

(The following is to be used as a guideline. The rest you need to look up on your own, but hopefully this will help. The original idea is credited to Mr. Samuel Goree in my period 5 class from 2009. Everyone thank him.)

How to use this document: The italicized text is an explanation of the name of the postulate or theorem. You may use that in proofs, or you can use the bolded part—the name of the postulate/theorem when applicable, or the actual statement of the theorem. Remember that you must cite a theorem by name or write it in a complete sentence!)

Basic Postulates:

- **Reflexive Property:** *Any quantity is equal/congruent to itself.*
- **Symmetric Property:** *If $a = b$, then $b = a$. Same holds for congruence.*
- **Transitive Property:** *If $a = b$ and $b = c$, then $a = c$. Same holds for congruence.*
- **Substitution Postulate:** *Any quantity can be substituted for its equal in any expression. Same holds for congruence.*
- **Partition Postulate:** *The whole is equal to the sum of its parts.*
- **Addition Postulate:** *If $a = b$ and $c = d$, then $a + c = b + d$. Alternately, when equal quantities are added to equal quantities, their sum is equal. Same holds for congruence.*
- **Subtraction Postulate:** *If $a = b$ and $c = d$, then $a - c = b - d$. Alternately, when equal quantities are subtracted from equal quantities, their difference is equal. Same holds for congruence.*
- **Multiplication Postulate:** *If $a = b$ and $c = d$, then $ac = bd$. Alternately, when equal quantities are multiplied with equal quantities, their product is equal.*
- **Division Postulate:** *If $a = b$ and $c = d$, then $\frac{a}{c} = \frac{b}{d}$. Alternately, when equal quantities are divided by equal quantities, their quotient is equal.*

Simple Angle Theorems:

- **“If two angles are congruent, then their complements/supplements are congruent.”**
- **“If two angles are complementary/supplementary to the same angle, then they are congruent.”**
- **“If two angles form a linear pair, then they are supplementary.”**
- **“If an angle is bisected, then the measure of the resulting angle is half as large as the original angle.”**
- **“If an angle is bisected, then the angle measures double that of the resulting angles.”**
- **“The midpoint of a line segment divides that segment into two segments that are half as long as the original segment.”**
- **“If a line segment is bisected, then it measures double that of the resulting line segments.”**
- **“If two lines intersect to form congruent adjacent angles, then the lines are perpendicular.”**
- **“If two angles are vertical angles, then they are congruent.”:** *Alternately, you could just claim that vertical angles are congruent.*
- **“All right angles are congruent.”:** *One of Euclid's Power Five—his original five postulates. Use well!*

Parallel Line Theorems:

The following theorems can only be used if *you know (by given or a conclusion you drew) that two lines are parallel*.

- **“If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.”**
- **“If two parallel lines are cut by a transversal, then the corresponding angles are congruent.”**
- **“If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.”**
- **“If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.”**

The following theorems are to be used *when you are trying to prove two lines are parallel*. Again, remember to satisfy all the conditions of a theorem before invoking it!

- “If two lines are cut by a transversal such that alternate interior angles are congruent, then the lines are parallel.”
- “If two lines are cut by a transversal such that alternate exterior angles are congruent, then the lines are parallel.”
- “If two lines are cut by a transversal such that corresponding angles are congruent, then the lines are parallel.”

The following theorems about parallel lines are miscellaneous.

- “If two lines are perpendicular to the same line, then the lines are parallel.”
- “If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.”
- “If two lines are each parallel to a third line, then the two lines are parallel.”
- **Euclid’s Fifth Postulate:** *Through a given point not on a given line, there exist exactly one line that can be drawn through the point parallel to the given line.*

Triangle Congruence Theorems

The following are the only ways of proving triangles are congruent.

- **SSS Postulate:** *Three pairs of corresponding sides are congruent.*
- **SAS Postulate:** *Two pairs of corresponding sides and the angle included (between) the sides are congruent. Make sure that the angle is between the sides!*
- **ASA Postulate:** *Two pairs of corresponding angles and the side included (between) the angles are congruent. Make sure that the side is between the angles!*
- **AAS Theorem:** *Two pairs of corresponding angles and a corresponding pair of sides opposite one of these angles are congruent. Make sure that the side is NOT between the angles!*
- **Hyp-Leg Theorem:** *In a right triangle, a pair of hypotenuses and a pair of corresponding legs are congruent. Make sure that the triangle is a right triangle, and declare this first!*
- “Corresponding parts of congruent triangles are congruent.”: *Do NOT abbreviate this! Write it out.*

Isosceles Triangle Theorems:

- “If two angles in a triangle are congruent, then the triangle is isosceles.”
- “If two sides in a triangle are congruent, then the triangle is isosceles.”
- “If two angles in a triangle are congruent, then the sides opposite them are congruent.”
- “If two sides in a triangle are congruent, then the angles opposite them are congruent.”
- “If a triangle is equiangular, then it is equilateral.”
- “If a triangle is equilateral, then it is equiangular.”

