}-\frac{1}{2}p}$ $\times P_{a+p-\frac{1}{2}}^{\frac{1}{2}-p}(\mu)$	$(1-e^{-t})^{-\frac{1}{2}}\{[\mu+(\mu^2-1)^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]^{a}$ $+[\mu-(\mu^2-1)^{\frac{1}{2}}(1-e^{-t})^{\frac{1}{2}}]^{a}\}$
(19)	$\dfrac{\pi^{\frac{1}{2}}\Gamma(2p)\Gamma(2\nu+1)}{2^{p+\nu-1}\Gamma(p+\nu+\frac{1}{2})}e^{-pa}$ $\times P_{\nu-p}^{-\nu-p}[(1-e^{-2a})^{\frac{1}{2}}]$ $\mathrm{Re}\,\nu>-\tfrac{1}{2},\quad a>0$	$0 \qquad 0<t<2a$ $e^{\nu a}(1-e^{-t})^{-\frac{1}{2}}[e^{-a}(1-e^{-t})^{\frac{1}{2}}$ $-e^{-\frac{1}{2}t}(1-e^{-2a})^{\frac{1}{2}}]^{2\nu} \qquad t>2a$

5.14. Bessel functions

(1)	$\pi e^{-ap}[\tfrac{1}{2}\pi\,Y_0(iap)$ $-J_0(iap)\log(\tfrac{1}{2}\gamma)] \qquad a>0$	$\dfrac{\log[4t(2a-t)/a^2]}{t^{\frac{1}{2}}(2a-t)^{\frac{1}{2}}} \qquad 0<t<2a$ $0 \qquad t>2a$

Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(2)	$\cos(ap)\,J_0(ap)+\sin(ap)\,Y_0(ap)$ $\operatorname{Re} a>0$	$\dfrac{2^{3/2}a}{\pi}\,\dfrac{[t+(t^2+4a^2)^{1/2}]^{-1/2}}{t^{1/2}(t^2+4a^2)^{1/2}}$
(3)	$\sin(ap)\,J_0(ap)-\cos(ap)\,Y_0(ap)$ $\operatorname{Re} a>0$	$\dfrac{2^{1/2}}{\pi}\,\dfrac{[t+(t^2+4a^2)^{1/2}]^{1/2}}{t^{1/2}(t^2+4a^2)^{1/2}}$
(4)	$\cos(ap)\,J_1(ap)+\sin(ap)\,Y_1(ap)$ $\operatorname{Re} a>0$	$-\dfrac{2^{5/2}a^2}{\pi}\,\dfrac{[t+(t^2+4a^2)^{1/2}]^{-3/2}}{t^{1/2}(t^2+4a^2)^{1/2}}$
(5)	$\sin(ap)\,J_1(ap)-\cos(ap)\,Y_1(ap)$ $\operatorname{Re} a>0$	$\dfrac{1}{2^{1/2}\pi a}\,\dfrac{[t+(t^2+4a^2)^{1/2}]^{3/2}}{t^{1/2}(t^2+4a^2)^{1/2}}$
(6)	$p^{-\nu}[\cos(ap)\,J_\nu(ap)$ $+\sin(ap)\,Y_\nu(ap)]$ $\operatorname{Re}\nu>-\tfrac{1}{2},\quad \operatorname{Re} a>0$	$-\dfrac{2t^{\nu-\frac{1}{2}}(t^2+4a^2)^{\frac{1}{2}\nu-\frac{1}{4}}}{\pi^{\frac{1}{2}}(2a)^\nu\Gamma(\nu+\frac{1}{2})}$ $\times\sin[(\nu-\tfrac{1}{2})\operatorname{ctn}^{-1}(\tfrac{1}{2}t/a)]$
(7)	$p^{-\nu}[\sin(ap)\,J_\nu(ap)$ $-\cos(ap)\,Y_\nu(ap)]$ $\operatorname{Re}\nu>-\tfrac{1}{2},\quad \operatorname{Re} a>0$	$\dfrac{2t^{\nu-\frac{1}{2}}(t^2+4a^2)^{\frac{1}{2}\nu-\frac{1}{4}}}{\pi^{\frac{1}{2}}(2a)^\nu\Gamma(\nu+\frac{1}{2})}$ $\times\cos[(\nu-\tfrac{1}{2})\operatorname{ctn}^{-1}(\tfrac{1}{2}t/a)]$
(8)	$p^{-\nu}[\cos(ap-\beta)\,J_\nu(ap)$ $+\sin(ap-\beta)\;Y_\nu(ap)]$ $\operatorname{Re}\nu>-\tfrac{1}{2},\quad \operatorname{Re} a>0$	$\dfrac{2t^{\nu-\frac{1}{2}}(t^2+4a^2]^{\frac{1}{2}\nu-\frac{1}{4}}}{\pi^{\frac{1}{2}}(2a)^\nu\Gamma(\nu+\frac{1}{2})}$ $\times\sin[(\tfrac{1}{2}-\nu)\operatorname{ctn}^{-1}(\tfrac{1}{2}t/a)+\beta]$
(9)	$p^{-\nu}e^{-iap}H_\nu^{(1)}(ap)\quad \operatorname{Re}\nu>-\tfrac{1}{2}$ $-\pi/2<\arg a<3\pi/2$	$-i\,\dfrac{2(t^2-2ait)^{\nu-\frac{1}{2}}}{\pi^{\frac{1}{2}}(2a)^\nu\Gamma(\nu+\frac{1}{2})}$

Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(10)	$\Gamma(\nu+\tfrac{1}{2})\, p^{-\nu} e^{iap} H_\nu^{(2)}(ap)$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}, \quad -3\pi/2<\arg a<\pi/2$	$i\,\pi^{-1/2}\, 2^{1-\nu} a^{-\nu}(t^2+2ait)^{\nu-1/2}$
(11)	$p^{-1} J_\nu(2a/p) \qquad \mathrm{Re}\,\nu>-1$	$J_\nu(2a^{1/2}t^{1/2})\, I_\nu(2a^{1/2}t^{1/2})$
(12)	$\Gamma(\nu+1)\,\Gamma(\lambda)\, p^{\nu-\lambda} J_\nu(4a/p)$ $\mathrm{Re}\,\lambda>0$	$(2a)^\nu t^{\lambda-1}\, {}_0F_3(\nu+1, \tfrac{1}{2}\lambda, \tfrac{1}{2}\lambda+\tfrac{1}{2}; -a^2t^2)$
(13)	$p^{-1} e^{(a^2-\beta^2)/p}\, J_\nu(2a\beta/p)$ $\mathrm{Re}\,\nu>-1$	$J_\nu(2\beta t^{1/2})\, I_\nu(2at^{1/2})$
(14)	$(p^2+1)^{-1/2} e^{-ap/(p^2+1)}$ $\times J_\nu\left(\dfrac{a}{p^2+1}\right) \qquad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$J_\nu(t)\, J_{2\nu}(2a^{1/2}t^{1/2})$
(15)	$p^{-\mu} J_\nu(p^{-1/2}) \qquad \mathrm{Re}(\mu+\tfrac{1}{2}\nu)>0$	$\dfrac{t^{\mu+\frac{1}{2}\nu-1}\, {}_0F_2(\mu+\tfrac{1}{2}\nu, \nu+1; -\tfrac{1}{4}t)}{2^\nu\,\Gamma(\mu+\tfrac{1}{2}\nu)\,\Gamma(\nu+1)}$
(16)	$(p^2+a^2)^{-\frac{1}{2}\nu} e^{ip} H_\nu^{(2)}[(p^2+a^2)^{1/2}]$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}$	$i\,2^{1/2}\pi^{-1/2} a^{1/2-\nu}(t^2+2it)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times J_{\nu-1/2}[a(t^2+2it)^{1/2}]$
(17)	$\Gamma(p+\tfrac{1}{2})\,(\tfrac{1}{2}a)^{-p} J_p(a)$	$\pi^{-1/2}(e^t-1)^{-1/2}\cos[a(1-e^{-t})^{1/2}]$
(18)	$\Gamma(p)\,(\tfrac{1}{2}a)^{-p} J_{p+\mu}(a)$ $\mathrm{Re}\,\mu>-1$	$(1-e^{-t})^{\frac{1}{2}\mu} J_\mu[a(1-e^{-t})^{1/2}]$
(19)	$p^{1/2}[J_{\nu+1/4}(ap)\, J_{\nu-1/4}(ap)$ $+ Y_{\nu+1/4}(ap)\, Y_{\nu-1/4}(ap)]$ $\mathrm{Re}\,a>0$	$(\tfrac{1}{2}\pi)^{-3/2}(t^3+4a^2t)^{-1/2}$ $\times e^{2\nu\sinh^{-1}(\frac{1}{2}t/a)}$

Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(20)	$p^{1/2}[J_{1/4+\nu}(ap)\,J_{1/4-\nu}(ap)$ $+Y_{1/4+\nu}(ap)\,Y_{1/4-\nu}(ap)]$ $\operatorname{Re} a>0$	$(\tfrac{1}{2}\pi)^{-3/2}(t^3+4a^2t)^{-1/2}$ $\times\{\cos[(\nu+\tfrac{1}{4})\pi]\,e^{-2\nu\sinh^{-1}(\frac{1}{2}t/a)}$ $+\sin[(\nu+\tfrac{1}{4})\pi]\,e^{2\nu\sinh^{-1}(\frac{1}{2}t/a)}\}$
(21)	$p^{1/2}[J_{\nu+1/4}(ap)\,Y_{\nu-1/4}(ap)$ $-J_{\nu-1/4}(ap)\,Y_{\nu+1/4}(ap)]$ $\operatorname{Re} a>0$	$(\tfrac{1}{2}\pi)^{-3/2}(t^3+4a^2t)^{-1/2}$ $\times e^{-2\nu\sinh^{-1}(\frac{1}{2}t/a)}$
(22)	$p^{1/2}[J_{1/4+\nu}(ap)\,Y_{1/4-\nu}(ap)$ $-J_{1/4-\nu}(ap)\,Y_{1/4+\nu}(ap)]$ $\operatorname{Re} a>0$	$(\tfrac{1}{2}\pi)^{-3/2}(t^3+4a^2t)^{-1/2}$ $\times\{\sinh[(\nu+\tfrac{1}{4})\pi]\,e^{-2\nu\sinh^{-1}(\frac{1}{2}t/a)}$ $-\cos[(\nu+\tfrac{1}{4})\pi]\,e^{2\nu\sinh^{-1}(\frac{1}{2}t/a)}\}$
(23)	$J_{\nu-p}(a)\,Y_{-\nu-p}(a)$ $-J_{-\nu-p}(a)\,Y_{\nu-p}(a)$ $\operatorname{Re} a>0,\quad \lvert\operatorname{Re}\nu\rvert<\tfrac{1}{2}$	$2\pi^{-2}\sin(2\nu\pi)K_{2\nu}[2a\sinh(\tfrac{1}{2}t)]$
(24)	$J_p(a)\dfrac{\partial Y_p(a)}{\partial p}-Y_p(a)\dfrac{\partial J_p(a)}{\partial p}$ $\operatorname{Re} a>0$	$-\dfrac{2}{\pi}K_0[2a\sinh(\tfrac{1}{2}t)]$
(25)	$p^{1/2}H^{(1)}_{1/8}\left(\dfrac{p^2}{a}\right)H^{(2)}_{1/8}\left(\dfrac{p^2}{a}\right)$ $a>0$	$a\cos\left(\dfrac{\pi}{8}\right)\dfrac{(2t)^{1/2}}{\pi^{1/2}}J_{1/8}\left(\dfrac{at^2}{16}\right)$ $\times J_{-1/8}\left(\dfrac{at^2}{16}\right)$
(26)	$p^{-1/2}H^{(1)}_{\nu}(\tfrac{1}{2}p/a)\,H^{(2)}_{\nu}(\tfrac{1}{2}p/a)$	$2a(2t/\pi)^{1/2}P^{1/4}_{\nu-1/4}[(1+a^2t^2)^{1/2}]$ $\times P^{-1/4}_{\nu-1/4}[(1+a^2t^2)^{1/2}]$

Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(27)	$p^{1/2} H^{(1)}_{1/2+\nu}(ap)\, H^{(2)}_{1/2-\nu}(ap)$ $\mathrm{Re}\, a>0$	$4[\pi^3 t(t^2+4a^2)]^{-1/2} e^{-\nu\pi i}$ $\times\{\cosh[2\nu \sinh^{-1}(\tfrac{1}{2}t/a)]$ $+i \sinh[2\nu \sinh^{-1}(\tfrac{1}{2}t/a)]\}$
(28)	$\pi\Gamma(2\lambda+2)e^{(\mu-\nu)\pi i} p^{-2\lambda}$ $\times H^{(1)}_{2\mu}(p/a)\, H^{(2)}_{2\nu}(p/a)$ $\mathrm{Re}\,\lambda>-\tfrac{1}{2}$	$2(2\lambda+1)\,a t^{2\lambda}$ $\times {}_4F_3(\tfrac{1}{2}+\mu+\nu, \tfrac{1}{2}-\mu+\nu, \tfrac{1}{2}+\mu-\nu, \tfrac{1}{2}-\mu-\nu;$ $\tfrac{1}{2}, \lambda+\tfrac{1}{2}, \lambda+1; -\tfrac{1}{4}a^2t^2)$ $+i\,4a^2(\mu^2-\nu^2)t^{2\lambda+1}$ $\times {}_4F_3(1+\mu+\nu, 1+\nu-\mu, 1-\mu-\nu, 1+\mu-\nu;$ $3/2, \lambda+1, \lambda+3/2; -\tfrac{1}{4}a^2t^2)$
(29)	$\Gamma(2\nu+\tfrac{1}{2})p^{-2\nu} H^{(1)}_{2\nu}(a^{1/2}p^{1/2})$ $\times H^{(2)}_{2\nu}(a^{1/2}p^{1/2})$ $\mathrm{Re}\,\nu>-\tfrac{1}{4}$	$2a^{-\nu-1/2} t^{3\nu-1/2} e^{\frac{1}{2}a/t} W_{\nu,\nu}(a/t)$
(30)	$p^{1/2}[H^{(1)}_{\nu}(a^{1/2}p^{1/2})\, H^{(2)}_{\nu+1}(a^{1/2}p^{1/2})$ $+H^{(1)}_{\nu+1}(a^{1/2}p^{1/2})\, H^{(2)}_{\nu}(a^{1/2}p^{1/2})]$	$a^{-1/2}\pi^{-3/2}(4\nu+2)\, e^{\frac{1}{2}a/t} W_{-1/2,\nu+1/2}(a/t)$

5.15. Modified Bessel functions of arguments kp and kp^2

(1)	$e^{-bp} I_0(bp)$ $b>0$	$\pi^{-1}(2bt-t^2)^{-1/2}$ $0<t<2b$ 0 $t>2b$
(2)	$\pi b e^{-bp} I_1(bp)$ $b>0$	$(b-t)(2bt-t^2)^{-1/2}$ $0<t<2b$ 0 $t>2b$
(3)	$e^{-\frac{1}{2}(a+b)p} I_n[\tfrac{1}{2}(b-a)p]$ $b>a\geq 0$	0 $0<t<a$ $\dfrac{\cos\left(n \cos^{-1}\frac{2t-a-b}{b-a}\right)}{\pi(t-a)^{1/2}(b-t)^{1/2}}$ $a<t<b$ 0 $t>b$

Modified Bessel functions of kp and kp^2 (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(4)	$\dfrac{\pi^{3/2}e^{-bp/2}p^{\nu}}{\Gamma(\nu+\frac{1}{2})\,b^{\nu}}I_{\nu}(\frac{1}{2}bp)$ $\operatorname{Re}\nu<\frac{1}{2},\quad b>0$	$\cos(2\pi\nu)(bt-t^2)^{-\nu-\frac{1}{2}} \quad 0<t<b$ $-\sin(2\nu\pi)(t^2-bt)^{-\nu-\frac{1}{2}} \quad t>b$
(5)	$\pi^{\frac{1}{2}}\Gamma(\nu+\frac{1}{2})e^{-\frac{1}{2}bp}b^{\nu}p^{-\nu}I_{\nu}(\frac{1}{2}bp)$ $\operatorname{Re}\nu>-\frac{1}{2},\quad b>0$	$(bt-t^2)^{\nu-\frac{1}{2}} \quad 0<t<b$ $0 \quad t>b$
(6)	$\Gamma(2\nu+n)e^{-\frac{1}{2}bp}b^{\nu}p^{-\nu}I_{\nu+n}(\frac{1}{2}bp)$ $\operatorname{Re}\nu>-\frac{1}{2},\quad b>0$	$\dfrac{(-1)^n n!\,\Gamma(\nu)\,2^{2\nu}}{\pi(bt-t^2)^{\frac{1}{2}-\nu}}C_n^{\nu}(2t/b-1)$ $0<t<b$ $0 \quad t>b$
(7)	$\Gamma(2\nu)\,p^{-\nu}\operatorname{csch}(ap)\,I_{\nu}(ap)$ $a>0,\quad \operatorname{Re}\nu>-\frac{1}{2}$	$\pi^{-1}2^{\nu}a^{-\nu}\Gamma(\nu)[2a(t-2ak)$ $-(t-2ak)^2]^{\nu-\frac{1}{2}} \quad k=0,1,2,\ldots$ $2ak<t<2a(k+1)$
(8)	$K_0(bp) \quad b>0$	$0 \quad 0<t<b$ $y^{-1} \quad t>b$
(9)	$p^{-1}K_0(bp) \quad b>0$	$0 \quad 0<t<b$ $\cosh^{-1}(t/b) \quad t>b$
(10)	$K_1(bp) \quad b>0$	$0 \quad 0<t<b$ $b^{-1}ty^{-1} \quad t>b$
(11)	$p^{-1}K_1(bp) \quad b>0$	$0 \quad 0<t<b$ $b^{-1}y \quad t>b$
(12)	$K_{\nu}(bp) \quad b>0$	$0 \quad 0<t<b$ $y^{-1}\cosh[\nu\cosh^{-1}(t/b)] \quad t>b$

$y=(t^2-b^2)^{\frac{1}{2}}$

Modified Bessel functions of kp and kp^2 (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(13)	$p^{-1}K_\nu(bp)$ $\qquad b>0$	$0 \qquad 0<t<b$ $\nu^{-1}\sinh[\nu\cosh^{-1}(t/b)] \qquad t>b$
(14)	$\Gamma(\nu+\tfrac{1}{2})p^{-\nu}K_\nu(bp)$ $b>0, \quad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$0 \qquad 0<t<b$ $2^{-\nu}\pi^{\frac{1}{2}}b^{-\nu}y^{2\nu-1} \qquad t>b$
(15)	$2^{2\mu}\Gamma(2\mu+\tfrac{1}{2})(p/b)^{-2\mu}K_{2\nu}(bp)$ $\mathrm{Re}\,\mu>-\tfrac{1}{4}, \quad b>0$	$0 \qquad 0<t<b$ $\pi^{\frac{1}{2}}y^{4\mu-1}$ $\times\,{}_2F_1(\mu-\nu,\mu+\nu;2\mu+\tfrac{1}{2};1-t^2/b^2)$ $t>b$
(16)	$e^{ap}K_0(ap)$ $\qquad \lvert\arg a\rvert<\pi$	$(t^2+2at)^{-\frac{1}{2}}$
(17)	$e^{ap}K_1(ap)$ $\qquad \lvert\arg a\rvert<\pi$	$a^{-1}(t+a)(t^2+2at)^{-\frac{1}{2}}$
(18)	$e^{ap}K_\nu(ap)$ $\qquad \lvert\arg a\rvert<\pi$	$(t^2+2at)^{-\frac{1}{2}}\cosh[\nu\cosh^{-1}(1+t/a)]$
(19)	$p^{-1}e^{ap}K_0(ap)$ $\qquad \lvert\arg a\rvert<\pi$	$\cosh^{-1}(1+t/a)$
(20)	$p^{-1}e^{ap}K_1(ap)$ $\qquad \lvert\arg a\rvert<\pi$	$a^{-1}(t^2+2a)^{-\frac{1}{2}}$
(21)	$p^{-1}e^{ap}K_\nu(ap)$ $\qquad \lvert\arg a\rvert<\pi$	$\nu^{-1}\sinh[\nu\cosh^{-1}(1+t/a)]$
(22)	$p^{-\nu}e^{ap}K_\nu(ap)$ $\mathrm{Re}\,\nu>-\tfrac{1}{2}, \quad \lvert\arg a\rvert<\pi$	$\pi^{\frac{1}{2}}[\Gamma(\nu+\tfrac{1}{2})]^{-1}(2a)^{-\nu}(t^2+2at)^{\nu-\frac{1}{2}}$
(23)	$p^{\mu}e^{ap}K_\nu(ap)$ $\mathrm{Re}\,\mu<\tfrac{1}{2}, \quad \lvert\arg a\rvert<\pi$	$2^{-\frac{1}{2}}\pi^{\frac{1}{2}}a^{-\frac{1}{2}}(t^2+2at)^{-\frac{1}{2}\mu-\frac{1}{4}}$ $\times\,P^{\mu+\frac{1}{2}}_{\nu-\frac{1}{2}}(1+t/a)$

$y=(t^2-b^2)^{\frac{1}{2}}$

Modified Bessel functions of kp and kp^2 (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(24)	$e^{ap^2}K_0(ap^2)$ $\qquad$ $\operatorname{Re} a>0$	$2^{-1/2}a^{-1/2}\pi^{1/2}\exp\left(-\frac{t^2}{16a}\right)I_0\left(\frac{t^2}{16a}\right)$
(25)	$p^{1/2}e^{ap^2}K_{1/4}(ap^2)$ $\qquad$ $\operatorname{Re} a>0$	$(2at)^{-1/2}\exp\left(-\frac{t^2}{8a}\right)$
(26)	$p^{-1/2}e^{ap^2}K_{1/4}(ap^2)$ $\qquad$ $\operatorname{Re} a>0$	$(8a)^{-1/4}\,\gamma\left(\frac{1}{4},\frac{t^2}{8a}\right)$
(27)	$\Gamma(4\nu+1)p^{-4\nu}e^{ap^2}K_{2\nu}(ap^2)$ $\operatorname{Re}\nu>-1/4,\quad \operatorname{Re} a>0$	$2^{3\nu+1}\pi^{1/2}a^{\nu}t^{2\nu-1}\exp\left(-\frac{t^2}{16a}\right)$ $\times M_{-3\nu,\nu}\left(\frac{t^2}{8a}\right)$

5.16. Modified Bessel functions of other arguments

(1)	$I_\nu(2a/p)$ $\qquad$ $\operatorname{Re}\nu>0$	$a^{1/2}t^{-1/2}Z_\nu^{(b)}(2a^{1/2}t^{1/2})$
(2)	$p\,I_\nu(2a/p)$ $\qquad$ $\operatorname{Re}\nu>1$	$a^2V_\nu^{(b)}(2a^{1/2}t^{1/2})$
(3)	$p^{-1}\,I_\nu(2a/p)$ $\qquad$ $\operatorname{Re}\nu>-1$	$X_\nu^{(b)}(2a^{1/2}t^{1/2})$
(4)	$p^{-2}\,I_\nu(2a/p)$ $\qquad$ $\operatorname{Re}\nu>-2$	$a^{-1/2}t^{1/2}W_\nu^{(b)}(2a^{1/2}t^{1/2})$
(5)	$p^{-\lambda}\,I_\nu\left(\frac{2a}{p}\right)$ $\operatorname{Re}(\lambda+\nu)>0$	$\frac{a^\nu t^{\lambda+\nu-1}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)}$ $\times\,{}_0F_3\left(\nu+1,\frac{\lambda+\nu}{2},\frac{\lambda+\nu+1}{2};\frac{1}{4}a^2t^2\right)$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(6)	$p^{-1/2}\, e^{a/p}\, I_{1/4}(a/p)$	$\pi^{-1}(2a)^{-1/4}\, t^{-3/4} \sinh[(8at)^{1/2}]$
(7)	$p^{-1/2}\, e^{-a/p}\, I_{1/4}(a/p)$	$\pi^{-1}(2a)^{-1/4}\, t^{-3/4} \sin[(8at)^{1/2}]$
(8)	$p^{-1/2}\, e^{a/p}\, I_{-1/4}(a/p)$	$\pi^{-1}(2a)^{-1/4}\, t^{-3/4} \cosh[(8at)^{1/2}]$
(9)	$p^{-1/2}\, e^{-a/p}\, I_{-1/4}(a/p)$	$\pi^{-1}(2a)^{-1/4}\, t^{-3/4} \cos[(8at)^{1/2}]$
(10)	$\pi p^{-1/2}\, e^{a/p}\, I_{3/4}(a/p)$	$2^{-1/4} a^{-1/4} t^{-3/4} \cosh[(8at)^{1/2}]$ $-2^{-7/4} a^{-3/4} t^{-5/4} \sinh[(8at)^{1/2}]$
(11)	$p^{-1/2}\, e^{-a/p}\, I_{3/4}(a/p)$	$2^{-7/4} a^{-3/4} t^{-5/4} \sin[(8at)^{1/2}]$ $-2^{-1/4} a^{-1/4} t^{-3/4} \cos[(8at)^{1/2}]$
(12)	$\pi p^{-1/2}\, e^{a/p}\, I_{-3/4}(a/p)$	$2^{-1/4} a^{-1/4} t^{-3/4} \sinh[(8at)^{1/2}]$ $-2^{-7/4} a^{-3/4} t^{-5/4} \cosh[(8at)^{1/2}]$
(13)	$\pi p^{-1/2}\, e^{-a/p}\, I_{-3/4}(a/p)$	$-2^{-1/4} a^{-1/4} t^{-3/4} \sin[(8at)]^{1/2}]$ $-2^{-7/4} a^{-3/4} t^{-5/4} \cos[(8at)^{1/2}]$
(14)	$p^{-1}\, e^{a/p}\, I_\nu(a/p) \qquad \operatorname{Re}\nu > -1$	$\{I_\nu[(2at)^{1/2}]\}^2$
(15)	$p^{-1}\, e^{-a/p}\, I_\nu(a/p) \qquad \operatorname{Re}\nu > -1$	$\{J_\nu[(2at)^{1/2}]\}^2$
(16)	$p^{-1/2}\, e^{a/p}\, I_\nu(a/p) \qquad \operatorname{Re}\nu > -1/2$	$(\pi t)^{-1/2}\, I_{2\nu}[(8at)^{1/2}]$
(17)	$p^{-1/2}\, e^{-a/p}\, I_\nu(a/p) \qquad \operatorname{Re}\nu > -1/2$	$(\pi t)^{-1/2}\, J_{2\nu}[(8at)^{1/2}]$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(18)	$p^{-\lambda} e^{a/p} I_\nu(a/p)$ $\qquad \operatorname{Re}(\lambda+\nu)>0$	$\dfrac{2^{-\nu} a^\nu t^{\lambda+\nu-1}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)}$ $\times {}_1F_2(\nu+\tfrac{1}{2}; 2\nu+1, \lambda+\nu; 2at)$
(19)	$p^{-\lambda} e^{-a/p} I_\nu(a/p)$ $\qquad \operatorname{Re}(\lambda+\nu)>0$	$\dfrac{2^{-\nu} a^\nu t^{\lambda+\nu-1}}{\Gamma(\nu+1)\Gamma(\lambda+\nu)}$ $\times {}_1F_2(\nu+\tfrac{1}{2}; 2\nu+1, \lambda+\nu; -2at)$
(20)	$p^{-\frac{1}{2}} \sinh(a/p)\, I_\nu(a/p)$ $\qquad \operatorname{Re}\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \{ I_{2\nu}[(8at)^{\frac{1}{2}}]$ $- J_{2\nu}[(8at)^{\frac{1}{2}}]\}$
(21)	$p^{-\frac{1}{2}} \cosh(a/p)\, I_\nu(a/p)$ $\qquad \operatorname{Re}\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}\pi^{-\frac{1}{2}} t^{-\frac{1}{2}} \{ I_{2\nu}[(8at)^{\frac{1}{2}}]$ $+ J_{2\nu}[(8at)^{\frac{1}{2}}]\}$
(22)	$p^{-1} e^{(a^2+\beta^2)/p} I_\nu(2a\beta/p)$ $\qquad \operatorname{Re}\nu>-1$	$I_\nu(2at^{\frac{1}{2}})\, I_\nu(2\beta t^{\frac{1}{2}})$
(23)	$p^{-1} e^{-(a^2+\beta^2)/p} I_\nu(2a\beta/p)$ $\qquad \operatorname{Re}\nu>-1$	$J_\nu(2at^{\frac{1}{2}})\, J_\nu(2\beta t^{\frac{1}{2}})$
(24)	$2p^{-1}\sinh[(a^2+\beta^2)/p]$ $\times I_\nu(2a\beta/p) \qquad \operatorname{Re}\nu>-1$	$I_\nu(2at^{\frac{1}{2}})\, I_\nu(2\beta t^{\frac{1}{2}})$ $- J_\nu(2at^{\frac{1}{2}})\, J_\nu(2\beta t^{\frac{1}{2}})$
(25)	$2p^{-1}\cosh[(a^2+\beta^2)/p]$ $\times I_\nu(2a\beta/p) \qquad \operatorname{Re}\nu>-1$	$I_\nu(2at^{\frac{1}{2}})\, I_\nu(2\beta t^{\frac{1}{2}})$ $+ J_\nu(2at^{\frac{1}{2}})\, J_\nu(2\beta t^{\frac{1}{2}})$

Modified Bessel functions of other arguments (cont'd)

	$g(p) = \int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(26)	$2^{1/2}\pi^{1/2} r^{-\nu} e^{-p} C_n^\nu(p/r) \times I_{\nu+n}(r) \qquad r = (p^2 + a^2)^{1/2}$	$(-1)^n a^{1/2-\nu}(2t-t^2)^{1/2\nu-1/4} C_n^\nu(t-1) \times I_{\nu-1/2}[a(2t-t^2)^{1/2}] \qquad 0<t<2$; $0 \qquad t>2$
(27)	$\pi e^{-p} I_0[(p^2-a^2)^{1/2}]$	$(2t-t^2)^{-1/2}\cos[a(2t-t^2)^{1/2}] \qquad 0<t<2$; $0 \qquad t>2$
(28)	$p^{1/2}[I_{\nu-1/4}(bp)\, I_{-\nu-1/4}(bp) - I_{\nu+1/4}(bp)\, I_{-\nu+1/4}(bp)]$	$\dfrac{2^{3/2}\cos[2\nu\cos^{-1}(\frac{1}{2}t/b)]}{\pi^{3/2}(4b^2t-t^3)^{1/2}} \qquad 0<t<2b$; $0 \qquad t>2b$
(29)	$p^{-1/2}\sinh(a/p)\, K_0(a/p)$	$\pi^{-1/2} t^{-1/2} K_0[(8at)^{1/2}] + \frac{1}{2}\pi^{1/2} t^{-1/2} Y_0[(8at)^{1/2}]$
(30)	$p^{-1/2}\cosh(a/p)\, K_0(a/p)$	$\pi^{-1/2} t^{-1/2} K_0[(8at)^{1/2}] - \frac{1}{2}\pi^{1/2} t^{-1/2} Y_0[(8at)^{1/2}]$
(31)	$a^{1/2} p^{-1/2} e^{a/p} K_{1/4}(a/p)$	$(2t)^{-1/4}\, e^{-(8at)^{1/2}}$
(32)	$\pi^{-1/2} p^{2\lambda} K_{2\nu}(2a/p) \qquad \mathrm{Re}(\lambda \pm \nu) < 0$	$2^{2\lambda} t^{-2\lambda-1} \times S_2(\nu-\frac{1}{2}, -\nu-\frac{1}{2}, \lambda+\frac{1}{2}, \lambda; \frac{1}{2}at)$
(33)	$p^{-1/2} e^{a/p} K_\nu(a/p) \qquad \lvert\mathrm{Re}\,\nu\rvert < \frac{1}{2}$	$2\pi^{-1/2} t^{-1/2}\cos(\nu\pi) K_{2\nu}[(8at)^{1/2}]$
(34)	$\pi^{-1/2} p^{-1/2} e^{-a/p} K_\nu(a/p) \qquad \lvert\mathrm{Re}\,\nu\rvert < \frac{1}{2}$	$-t^{-1/2}\sin(\nu\pi) J_{2\nu}[(8at)^{1/2}] - t^{-1/2}\cos(\nu\pi) Y_{2\nu}[(8at)^{1/2}]$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(35)	$K_0(a^{1/2}p^{1/2})$ Re $a\geq 0$, $a\neq 0$	$\frac{1}{2}t^{-1}e^{-\frac{1}{4}a/t}$
(36)	$a^{1/2}p^{-1/2}K_1(a^{1/2}p^{1/2})$ Re $a\geq 0$	$e^{-\frac{1}{4}a/t}$
(37)	$a^{-1/2}p^{1/2}K_1(a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{4}t^{-2}e^{-\frac{1}{4}a/t}$
(38)	$p^{-1/2}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}\pi^{-1/2}t^{-1/2}e^{-\frac{1}{2}a/t}K_{\frac{1}{2}\nu}(\frac{1}{2}a/t)$
(39)	$a^{\frac{1}{2}\nu}p^{-\frac{1}{2}\nu}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}t^{\nu-1}e^{-a/t}$
(40)	$a^{-\frac{1}{2}\nu}p^{\frac{1}{2}\nu}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}t^{-\nu-1}e^{-a/t}$
(41)	$p^{\frac{1}{2}\nu-1}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}a^{-\frac{1}{2}\nu}\Gamma(\nu, a/t)$
(42)	$p^{\frac{1}{2}\nu+n}K_\nu(2a^{1/2}p^{1/2})$ Re $a>0$	$\frac{1}{2}(-1)^n n!\,a^{\frac{1}{2}\nu}t^{-n}e^{-a/t}L_n^\nu(a/t)$
(43)	$2a^{1/2}p^{\mu-1}K_{2\nu}(2a^{1/2}p^{1/2})$ Re $a>0$	$t^{\frac{1}{2}-\mu}e^{-\frac{1}{2}a/t}W_{\mu-\frac{1}{2},\nu}(a/t)$
(44)	$r^{-1}K_1(br)$ $b>0$	0 $0<t<b$ $a^{-1}b^{-1}\sin a\gamma$ $t>b$
(45)	$2^{1/2}\pi^{-1/2}r^{-\nu}K_\nu(br)$ Re $\nu>-\frac{1}{2}$	0 $0<t<b$ $a^{\frac{1}{2}-\nu}b^{-\nu}\gamma^{\nu-\frac{1}{2}}J_{\nu-\frac{1}{2}}(a\gamma)$ $t>b$
(46)	$2^{1/2}\pi^{-1/2}r^{-\nu}e^{\beta p}K_\nu(\beta r)$ Re $\nu>-\frac{1}{2}$, $\lvert\arg\beta\rvert<\pi$	$a^{\frac{1}{2}-\nu}\beta^{-\nu}(t^2+2\beta t)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times J_{\nu-\frac{1}{2}}[a(t^2+2\beta t)^{1/2}]$

$r=(p^2+a^2)^{1/2}$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(47)	$2^{1/2}\pi^{-1/2}s^{-\nu}K_\nu(bs)$ $\quad \mathrm{Re}\,\nu>-\frac{1}{2}$	$0 \qquad 0<t<b$ $a^{1/2-\nu}b^{-\nu}y^{\nu-1/2}I_{\nu-1/2}(ay) \qquad t>b$
(48)	$2^{1/2}\pi^{-1/2}s^{-\nu}e^{\beta p}K_\nu(\beta s)$ $\mathrm{Re}\,\nu>-\frac{1}{2},\quad \lvert\arg\beta\rvert<\pi$	$a^{1/2-\nu}\beta^{-\nu}(t^2+2\beta t)^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times I_{\nu-1/2}[a(t^2+2\beta t)^{1/2}]$
(49)	$\dfrac{(\frac{1}{2}c)^p K_p(c)}{\Gamma(p+\frac{1}{2})} \qquad c>0$	$\dfrac{\cos[c(e^t-1)^{1/2}]}{2\pi^{1/2}(1-e^{-t})^{1/2}}$
(50)	$\dfrac{a^p K_{\nu-p}(a)}{\Gamma(p+1)}$ $\mathrm{Re}\,\nu>-1,\quad a>0$	$\frac{1}{2}(e^t-1)^{\frac{1}{2}\nu}J_\nu[2a(e^t-1)^{1/2}]$
(51)	$J_\nu(a^{1/2}p^{1/2})K_\nu(a^{1/2}p^{1/2}) \qquad a>0$	$\frac{1}{2}t^{-1}J_\nu(\frac{1}{2}a/t)$
(52)	$Y_\nu(a^{1/2}p^{1/2})K_\nu(a^{1/2}p^{1/2}) \qquad a>0$	$\frac{1}{2}t^{-1}Y_\nu(\frac{1}{2}a/t)$
(53)	$H_\nu^{(1)}(a^{1/2}p^{1/2})K_\nu(a^{1/2}p^{1/2}) \qquad a>0$	$\frac{1}{2}t\,H_\nu^{(1)}(\frac{1}{2}a/t)$
(54)	$H_\nu^{(2)}(a^{1/2}p^{1/2})K_\nu(a^{1/2}p^{1/2}) \qquad a>0$	$\frac{1}{2}t^{-1}H_\nu^{(2)}(\frac{1}{2}a/t)$
(55)	$p^{1/2}I_n(bp)K_{n+1/2}(bp) \qquad b>0$	$\dfrac{(-1)^n\cos[(2n+\frac{1}{2})\cos^{-1}(\frac{1}{2}t/b)]}{[\frac{1}{2}\pi(4b^2t-t^3)]^{1/2}} \qquad 0<t<2b$ $0 \qquad t>2b$
(56)	$K_\nu(a^{1/2}p^{1/2}+\beta^{1/2}p^{1/2})$ $\times I_\nu(a^{1/2}p^{1/2}-\beta^{1/2}p^{1/2})$ $\mathrm{Re}\,a>0,\quad \mathrm{Re}\,\beta>0$	$\frac{1}{2}t^{-1}e^{-\frac{1}{2}(a+\beta)/t}I_\nu[\frac{1}{2}(a-\beta)/t]$

$s=(p^2-a^2)^{1/2},\qquad y=(t^2-b^2)^{1/2}$

Modified Bessel functions of other arguments (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(57)	$I_\nu[\tfrac{1}{2}b(r-p)]K_\nu[\tfrac{1}{2}b(r+p)]$ $r=(p^2+a^2)^{\frac{1}{2}},\quad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$0 \quad 0<t<b$ $(t^2-b^2)^{-\frac{1}{2}}J_{2\nu}[a(t^2-b^2)^{\frac{1}{2}}] \quad t>b$
(58)	$I_{\nu+p}(c)K_{\nu-p}(c)$ $c>0,\quad \mathrm{Re}\,\nu>-\tfrac{1}{2}$	$\tfrac{1}{2}J_{2\nu}(2c\sinh\tfrac{1}{2}t)$
(59)	$p^{2\nu}[K_{2\nu}(a^{\frac{1}{2}}p^{\frac{1}{2}})]^2 \quad \mathrm{Re}\,a>0$	$\tfrac{1}{2}\pi^{\frac{1}{2}}a^{\nu-\frac{1}{2}}t^{-3\nu-\frac{1}{2}}e^{-\frac{1}{2}a/t}W_{\nu,\nu}(a/t)$
(60)	$e^{\frac{1}{2}(a+\beta)p}K_{2\nu}(\tfrac{1}{2}ap)K_{2\nu}(\tfrac{1}{2}\beta p)$ $\lvert\arg a\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$	$\pi(a\beta)^{\nu-\frac{1}{4}}(a+t)^{-\nu-\frac{1}{4}}(\beta+t)^{-\nu-\frac{1}{4}}$ $\times P_{2\nu-\frac{1}{2}}[2a^{-1}\beta^{-1}(a+t)(\beta+t)-1]$
(61)	$p^{\frac{1}{2}}K_{\nu+\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})K_{\nu-\frac{1}{2}}(a^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,a>0$	$\tfrac{1}{2}(2a)^{-\frac{1}{2}}\pi^{\frac{1}{2}}t^{-1}e^{-\frac{1}{2}a/t}W_{\frac{1}{2},\nu}(a/t)$
(62)	$p^{\frac{1}{2}}K_{\nu+\frac{1}{4}}(bp)K_{\nu-\frac{1}{4}}(bp) \quad b>0$	$0 \quad 0<t<2b$ $\dfrac{2^{\frac{1}{2}}\pi^{\frac{1}{2}}\cosh[2\nu\cosh^{-1}(\tfrac{1}{2}t/b)]}{(t^3-4b^2t)^{\frac{1}{2}}}$ $t>2b$
(63)	$p^{\frac{1}{2}}e^{2ap}K_{\nu+\frac{1}{4}}(ap)K_{\nu-\frac{1}{4}}(ap)$ $\lvert\arg a\rvert<\pi$	$(2\pi)^{\frac{1}{2}}[t(t+2a)(t+4a)]^{-\frac{1}{2}}$ $\times\cosh[2\nu\cosh^{-1}(1+\tfrac{1}{2}t/a)]$
(64)	$K_\nu(a^{\frac{1}{2}}p^{\frac{1}{2}}+\beta^{\frac{1}{2}}p^{\frac{1}{2}})$ $\times K_\nu(a^{\frac{1}{2}}p^{\frac{1}{2}}-\beta^{\frac{1}{2}}p^{\frac{1}{2}})$ $\mathrm{Re}\,a>0,\quad \mathrm{Re}\,\beta>0$	$\tfrac{1}{2}t^{-1}e^{-\frac{1}{2}(a+\beta)/t}K_\nu[\tfrac{1}{2}(a-\beta)/t]$
(65)	$K_\nu[p^{\frac{1}{2}}+(p-1)^{\frac{1}{2}}]K_\nu[p^{\frac{1}{2}}-(p-1)^{\frac{1}{2}}]$	$\tfrac{1}{2}t^{-1}e^{\frac{1}{2}t-1/t}K_\nu(\tfrac{1}{2}t)$
(66)	$K_\nu[(\lambda S/a)^{\frac{1}{2}}]K_\nu[(\lambda a/S)^{\frac{1}{2}}]$ $S=(p^2-a^2)^{\frac{1}{2}}+p$ $\mathrm{Re}(\lambda/a)>0$	$\tfrac{1}{2}t^{-1}e^{-\frac{1}{2}\lambda a^{-1}t^{-1}}K_\nu(a\lambda t)$

5.17. Functions related to Bessel functions

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(1)	$\mathbf{J}_\nu(p)-J_\nu(p)$	$\pi^{-1}\sin(\nu\pi)(t^2+1)^{-1/2}[(t^2+1)^{1/2}-t]^\nu$
(2)	$\csc(\pi p)[\mathbf{J}_p(a)-J_p(a)]$ $\operatorname{Re} a \geq 0$	$\pi^{-1}e^{-a\sinh t}$
(3)	$\mathbf{E}_\nu(p)+Y_\nu(p)$	$-(t^2+1)^{-1/2}\{[(t^2+1)^{1/2}+t]^\nu$ $+\cos(\nu\pi)[(t^2+1)^{1/2}-t]^\nu\}$
(4)	$p^{-1}[\mathbf{H}_0(ap)-Y_0(ap)]$	$2\pi^{-1}\sinh^{-1}(t/a)$
(5)	$\frac{1}{2}\pi[\mathbf{H}_1(ap)-Y_1(ap)]-1$ $\lvert\arg a\rvert<\frac{1}{2}\pi$	$a^{-1}t(t^2+a^2)^{-1/2}$
(6)	$p^{-\nu}[\mathbf{H}_\nu(ap)-Y_\nu(ap)]$ $\operatorname{Re} a>0$	$\dfrac{2^{1-\nu}a^{-\nu}}{\pi^{1/2}\Gamma(\nu+\frac{1}{2})}(t^2+a^2)^{\nu-1/2}$
(7)	$p^{1/2}[\mathbf{H}_{1/4}(p^2/a)-Y_{1/4}(p^2/a)]$ $a>0$	$a\pi^{-1/2}t^{1/2}J_{-1/4}(\frac{1}{4}at^2)$
(8)	$p^{1/2}[\mathbf{H}_{-1/4}(p^2/a)-Y_{-1/4}(p^2/a)]$ $a>0$	$a\pi^{-1/2}t^{1/2}J_{1/4}(\frac{1}{4}at^2)$
(9)	$p^{3/2}[\mathbf{H}_{-3/4}(p^2/a)-Y_{-3/4}(p^2/a)]$ $a>0$	$-\frac{1}{2}a^2\pi^{-1/2}t^{3/2}J_{-1/4}(\frac{1}{4}at^2)$
(10)	$p^{3/2}[\mathbf{H}_{-1/4}(p^2/a)-Y_{-1/4}(p^2/a)]$ $a>0$	$\frac{1}{2}a^2\pi^{-1/2}t^{3/2}J_{-3/4}(\frac{1}{4}at^2)$
(11)	$p^{-\lambda}\mathbf{H}_\nu\left(\dfrac{2a}{p}\right)$ $\operatorname{Re}(\lambda+\nu)>-1$	$\dfrac{2\pi^{-1/2}a^{\nu+1}t^{\lambda+\nu}}{\Gamma(\nu+3/2)\Gamma(\lambda+\nu+1)}$ $\times{}_1F_4\left(1;\frac{3}{2},\nu+\frac{3}{2},\frac{\lambda+\nu+1}{2},\frac{\lambda+\nu}{2}+1;-\frac{a^2t^2}{4}\right)$

Functions related to Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(12)	$p^{-1/2}[\mathbf{H}_0(2ap^{1/2})-Y_0(2ap^{1/2})]$	$2\pi^{-3/2}t^{-1/2}e^{\frac{1}{2}a^2/t}K_0(\frac{1}{2}a^2/t)$
(13)	$p^{-\frac{1}{2}\nu}[\mathbf{H}_{-\nu}(ap^{1/2})-Y_{-\nu}(ap^{1/2})]$ $\operatorname{Re}\nu>-\frac{1}{2}$	$2^{\nu}\pi^{-1}a^{-\nu}\cos(\nu\pi)t^{\nu-1}\exp(\frac{1}{4}a^2t^{-1})$ $\times\operatorname{Erfc}(\frac{1}{2}at^{-1/2})$
(14)	$p^{-\frac{1}{2}\nu-\frac{1}{2}}[\mathbf{H}_{\nu}(ap^{1/2})-Y_{\nu}(ap^{1/2})]$	$2\pi^{-1/2}a^{-1}[\Gamma(\frac{1}{2}+\nu)]^{-1}t^{-\frac{1}{2}\nu}$ $\times\exp\left(\frac{a^2}{8t}\right)W_{\frac{1}{2}\nu,\frac{1}{2}\nu}\left(\frac{a^2}{4t}\right)$
(15)	$\Gamma(p+\frac{1}{2})2^pa^{-p}\mathbf{H}_p(a)$	$\pi^{-1/2}(e^t-1)^{-1/2}\sin[a(1-e^{-t})^{1/2}]$
(16)	$p^{-1}[I_0(bp)-\mathbf{L}_0(bp)]$ $b>0$	$2\pi^{-1}\sin^{-1}(t/b)$ $0<t<b$ 0 $t>b$
(17)	$\frac{1}{2}\pi[\mathbf{L}_1(bp)-I_1(bp)]+1$ $b>0$	$b^{-1}t(b^2-t^2)^{-1/2}$ $0<t<b$ 0 $t>b$
(18)	$\pi^{1/2}\Gamma(\nu+\frac{1}{2})p^{-\nu}[I_{\nu}(bp)-\mathbf{L}_{\nu}(bp)]$ $\operatorname{Re}\nu>-\frac{1}{2},\quad b>0$	$2^{1-\nu}b^{-\nu}(b^2-t^2)^{\nu-\frac{1}{2}}$ $0<t<b$ 0 $t>b$
(19)	$\pi^{1/2}(2b)^{\nu}\Gamma(\nu+\frac{1}{2})p^{-\nu}e^{-bp}\mathbf{L}_{\nu}(bp)$ $\operatorname{Re}\nu>-\frac{1}{2},\quad b>0$	$(2bt-t^2)^{\nu-\frac{1}{2}}$ $0<t<b$ $-(2bt-t^2)^{\nu-\frac{1}{2}}$ $b<t<2b$ 0 $t>2b$
(20)	$\frac{1}{2}\pi^{1/2}\Gamma(\nu+\frac{1}{2})p^{-\nu}\operatorname{csch}p\,\mathbf{L}_{\nu}(p)$ $\operatorname{Re}\nu>-\frac{1}{2}$	$[2(t-2k)-(t-2k)^2]^{\nu-\frac{1}{2}}$ $2k<t<2k+1$ $-[2(t-2k)-(t-2k)^2]^{\nu-\frac{1}{2}}$ $2k+1<t<2k+2$

Functions related to Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(21)	$\Gamma(\nu+\tfrac{1}{2})p^{-\nu}\operatorname{csch}(\tfrac{1}{2}p)\,[I_\nu(p) - \mathbf{L}_\nu(p)]$ $\operatorname{Re}\nu>-\tfrac{1}{2}$	$0 \qquad 0<t<1/2$ $4\pi^{-1/2}[\tfrac{3}{4}+t-k-(t-k)^2]^{\nu-1/2}$ $k+1/2<t<k+3/2$ $k=0,1,2,\dots$
(22)	$p^{-\lambda}\,\mathbf{L}_\nu(2a/p)$ $\operatorname{Re}(\lambda+\nu)>-1$	$\dfrac{2\pi^{-1/2}a^{\nu+1}t^{\lambda+\nu}}{\Gamma(\nu+3/2)\Gamma(\lambda+\nu+1)}$ $\times {}_1F_4\left(1;\frac{3}{2},\nu+\frac{3}{2},\frac{\lambda+\nu+1}{2},\frac{\lambda+\nu}{2}+1;\frac{a^2t^2}{4}\right)$
(23)	$p^{-1/2}[I_0(2ap^{1/2})-\mathbf{L}_0(2ap^{1/2})$ $\operatorname{Re}a>0$	$(\pi t)^{-1/2}e^{-\frac{1}{2}a^2/t}\,I_0(\tfrac{1}{2}a^2/t)$
(24)	$p^{-\frac{1}{2}\nu}[\mathbf{L}_{-\nu}(ap^{1/2})-I_\nu(ap^{1/2})]$ $\operatorname{Re}\nu>-\tfrac{1}{2}$	$i\pi^{-1}2^\nu a^{-\nu}\cos(\nu\pi)\,t^{\nu-1}$ $\times\exp(\tfrac{1}{4}a^2t^{-1})\operatorname{Erf}(\tfrac{1}{2}iat^{-1/2})$
(25)	$\Gamma(\tfrac{1}{2}-p)(\tfrac{1}{2}b)^p[I_p(b)-\mathbf{L}_{-p}(b)]$ $b>0$	$\pi^{-1/2}(1-e^{-t})^{-1/2}\sin[b(e^t-1)^{1/2}]$
(26)	$S_{0,\nu}(p)$	$(1+t^2)^{-1/2}\cosh(\nu\sinh^{-1}t)$
(27)	$S_{-1,\nu}(p)$	$\nu^{-1}(1+t^2)^{-1/2}\sinh(\nu\sinh^{-1}t)$
(28)	$p^{-1}S_{0,\nu}(p)$	$\nu^{-1}\sinh(\nu\sinh^{-1}t)$
(29)	$p^{-1}S_{1,\nu}(p)$	$\cosh(\nu\sinh^{-1}t)$
(30)	$p^{1-2\lambda-\mu}S_{\mu,\nu}\left(\frac{p}{a}\right)$ $\operatorname{Re}\lambda>0,\quad\operatorname{Re}a>0$	$\dfrac{a^{1-\mu}t^{2\lambda-1}}{\Gamma(2\lambda)}$ $\times {}_3F_2\left(1,\frac{1-\mu+\nu}{2};\frac{1-\mu-\nu}{2};\lambda,\lambda+\frac{1}{2};-a^2t^2\right)$

Functions related to Bessel functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(31)	$p^{1/2} S_{-\mu-1,1/4}(\frac{1}{2}p^2)$ Re $\mu > -\frac{3}{4}$	$2^{2\mu+1}[\Gamma(2\mu+3/2)]^{-1} t^{1/2}$ $\times s_{\mu,1/4}(\frac{1}{2}t^2)$
	Further similar formulas may be found in *Nederl. Akad. Wetensch. Proc.*, 1935: 38, Part II, p. 629.	
(32)	$p^{-\mu-1/2} S_{2\mu,2\nu}(2a^{1/2}p^{1/2})$ $\operatorname{Re}(\mu\pm\nu) > -\frac{1}{2}$, $\lvert\arg a\rvert < \pi$	$2^{2\mu-1} a^{-1/2} t^{\mu} e^{1/2 a/t} W_{\mu,\nu}(a/t)$
(33)	$p^{-1/2\nu} S_{\mu,\nu}(2a^{1/2}p^{1/2})$ $\operatorname{Re}(\mu-\nu) < 1$, $\lvert\arg a\rvert < \pi$	$2^{\mu-1} a^{-1/2\nu} t^{\nu-1} e^{a/t}$ $\times \Gamma(\frac{1}{2}\mu+\frac{1}{2}\nu+\frac{1}{2},\ a/t)$
(34)	$p^{-1} S_{2,\nu}(ap) - a$	$(\nu-1/\nu)\sinh[\nu\sinh^{-1}(t/a)]$
(35)	$2\,O_n(p)$	$[t+(1+t^2)^{1/2}]^n + [t-(1+t^2)^{1/2}]^n$
(36)	$S_n(p)$	$\dfrac{[t+(1+t^2)^{1/2}]^n - [t-(1+t^2)^{1/2}]^n}{(1+t^2)^{1/2}}$

5.18. Parabolic cylinder functions

(1)	$\Gamma(\nu)e^{1/4 a^2p^2} D_{-\nu}(ap)$ Re $\nu > 0$, $\lvert\arg a\rvert < \frac{1}{4}\pi$	$a^{-\nu} t^{\nu-1} e^{-1/2 t^2/a^2}$
(2)	$\Gamma(2\nu)\,p^{-1} e^{1/4 a^2p^2} D_{-2\nu}(ap)$ Re $\nu > 0$, $\lvert\arg a\rvert < \frac{1}{4}\pi$	$2^{\nu+1}\gamma(\nu, \frac{1}{2}t^2/a^2)$
(3)	$D_{-2\nu}(2b^{1/2}p^{1/2})$ Re $\nu > 0$, $b > 0$	0 $\quad 0 < t < b$ $\dfrac{2^{1/2-\nu}b^{1/2}}{\Gamma(\nu)}\ \dfrac{(t-b)^{\nu-1}}{(t+b)^{\nu+1/2}}$ $\quad t > b$

Parabolic cylinder functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(4)	$p^{-1/2}D_{1-2\nu}(2b^{1/2}p^{1/2})$ $\mathrm{Re}\,\nu>0,\quad b>0$	$0 \qquad 0<t<b$ $\dfrac{2^{1/2-\nu}(t-b)^{\nu-1}}{\Gamma(\nu)(t+b)^{\nu-1/2}} \qquad t>b$
(5)	$\Gamma(\nu)e^{1/2ap}D_{-2\nu}(2^{1/2}a^{1/2}p^{1/2})$ $\mathrm{Re}\,\nu>0,\quad \lvert\arg a\rvert<\pi$	$2^{-\nu}a^{1/2}t^{\nu-1}(t+a)^{-\nu-1/2}$
(6)	$\Gamma(\nu)p^{-1/2}e^{1/2ap}D_{1-2\nu}(2^{1/2}a^{1/2}p^{1/2})$ $\mathrm{Re}\,\nu>0,\quad \lvert\arg a\rvert<\pi$	$2^{1/2-\nu}t^{\nu-1}(t+a)^{1/2-\nu}$
(7)	$\Gamma(2\nu)p^{-\nu}e^{1/4a/p}D_{-2\nu}(a^{1/2}p^{-1/2})$ $\mathrm{Re}\,\nu>0$	$(2t)^{\nu-1}e^{-2^{1/2}a^{1/2}t^{1/2}}$
(8)	$p^{-\nu}e^{-1/4a/p}D_{2\nu-1}(a^{1/2}p^{-1/2})$ $\mathrm{Re}\,\nu>0$	$2^{\nu+1/2}\pi^{-1/2}t^{\nu-1}\sin(\nu\pi-2^{1/2}a^{1/2}t^{1/2})$
(9)	$2^{p+\nu}\Gamma(p+\nu)D_{-2p}(a)$ $\lvert\arg a\rvert<\tfrac14\pi$	$e^{1/2t}(e^t-1)^{-\nu-1/2}\exp\left[-\dfrac{a^2e^{-t}}{4(1-e^{-t})}\right]$ $\times D_{2\nu}\left[\dfrac{a}{(1-e^{-t})^{1/2}}\right]$
(10)	$\Gamma(\nu+1)D_{-\nu-1}(pe^{1/4\pi i})$ $\times D_{-\nu-1}(pe^{-1/4\pi i})\quad \mathrm{Re}\,\nu>-1$	$\pi^{1/2}J_{\nu+1/2}(\tfrac12 t^2)$
(11)	$2^{1/2}\Gamma(\nu)D_{-\nu}(2^{1/2}p^{1/2}e^{1/4\pi i})$ $\times D_{-\nu}(2^{1/2}p^{1/2}e^{-1/4\pi i})\quad \mathrm{Re}\,\nu>0$	$t^{\nu-1}(1+t^2)^{-1/2}[1+(1+t^2)^{1/2}]^{1/2-\nu}$
(12)	$e^pD_{\nu-1/2}(2^{1/2}p^{1/2})D_{-\nu-1/2}(2^{1/2}p^{1/2})$	$\dfrac{\cos\{\nu\cos^{-1}[(1+t)^{-1}]\}}{[\pi t(t+1)(t+2)]^{1/2}}$

Parabolic cylinder functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(13)	$p^{-1/2}e^{1/2(\alpha+\beta)p}D_{4\mu}(2^{1/2}\alpha^{1/2}p^{1/2})$ $\times D_{4\nu}(2^{1/2}\beta^{1/2}p^{1/2})$ $\mathrm{Re}\,(\mu+\nu)<1/4$ $\lvert\arg\alpha\rvert<\pi,\quad\lvert\arg\beta\rvert<\pi$	$2^{-1/2}t^{-\mu-\nu-1/4}(t+\alpha)^{\mu-\nu-1/4}$ $\times(t+\beta)^{\nu-\mu-1/4}(-t-\alpha-\beta)^{\mu+\nu+1/4}$ $\times P_{2\nu-2\mu-1/2}^{\nu+\mu+1/4}[\alpha^{1/2}\beta^{1/2}(t+\alpha)^{-1/2}(t+\beta)^{-1/2}]$

5.19. Gauss' hypergeometric function

(1)	$F(\alpha,\beta;\gamma;1/2-p/\lambda)$ $\mathrm{Re}\,\alpha>0,\quad\mathrm{Re}\,\beta>0$	$\dfrac{\lambda\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\beta)}(\lambda t)^{1/2(\alpha+\beta-3)}$ $\times W_{1/2(\alpha+\beta+1)-\gamma,\,1/2(\alpha-\beta)}(\lambda t)$
(2)	$\Gamma(\alpha)p^{-\alpha}F(\alpha,\beta;\gamma;\lambda/p)$ $\mathrm{Re}\,\alpha>0$	$\lambda^{-1/2\gamma}t^{\alpha-1/2\gamma-1}M_{1/2\gamma-\beta,\,1/2\gamma-1/2}(\lambda t)$
(3)	$p^{\gamma-1}(p-1)^nF[-n,\alpha;\gamma;p/(p-1)]$ $\mathrm{Re}\,\gamma<1-n$ $\mathrm{Re}(\alpha-\gamma)>n-1$	$n!\,[\Gamma(1-\gamma)]^{-1}t^{-\gamma-n}L_n^{\alpha-\gamma-n}(t)$
(4)	$p^{m+n}(1+p)^{-m-n-2}$ $\times F(-m,-n;\,2;\,p^{-2})$	$(-1)^{m+n}t^{-1}k_{2m+2}(1/2\,t)k_{2n+2}(1/2\,t)$
(5)	$(p+1)^{-2\alpha}F\left[-n,\alpha;\dfrac{1}{2}-\nu;\left(\dfrac{p-1}{p+1}\right)^2\right]$ $\mathrm{Re}\,\alpha>0$	$\dfrac{(n!)^2\pi 2^{1-\alpha}}{\Gamma(\alpha)\Gamma(1/2+n)}t^{2\alpha-1}[L_n^{\alpha-1/2}(t)]^2$
(6)	$(p^2+a^2)^{-\lambda}$ $\times F\left(\lambda,\mu;\lambda+\mu+\dfrac{1}{2};\dfrac{a^2}{p^2+a^2}\right)$ $\mathrm{Re}\,\lambda>0$	$\dfrac{\Gamma(\lambda+\mu+1/2)}{\Gamma(2\lambda)}\left(\dfrac{a}{2}\right)^{1/2-\lambda-\mu}$ $\times t^{\lambda-\mu-1/2}J_{\lambda+\mu-1/2}(at)$

Gauss' hypergeometric function (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(7)	$(p-\alpha)^n(p-\beta)^m p^{-m-n-2}$ $\times F\left[-m,-n;-m-n-1;\ \frac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$	$\frac{(m+1)!(n+1)!}{(m+n+1)!}\frac{(-1)^{m+n}}{\alpha\beta t}e^{\frac{1}{2}(\alpha+\beta)t}$ $\times k_{2n+2}(\tfrac{1}{2}\alpha t)\,k_{2m+2}(\tfrac{1}{2}\beta t)$
(8)	$(p-\alpha)^n(p-\beta)^m p^{-m-n-\frac{1}{2}}$ $\times F\left[-m,-n;-m-n+\tfrac{1}{2};\ \frac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$	$\frac{(-2)^{m+n}(m+n)!}{(2m+2n)!\,\pi^{\frac{1}{2}}t^{\frac{1}{2}}}e^{\frac{1}{2}(\alpha+\beta)t}$ $\times D_{2n}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}})\,D_{2m}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$
(9)	$(p-\alpha)^n(p-\beta)^m p^{-m-n-3/2}$ $\times F\left[-m,-n;-m-n-\tfrac{1}{2};\ \frac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$	$-\frac{(-1)^{m+n}(-2)^{m+n+1}(m+n+1)!}{(2m+2n+2)!\,(\pi\alpha\beta t)^{\frac{1}{2}}}$ $\times e^{\frac{1}{2}(\alpha+\beta)t}D_{2n+1}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$ $\times D_{2m+1}(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}t^{\frac{1}{2}})$
(10)	$(p-\alpha)^n(p-\beta)^m p^{-m-n-\lambda-1}$ $\times F\left[-m,-n;-m-n-\lambda;\ \frac{p(p-\alpha-\beta)}{(p-\alpha)(p-\beta)}\right]$ Re $\lambda>-1$	$\frac{m!\,n!\,t^{\lambda}}{\Gamma(m+n+\lambda+1)}L_n^{\lambda}(\alpha t)\,L_m^{\lambda}(\beta t)$
(11)	$\mathrm{B}(p,\gamma)F(\alpha,\beta;\gamma+p;z)$ Re $\gamma>0$, $\lvert\arg(z-1)\rvert<\pi$	$(1-e^{-t})^{\gamma-1}F[\alpha,\beta;\gamma;z(1-e^{-t})]$
(12)	$\frac{\Gamma(p)}{\Gamma(p+\frac{1}{2})}$ $\times F(-\mu-\nu,\tfrac{1}{2}-\mu+\nu;p+\tfrac{1}{2};z^2)$ $\lvert z\rvert<1$	$\frac{\Gamma(\frac{1}{2}-\mu-\nu)\Gamma(\frac{1}{2}-\mu+\nu)}{2^{2\mu+1}\pi(1-e^{-t})^{\frac{1}{2}}}$ $\times(1-z^2+z^2e^{-t})^{\mu}$ $\times\{P_{2\nu}^{2\mu}[z(1-e^{-t})^{\frac{1}{2}}]$ $+P_{2\nu}^{2\mu}[-z(1-e^{-t})^{\frac{1}{2}}]\}$

Gauss' hypergeometric function (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(13)	$\dfrac{\Gamma(p)}{\Gamma(p+3/2)}$ $\times F(\tfrac{1}{2}-\mu-\nu, 1-\mu+\nu; p+3/2; z^2)$ $\lvert z\rvert < 1$	$-\dfrac{\Gamma(-\mu-\nu)\Gamma(\tfrac{1}{2}-\mu+\nu)}{4^{\mu+\frac{1}{2}}\,\pi z}$ $\times(1-z^2+z^2e^{-t})^{\mu}$ $\times\{P_{2\nu}^{2\mu}[z(1-e^{-t})^{\frac{1}{2}}]$ $-P_{2\nu}^{2\mu}[-z(1-e^{-t})^{\frac{1}{2}}]\}$
(14)	$B(p,\nu)F(a,p;p+\nu;z)$ $\mathrm{Re}\,\nu>0,\quad \lvert\arg(z-1)\rvert<\pi$	$(1-e^{-t})^{\nu-1}(1-ze^{-t})^{-a}$

5.20. Confluent hypergeometric functions

(1)	$p^{-\mu-\frac{1}{2}}e^{-\frac{1}{2}(a+b)p}M_{\kappa,\mu}[(b-a)p]$ $\mathrm{Re}(\mu\pm\kappa)>-\tfrac{1}{2},\quad b>a\geq 0$	0 $\quad 0<t<a$ $\dfrac{(b-a)^{\frac{1}{2}-\mu}}{B(\tfrac{1}{2}+\kappa+\mu,\tfrac{1}{2}-\kappa+\mu)}\dfrac{(t-a)^{\kappa+\mu-\frac{1}{2}}}{(b-t)^{\kappa-\mu+\frac{1}{2}}}$ $\quad a<t<b$ 0 $\quad t>b$
(2)	$p^{\kappa}e^{\frac{1}{2}a/p}M_{\kappa,\mu}(a/p)$ $\mathrm{Re}(\kappa-\mu)<\tfrac{1}{2}$	$\dfrac{a^{\frac{1}{2}}\Gamma(2\mu+1)}{\Gamma(\mu-\kappa+\tfrac{1}{2})}t^{-\kappa-\frac{1}{2}}I_{2\mu}(2a^{\frac{1}{2}}t^{\frac{1}{2}})$
(3)	$p^{\frac{1}{2}}M_{\frac{1}{4},\nu}(a/p)M_{-\frac{1}{4},\nu}(a/p)$ $\mathrm{Re}\,\nu>-\tfrac{1}{4}$	$2^{2\nu}at^{-\frac{1}{2}}\dfrac{[\Gamma(2\nu+1)]^2}{\Gamma(2\nu+\tfrac{1}{2})}$ $\times J_{2\nu}[e^{\frac{1}{4}\pi i}(2at)^{\frac{1}{2}}]$ $\times J_{2\nu}[e^{-\frac{1}{4}\pi i}(2at)^{\frac{1}{2}}]$
(4)	$p^{-\mu-\frac{1}{2}}W_{\kappa,\mu}(p)$ $\mathrm{Re}(\mu-\kappa)>\tfrac{1}{2}$	0 $\quad 0<t<\tfrac{1}{2}$ $\dfrac{1}{\Gamma(\mu+\tfrac{1}{2}-\kappa)}\dfrac{(t-\tfrac{1}{2})^{\mu-\kappa-\frac{1}{2}}}{(t+\tfrac{1}{2})^{-\mu-\kappa+\frac{1}{2}}}$ $\quad t>\tfrac{1}{2}$

Confluent hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(5)	$p^{-1} e^{\frac{1}{2}ap} W_{\kappa,\mu}(ap)$ $\lvert\arg a\rvert < \pi, \quad \mathrm{Re}\,\kappa < 1$	$(1+at^{-1})^{\frac{1}{2}\kappa} P^{\kappa}_{\mu-\frac{1}{2}}(1+2t/a)$
(6)	$p^{-\mu-\frac{1}{2}} e^{\frac{1}{2}ap} W_{\kappa,\mu}(ap)$ $\lvert\arg \lvert a\rvert < \pi$ $\mathrm{Re}(\frac{1}{2}-\kappa+\mu) > 0$	$\dfrac{a^{\frac{1}{2}-\mu} t^{\mu-\kappa-\frac{1}{2}} (a+t)^{\mu+\kappa-\frac{1}{2}}}{\Gamma(\frac{1}{2}-\kappa+\mu)}$
(7)	$p^{\kappa-\frac{1}{2}} e^{\frac{1}{2}p} W_{\kappa,\mu}(p)$ $\mathrm{Re}\,\kappa < \frac{1}{4}$	$2^{-2\kappa-\frac{1}{2}} t^{-\kappa-\frac{1}{4}} (1+t)^{-\frac{1}{2}}$ $\times P^{2\kappa+\frac{1}{2}}_{2\mu-\frac{1}{2}}[(1+t)^{\frac{1}{2}}]$
(8)	$p^{\kappa-1} e^{\frac{1}{2}p} W_{\kappa,\mu}(p) \qquad \mathrm{Re}\,\kappa < \frac{3}{4}$	$2^{-2\kappa+\frac{1}{2}} t^{-\kappa+\frac{1}{4}} P^{2\kappa-\frac{1}{2}}_{2\mu-\frac{1}{2}}[(1+t)^{\frac{1}{2}}]$
(9)	$p^{-\sigma} e^{\frac{1}{2}p/a} W_{\kappa,\mu}(p/a)$ $\lvert\arg a\rvert < \pi, \quad \mathrm{Re}(\sigma-\kappa) > 0$	$a^{-\kappa}[\Gamma(\sigma-\kappa)]^{-1} t^{\sigma-\kappa-1}$ $\times {}_2F_1(\frac{1}{2}-\kappa+\mu, \frac{1}{2}-\kappa-\mu; \sigma-\kappa; -at)$
(10)	$p^{-2\mu-1} e^{\frac{1}{2}ap^2} W_{-3\mu,\mu}(ap^2)$ $\mathrm{Re}\,a > 0, \quad \mathrm{Re}\,\mu > -1/8$	$2^{8\mu} a^{-\mu} \dfrac{\Gamma(2\mu+1)}{\Gamma(8\mu+1)} t^{4\mu}$ $\times \exp\left(-\dfrac{t^2}{8a}\right) I_{2\mu}\left(\dfrac{t^2}{8a}\right)$
(11)	$p^{-2\mu-1} e^{\frac{1}{2}ap^2} W_{\kappa,\mu}(ap^2)$ $\mathrm{Re}\,a > 0, \quad \mathrm{Re}(\kappa-\mu) < \frac{1}{2}$	$\dfrac{2^{1-\kappa+\mu} a^{\frac{1}{2}(\mu+\kappa+1)}}{\Gamma(1-2\kappa+2\mu)} t^{\mu-\kappa-1} \exp\left(-\dfrac{t^2}{8a}\right)$ $\times M_{-\frac{1}{2}(\kappa+3\mu), \frac{1}{2}(\mu-\kappa)}\left(\dfrac{t^2}{4a}\right)$
(12)	$p^{\kappa} W_{\kappa,\mu}(a/p) \qquad \mathrm{Re}(\kappa \pm \mu) < \frac{1}{2}$	$\dfrac{2a^{\frac{1}{2}} t^{-\kappa-\frac{1}{2}} K_{2\mu}(2a^{\frac{1}{2}} t^{\frac{1}{2}})}{\Gamma(\frac{1}{2}-\kappa+\mu)\Gamma(\frac{1}{2}-\kappa-\mu)}$

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Confluent hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(20)	$p^{1/2} W_{1/4,\nu}(i\,a/p)\, W_{1/4,\nu}(-i\,a/p)$ $\lvert\mathrm{Re}\,\nu\rvert < 1/4$	$-\dfrac{4a(\tfrac{1}{2}\pi/t)^{1/2} K_{2\nu}[(2at)^{1/2}]}{\Gamma(1/4+\nu)\,\Gamma(1/4-\nu)}$ $\times\{J_{2\nu}[(2at)^{1/2}]\sin[(\nu-1/4)\pi]$ $+Y_{2\nu}[(2at)^{1/2}]\cos[(\nu-1/4)\pi]\}$
(21)	$p^{-3/2} W_{\kappa,1/8}(\tfrac{1}{4}ip^2/a)$ $\times W_{\kappa,1/8}(-\tfrac{1}{4}ip^2/a)$ $\mathrm{Re}\,\kappa < 3/8, \quad a > 0$	$(\tfrac{1}{2}\pi^3 t)^{1/2}\,\dfrac{J_{-\kappa+1/8}(\tfrac{1}{2}at^2)\,J_{-\kappa-1/8}(\tfrac{1}{2}at^2)}{\Gamma(3/8-\kappa)\,\Gamma(5/8-\kappa)}$
(22)	$\dfrac{\Gamma(\tfrac{1}{2}-\kappa+\mu+p)}{\Gamma(1+2\mu+p)}\, a^{-\frac{1}{2}(1+2\mu+p)}$ $\times M_{\kappa-\frac{1}{2}p,\mu+\frac{1}{2}p}(a)$ $\mathrm{Re}(\tfrac{1}{2}+\kappa+\mu) > 0$	$\dfrac{e^{-(\frac{1}{2}+\kappa+\mu)t}}{\Gamma(\tfrac{1}{2}+\kappa+\mu)}\,(1-e^{-t})^{\kappa+\mu-\frac{1}{2}}$ $\times\exp[-a(\tfrac{1}{2}-e^{-t})]$
(23)	$\Gamma(\tfrac{1}{2}-\kappa-\mu+p)\, W_{\kappa-p,\mu}(a)$ $\mathrm{Re}\,a > 0$	$a^{\frac{1}{2}-\mu}(e^t-1)^{2\mu-1}\exp[-\tfrac{1}{2}a$ $+(\tfrac{1}{2}-\kappa-\mu)t-a/(e^t-1)]$
(24)	$\dfrac{\Gamma(\tfrac{1}{2}+\mu+p)\,\Gamma(\tfrac{1}{2}-\mu+p)}{\Gamma(1-\kappa+p)}\, W_{-p,\mu}(a)$ $\lvert\arg a\rvert < \pi$	$(1-e^{-t})^{-\kappa}\exp\left[-\dfrac{\tfrac{1}{2}a}{(1-e^{-t})}\right]$ $\times W_{\kappa,\mu}\left[\dfrac{a}{(e^t-1)}\right]$
(25)	$\dfrac{\Gamma(\tfrac{1}{2}-\kappa-\mu+p)}{\Gamma(1+p)}\, W_{\kappa-\frac{1}{2}p,\mu-\frac{1}{2}p}(a)$ $\mathrm{Re}\,\mu > -\tfrac{1}{2}$	$\dfrac{1}{\Gamma(2\mu+1)}\,(e^t-1)^{\mu-\frac{1}{2}}\exp(-\tfrac{1}{2}ae^t)$ $\times M_{-\kappa,\mu}[a(e^t-1)]$
(26)	$a^{\mu-\frac{1}{2}+\frac{1}{2}p} W_{\kappa-\frac{1}{2}p,\mu+\frac{1}{2}p}(a)$ $\mathrm{Re}(\mu+\kappa) < \tfrac{1}{2}, \quad \mathrm{Re}\,a > 0$	$[\Gamma(\tfrac{1}{2}-\mu-\kappa)]^{-1}(e^t-1)^{-\frac{1}{2}-\mu-\kappa}$ $\times\exp[-(\tfrac{1}{2}-\mu+\kappa)t-a(e^t-\tfrac{1}{2})]$

Confluent hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(27)	$\Gamma(\tfrac{1}{2}+\mu+p)M_{p,\mu}(\alpha)W_{-p,\mu}(\beta)$ $\operatorname{Re}\alpha>0,\quad \operatorname{Re}\beta>0$	$\tfrac{1}{2}\Gamma(2\mu+1)\alpha^{1/2}\beta^{1/2}\operatorname{csch}(\tfrac{1}{2}t)$ $\times e^{\frac{1}{2}(\alpha-\beta)\operatorname{ctnh}(\frac{1}{2}t)}$ $\times J_{2\mu}[\alpha^{1/2}\beta^{1/2}\operatorname{csch}(\tfrac{1}{2}t)]$
(28)	$\Gamma(\tfrac{1}{2}+\mu+p)\Gamma(\tfrac{1}{2}-\mu+p)$ $\times W_{-p,\mu}(\alpha)W_{+p,\mu}(\beta)$ $\operatorname{Re}\alpha>0,\quad \operatorname{Re}\beta>0$	$\tfrac{1}{2}\alpha^{1/2}\beta^{1/2}\operatorname{csch}(\tfrac{1}{2}t)e^{-\frac{1}{2}(\alpha+\beta)\operatorname{ctnh}(\frac{1}{2}t)}$ $\times K_{2\mu}[\alpha^{1/2}\beta^{1/2}\operatorname{csch}(\tfrac{1}{2}t)]$
(29)	$p^{-\nu}e^{-1/(8p)}\,k_{2n}(2^{-3}p^{-1})$	$\tfrac{1}{2}(-1)^{n-1}(t^{n-1/2}/n!)\,J_1(t^{1/2})$
(30)	$\Gamma(\nu+1)p^{-\nu}e^{-\frac{1}{2}\alpha/p}$ $\times k_{-2\nu}(\tfrac{1}{2}e^{\pi i}\alpha/p)\quad \operatorname{Re}\nu>0$	$-i\,\sin(\nu\pi)t^{\nu-1/2}H_1^{(2)}(2\alpha^{1/2}t^{1/2})$
(31)	$\Gamma(\nu+1)p^{-\nu}e^{-\frac{1}{2}\alpha/p}$ $\times k_{-2\nu}(\tfrac{1}{2}e^{-\pi i}\alpha/p)\quad \operatorname{Re}\nu>0$	$i\,\sin(\nu\pi)t^{\nu-1/2}H_1^{(1)}(2\alpha^{1/2}t^{1/2})$

5.21. Generalized hypergeometric functions

(1)	$\Gamma(\sigma)p^{-\sigma}$ $\times {}_mF_n(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n;\lambda/p)$ $m\le n+1,\quad \operatorname{Re}\sigma>0$	$t^{\sigma-1}\,{}_mF_{n+1}(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n,\sigma;\lambda t)$
(2)	$\Gamma(2\sigma)p^{-2\sigma}$ $\times {}_mF_n(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n;\lambda^2/p^2)$ $m\le n+1)\quad \operatorname{Re}\sigma>0$	$t^{2\sigma-1}$ $\times {}_mF_{n+2}(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n,\sigma,\sigma+\tfrac{1}{2};$ $\tfrac{1}{4}\lambda^2t^2)$
(3)	$\Gamma(k\sigma)p^{-k\sigma}$ $\times {}_mF_n(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n;\lambda^k/p^k)$ $m\le n+1,\quad \operatorname{Re}\sigma>0$	$t^{k\sigma-1}\,{}_mF_{n+k}(a_1,\ldots,a_m;$ $\rho_1,\ldots,\rho_n,\sigma,\sigma+1/k,\ldots,\sigma+k-1/k;$ $\lambda^kt^k/k^k)$

Generalized hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(4)	$p^{-1/2}$ $\times {}_mF_n(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n;-\lambda p^{1/2})$ $m\leq n$	$\pi^{-1/2}t^{-1/2}\,{}_{2m}F_{2n}(1/2 a_1, 1/2 a_1+1/2,\ldots,1/2 a_m,$ $1/2 a_m+1/2; 1/2\rho_1, 1/2\rho_1+1/2,\ldots,1/2\rho_n,$ $1/2\rho_n+1/2;-2^{m-n-2}\lambda^2/t)$
(5)	$\mathrm{B}(p,\lambda)$ $\times {}_3F_2(\alpha,\beta,p;\gamma,p+\lambda;z)$ $\mathrm{Re}\,\lambda>0,\quad \lvert\arg(z-1)\rvert<\pi$	$(1-e^{-t})^{\lambda-1}F(\alpha,\beta;\gamma;ze^{-t})$
(6)	$\mathrm{B}(p,\lambda)$ $\times {}_3F_2(\alpha,\beta,\lambda;\gamma,p+\lambda;z)$ $\mathrm{Re}\,\lambda>0,\quad \lvert\arg(z-1)\rvert<\pi$	$(1-e^{-t})^{\lambda-1}F(\alpha,\beta;\gamma;z(1-e^{-t})]$
(7)	$2^{2p+\alpha}\mathrm{B}(p,p+\alpha)$ $\times {}_4F_3(-n,n+1,p+\alpha;1,2p+\alpha;1)$	$\theta^{-1}[(1-\theta)^{\alpha}+(-1)^n(1+\theta)^{\alpha}]\,P_n(\theta)$ $\theta=(1-e^{-t})^{1/2}$
(8)	$\mathrm{B}(p,\sigma)$ $\times {}_{m+1}F_{n+1}(a_1,\ldots,a_m,p;$ $\rho_1,\ldots,\rho_n,p+\sigma;z)$ $\mathrm{Re}\,\sigma>0,\quad m\leq n+1$ $\lvert z\rvert<1$ if $m=n+1$	$(1-e^{-t})^{\sigma-1}$ $\times {}_mF_n(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n;ze^{-t})$
(9)	$\mathrm{B}(p,\sigma)$ $\times {}_{m+1}F_{n+1}(a_1,\ldots,a_m,\sigma;$ $\rho_1,\ldots,\rho_n,p+\sigma;z)$ $\mathrm{Re}\,\sigma>0,\quad m\leq n+1$ $\lvert z\rvert<1$ if $m=n+1$	$(1-e^{-t})^{\sigma-1}$ $\times {}_mF_n(a_1,\ldots,a_m;\rho_1,\ldots,\rho_n;z(1-e^{-t})]$
(10)	$p^{-\lambda-\mu}$ $\times E(-\nu,\nu+1,\lambda+\mu:\mu+1:2p)$ $\mathrm{Re}(\lambda+\mu)>0$	$-\pi\csc(\nu\pi)\,t^{\lambda+1/2\mu-1}(t+2)^{1/2\mu}$ $\times P_\nu^{-\mu}(t+1)$