Equidistance Theorems:

- “If a point lies on the perpendicular bisector of a line segment, then it is equidistant from the endpoints of the line segment.”
- “If a point is equidistant from two points, then it lies on the perpendicular bisector of the line segment.”
- “If a point lies on an angle bisector, then it is equidistant from the sides of that angle.”
- “If a point is equidistant from the sides of an angle, then it lies on the angle’s bisector.”

Inequality Postulates/Theorems

- “The whole is greater than any of its parts.”
- **The Trichotomy Postulate:** “Given two numbers, a and b , exactly one of the following is true— $a > b$, $a < b$, or $a = b$.”
- **Transitive Property:** “If $a > b$ and $b > c$, then $a > c$.”

- **The Addition Postulate of Inequality:** “If $a > b$ and $c \geq d$, then $a + c > b + d$. The same is true if the signs are reversed.
- **The Subtraction Postulate of Inequality:** “If $a > b$ and $c = d$, then $a - c > b - d$. The same is true if the signs are reversed.
- **The Multiplication Postulate of Inequality:** If $a > b$ and $c > 0$, then $ac > bc$. Similarly, if $a > b$ and $c < 0$, then $ac < bc$.
- **The Triangle Inequality:** “The sum of the lengths of two sides of a triangle is greater than that of the third.”
- “The measure of an exterior angle of a triangle is greater than the measure of either of the two remote interior angles.”
- “If the lengths of two sides of a triangle are unequal, then the larger angle is opposite the longer side, and vice versa.”
- “If the measures of two angles of a triangle are unequal, then the longer side is opposite the larger angle, and vice versa.”

Quadrilaterals

1) The Parallelogram

Properties:

- Opposite sides of a parallelogram are parallel.
- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Consecutive angles of a parallelogram are supplementary.
- A diagonal divides a parallelogram into two congruent triangles.
- The diagonals of a parallelogram bisect each other.

The following theorems are to be used to show a quadrilateral is a parallelogram.

- “If a quadrilateral has two pairs of opposite parallel sides, then it is a parallelogram.”
- “If a quadrilateral has two pairs of opposite congruent sides, then it is a parallelogram.”
- “If a quadrilateral has two pairs of opposite congruent angles, then it is a parallelogram.”
- “If a quadrilateral has one pair of opposite, parallel, and congruent sides, then it is a parallelogram.”
- “If a quadrilateral has diagonals that bisect each other, then it is a parallelogram.”

2) The Rectangle

Properties:

- Has all properties of the parallelogram.
- All angles in a rectangle are right angles.
- All angles in a rectangle are congruent.
- The diagonals of a rectangle are congruent.

The following theorems are to be used to show a quadrilateral is a rectangle. *Make sure to show that the quadrilateral is a parallelogram first!*

- “If a parallelogram has a right angle, then it is a rectangle.”
- “If a parallelogram has congruent diagonals, then it is a rectangle.”

3) The Rhombus

Properties:

- Has all properties of a parallelogram.
- All sides of a rhombus are congruent.
- The diagonals of a rhombus are perpendicular.

- **The diagonals of a rhombus bisect its angles.**

The following theorems are to be used to show a quadrilateral is a rhombus. *Make sure to show that the quadrilateral is a parallelogram first!*

- **“If a parallelogram has two congruent consecutive sides, then it is a rhombus.”**
- **“If a parallelogram has perpendicular diagonals, then it is a rhombus.”**
- **“If a parallelogram has a diagonal that bisects two opposite angles, then it is a rhombus.”**

4) The Square

Properties:

- **Has all properties of a rhombus.**
- **Has all properties of a rectangle.**

The following theorems are to be used to show a quadrilateral is a square.

- **“If a rectangle has a pair of congruent consecutive sides, then it is a square.”**
- **“If a rhombus has a right angle, then it is a square.”**

5) The Trapezoid

Properties:

- **A trapezoid has exactly one pair of parallel sides.**

6) The Isosceles Trapezoid

Properties:

- **Has all properties of a trapezoid.**
- **The diagonals of an isosceles trapezoid are congruent.**
- **The base angles of an isosceles trapezoid are congruent.**
- **The legs of an isosceles trapezoid are congruent.**

The following theorems are to be used to show a trapezoid is an isosceles trapezoid.