Generalized hypergeometric functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(11)	$p^{-\lambda}E(\mu+\nu+1,\mu-\nu,\lambda:\mu+1:2p)$ $\operatorname{Re}\lambda>0$	$2^{\mu}\Gamma(\mu+\nu+1)\Gamma(\mu-\nu)t^{\lambda-\frac{1}{2}\mu-1}$ $\times(t+2)^{-\frac{1}{2}\mu}P_{\nu}^{-\mu}(t+1)$
(12)	$p^{-\gamma}E(a,\beta,\gamma:\delta:p) \quad \operatorname{Re}\gamma>0$	$\dfrac{\Gamma(a)\Gamma(\beta)}{\Gamma(\delta)}t^{\gamma-1}F(a,\beta;\delta;-t)$
(13)	$p^{-a_m}E(m;a_r:n;\beta_s:p)$ $\operatorname{Re}a_m>0$	$t^{a_m-1}E(m-1;a_r:n;\beta_s:1/t)$
(14)	$\Gamma(p-a_m)$ $\times E(m;a_r:n+1;\beta_1,\ldots,\beta_n,p:z)$ $\operatorname{Re}a_m>0$	$(e^t-1)^{a_m}$ $\times E[m-1;a_1,\ldots,a_{m-1}:n;$ $\beta_1,\ldots,\beta_n:z/(1-e^{-t})$

5.22. Elliptic functions and theta functions

(1)	$p^{-1}K(a/p)$	$\frac{1}{2}\pi I_0^2(\frac{1}{2}at)$
(2)	$K(a/p)-\frac{1}{2}\pi$	$\frac{1}{2}\pi a\, I_0(\frac{1}{2}\pi t)\, I_1(\frac{1}{2}at)$
(3)	$pK(a/p)-\frac{1}{2}\pi p$	$\frac{1}{4}\pi a^2\{\frac{1}{2}[I_0(\frac{1}{2}at)]^2+[I_1(\frac{1}{2}at)]^2$ $+\frac{1}{2}I_0[\frac{1}{2}at)\,I_2(\frac{1}{2}at)\}$
(4)	$\frac{1}{2}\pi p-pE(a/p)$	$\frac{1}{2}\pi a t^{-1}\,I_0(\frac{1}{2}at)\,I_1(\frac{1}{2}at)$
(5)	$p[K(a/p)-E(a/p)]$	$\frac{1}{4}\pi a^2[I_0^2(\frac{1}{2}at)+I_1^2(\frac{1}{2}at)]$
(6)	$p(p^2-a^2)^{-1}E(a/p)$	$\frac{1}{2}\pi\, I_0(\frac{1}{2}at)[I_0(\frac{1}{2}at)$ $+at\,I_1(\frac{1}{2}at)]$

Elliptic functions and theta functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt} f(t)\,dt$	$f(t)$
(7)	$r^{-1} E(a/r)$	$\tfrac{1}{2}\pi J_0(\tfrac{1}{2}at)[J_0(\tfrac{1}{2}at) - at\, J_1(\tfrac{1}{2}at)]$
(8)	$r^{-1} K(a/r)$	$\tfrac{1}{2}\pi J_0^2(\tfrac{1}{2}at)$
(9)	$(1-\tfrac{1}{2}a^2 r^{-2})K(a/r) - E(a/r)$	$\tfrac{1}{4}\pi a^2 J_1^2(\tfrac{1}{2}at)$
(10)	$r^{-1}[K(a/r) - E(a/r)]$	$\tfrac{1}{2}\pi a t\, J_0(\tfrac{1}{2}at)\, J_1(\tfrac{1}{2}at)$
(11)	$p^{-\frac{1}{2}}\, \theta_1(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} (-1)^n J_0[2(a+n-\tfrac{1}{2})t^{\frac{1}{2}}]$
(12)	$p^{-\frac{1}{2}}\, \theta_2(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} (-1)^n J_0[2(a+n)t^{\frac{1}{2}}]$
(13)	$p^{-\frac{1}{2}}\, \theta_3(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} J_0[2(a+n)t^{\frac{1}{2}}]$
(14)	$p^{-\frac{1}{2}}\, \theta_4(a, i\pi p)$	$\pi^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} J_0[2(a+n+\tfrac{1}{2})t^{\frac{1}{2}}]$
(15)	$p^{-1}\, \theta_1(a, i\pi p)$ $\quad \lvert \mathrm{Re}\, a \rvert < \tfrac{1}{2}$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \dfrac{(-1)^n \sin[2(a+n-\tfrac{1}{2})t^{\frac{1}{2}}]}{a+n-\tfrac{1}{2}}$
(16)	$p^{-1}\, \theta_2(a, i\pi p)$ $\quad 0 < \mathrm{Re}\, a < 1$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \dfrac{(-1)^n \sin[2(a+n)t^{\frac{1}{2}}]}{a+n}$
(17)	$p^{-1}\, \theta_3(a, i\pi p)$ $\quad 0 < \mathrm{Re}\, a < 1$	$\pi^{-1} \sum_{n=-\infty}^{\infty} \dfrac{\sin[2(a+n)t^{\frac{1}{2}}]}{a+n}$

$r = (p^2 + a^2)^{\frac{1}{2}}$

Elliptic functions and theta functions (cont'd)

	$g(p)=\int_0^\infty e^{-pt}f(t)\,dt$	$f(t)$
(18)	$p^{-1}\,\theta_4(a, i\pi p)$ $\quad \lvert\mathrm{Re}\,a\rvert < \frac{1}{2}$	$\pi^{-1}\sum_{n=-\infty}^{\infty}\dfrac{\sin[2(a+n+\frac{1}{2})t^{1/2}]}{a+n+\frac{1}{2}}$
(19)	$p^{-1}\,\theta_2(0, i\pi p)$	$0 \quad 0<t<\frac{1}{4}\pi^2$ $2n+2 \quad \pi^2(n+1/2)^2<t<\pi^2(n+3/2)^2$
(20)	$p^{-1}\,\theta_3(0, i\pi p)$	$2n+1 \quad \pi^2 n^2<t<\pi^2(n+1)^2$
(21)	$p^{-1}\,\theta_4(0, i\pi p)$	$1 \quad (2k)^2\pi^2<t<(2k+1)^2\pi^2$ $-1 \quad (2k+1)^2\pi^2<t<(2k+2)^2\pi^2$
(22)	$p^{-\nu}\,\theta_1(a, i\pi p)$ $\mathrm{Re}\,\nu \geq \frac{1}{2}, \quad \lvert\mathrm{Re}\,a\rvert < \frac{1}{2}$	$\pi^{-1/2}t^{\frac{1}{2}\nu-\frac{1}{4}}$ $\times\sum_{n=-\infty}^{\infty}\dfrac{(-1)^n J_{\nu-\frac{1}{2}}[2(a+n-\frac{1}{2})t^{1/2}]}{(a+n-\frac{1}{2})^{\nu-\frac{1}{2}}}$
(23)	$p^{-\nu}\,\theta_2(a, i\pi p)$ $\mathrm{Re}\,\nu \geq \frac{1}{2}, \quad 0<\mathrm{Re}\,a<1$	$\pi^{-1/2}t^{\frac{1}{2}\nu-\frac{1}{4}}\sum_{n=-\infty}^{\infty}\dfrac{(-1)^n J_{\nu-\frac{1}{2}}[2(a+n)t^{1/2}]}{(a+n)^{\nu-\frac{1}{2}}}$
(24)	$p^{-\nu}\,\theta_3(a, i\pi p)$ $\mathrm{Re}\,\nu \geq \frac{1}{2}, \quad 0<\mathrm{Re}\,a<1$	$\pi^{-1/2}t^{\frac{1}{2}\nu-\frac{1}{4}}\sum_{n=-\infty}^{\infty}\dfrac{J_{\nu-\frac{1}{2}}[2(a+n)t^{1/2}]}{(a+n)^{\nu-\frac{1}{2}}}$
(25)	$p^{-\nu}\,\theta_4(a, i\pi p)$ $\mathrm{Re}\,\nu > \frac{1}{2}, \quad \lvert\mathrm{Re}\,a\rvert < \frac{1}{2}$	$\pi^{-1/2}t^{\frac{1}{2}\nu-\frac{1}{4}}\sum_{n=-\infty}^{\infty}\dfrac{J_{\nu-\frac{1}{2}}[2(a+n+\frac{1}{2})t^{1/2}]}{(a+n+\frac{1}{2})^{\nu-\frac{1}{2}}}$

MELLIN TRANSFORMS

We call

$$g(s) = \mathfrak{M}\{f(x); s\} = \int_0^\infty f(x)\, x^{s-1}\, dx$$

the *Mellin transform* of $f(x)$, and regard s as a complex variable. The function $f(x)$ is called the *inverse Mellin transform* of $g(s)$. Under certain conditions, the inverse Mellin transform may be represented as an integral, see 6.1(1). We give tables of both Mellin transforms and inverse Mellin transforms. In chapter VI transform pairs are classified according to $f(x)$, in chapter VII according to $g(s)$.

Mellin transforms are virtually two-sided Laplace transforms and may be expressed either as exponential Fourier transforms in the complex domain, or as combinations of Laplace transforms.

$$\mathfrak{M}\{f(x); s\} = \mathfrak{F}\{f(e^x); is\} = \mathfrak{L}\{f(e^t); -s\} + \mathfrak{L}\{f(e^{-t}); s\}.$$

Accordingly, information about Mellin transforms is found in several works on Fourier and Laplace transforms. The two most important sources are Doetsch (1950) and Titchmarsh (1937).

For the same reason it seems sufficient to give comparatively brief tables of Mellin and inverse Mellin transforms. Further transform pairs may be obtained by means of the methods indicated in the introduction to this volume, by means of the general formulas given in sections 6.1 and 7.1, and from tables of Fourier and Laplace transforms by means of the formulas given above.

REFERENCES

See also under Fourier and Laplace transforms.

Doetsch, Gustav, 1950: *Handbuch der Laplace Transformation*, vol. I, Birkhäuser, Basel.

Titchmarsh, E. G., 1937: *Introduction to the theory of Fourier integrals*, Oxford.

CHAPTER VI

MELLIN TRANSFORMS

6.1. General formulas

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(1)	$(2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty} g(s)\,x^{-s}ds$	$g(s)$
(2)	$f(ax)$ $\qquad a>0$	$a^{-s}g(s)$
(3)	$x^{\alpha}f(x)$	$g(s+\alpha)$
(4)	$f(1/x)$	$g(-s)$
(5)	$f(x^h)$ $\qquad h>0$	$h^{-1}g(s/h)$
(6)	$f(x^{-h})$ $\qquad h>0$	$h^{-1}g(-s/h)$
(7)	$x^{\beta}f(ax^h)$ $\qquad a>0,\quad h>0$	$h^{-1}a^{-(s+\beta)/h}\,g[(s+\beta)/h]$
(8)	$x^{\beta}f(ax^{-h})$ $\qquad a>0,\quad h>0$	$h^{-1}a^{(s+\beta)/h}\,g[-(s+\beta)/h]$
(9)	$f'(x)$	$-(s-1)g(s-1)$
(10)	$f^{(n)}(x)$	$(-1)^n(s-n)_n\,g(s-n)$
(11)	$\left(x\dfrac{d}{dx}\right)^n f(x)$	$(-1)^n s^n g(s)$

General formulas (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(12)	$\left(\frac{d}{dx}x\right)^n f(x)$	$(-1)^n(s-1)^n g(s)$
(13)	$x^\alpha\int_0^\infty \xi^\beta f_1(x\xi)f_2(\xi)\,d\xi$	$g_1(s+\alpha)g_2(1-s-\alpha+\beta)$
(14)	$x^\alpha\int_0^\infty \xi^\beta f_1(x/\xi)f_2(\xi)\,d\xi$	$g_1(s+\alpha)g_2(s+\alpha+\beta+1)$

6.2. Algebraic functions and powers with arbitrary index

(1)	x $\quad 0<x<1$ $2-x$ $\quad 1<x<2$ 0 $\quad x>2$	$2s^{-1}(s+1)^{-1}(2^s-1)$ $\quad s\neq 0$ $2\log 2$ $\quad s=0$ $\operatorname{Re} s>-1$
(2)	$(1+x)^{-1}$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$	$\tfrac12\psi(\tfrac12+\tfrac12 s)-\tfrac12\psi(\tfrac12 s)$ $\quad \operatorname{Re} s>0$
(3)	$(a+x)^{-1}$ $\quad \lvert\arg a\rvert<\pi$	$\pi a^{s-1}\csc(\pi s)$ $\quad 0<\operatorname{Re} s<1$
(4)	$(a-x)^{-1}$ $\quad a>0$	$\pi a^{s-1}\operatorname{ctn}(\pi s)$ $\quad 0<\operatorname{Re} s<1$ The integral is a Cauchy Principal Value
(5)	$(1+ax)^{-1}$ $\quad 0<x<b$ 0 $\quad x>b$ $\lvert\arg(1-ab)\rvert<\pi$	$b^s s^{-1}\,{}_2F_1(1,s;1+s;-ab)$ $\operatorname{Re} s>0$
(6)	$(1+ax)^{-n-1}$ $\quad \lvert\arg a\rvert<\pi$	$(-1)^n\pi a^{-s}\csc(\pi s)\binom{s-1}{n}$ $0<\operatorname{Re} s<n+1$

Algebraic functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(7)	$(a+x)^{-1}(\beta+x)^{-1}$ $\lvert\arg a\rvert<\pi, \quad \lvert\arg\beta\rvert<\pi$	$\pi\csc(\pi s)(a^{s-1}-\beta^{s-1})/(\beta-a)$ $0<\operatorname{Re} s<2$
(8)	$(a+x)^{-1}(b-x)^{-1}$ $\lvert\arg a\rvert<\pi, \quad b>0$	$\pi(a+b)^{-1}[a^{s-1}\csc(\pi s)+b^{s-1}\operatorname{ctn}(\pi s)]$ $0<\operatorname{Re} s<2$ The integral is a Cauchy Principal Value
(9)	$(a-x)^{-1}(b-x)^{-1}$ $\quad a>b>0$	$\pi\operatorname{ctn}(\pi s)\left(\dfrac{a^{s-1}-b^{s-1}}{b-a}\right)$ $0<\operatorname{Re} s<2$ The integral is a Cauchy Principal Value
(10)	$(x+a)(x+\beta)^{-1}(x+\gamma)^{-1}$ $\lvert\arg\beta\rvert<\pi$ $\lvert\arg\gamma\rvert<\pi$	$\dfrac{\pi}{\sin(\pi s)}\left[\left(\dfrac{\beta-a}{\beta-\gamma}\right)\beta^{s-1}+\left(\dfrac{\gamma-a}{\gamma-\beta}\right)\gamma^{s-1}\right]$ $0<\operatorname{Re} s<1$
(11)	$(x^2+a^2)^{-1}$ $\quad \operatorname{Re} a>0$	$\tfrac{1}{2}\pi a^{s-2}\csc(\tfrac{1}{2}\pi s)$ $\quad 0<\operatorname{Re} s<2$
(12)	$(x^2+2ax\cos\theta+a^2)^{-1}$ $a>0, \quad -\pi<\theta<\pi$	$-\pi a^{s-2}\csc\theta\csc(\pi s)\sin[(s-1)\theta]$ $0<\operatorname{Re} s<2$
(13)	$(x^2+2ax\cos\theta+a^2)^{-2}$ $a>0, \quad -\pi<\theta<\pi$	$\tfrac{1}{2}\pi a^{s-4}\csc(\pi s)\,(\csc\theta)^3$ $\times\{(s-1)\sin\theta\cos[(s-2)\theta]$ $-\sin[(s-1)\theta]\}$ $\quad 0<\operatorname{Re} s<4$
(14)	$(a+x^2)^{-1}(\beta+x^2)^{-1}$ $\lvert\arg a\rvert<\pi, \quad \lvert\arg\beta\rvert<\pi$	$\tfrac{1}{2}\pi\csc(\tfrac{1}{2}\pi s)(a^{\frac{1}{2}s-1}-\beta^{\frac{1}{2}s-1})/(\beta-a)$ $0<\operatorname{Re} s<4$
(15)	$(a+x^n)^{-1}$ $\quad \lvert\arg a\rvert<\pi$	$(\pi/n)\csc(\pi s/n)\,a^{s/n-1}$ $\quad 0<\operatorname{Re} s<n$
(16)	$(1-x)(1-x^m)^{-1}$	$(\pi/m)\sin(\pi/m)\csc(\pi s/m)$ $\times\csc[(\pi s+\pi)/m]$ $\quad 0<\operatorname{Re} s<m-1$

Algebraic functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(17)	$(1+2x\cos\theta+x^2)^{-1/2}$ $-\pi<\theta<\pi$	$\pi\csc(\pi s)\,\mathrm{P}_{s-1}(\cos\theta)$ $0<\mathrm{Re}\,s<1$
(18)	x^{ν} $\quad 0<x<1$ 0 $\quad x>1$	$(s+\nu)^{-1}$ $\quad \mathrm{Re}\,s>-\mathrm{Re}\,\nu$
(19)	$(1+ax)^{-\nu}$ $\quad \lvert\arg a\rvert<\pi$	$a^{-s}\mathrm{B}(s,\nu-s)$ $\quad 0<\mathrm{Re}\,s<\mathrm{Re}\,\nu$
(20)	$(1+ax)^{-\nu}$ $\quad 0<x<b$ 0 $\quad x>b$ $\lvert\arg(1+ab)\rvert<\pi$	$s^{-1}b^{s}\,{}_2F_1(\nu,s;1+s;-ab)$ $\mathrm{Re}\,s>0$
(21)	0 $\quad 0<x<b$ $(1+ax)^{-\nu}$ $\quad x>b$	$a^{-\nu}b^{s-\nu}(\nu-s)^{-1}$ $\times\,{}_2F_1(\nu,\nu-s;\nu-s+1;-a^{-1}b^{-1})$ $\mathrm{Re}\,s<\mathrm{Re}\,\nu$
(22)	$(1+2x\cos\theta+x^2)^{-\nu}$ $-\pi<\theta<\pi$	$2^{\nu-1/2}(\sin\theta)^{\nu-1/2}\Gamma(\tfrac{1}{2}+\nu)$ $\times\mathrm{B}(s,2\nu-s)\,\mathrm{P}^{1/2-\nu}_{s-\nu-1/2}(\cos\theta)$ $0<\mathrm{Re}\,s<\mathrm{Re}\,2\nu$
(23)	$(1+x)^{\nu}(1+ax)^{\mu}$ $\quad \lvert\arg a\rvert<\pi$	$\mathrm{B}(s,-\mu-\nu-s)$ $\times\,{}_2F_1(-\mu,s;-\mu-\nu;1-a)$ $0<\mathrm{Re}\,s<-\mathrm{Re}(\mu+\nu)$
(24)	$(1-x)^{\nu}(1+ax)^{\mu}$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$ $\mathrm{Re}\,\nu>-1,\quad \lvert\arg(1+a)\rvert<\pi$	$\mathrm{B}(\nu+1,s)$ $\times\,{}_2F_1(-\mu,s,\nu+s+1;-a)$ $\mathrm{Re}\,s>0$
(25)	$(1+x^2)^{-1/2}[(1+x^2)^{1/2}+a]^{\nu}$ $\mathrm{Re}\,a>-1$	$2^{\frac{1}{2}s-1}(a^2-1)^{\frac{1}{2}\nu+\frac{1}{4}s}\Gamma(\tfrac{1}{2}s)$ $\times\Gamma(1-\nu-s)\,P^{\nu+\frac{1}{2}s}_{\frac{1}{2}s-1}(a)$ $0<\mathrm{Re}\,s<1-\mathrm{Re}\,\nu$

Algebraic functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$
(26)	$(1+x^2)^{-1/2}[(a^2-1)^{1/2}(1+x^2)^{1/2}+a]^\nu$ $\mathrm{Re}\,\nu<0,\quad \mathrm{Re}\,a>1$	$2^{1/2 s-1/2}\pi^{-1/2}e^{-1/2 i\pi(s-1)}$ $\times\Gamma(1-\nu-s)\,\Gamma(\tfrac{1}{2}s)[\Gamma(-\nu)]^{-1}$ $\times(a^2-1)^{-1/4 s+1/4}\,Q^{1/2 s-1/2}_{-\nu-1/2 s-1/2}(a)$ $\mathrm{Re}\,s<1-\mathrm{Re}\,\nu$
(27)	$(1+x^2)^{-1/2}[\cos\theta\pm i\sin\theta(1+x^2)^{1/2}]^\nu$ $\mathrm{Re}\,\nu<0,\quad -\pi<\theta<\pi$ With either upper or lower signs on both sides	$2^{1/2 s-1/2}(\sin\theta)^{1/2-1/2 s}\dfrac{\Gamma(\tfrac{1}{2}s)\,\Gamma(1-s-\nu)}{\Gamma(-\nu)}$ $\times[\pi^{-1/2}\,Q^{1/2 s-1/2}_{-1/2 s-1/2-\nu}(\cos\theta)$ $\mp\tfrac{1}{2}i\,\pi^{1/2}\,P^{1/2 s-1/2}_{-1/2 s-1/2-\nu}(\cos\theta)]$ $\mathrm{Re}\,s>0$
(28)	$(a^2+x^2)^{-1/2}[(a^2+x^2)^{1/2}+x]^\nu$ $\mathrm{Re}\,a>0$	$2^{-s}a^{\nu+s-1}\,\mathrm{B}(s,\tfrac{1}{2}-\tfrac{1}{2}s-\tfrac{1}{2}\nu)$ $0<\mathrm{Re}\,s<-\mathrm{Re}\,\nu+1$
(29)	$0 \qquad 0<x<1$ $(x^2-1)^{-1/2}\{[x-(x^2-1)^{1/2}]^\nu$ $+[x-(x^2-1)^{1/2}]^{-\nu}\} \qquad 1<x<\infty$	$2^{-s}\mathrm{B}(\tfrac{1}{2}-\tfrac{1}{2}s+\tfrac{1}{2}\nu,\tfrac{1}{2}-\tfrac{1}{2}s-\tfrac{1}{2}\nu)$ $\mathrm{Re}\,s<\mathrm{Re}\,\nu+1$
(30)	$(1+ax^h)^{-\nu} \qquad \lvert\arg a\rvert<\pi,\quad h>0$	$h^{-1}a^{-s/h}\,\mathrm{B}(s/h,\nu-s/h)$ $0<\mathrm{Re}\,s<h\,\mathrm{Re}\,\nu$
(31)	$(1-x^h)^{\nu-1} \qquad 0<x<1$ $0 \qquad x\geq 1$ $h>0,\quad \mathrm{Re}\,\nu>0$	$h^{-1}\mathrm{B}(\nu,s/h) \qquad \mathrm{Re}\,s>0$
(32)	$0 \qquad 0<x\leq 1$ $(x^h-1)^{\nu-1} \qquad x\geq 1$ $h>0,\quad \mathrm{Re}\,\nu>0$	$h^{-1}\mathrm{B}(1-\nu-s/h,\nu)$ $\mathrm{Re}\,s<h-h\,\mathrm{Re}\,\nu$
(33)	$(1-x^a)(1-x^{na})^{-1}$	$\dfrac{\pi}{na}\sin\!\left(\dfrac{\pi}{n}\right)\csc\!\left(\dfrac{\pi s}{na}\right)\csc\!\left(\dfrac{\pi s+\pi a}{na}\right)$ $0<\mathrm{Re}\,s<(n-1)a$

Algebraic functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(34)	$(1+x^{2\nu})^{-1}(1+x^{3\nu})^{-1}$ $\quad \operatorname{Re}\nu>0$	$-\dfrac{\pi}{8\nu}\;\dfrac{\csc(\pi\nu^{-1}s/3)}{1-4\cos^2(\pi\nu^{-1}s/3)}$ $\quad 0<\operatorname{Re}s<5\operatorname{Re}\nu$
(35)	$(1+\alpha x^h)^{-\mu}(1+\beta x^h)^{-\nu}\quad h>0$ $\quad \lvert\arg\alpha\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$	$h^{-1}\alpha^{-s/h}\mathrm{B}(s/h,\mu+\nu-s/h)$ $\times{}_2F_1(\nu,s/h;\mu+\nu;1-\beta/\alpha)$ $\quad 0<\operatorname{Re}s<2\operatorname{Re}(\mu+\nu)$

6.3. Exponential functions

(1)	$e^{-\alpha x}$ $\quad \operatorname{Re}\alpha>0$	$\alpha^{-s}\Gamma(s)$ $\quad \operatorname{Re}s>0$
(2)	$e^{-\beta x}\quad 0<x<a$ $0\quad x>a$	$\beta^{-s}\gamma(s,\beta a)$ $\quad \operatorname{Re}s>0$
(3)	$0\quad 0<x<a$ $e^{-\beta x}\quad a<x<\infty$ $\operatorname{Re}\beta>0$	$\beta^{-s}\Gamma(s,\beta a)$
(4)	$(x+\beta)^{-1}e^{-\alpha x}$ $\quad \operatorname{Re}\alpha>0,\quad \lvert\arg\beta\rvert<\pi$	$\Gamma(s)\beta^{s-1}e^{\alpha\beta}\Gamma(1-s,\alpha\beta)$ $\quad \operatorname{Re}s>0$
(5)	$\dbinom{x}{n}e^{-\alpha x}$ $\quad \operatorname{Re}\alpha>0$	$\alpha^{-s}\Gamma(s)\Phi_n(s,\alpha^{-1})$ where $\sum_{n=0}^{\infty}h^n\Phi_n(\nu,z)=[1-z\log(h+1)]^{-\nu}$ $\operatorname{Re}s>0$
(6)	$(e^{\alpha x}+1)^{-1}$ $\quad \operatorname{Re}\alpha>0$	$\alpha^{-s}\Gamma(s)(1-2^{1-s})\,\zeta(s)$ $\quad \operatorname{Re}s>0$
(7)	$(e^{\alpha x}-1)^{-1}$ $\quad \operatorname{Re}\alpha>0$	$\alpha^{-s}\Gamma(s)\,\zeta(s)$ $\quad \operatorname{Re}s>1$

Exponential functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$
(8)	$e^{-\alpha x}(1-e^{-x})^{-1}$ Re $\alpha>0$	$\Gamma(s)\,\zeta(s,\alpha)$ Re $s>1$
(9)	$e^{-\alpha x}(1-\beta e^{-x})^{-1}$ Re $\alpha>0$, $\lvert\arg(1-\beta)\rvert<\pi$	$\Gamma(s)\,\Phi(\beta,s,\alpha)$ Re $s>0$
(10)	$(e^{x}-1)^{-2}$	$\Gamma(s)[\zeta(s-1)-\zeta(s)]$ Re $s>2$
(11)	$e^{-\alpha x}(1-e^{-x})^{-2}$ Re $\alpha>0$	$\Gamma(s)[\zeta(s-1,\alpha-1)$ $-(\alpha-1)\,\zeta(s,\alpha-1)]$ Re $s>2$
(12)	$e^{-\alpha x}(1-\beta e^{-x})^{-2}$ Re $\alpha>0$, $\lvert\arg(1-\beta)\rvert<\pi$	$\Gamma(s)\,[\Phi(\beta,s-1,\alpha-1)$ $-(\alpha-1)\Phi(\beta,s,\alpha-1)]$ Re $s>0$
(13)	$e^{-\alpha x^2-\beta x}$ Re $\alpha>0$	$(2\alpha)^{-\frac{1}{2}s}\,\Gamma(s)\exp\left(\dfrac{\beta^2}{8\alpha}\right)$ $\times D_{-s}[\beta(2\alpha)^{-\frac{1}{2}}]$ Re $s>0$
(14)	$(1+x)^{-\frac{1}{2}}\exp[-\alpha(1+x)^{\frac{1}{2}}]$ Re $\alpha>0$	$2\pi^{-\frac{1}{2}}(\tfrac{1}{2}\alpha)^{\frac{1}{2}-s}\,\Gamma(s)K_{\frac{1}{2}-s}(\alpha)$ Re $s>0$
(15)	$\exp(-\alpha x^{h})$ Re $\alpha>0$, $h>0$	$h^{-1}\,\alpha^{-s/h}\Gamma(s/h)$ Re $s>0$
(16)	$\exp(-\alpha x^{-h})$ Re $\alpha>0$, $h>0$	$h^{-1}\,\alpha^{s/h}\,\Gamma(-s/h)$ Re $s<0$
(17)	$\exp(-\alpha x^{h}-\beta x^{-h})$ $h>0$ Re $\alpha>0$, Re $\beta>0$	$2h^{-1}(\beta/\alpha)^{\frac{1}{2}s/h}\,K_{s/h}(2\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}})$
(18)	$1-\exp(-\alpha x^{h})$ Re $\alpha>0$, $h>0$	$-h^{-1}\,\alpha^{-s/h}\,\Gamma(s/h)$ $-h<$ Re $s<0$

Exponential functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(19)	$1-\exp(-ax^{-h})$ $\mathrm{Re}\,a>0,\quad h>0$	$-h^{-1}a^{s/h}\Gamma(-s/h)\quad 0<\mathrm{Re}\,s<h$

6.4. Logarithmic functions

(1)	$\log x \qquad 0<x<a$ $0 \qquad x>a$	$s^{-1}a^{-s}(\log a-s^{-1})\qquad \mathrm{Re}\,s>0$
(2)	$(x+a)^{-1}\log x \qquad \lvert\arg a\rvert<\pi$	$\pi a^{s-1}\csc(\pi s)[\log a-\pi\,\mathrm{ctn}(\pi s)]$ $0<\mathrm{Re}\,s<1$
(3)	$(x+1)^{-1}\log x \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$\tfrac{1}{4}[\psi'(\tfrac{1}{2}+\tfrac{1}{2}s)-\psi'(\tfrac{1}{2}s)]$ $\mathrm{Re}\,s>0$
(4)	$(x-1)^{-1}\log x \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$\psi'(s)\qquad \mathrm{Re}\,s>0$
(5)	$(a-x)^{-1}\log x \qquad a>0$	$\pi a^{s-1}[\mathrm{ctn}(\pi s)\log a-\pi\csc^2(\pi s)]$ $0<\mathrm{Re}\,s<1$ The integral is a Cauchy Principal Value
(6)	$(x+a)^{-1}(x+\beta)^{-1}\log x$ $\lvert\arg a\rvert<\pi,\quad \lvert\arg\beta\rvert<\pi$	$\pi(\beta-a)^{-1}\csc(\pi s)[a^{s-1}\log a$ $-\beta^{s-1}\log\beta-\pi\,\mathrm{ctn}(\pi s)(a^{s-1}-\beta^{s-1})]$ $0<\mathrm{Re}\,s<1$
(7)	$x^{\nu}\log x \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$-(s+\nu)^{-2}\qquad \mathrm{Re}\,s>-\mathrm{Re}\,\nu$
(8)	$(1-x)^{\nu-1}\log x \qquad 0<x<1$ $0 \qquad 1<x<\infty$ $\mathrm{Re}\,\nu>0$	$\mathrm{B}(\nu,s)[\psi(s)-\psi(\nu+s)]$ $\mathrm{Re}\,s>0$

Logarithmic functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(9)	$x^{\nu}e^{-ax}\log x$ $\quad \operatorname{Re} a>0$	$a^{-s-\nu}\Gamma(s+\nu)[\psi(s+\nu)-\log a]$ $\quad \operatorname{Re} s>-\operatorname{Re}\nu$
(10)	$(1+x)^{-1}(\log x)^2$	$\pi^3\csc^3(\pi s)\,[2-\sin^2(\pi s)]$ $\quad 0<\operatorname{Re} s<1$
(11)	$(1-x)^{\nu-1}(\log x)^2$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$ $\operatorname{Re}\nu>0$	$B(s,\nu)\{[\psi(s)-\psi(\nu+s)]^2$ $+\psi'(s)-\psi'(s+\nu)\}$ $\quad \operatorname{Re} s>0$
(12)	$(a^2+2ax\cos\theta+x^2)^{-1}(\log x)^n$ $a>0,\quad -\pi<\theta<\pi$	$-\pi\cos\theta$ $\times\frac{d^n}{ds^n}[a^{s-2}\csc(s\pi)\sin(s-1)\theta]$ $0<\operatorname{Re} s<2$
(13)	$e^{-x}(\log x)^n$	$\frac{d^n}{ds^n}\Gamma(s)$ $\quad \operatorname{Re} s>0$
(14)	$(\log x)^{\nu-1}$ $\quad 0<x<1$ 0 $\quad x>1$ $\operatorname{Re}\nu>0$	$(-s)^{-\nu}\Gamma(\nu)$ $\quad \operatorname{Re} s<0$
(15)	$\log(1+ax)$ $\quad \lvert\arg a\rvert<\pi$	$\pi s^{-1}a^{-s}\csc(\pi s)$ $\quad -1<\operatorname{Re} s<0$
(16)	$\log(1+x)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$	$s^{-1}[\log 2-\tfrac12\psi(\tfrac12 s+1)$ $+\tfrac12\psi(\tfrac12 s+\tfrac12)]$ $\quad \operatorname{Re} s>-1$
(17)	0 $\quad 0<x<1$ $\log(1+x)$ $\quad 1<x<\infty$	$s^{-1}[\tfrac12\psi(-\tfrac12 s)-\tfrac12\psi(\tfrac12-\tfrac12 s)$ $-\log 2]$
(18)	$\log\lvert 1-x\rvert$	$\pi s^{-1}\operatorname{ctn}(\pi s)$ $\quad -1<\operatorname{Re} s<0$

Logarithmic functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(19)	$\log(1-x) \quad 0<x<1$ $0 \quad x>1$	$-s^{-1}[\psi(s+1)-\psi(1)] \quad \mathrm{Re}\, s>-1$
(20)	$0 \quad 0<x<1$ $\log(x-1) \quad x>1$	$s^{-1}[\pi \operatorname{ctn}(\pi s)+\psi(s+1)-\psi(1)]$ $\mathrm{Re}\, s<0$
(21)	$(1+x)^{-1}\log(1+x)$	$\pi \csc(\pi s)[C+\psi(1-s)]$ $-1<\mathrm{Re}\, s<1$
(22)	$(1-x)^{\nu-1}\log(1-x) \quad 0<x<1$ $0 \quad 1<x<\infty$ $\mathrm{Re}\,\nu>0$	$\mathrm{B}(s,\nu)[\psi(\nu)-\psi(\nu+s)]$ $\mathrm{Re}\, s>0$
(23)	$(a+x)^{-\nu}\log(a+x). \quad \lvert\arg a\rvert<\pi$	$a^{s-\nu}\mathrm{B}(s,\nu-s)[\psi(\nu)$ $-\psi(\nu-s)+\log a]$ $0<\mathrm{Re}\, s<\mathrm{Re}\,\nu$
(24)	$\log\left\lvert\dfrac{1+x}{1-x}\right\rvert$	$\pi s^{-1}\tan(\tfrac{1}{2}\pi s) \quad -1<\mathrm{Re}\, s<1$
(25)	$(1+x)^{-1}\log(1+x^2)$	$\tfrac{1}{2}\pi\csc(\pi s)\{\log 4+(1-s)\sin(\tfrac{1}{2}\pi s)$ $\times[\psi(\tfrac{3}{4}-\tfrac{1}{4}s)-\psi(\tfrac{1}{4}-\tfrac{1}{4}s)]$ $-(2-s)\cos(\tfrac{1}{2}\pi s)$ $\times[\psi(1-\tfrac{1}{4}s)-\psi(\tfrac{1}{2}-\tfrac{1}{4}s)]\}$ $-2<\mathrm{Re}\, s<1$
(26)	$\log[x+(1+x^2)^{\frac{1}{2}}]$	$-\tfrac{1}{2}s^{-1}a^{-s}\mathrm{B}(\tfrac{1}{2}s+\tfrac{1}{2},-\tfrac{1}{2}s)$ $-1<\mathrm{Re}\, s<0$
(27)	$\log(1+2x\cos\theta+x^2)$ $-\pi<\theta<\pi$	$2\pi s^{-1}\cos(\theta s)\csc(\pi s)$ $-1<\mathrm{Re}\, s<0$

Logarithmic functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(28)	$\log(1-2ae^{-x}\cos\theta+a^2e^{-2x})$ $0<a<1,\quad -\pi<\theta<\pi$	$-2\Gamma(s)\sum_{n=1}^{\infty}\frac{a^n\cos(n\theta)}{n+1}\quad \mathrm{Re}\, s>0$
(29)	For other integrands of the form $f(x)\log[g(x)]$ see Gröbner, W. and N. Hofreiter, 1950: *Integraltafel. Zweiter Teil, Bestimmte Integrale*, p. 69-90, Springer.	

6.5. Trigonometric and inverse trigonometric functions

(1)	$\sin(ax)\qquad a>0$	$a^{-s}\Gamma(s)\sin(\frac{1}{2}\pi s)$ $-1<\mathrm{Re}\, s<1$
(2)	$\sin(\beta x)\qquad 0<x<c$ $0\qquad c<x<\infty$	$\frac{1}{2}i(i\beta)^{-s}\gamma(s,i\beta c)$ $-\frac{1}{2}i(-i\beta)^{-s}\gamma(s,-i\beta c)$ $\mathrm{Re}\, s>-1$
(3)	$0\qquad 0<x<c$ $\sin ax\qquad c<x<\infty$ $a>0$	$\frac{1}{2}e^{\frac{1}{2}\pi i(s-1)}a^{-s}\Gamma(s,-iac)$ $+\frac{1}{2}e^{\frac{1}{2}\pi i(1-s)}a^{-s}\Gamma(s,iac)$ $\mathrm{Re}\, s<1$
(4)	$(1+x^2)^{-1}\sin(ax)\qquad a>0$	$\frac{1}{2}\pi\sec(\frac{1}{2}s\pi)\sinh a$ $+\frac{1}{2}\Gamma(s)\sin(\frac{1}{2}\pi s)$ $\times[e^{-a}e^{i(1-s)\pi}\gamma(1-s,-a)$ $-e^{a}\gamma(1-s,a)]\quad -1<\mathrm{Re}\, s<3$
(5)	$(1-x)^{\nu-1}\sin(ax)\qquad 0<x<1$ $0\qquad 1<x<\infty$ $\mathrm{Re}\,\nu>0$	$-\frac{1}{2}i\,\mathrm{B}(s,\nu)\,[{}_1F_1(s;s+\nu;ia)$ $-{}_1F_1(s;s+\nu;-ia)]\qquad \mathrm{Re}\, s>0$

Trigonometric functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(6)	$(x^2+a^2)^\nu \sin(bx)$	see under Fourier-transforms
(7)	$e^{-ax}\sin(\beta x)$ $\quad \operatorname{Re} a > \lvert\operatorname{Im}\beta\rvert$	$(a^2+\beta^2)^{-\frac{1}{2}s}\,\Gamma(s)\sin[s\tan^{-1}(\beta/a)]$ $\quad \operatorname{Re} s > -1$
(8)	$e^{-ax}\sin(\beta x)$ $\quad 0<x<c$ 0 $\quad c<x<\infty$	$\frac{1}{2}i(a+i\beta)^{-s}\gamma[s,(a+i\beta)c]$ $-\frac{1}{2}i(a-i\beta)^{-s}\gamma[s,(a-i\beta)c]$ $\quad \operatorname{Re} s > -1$
(9)	0 $\quad 0<x<c$ $e^{-ax}\sin(\beta x)$ $\quad c<x<\infty$ $\operatorname{Re} a > \lvert\operatorname{Im}\beta\rvert$	$\frac{1}{2}i(a+i\beta)^{-s}\Gamma[s,(a+i\beta)c]$ $-\frac{1}{2}i(a-i\beta)^{-s}\Gamma[s,(a-i\beta)c]$
(10)	$e^{-ax^2}\sin(\beta x)$ $\quad \operatorname{Re} a>0$	$\frac{1}{2}\beta a^{-\frac{1}{2}-\frac{1}{2}s}\Gamma(1/2+s/2)\,e^{-\frac{1}{4}\beta^2/a}$ $\times {}_1F_1(-s/2;3/2;\frac{1}{4}\beta^2/a)$ $\quad \operatorname{Re} s > -1$
(11)	$e^{-\frac{1}{2}ax^2-\beta x}\sin(\gamma x)$ $\quad \operatorname{Re} a>0$	$-\frac{1}{2}i\Gamma(s)\,a^{-\frac{1}{2}s}\,e^{\frac{1}{4}a^{-1}(\beta^2-\gamma^2)}$ $\times\{e^{-\frac{1}{2}i\,a^{-1}\beta\gamma}D_{-s}[a^{-\frac{1}{2}}(\beta-i\gamma)]$ $-e^{\frac{1}{2}i\,a^{-1}\beta\gamma}D_{-s}[a^{-\frac{1}{2}}(\beta+i\gamma)]\}$ $\quad \operatorname{Re} s > -1$
(12)	$e^{-\frac{1}{4}a^2x^{-1}}\sin(bx)$ $\operatorname{Re} a>0, \quad b>0$	$i\,2^{-s}a^s b^{\frac{1}{2}s}[e^{-\frac{1}{4}\pi is}K_s(ae^{\frac{1}{4}\pi i}b^{\frac{1}{2}})$ $-e^{\frac{1}{4}i\pi s}K_s(ae^{-\frac{1}{4}\pi i}b^{\frac{1}{2}})]$ $\quad \operatorname{Re} s < 1$
(13)	$\log x\,\sin(ax)$ $\quad a>0$	$a^{-s}\Gamma(s)\sin(\frac{1}{2}\pi s)\,[\psi(s)-\log a$ $+\frac{1}{2}\pi\operatorname{ctn}(\frac{1}{2}\pi s)]$ $\quad -1<\operatorname{Re} s<1$
(14)	$e^{-ax}\log x\,\sin(\beta x)$ $\operatorname{Re} a > \lvert\operatorname{Im}\beta\rvert$	$(a^2+\beta^2)^{-\frac{1}{2}s}\Gamma(s)\sin[s\tan^{-1}(\beta/a)]$ $\times\{\psi(s)-\frac{1}{2}\log(a^2+\beta^2)$ $+\tan^{-1}(\beta/a)\operatorname{ctn}[s\tan^{-1}(\beta/a)]\}$ $\quad \operatorname{Re} s > -1$

Trigonometric functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(15)	$\sin^2(ax)$ $\quad a>0$	$-2^{-s-1}a^{-s}\Gamma(s)\cos(\frac{1}{2}\pi s)$ $\quad -2<\operatorname{Re} s<0$
(16)	$\sin[a(x-b^2x^{-1})]$ $\quad a, b>0$	$2b^s K_s(2ab)\sin(\frac{1}{2}s\pi)$ $\quad \lvert\operatorname{Re} s\rvert<1$
(17)	$\sin[a(x+b^2x^{-1})]$ $\quad a, b>0$	$\pi b^s[J_s(2ab)\cos(\frac{1}{2}\pi s)$ $-Y_s(2ab)\sin(\frac{1}{2}\pi s)]$ $\quad \lvert\operatorname{Re} s\rvert<1$
(18)	$e^{-x}\sin(x+ax^2)$ $\quad a>0$	$(2a)^{-\frac{1}{2}s}\Gamma(s)e^{-2a^{-1}}\sin(\frac{1}{4}\pi s)$ $\times D_{-s}(a^{-\frac{1}{2}})$ $\quad \operatorname{Re} s>-1$
(19)	$\sin(a\log x)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$	$-a(a^2+s^2)^{-1}$ $\quad \operatorname{Re} s>\lvert\operatorname{Im} a\rvert$
(20)	$e^{-x}\sin(a\log x)$	$\lvert\Gamma(s+ia)\rvert\sin[\arg\Gamma(s+ia)]$ $\quad \operatorname{Re} s>\lvert\operatorname{Im} a\rvert$
(21)	$\cos(ax)$ $\quad a>0$	$a^{-s}\Gamma(s)\cos(\frac{1}{2}\pi s)$ $\quad 0<\operatorname{Re} s<1$
(22)	$\cos(\beta x)$ $\quad 0<x<c$ 0 $\quad c<x<\infty$	$\frac{1}{2}(i\beta)^{-s}\gamma(s,i\beta c)$ $+\frac{1}{2}(-i\beta)^{-s}\gamma(s,-i\beta c)$ $\quad \operatorname{Re} s>0$
(23)	0 $\quad 0<x<c$ $\cos(ax)$ $\quad c<x<\infty$ $a>0$	$\frac{1}{2}e^{\frac{1}{2}i\pi s}a^{-s}\Gamma(s,-iac)$ $+\frac{1}{2}e^{-\frac{1}{2}i\pi s}a^{-s}\Gamma(s,iac)$
(24)	$(1+x^2)^{-1}\cos(ax)$ $\quad a>0$	$\frac{1}{2}\pi\cosh a\,\csc(\frac{1}{2}\pi s)$ $+\frac{1}{2}\Gamma(s)\cos(\frac{1}{2}\pi s)[e^{-a}e^{i(1-s)\pi}$ $\times\gamma(1-s,-a)-e^{a}\gamma(1-s,a)]$ $\quad 0<\operatorname{Re} s<3$

Trigonometric functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(25)	$(1-x)^{\nu-1}\cos(ax)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$ $\mathrm{Re}\,\nu>0$	$\tfrac{1}{2}\mathrm{B}(s,\nu)\,[{}_1F_1(s;s+\nu;ia)$ $-{}_1F_1(s;s+\nu;-ia)]$ $\quad \mathrm{Re}\,s>0$
(26)	$(x^2+a^2)^\nu\cos(bx)$	see under Fourier transforms
(27)	$e^{-\alpha x}\cos(\beta x)$ $\quad \mathrm{Re}\,\alpha>\lvert\mathrm{Im}\,\beta\rvert$	$(\alpha^2+\beta^2)^{-\frac{1}{2}s}\,\Gamma(s)\cos[s\tan^{-1}(\beta/\alpha)]$ $\mathrm{Re}\,s>0$
(28)	$e^{-\alpha x}\cos(\beta x)$ $\quad 0<x<c$ 0 $\quad c<x<\infty$	$\tfrac{1}{2}(\alpha+i\beta)^{-s}\,\gamma[s,(\alpha+i\beta)c]$ $+\tfrac{1}{2}(\alpha-i\beta)^{-s}\,\gamma[s,(\alpha-i\beta)c]$ $\mathrm{Re}\,s<0$
(29)	0 $\quad 0<x<c$ $e^{-\alpha x}\cos(\beta x)$ $\quad c<x<\infty$ $\mathrm{Re}\,\alpha>\lvert\mathrm{Im}\,\beta\rvert$	$\tfrac{1}{2}(\alpha+i\beta)^{-s}\,\Gamma[s,(\alpha+i\beta)c]$ $+\tfrac{1}{2}(\alpha-i\beta)^{-s}\,\Gamma[s,(\alpha-i\beta)c]$
(30)	$e^{-\alpha x^2}\cos(\beta x)$ $\quad \mathrm{Re}\,\alpha>0$	$\tfrac{1}{2}\alpha^{-\frac{1}{2}s}\,\Gamma(\tfrac{1}{2}s)\,e^{-\frac{1}{4}\beta^2/\alpha}$ $\times\,{}_1F_1(-\tfrac{1}{2}s+\tfrac{1}{2};\tfrac{1}{2};\tfrac{1}{4}\beta^2/\alpha)$ $\mathrm{Re}\,s>0$
(31)	$e^{-\frac{1}{2}\alpha x^2-\beta x}\cos(\gamma x)$ $\quad \mathrm{Re}\,\alpha>0$	$\tfrac{1}{2}\alpha^{-\frac{1}{2}s}\,\Gamma(s)\,e^{\frac{1}{4}\alpha^{-1}(\beta^2-\gamma^2)}$ $\times\{e^{-\frac{1}{2}i\beta\gamma\alpha^{-1}}D_{-s}[\alpha^{-\frac{1}{2}}(\beta-i\gamma)]$ $+e^{\frac{1}{2}i\beta\gamma\alpha^{-1}}D_{-s}[\alpha^{-\frac{1}{2}}(\beta+i\gamma)]\}$ $\mathrm{Re}\,s>0$
(32)	$e^{-\frac{1}{4}a^2x^{-1}}\cos(bx)$ $\mathrm{Re}\,a>0,\quad b>0$	$2^{-s}a^s b^{-\frac{1}{2}s}[e^{-\frac{1}{4}\pi is}K_s(ae^{\frac{1}{4}\pi i}b^{\frac{1}{2}})$ $+e^{\frac{1}{4}\pi is}K_s(ae^{-\frac{1}{4}\pi i}b^{\frac{1}{2}})]$ $\mathrm{Re}\,s<1$

Trigonometric functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(33)	$\log x\ \cos(ax)$ $\quad a>0$	$a^{-s}\,\Gamma(s)\cos(\tfrac{1}{2}\pi s)[\psi(s)-\log a$ $-\tfrac{1}{2}\pi\tan(\tfrac{1}{2}\pi s)]$ $\quad 0<\mathrm{Re}\,s<1$
(34)	$e^{-\alpha x}\log x\ \cos(\beta x)$ $\quad \mathrm{Re}\,\alpha>\lvert\mathrm{Im}\,\beta\rvert$	$(\alpha^2+\beta^2)^{-\frac{1}{2}s}\,\Gamma(s)\{[\psi(s)$ $-\log(\alpha^2+\beta^2)^{\frac{1}{2}}]\cos[s\tan^{-1}(\beta/\alpha)]$ $-\tan^{-1}(\beta/\alpha)\sin[s\tan^{-1}(\beta/\alpha)]\}$ $\quad \mathrm{Re}\,s>0$
(35)	$\cos[a(x+b^2x^{-1})]$ $\quad a>0,\quad b>0$	$-\pi b^s[J_s(2ab)\sin(\tfrac{1}{2}\pi s)$ $+Y_s(2ab)\cos(\tfrac{1}{2}\pi s)]$ $\quad -1<\mathrm{Re}\,s<1$
(36)	$\cos[a(x-b^2x^{-1})]$ $\quad a,\ b>0$	$2b^s K_s(2ab)\cos(\tfrac{1}{2}\pi s)$ $\quad -1<\mathrm{Re}\,s<1$
(37)	$e^{-x}\cos(x+ax^2)$ $\quad a>0$	$(2a)^{-\frac{1}{2}s}\,\Gamma(s)\,e^{-2a^{-1}}$ $\times\cos(\tfrac{1}{2}\pi s)\,D_{-s}(a^{-\frac{1}{2}})$ $\quad \mathrm{Re}\,s>0$
(38)	$\cos(a\log x)$ $\quad 0<x<1$; 0 $\quad 1<x<\infty$	$s(a^2+s^2)^{-1}$ $\quad \mathrm{Re}\,s>\lvert\mathrm{Im}\,a\rvert$
(39)	$e^{-x}\cos(a\log x)$	$\lvert\Gamma(s+ia)\rvert\ \cos[\arg\Gamma(s+ia)]$ $\quad \arg\Gamma(x)=-i\log\dfrac{\Gamma(x)}{\lvert\Gamma(x)\rvert}$ $\quad \mathrm{Re}\,s>\lvert\mathrm{Im}\,a\rvert$
(40)	$\sin(ax)\sin(bx)$ $\quad a,\ b>0,\quad a\neq b$	$\tfrac{1}{2}\Gamma(s)\cos(\tfrac{1}{2}\pi s)[\lvert b-a\rvert^{-s}$ $-(b+a)^{-s}]$ $\quad -2<\mathrm{Re}\,s<1$
(41)	$\sin(ax)\cos(bx)$ $\quad a,\ b>0$	$\tfrac{1}{2}\Gamma(s)\sin(\tfrac{1}{2}\pi s)[(a+b)^{-s}$ $+\mathrm{sgn}(a-b)\lvert a-b\rvert^{-s}]$ $\quad -1<\mathrm{Re}\,s<1$

Trigonometric functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(42)	$\sin(b^2x^{-1})\ \sin(ax) \qquad a,\ b>0$	$\frac{1}{4}\pi b^s a^{-\frac{1}{2}s}\csc(\frac{1}{2}\pi s)\,[J_s(2ba^{\frac{1}{2}}) - J_{-s}(2ba^{\frac{1}{2}})+I_{-s}(2ba^{\frac{1}{2}}) - I_s(2ba^{\frac{1}{2}})] \qquad \lvert\mathrm{Re}\,s\rvert<1$
(43)	$\sin(b^2x^{-1})\cos(ax) \qquad a,\ b>0$	$\frac{1}{4}\pi b^s a^{-\frac{1}{2}s}\sec(\frac{1}{2}s\pi)\,[J_s(2ba^{\frac{1}{2}}) + J_{-s}(2ba^{\frac{1}{2}})+I_s(2ba^{\frac{1}{2}}) - I_{-s}(2ba^{\frac{1}{2}})] \qquad \lvert s\rvert<1$
(44)	$\cos(b^2x^{-1})\cos(ax) \qquad a,\ b>0$	$\frac{1}{4}\pi b^s a^{-\frac{1}{2}s}\csc(\frac{1}{2}\pi s)\,[J_{-s}(2ba^{\frac{1}{2}}) - J_s[2ba^{\frac{1}{2}})+I_{-s}(2ba^{\frac{1}{2}}) - I_s(2ba^{\frac{1}{2}})] \qquad -1<\mathrm{Re}\,s<1$
(45)	$\tan^{-1}x \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$\frac{1}{4}s^{-1}[\pi+\psi(\frac{1}{4}s+\frac{1}{4})-\psi(\frac{1}{4}s+\frac{3}{4})]$ $\mathrm{Re}\,s>-1$
(46)	$\tan^{-1}x$	$-\frac{1}{2}\pi s^{-1}\sec(\frac{1}{2}\pi s) \qquad -1<\mathrm{Re}\,s<0$
(47)	$\tan^{-1}(ae^{-x})$	$2^{-s-1}\,\Gamma(s)\,a\,\Phi(-a^2,s+1,\frac{1}{2})$
(48)	$\mathrm{ctn}^{-1}x \qquad 0<x<1$ $0 \qquad 1<x<\infty$	$\frac{1}{4}s^{-1}[\pi-\psi(\frac{1}{4}s+\frac{1}{4})+\psi(\frac{1}{4}s+\frac{3}{4})]$ $\mathrm{Re}\,s>0$
(49)	$\mathrm{ctn}^{-1}x$	$\frac{1}{2}\pi s^{-1}\sec(\frac{1}{2}\pi s) \qquad 0<\mathrm{Re}\,s<1$

6.6. Hyperbolic and inverse hyperbolic functions

(1)	$\mathrm{sech}(ax) \qquad \mathrm{Re}\,a>0$	$a^{-s}2^{1-s}\Gamma(s)\,\Phi(-1,s,\frac{1}{2})$ $\mathrm{Re}\,s>0$

Hyperbolic functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(2)	$\operatorname{csch}(ax)$ $\quad \operatorname{Re} a>0$	$a^{-s}2(1-2^{-s})\Gamma(s)\,\zeta(s)$ $\quad \operatorname{Re} s>1$
(3)	$\operatorname{sech}^2(ax)$ $\quad \operatorname{Re} a>0$	$4a^{-s}(1-2^{2-s})\Gamma(s)2^{-s}\zeta(s-1)$ $\quad \operatorname{Re} s>0$
(4)	$\operatorname{csch}^2(ax)$ $\quad \operatorname{Re} a>0$	$4a^{-s}\Gamma(s)2^{-s}\zeta(s-1)$ $\quad \operatorname{Re} s>2$
(5)	$(\cosh x-\cos\theta)^{-1}$ $\quad 0<\theta<2\pi$	$i\csc\theta\,\Gamma(s)\,[e^{-i\theta}\Phi(e^{-i\theta},s,1)$ $-e^{i\theta}\Phi(e^{i\theta},s,1)]$ $\quad \operatorname{Re} s>0$
(6)	$e^{-x}\operatorname{sech} x$	$(1-2^{1-s})\Gamma(s)2^{1-s}\zeta(s)$ $\quad \operatorname{Re} s>0$
(7)	$e^{-x}\operatorname{csch} x$	$2^{1-s}\Gamma(s)\,\zeta(s)$ $\quad \operatorname{Re} s>1$
(8)	$e^{-ax}(\cosh x-\cos\theta)^{-1}$ $\quad \operatorname{Re} a>-1,\quad 0<\theta<2\pi$	$i\csc\theta\,\Gamma(s)\,[e^{-i\theta}\Phi(e^{-i\theta},s,a+1)$ $-e^{i\theta}\Phi(e^{i\theta},s,a+1)]$ $\quad \operatorname{Re} s>0$
(9)	$e^{-ax}(e^{x}-\cos\theta)$ $\times(\cosh x-\cos\theta)^{-1}$ $\quad \operatorname{Re} a>0,\quad 0<\theta<2\pi$	$\Gamma(s)\,[\Phi(e^{i\theta},s,a)$ $+\Phi(e^{-i\theta},s,a)]$ $\quad \operatorname{Re} s>0$
(10)	$\sinh(ax)\operatorname{csch}(\beta x)$ $\quad \operatorname{Re}\beta>\lvert\operatorname{Re} a\rvert$	$(2\beta)^{-s}\Gamma(s)\{\zeta[s,\tfrac{1}{2}(1-a/\beta)]$ $-\zeta[s,\tfrac{1}{2}(1+a/\beta)]\}$ $\quad \operatorname{Re} s>-1$
(11)	$\operatorname{csch}(ax)\operatorname{sech}(\beta x)$ $\quad \operatorname{Re}\beta>\lvert\operatorname{Re} a\rvert$	$(2\beta)^{-s}\Gamma(s)\{\Phi[-1,s,\tfrac{1}{2}(1+a/\beta)]$ $+\Phi[-1,s,\tfrac{1}{2}(1-a/\beta)]\}$ $\quad \operatorname{Re} s>0$
(12)	$\cosh(ax)\operatorname{csch}(\beta x)$ $\quad \operatorname{Re}\beta>\lvert\operatorname{Re} a\rvert$	$(2\beta)^{-s}\Gamma(s)\{\zeta[s,\tfrac{1}{2}(1-a/\beta)]$ $+\zeta[s,\tfrac{1}{2}(1+a/\beta)]\}$ $\quad \operatorname{Re} s>-1$
(13)	$\sinh^{-1}(ax)$	$-\tfrac{1}{2}s^{-1}a^{-s}\mathrm{B}(\tfrac{1}{2}s+\tfrac{1}{2},-\tfrac{1}{2}s)$ $\quad -1<\operatorname{Re} s<0$

Hyperbolic functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(14)	$(1+x^2)^{-1/2}\sinh(\nu\sinh^{-1}x)$	$\pi^{-1}2^{-s}\Gamma(s)\sin(\frac{1}{2}\pi s)\sin(\frac{1}{2}\pi\nu)$ $\times\Gamma(\frac{1}{2}-\frac{1}{2}s-\frac{1}{2}\nu)\Gamma(\frac{1}{2}-\frac{1}{2}s+\frac{1}{2}\nu)$ $-1<\mathrm{Re}\,s<1-\lvert\mathrm{Re}\,\nu\rvert$
(15)	$(1+x^2)^{-1/2}\cosh(\nu\sinh^{-1}x)$	$\pi^{-1}2^{-s}\cos(\frac{1}{2}\pi s)\cos(\frac{1}{2}\pi\nu)$ $\times\Gamma(\frac{1}{2}-\frac{1}{2}s-\frac{1}{2}\nu)\Gamma(\frac{1}{2}-\frac{1}{2}s+\frac{1}{2}\nu)$ $0<\mathrm{Re}\,s<1-\lvert\mathrm{Re}\,\nu\rvert$

6.7. Orthogonal polynomials, gamma functions, Legendre functions and related functions

(1)	$P_{n-1}[(1-x)(1+x)^{-1}](1+x)^{-n}$	$\pi^{-1}\sin(\pi s)[\mathrm{B}(s,n-s)]^2$ $0<\mathrm{Re}\,s<n$
(2)	$(1-x^2)^{-1/2}T_n(x)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$	$\pi s^{-1}2^{-s}$ $\times[\mathrm{B}(\frac{1}{2}+\frac{1}{2}s+\frac{1}{2}n,\frac{1}{2}+\frac{1}{2}s-\frac{1}{2}n)]^{-1}$ $\mathrm{Re}\,s>0$
(3)	$e^{-ax}[\mathrm{He}_n(x^{1/2})]^2$ $\quad \mathrm{Re}\,a>0$	$h_n^s(a)\,n!\,\Gamma(s)$ where $\sum_{n=0}^{\infty}h_n^s(a)t^n$ $=(1-t^2)^{-1/2}[a-t(1+t)^{-1}]^{-s}$ $\mathrm{Re}\,s>0$
(4)	$e^{-ax}L_n^\nu(\beta x)$ $\quad \mathrm{Re}\,a>0$	$\Gamma(s+n)(n!)^{-1}(a-\beta)^n a^{-s-n}$ $\times{}_2F_1[-n,\nu-s+1;-s-n+1;$ $a(a-\beta)^{-1}]$ $\quad \mathrm{Re}\,s>0$
(5)	$\psi(x+1)+\log\gamma$	$\pi\csc(\pi s)\,\zeta(2-s)$ $\quad 0<\mathrm{Re}\,s<1$
(6)	$\psi(1+x)-\log(1+x)$	$-\pi\csc(\pi s)[\zeta(1-s)+s^{-1}]$ $0<\mathrm{Re}\,s<1$

Orthogonal polynomials etc. (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(7)	$\psi'(x+1)$	$\pi(1-s)\csc(\pi s)\,\zeta(2-s)$ $\quad 0<\operatorname{Re} s<2$
(8)	$\operatorname{Erfc}(x)$	$\pi^{-\frac{1}{2}}s^{-1}\Gamma(\frac{1}{2}s+\frac{1}{2})$ $\quad \operatorname{Re} s>0$
(9)	$e^{\frac{1}{2}x^2}\operatorname{Erfc}(2^{-\frac{1}{2}}x)$	$2^{\frac{1}{2}s-1}\sec(\frac{1}{2}\pi s)\Gamma(\frac{1}{2}s)$ $\quad 0<\operatorname{Re} s<1$
(10)	$\operatorname{Ei}(-x)$	$-s^{-1}\Gamma(s)$ $\quad \operatorname{Re} s>0$
(11)	$\operatorname{Si}(x)$	$-s^{-1}\sin(\frac{1}{2}\pi s)\Gamma(s)$ $\quad -1<\operatorname{Re} s<0$
(12)	$\operatorname{si}(x)$	$-4s^{-1}\sin(\frac{1}{2}\pi s)\Gamma(s)$ $\quad -1<\operatorname{Re} s<0$
(13)	$\operatorname{Ci}(x)$	$-s^{-1}\cos(\frac{1}{2}\pi s)\Gamma(s)$ $\quad 0<\operatorname{Re} s<1$
(14)	$S(x)$	$-(2\pi)^{-\frac{1}{2}}s^{-1}\sin(\frac{1}{2}\pi s+\frac{1}{4}\pi)$ $\times\Gamma(s+\frac{1}{2})$ $\quad -3/2<\operatorname{Re} s<0$
(15)	$C(x)$	$-(2\pi)^{-\frac{1}{2}}s^{-1}\cos(\frac{1}{2}\pi s+\frac{1}{4}\pi)$ $\times\Gamma(s+\frac{1}{2})$ $\quad -\frac{1}{2}<\operatorname{Re} s<0$
(16)	$e^{-\beta x}\,\Gamma(a,x)$ $\quad \operatorname{Re}\beta>-1$	$s^{-1}(1+\beta)^{-s-a}\Gamma(s+a)$ $\times{}_2F_1[1,a+s;s+1;\beta(1+\beta)^{-1}]$ $\quad \operatorname{Re} s>0$
(17)	$e^{-\beta x}\,\gamma(a,x)$ $\quad \operatorname{Re}\beta>0$	$a^{-1}(1+\beta)^{-a-s}\Gamma(a+s)$ $\times{}_2F_1[1,a+s;a+1;(1+\beta)^{-1}]$ $\quad \operatorname{Re} s>-\operatorname{Re} a$

Orthogonal polynomials etc. (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(18)	$P_\nu(x)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$	$\pi^{1/2}2^{-s}\Gamma(s)$ $\times[\Gamma(1/2+1/2s-1/2\nu)\Gamma(1+1/2s+1/2\nu)]^{-1}$ $\operatorname{Re} s>0$
(19)	$(1-x^2)^{1/2m}P_\nu^m(x)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$	$(-1)^m 2^{-m-s}\Gamma(s)\Gamma(1+m+\nu)\pi^{1/2}$ $\times[\Gamma(1-m+\nu)]^{-1}$ $\times[\Gamma(1/2+1/2s+1/2m-1/2\nu)$ $\times\Gamma(1+1/2s+1/2m+1/2\nu)]^{-1}$ $\operatorname{Re} s>0$
(20)	$(1-x^2)^{-1/2\mu}P_\nu^\mu(x)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$ $\operatorname{Re}\mu<1$	$\pi^{1/2}2^{\mu-s}\Gamma(s)[\Gamma(1/2s+1/2-1/2\nu-1/2\mu)$ $\times\Gamma(1+1/2s+1/2\nu-1/2\mu)]^{-1}$ $\operatorname{Re} s>0$
(21)	$(1+x^2)^{-1/2\nu}P_{\nu-1}[x(1+x^2)^{-1/2}]$	$\dfrac{2^{\nu-s}B(s,1/2\nu-1/2s)}{1/2(s-\nu)B(\nu,1/2s-1/2\nu)}$ $\operatorname{Re} s>0,\ -\operatorname{Re}\nu$

6.8. Bessel functions and related functions

(1)	$J_\nu(ax)$ $\quad a>0$	$\dfrac{2^{s-1}\Gamma(1/2s+1/2\nu)}{a^s\Gamma(1/2\nu-1/2s+1)}$ $-\operatorname{Re}\nu<\operatorname{Re}s<3/2$
(2)	$J_\nu(ax)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$	$\dfrac{a^\nu}{2^\nu(s+\nu)\Gamma(\nu+1)}$ $\times{}_1F_2\left(\dfrac{s+\nu}{2};\nu+1,\dfrac{s+\nu}{2}+1;-\dfrac{a^2}{4}\right)$ $\operatorname{Re}s>-\operatorname{Re}\nu$

Bessel functions and related functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(3)	$(x^2+\beta^2)^{-\mu-1}J_\nu(ax)$ $a>0,\quad \operatorname{Re}\beta>0$	$\dfrac{2^{-\nu-1}a^\nu \mathrm{B}(\tfrac12 s+\tfrac12\nu,\ \mu+1-\tfrac12 s-\tfrac12\nu)}{\beta^{2\mu+2-s-\nu}\Gamma(\nu+1)}$ $\times{}_1F_2\left(\dfrac{s+\nu}{2};\dfrac{s+\nu}{2}-\mu,\ \nu+1;\dfrac{a^2\beta^2}{4}\right)$ $+\dfrac{a^{2\mu+2-s}\,\Gamma(\tfrac12 s+\tfrac12\nu-\mu-1)}{2^{2\mu+3-s}\Gamma(\mu+2+\tfrac12\nu-\tfrac12 s)}$ $\times{}_1F_2\left(1+\mu;2+\mu+\dfrac{\nu-s}{2},\right.$ $\left.2+\mu-\dfrac{\nu-s}{2};\dfrac{a^2\beta^2}{4}\right)$ $-\operatorname{Re}\nu<\operatorname{Re}s<2\operatorname{Re}\mu+7/2$
(4)	$(1-x^2)^\lambda J_\nu(ax)\qquad 0<x<1$ $0\qquad 1<x<\infty$ $a>0,\quad \operatorname{Re}\lambda>-1$	$\dfrac{a^\nu\,\mathrm{B}(\lambda+1,\tfrac12 s+\tfrac12\nu)}{2^{\nu+1}\Gamma(\nu+1)}$ $\times{}_1F_2\left(\dfrac{s+\nu}{2};\nu+1,\dfrac{s+\nu}{2}+1+\lambda;-\dfrac{a^2}{4}\right)$ $\operatorname{Re}s>-\operatorname{Re}\nu$
(5)	$e^{-ax\cos\phi}J_\nu(ax\sin\phi)$ $\operatorname{Re}(ae^{\pm i\phi})>0,\quad 0<\phi<\pi$	$a^{-s}\,\Gamma(s+\nu)\,\mathrm{P}_{s-1}^{-\nu}(\cos\phi)$ $\operatorname{Re}s>-\operatorname{Re}\nu$
(6)	$e^{-ax}J_\nu(\beta x)\qquad \operatorname{Re}a>\lvert\operatorname{Im}\beta\rvert$	$\dfrac{\beta^\nu\Gamma(s+\nu)}{2^\nu a^{\nu+s}\,\Gamma(1+\nu)}$ $\times{}_2F_1\left(\dfrac{s+\nu}{2},\dfrac{s+\nu+1}{2};\nu+1;-\dfrac{\beta^2}{a^2}\right)$ $\operatorname{Re}s>-\operatorname{Re}\nu$

Bessel functions and related functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$
(7)	$(1-x)^{\mu}\,e^{\pm i\alpha x}\,J_\nu(ax)$ $\quad 0<x<1$ $0 \quad 1<x<\infty$ $\operatorname{Re}\mu>-1$	$\dfrac{a^\nu \mathrm{B}(s+\nu,\mu+1)}{2^\nu\Gamma(\nu+1)}$ $\times\,{}_2F_2(\nu+\tfrac{1}{2},\,s+\nu;\,s+\mu+\nu+1,\,2\nu+1;\,\pm 2ia)$ $\operatorname{Re}s>-\operatorname{Re}\nu$
(8)	$e^{-\beta^2x^2}\,J_\nu(ax)$ $\quad \lvert\arg\beta\rvert<\pi/4$	$\dfrac{\Gamma(\tfrac{1}{2}\nu+\tfrac{1}{2}s)}{a\beta^{s-1}\Gamma(\nu+1)}$ $\times\exp\left(-\dfrac{a^2}{8\beta^2}\right)\,M_{\frac{1}{2}s-\frac{1}{2},\,\frac{1}{2}\nu}\left(\dfrac{a^2}{4\beta^2}\right)$ $\operatorname{Re}s>-\operatorname{Re}\nu$
(9)	$\log x\,J_\nu(ax)$ $\quad a>0$	$\dfrac{2^{s-2}\Gamma(\tfrac{1}{2}s+\tfrac{1}{2}\nu)}{a^s\,\Gamma(\tfrac{1}{2}\nu-\tfrac{1}{2}s+1)}$ $\times\left[\psi\left(\dfrac{s+\nu}{2}\right)+\psi\left(\dfrac{\nu-s}{2}+1\right)-\log\dfrac{a^2}{4}\right]$ $-\operatorname{Re}\nu<\operatorname{Re}s<3/2$
(10)	$\sin(ax)\,J_\nu(ax)$ $\quad a>0$	$\dfrac{2^{\nu-1}\Gamma(\tfrac{1}{2}-s)\,\Gamma(\tfrac{1}{2}+\tfrac{1}{2}\nu+\tfrac{1}{2}s)}{a^s\Gamma(1+\nu-s)\,\Gamma(1-\tfrac{1}{2}\nu-\tfrac{1}{2}s)}$ $-1<\operatorname{Re}\nu<\operatorname{Re}s<\tfrac{1}{2}$
(11)	$\cos(ax)\,J_\nu(ax)$ $\quad a>0$	$\dfrac{2^{\nu-1}\Gamma(\tfrac{1}{2}-s)\,\Gamma(\tfrac{1}{2}\nu+\tfrac{1}{2}s)}{a^s\Gamma(\tfrac{1}{2}-\tfrac{1}{2}\nu-\tfrac{1}{2}s)\,\Gamma(1+\nu-s)}$ $-\operatorname{Re}\nu<\operatorname{Re}s<\tfrac{1}{2}$
(12)	$(x^2+\beta^2)^{-\frac{1}{2}\nu}\,J_\nu[a(x^2+\beta^2)^{\frac{1}{2}}]$ $a>0$	$2^{\frac{1}{2}s-1}\,a^{-\frac{1}{2}s}\,\beta^{\frac{1}{2}s-\nu}\,\Gamma(\tfrac{1}{2}s)\,J_{\nu-\frac{1}{2}s}(a\beta)$ $0<\operatorname{Re}s<\operatorname{Re}\nu+3/2$

Bessel functions and related functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$
(13)	$(a^2-x^2)^{\frac{1}{2}\nu}\,J_\nu[\beta(a^2-x^2)^{\frac{1}{2}}]$ $\quad 0<x<a$ $0 \quad a<x<\infty$ $\operatorname{Re}\nu>-1$	$2^{\frac{1}{2}s-1}\,\Gamma(\tfrac{1}{2}s)\,\beta^{-\frac{1}{2}s}\,a^{\nu+\frac{1}{2}s}\,J_{\nu+\frac{1}{2}s}(a\beta)$ $\operatorname{Re}s>0$
(14)	$(a^2-x^2)^{-\frac{1}{2}\nu}\,J_\nu[\beta(a^2-x^2)^{\frac{1}{2}}]$ $\quad 0<x<a$ $0 \quad a<x<\infty$	$2^{1-\nu}[\Gamma(\nu)]^{-1}\,a^{\frac{1}{2}s-\nu}\,\beta^{-\frac{1}{2}s}$ $\times s_{\nu+\frac{1}{2}s-1,\frac{1}{2}s-\nu}(a\beta) \quad \operatorname{Re}s>0$
(15)	$(1-x)^\nu e^{\mp iax}\,J_\nu[a(1-x)]$ $\quad 0<x<1$ $0 \quad 1<x<\infty$ $\operatorname{Re}\nu>-\tfrac{1}{2}$	$\pi^{-\frac{1}{2}}(2a)^\nu\,\Gamma(s)\,e^{\mp ia}\,\dfrac{\Gamma(\nu+\frac{1}{2})}{\Gamma(2\nu+s+1)}$ $\times{}_1F_1(\nu+\tfrac{1}{2};\,2\nu+s+1;\,\pm 2ia)$ $\operatorname{Re}s>0$
(16)	$(1+x^2)^{-1}\exp\{a[(1-x^2)$ $\times(1+x^2)^{-1}]\}\,J_\nu[2bx(1+x^2)^{-1}]$	$\dfrac{\Gamma(\frac{1}{2}s+\frac{1}{2}\nu)\,\Gamma(1-\frac{1}{2}s+\frac{1}{2}\nu)}{2b\,[\Gamma(\nu+1)]^2}$ $\times M_{\frac{1}{2}s-\frac{1}{2},\frac{1}{2}\nu}[a+(a^2-b^2)^{\frac{1}{2}}]$ $\times M_{\frac{1}{2}s-\frac{1}{2},\frac{1}{2}\nu}[a-(a^2-b^2)^{\frac{1}{2}}]$ $-\operatorname{Re}\nu<\operatorname{Re}s<\operatorname{Re}\nu+2$
	For more integrals containing $J_\nu(ax)$ see also Hankel transforms and Fourier transforms.	
(17)	$Y_\nu(ax) \quad a>0$	$-2^{s-1}a^{-s}\pi^{-1}\,\Gamma(\tfrac{1}{2}s+\tfrac{1}{2}\nu)$ $\times\Gamma(\tfrac{1}{2}s-\tfrac{1}{2}\nu)\cos[\tfrac{1}{2}(s-\nu)\pi]$ $\lvert\operatorname{Re}\nu\rvert<\operatorname{Re}s<3/2$
(18)	$\sin(ax)\,Y_\nu(ax)$	$2^{s-1}\pi^{-\frac{1}{2}}a^{-s}\sin[\tfrac{1}{2}(s-\nu)\pi]$ $\times\dfrac{\Gamma(\frac{1}{2}s+\frac{1}{2}\nu+\frac{1}{2})\,\Gamma(\frac{1}{2}s-\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(1-\frac{1}{2}s-\frac{1}{2}\nu)\,\Gamma(1+\frac{1}{2}\nu-\frac{1}{2}s)}$ $\lvert\operatorname{Re}\nu\rvert-1<\operatorname{Re}s<\tfrac{1}{2}$

Bessel functions and related functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$
(19)	$\cos(ax)\,Y_\nu(ax)$	$2^{s-1}\pi^{-1/2}a^{-s}\sin[\frac{1}{2}(s-\nu-1)\pi]$ $\times\dfrac{\Gamma(\frac{1}{2}s+\frac{1}{2}\nu)\,\Gamma(\frac{1}{2}s-\frac{1}{2}\nu)}{\Gamma(\frac{1}{2}-\frac{1}{2}s-\frac{1}{2}\nu)\,\Gamma(\frac{1}{2}-\frac{1}{2}s+\frac{1}{2}\nu)}$ $\lvert\mathrm{Re}\,\nu\rvert<\mathrm{Re}\,s<\frac{1}{2}$
(20)	$H_\nu^{(1)}(ax)$ $\quad a>0$	$\dfrac{(1-i)\,2^{s-1}\Gamma(\frac{1}{2}s+\frac{1}{2}\nu)}{a^s\Gamma(\frac{1}{2}\nu-\frac{1}{2}s+1)}$ For $H_\nu^{(2)}$, change $-i$ into $+i$ $\lvert\mathrm{Re}\,\nu\rvert<\mathrm{Re}\,s<3/2$
(21)	$(x^2+\beta^2)^{-\frac{1}{2}\nu}H_\nu^{(k)}[a(x^2+\beta^2)^{\frac{1}{2}}]$ $k=1,2,\quad a>0,\quad \mathrm{Re}\,\beta>0$	$2^{\frac{1}{2}s-1}a^{-\frac{1}{2}s}\beta^{\frac{1}{2}s-\nu}\Gamma(\frac{1}{2}s)\,H_{\nu-\frac{1}{2}s}^{(k)}(a\beta)$ $0<\mathrm{Re}\,s<\mathrm{Re}\,\nu+3/2$
(22)	$e^{-ax}I_\nu(ax)$ $\quad \mathrm{Re}\,a>0$	$\dfrac{\Gamma(\frac{1}{2}-s)\,\Gamma(s+\nu)}{2^s a^s\pi^{\frac{1}{2}}\Gamma(1+\nu-s)}$ $-\mathrm{Re}\,\nu<\mathrm{Re}\,s<\frac{1}{2}$
(23)	$e^{-ax}I_\nu(\beta x)$ $\quad \mathrm{Re}\,a>\lvert\mathrm{Re}\,\beta\rvert$	$\dfrac{2^{s-1}\beta^\nu\Gamma(\frac{1}{2}s+\frac{1}{2}\nu)\,\Gamma[\frac{1}{2}(s+\nu+1)]}{\pi^{\frac{1}{2}}a^{s+\nu}\Gamma(\nu+1)}$ $\times{}_2F_1\left(\dfrac{s+\nu+1}{2},\dfrac{s+\nu}{2};\nu+1;\dfrac{\beta^2}{a^2}\right)$ $\mathrm{Re}\,s>-\mathrm{Re}\,\nu$
(24)	$e^{-ax}I_\nu(\beta x)$ $\quad \mathrm{Re}\,a>\lvert\mathrm{Re}\,\beta\rvert$	$(\frac{1}{2}\pi\beta)^{-\frac{1}{2}}e^{-i\pi(s-\frac{1}{2})}(a^2-\beta^2)^{-\frac{1}{2}s+\frac{1}{4}}$ $\times Q_{\nu-\frac{1}{2}}^{s-\frac{1}{2}}(a\beta^{-1})$ $\quad \mathrm{Re}\,s>-\mathrm{Re}\,\nu$
(25)	$\dfrac{1}{(1+x^2)}I_\nu\left[\dfrac{ax}{(1+x^2)}\right]$ $-\mathrm{Re}\,\nu<\mathrm{Re}\,s<\mathrm{Re}\,\nu+2$	$\dfrac{\Gamma(\frac{1}{2}s+\frac{1}{2}\nu)\,\Gamma(1-\frac{1}{2}s+\frac{1}{2}\nu)}{a[\Gamma(\nu+1)]^2}$ $\times M_{\frac{1}{2}s-\frac{1}{2},\frac{1}{2}\nu}(a)\,M_{-\frac{1}{2}s+\frac{1}{2},\frac{1}{2}\nu}(a)$