- **“If a trapezoid has congruent legs, then it is isosceles.”**
- **“If a trapezoid has congruent base angles, then it is isosceles.”**
- **“If a trapezoid has congruent diagonals, then it is isosceles.”**

Similarity Theorems:

- **“If two polygons are similar, then the ratio of their perimeters is equal to that of their scale factor.”**
- **“If two polygons are similar, then the ratio of their areas is equal to the square of their scale factor.”**
- **The Triangle Proportionality Theorem:** *“A line connecting two sides of a triangle is parallel to the third side if and only if it divides both of those sides proportionally.”*
- **The Angle Bisector Theorem:** *“An angle bisector of a triangle divides one side of a triangle into two segments that are proportional to the other two sides of that triangle.”*
- **“The segment that joins the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.”** (This is the Triangle Midline Theorem.)
- **The Right Triangle Altitude Theorem:** *“If an altitude is drawn to the hypotenuse of a right triangle, then:*
 - 1. The two triangles formed are similar to each other and the large triangle.*
 - 2. The altitude is the geometric mean of the segments of the hypotenuse.*
 - 3. Each leg is the geometric mean of the entire hypotenuse and the segment of the hypotenuse closest to that leg (the shadow).*

The Circle:

- “All radii of a circle are congruent.”
- “In a circle, congruent central angles intercept congruent arcs.”
- “In a circle, congruent arcs have congruent chords. Conversely, congruent chords cut off congruent arcs.”
- “A diameter perpendicular to a chord bisects that chord and its arcs.”
- “Congruent chords in the same circle are equidistant from the center of the circle.”
- “Inscribed angles that intercept congruent (or the same) chords are congruent.”
- “An angle inscribed in a semicircle (that is, intercepts a semicircle) is a right angle.”
- “Parallel lines intercept congruent arcs.”
- “If a line is tangent to a circle, then it is perpendicular to a radius drawn to that point.”
- “Two tangents drawn to the same circle from the same external point are congruent.” *This is more affectionately known as the “Dunce Cap Theorem.”*
- “The measure of an inscribed angle is one-half of the measure of the arc that it intercepts.”
- “The measure of an angle formed by a chord and a tangent is one-half of the measure of the arc that it intercepts.”
- “The measure of an angle formed by two chords is equal to one-half of the sum of the measures of the arcs that the chords intercept.”
- “The measure of an angle formed outside of the circle (by two tangents, two secants, or a tangent and a secant) is equal to one-half of the difference of the measures of the arcs that the chords intercept.”
- **The Power of a Point Theorem:** *If two chords intersect in a circle, then the product of the lengths of the pieces of one is equal to the product of the lengths of the pieces of the other.*
- **The (Extended) Power of a Point Theorem:** *If two lines outside of a circle (two tangents, two secants, or a tangent and a secant) intersect at a common point, then the product of the length of the entire segment and the part of the segment outside of the circle of one of the segments is equal to the product of the length of the entire segment and the part of the segment outside of the circle of the other.*

Definitions:

Remember that these are biconditional—that means that they can be used in both directions. You need not cite which direction you are using in your definition, but please stay true to the definition—you can’t justify with the definition anything that a definition does not claim!

- Two segments/angles are **congruent** iff they have the same measure.
- A point on a line segment is a **midpoint** iff it divides the segment into two congruent segments.
- A segment/line/ray is an **angle bisector** iff it divides the angle into two congruent angles.
- A segment/line/ray is a **segment bisector** iff it intersects a segment at its midpoint.
- Two angles are **complementary** iff their measures sum to 90 degrees.
- Two angles are **supplementary** iff their measure sum to 180 degrees.
- Two lines are **perpendicular** iff they intersect to form right angles.
- Two lines are **parallel** iff they are coplanar lines that do not intersect.
- Two lines are **skew** iff they are not parallel but do not intersect. *In three dimensions only.*
- Two polygons are **congruent** iff their corresponding sides and corresponding angles are congruent.
- A segment in a triangle is a **median** iff it connects a vertex to the midpoint of the opposite side.
- A segment in a triangle is an **altitude** iff it is drawn from a vertex perpendicular to the opposite side.
- A segment in a triangle is an **angle bisector** iff it is drawn bisecting an angle of the triangle.
- Two polygons are **similar** iff their corresponding sides are in proportion and their corresponding angles are congruent.