Bessel functions and related functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$
(26)	$K_\nu(ax)$ $\quad$ $\mathrm{Re}\, a>0$	$a^{-s}2^{s-2}\Gamma(\tfrac12 s-\tfrac12\nu)\Gamma(\tfrac12 s+\tfrac12\nu)$ $\quad$ $\mathrm{Re}\, s>\lvert\mathrm{Re}\,\nu\rvert$
(27)	$K_{ia}(x)$	$2^{s-2}[\Gamma(\tfrac12 s)]^2\{\prod_{n=0}^{\infty}[1+a^2(s+2n)^{-2}]\}^{-1}$ $\quad$ $\mathrm{Re}\, s>0$
(28)	$e^{-ax}K_\nu(ax)$ $\quad$ $\mathrm{Re}\, a>0$	$\dfrac{\pi^{1/2}\Gamma(s+\nu)\Gamma(s-\nu)}{2^s a^s\,\Gamma(s+\tfrac12)}$ $\quad$ $\mathrm{Re}\, s>\lvert\mathrm{Re}\,\nu\rvert$
(29)	$e^{-ax}K_\nu(\beta x)$ $\quad$ $\mathrm{Re}(a+\beta)>0$	$\dfrac{\pi^{1/2}\beta^\nu\Gamma(s+\nu)\Gamma(s-\nu)}{2^s a^{s+\nu}\,\Gamma(s+\tfrac12)}\times{}_2F_1\left(\dfrac{s+\nu+1}{2},\dfrac{s+\nu}{2};s+\dfrac12;1-\dfrac{\beta^2}{a^2}\right)$ $\quad$ $\mathrm{Re}\, s>\lvert\mathrm{Re}\,\nu\rvert$
(30)	$e^{-\beta^2x^2}K_\nu(ax)$ $\quad$ $\lvert\arg\beta\rvert<\pi/4$	$\dfrac{\Gamma(\tfrac12 s-\tfrac12\nu)\Gamma(\tfrac12 s+\tfrac12\nu)}{2\beta^{s-1}a}\times\exp\left(\dfrac{a^2}{8\beta^2}\right)W_{\frac12-\frac12 s,\frac12\nu}\left(\dfrac{a^2}{4\beta^2}\right)$ $\quad$ $\mathrm{Re}\, s>\lvert\mathrm{Re}\,\nu\rvert$
(31)	$2\pi^{-1}K_0(x)-Y_0(x)$	$2^s\pi^{-1}[\Gamma(\tfrac12 s)]^2\cos^2(\tfrac14 s\pi)$ $\quad$ $0<\mathrm{Re}\, s<3/2$
(32)	$(x^2+\beta^2)^{-\frac12\nu}K_\nu[a(x^2+\beta^2)^{\frac12}]$ $\quad$ $\mathrm{Re}\, a,\ \ \mathrm{Re}\,\beta>0$	$a^{-\frac12 s}2^{\frac12 s-1}\beta^{\frac12 s-\nu}\Gamma(\tfrac12 s)K_{\nu-\frac12 s}(a\beta)$ $\quad$ $\mathrm{Re}\, s>0$
(33)	$J_\mu(ax)\,J_\nu(ax)$ $\quad$ $a>0$	$\dfrac{2^{s-1}a^{-s}\mathrm{B}(1-s,\tfrac12\mu+\tfrac12\nu+\tfrac12 s)}{\Gamma(\tfrac12\nu-\tfrac12\mu-\tfrac12 s+1)\,\Gamma(\tfrac12\mu-\tfrac12\nu-\tfrac12 s+1)}$ $\quad$ $-\mathrm{Re}(\mu+\nu)<\mathrm{Re}\, s<1$

Bessel functions and related functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(34)	$e^{-\beta^2x^2}J_\mu(ax)J_\nu(ax)$ $\qquad \lvert\arg\beta\rvert<\pi/4$	$\dfrac{a^{\mu+\nu}\Gamma[\tfrac{1}{2}(s+\mu+\nu)]}{2^{\mu+\nu+1}\beta^{s+\mu+\nu}[\Gamma(\nu+1)]^2}$ $\times{}_3F_3\left(\dfrac{\mu+\nu+1}{2},\dfrac{\mu+\nu+2}{2},\dfrac{s+\mu+\nu}{2};\ \mu+1,\nu+1,\mu+\nu+1;-\dfrac{a^2}{\beta^2}\right)$ $\quad \mathrm{Re}\,s>-\mathrm{Re}(\mu+\nu)$
(35)	$(a-x)^{\lambda-1}J_\mu(x)J_\nu(a-x)$ see Bailey, W. N., 1930: *Proc. London Math. Soc.* 31, 201-208.	
(36)	$J_\nu(ax)Y_\mu(ax)$ $\qquad a>0$	$2^{s-1}a^{-s}\pi^{-1}\sin[\tfrac{1}{2}(\nu-\mu+s-1)\pi]$ $\times\dfrac{\Gamma[\tfrac{1}{2}(s+\mu+\nu)]\Gamma[\tfrac{1}{2}(s-\mu+\nu)]}{\Gamma[\tfrac{1}{2}(\nu-\mu-s+2)]\Gamma[\tfrac{1}{2}(\nu+\mu-s+2)]}$ $\quad \mathrm{Re}(-\nu\pm\mu)<\mathrm{Re}\,s<1$
(37)	$J_\nu(bx)Y_\mu(ax)$ $\qquad a>b>0$	$\dfrac{\sin[\tfrac{1}{2}(\nu-\mu+s-1)\pi]}{2^{1-s}\pi\Gamma(1+\nu)b^{-\nu}a^{\nu+s}}$ $\times\Gamma[\tfrac{1}{2}(s+\mu+\nu)]\Gamma[\tfrac{1}{2}(s-\mu+\nu)]$ $\times{}_2F_1\left(\dfrac{s+\mu+\nu}{2},\dfrac{s-\mu+\nu}{2};1+\nu;\dfrac{b^2}{a^2}\right)$ $\quad \mathrm{Re}(-\nu\pm\mu)<\mathrm{Re}\,s<2$
(38)	$J_\nu(bx)Y_\mu(ax)$ $\qquad b>a>0$	$-\int_0^\infty\{J_\mu(ax)Y_\nu(bx)$ $+4\pi^{-2}\cos[\tfrac{1}{2}(1-s+\nu+\mu)\pi]$ $\times K_\nu(bx)K_\mu(ax)\}x^{-s}\,dx$ $\quad \mathrm{Re}(-\nu\pm\mu)<\mathrm{Re}\,s<2$

Bessel functions and related functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}dx$
(39)	$Y_\mu(ax)\,Y_\nu(bx)$ $\quad a>b>0$	$\int_0^\infty \{J_\mu(ax)\,J_\nu(bx)$ $+4\pi^{-2}\sin[\tfrac{1}{2}(1-s+\mu+\nu)\pi]$ $\times K_\mu(ax)K_\nu(bx)\}x^{s-1}dx$ $\lvert\mathrm{Re}(\mu\pm\nu\rvert<\mathrm{Re}\,s<2$
(40)	$H_\mu^{(2)}(ax)\,H_\nu^{(2)}(bx)$ $\quad a,\ b>0$	$\dfrac{\Gamma[\frac{1}{2}(s+\mu+\nu)]\,\Gamma[\frac{1}{2}(s-\mu+\nu)]}{2^{1-s}e^{-\frac{1}{2}\pi i(\mu+\nu+s+2)}}$ $\times\dfrac{\Gamma[\frac{1}{2}(s+\mu-\nu)]\,\Gamma[\frac{1}{2}(s-\mu-\nu)]}{\pi^2\Gamma(s)\,a^{s+\nu}b^{-\nu}}$ $\times{}_2F_1\left(\dfrac{s+\mu+\nu}{2},\dfrac{s-\mu+\nu}{2};s;1-\dfrac{b^2}{a^2}\right)$ For $H_\nu^{(1)}\ H_\mu^{(1)}$ change i into $-i$ $\lvert\mathrm{Re}\,\mu\rvert+\lvert\mathrm{Re}\,\nu\rvert<\mathrm{Re}\,s<1$
(41)	$J_\nu(ax)\,K_\nu(ax)$ $\quad \lvert\arg a\rvert<\pi/4$	$\dfrac{2^{s-2}\Gamma(\frac{1}{2}s)\,\Gamma(\frac{1}{4}s+\frac{1}{2}\nu)}{a^s\,\Gamma(1-\frac{1}{4}s+\frac{1}{2}\nu)}$ $\mathrm{Re}\,s>-2\,\mathrm{Re}\,\nu,\ 0$
(42)	$K_\mu(ax)\ I_\mu(\beta x)$ $\quad \mathrm{Re}\,a>\mathrm{Re}\,\beta$	$2^{s-2}(\pi a\beta)^{-\frac{1}{2}}\,\Gamma(\tfrac{1}{2}s)\,e^{-\frac{1}{2}\pi i(s-1)}$ $\times(a^2-\beta^2)^{-\frac{1}{2}s+\frac{1}{2}}$ $\times Q_{\mu-\frac{1}{2}}^{\frac{1}{2}s+\frac{1}{2}}[(\beta^2+a^2)(2\beta a)^{-1}]$ $\mathrm{Re}\,s>0,\ -2\,\mathrm{Re}\,\mu$
(43)	$I_\nu(ax)\,K_\mu(ax)$ $\quad \mathrm{Re}\,a>0$	$\dfrac{\Gamma[\frac{1}{2}(s+\mu+\nu)]\,\mathrm{B}(1-s,\frac{1}{2}s-\frac{1}{2}\mu+\frac{1}{2}\nu)}{2^{2-s}a^s\,\Gamma(\frac{1}{2}\nu+\frac{1}{2}\mu-\frac{1}{2}s+1)}$ $\mathrm{Re}(-\nu\mp\mu)<\mathrm{Re}\,s<1$

Bessel functions and related functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(44)	$K_\mu(\alpha x)\, I_\nu(\beta x)$ $\quad \operatorname{Re}\alpha > \lvert\operatorname{Re}\beta\rvert$	$\dfrac{\Gamma[\frac{1}{2}(s+\mu+\nu)]\,\Gamma[\frac{1}{2}(s-\mu+\nu)]}{2^{2-s}\alpha^{s+\nu}\beta^{-\nu}\Gamma(\nu+1)}$ $\times {}_2F_1\left(\dfrac{s+\mu+\nu}{2}, \dfrac{s-\mu+\nu}{2}; \nu+1; \dfrac{\beta^2}{\alpha^2}\right)$ $\operatorname{Re}s > \operatorname{Re}(-\nu+\mu)$
(45)	$[K_\mu(\alpha x)]^2$ $\quad \operatorname{Re}\alpha > 0$	$2^{s-3}\alpha^{-s}[\Gamma(\frac{1}{2}s)]^2$ $\times \mathrm{B}(\frac{1}{2}s+\mu, \frac{1}{2}s-\mu)$ $\operatorname{Re}s > 2\lvert\operatorname{Re}\mu\rvert$
(46)	$K_\mu(\alpha x)\, K_\mu(\beta x)$ $\quad \operatorname{Re}(\alpha+\beta) > 0$	$\dfrac{\frac{1}{4}\pi^{\frac{1}{2}}\,\Gamma(\frac{1}{2}s)\,\Gamma(\frac{1}{2}s+\mu)\,\Gamma(\frac{1}{2}s-\mu)}{(\alpha\beta)^{s-\frac{1}{2}}(\beta^2-\alpha^2)^{\frac{1}{2}s-\frac{1}{2}}}$ $\times P_{\mu-\frac{1}{2}}^{-\frac{1}{2}s+\frac{1}{2}}\left(\dfrac{\beta^2+\alpha^2}{2\alpha\beta}\right)$ $\operatorname{Re}s > \lvert 2\operatorname{Re}\mu\rvert$
(47)	$K_\mu(\alpha x)\, K_\nu(\beta x)$ $\quad \operatorname{Re}(\alpha+\beta) > 0$	$2^{s-3}\alpha^{-s-\nu}\beta^{\nu}[\Gamma(s)]^{-1}$ $\times\Gamma[\frac{1}{2}(s+\mu+\nu)]\,\Gamma[\frac{1}{2}(s-\mu+\nu)]$ $\times\Gamma[\frac{1}{2}(s+\mu-\nu)]\,\Gamma[\frac{1}{2}(s-\mu-\nu)]$ $\times {}_2F_1\left(\dfrac{s+\mu+\nu}{2}, \dfrac{s-\mu+\nu}{2}; s; 1-\dfrac{\beta^2}{\alpha^2}\right)$ $\operatorname{Re}s > \lvert\operatorname{Re}\mu\rvert + \lvert\operatorname{Re}\nu\rvert$
(48)	$K_\mu(\alpha x)\, K_\nu(\alpha x)$ $\quad \operatorname{Re}\alpha > 0$	$2^{s-3}\alpha^{-s}[\Gamma(s)]^{-1}$ $\times\Gamma[\frac{1}{2}(s+\mu+\nu)]\,\Gamma[\frac{1}{2}(s-\mu+\nu)]$ $\times\Gamma[\frac{1}{2}(s+\mu-\nu)]\,\Gamma[\frac{1}{2}(s-\mu-\nu)]$ $\operatorname{Re}s > \lvert\operatorname{Re}\mu\rvert + \lvert\operatorname{Re}\nu\rvert$

Bessel functions and related functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(49)	$J_\nu^2(ax)\,K_\mu^2(bx)$	$\dfrac{a^{2\nu}\Gamma(\nu+\frac{1}{2})\,\Gamma(\nu+\mu+\frac{1}{2}s)}{4b^{s+2\nu}\Gamma(\nu+1)\,\Gamma(2\nu+1)\,\Gamma(\nu+\frac{1}{2}s+\frac{1}{2})}$ $\times\Gamma(\nu+\frac{1}{2}s)\,\Gamma(\nu-\mu+\frac{1}{2}s)$ $\times{}_4F_3\left(\nu+\dfrac{1}{2},\nu+\mu+\dfrac{s}{2},\nu+\dfrac{s}{2},\nu-\mu+\dfrac{s}{2};\right.$ $\left.\nu+1,2\nu+1,\nu+\dfrac{s+1}{2};-\dfrac{a^2}{b^2}\right)$ $\operatorname{Re}s>-2\operatorname{Re}(\nu\pm\mu)$
(50)	For more integrands of the form $J_\mu(ax)\,J_\nu(bx),\quad K_\mu(ax)\,J_\nu(bx)$ see Hankel transforms.	
(51)	$e^{-\alpha^2x^2}\,\mathbf{H}_\nu(\beta x)$ $\quad|\arg\alpha|<\pi/4$	$\dfrac{\beta^{\nu+1}\Gamma(1/2+s/2+\nu/2)}{2^{\nu+1}\pi^{\frac{1}{2}}\alpha^{\frac{1}{2}\nu+\frac{1}{2}s+\frac{1}{2}}\Gamma(\nu+3/2)}$ $\times{}_2F_2\left(1,\dfrac{\nu+s+1}{2};\dfrac{3}{2},\nu+\dfrac{3}{2};-\dfrac{\beta^2}{4\alpha^2}\right)$ $\operatorname{Re}s>-\operatorname{Re}\nu-1$
(52)	$\mathbf{H}_\nu(ax)$ $\quad a>0$	$\dfrac{2^{s-1}\Gamma(\frac{1}{2}s+\frac{1}{2}\nu)}{a^s\Gamma(\frac{1}{2}\nu-\frac{1}{2}s+1)}\tan[\tfrac{1}{2}(s+\nu)\pi]$ $-1-\operatorname{Re}\nu<\operatorname{Re}s<\min(3/2,1-\operatorname{Re}\nu)$
(53)	$K_\nu(\alpha x)\,\mathbf{H}_\mu(\beta x)$ $\quad\operatorname{Re}\alpha>|\operatorname{Im}\beta|$	$\dfrac{\Gamma[(1+s+\mu-\nu)/2]\,\Gamma[(1+s+\mu+\nu)/2]}{\pi^{\frac{1}{2}}2^{1-s}\alpha^{s+\mu+1}\beta^{-\mu-1}\Gamma(\mu+3/2)}$ $\times{}_3F_2\left(\dfrac{1+s+\mu-\nu}{2},\dfrac{1+s+\mu+\nu}{2},1;\right.$ $\left.\dfrac{3}{2},\mu+\dfrac{3}{2};-\dfrac{\beta^2}{\alpha^2}\right)$ $\operatorname{Re}s>|\operatorname{Re}\nu|-\operatorname{Re}\mu-1$

6.9. Other higher transcendental functions

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(1)	$e^{-\frac{1}{4}x^2} D_{-\nu}(x)$	$\dfrac{\pi^{\frac{1}{2}}\,\Gamma(s)}{2^{\frac{1}{2}s+\frac{1}{2}\nu}\,\Gamma(\frac{1}{2}s+\frac{1}{2}\nu+\frac{1}{2})}$ $\quad \mathrm{Re}\, s>0$
(2)	$e^{-\alpha x} D_{-2\nu}[2(\beta x)^{\frac{1}{2}}]$ $\quad \mathrm{Re}(\alpha+\beta)>0$	$\dfrac{\pi^{1/2}\,2^{3/2-\nu-2s}\,\beta^{1/2}\Gamma(2s)}{\Gamma(s+\nu+\frac{1}{2})(\alpha+\beta)^{s+1/2}}$ $\times {}_2F_1\left(s+\dfrac{1}{2},\nu+\dfrac{1}{2};s+\nu+\dfrac{1}{2};\dfrac{\alpha-\beta}{\alpha+\beta}\right)$ $\quad \mathrm{Re}\, s>0$
(3)	${}_2F_1(\alpha,\beta;\gamma;-x)$	$\dfrac{\mathrm{B}(s,\alpha-s)\,\mathrm{B}(s,\beta-s)}{\mathrm{B}(s,\gamma-s)}$ $\quad 0<\mathrm{Re}\, s<\min(\mathrm{Re}\,\alpha,\ \mathrm{Re}\,\beta)$
(4)	$(1-x^2)^\nu\ {}_2F_1(-n,a;b;x^2)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$ $\mathrm{Re}\,\nu>-1$	$\frac{1}{2}\mathrm{B}(\nu+1,\frac{1}{2}s)$ $\times {}_3F_2(-n,a,\frac{1}{2}s;b,\nu+1+\frac{1}{2}s;1)$ $\quad \mathrm{Re}\, s>0$
(5)	$e^{-\frac{1}{2}x} M_{\kappa,\mu}(x)$	$\dfrac{\Gamma(2\mu+1)}{\Gamma(\mu+\frac{1}{2}-s)}\,\mathrm{B}(\mu+\frac{1}{2}+s,\kappa-s)$ $\quad -\frac{1}{2}-\mathrm{Re}\,\mu<\mathrm{Re}\, s<\mathrm{Re}\,\kappa$
(6)	$e^{-\alpha x} M_{\kappa,\mu}(\beta x)$ $\quad \mathrm{Re}\,\alpha>\frac{1}{2}\lvert\mathrm{Re}\,\beta\rvert$	$\beta^{\mu+\frac{1}{2}}\,\Gamma(\mu+s+\frac{1}{2})(\alpha+\frac{1}{2}\beta)^{-\mu-s-\frac{1}{2}}$ $\times {}_2F_1\left(\mu+s+\dfrac{1}{2},\mu-\kappa+\dfrac{1}{2};2\mu+1;\dfrac{\beta}{\alpha+\frac{1}{2}\beta}\right)$ $\quad \mathrm{Re}\, s>-\frac{1}{2}-\mathrm{Re}\,\mu$
(7)	$e^{\frac{1}{2}x} W_{\kappa,\mu}(x)$	$\dfrac{\Gamma(\mu+s-\frac{1}{2})}{\Gamma(\mu-\kappa+\frac{1}{2})}\,\mathrm{B}(s-\mu+\frac{1}{2},-s-\kappa)$ $\quad \lvert\mathrm{Re}\,\mu\rvert-\frac{1}{2}<\mathrm{Re}\, s<-\mathrm{Re}\,\kappa$

Higher functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$
(8)	$e^{-ax}\,W_{\kappa,\mu}(\beta x)$ $\operatorname{Re}(a+\tfrac{1}{2}\beta)>0$	$\dfrac{\beta^{\mu+\frac{1}{2}}\,\Gamma(\mu+s+\frac{1}{2})\,\Gamma(-\mu+s+\frac{1}{2})}{\Gamma(s-\kappa+1)\,(a+\frac{1}{2}\beta)^{\mu+s+\frac{1}{2}}}$ $\times {}_2F_1\left[\mu+s+\dfrac{1}{2},\,\mu-\kappa+\dfrac{1}{2};\,s-\kappa+1;\,\dfrac{(2a-\beta)}{(2a+\beta)}\right]$ $\operatorname{Re}s>\lvert\operatorname{Re}\mu\rvert-\tfrac{1}{2}$
(9)	$e^{-ax}\;{}_1F_1(\beta;\rho;\lambda x)$ $\operatorname{Re}a>0,\ \operatorname{Re}\lambda$	$a^{-s}\,\Gamma(s)\;{}_2F_1(\beta,s;\rho;\lambda a^{-1})$ $\operatorname{Re}s>0$
(10)	$(1-x)^{\nu-1}$ $\times {}_pF_q(a_1,\dots,a_p;b_1,\dots,b_q,ax)$ $0<x<1$ $0 \qquad 1<x<\infty$ $\operatorname{Re}\nu>0$	$\mathrm{B}(\nu,s)$ $\times {}_{p+1}F_{q+1}(a_1,\dots,a_p;s;b_1,\dots,b_q;s+\nu;a)$ $\operatorname{Re}s>0$
(11)	$e^{-x}\,{}_pF_q(a_1,\dots,a_p;b_1,\dots,b_q;ax)$ $p<q$	$\Gamma(s)\;{}_{p+1}F_q(s,a_1,\dots,a_p;b_1,\dots,b_q;a)$ $\operatorname{Re}s>0$
(12)	$K_\nu(2x^{\frac{1}{2}})$ $\times {}_pF_q(a_1,\dots,a_p;b_1,\dots,b_q;ax)$ $p<q-1$	$\tfrac{1}{2}\Gamma(s+\tfrac{1}{2}\nu)\,\Gamma(s-\tfrac{1}{2}\nu)$ $\times {}_{p+2}F_q(s+\tfrac{1}{2}\nu,s-\tfrac{1}{2}\nu,a_1,\dots,a_p;$ $b_1,\dots,b_q;a)$ $\operatorname{Re}s>\tfrac{1}{2}\lvert\operatorname{Re}\nu\rvert$
(13)	$K_\lambda(ax)\,K_\mu(ax)$ $\times {}_pF_q(a_1,a_2,\dots,a_p;b_1,\dots,b_q;bx^2)$	see Sinha, S., 1943: *Bull. Calcutta Math. Soc.* 35, p. 37-42.
(14)	$G^{m,n}_{p,q}\left(x\,\middle\vert\,\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_p\end{matrix}\right)$ $0\le m\le q,\quad 0\le n\le p$ $p+q<2(m+n)$	$\dfrac{\prod_{j=1}^{m}\Gamma(b_j+s)\,\prod_{j=1}^{n}\Gamma(1-a_j-s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j-s)\,\prod_{j=n+1}^{p}\Gamma(a_j+s)}$ $-\min_{1\le j\le m}\operatorname{Re}b_j<\operatorname{Re}s<1-\max_{1\le j\le n}\operatorname{Re}a_j$

Higher functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(15)	$(1-x)^{\beta-1}G^{m,n}_{p,q}\left(ax\,\middle\|\,\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$ $0<x<1$ $1<x<\infty$ $p+q<2(m+n),\quad \operatorname{Re}\beta>0$ $\|\arg\alpha\|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi$	$\Gamma(\beta)G^{m,n+1}_{p+1,q+1}\left(a\,\middle\|\,\begin{matrix}1-s,a_1,\dots,a_p\\ b_1,\dots,b_q,1-s-\beta\end{matrix}\right)$ $\operatorname{Re}s>-\min\limits_{1\le h\le m}\operatorname{Re}b_h$
(16)	$e^{-x}G^{m,n}_{p,q}\left(ax\,\middle\|\,\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$ $p+q<2(m+n)$ $\|\arg\alpha\|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi$	$G^{m,n+1}_{p+1,q}\left(a\,\middle\|\,\begin{matrix}1-s,a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$ $\operatorname{Re}s>-\min\limits_{1\le h\le m}\operatorname{Re}b_j$
(17)	$J_\nu(2x^{1/2})G^{m,n}_{p,q}\left(ax\,\middle\|\,\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$ $p+q<2(m+n)$ $\|\arg\alpha\|<(p+q-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi$	$G^{m,n+1}_{p+2,q}\left(a\,\middle\|\,\begin{matrix}1-s-\tfrac{1}{2}\nu,a_1,\dots,a_p,1-s+\tfrac{1}{2}\nu\\ b_1,\dots,b_q\end{matrix}\right)$ $-\tfrac{1}{2}\operatorname{Re}\nu-\min\limits_{1\le h\le m}\operatorname{Re}b_h$ $<\operatorname{Re}s<5/4-\max\limits_{1\le j\le n}\operatorname{Re}a_j$
(18)	$K_\nu(2x^{1/2})G^{m,n}_{p,q}\left(ax\,\middle\|\,\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$ $p+q<2(m+n)$ $\|\arg\alpha\|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi$	$\tfrac{1}{2}G^{m,n+2}_{p+2,q}\left(a\,\middle\|\,\begin{matrix}1-s-\tfrac{1}{2}\nu,1-s+\tfrac{1}{2}\nu,a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$ $\operatorname{Re}s>\tfrac{1}{2}\|\operatorname{Re}\nu\|-\min\limits_{1\le h\le m}\operatorname{Re}b_h$
(19)	$(1+x)^{-\beta}G^{m,n}_{p,q}\left(\dfrac{ax}{1+x}\,\middle\|\,\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$ $(p+q)<2(m+n)$ $\|\arg\alpha\|<(m+n-\tfrac{1}{2}p-\tfrac{1}{2}q)\pi$	$\Gamma(\beta-s)G^{m,n+1}_{p+1,q+1}\left(a\,\middle\|\,\begin{matrix}1-s,a_1,\dots,a_p\\ b_1,\dots,b_q,1-\beta\end{matrix}\right)$ $-\min\limits_{1<h<m}\operatorname{Re}b_h<\operatorname{Re}s<\operatorname{Re}\beta$

Higher functions (cont'd)

	$f(x)$	$g(s)=\int_0^\infty f(x)x^{s-1}\,dx$
(20)	$\theta_2(0\|ix^2)$	$2^s(1-2^{-s})\pi^{-\frac{1}{2}s}\Gamma(\tfrac{1}{2}s)\zeta(s)$ Re $s>2$
(21)	$\theta_3(0\|ix^2)-1$	$\pi^{-\frac{1}{2}s}\Gamma(\tfrac{1}{2}s)\zeta(s)$ Re $s>2$
(22)	$1-\theta_4(0\|ix^2)$	$(1-2^{1-s})\pi^{-\frac{1}{2}s}\Gamma(\tfrac{1}{2}s)\zeta(s)$ Re $s>2$
(23)	$e^{-\pi a^2 x}\,\theta_3(b+iax\|ix)$	$\pi^{-s}\Gamma(s)[\Phi(b,a,2s)$ $+e^{-2\pi ib}\phi(-b,1-a,2s)]$
(24)	$\theta_4(0\|ix^2)+\theta_2(0\|ix^2)$ $-\theta_3(0\|ix^2)$	$-(2^s-1)(2^{1-s}-1)\pi^{-\frac{1}{2}s}\Gamma(\tfrac{1}{2}s)\zeta(s)$

CHAPTER VII

INVERSE MELLIN TRANSFORMS

For general formulas see 6.1

7.1. Algebraic functions and powers with arbitrary index

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$		$f(x)$	
(1)	s^{-1}	$\operatorname{Re} s > 0$	1 0	$0 < x < 1$ $1 < x < \infty$
(2)	s^{-1}	$\operatorname{Re} s < 0$	0 -1	$0 < x < 1$ $1 < x < \infty$
(3)	$(s+\alpha)^{-1}$	$\operatorname{Re} s > -\operatorname{Re} \alpha$	x^{α} 0	$0 < x < 1$ $1 < x < \infty$
(4)	$(s+\alpha)^{-1}$	$\operatorname{Re} s < -\operatorname{Re} \alpha$	0 $-x^{\alpha}$	$0 < x < 1$ $1 < x < \infty$
(5)	$(s+\alpha)^{-2}$	$\operatorname{Re} s > -\operatorname{Re} \alpha$	$-x^{\alpha} \log x$ 0	$0 < x < 1$ $1 < x < \infty$
(6)	$(s+\alpha)^{-2}$	$\operatorname{Re} s < -\operatorname{Re} \alpha$	0 $x^{\alpha} \log x$	$0 < x < 1$ $1 < x < \infty$
(7)	$(s+\alpha)^{-1}(s+\beta)^{-1}$	$\operatorname{Re} s > -\operatorname{Re} \alpha,\ -\operatorname{Re} \beta$	$(\beta-\alpha)^{-1}(x^{\alpha}-x^{\beta})$ 0	$0 < x < 1$ $1 < x < \infty$

Algebraic functions (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$	
(8)	$(s+\alpha)^{-1}(s+\beta)^{-1}$ $-\mathrm{Re}\,\alpha<\mathrm{Re}\,s<-\mathrm{Re}\,\beta$	$(\beta-\alpha)^{-1}x^{\alpha}$ $(\beta-\alpha)^{-1}x^{\beta}$	$0<x<1$ $1<x<\beta$
(9)	$(s+\alpha)^{-1}(s+\beta)^{-1}$ $\mathrm{Re}\,s<-\mathrm{Re}\,\alpha,\ -\mathrm{Re}\,\beta$	0 $(\beta-\alpha)^{-1}(x^{\beta}-x^{\alpha})$	$0<x<1$ $1<x<\infty$
(10)	$[(s+\alpha)^2+\beta^2]^{-1}$ $\mathrm{Re}\,(s+\alpha)>\lvert\mathrm{Im}\,\beta\rvert$	$\dfrac{1}{\beta}x^{\alpha}\sin\left(\beta\log\dfrac{1}{x}\right)$ 0	$0<x<1$ $1<x<\infty$
(11)	$[(s+\alpha)^2+\beta^2]^{-1}$ $-\mathrm{Im}\,\beta<\mathrm{Re}\,(s+\alpha)<\mathrm{Im}\,\beta$	$-\dfrac{i}{2\beta}\,x^{\alpha-i\beta}$ $-\dfrac{i}{2\beta}\,x^{\alpha+i\beta}$	$0<x<1$ $1<x<\infty$
(12)	$(s+\alpha)\,[(s+\alpha)^2+\beta^2]^{-1}$ $\mathrm{Re}\,(s+\alpha)>\lvert\mathrm{Im}\,\beta\rvert$	$x^{\alpha}\cos(\beta\log x)$ 0	$0<x<1$ $1<x<\infty$
(13)	$(s+\alpha)\,[(s+\alpha)^2+\beta^2]^{-1}$ $-\mathrm{Im}\,\beta<\mathrm{Re}\,(s+\alpha)<\mathrm{Im}\,\beta$	$\tfrac{1}{2}x^{\alpha-i\beta}$ $-\tfrac{1}{2}x^{\alpha+i\beta}$	$0<x<1$ $1<x<\infty$
(14)	$(s^2-\alpha^2)^{1/2}-s \quad \mathrm{Re}\,s>\lvert\mathrm{Re}\,\alpha\rvert$	$\alpha y^{-1}I_1(\alpha y)$ 0	$0<x<1$ $1<x<\infty$
(15)	$\left(\dfrac{s+\alpha}{s-\alpha}\right)^{1/2}-1$ $\mathrm{Re}\,s>\lvert\mathrm{Re}\,\alpha\rvert$	$\alpha[I_0(\alpha y)+I_1(\alpha y)]$ 0	$0<x<1$ $1<x<\infty$

$y=-\log x$

Algebraic functions (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(16)	$\Gamma(\nu)\,(s+a)^{-\nu}$ $\operatorname{Re}\nu>0,\quad \operatorname{Re}s>-\operatorname{Re}a$	$x^{a}(-\log x)^{\nu-1} \quad 0<x<1$ $0 \quad 1<x<\infty$
(17)	$s^{-1}(s+a)^{-\nu} \quad \operatorname{Re}\nu>0$ $\operatorname{Re}s>0,\ -\operatorname{Re}a$	$a^{-\nu}\gamma(\nu,a y)/\Gamma(\nu) \quad 0<x<1$ $0 \quad 1<x<\infty$
(18)	$s^{-1}(s+a)^{-\nu} \quad \operatorname{Re}\nu>0$ $-\operatorname{Re}a<\operatorname{Re}s<0$	$-a^{-\nu}\Gamma(\nu,a y)/\Gamma(\nu) \quad 0<x<1$ $-a^{-\nu} \quad 1<x<\infty$
(19)	$[(s+a)^{1/2}+\beta^{1/2}]^{\nu}$ $\lvert\arg\beta\rvert\le\pi,\quad \operatorname{Re}\nu<0$ $\operatorname{Re}s>-\operatorname{Re}a$	$-2^{1/2}\pi^{-1/2}\nu(2y)^{-1/2\nu-1}$ $\times x^{a-1/2\beta}D_{\nu-1}[(2\beta y)^{1/2}] \quad 0<x<1$ $0 \quad 1<x<\infty$
(20)	$(s+a)^{-1/2}[(s+a)^{1/2}+\beta^{1/2}]^{\nu}$ $\lvert\arg\beta\rvert\le\pi,\quad \operatorname{Re}\nu<1$ $\operatorname{Re}s>-\operatorname{Re}a$	$2^{1/2}\pi^{-1/2}(2y)^{-1/2\nu-1/2}$ $\times x^{a-1/2\beta}D_{\nu}[(2\beta y)^{1/2}] \quad 0<x<1$ $0 \quad 1<x<\infty$
(21)	$\pi^{-1/2}\Gamma(\nu)\,(s^2-a^2)^{-\nu}$ $\operatorname{Re}s>\lvert\operatorname{Re}a\rvert,\quad \operatorname{Re}\nu>0$	$(\tfrac12 y/a)^{\nu-1/2}I_{\nu-1/2}(a y) \quad 0<x<1$ $0 \quad 1<x<\infty$
(22)	$\pi^{1/2}\Gamma(\nu)\,(a^2-s^2)^{-\nu} \quad \operatorname{Re}\nu>0$ $-\operatorname{Re}a<\operatorname{Re}s<\operatorname{Re}a$	$(\tfrac12 y/a)^{\nu-1/2}K_{\nu-1/2}(a y) \quad 0<x<1$ $(-\tfrac12 y/a)^{\nu-1/2}K_{\nu-1/2}(-a y) \quad 1<x<\infty$
(23)	$\Gamma(2\nu-2\lambda)\,(s-a)^{2\lambda}(s-\beta)^{-2\nu}$ $\operatorname{Re}(\nu-\lambda)>0$ $\operatorname{Re}s>\operatorname{Re}a,\ \operatorname{Re}\beta$	$(a-\beta)^{\lambda-\nu}x^{-1/2a-1/2\beta}(y)^{\nu-\lambda-1}$ $\times M_{\lambda+\nu,\nu-\lambda-1/2}[(a-\beta)y]$ $0<x<1$ $0 \quad 1<x<\infty$

$y=-\log x$

Algebraic functions (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(24)	$(s-\alpha)^{2\lambda}(\beta-s)^{-2\nu}$ $\mathrm{Re}(\nu-\lambda)>0$ $\mathrm{Re}\,\alpha<\mathrm{Re}\,s<\mathrm{Re}\,\beta$	$[\Gamma(-2\lambda)]^{-1}(\beta-\alpha)^{\lambda-\nu}y^{\nu-\lambda-1}$ $\times x^{-\frac{1}{2}\alpha-\frac{1}{2}\beta}W_{-\lambda-\nu,\lambda-\nu+\frac{1}{2}}[(\beta-\alpha)y]$ $0<x<1$ $[\Gamma(2\nu)]^{-1}(\beta-\alpha)^{\lambda-\nu}(-y)^{\nu-\lambda-1}$ $\times x^{-\frac{1}{2}\alpha-\frac{1}{2}\beta}W_{\lambda+\nu,\lambda-\nu+\frac{1}{2}}[(\alpha-\beta)y]$ $1<x<\infty$

7.2. Other elementary functions

(1)	$e^{\alpha s^2}$ $\quad\mathrm{Re}\,\alpha\geq 0$	$\frac{1}{2}\pi^{-\frac{1}{2}}\alpha^{-\frac{1}{2}}e^{-\frac{1}{4}\alpha^{-1}y^2}$
(2)	$s^{\nu}e^{\alpha s^2}$ $\quad\mathrm{Re}\,s>0,\quad\mathrm{Re}\,\alpha>0$	$2^{-\frac{1}{2}\nu}\pi^{-\frac{1}{2}}\alpha^{-\frac{1}{2}\nu-\frac{1}{2}}$ $\times\exp\left(-\frac{y^2}{8\alpha}\right)D_\nu\left(\frac{-y}{2^{\frac{1}{2}}\alpha^{\frac{1}{2}}}\right)$
(3)	$s^{-\nu}e^{-\alpha/s}$ $\quad\alpha>0$ $\mathrm{Re}\,\nu>0,\quad\mathrm{Re}\,s>0$	$(y/\alpha)^{\frac{1}{2}\nu-\frac{1}{2}}J_{\nu-1}(2\alpha^{\frac{1}{2}}y^{\frac{1}{2}})$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$
(4)	$e^{-\alpha^{\frac{1}{2}}s^{\frac{1}{2}}}$ $\quad\mathrm{Re}\,\alpha>0,\quad\mathrm{Re}\,s>0$	$\frac{1}{2}\alpha^{1/2}\pi^{-1/2}y^{-3/2}e^{-\frac{1}{4}\alpha/y}$ $0<x<1$ 0 $\quad 1<x<\infty$
(5)	$s^{-1}e^{-\alpha s^{\frac{1}{2}}}$ $\lvert\arg\alpha\rvert<\pi/4,\quad\mathrm{Re}\,s>0$	$\mathrm{Erfc}(\frac{1}{2}\alpha y^{-\frac{1}{2}})$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$
(6)	$s^{-1}e^{-\alpha s^{\frac{1}{2}}}-s^{-1}$ $\lvert\arg\alpha\rvert<\pi/4,\quad\mathrm{Re}\,s>0$	$-\mathrm{Erf}(\frac{1}{2}\alpha y^{-\frac{1}{2}})$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$

$y=-\log x$

Elementary functions (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(7)	$\dfrac{e^{-as^{1/2}}}{s+\beta}$ $\quad \lvert\arg a\rvert<\pi/4$ $\mathrm{Re}\,s>0,\ -\mathrm{Re}\,\beta$	$\tfrac{1}{2}x^{\beta}[e^{-i\beta^{1/2}a}\,\mathrm{Erfc}\,(\tfrac{1}{2}ay^{-1/2}-i\beta^{1/2}y^{1/2})$ $+e^{i\beta^{1/2}a}\,\mathrm{Erfc}\,(\tfrac{1}{2}ay^{-1/2}+i\beta^{1/2}y^{1/2})]$ $\quad 0<x<1$ $0 \quad 1<x<\infty$
(8)	$s^{1/2}\,e^{-as^{1/2}}$ $\lvert\arg a\rvert<\pi/4,\quad \mathrm{Re}\,s>0$	$\pi^{-1/2}(\tfrac{1}{4}a^2y^{-1}-\tfrac{1}{2})y^{-3/2}e^{-\frac{1}{4}a^2/y}$ $\quad 0<x<1$ $0 \quad 1<x<\infty$
(9)	$s^{-1/2}\,e^{-as^{1/2}}$ $\lvert\arg a\rvert<\pi/4,\quad \mathrm{Re}\,s>0$	$\pi^{-1/2}\,y^{-1/2}\,e^{-\frac{1}{4}a^2/y}$ $\quad 0<x<1$ $0 \quad 1<x<\infty$
(10)	$\log\dfrac{s+a}{s+\beta}$ $\mathrm{Re}\,s>-\mathrm{Re}\,a,\ -\mathrm{Re}\,\beta$	$\dfrac{x^{a}-x^{\beta}}{\log x}$ $\quad 0<x<1$ $0 \quad 1<x<\infty$
(11)	$s^{-\nu}\log s$ $\mathrm{Re}\,\nu>0,\quad \mathrm{Re}\,s>0$	$y^{\nu-1}\,\dfrac{\psi(\nu)-\log y}{\Gamma(\nu)}$ $\quad 0<x<1$ $0 \quad 1<x<\infty$
(12)	$\pi\csc(\pi s)$ $\quad 0<\mathrm{Re}\,s<1$	$1/(1+x)$
(13)	$\pi\csc(\pi s)$ $\quad -n<\mathrm{Re}\,s<1-n$ $n=\dots,-1,0,1,\dots$	$(-1)^n\dfrac{x^n}{1+x}$
(14)	$\pi\sec(\pi s)$ $\quad -\tfrac{1}{2}<\mathrm{Re}\,s<\tfrac{1}{2}$	$x^{1/2}/(1+x)$
(15)	$\pi\sec(\pi s)$ $n-\tfrac{1}{2}<\mathrm{Re}\,s<n+\tfrac{1}{2}$ $n=\dots,-1,0,1,2,\dots$	$(-1)^n x^{1/2-n}/(1+x)$

$y=-\log x$

Elementary functions (cont'd)

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(16)	$\pi \tan(\pi s) \qquad -\tfrac{1}{2} < \operatorname{Re} s < \tfrac{1}{2}$	$\dfrac{x^{1/2}}{1-x}$ The integral is a Cauchy Principal Value
(17)	$\pi \tan(\pi s)$ $n - \tfrac{1}{2} < \operatorname{Re} s < n + \tfrac{1}{2}$ $n = \ldots, -1, 0, 1, 2, \ldots$	$\dfrac{x^{1/2-n}}{1-x}$ The integral is a Cauchy Principal Value
(18)	$\pi \operatorname{ctn}(\pi s) \qquad 0 < \operatorname{Re} s < 1$	$\dfrac{1}{1-x}$ The integral is a Cauchy Principal Value
(19)	$\pi \operatorname{ctn}(\pi s) \qquad -n < \operatorname{Re} s < 1 - n$ $n = \ldots, -1, 0, 1, 2, \ldots$	$\dfrac{x^n}{1-x}$ The integral is a Cauchy Principal Value
(20)	$\pi^2 \csc^2(\pi s) \qquad 0 < \operatorname{Re} s < 1$	$(x-1)^{-1} \log x$
(21)	$\pi^2 \csc^2(\pi s) \qquad n < \operatorname{Re} s < n + 1$ $n = \ldots, -1, 0, 1, 2, \ldots$	$x^{-n} (x-1)^{-1} \log x$
(22)	$2\pi^3 \csc^3(\pi s) \qquad 0 < \operatorname{Re} s < 1$	$[\pi^2 + (\log x)^2]/(1+x)$
(23)	$2\pi^3 \csc^3(\pi s)$ $n < \operatorname{Re} s < n + 1$ $n = \ldots, -1, 0, 1, 2, \ldots$	$(-x)^{-n} [\pi^2 + (\log x)^2]/(1+x)$

Elementary functions (cont'd)

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(24)	$\csc\left(\frac{\pi s}{n}\right) \csc\left(\frac{\pi s + \pi}{n}\right)$ n integer, $0 < \operatorname{Re} s < n - 1$	$\frac{n}{\pi} \csc\left(\frac{\pi}{n}\right) \frac{1-x}{1-x^n}$
(25)	$s^{-1} \cos(\theta s) \csc(\pi s)$ $-1 < \operatorname{Re} s < 0, \quad -\pi < \theta < \pi$	$\tfrac{1}{2}\pi^{-1} \log(1 + 2x \cos\theta + x^2)$
(26)	$\sin(s^2/a) \qquad a > 0$	$\tfrac{1}{2}\pi^{-\frac{1}{2}} a^{\frac{1}{2}} \sin(\tfrac{1}{4} a y^2 - \tfrac{1}{4}\pi)$
(27)	$\cos(s^2/a) \qquad a > 0$	$\tfrac{1}{2}\pi^{-\frac{1}{2}} a^{\frac{1}{2}} \cos(\tfrac{1}{4} a y^2 - \tfrac{1}{4}\pi)$
(28)	$\tan^{-1}\left(\frac{a}{s+\beta}\right)$ $\operatorname{Re} s > -\operatorname{Re} \beta$	$x^\beta y^{-1} \sin(a y) \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$

7.3. Gamma function and related functions; Riemann's zeta function

In this section $\sum_{n=0}^{h-1}$ is to be interpreted as zero if $h \le 0$, and similarly for other sums.

(1)	$\Gamma(s) \qquad \operatorname{Re} s > 0$	e^{-x}
(2)	$\Gamma(s) \qquad -1 < \operatorname{Re} s < 0$	$e^{-x} - 1$
(3)	$e^{-i\alpha s}\, \Gamma(s)$ $\operatorname{Re} s > 0, \quad -\tfrac{1}{2}\pi < \operatorname{Re} \alpha < \tfrac{1}{2}\pi$	$\exp(-x e^{i\alpha})$
(4)	$e^{-i\alpha s}\, \Gamma(s) \qquad -1 < \operatorname{Re} s < 0$ $-\tfrac{1}{2}\pi \le \operatorname{Re} \alpha \le \tfrac{1}{2}\pi$	$\exp(-x e^{i\alpha}) - 1$

Gamma function etc. (cont'd)

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(5)	$e^{-ias}\,\Gamma(s)$ $-\tfrac{1}{2}\pi \leq \mathrm{Re}\, a \leq \tfrac{1}{2}\pi$ $-m < \mathrm{Re}\, s < 1 - m$ $m = 1, 2, \ldots$	$\exp(-xe^{ia}) - \sum_{r=0}^{m-1} (-xe^{ia})^r / r!$
(6)	$\sin(\tfrac{1}{2}\pi s)\,\Gamma(s) \quad -1 < \mathrm{Re}\, s < 1$	$\sin x$
(7)	$\cos(\tfrac{1}{2}\pi s)\,\Gamma(s) \quad 0 < \mathrm{Re}\, s < 1$	$\cos x$
(8)	$\cos(\tfrac{1}{2}\pi s)\,\Gamma(s)$ $-2 < \mathrm{Re}\, s < 0$	$-2\sin^2(\tfrac{1}{2}x)$
(9)	$\sin(as)\,\Gamma(s) \quad \mathrm{Re}\, s > -1$ $-\tfrac{1}{2}\pi < \mathrm{Re}\, a < \tfrac{1}{2}\pi$	$e^{-x\cos a} \sin(x \sin a)$
(10)	$\sin(as)\,\Gamma(s)$ $-\tfrac{1}{2}\pi \leq \mathrm{Re}\, a \leq \tfrac{1}{2}\pi$ $-m < \mathrm{Re}\, s < 1 - m$ $m = 2, 3, \ldots$	$e^{-x\cos a} \sin(x \sin a)$ $+ \sum_{r=1}^{m-1} (-1)^r \sin(ar)\, x^r / r!$
(11)	$\cos(as)\,\Gamma(s) \quad \mathrm{Re}\, s > 0$ $-\tfrac{1}{2}\pi < \mathrm{Re}\, a < \tfrac{1}{2}\pi$	$e^{-x\cos a} \cos(x \sin a)$
(12)	$\cos(as)\,\Gamma(s)$ $-\tfrac{1}{2}\pi \leq \mathrm{Re}\, a \leq \tfrac{1}{2}\pi$ $-m < \mathrm{Re}\, s < 1 - m$ $m = 1, 2, \ldots$	$e^{-x\cos a} \cos(x \sin a)$ $- \sum_{r=0}^{m-1} (-1)^r \cos(ar)\, x^r / r!$
(13)	$\sec(\pi s)\,\Gamma(s) \quad 0 < \mathrm{Re}\, s < \tfrac{1}{2}$	$e^x \,\mathrm{Erfc}(x^{1/2})$
(14)	$\sec(\pi s)\,\Gamma(s) \quad n = 1, 2, \ldots$ $n - \tfrac{1}{2} < \mathrm{Re}\, s < n + \tfrac{1}{2}$	$e^x \,\mathrm{Erfc}(x^{1/2})$ $-\pi^{-1} \sum_{m=0}^{n-1} (-1)^m\, \Gamma(m + \tfrac{1}{2})\, x^{-m-1/2}$

Empty sums are interpreted as zero.

Gamma function etc. (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(15)	$\Gamma(\alpha+s)\,\Gamma(\beta-s)$ $\mathrm{Re}(\alpha+\beta)>0$ $-\mathrm{Re}\,\alpha<s<\mathrm{Re}\,\beta$	$\Gamma(\alpha+\beta)\,x^{\alpha}\,(1+x)^{-\alpha-\beta}$
(16)	$\Gamma(\alpha+s)\,\Gamma(\beta-s)$ $h,\ k$ integers $0<\mathrm{Re}(\alpha+s)+h<1$ $-1<\mathrm{Re}(s-\beta)-k<0$	$\Gamma(\alpha+\beta)\,x^{\alpha}\,(1+x)^{-\alpha-\beta}$ $-\sum_{m=0}^{h-1}(-1)^m\,\Gamma(\alpha+\beta+m)\,x^{\alpha+m}/m!$ $-\sum_{n=0}^{k-1}(-1)^n\,\Gamma(\alpha+\beta+n)\,x^{-\beta-n}/n!$
(17)	$\Gamma(2\alpha+s)\,\Gamma(2\beta+s)$ $\mathrm{Re}\,s>-2\,\mathrm{Re}\,\alpha,\quad -2\,\mathrm{Re}\,\beta$	$2x^{\alpha+\beta}K_{\alpha-\beta}(2x^{1/2})$
(18)	$\Gamma(2\alpha+s)\,\Gamma(2\beta+s)$ $h,\ k$ integers $0<\mathrm{Re}(2\alpha+s)+h<1$ $0<\mathrm{Re}(2\beta+s)+k<1$	$2x^{\alpha+\beta}K_{\alpha-\beta}(2x^{1/2})$ $-\sum_{m=0}^{h-1}(-1)^m\,\Gamma(2\beta-2\alpha-m)\,x^{2\alpha+m}/m!$ $-\sum_{n=0}^{k-1}(-1)^n\,\Gamma(2\alpha-2\beta-n)\,x^{2\beta+n}/n!$
(19)	$\Gamma(s+\nu)\,\Gamma(s-\nu)\cos[\pi(s-\nu)]$ $\lvert\mathrm{Re}\,\nu\rvert<\mathrm{Re}\,s<3/4$	$-\pi\,Y_{2\nu}(2x^{1/2})$
(20)	$\dfrac{\Gamma(s)}{\Gamma(s+\nu)}$ $\mathrm{Re}\,\nu>0,\quad \mathrm{Re}\,s>0$	$\dfrac{(1-x)^{\nu-1}}{\Gamma(\nu)}\qquad 0<x<1$ $0\qquad 1<x<\infty$
(21)	$\dfrac{\Gamma(s)\Gamma(\nu)}{\Gamma(s+\nu)}\qquad \mathrm{Re}\,\nu>0$ $-n<\mathrm{Re}\,s<1-n$ n integer	$(1-x)^{\nu-1}-\sum_{m=0}^{n-1}\dfrac{(1-\nu)_m}{m!}x^m\qquad 0<x<1$ $-\sum_{m=0}^{n-1}\dfrac{(1-\nu)_m}{m!}x^m\qquad 1<x<\infty$

Empty sums are interpreted as zero.

Gamma functions etc. (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(22)	$\dfrac{\Gamma(1-\nu-s)}{\Gamma(1-s)}$ $\operatorname{Re}\nu>0, \quad \operatorname{Re}s<1-\operatorname{Re}\nu$	$0 \qquad 0<x<1$ $\dfrac{(x-1)^{\nu-1}}{\Gamma(\nu)} \qquad 1<x<\infty$
(23)	$\dfrac{\Gamma(s)}{\Gamma(\nu-s+1)}$ $0<\operatorname{Re}s<\tfrac{1}{2}\operatorname{Re}\nu+\tfrac{1}{4}$	$x^{-\frac{1}{2}\nu}J_\nu(2x^{\frac{1}{2}})$
(24)	$\dfrac{\Gamma(s)}{\Gamma(\nu-s+1)} \qquad \operatorname{Re}\nu>0$ $-n<\operatorname{Re}s<1-n$ $n=1,2,3,\ldots$	$x^{-\frac{1}{2}\nu}J_\nu(2x^{\frac{1}{2}})-\sum_{m=0}^{n-1}\dfrac{(-1)^m x^m}{m!\,\Gamma(\nu+m+1)}$
(25)	$\dfrac{\Gamma(s+\nu)\,\Gamma(s-\nu)}{\Gamma(s+\frac{1}{2})}$ $\operatorname{Re}s>\lvert\operatorname{Re}\nu\rvert$	$\pi^{-\frac{1}{2}}e^{-\frac{1}{2}x}K_\nu(\tfrac{1}{2}x)$
(26)	$\dfrac{\Gamma(\frac{1}{2}-s)\,\Gamma(s+\nu)}{\Gamma(1+\nu-s)}$ $-\operatorname{Re}\nu<\operatorname{Re}s<\tfrac{1}{2}$	$\pi^{\frac{1}{2}}e^{-\frac{1}{2}x}I_\nu(\tfrac{1}{2}x)$
(27)	$\dfrac{\Gamma(\frac{1}{2}-s)\,\Gamma(s+\nu)}{\Gamma(1+\nu-s)}$ $n-\tfrac{1}{2}<\operatorname{Re}s<n+\tfrac{1}{2}$ $\operatorname{Re}\nu>\tfrac{1}{2}-n, \quad n=1,2,3,\ldots$	$\pi^{\frac{1}{2}}e^{-\frac{1}{2}x}I_\nu(\tfrac{1}{2}x)$ $-\sum_{m=0}^{n-1}\dfrac{(-1)^m\,\Gamma(\frac{1}{2}+\nu+m)}{m!\,\Gamma(\frac{1}{2}+\nu-m)}x^{-\frac{1}{2}-m}$
(28)	$\dfrac{\Gamma(\frac{1}{2}-s)\,\Gamma(s+\nu)}{\Gamma(1+\nu-s)}$ $-n<\operatorname{Re}(s+\nu)<1-n$ $\operatorname{Re}\nu>\tfrac{1}{2}-n, \quad n=1,2,3,\ldots$	$\pi^{\frac{1}{2}}e^{-\frac{1}{2}x}I_\nu(\tfrac{1}{2}x)$ $-\sum_{m=0}^{n-1}\dfrac{(-1)^m\,\Gamma(\frac{1}{2}+\nu+m)}{m!\,\Gamma(1+2\nu+m)}x^{\nu+m}$

Empty sums are interpreted as zero.

Gamma function etc. (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(29)	$[\Gamma(s)/\Gamma(\frac{1}{2}-s)]^2$ $0<\mathrm{Re}\,s<9/8$	$2\pi^{-1}K_0(4x^{1/4})-Y_0(4x^{1/4})$
(30)	$\dfrac{\Gamma(2s)\,\Gamma(s+\nu)}{\Gamma(1-s+\nu)}$ $\mathrm{Re}\,s>-\mathrm{Re}\,\nu,\quad 0$	$2J_{2\nu}(2x^{1/4})\,K_{2\nu}(2x^{1/4})$
(31)	$\dfrac{\Gamma(2s)\,\Gamma(\nu-s)}{\Gamma(\nu+s)}$ $0<\mathrm{Re}\,s<\mathrm{Re}\,\nu$	$\dfrac{[(4+x)^{1/2}+x^{1/2}]^{1-2\nu}}{2^{1-2\nu}(4+x)^{1/2}}$
(32)	$\dfrac{\Gamma(2s)}{\Gamma(s+\mu)\,\Gamma(s+\nu)}$ $\mathrm{Re}\,s>0,\quad \mathrm{Re}(\mu+\nu)>\frac{1}{2}$	$\frac{1}{2}\pi^{-1/2}(4-x)^{\mu/2+\nu/2-3/4}$ $\times P^{3/2-\mu-\nu}_{\mu-\nu-1/2}(\frac{1}{2}x^{1/2})\qquad 0<x<2$ $0\qquad 2<x<\infty$
(33)	$\dfrac{\Gamma(s)}{\Gamma(\frac{1}{2}+\frac{1}{2}s+\frac{1}{2}n)\,\Gamma(\frac{1}{2}+\frac{1}{2}s-\frac{1}{2}n)}$ $\mathrm{Re}\,s>0$	$2\pi^{-1}(4-x^2)^{-1/2}\,T_n(\frac{1}{2}x)\qquad 0<x<2$ $0\qquad 2<x<\infty$
(34)	$\dfrac{\Gamma(\frac{1}{2}-s+\nu)\,\Gamma(\frac{1}{2}+\frac{1}{2}s)}{\Gamma(1-s+2\nu)\,\Gamma(1-\frac{1}{2}s)}$ $-1<\mathrm{Re}\,s<\frac{1}{2}+\mathrm{Re}\,\nu$	$2^{1-\nu}x^{-\nu}\sin x\,J_\nu(x)$
(35)	$\dfrac{\Gamma(\frac{1}{2}s)\,\Gamma(\frac{1}{2}-s+\nu)}{\Gamma(1-s+2\nu)\,\Gamma(\frac{1}{2}-\frac{1}{2}s)}$ $0<\mathrm{Re}\,s<\frac{1}{2}+\mathrm{Re}\,\nu$	$2^{1-\nu}x^{-\nu}\cos x\,J_\nu(x)$

Gamma function etc. (cont'd)

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(36)	$\sin[\pi(s+\nu-\mu-\tfrac{1}{2})] \times \dfrac{\Gamma(s+\mu+\nu)\,\Gamma(s-\mu+\nu)\,\Gamma(1-2s)}{\Gamma(\nu-\mu-s+1)\,\Gamma(\nu+\mu-s+1)}$ $\mathrm{Re}(-\nu \pm \mu) < \mathrm{Re}\, s < \tfrac{1}{2}$	$\pi\, J_{2\nu}(2x^{1/2})\, Y_{2\mu}(2x^{1/2})$
(37)	$\dfrac{\Gamma(s)\,\Gamma(s+\mu)\,\Gamma(s-\mu)}{\Gamma(s+\tfrac{1}{2})}$ $\mathrm{Re}\, s > \lvert\mathrm{Re}\,\mu\rvert$	$2\pi^{-1/2}\,[K_\mu(x^{1/2})]^2$
(38)	$\dfrac{\Gamma(s)\,\Gamma(\tfrac{1}{2}-s-\nu)\,\Gamma(\tfrac{1}{2}-s+\nu)}{\Gamma(\tfrac{1}{2}-s)}$ $0 < \mathrm{Re}\, s < \tfrac{1}{2} - \lvert\mathrm{Re}\,\nu\rvert$	$\pi^{1/2}[\sec(\pi\nu)](1+x)^{-1/2}$ $\times \cosh[2\nu \sinh^{-1}(x^{1/2})]$
(39)	$\dfrac{\Gamma(s+\tfrac{1}{2})\,\Gamma(\tfrac{1}{2}-s-\nu)\,\Gamma(\tfrac{1}{2}-s+\nu)}{\Gamma(1-s)}$ $-\tfrac{1}{2} < \mathrm{Re}\, s < \tfrac{1}{2} - \lvert\mathrm{Re}\,\nu\rvert$	$\pi^{1/2}[\csc(\pi\nu)](1+x)^{-1/2}$ $\times \sinh[2\nu \sinh^{-1}(2x^{1/2})]$
(40)	$\dfrac{\Gamma(2s)\,\Gamma(\nu-s)}{\Gamma(s-\nu+1)}$ $0 < \mathrm{Re}\, s < \mathrm{Re}\,\nu$	$\tfrac{1}{2}\,\Gamma(\nu)\,(4+x)^{-\nu}$ $\times \mathrm{P}_{-2\nu}[(1+4x^{-1})^{-1/2}]$
(41)	$\dfrac{\Gamma(s)\,[\Gamma(\nu-s)]^2}{\Gamma(1-s)}$ $0 < \mathrm{Re}\, s < \mathrm{Re}\,\nu$	$[\Gamma(\nu)]^2\,(1+x)^{-\nu}\, P_{\nu-1}\left(\dfrac{1-x}{1+x}\right)$
(42)	$\dfrac{\Gamma(s)\,\Gamma(\alpha-s)\,\Gamma(\beta-s)}{\Gamma(\gamma-s)}$ $0 < \mathrm{Re}\, s < \min(\mathrm{Re}\,\alpha, \mathrm{Re}\,\beta)$	$\dfrac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\gamma)}\; {}_2F_1(\alpha, \beta; \gamma; -x)$

Gamma function etc. (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(43)	$\dfrac{\prod_{j=1}^{m}\Gamma(b_j+s)\prod_{j=1}^{n}\Gamma(1-a_j-s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j-s)\prod_{j=n+1}^{p}\Gamma(a_j+s)}$ $\max_{1\le j\le m}\operatorname{Re} b_j<\operatorname{Re} s$ $<\min_{1\le j\le n}\operatorname{Re}(1-a_j)$	$G_{pq}^{mn}\left(x\,\middle\vert\,\begin{matrix}a_1,\dots,a_p\\ b_1,\dots,b_q\end{matrix}\right)$
(44)	$s^{-1}[\psi(s+1)+\log\gamma]$ $\quad\operatorname{Re} s>-1$	$-\log(1-x)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$
(45)	$s^{-1}[\psi(s+1)+\log\gamma$ $+\tfrac{1}{2}\pi\operatorname{ctn}(\pi s)]$ $\quad -1<\operatorname{Re} s<0$	$\lvert\log\lvert 1-x\rvert\,\rvert$
(46)	$s^{-1}[\psi(s+1)+\log\gamma$ $+\pi\operatorname{ctn}(\pi s)]$ $\quad\operatorname{Re} s<0$	0 $\quad 0<x<1$ $\log(x-1)$ $\quad 1<x<\infty$
(47)	$\pi\csc(\pi s)\,\psi(1+s)$ $\quad -1<\operatorname{Re} s<0$	$(1+x^{-1})^{-1}\log[\gamma(1+x^{-1})]$
(48)	$\pi\csc(\pi s)\,[\psi(1+s)+\log\gamma]$ $-1<\operatorname{Re} s<1$	$(1+x^{-1})^{-1}\log(1+x^{-1})$
(49)	$\psi(s+\alpha)-\psi(s+\beta)$ $\operatorname{Re} s>-\operatorname{Re}\alpha,\quad -\operatorname{Re}\beta$	$(x^{\beta}-x^{\alpha})/(1-x)$ $\quad 0<x<1$ 0 $\quad 1<x<\infty$
(50)	$\psi(s+\alpha)-\psi(s+\beta)$ $-h<\operatorname{Re}(s+\alpha)<1-h$ $-k<\operatorname{Re}(s+\beta)<1+k$ $h,\ k=0,1,2,\dots$	$(x^{\beta+k}-x^{\alpha+h})/(1-x)$ $0<x<1$ $(x^{\alpha}-x^{\alpha+h}-x^{\beta}+x^{\beta+k})/(1-x)$ $1<x<\infty$

An empty product is to be interpreted as unity.

Gamma function etc. (cont'd)

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(51)	$\Gamma(s)\,\psi(s) \qquad \operatorname{Re} s > 0$	$e^{-x} \log x$
(52)	$B(s, a)\,[\psi(a) - \psi(a+s)]$ $\operatorname{Re} s > -1, \quad \operatorname{Re} a > 0$	$(1-x)^{a-1} \log(1-x) \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$
(53)	$B(-s, \nu + s)\,\psi(\nu + s)$ $-\operatorname{Re} \nu < \operatorname{Re} s < 0$	$[\psi(\nu) - \nu \log(1+x^{-1})]\,(1+x^{-1})^{-\nu}$
(54)	$B(-s, \nu+s)\,[\psi(\nu+s) - \psi(\nu)]$ $-\operatorname{Re} \nu < \operatorname{Re} s < 1$	$-\nu\,(1+x^{-1})^{-\nu} \log(1+x^{-1})$
(55)	$B(s, a)\,[\psi(s) - \psi(s+a)]$ $\operatorname{Re} a > -1, \quad \operatorname{Re} s > 0$	$(1-x)^{a-1} \log x \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$
(56)	$B(s, a)\{[\psi(s) - \psi(s+a)]^2$ $+ \psi'(s) - \psi'(s+a)\}$ $\operatorname{Re} s > 0, \quad \operatorname{Re} a > -2$	$(1-x)^{a-1} (\log x)^2 \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$
(57)	$\dfrac{\Gamma(\frac{1}{2}s + \frac{1}{2}\nu)}{\Gamma(\frac{1}{2}\nu - \frac{1}{2}s + 1)}$ $\times [\psi(\frac{1}{2}s + \frac{1}{2}\nu) + \psi(\frac{1}{2}\nu - \frac{1}{2}s + 1)]$ $-\operatorname{Re} \nu < \operatorname{Re} s < 3/2$	$4 \log x\, J_\nu(x)$
(58)	$\sin(\frac{1}{2}\pi s)\,\Gamma(s)\,\psi(s)$ $0 < \operatorname{Re} s < 1$	$\log x \sin x - \frac{1}{2}\pi \cos x$
(59)	$\cos(\frac{1}{2}\pi s)\,\Gamma(s)\,\psi(s)$ $0 < \operatorname{Re} s < 1$	$\log x \cos x + \frac{1}{2}\pi \sin x$
(60)	$\Gamma(s, a) \qquad a > 0$	$0 \qquad 0 < x < a$ $e^{-x} \qquad a < x < \infty$

Gamma function etc. (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(61)	$\Gamma(s)\,\Gamma(1-s,a)$ $\mathrm{Re}\,s>0,\quad \mathrm{Re}\,a>0$	$(x+1)^{-1}\,e^{-a(x+1)}$
(62)	$\Gamma(1-\nu)\,\Gamma(\nu,as)$ $a>0,\quad \mathrm{Re}\,\nu<1,\quad \mathrm{Re}\,s>0$	$(\log x^{-1})^{-1}(a^{-1}\log x^{-1}-1)^{-\nu}\qquad 0<x<e^{-a}$ $0\qquad e^{-a}<x<\infty$
(63)	$\gamma(s,a)\qquad a>0,\quad \mathrm{Re}\,s>0$	$e^{-x}\qquad 0<x<a$ $0\qquad a<x<\infty$
(64)	$s^{-\nu}\gamma(\nu,as)$ $a>0,\quad \mathrm{Re}\,\nu>0$	$0\qquad 0<x<e^{-a}$ $(-\log x)^{\nu-1}\qquad e^{-a}<x<1$ $0\qquad 1<x<\infty$
(65)	$\pi\csc(\pi s)\,\zeta(s)\qquad 1<\mathrm{Re}\,s<2$	$-x^{-2}[\psi(x^{-1}+1)-\log\gamma]$
(66)	$\pi\csc(\pi s)\,[\zeta(1+s)-s^{-1}]$ $-1<\mathrm{Re}\,s<0$	$\psi(x^{-1}+1)-\log(x^{-1}+1)$
(67)	$\Gamma(s)\,\zeta(s)\qquad \mathrm{Re}\,s>1$	$(e^x-1)^{-1}$
(68)	$\Gamma(s+1)\,\zeta(s)\qquad \mathrm{Re}\,s>1$	$\tfrac14(\sinh\tfrac12 x)^{-2}$
(69)	$\Gamma(\tfrac12 s)\,\zeta(s)\qquad \mathrm{Re}\,s>2$	$\theta_3(0\vert ix^2/\pi)-1$
(70)	$\Gamma(s)\,\zeta(s,a)$ $\mathrm{Re}\,a>0,\quad \mathrm{Re}\,s>1$	$e^{-ax}(1-e^{-x})^{-1}$

7.4. Bessel functions

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(1)	$J_0[a(\beta^2 - s^2)^{1/2}] \qquad a > 0$	$0 \qquad 0 < x < e^{-a}$ $\dfrac{\cos[\beta(a^2 - y^2)^{1/2}]}{\pi(a^2 - y^2)^{1/2}} \qquad e^{-a} < x < e^{a}$ $0 \qquad e^{a} < x < \infty$
(2)	$\dfrac{J_1[a(\beta^2 - s^2)^{1/2}]}{(\beta^2 - s^2)^{1/2}} \qquad a > 0$	$0 \qquad 0 < x < e^{-a}$ $(\pi a \beta)^{-1} \sin[\beta(a^2 - y^2)^{1/2}] \qquad e^{-a} < x < e^{a}$ $0 \qquad e^{a} < x < \infty$
(3)	$(\beta^2 - s^2)^{-\frac{1}{2}\nu} J_\nu[a(\beta^2 - s^2)^{1/2}]$ $\operatorname{Re} \nu > -\tfrac{1}{2}, \quad a > 0$	$0 \qquad 0 < x < e^{-a}$ $\dfrac{(a^2 - y^2)^{\frac{1}{2}\nu - \frac{1}{4}} J_{\nu - \frac{1}{2}}[\beta(a^2 - y^2)^{1/2}]}{(2\pi)^{1/2} a^\nu \beta^{\nu - \frac{1}{2}}} \qquad e^{-a} < x < e^{a}$ $0 \qquad e^{a} < x < \infty$
(4)	$(\beta^2 - s^2)^{\frac{1}{2}\nu} J_\nu[a(\beta^2 - s^2)^{1/2}]$ $a > 0, \quad \operatorname{Re} \nu < \tfrac{1}{2}$ $\operatorname{Re} s > \operatorname{Re} \beta > 0$	$-\dfrac{2^{1/2} \sin(\pi\nu)}{\pi^{3/2} a^{-\nu} \beta^{-\nu - 1/2}}$ $\times (y^2 - a^2)^{-\nu/2 - 1/4}$ $\times K_{\nu + \frac{1}{2}}[\beta(y^2 - a^2)^{1/2}]$ $0 < x < e^{-a} \quad$ and $\quad e^{a} < x < \infty$ $-\dfrac{(a^2 - y^2)^{-\frac{1}{2}\nu - \frac{1}{4}}}{(2\pi)^{1/2} a^{-\nu} \beta^{-\nu - \frac{1}{2}}}$ $\times Y_{\nu + \frac{1}{2}}[\beta(a^2 - y^2)^{1/2}] \qquad e^{-a} < x < e^{a}$
(5)	$\Gamma(\tfrac{1}{2}s)\, J_{\nu + \frac{1}{2}s}(2a^2) \qquad a > 0$ $\operatorname{Re} \nu > -1, \quad \operatorname{Re} s > 0$	$2a^{-\nu}(a^2 - x^2)^{\frac{1}{2}\nu} J_\nu[2a(a^2 - x^2)^{1/2}] \qquad 0 < x < a$ $0 \qquad a < x < \infty$

$y = -\log x$

Bessel functions (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(6)	$\Gamma(\tfrac12 s)\,J_{\nu-\frac12 s}(2a^2)$ $\quad a>0$ $0<\operatorname{Re} s<\operatorname{Re}\nu+3/2$	$2a^{\nu}(x^2+a^2)^{-\frac12\nu}\,J_{\nu}[2a(x^2+a^2)^{\frac12}]$
(7)	$\Gamma(\tfrac12 s)\,Y_{\nu-s}(2a^2)$ $\quad a>0$ $0<\operatorname{Re} s<\operatorname{Re}\nu+3/2$	$2a^{\nu}(x^2+a^2)^{-\frac12\nu}\,Y_{\nu}(2a(x^2+a^2)^{\frac12}]$
(8)	$J_s(a)\sin(\tfrac12\pi s)$ $+\,Y_s(a)\cos(\tfrac12\pi s)$ $a>0,\quad -1<\operatorname{Re} s<1$	$-\pi^{-1}\cos[\tfrac12 a(x+x^{-1})]$
(9)	$J_s(a)\cos(\tfrac12\pi s)$ $-\,Y_s(a)\sin(\tfrac12\pi s)$ $a>0,\quad \lvert\operatorname{Re} s\rvert<1$	$\pi^{-1}\sin[\tfrac12 a(x+x^{-1})]$
(10)	$H_s^{(1)}(a)\,e^{\frac12\pi i s}$ $a>0,\quad -1<\operatorname{Re} s<1$	$-i\pi^{-1}\exp[\tfrac12 ai(x+x^{-1})]$
(11)	$H_s^{(2)}(a)\,e^{-\frac12\pi i s}$ $a>0,\quad -1<\operatorname{Re} s<1$	$i\pi^{-1}\exp[-\tfrac12 ai(x+x^{-1})]$
(12)	$\Gamma(\tfrac12 s)\,H^{(1)}_{\nu-s/2}(2a^2)$ $\operatorname{Re} s>0,\quad 0<\arg a<\tfrac12\pi$	$2a^{\nu}(x^2+a^2)^{-\frac12\nu}\,H_{\nu}^{(1)}[2a(x^2+a^2)^{\frac12}]$
(13)	$\Gamma(\tfrac12 s)\,H^{(2)}_{\nu-s/2}(2a^2)$ $-\tfrac12\pi<\arg a<0,\quad \operatorname{Re} s>0$	$2a^{\nu}(x^2+a^2)^{-\frac12\nu}\,H_{\nu}^{(2)}[2a(x^2+a^2)^{\frac12}]$
(14)	$s^{-1}\,I_0(s)$ $\quad \operatorname{Re} s>0$	1 $\quad 0<x<e^{-1}$ $\pi^{-1}\cos^{-1}(-y)$ $\quad e^{-1}<x<e$ 0 $\quad e<x<\infty$

$y=-\log x$

Bessel functions (cont'd)

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(15)	$I_\nu(s)$ $\operatorname{Re} s > 0$	$-\dfrac{2^\nu \sin(\nu\pi)}{\pi(y^2-1)^{1/2}}[(y-1)^{1/2}+(y+1)^{1/2}]^{-2\nu}$ $0 < x < e^{-1}$ $\dfrac{\cos[\nu \cos^{-1}(-y)]}{\pi(1-y^2)^{1/2}}$ $e^{-1} < x < e$ 0 $e < x < \infty$
(16)	$s^{-1} I_\nu(s)$ $\operatorname{Re} s > 0$	$2^\nu(\pi\nu)^{-1} \sin(\pi\nu)$ $\times[(y-1)^{1/2} + (y+1)^{1/2}]^{-2\nu}$ $0 < x < e^{-1}$ $(\pi\nu)^{-1} \sin[\nu \cos^{-1}(-y)]$ $e^{-1} < x < e$ 0 $e < x < \infty$
(17)	$s^{-\nu} I_\nu(s)$ $\operatorname{Re} \nu > -\frac{1}{2}$	0 $0 < x < e^{-1}$ $\dfrac{(1-y^2)^{\nu-1/2}}{\pi^{1/2}\, 2^\nu \Gamma(\nu+\frac{1}{2})}$ $e^{-1} < x < e$ 0 $e < x < \infty$
(18)	$\dfrac{1}{s^{1/2}} \exp\left(\dfrac{a}{2s}\right) I_\nu\left(\dfrac{a}{2s}\right)$ $\operatorname{Re} \nu > -\frac{1}{2}, \quad \operatorname{Re} s > 0$	$(\pi y)^{-1/2} I_{2\nu}(2a^{1/2} y^{1/2})$ $0 < x < 1$ 0 $1 < x < \infty$
(19)	$\dfrac{1}{s} \exp\left(\dfrac{a+\beta}{2s}\right) I_\nu\left(\dfrac{a-\beta}{2s}\right)$ $\operatorname{Re} \nu > -1, \quad \operatorname{Re} s > 0$	$I_\nu[(a^{1/2}+\beta^{1/2})y^{1/2}]\, I_\nu[(a^{1/2}-\beta^{1/2})y^{1/2}]$ $0 < x < 1$ 0 $1 < x < \infty$

$y = -\log x$

Bessel functions (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(20)	$\dfrac{I_\nu[(s+2\alpha)^{1/2}(s+2\beta)^{1/2}]}{(s+2\alpha)^{1/2\nu}(s+2\beta)^{1/2\nu}}$ $\operatorname{Re}\nu>-\tfrac{1}{2}$	$0 \qquad 0<x<e^{-1}$ $\dfrac{(1-y^2)^{1/2\nu-1/4}\,x^{\alpha+\beta}}{2^{1/2}\pi^{1/2}(\alpha-\beta)^{\nu-1/2}}$ $\times J_\nu[(\alpha-\beta)(1-y^2)^{1/2}]$ $e^{-1}<x<e$ $0 \qquad e<x<\infty$
(21)	$I_\nu\{\tfrac{1}{4}[(s+2\alpha)^{1/2}+(s+2\beta)^{1/2}]^2\}$ $\times I_\nu\{\tfrac{1}{4}[(s+2\alpha)^{1/2}-(s+2\beta)^{1/2}]^2\}$ $\operatorname{Re}\nu>-\tfrac{1}{2}$	$0 \qquad 0<x<e^{-1}$ $x^{\alpha+\beta}\,\dfrac{J_{2\nu}[(\alpha-\beta)(1-y^2)^{1/2}]}{\pi(1-y^2)^{1/2}}$ $e^{-1}<x<e$ $0 \qquad e<x<\infty$
(22)	$s^{1/2}[I_{\nu-1/4}(\tfrac{1}{2}s)\,I_{-\nu-1/4}(\tfrac{1}{2}s)$ $-\,I_{\nu+1/4}(\tfrac{1}{2}as)\,I_{-\nu+1/4}(\tfrac{1}{2}as)]$	$0 \qquad 0<x<e^{-1}$ $(\tfrac{1}{2}\pi)^{-3/2}y^{-1/2}(1-y^2)^{-1/2}$ $\times\cos(2\nu\cos^{-1}y) \qquad e^{-1}<x<1$ $0 \qquad 1<x<\infty$
(23)	$s^{-1}K_0(s) \qquad \operatorname{Re}s>0$	$\cosh^{-1}y \qquad 0<x<e^{-1}$ $0 \qquad e^{-1}<x<\infty$
(24)	$s^{-1}K_1(s) \qquad \operatorname{Re}s>0$	$(y^2-1)^{1/2} \qquad 0<x<e^{-1}$ $0 \qquad e^{-1}<x<\infty$
(25)	$K_\nu(s) \qquad \operatorname{Re}s>0$	$(y^2-1)^{-1/2}\cosh(\nu\cosh^{-1}y)$ $0<x<e^{-1}$ $0 \qquad e^{-1}<x<\infty$

$y=-\log x$

Bessel functions (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(26)	$s^{-1}K_\nu(s)$ $\qquad$ $\operatorname{Re} s>0$	$\nu^{-1}\sinh(\nu\cosh^{-1}y)$ $\quad 0<x<e^{-1}$ 0 $\quad e^{-1}<x<\infty$
(27)	$s^{-\nu}K_\nu(s)$ $\operatorname{Re}\nu>-\tfrac{1}{2},\quad \operatorname{Re} s>0$	$\dfrac{2^{-\nu}\pi^{1/2}}{\Gamma(\nu+\frac{1}{2})}(y^2-1)^{\nu-1/2}$ $\quad 0<x<e^{-1}$ 0 $\quad e^{-1}<x<\infty$
(28)	$s^{1/2}K_{\nu+1/4}(\tfrac{1}{2}s)\,K_{\nu-1/4}(\tfrac{1}{2}s)$	$\dfrac{2^{1/2}\pi^{1/2}}{y^{1/2}(y^2-1)^{1/2}}\cosh(2\nu\cosh^{-1}y)$ $\quad 0<x<e^{-1}$ 0 $\quad e^{-1}<x<\infty$
(29)	$K_0[\beta(a^2-s^2)^{1/2}]$ $-\operatorname{Re} a<\operatorname{Re} s<\operatorname{Re} a,\quad \operatorname{Re}\beta>0$	$\tfrac{1}{2}(\beta^2+y^2)^{-1/2}\exp[-a(\beta^2+y^2)^{1/2}]$
(30)	$(a^2-s^2)^{-1/2}K_1[\beta(a^2-s^2)^{1/2}]$ $-\operatorname{Re} a<\operatorname{Re} s<\operatorname{Re} a$ $\operatorname{Re}\beta>0$	$(2a\beta)^{-1}\exp[-a(\beta^2+y^2)^{1/2}]$
(31)	$(a^2-s^2)^{1/2}K_1[\beta(a^2-s^2)^{1/2}]$ $-\operatorname{Re} a<\operatorname{Re} s<\operatorname{Re} a$ $\operatorname{Re}\beta>0$	$\tfrac{1}{2}\beta(\beta^2+y^2)^{-3/2}[1+a(\beta^2+y^2)^{1/2}]$ $\times\exp[-a(\beta^2+y^2)^{1/2}]$
(32)	$(a^2-s^2)^{-1/2\nu}K_\nu[\beta(a^2-s^2)^{1/2}]$ $-\operatorname{Re} a<\operatorname{Re} s<\operatorname{Re} a$ $\operatorname{Re}\beta>0$	$(2\pi)^{-1/2}a^{1/2-\nu}\beta^{-\nu}(\beta^2+y^2)^{1/2\nu-1/4}$ $\times K_{\nu-1/2}[a(\beta^2+y^2)^{1/2}]$

$y=-\log x$

Bessel functions (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(33)	$\dfrac{K_\nu[(s+2\alpha)^{1/2}(s+2\beta)^{1/2}]}{(s+2\alpha)^{1/2\nu}(s+2\beta)^{1/2\nu}}$ $\operatorname{Re}\nu>-\tfrac12$ $\operatorname{Re}s>-2\operatorname{Re}\alpha,\quad -2\operatorname{Re}\beta$	$(\tfrac12\pi)^{1/2}(\alpha-\beta)^{1/2-\nu}x^{\alpha+\beta}(y^2-1)^{1/2\nu-1/4}$ $\times I_{\nu-1/2}[(\alpha-\beta)(y^2-1)^{1/2}]$ $0<x<e^{-1}$ $0 \qquad e^{-1}<x<\infty$
(34)	$s^{1/2}\, I_n(\tfrac12 s)\, K_{n+1/2}(\tfrac12 s)$	$0 \qquad 0<x<e^{-1}$ $(-1)^n(\tfrac12\pi y)^{-1/2}(1-y^2)^{-1/2}$ $\times\cos[(2n+\tfrac12)\cos^{-1}y]$ $e^{-1}<x<1$ $0 \qquad 1<x<\infty$
(35)	$K_\nu[(\alpha^{1/2}+\beta^{1/2})s^{1/2}]$ $\times I_\nu[(\alpha^{1/2}-\beta^{1/2})s^{1/2}]\quad \operatorname{Re}\alpha>0$ $\operatorname{Re}\beta>0,\quad \operatorname{Re}s>0$	$\dfrac{1}{2y}\exp\left(-\dfrac{\alpha+\beta}{2y}\right)I_\nu\left(\dfrac{\alpha-\beta}{2y}\right)$ $0<x<1$ $0 \qquad 1<x<\infty$
(36)	$K_s(\alpha) \qquad \operatorname{Re}\alpha>0$	$\tfrac12\exp[-\tfrac12\alpha(x+x^{-1})]$
(37)	$\sin(\tfrac12\pi s)\,K_s(a)$ $a>0,\quad \lvert\operatorname{Re}s\rvert<1$	$\tfrac12\sin[\tfrac12 a(x-x^{-1})]$
(38)	$\cos(\tfrac12\pi s)\,K_s(a)$ $a>0,\quad \lvert\operatorname{Re}s\rvert<1$	$\tfrac12\cos[\tfrac12 a(x-x^{-1})]$
(39)	$e^{i\theta s}K_s(\alpha) \qquad -\tfrac12\pi<\theta<\tfrac12\pi$ $\lvert\arg\alpha\rvert<\tfrac12\pi-\lvert\theta\rvert$	$\tfrac12\exp[-\tfrac12\alpha(e^{-i\theta}x+e^{i\theta}x^{-1})]$
(40)	$\Gamma(\tfrac12 s)K_{\nu-s/2}(2\alpha^2)$ $\operatorname{Re}\alpha>0,\quad \operatorname{Re}s>0$	$2\alpha^\nu(x^2+\alpha^2)^{-1/2\nu}K_\nu[2\alpha(x^2+\alpha^2)^{1/2}]$

$y=-\log x$

7.5. Other higher transcendental functions

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(1)	$\exp(\tfrac{1}{4}as^2)\,D_{-\nu}(a^{\frac{1}{2}}s)$ $\operatorname{Re} a>0,\quad \operatorname{Re}\nu>0$	$\dfrac{y^{\nu-1}}{a^{\frac{1}{2}\nu}\Gamma(\nu)}\exp\left(-\dfrac{y^2}{2a}\right)\quad 0<x<1$ $0\quad 1<x<\infty$
(2)	$D_{2\nu}(2s^{\frac{1}{2}})$ $\operatorname{Re}\nu<0,\quad \operatorname{Re}s>0$	$\dfrac{2^{\nu+\frac{1}{2}}(y+1)^{\nu-\frac{1}{2}}}{\Gamma(-\nu)(y-1)^{\nu+1}}\quad 0<x<e^{-1}$ $0\quad e^{-1}<x<\infty$
(3)	$2^{-\nu-\frac{1}{2}}\Gamma(-\nu)\,D_{2\nu}(-2s^{\frac{1}{2}})$ $-\tfrac{1}{2}<\operatorname{Re}\nu<0,\quad \operatorname{Re}s>0$	$-(y+1)^{\nu-\frac{1}{2}}(y-1)^{-\nu-1}$ $0<x<e^{-1}$ $2\cos(\pi\nu)(1+y)^{\nu-\frac{1}{2}}(1-y)^{-\nu-1}$ $e^{-1}<x<e$ $0\quad e<x<\infty$
(4)	$s^{-\frac{1}{2}}D_{2\nu+1}(2s^{\frac{1}{2}})$ $\operatorname{Re}\nu<0,\quad \operatorname{Re}s>0$	$\dfrac{2^{\nu+\frac{1}{2}}(y+1)^{\nu+\frac{1}{2}}}{\Gamma(-\nu)(y-1)^{\nu+1}}\quad 0<x<e^{-1}$ $0\quad e^{-1}<x<\infty$
(5)	$2^{-\nu-\frac{1}{2}}\Gamma(-\nu)\,s^{-\frac{1}{2}}D_{2\nu+1}(-2s^{\frac{1}{2}})$ $-3/2<\operatorname{Re}\nu<0,\quad \operatorname{Re}s>0$	$(y+1)^{\nu+\frac{1}{2}}(y-1)^{-\nu-1}\quad 0<x<e^{-1}$ $-2\cos(\pi\nu)(1+y)^{\nu+\frac{1}{2}}(1-y)^{-\nu-1}$ $e^{-1}<x<e$ $0\quad e<x<\infty$
(6)	$s^{-\frac{1}{2}\nu}\exp(\tfrac{1}{4}as^{-1})\,D_{-\nu}(a^{\frac{1}{2}}s^{-\frac{1}{2}})$ $\operatorname{Re}\nu>0,\quad \operatorname{Re}s>0$	$[\Gamma(\nu)]^{-1}(2y)^{\frac{1}{2}\nu-1}\exp[-(2ay)^{\frac{1}{2}}]$ $0<x<1$ $0\quad 1<x<\infty$
(7)	$\Gamma(s)\,D_{-s}(2a)\quad \operatorname{Re}s>0$	$\exp(-a^2-2ax-\tfrac{1}{2}x^2)$

Higher functions (cont'd)

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(8)	$\sin(\tfrac14 \pi s)\, \Gamma(s)\, D_{-s}(2^{\frac12} a)$ $-\tfrac14 \pi < \arg a < \tfrac14 \pi$ $\operatorname{Re} s > -1$	$e^{-\frac12 a^2 - ax} \sin(ax + \tfrac12 x^2)$
(9)	$\cos(\tfrac14 \pi s)\, \Gamma(s)\, D_{-s}(2^{\frac12} a)$ $-\tfrac14 \pi < \arg a < \tfrac14 \pi, \quad \operatorname{Re} s > 0$	$e^{-\frac12 a^2 - ax} \cos(ax + \tfrac12 x^2)$
(10)	$e^{i\theta s}\, \Gamma(s)\, D_{-s}(2a)$ $-\tfrac14 \pi < \operatorname{Re} \theta < \tfrac14 \pi, \quad \operatorname{Re} s > 0$	$\exp(-a^2 - 2axe^{-i\theta} - \tfrac12 x^2 e^{-2i\theta})$
(11)	${}_2F_1(s, \nu; s+1; -a)$ $\lvert\arg(1+a)\rvert < \pi, \quad \operatorname{Re} s > -1$	$a\nu x(1+ax)^{-\nu-1} \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$
(12)	${}_2F_1(-s, \nu; -s+1; -a)$ $\lvert\arg(1+a)\rvert < \pi, \quad \operatorname{Re} s < 1$	$0 \qquad 0 < x < 1$ $-a\nu x^{-1}(a/x+1)^{-\nu-1} \qquad 1 < x < \infty$
(13)	$\Gamma(s)\, {}_2F_1(s, \alpha; \beta; \lambda)$ $\operatorname{Re} s > 0, \quad \operatorname{Re} \lambda < 1$	$e^{-x}\, {}_1F_1(\alpha; \beta; \lambda x)$
(14)	$\Gamma(2s)\, {}_2F_1(s, s+\tfrac12; \nu+1; -a^2)$ $\lvert\operatorname{Im} a\rvert < 1, \quad \operatorname{Re} s > 0$	$2^{\nu-1}\, \Gamma(\nu+1)\, a^{-\nu} x^{-\frac12 \nu}$ $\times \exp(-x^{\frac12})\, J_\nu(ax^{\frac12})$
(15)	$\mathrm{B}(s, \nu - s)\, {}_2F_1(\mu, s; \nu; 1-a)$ $0 < \operatorname{Re} s < \operatorname{Re} \nu, \quad \lvert\arg a\rvert < \pi$	$(1+x)^{\mu-\nu}(1+ax)^{-\mu}$
(16)	$\mathrm{B}(\nu, s)\, {}_2F_1(\mu, s, \nu+s; -a)$ $\operatorname{Re} s > 0, \quad \operatorname{Re} \nu > 0$ $\lvert\arg(1+a)\rvert < \pi$	$(1-x)^{\nu-1}(1+ax)^{-\mu} \qquad 0 < x < 1$ $0 \qquad 1 < x < \infty$
(17)	$\mathrm{B}(\lambda, s)\, {}_2F_1(\mu, \nu; \lambda+s; a)$ $\operatorname{Re} \lambda > 0, \quad \operatorname{Re} s > 0$ $\lvert\arg(1-a)\rvert < \pi$	$(1-x)^{\lambda-1}\, {}_2F_1[\mu, \nu; \lambda; a(1-x)]$ $0 < x < 1$ $0 \qquad 1 < x < \infty$

Higher functions (cont'd)

	$g(s)=\int_0^\infty f(x)\,x^{s-1}\,dx$	$f(x)$
(18)	$\dfrac{\Gamma(s+\nu)\,\Gamma(s-\nu)}{\Gamma(s+\frac{1}{2})} \times {}_2F_1\left(\dfrac{s+\nu}{2},\dfrac{s+\nu+1}{2};s+\dfrac{1}{2};1-a^2\right)$ $\operatorname{Re} a>-1, \quad \operatorname{Re} s>\lvert\operatorname{Re}\nu\rvert$	$\pi^{-\frac{1}{2}}\,a^{-\nu}\,e^{-\frac{1}{2}x}\,K_\nu(\tfrac{1}{2}ax)$
(19)	$\Gamma(s+\nu+\tfrac{1}{2})\,M_{s,\nu}(a)$ $\operatorname{Re}(s+\nu)>-\tfrac{1}{2}$	$a^{\frac{1}{2}}\,\Gamma(2\nu+1)\,e^{-x+\frac{1}{2}a}$ $\times J_{2\nu}(2a^{\frac{1}{2}}x^{\frac{1}{2}})$
(20)	$\Gamma(s+\nu+\tfrac{1}{2})\,\Gamma(s-\nu+\tfrac{1}{2})\,W_{-s,\nu}(a)$ $\operatorname{Re} s>\lvert\operatorname{Re}\nu\rvert-\tfrac{1}{2}$	$2a^{\frac{1}{2}}\,e^{-x+\frac{1}{2}a}K_{2\nu}(2a^{\frac{1}{2}}x^{\frac{1}{2}})$
(21)	$\Gamma(\tfrac{1}{2}-s+\nu)\,\Gamma(\tfrac{1}{2}+s+\nu)$ $\times M_{s,\nu}(a)\,M_{s,\nu}(\beta)$ $\lvert\operatorname{Re} s+\tfrac{1}{2}\rvert<\lvert\operatorname{Re}\nu\rvert$	$[\Gamma(2\nu+1)]^2\,\dfrac{(a\beta x)^{\frac{1}{2}}}{1+x}$ $\times\exp\left(\dfrac{a+\beta}{2}\,\dfrac{1-x}{1+x}\right)$ $\times J_{2\nu}\left[\dfrac{2(a\beta x)^{\frac{1}{2}}}{1+x}\right]$
(22)	$\Gamma(\tfrac{1}{2}+s)\;{}_1F_1(-s;\tfrac{1}{2};a) \quad \operatorname{Re} s>\tfrac{1}{2}$	$x^{\frac{1}{2}}\,e^{a-x}\cos(2a^{\frac{1}{2}}x^{\frac{1}{2}})$
(23)	$\Gamma(\tfrac{1}{2}+s)\;{}_1F_1(1-s;3/2;a)$ $\operatorname{Re} s>-\tfrac{1}{2}$	$\tfrac{1}{2}a^{-\frac{1}{2}}\,e^{a-x}\sin(2a^{\frac{1}{2}}x^{\frac{1}{2}})$
(24)	$\Gamma(s)\;{}_1F_1(s;\nu+1;a) \qquad \operatorname{Re} s>0$	$\Gamma(\nu+1)\,(ax)^{-\frac{1}{2}\nu}\,e^{-x}$ $\times I_\nu(2a^{\frac{1}{2}}x^{\frac{1}{2}})$
(25)	$\mathrm{B}(\nu,s)\;{}_1F_1(s;s+\nu;a)$ $\operatorname{Re}\nu>0, \quad \operatorname{Re} s>0$	$(1-x)^{\nu-1}\,e^{ax} \qquad 0<x<1$ $0 \qquad 1<x<\infty$

Higher functions (cont'd)

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(26)	$B(\mu, s)\ {}_1F_2(s; s+\mu, \nu+1; a)$ $\operatorname{Re}\mu > 0, \quad \operatorname{Re} s > 0$	$\Gamma(\nu+1)\,(ax)^{-\frac{1}{2}\nu}\,(1-x)^{\mu-1} \times I_\nu(2a^{\frac{1}{2}}x^{\frac{1}{2}}) \quad 0 < x < 1$ $0 \quad 1 < x < \infty$
(27)	$\Gamma(s)\ {}_{p+1}F_q(s, a_1, \ldots, a_p; b_1, \ldots, b_q; a)$ $p < q, \quad \operatorname{Re} s > 0$	$e^{-x}\ {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; ax)$
(28)	$\Gamma(s+\nu)\,\Gamma(s-\nu) \times {}_{p+2}F_q(s+\nu, s-\nu, a_1, \ldots, a_p; b_1, \ldots, b_q; a)$ $p < q-1, \quad \operatorname{Re} s > \lvert\operatorname{Re}\nu\rvert$	$2K_{2\nu}(2x^{\frac{1}{2}}) \times {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; ax)$
(29)	$B(s, \nu)\ {}_{p+1}F_{q+1}(s, a_1, \ldots, a_p; s+\nu, b_1, \ldots, b_q; a)$ $\operatorname{Re}\nu > 0, \quad \operatorname{Re} s > 0$	$(1-x)^{\nu-1} \times {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; ax) \quad 0 < x < 1$ $0 \quad 1 < x < \infty$
(30)	$G^{m,n+1}_{p+1,q}\left(a \,\middle\vert\, \begin{matrix} 1-s, a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right)$ $p+q < 2(m+n)$ $\lvert\arg a\rvert < (m+n-\frac{1}{2}p-\frac{1}{2}q)\pi$ $\operatorname{Re}(s+b_j) > 0, \quad j = 1, \ldots, m$	$e^{-x}\, G^{m,n}_{p,q}\left(ax \,\middle\vert\, \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right)$
(31)	$G^{m,n+1}_{p+2,q}\left(a \,\middle\vert\, \begin{matrix} 1-s-\nu, a_1, \ldots, a_p, 1-s+\nu \\ b_1, \ldots, b_q \end{matrix}\right)$ $p+q < 2m+2n$ $\lvert\arg a\rvert < (m+n-\frac{1}{2}p-\frac{1}{2}q)\pi$ $\operatorname{Re}(s+\nu+b_j) > 0, \quad j = 1, \ldots, m$ $\operatorname{Re}(s+a_j) < 5/4, \quad j = 1, \ldots, n$	$J_{2\nu}(2x^{\frac{1}{2}})\, G^{m,n}_{p,q}\left(ax \,\middle\vert\, \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right)$

Higher functions (cont'd)

	$g(s) = \int_0^\infty f(x)\, x^{s-1}\, dx$	$f(x)$
(32)	$G^{m,n+2}_{p+2,q}\left(a \middle\vert \begin{matrix} 1-s+\nu, 1-s-\nu, a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right)$ $p + q < 2m + 2n$ $\vert\arg a\vert < (m + n - \tfrac{1}{2}p - \tfrac{1}{2}q)\pi$ $\mathrm{Re}(s + b_j) > \vert\mathrm{Re}\,\nu\vert, \quad j = 1, \ldots, m$	$K_{2\nu}(2x^{1/2})\, G^{m,n}_{p,q}\left(ax \middle\vert \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right)$

APPENDIX

NOTATIONS AND DEFINITIONS OF HIGHER TRANSCENDENTAL FUNCTIONS

H.T.F. I refers to volume I, and H.T.F. II to volume II, of *Higher transcendental functions* by the same authors as the present work.

Miscellaneous notations

Ad hoc notations are explained where they occur. Notations occurring several times on a page are explained at the bottom of the page.

In general, real variables and parameters are denoted by Latin letters, and complex variables and parameters by Greek letters. Exceptions are made to preserve traditional notations (such as p in chapters IV and V). The letters m, n denote integers mostly.

Re z, Im z. Real and imaginary parts of a complex quantity z.

$|z|$, arg z. Modulus and argument (phase) of a complex quantity.

Cauchy Principal Value. If the integrand has a singularity at c, $a < c < b$, the Cauchy Principal Value of

$$\int_a^b f(x)\,dx$$

is

$$\lim\left[\int_a^{c-\epsilon} f(x)\,dx + \int_{c-\epsilon}^{b} f(x)\,dx\right] \qquad \epsilon > 0, \quad \epsilon \to 0.$$

Empty sums are to be interpreted as zero, and empty products as unity. $\sum_{n=a}^{b}$, $\prod_{n=a}^{b}$ are empty if $b < a$.

$[x]$ largest integer $\leq x$.

$$(a)_\nu = \Gamma(a+\nu)/\Gamma(a)$$

$$(a)_0 = 1$$

$$(a)_n = a(a+1)\cdots(a+n-1) \qquad n = 1, 2, \ldots$$

$$(a)_n = (-1)^n (1-a-n)_n \qquad n \text{ integer}$$

$$(a)_{-n} = (-1)^n/(1-a)_n \qquad n \text{ integer}$$

Binomial coefficient

$$\binom{a}{\beta} = \frac{\Gamma(a+1)}{\Gamma(\beta+1)\,\Gamma(a-\beta+1)}.$$

$$\operatorname{sgn} x = \begin{cases} -1 & x<0 \\ 0 & x=0 \\ 1 & x>0 \end{cases}$$

Euler-Mascheroni constant.

$$C = \lim_{m\to\infty} \left(\sum_{n=1}^{m} 1/n - \log m\right) = 0.5772156649 \ldots$$

$$\gamma = e^C.$$

Orthogonal polynomials

See also H.F.T. II Chapter X.

Legendre polynomial

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n.$$

Gegenbauer polynomial

$$C_n^\nu(x) = \frac{(-2)^n (\nu)_n}{n!(n+2\nu)_n} (1-x^2)^{\frac{1}{2}-\nu} \frac{d^n}{dx^n} (1-x^2)^{n+\nu-\frac{1}{2}}.$$

Tchebichef polynomials

$$T_n(x) = \cos(n \cos^{-1} x)$$

$$U_n(x) = \frac{\sin[(n+1)\cos^{-1} x]}{\sin(\cos^{-1} x)}.$$

Jacobi polynomial

$$P_n^{(\alpha,\beta)}(x) = \frac{(-1)^n}{2^n n!} (1-x)^{-\alpha} (1+x)^{-\beta} \frac{d^n}{dx^n} [(1-x)^{n+\alpha} (1+x)^{n+\beta}].$$

Laguerre polynomial

$$L_n^{\alpha}(z) = \frac{e^z z^{-\alpha}}{n!} \frac{d^n}{dz^n}(e^{-z} z^{n+\alpha})$$

$$L_n^0(z) = L_n(z).$$

Hermite polynomials

$$\mathrm{He}_n(x) = (-1)^n e^{\frac{1}{2}x^2} \frac{d^n}{dx^n}(e^{-\frac{1}{2}x^2})$$

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}.$$

Charlier polynomial

$$p_n(x;\, a) = n!\, a^{-n} L_n^{x-n}(a).$$

The gamma function and related functions

See also H.T.F. I Chapter I.

Gamma function

$$\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1}\, dt \qquad \mathrm{Re}\ z > 0.$$

Logarithmic derivative of the gamma function

$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}, \qquad \psi'(z) = \frac{d\psi}{dz}, \qquad \text{etc.}$$

Beta function

$$\mathrm{B}(x,\, y) = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}.$$

Euler's dilogarithm

$$L_2(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^2} = -\int_0^z \frac{\log(1-z)}{z}\, dz.$$

Incomplete gamma functions. See under Confluent hypergeometric functions.

Incomplete beta function. See under Hypergeometric functions.

Riemann's zeta function and related functions

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$$

$$\xi(t) = -\tfrac{1}{2}(t^2 + \tfrac{1}{4})\, \pi^{-\frac{1}{2}it-\frac{1}{4}}\, \Gamma(\tfrac{1}{2}it + \tfrac{1}{4})\, \zeta(it + \tfrac{1}{2})$$

$$\zeta(z, a) = \sum_{n=0}^{\infty} (n + a)^{-z}, \quad \Phi(z, s, v) = \sum_{n=0}^{\infty} \frac{z^n}{(v + n)^s} .$$

Legendre functions

See also H.T.F. I Chapter III.

$$P_\nu^\mu(z) = \frac{1}{\Gamma(1-\mu)} \left(\frac{z+1}{z-1}\right)^{\frac{1}{2}\mu} {}_2F_1(-\nu, \nu + 1; 1 - \mu; \tfrac{1}{2} - \tfrac{1}{2}z)$$

$$Q_\nu^\mu(z) = \frac{e^{\mu\pi i}\pi^{\frac{1}{2}}\, \Gamma(\mu + \nu + 1)}{2^{\nu+1}\, \Gamma(\nu + 3/2)}\, z^{-\mu-\nu-1}\, (z^2 - 1)^{\frac{1}{2}\mu}$$

$$\times\, {}_2F_1\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2}; \nu + \frac{3}{2}; \frac{1}{z^2}\right)$$

z in the complex plane cut along the real axis from -1 to 1.

$$\mathrm{P}_\nu^\mu(x) = \frac{1}{\Gamma(1-\mu)} \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}\mu} {}_2F_1(-\nu, \nu + 1; 1 - \mu; \tfrac{1}{2} - \tfrac{1}{2}x)$$

$$-1 < x < 1$$

$$\mathrm{Q}_\nu^\mu(x) = \tfrac{1}{2} e^{-i\mu\pi} [e^{-\frac{1}{2}\mu\pi i} Q_\nu^\mu(x + i0) + e^{\frac{1}{2}\mu\pi i} Q_\nu^\mu(x - i0)]$$

$$-1 < x < 1$$

$$P_\nu(z) = P_\nu^0(z), \quad Q_\nu(z) = Q_\nu^0(z).$$

Bessel functions and related functions

See also H.T.F. II Chapter VII.

Bessel functions

$$J_\nu(z) = \sum_{m=0}^{\infty} \frac{(-1)^m\, (\tfrac{1}{2}z)^{\nu+2m}}{m!\, \Gamma(\nu + m + 1)}$$

$$Y_\nu(z) = \operatorname{cosec} \nu\pi\, [J_\nu(z) \cos \nu\pi - J_{-\nu}(z)]$$

$$H_\nu^{(1)}(z) = J_\nu(z) + i\,Y_\nu(z)$$

$$H_\nu^{(2)}(z) = J_\nu(z) - i\,Y_\nu(z)$$

$$Ji_\nu(x) = \int_\infty^x J_\nu(t)\,\frac{dt}{t}\,.$$

Modified Bessel functions

$$I_\nu(z) = \sum_{m=0}^{\infty} \frac{(\tfrac{1}{2}z)^{\nu+2m}}{m!\,\Gamma(\nu+m+1)}$$

$$K_\nu(z) = \frac{\pi}{2}\,\frac{I_{-\nu}(z) - I_\nu(z)}{\sin \nu\pi}\,.$$

Kelvin's and related functions

$$\mathrm{ber}_\nu(z) + i\,\mathrm{bei}_\nu(z) = J_\nu(z e^{\frac{3}{4}\pi i})$$

$$\mathrm{ber}_\nu(z) - i\,\mathrm{bei}_\nu(z) = J_\nu(z e^{-\frac{3}{4}\pi i})$$

$$\mathrm{ker}_\nu(z) + i\,\mathrm{kei}_\nu(z) = K_\nu(z e^{\frac{1}{4}\pi i})$$

$$\mathrm{ker}_\nu(z) - i\,\mathrm{kei}_\nu(z) = K_\nu(z e^{-\frac{1}{4}\pi i})$$

$$\mathrm{ber}(z) = \mathrm{ber}_0(z), \qquad \mathrm{bei}(z) = \mathrm{bei}_0(z),$$

$$\mathrm{ker}(z) = \mathrm{ker}_0(z), \qquad \mathrm{kei}(z) = \mathrm{kei}_0(z).$$

Note that the definition of $\mathrm{ker}_\nu(z)$ and $\mathrm{kei}_\nu(z)$ differs from that given in H.T.F. II sec. 7.2.3.

$$X_\nu^{(b)}(z) = \mathrm{ber}_\nu^2(z) + \mathrm{bei}_\nu^2(z)$$

$$V_\nu^{(b)}(z) = [\mathrm{ber}_\nu'(z)]^2 + [\mathrm{bei}_\nu'(z)]^2$$

$$W_\nu^{(b)}(z) = \mathrm{ber}_\nu(z)\,\mathrm{bei}_\nu'(z) - \mathrm{bei}_\nu(z)\,\mathrm{ber}_\nu'(z)$$

$$\tfrac{1}{2}Z_\nu^{(b)}(z) = \mathrm{ber}_\nu(z)\,\mathrm{bei}_\nu'(z) + \mathrm{bei}_\nu(z)\,\mathrm{ber}_\nu'(z).$$

Neumann polynomials

$$O_0(x) = \frac{1}{x}; \qquad O_n(x) = \frac{1}{4}\sum_{m=0}^{\le \frac{1}{2}n} \frac{n\,(n-m-1)!}{m!\,(\tfrac{1}{2}x)^{n-2m+1}} \qquad n = 1, 2, \ldots$$

$$O_{-n}(x) = (-1)^n\,O_n(x). \qquad n = 1, 2, \ldots$$

Anger-Weber functions

$$\mathbf{J}_\nu(z) = \pi^{-1} \int_0^\pi \cos(\nu\theta - z \sin\theta)\, d\theta$$

$$\mathbf{E}_\nu(z) = \pi^{-1} \int_0^\pi \sin(\nu\theta - z \sin\theta)\, d\theta.$$

Struve's functions

$$\mathbf{H}_\nu(z) = \sum_{m=0}^{\infty} \frac{(-1)^m (z/2)^{\nu+2m+1}}{\Gamma(m+3/2)\,\Gamma(\nu+m+3/2)}$$

$$= \frac{(z/2)^{\nu+1}}{\Gamma(3/2)\,\Gamma(\nu+3/2)}\, {}_1F_2(1;\, 3/2,\, \nu+3/2;\, -z^2/4)$$

$$= 2^{1-\nu}\, \pi^{-1/2}\, [\Gamma(\nu+\tfrac{1}{2})]^{-1}\, s_{\nu,\nu}(z).$$

$$\mathbf{L}_\nu(z) = e^{-\frac{1}{2}(\nu+1)\pi i}\, \mathbf{H}_\nu(z e^{\frac{1}{2} i\pi}).$$

Lommel's functions

$$s_{\mu,\nu}(z) = \frac{z^{\mu+1}}{(\mu-\nu+1)(\mu+\nu+1)}\, {}_1F_2\left(1;\, \frac{\mu-\nu+3}{2},\, \frac{\mu+\nu+3}{2};\, -\frac{z^2}{4}\right)$$

$$S_{\mu,\nu}(z) = s_{\mu,\nu}(z) + 2^{\mu-1}\, \Gamma\left(\frac{\mu-\nu+1}{2}\right) \Gamma\left(\frac{\mu+\nu+1}{2}\right).$$

$$\times \left[\sin\left(\frac{\mu-\nu}{2}\pi\right) J_\nu(z) - \cos\left(\frac{\mu-\nu}{2}\pi\right) Y_\nu(z)\right].$$

Lommel's functions of two variables

$$U_\nu(w, z) = \sum_{m=0}^{\infty} (-1)^m \left(\frac{w}{z}\right)^{\nu+2m} J_{\nu+2m}(z)$$

$$V_\nu(w, z) = \cos\left(\frac{w}{2} + \frac{z^2}{2w} + \frac{\nu\pi}{2}\right) + U_{2-\nu}(w, z).$$

Hypergeometric functions

See also H.T.F. I Chapters II, IV, V.

Generalized hypergeometric series

$$ {}_mF_n(a_1, \dots, a_m; \gamma_1, \dots, \gamma_n; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k \cdots (a_m)_k}{(\gamma_1)_k \cdots (\gamma_n)_k} \frac{z^k}{k!}. $$

${}_2F_1(a, b; c; z)$ is Gauss' hypergeometric series and is often (for instance in H.T.F. I Chapter II) denoted by $F(a, b; c; z)$.

${}_1F_1(a; c; z)$ is Kummer's confluent hypergeometric series and is sometimes (for instance in H.T.F. I Chapter VI) denoted by $\Phi(a; c; z)$.

${}_mF_n(a_1, \dots, a_m; \gamma_1, \dots, \gamma_n; z)$ is sometimes written as

$$ {}_mF_n\left[\begin{matrix} a_1, \dots, a_m; z \\ \gamma_1, \dots, \gamma_n \end{matrix}\right]. $$

Incomplete beta function

$$ B_x(p, q) = \int_0^x t^{p-1}(1-t)^{q-1}\,dt = p^{-1}x^p\, {}_2F_1(p, 1-q; p+1; x). $$

$$ S_n(b_1, b_2, b_3, b_4; z) = \sum_{h=1}^{n} \frac{\prod_{j=1}^{n}{}' \Gamma(b_j - b_h)}{\prod_{j=n+1}^{4} \Gamma(1 + b_h - b_j)} z^{1+2b_h} $$

$$ \times\, {}_0F_3(1 + b_h - b_1, \dots, *, \dots, 1 + b_h - b_4; (-1)^n z^2) $$

The prime in Π' and the asterisk in ${}_0F_3$ mean that the term containing $b_h - b_h$ is to be omitted. For $n = 1$ the product Π in the numerator, for $n = 4$ that in the denominator is to be replaced by unity.

MacRobert's E-function.

If $p \geq q + 1$,

$$ E(p; a_r: q; \rho_s: x) = \sum_{r=1}^{p} \frac{\prod_{s=1}^{p}{}' \Gamma(a_s - a_r)}{\prod_{t=1}^{q} \Gamma(\rho_t - a_r)} \Gamma(a_r)\, x^{a_r} $$

$$ \times\, {}_{q+1}F_{p-1}(a_r, a_r - \rho_1 + 1, \dots, a_r - \rho_q + 1; a_r - a_1 + 1, \dots, *, \dots, a_r - a_p + 1; (-1)^{p+q} x) $$

where $|x| < 1$ when $p = q + 1$.

If $p \leq q + 1$,

$$E(p;\, a_r:\, q;\, \rho_s:\, x) = \frac{\prod_{r=1}^{p} \Gamma(a_r)}{\prod_{s=1}^{q} \Gamma(\rho_s)}\, {}_pF_q(a_1, \ldots, a_p;\, \rho_1, \ldots, \rho_q;\, -1/x)$$

where $x \neq 0$ and $|x| > 1$ if $p = q + 1$. If $p > q + 1$, the last relation gives the asymptotic expansion of the E-function for large x.

Meijer's G-function

$$G^{m,n}_{p,q}\left(x \,\middle|\, \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right)$$

$$= \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^{m} \Gamma(b_j - s) \prod_{j=1}^{n} \Gamma(1 - a_j + s)}{\prod_{j=m+1}^{q} \Gamma(1 - b_j + s) \prod_{j=n+1}^{p} \Gamma(a_j - s)}\, x^s ds$$

where L is a path separating the poles of $\Gamma(b_1 - s) \cdots \Gamma(b_m - s)$ from those of $\Gamma(1 - a_1 + s) \cdots \Gamma(1 - a_n + s)$. For a more detailed definition see H.T.F. I sec. 5.3.

Formulas involving the G-function may be used as key formulas from which many integrals with Bessel functions, Legendre functions, and other higher transcendental functions follow by specializing parameters. The following two lists give expressions of certain special G-functions in terms of well-known higher transcendental functions, and, conversely, expressions for higher transcendental functions in terms of G-functions. The list is not complete. See also H.T.F. I sec. 5.6.

Particular cases of the G-function

$$G^{10}_{02}(x \,|\, a, b) = x^{\frac{1}{2}(a+b)} J_{a-b}(2x^{\frac{1}{2}})$$

$$G^{20}_{02}(x \,|\, a, b) = 2x^{\frac{1}{2}(a+b)} K_{a-b}(2x^{\frac{1}{2}})$$

$$G^{11}_{12}\left(x \,\middle|\, \begin{matrix} \frac{1}{2} \\ b, -b \end{matrix}\right) = \pi^{\frac{1}{2}} e^{-\frac{1}{2}x} I_b(\tfrac{1}{2}x)$$

$$G_{12}^{11}\left(x\,\middle|\,\begin{matrix}a\\ b,\ c\end{matrix}\right)=\frac{\Gamma(1-a+b)}{\Gamma(1+b-c)}\,x^{b}\,{}_1F_1(1-a+b;\,1+b-c;\,-x)$$

$$G_{12}^{20}\left(x\,\middle|\,\begin{matrix}\tfrac12\\ b,\ -b\end{matrix}\right)=\pi^{-\frac12}\,e^{-\frac12 x}\,K_b(\tfrac12 x)$$

$$G_{12}^{20}\left(x\,\middle|\,\begin{matrix}a\\ b,\ c\end{matrix}\right)=x^{\frac12(b+c-1)}\,e^{-\frac12 x}\,W_{k,m}(x)$$

$$k=\tfrac12(1+b+c)-a,\qquad m=\tfrac12 b-\tfrac12 c$$

$$G_{12}^{21}\left(x\,\middle|\,\begin{matrix}\tfrac12\\ b,\ -b\end{matrix}\right)=\frac{\pi^{\frac12}}{\cos b\pi}\,e^{\frac12 x}\,K_b(\tfrac12 x)$$

$$G_{12}^{21}\left(x\,\middle|\,\begin{matrix}a\\ b,\ c\end{matrix}\right)=\Gamma(b-a+1)\,\Gamma(c-a+1)\,x^{\frac12(b+c-1)}\,e^{\frac12 x}\,W_{k,m}(x)$$

$$k=a-\tfrac12(b+c+1),\qquad m=\tfrac12 b-\tfrac12 c$$

$$G_{04}^{10}(x\,|\,a,\ b,\ 2b-a,\ b+\tfrac12)=\pi^{-\frac12}x^{b}\,I_{2(a-b)}(2^{3/2}x^{1/4})\,J_{2(a-b)}(2^{3/2}x^{1/4})$$

$$\begin{aligned}G_{04}^{10}(x\,|\,a,\ a+\tfrac12,\ b,\ 2a-b)&=\tfrac12\pi^{-\frac12}\sec(b-a)\pi\\&\times x^{a}[J_{2(a-b)}(2^{3/2}x^{1/4})\,I_{2(b-a)}(2^{3/2}x^{1/4})\\&+I_{2(a-b)}(2^{3/2}x^{1/4})\,J_{2(b-a)}(2^{3/2}x^{1/4})]\end{aligned}$$

$$\begin{aligned}G_{04}^{10}(x\,|\,a+\tfrac12,\ a,\ b,\ 2a-b)&=\tfrac12\pi^{-\frac12}[\sin(a-b)\pi]^{-1}\\&\times x^{a}[J_{2(a-b)}(2^{3/2}x^{1/4})\,I_{2(b-a)}(2^{3/2}x^{1/4})\\&-I_{2(a-b)}(2^{3/2}x^{1/4})\,J_{2(b-a)}(2^{3/2}x^{1/4})]\end{aligned}$$

$$G_{04}^{20}(x\,|\,a,\ a+\tfrac12,\ b,\ b+\tfrac12)=x^{\frac12(a+b)}\,J_{2(a-b)}(4x^{1/4})$$

$$\begin{aligned}G_{04}^{20}(x\,|\,a,\ -a,\ 0,\ \tfrac12)&=-\pi^{\frac12}(\sin 2a\pi)^{-1}[J_{2a}(ze^{\pi i/4})\,J_{2a}(ze^{-\pi i/4})\\&-J_{-2a}(ze^{\pi i/4})\,J_{-2a}(ze^{-\pi i/4})]\qquad z=2^{3/2}x^{1/4}\end{aligned}$$

$$G_{04}^{20}(x|0, \tfrac{1}{2}, a, -a) = \pi^{\frac{1}{2}} i^{-1} (\sin 2a\pi)^{-1}$$
$$\times [e^{2a\pi i} J_{2a}(ze^{-\pi i/4}) J_{-2a}(ze^{\pi i/4}) - e^{-2a\pi i} J_{2a}(ze^{\pi i/4})$$
$$\times J_{-2a}(ze^{-\pi i/4})] \qquad z = 2^{3/2} x^{1/4}$$

$$G_{04}^{30}(x|3a - \tfrac{1}{2}, a, -a - \tfrac{1}{2}, a - \tfrac{1}{2}) = 2\pi^{\frac{1}{2}} (\cos 2a\pi)^{-1}$$
$$\times x^{a-1/2} K_{4a}(2^{3/2} x^{1/4}) [J_{4a}(2^{3/2} x^{1/4}) + J_{-4a}(2^{3/2} x^{1/4})]$$

$$G_{04}^{30}(x|0, a - \tfrac{1}{2}, -a - \tfrac{1}{2}, -\tfrac{1}{2}) = 4\pi^{\frac{1}{2}} x^{-\frac{1}{2}}$$
$$\times K_{2a}(2^{3/2} x^{1/4}) [J_{2a}(2^{3/2} x^{1/4}) \cos a\pi - Y_{2a}(2^{3/2} x^{1/4}) \sin a\pi]$$

$$G_{04}^{30}(x|-\tfrac{1}{2}, a - \tfrac{1}{2}, -a - \tfrac{1}{2}, 0) = -4\pi^{\frac{1}{2}} x^{-\frac{1}{2}}$$
$$\times K_{2a}(2^{3/2} x^{1/4}) [J_{2a}(2^{3/2} x^{1/4}) \sin a\pi + Y_{2a}(2^{3/2} x^{1/4}) \cos a\pi]$$

$$G_{04}^{30}(x|a, b + \tfrac{1}{2}, b, 2b - a) = \pi^{\frac{1}{2}} 2^{\frac{1}{2}} x^{b} K_{2(a-b)}(2^{3/2} x^{1/4})$$
$$\times J_{2(a-b)}(2^{3/2} x^{1/4})$$

$$G_{04}^{40}(x|a, a + \tfrac{1}{2}, b, b + \tfrac{1}{2}) = 4\pi x^{\frac{1}{2}(a+b)} K_{2(a-b)}(4x^{\frac{1}{4}})$$

$$G_{04}^{40}(x|a, a + \tfrac{1}{2}, b, 2a - b) = 2^{3} \pi^{\frac{1}{2}} x^{a}$$
$$\times K_{2(b-a)}(2^{3/2} x^{1/4} e^{\pi i/4}) K_{2(b-a)}(2^{3/2} x^{1/4} e^{-\pi i/4})$$

$$G_{04}^{n0}(x|a, b, c, d) = x^{-\frac{1}{2}} S_{n}(a, b, c, d; x^{\frac{1}{2}}) \qquad n = 1, 2, 3, 4$$

$$G_{13}^{11}\left(x \,\middle|\, \begin{matrix} \tfrac{1}{2} \\ a, 0, -a \end{matrix}\right) = \pi^{\frac{1}{2}} J_{a}^{2}(x^{\frac{1}{2}})$$

$$G_{13}^{11}\left(x \,\middle|\, \begin{matrix} \tfrac{1}{2} \\ 0, a, -a \end{matrix}\right) = \pi^{\frac{1}{2}} J_{a}(x^{\frac{1}{2}}) J_{-a}(x^{\frac{1}{2}})$$

$$G_{13}^{11}\left(x \,\middle|\, \begin{matrix} a \\ a, b, a - \tfrac{1}{2} \end{matrix}\right) = x^{\frac{1}{2}a + \frac{1}{2}b - \frac{1}{4}} \mathbf{H}_{a-b-\frac{1}{2}}(2x^{\frac{1}{2}})$$

$$G_{13}^{20}\left(x \,\middle|\, \begin{matrix} a - \tfrac{1}{2} \\ a, b, a - \tfrac{1}{2} \end{matrix}\right) = x^{\frac{1}{2}(a+b)} Y_{b-a}(2x^{\frac{1}{2}})$$

$$G_{13}^{20}\left(x\,\middle|\,\begin{matrix}a+\tfrac{1}{2}\\ b,\ a,\ 2a-b\end{matrix}\right)=-\pi^{\frac{1}{2}}\,x^{a}\,J_{b-a}(x^{\frac{1}{2}})\,Y_{b-a}(x^{\frac{1}{2}})$$

$$G_{13}^{20}\left(x\,\middle|\,\begin{matrix}\tfrac{1}{2}\\ a,\ -a,\ 0\end{matrix}\right)=\pi^{\frac{1}{2}}\,2^{-1}\,(\sin a\pi)^{-1}\,[J_{-a}^{2}(x^{\frac{1}{2}})-J_{a}^{2}(x^{\frac{1}{2}})]$$

$$G_{13}^{21}\left(x\,\middle|\,\begin{matrix}\tfrac{1}{2}\\ a,\ 0,\ -a\end{matrix}\right)=2\pi^{\frac{1}{2}}\,I_{a}(x^{\frac{1}{2}})\,K_{a}(x^{\frac{1}{2}})$$

$$G_{13}^{21}\left(x\,\middle|\,\begin{matrix}\tfrac{1}{2}\\ a,\ -a,\ 0\end{matrix}\right)=\pi^{3/2}\,(\sin 2a\pi)^{-1}\,[I_{-a}^{2}(x^{\frac{1}{2}})-I_{a}^{2}(x^{\frac{1}{2}})]$$

$$G_{13}^{21}\left(x\,\middle|\,\begin{matrix}a+\tfrac{1}{2}\\ a+\tfrac{1}{2},\ b,\ a\end{matrix}\right)=\frac{\pi x^{\frac{1}{2}(a+b)}}{\cos(a-b)\pi}\,[I_{b-a}(2x^{\frac{1}{2}})-\mathbf{L}_{a-b}(2x^{\frac{1}{2}})]$$

$$G_{13}^{21}\left(x\,\middle|\,\begin{matrix}a+\tfrac{1}{2}\\ a,\ a+\tfrac{1}{2},\ b\end{matrix}\right)=\pi x^{\frac{1}{2}(a+b)}\,[I_{a-b}(2x^{\frac{1}{2}})-\mathbf{L}_{a-b}(2x^{\frac{1}{2}})]$$

$$G_{13}^{30}\left(x\,\middle|\,\begin{matrix}a+\tfrac{1}{2}\\ a+b,\ a-b,\ a\end{matrix}\right)=2\pi^{-\frac{1}{2}}\,x^{a}\,K_{b}^{2}(x^{\frac{1}{2}})$$

$$G_{13}^{31}\left(x\,\middle|\,\begin{matrix}a+\tfrac{1}{2}\\ a+\tfrac{1}{2},\ -a,\ a\end{matrix}\right)=\frac{\pi^{2}}{\cos 2a\pi}\,[\mathbf{H}_{2a}(2x^{\frac{1}{2}})-Y_{2a}(2x^{\frac{1}{2}})]$$

$$G_{13}^{31}\left(x\,\middle|\,\begin{matrix}a\\ a,\ b,\ -b\end{matrix}\right)=2^{-2a+2}\,\Gamma(1-a-b)\,\Gamma(1-a+b)\,S_{2a-1,\,2b}(2x^{\frac{1}{2}})$$

$$G_{13}^{31}\left(x\,\middle|\,\begin{matrix}a+\tfrac{1}{2}\\ b,\ 2a-b,\ a\end{matrix}\right)=\pi^{5/2}\,2^{-1}\,[\cos(b-a)\pi]^{-1}\times x^{a}\,H_{b-a}^{(1)}(x^{\frac{1}{2}})\,H_{b-a}^{(2)}(x^{\frac{1}{2}})$$

$$G_{22}^{12}\left(x\,\middle|\,\begin{matrix}-c_{1},\,-c_{2}\\ a-1,\,-b\end{matrix}\right)=\frac{\Gamma(a+c_{1})\,\Gamma(a+c_{2})}{\Gamma(a+b)}\times x^{a-1}\,{}_{2}F_{1}(a+c_{1},\,a+c_{2};\,a+b;\,-x)$$

$$G_{24}^{12}\left(x\,\middle|\,\begin{matrix}a+\tfrac{1}{2},\ a\\ b+a,\ a-c,\ a+c,\ a-b\end{matrix}\right)=\pi^{\frac{1}{2}}\,x^{a}\,J_{b+c}(x^{\frac{1}{2}})\,J_{b-c}(x^{\frac{1}{2}})$$

$$G_{24}^{22}\left(x\,\middle|\,\begin{matrix}a,\ a+\tfrac{1}{2}\\ b,\ c,\ 2a-c,\ 2a-b\end{matrix}\right)=2\pi^{1/2}\,x^{a}\,I_{b+c-2a}(x^{1/2})\,K_{b-c}(x^{1/2})$$

$$G_{24}^{30}\left(x\,\middle|\,\begin{matrix}0,\ \tfrac{1}{2}\\ a,\ b,\ -b,\ -a\end{matrix}\right)=i\,2^{-2}\,\pi^{1/2}$$
$$\times\left[H_{a-b}^{(1)}(x^{1/2})\,H_{a+b}^{(1)}(x^{1/2})-H_{a-b}^{(2)}(x^{1/2})\,H_{a+b}^{(2)}(x^{1/2})\right]$$

$$G_{24}^{31}\left(x\,\middle|\,\begin{matrix}\tfrac{1}{2}+a,\ \tfrac{1}{2}-a\\ 0,\ \tfrac{1}{2},\ b,\ -b\end{matrix}\right)=\frac{\pi^{1/2}\,\Gamma(\tfrac{1}{2}-a+b)\,x^{-1/4}}{\Gamma(1+2a)}W_{a,b}(2x^{1/2})\,M_{-a,b}(2x^{1/2})$$

$$G_{24}^{40}\left(x\,\middle|\,\begin{matrix}\tfrac{1}{2}+a,\ \tfrac{1}{2}-a\\ 0,\ \tfrac{1}{2},\ b,\ -b\end{matrix}\right)=\pi^{1/2}\,x^{-1/4}\,W_{a,b}(2x^{1/2})\,W_{-a,b}(2x^{1/2})$$

$$G_{24}^{40}\left(x\,\middle|\,\begin{matrix}a,\ a+\tfrac{1}{2}\\ b+c,\ b-c,\ b+\tfrac{1}{2}+c,\ b+\tfrac{1}{2}-c\end{matrix}\right)$$
$$=\pi^{1/2}\,2^{-k}\,x^{b-1/4}\,e^{-x^{1/2}}\,W_{k,2c}(2x^{1/2})\qquad k=\tfrac{1}{2}+2b-2c$$

$$G_{24}^{40}\left(x\,\middle|\,\begin{matrix}a,\ a+\tfrac{1}{2}\\ a+b,\ a+c,\ a-c,\ a-b\end{matrix}\right)=2\pi^{-1/2}\,x^{a}\,K_{b+c}(x^{1/2})\,K_{b-c}(x^{1/2})$$

$$G_{24}^{41}\left(x\,\middle|\,\begin{matrix}0,\ \tfrac{1}{2}\\ a,\ b,\ -b,\ -a\end{matrix}\right)=\frac{-2^{-2}\ \pi^{5/2}}{i\ \sin a\pi\ \sin b\pi}$$
$$\times\left[e^{-b\pi i}H_{a-b}^{(1)}(x^{1/2})\,H_{a+b}^{(2)}(x^{1/2})-e^{b\pi i}H_{a+b}^{(1)}(x^{1/2})\,H_{a-b}^{(2)}(x^{1/2})\right]$$

$$G_{24}^{41}\left(x\,\middle|\,\begin{matrix}\tfrac{1}{2},\ 0\\ a,\ b,\ -b,\ -a\end{matrix}\right)=\frac{2^{-2}\,\pi^{5/2}}{\cos a\pi\ \cos b\pi}$$
$$\times\left[e^{-b\pi i}H_{a-b}^{(1)}(x^{1/2})\,H_{a+b}^{(2)}(x^{1/2})+e^{b\pi i}H_{a+b}^{(1)}(x^{1/2})\,H_{a-b}^{(2)}(x^{1/2})\right]$$

$$G_{24}^{41}\left(x\,\middle|\,\begin{matrix}\tfrac{1}{2}+a,\ \tfrac{1}{2}-a\\ 0,\ \tfrac{1}{2},\ b,\ -b\end{matrix}\right)=x^{-1/4}\,\pi^{1/2}\,\Gamma(\tfrac{1}{2}+b-a)\,\Gamma(\tfrac{1}{2}-b-a)$$
$$\times W_{a,b}(2ix^{1/2})\,W_{a,b}(-2ix^{1/2})$$

$$G_{24}^{42}\left(x \,\middle|\, \begin{matrix} a,\ a+\tfrac{1}{2} \\ b+c,\ b-c,\ b+\tfrac{1}{2}+c,\ b+\tfrac{1}{2}-c \end{matrix}\right)$$
$$= 2^{k+1}\,\pi^{3/2}\,\Gamma(1-2a+2b+2c)\,\Gamma(1-2a+2b-2c)$$
$$\times\, x^{b-1/4}\, e^{x^{1/2}}\, W_{k,2c}(2x^{1/2}) \qquad k = 2a-2b-\tfrac{1}{2}$$

$$G_{44}^{14}\left(x \,\middle|\, \begin{matrix} a-1,\ -c_1,\ -c_2,\ -c_3 \\ -b_1,\ -b_2,\ -b_3,\ -b_4 \end{matrix}\right) = \frac{\prod_{h=1}^{4}\Gamma(a+b_h)}{\prod_{h=1}^{3}\Gamma(a+c_h)}\, x^{a-1}$$
$$\times\, {}_4F_3(a+b_1,\ a+b_2,\ a+b_3,\ a+b_4;\ a+c_1,\ a+c_2,\ a+c_3;\ -x)$$

$$G_{pq}^{1p}\left(x \,\middle|\, \begin{matrix} a_1,\ \ldots,\ a_p \\ b_1,\ \ldots,\ b_q \end{matrix}\right) = \frac{\prod_{j=1}^{p}\Gamma(1+b_1-a_j)}{\prod_{j=2}^{q}\Gamma(1+b_1-b_j)}\, x^{b_1}$$
$$\times\, {}_pF_{q-1}(1+b_1-a_1,\ \ldots,\ 1+b_1-a_p;$$
$$1+b_1-b_2,\ \ldots,\ 1+b_1-b_q;\ -x) \qquad p \le q$$

$$G_{pq}^{1n}\left(x \,\middle|\, \begin{matrix} a_1,\ \ldots,\ a_p \\ b_1,\ \ldots,\ b_q \end{matrix}\right) = \frac{\prod_{j=1}^{n}\Gamma(1+b_1-a_j)\, x^{b_1}}{\prod_{j=2}^{q}\Gamma(1+b_1-b_j)\ \prod_{j=n+1}^{p}\Gamma(a_j-b_1)}$$
$$\times\, {}_pF_{q-1}(1+b_1-a_1,\ \ldots,\ 1+b_1-a_p;$$
$$1+b_1-b_2,\ \ldots,\ 1+b_1-b_q;\ -x) \qquad p \le q$$

$$G_{pq}^{q1}\left(x \,\middle|\, \begin{matrix} a_1,\ \ldots,\ a_p \\ b_1,\ \ldots,\ b_q \end{matrix}\right)$$
$$\times\, x^{a_1-1}\, E(1-a_1+b_1, \ldots, 1-a_1+b_q : 1-a_1+a_2, \ldots, 1-a_1+a_p : x)$$

Functions expressible in terms of the G-function

$$x^\mu J_\nu(x) = 2^\mu G^{10}_{02}(\tfrac{1}{4}x^2 | \tfrac{1}{2}\nu + \tfrac{1}{2}\mu, \tfrac{1}{2}\mu - \tfrac{1}{2}\nu)$$

$$x^\mu J_\nu(x) = 4^\mu G^{20}_{04}(4^{-4}x^4 | \tfrac{1}{4}\nu + \tfrac{1}{4}\mu, \tfrac{1}{4}\nu + \tfrac{1}{4}\mu + \tfrac{1}{2}, \tfrac{1}{4}\mu - \tfrac{1}{4}\nu, \tfrac{1}{2} + \tfrac{1}{4}\mu - \tfrac{1}{4}\nu)$$

$$x^\mu Y_\nu(x) = 2^\mu G^{20}_{13}\left(\tfrac{1}{4}x^2 \middle| \begin{matrix} \tfrac{1}{2}\mu - \tfrac{1}{2}\nu - \tfrac{1}{2} \\ \tfrac{1}{2}\mu - \tfrac{1}{2}\nu, \tfrac{1}{2}\mu + \tfrac{1}{2}\nu, \tfrac{1}{2}\mu - \tfrac{1}{2}\nu - \tfrac{1}{2} \end{matrix}\right)$$

$$x^\mu K_\nu(x) = 2^{\mu-1} G^{20}_{02}(\tfrac{1}{4}x^2 | \tfrac{1}{2}\mu + \tfrac{1}{2}\nu, \tfrac{1}{2}\mu - \tfrac{1}{2}\nu)$$

$$x^\mu K_\nu(x) = 4^{\mu-1}\pi^{-1}$$
$$\times G^{40}_{04}(4^{-4}x^4 | \tfrac{1}{4}\nu + \tfrac{1}{4}\mu, \tfrac{1}{2} + \tfrac{1}{4}\nu + \tfrac{1}{4}\mu, -\tfrac{1}{4}\nu + \tfrac{1}{4}\mu, \tfrac{1}{2} - \tfrac{1}{4}\nu + \tfrac{1}{4}\mu)$$

$$e^{-x} I_\nu(x) = \pi^{-1/2} G^{11}_{12}\left(2x \middle| \begin{matrix} \tfrac{1}{2} \\ \nu, -\nu \end{matrix}\right)$$

$$e^{-x} K_\nu(x) = \pi^{1/2} G^{20}_{12}\left(2x \middle| \begin{matrix} \tfrac{1}{2} \\ \nu, -\nu \end{matrix}\right)$$

$$e^{x} K_\nu(x) = \pi^{-1/2} \cos \nu\pi \, G^{21}_{12}\left(2x \middle| \begin{matrix} \tfrac{1}{2} \\ \nu, -\nu \end{matrix}\right)$$

$$x^\mu \mathbf{H}_\nu(x) = 2^\mu G^{11}_{13}\left(\tfrac{1}{4}x^2 \middle| \begin{matrix} \tfrac{1}{2} + \tfrac{1}{2}\nu + \tfrac{1}{2}\mu \\ \tfrac{1}{2} + \tfrac{1}{2}\nu + \tfrac{1}{2}\mu, \tfrac{1}{2}\mu - \tfrac{1}{2}\nu, \tfrac{1}{2}\mu + \tfrac{1}{2}\nu \end{matrix}\right)$$

$$\mathbf{H}_\nu(x) - Y_\nu(x) = \pi^{-2} \cos \nu\pi \, G^{31}_{13}\left(\tfrac{1}{4}x^2 \middle| \begin{matrix} \tfrac{1}{2} + \tfrac{1}{2}\nu \\ \tfrac{1}{2} + \tfrac{1}{2}\nu, -\tfrac{1}{2}\nu, \tfrac{1}{2}\nu \end{matrix}\right)$$

$$x^\mu[I_\nu(x) - \mathbf{L}_\nu(x)] = \pi^{-1} 2^\mu G^{21}_{13}\left(\tfrac{1}{4}x^2 \middle| \begin{matrix} \tfrac{1}{2}\mu + \tfrac{1}{2}\nu + \tfrac{1}{2} \\ \tfrac{1}{2}\mu + \tfrac{1}{2}\nu, \tfrac{1}{2}\mu + \tfrac{1}{2}\nu + \tfrac{1}{2}, \tfrac{1}{2}\mu - \tfrac{1}{2}\nu \end{matrix}\right)$$

$$x^{\mu}\,[I_{-\nu}(x) - \mathbf{L}_{\nu}(x)]$$
$$= \pi^{-1}\, 2^{\mu} \cos \nu\pi\, G^{21}_{13}\left(\tfrac{1}{4}x^2 \,\middle|\, \begin{matrix} \tfrac{1}{2} + \tfrac{1}{2}\nu + \tfrac{1}{2}\mu \\ \tfrac{1}{2} + \tfrac{1}{2}\nu + \tfrac{1}{2}\mu,\ \tfrac{1}{2}\mu - \tfrac{1}{2}\nu,\ \tfrac{1}{2}\mu + \tfrac{1}{2}\nu \end{matrix}\right)$$

$$S_{\mu,\nu}(x) = 2^{\mu-1} \frac{1}{\Gamma(\tfrac{1}{2} - \tfrac{1}{2}\mu - \tfrac{1}{2}\nu)\,\Gamma(\tfrac{1}{2} - \tfrac{1}{2}\mu + \tfrac{1}{2}\nu)}$$
$$\times G^{31}_{13}\left(\tfrac{1}{4}x^2 \,\middle|\, \begin{matrix} \tfrac{1}{2} + \tfrac{1}{2}\mu \\ \tfrac{1}{2} + \tfrac{1}{2}\mu,\ \tfrac{1}{2}\nu,\ -\tfrac{1}{2}\nu \end{matrix}\right)$$

$$J^2_{\nu}(x) = \pi^{-1/2}\, G^{11}_{13}\left(x^2 \,\middle|\, \begin{matrix} \tfrac{1}{2} \\ \nu,\ 0,\ -\nu \end{matrix}\right)$$

$$J_{\nu}(x)\, J_{-\nu}(x) = \pi^{-1/2}\, G^{11}_{13}\left(x^2 \,\middle|\, \begin{matrix} \tfrac{1}{2} \\ 0,\ \nu,\ -\nu \end{matrix}\right)$$

$$x^{\sigma} J_{\mu}(x)\, J_{\nu}(x)$$
$$= \pi^{-1/2} G^{12}_{24}\left[x^2 \,\middle|\, \begin{matrix} \tfrac{1}{2} + \tfrac{1}{2}\sigma,\ \tfrac{1}{2}\sigma \\ \tfrac{1}{2}(\mu + \nu + \sigma),\ \tfrac{1}{2}(\nu + \sigma - \mu),\ \tfrac{1}{2}(\mu + \sigma - \nu),\ \tfrac{1}{2}(\sigma - \mu - \nu) \end{matrix}\right]$$

$$x^{\mu} I_{\nu}(x)\, J_{\nu}(x) = \pi^{1/2}\, 2^{3\mu/2}\, G^{10}_{04}\left(\frac{x^4}{64} \,\middle|\, \tfrac{1}{4}\mu + \tfrac{1}{2}\nu,\ \tfrac{1}{4}\mu - \tfrac{1}{2}\nu,\ \tfrac{1}{4}\mu,\ \tfrac{1}{4}\mu + \tfrac{1}{2}\right)$$

$$I_{\nu}(x)\, J_{-\nu}(x) = \pi^{1/2} \cos(\tfrac{1}{2}\nu\pi)\, G^{10}_{04}\left(\frac{x^4}{64} \,\middle|\, 0,\ \tfrac{1}{2},\ \tfrac{1}{2}\nu,\ -\tfrac{1}{2}\nu\right)$$
$$- \pi^{1/2} \sin(\tfrac{1}{2}\nu\pi)\, G^{10}_{04}\left(\frac{x^4}{64} \,\middle|\, \tfrac{1}{2},\ 0,\ \tfrac{1}{2}\nu,\ -\tfrac{1}{2}\nu\right)$$

$$x^{\mu} J_{\nu}(x)\, Y_{\nu}(x) = -\pi^{-1/2}\, G^{20}_{13}\left(x^2 \,\middle|\, \begin{matrix} \tfrac{1}{2} + \tfrac{1}{2}\mu \\ \nu + \tfrac{1}{2}\mu,\ \tfrac{1}{2}\mu,\ \tfrac{1}{2}\mu - \nu \end{matrix}\right)$$

$$I_{\nu}(x)\, K_{\nu}(x) = 2^{-1}\, \pi^{-1/2}\, G^{21}_{13}\left(x^2 \,\middle|\, \begin{matrix} \tfrac{1}{2} \\ \nu,\ 0,\ -\nu \end{matrix}\right)$$

$$x^{\mu} K_{\nu}(x) J_{\nu}(x) = \pi^{-1/2} 2^{3\mu/2-1/2} \times G^{30}_{04}\left(\frac{1}{64} x^4 \middle| \tfrac14\mu + \tfrac12\nu, \tfrac14\mu + \tfrac12, \tfrac14\mu, \tfrac14\mu - \tfrac12\nu\right)$$

$$x^{\sigma} I_{\nu}(x) K_{\mu}(x) = 2^{-1} \pi^{-1/2} \times G^{22}_{24}\left[x^2 \middle| \begin{matrix} \tfrac12\sigma, \tfrac12\sigma + \tfrac12 \\ \tfrac12(\nu+\mu+\sigma), \tfrac12(\nu+\sigma-\mu), \tfrac12(\mu+\sigma-\nu), \tfrac12(\sigma-\nu-\mu) \end{matrix}\right]$$

$$x^{\mu} H^{(1)}_{\nu}(x) H^{(2)}_{\nu}(x) = \pi^{-5/2} 2 \cos\nu\pi \times G^{31}_{13}\left(x^2 \middle| \begin{matrix} \tfrac12 + \tfrac12\mu \\ \tfrac12\mu + \nu, \tfrac12\mu - \nu, \tfrac12\mu \end{matrix}\right)$$

$$x^{\mu} K^2_{\nu}(x) = 2^{-1} \pi^{1/2} G^{30}_{13}\left(x^2 \middle| \begin{matrix} \tfrac12 + \tfrac12\mu \\ \nu + \tfrac12\mu, -\nu + \tfrac12\mu, \tfrac12\mu \end{matrix}\right)$$

$$x^{\sigma} K_{\nu}(x) K_{\mu}(x) = 2^{-1} \pi^{1/2} \times G^{40}_{24}\left[x^2 \middle| \begin{matrix} \tfrac12\sigma, \tfrac12\sigma + \tfrac12 \\ \tfrac12(\nu+\mu+\sigma), \tfrac12(\nu+\sigma-\mu), \tfrac12(\mu+\sigma-\nu), \tfrac12(\sigma-\nu-\mu) \end{matrix}\right]$$

$$x^{2\mu} K_{2\nu}(xe^{\pi i/4}) K_{2\nu}(xe^{-\pi i/4}) = 2^{3\mu-3} \pi^{-1/2} \times G^{40}_{04}\left(\frac{1}{64} x^4 \middle| \tfrac12\mu, \tfrac12\mu + \tfrac12, \tfrac12\mu + \nu, \tfrac12\mu - \nu\right)$$

$$x^{l} e^{-\frac12 x} W_{k,m}(x) = G^{20}_{12}\left(x \middle| \begin{matrix} l - k + 1 \\ m + l + \tfrac12, l - m + \tfrac12 \end{matrix}\right)$$

$$x^{l} e^{\frac12 x} W_{k,m}(x) = \frac{1}{\Gamma(\tfrac12 + m - k)\,\Gamma(\tfrac12 - m - k)} \times G^{21}_{12}\left(x \middle| \begin{matrix} k + l + 1 \\ l - m + \tfrac12, m + l + \tfrac12 \end{matrix}\right)$$

$$e^{-\frac{1}{2}x}\, W_{k,m}(x) = \pi^{-\frac{1}{2}}\, x^{\frac{1}{2}}\, 2^{k-\frac{1}{2}}$$

$$\times\, G^{40}_{24}\left(2^{-2}x^2 \,\middle|\, \begin{matrix} \frac{1}{4}-\frac{1}{2}k,\ \frac{3}{4}-\frac{1}{2}k \\ \frac{1}{2}+\frac{1}{2}m,\ \frac{1}{2}-\frac{1}{2}m,\ \frac{1}{2}m,\ -\frac{1}{2}m \end{matrix}\right)$$

$$e^{x}\, W_{k,m}(2x) = \frac{x^{\frac{1}{2}}\, 2^{-(k+1)}\, \pi^{-3/2}}{\Gamma(\frac{1}{2}+m-k)\,\Gamma(\frac{1}{2}-m-k)}$$

$$\times\, G^{42}_{24}\left(x^2 \,\middle|\, \begin{matrix} \frac{1}{4}+\frac{1}{2}k,\ \frac{3}{4}+\frac{1}{2}k \\ \frac{1}{2}m,\ \frac{1}{2}+\frac{1}{2}m,\ -\frac{1}{2}m,\ \frac{1}{2}-\frac{1}{2}m \end{matrix}\right)$$

$$W_{k,m}(x)\, M_{-k,m}(x) = \frac{\pi^{-\frac{1}{2}}\,\Gamma(1+2m)}{\Gamma(\frac{1}{2}-k+m)}\; G^{31}_{24}\left(\tfrac{1}{4}x^2 \,\middle|\, \begin{matrix} 1+k,\ 1-k \\ \frac{1}{2},\ 1,\ \frac{1}{2}+m,\ \frac{1}{2}-m \end{matrix}\right)$$

$$x^{l}\, W_{k,m}(2ix)\, W_{k,m}(-2ix) = \frac{x\pi^{-\frac{1}{2}}}{\Gamma(\frac{1}{2}+m-k)\,\Gamma(\frac{1}{2}-m-k)}$$

$$\times\, G^{41}_{24}\left(x^2 \,\middle|\, \begin{matrix} \frac{1}{2}+\frac{1}{2}l+k,\ \frac{1}{2}+\frac{1}{2}l-k \\ \frac{1}{2}l,\ \frac{1}{2}+\frac{1}{2}l,\ \frac{1}{2}l+m,\ \frac{1}{2}l-m \end{matrix}\right)$$

$$W_{k,m}(x)\, W_{-k,m}(x)$$

$$= \pi^{-\frac{1}{2}}\, G^{40}_{24}\left(\tfrac{1}{4}x^2 \,\middle|\, \begin{matrix} k+1,\ -k+1 \\ \frac{1}{2},\ 1,\ m+\frac{1}{2},\ -m+\frac{1}{2} \end{matrix}\right)$$

$${}_2F_1(a,\, b;\, c;\, -x) = \frac{\Gamma(c)\, x}{\Gamma(a)\,\Gamma(b)}\quad G^{12}_{22}\left(x \,\middle|\, \begin{matrix} -a,\ -b \\ -1,\ -c \end{matrix}\right)$$

$${}_4F_3(a,\, b,\, c,\, d;\, e,\, f,\, l;\, -x) = \frac{\Gamma(e)\,\Gamma(f)\,\Gamma(l)}{\Gamma(a)\,\Gamma(b)\,\Gamma(c)\,\Gamma(d)}\, x$$

$$\times\, G^{14}_{44}\left(x \,\middle|\, \begin{matrix} -a,\ -b,\ -c,\ -d \\ -1,\ -e,\ -f,\ -l \end{matrix}\right)$$

$$ {}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; -x) = \frac{\prod_{j=1}^{q} \Gamma(b_j)}{\prod_{j=1}^{p} \Gamma(a_j)} $$

$$ \times x\, G^{1,p}_{p,q+1}\left(x \,\middle|\, \begin{matrix} -a_1, \ldots, -a_p \\ -1, -b_1, \ldots, -b_q \end{matrix}\right) \qquad p \le q+1 $$

$$ E(p;\, \alpha_r :\, q;\, \beta_s :\, x) = G^{p,1}_{q+1,p}\left(x \,\middle|\, \begin{matrix} 1, \beta_1, \ldots, \beta_q \\ \alpha_1, \ldots, \alpha_p \end{matrix}\right) $$

For further special functions expressible in terms of the G-function, in particular for combinations of Legendre functions, and also combinations of generalized hypergeometric series, see C.S. Meijer, *Nederl. Akad. Wetensch., Proc.* 43 (1940), 198-210 and 366-378; 44 (1941), 82-92, 186-194, 298-307, 435-451, 590-605, 1062-1070; 49 (1946), 227-235, 344-356, 457-469, 632-641, 765-772, 936-943, 1063-1072, 1164-1175; 55 (1952), 369-379, 483-487; 56 (1953), 43-49, 187-193.

Hypergeometric series of two variables. In all double sums m and n run from 0 to ∞.

$$ F_1(\alpha; \beta, \beta'; \gamma; x, y) = \sum \frac{(\alpha)_{m+n} (\beta)_m (\beta')_n}{(\gamma)_{m+n}\, m!\, n!} x^m y^n $$

$$ F_2(\alpha; \beta, \beta'; \gamma, \gamma'; x, y) = \sum \frac{(\alpha)_{m+n} (\beta)_m (\beta')_n}{(\gamma)_m (\gamma')_n\, m!\, n!} x^m y^n $$

$$ F_3(\alpha, \alpha', \beta, \beta'; \gamma; x, y) = \sum \frac{(\alpha)_m (\alpha')_n (\beta)_m (\beta')_n}{(\gamma)_{m+n}\, m!\, n!} x^m y^n $$

$$ F_4(\alpha, \beta; \gamma, \gamma'; x, y) = \sum \frac{(\alpha)_{m+n} (\beta)_{m+n}}{(\gamma)_m (\gamma')_n\, m!\, n!} x^m y^n $$

$$ \Phi_1(\alpha, \beta, \gamma; x, y) = \sum \frac{(\alpha)_{m+n} (\beta)_m}{(\gamma)_{m+n}\, m!\, n!} x^m y^n $$

$$\Phi_2(\beta, \beta'; \gamma; x, y) = \sum \frac{(\beta)_m (\beta')_n}{(\gamma)_{m+n}\, m!\, n!} x^m y^n$$

$$\Phi_3(\beta, \gamma; x, y) = \sum \frac{(\beta)_m}{(\gamma)_{m+n}\, m!\, n!} x^m y^n$$

$$\Psi_1(\alpha, \beta, \gamma, \gamma'; x, y) = \sum \frac{(\alpha)_{m+n} (\beta)_m}{(\gamma)_m (\gamma')_n\, m!\, n!} x^m y^n$$

$$\Psi_2(\alpha, \gamma, \gamma'; x, y) = \sum \frac{(\alpha)_{m+n}}{(\gamma)_m (\gamma')_n\, m!\, n!} x^m y^n$$

$$\Xi_1(\alpha, \alpha', \beta, \gamma; x, y) = \sum \frac{(\alpha)_m (\alpha')_n (\beta)_m}{(\gamma)_{m+n}\, m!\, n!} x^m y^n$$

$$\Xi_2(\alpha, \beta, \gamma; x, y) = \sum \frac{(\alpha)_m (\beta)_m}{(\gamma)_{m+n}\, m!\, n!} x^m y^n$$

For other hypergeometric series of two variables see H.T.F. I sec. 5.7.1.

Hypergeometric series of several variables. All summations run from 0 to ∞.

$$F_A(\alpha; \beta_1, \ldots, \beta_n; \gamma_1, \ldots, \gamma_n; z_1, \ldots, z_n)$$
$$= \sum \frac{(\alpha)_{m_1+\cdots+m_n} (\beta_1)_{m_1} \cdots (\beta_n)_{m_n}}{(\gamma_1)_{m_1} \cdots (\gamma_n)_{m_n}\, m_1! \cdots m_n!} z_1^{m_1} \cdots z_n^{m_n}$$

$$\Phi_2(\beta_1, \ldots, \beta_n; \gamma; z_1, \ldots, z_n) = \sum \frac{(\beta_1)_{m_1} \cdots (\beta_n)_{m_n}}{(\gamma)_{m_1+\cdots+m_n}\, m_1! \cdots m_n!} z_1^{m_1} \cdots z_n^{m_n}$$

$$\Psi_2(\alpha; \gamma_1, \ldots, \gamma_n; z_1, \ldots, z_n) = \sum \frac{(\alpha)_{m_1+\cdots+m_n}}{(\gamma_1)_{m_1} \cdots (\gamma_n)_{m_n}\, m_1! \cdots m_n!} z_1^{m_1} \cdots z_n^{m_n}$$

Confluent hypergeometric functions

See also H.T.F. I Chapter VI and H.T.F. II Chapters VIII and IX. See also under Hypergeometric functions, Orthogonal polynomials.

Whittaker's functions

$$M_{\kappa,\mu}(z) = z^{\frac{1}{2}+\mu} e^{-\frac{1}{2}z} \,{}_1F_1(\tfrac{1}{2}+\mu-\kappa;\, 2\mu+1;\, z)$$

$$W_{\kappa,\mu}(z) = \frac{\Gamma(-2\mu)\, M_{\kappa,\mu}(z)}{\Gamma(\frac{1}{2}-\mu-\kappa)} + \frac{\Gamma(2\mu)\, M_{\kappa,-\mu}(z)}{\Gamma(\frac{1}{2}+\mu-\kappa)}.$$

Parabolic cylinder functions

$$D_\nu(z) = 2^{\frac{1}{2}\nu+\frac{1}{4}} z^{-\frac{1}{2}} W_{\frac{1}{2}\nu+\frac{1}{4},\frac{1}{4}}(\tfrac{1}{2}z^2)$$

$$D_n(z) = (-1)^n e^{\frac{1}{4}z^2} \frac{d^n}{dz^n}(e^{-\frac{1}{2}z^2}).$$

Bateman's function

$$k_{2\nu}(z) = \frac{1}{\Gamma(\nu+1)} W_{\nu,\frac{1}{2}}(2z).$$

The exponential integral and related functions

$$-\mathrm{Ei}(-x) = E_1(x) = \int_x^\infty e^{-t}\frac{dt}{t} = \Gamma(0,x) \qquad -\pi < \arg x < \pi$$

$$\mathrm{Ei}^+(x) = \mathrm{Ei}(x+i0), \quad \mathrm{Ei}^-(x) = \mathrm{Ei}(x-i0) \qquad x>0$$

$$\overline{\mathrm{Ei}}(x) = \tfrac{1}{2}[\mathrm{Ei}^+(x) + \mathrm{Ei}^-(x)] \qquad x>0.$$

The last function is denoted by $E^*(x)$ in H.T.F. II sec. 9.7.

$$\mathrm{li}(z) = \int_0^z \frac{dt}{\log t} = \mathrm{Ei}(\log z)$$

$$\mathrm{si}(x) = -\int_x^\infty \frac{\sin t}{t}\,dt = \frac{1}{2i}[\mathrm{Ei}(ix) - \mathrm{Ei}(-ix)]$$

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt = \tfrac{1}{2}\pi + \mathrm{si}(x)$$

$$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos t}{t}\,dt = -\,\mathrm{ci}(x) = \tfrac{1}{2}[\mathrm{Ei}(ix) + \mathrm{Ei}(-ix)]$$

Error functions and related functions

$$\mathrm{Erf}(x) = 2\pi^{-\frac{1}{2}} \int_0^x e^{-t^2}\, dt = \frac{2x}{\sqrt{\pi}}\ {}_1F_1\left(\frac{1}{2}, \frac{3}{2}; -x^2\right)$$

$$\mathrm{Erfc}(x) = 2\pi^{-\frac{1}{2}} \int_x^\infty e^{-t^2}\, dt = 1 - \mathrm{Erf}(x) = (\pi x)^{-\frac{1}{2}}\, e^{-\frac{1}{2}x^2}\, W_{-\frac{1}{4},\frac{1}{4}}(x^2).$$

These functions differ by the factor $2\pi^{-\frac{1}{2}}$ from the functions introduced in H.T.F. II sec. 9.9.

$$C(x) = 2^{-\frac{1}{2}}\, \pi^{-\frac{1}{2}} \int_0^x t^{-\frac{1}{2}} \cos t\, dt$$

$$S(x) = 2^{-\frac{1}{2}}\, \pi^{-\frac{1}{2}} \int_0^x t^{-\frac{1}{2}} \sin t\, dt.$$

Incomplete gamma functions

$$\gamma(a, x) = \int_0^x e^{-t} t^{\alpha-1}\, dt = a^{-1} x^\alpha\ {}_1F_1(a; a+1; -x)$$

$$\Gamma(a, x) = \int_x^\infty e^{-t} t^{\alpha-1}\, dt = \Gamma(a) - \gamma(a, x)$$

$$= x^{\frac{1}{2}(\alpha-1)} e^{-\frac{1}{2}x}\, W_{\frac{1}{2}(\alpha-1), \frac{1}{2}\alpha}(x).$$

Elliptic functions and integrals

See also H.T.F. II Chapter XIII.
Complete elliptic integrals

$$K(k) = \int_0^{\frac{1}{2}\pi} (1 - k^2 \sin^2\phi)^{-\frac{1}{2}}\, d\phi = \tfrac{1}{2}\pi\ {}_2F_1(\tfrac{1}{2}, \tfrac{1}{2}; 1; k^2)$$

$$E(k) = \int_0^{\frac{1}{2}\pi} (1 - k^2 \sin^2\phi)^{\frac{1}{2}}\, d\phi = \tfrac{1}{2}\pi\ {}_2F_1(-\tfrac{1}{2}, \tfrac{1}{2}; 1; k^2).$$

Theta functions

$$\theta_0(v|\tau) = (-i\tau)^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} e^{-i\pi(v-\frac{1}{2}+n)^2/\tau}$$

$$\theta_1(v|\tau) = (-i\tau)^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} (-1)^n e^{-i\pi(v-\frac{1}{2}+n)^2/\tau}$$

$$\theta_2(v|\tau) = (-i\tau)^{-1/2} \sum_{n=-\infty}^{\infty} (-1)^n e^{-i\pi(v+n)^2/\tau}$$

$$\theta_3(v|\tau) = (-i\tau)^{-1/2} \sum_{n=-\infty}^{\infty} e^{-i\pi(v+n)^2/\tau}$$

$$\theta_4(v|\tau) = \theta_0(v|\tau).$$

The series given here are connected with the definitions given in H.T.F. II equations 13.19(10) to (13) by means of Jacobi's imaginary transformation, see H.T.F. II equations 13.22(8).

Modified theta functions

$$\hat{\theta}_0(v|\tau) = (-i\tau)^{-1/2} \left[\sum_{n=0}^{\infty} e^{-i\pi(v+1/2+n)^2/\tau} - \sum_{n=-1}^{-\infty} e^{-i\pi(v+1/2+n)^2/\tau} \right]$$

$$\hat{\theta}_2(v|\tau) = (-i\tau)^{-1/2} \left[\sum_{n=0}^{\infty} (-1)^n e^{-i\pi(v+n)^2/\tau} - \sum_{n=-1}^{-\infty} (-1)^n e^{-i\pi(v+n)^2/\tau} \right]$$

$$\hat{\theta}_3(v|\tau) = (-i\tau)^{-1/2} \left[\sum_{n=0}^{\infty} e^{-i\pi(v+n)^2/\tau} - \sum_{n=-1}^{-\infty} e^{-i\pi(v+n)^2/\tau} \right].$$

Miscellaneous functions

See also H.T.F. III Chapter XVIII.

$$\mu(x, a) = \int_0^\infty \frac{x^s s^a}{\Gamma(s+1)} ds$$

$$\nu(x) = \int_0^\infty \frac{x^s}{\Gamma(s+1)} ds$$

$$\nu(x, a) = \int_0^\infty \frac{x^{s+a}}{\Gamma(s+a+1)} ds = \int_a^\infty \frac{x^s}{\Gamma(s+1)} ds.$$

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$\Gamma(z)$ Gamma function, 369
$\gamma(\alpha, x)$, $\Gamma(\alpha, x)$ Incomplete gamma functions, 387
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sgn x 368
$[x]$ largest integer $\leq x$
Re z real part of z (complex)
Im z imaginary part of z (complex)
$|z|$ modulus of z (complex)
arg z argument (or phase) of z (complex)