Algebra 2 – Things to Remember!

Exponents:

$x^0 = 1$ $\qquad$ $x^{-m} = \dfrac{1}{x^m}$

$x^m \bullet x^n = x^{m+n}$ $\qquad$ $(x^n)^m = x^{n \bullet m}$

$\dfrac{x^m}{x^n} = x^{m-n}$ $\qquad$ $\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$(xy)^n = x^n \bullet y^n$

Complex Numbers:

$\sqrt{-1} = i$ $\qquad$ $\sqrt{-a} = i\sqrt{a}; a \ge 0$

$i^2 = -1$ $\qquad$ $i^{14} = i^2 = -1$ divide exponent by 4, use remainder, solve.

$(a+bi)$ conjugate $(a-bi)$

$(a+bi)(a-bi) = a^2 + b^2$

$|a+bi| = \sqrt{a^2+b^2}$ absolute value=magnitude

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\ln x = \log_e x$ natural log

$e = 2.71828\ldots$

$\log x = \log_{10} x$ common log

Change of base formula:

$\log_b a = \dfrac{\log a}{\log b}$

Properties of Logs:

$\log_b b = 1$ $\qquad$ $\log_b 1 = 0$

$\log_b(m \bullet n) = \log_b m + \log_b n$

$\log_b\left(\dfrac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^r) = r\log_b m$

Domain: $\log_b x$ is $x > 0$

Factoring:

Look to see if there is a GCF (greatest common factor) first. $ab + ac = a(b+c)$

$x^2 - a^2 = (x-a)(x+a)$

$(x+a)^2 = x^2 + 2ax + a^2$

$(x-a)^2 = x^2 - 2ax + a^2$

Factor by Grouping:

$x^3 + 2x^2 - 3x - 6$

$(x^3 + 2x^2) - (3x + 6)$ group

$x^2(x+2) - 3(x+2)$ factor each

$(x^2 - 3)(x+2)$ factor

Exponentials $\quad e^x = \exp(x)$

$b^x = b^y \to x = y \quad (b > 0$ and $b \ne 1)$

If the bases are the same, set the exponents equal and solve.

Solving exponential equations:

1. Isolate exponential expression.
2. Take *log* or *ln* of both sides.
3. Solve for the variable.

$\ln(x)$ and e^x are inverse functions

$\ln e^x = x$ $\qquad$ $e^{\ln x} = x$

$\ln e = 1$ $\qquad$ $e^{\ln 4} = 4$

$e^{2\ln 3} = e^{\ln 3^2} = 9$

Quadratic Equations: $\quad ax^2 + bx + c = 0 \quad$ (Set = 0.)

Solve by factoring, completing the square, quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac > 0$ two real unequal roots

$b^2 - 4ac = 0$ repeated real roots

$b^2 - 4ac < 0$ two complex roots

Square root property: If $x^2 = m$, then $x = \pm\sqrt{m}$

Completing the square: $\quad x^2 - 2x - 5 = 0$

1. If other than one, divide by coefficient of x^2
2. Move constant term to other side $\quad x^2 - 2x = 5$
3. Take half of coefficient of x, square it, add to both sides

   $x^2 - 2x + \boxed{1} = 5 + \boxed{1}$
4. Factor perfect square on left side. $\quad (x-1)^2 = 6$
5. Use square root property to solve and get two answers. $\quad x = 1 \pm \sqrt{6}$

Sum of roots: $r_1 + r_2 = -\dfrac{b}{a}$ $\qquad$ **Product of roots**: $r_1 \bullet r_2 = \dfrac{c}{a}$

Variation: always involves the constant of proportionality, k. Find k, and then proceed.

Direct variation: $\quad y = kx$

Inverse variation: $\quad y = \dfrac{k}{x}$

Varies jointly: $\quad y = kxj$

Combo: Sales vary directly with advertising and inversely with candy cost. $\qquad y = \dfrac{ka}{c}$

Absolute Value: $\quad |a| > 0$

$$|a| = \begin{cases} a; & a \ge 0 \\ -a; & a < 0 \end{cases}$$

$|m| = b \Rightarrow m = -b$ or $m = b$

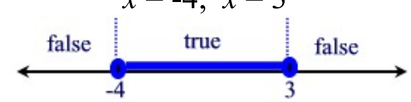
$|m| < b \Rightarrow -b < m < b$

$|m| > b \Rightarrow m > b$ or $m < -b$

Inequalities: $\quad x^2 + x - 12 \le 0$ Change to =, factor, locate critical points on number line, check each section.

$(x + 4)(x - 3) = 0$

$x = -4; \; x = 3$



ANSWER: $\quad -4 \le x \le 3 \quad$ or $\quad [-4, 3] \quad$ (in interval notation)

Radicals: Remember to use fractional exponents.

$$\sqrt[a]{x} = x^{\frac{1}{a}} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\sqrt[n]{a^n} = a \qquad \sqrt[n]{ab} = \sqrt[n]{a}\bullet\sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Simplify: look for perfect powers.

$$\sqrt{x^{12}y^{17}} = \sqrt{x^{12}y^{16}y} = x^6y^8\sqrt{y}$$

$$\sqrt[3]{72x^9y^8z^3} = \sqrt[3]{8\bullet 9x^8xy^8z^3} = 2x^2y^2z\sqrt[3]{9x}$$

Use conjugates to rationalize denominators:

$$\frac{5}{2+\sqrt{3}}\bullet\frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{10-5\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-\sqrt{9}} = 10-5\sqrt{3}$$

Equations: isolate the radical; square both sides to eliminate radical; combine; solve.

$$2x-5\sqrt{x}-3=0 \quad \rightarrow \quad (2x-3)^2 = (5\sqrt{x})^2$$

$$4x^2-12x+9=25x \quad \rightarrow \quad solve: x=9;\ x=1/4$$

CHECK ANSWERS. Answer only $x = 9$.

Functions: A function is a set of ordered pairs in which each x-element has only ONE y-element associated with it.

Vertical Line Test: is this graph a function?
Domain: x-values used; **Range:** y-values used
Onto: all elements in B used.
1-to-1: no element in B used more than once.
Composition: $(f\circ g)(x) = f(g(x))$
Inverse functions f & g: $f(g(x)) = g(f(x)) = x$
Horizontal line test: will inverse be a function?

Transformations:
$-f(x)$ over x-axis; $f(-x)$ over y-axis
$f(x+a)$ horizontal shift; $f(x)+a$ vertical shift
$f(ax)$ stretch horizontal; $af(x)$ stretch vertical

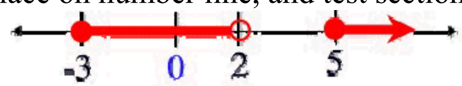
Working with Rationals (Fractions):
Simplify:
remember to look for a factoring of -1:

$$\frac{3x-1}{1-3x} = \frac{-1(\cancel{-3x+1})}{\cancel{1-3x}} = -1$$

Add: Get the common denominator.
Factor first if possible:
Multiply and Divide: Factor First

Rational Inequalities

$\frac{x^2-3x-15}{x-2} \geq 0$ The critical values from factoring the numerator are -3, 5. The denominator is zero at $x = 2$. Place on number line, and test sections.



Solving Rational Equations:
Get rid of the denominators by mult. all terms by common denominator.

$$\frac{22}{2x^2-9x-5} - \frac{3}{2x+1} = \frac{2}{x-5}$$

multiply all by $2x^2-9x-5$ *and get*

$$22-3(x-5) = 2(2x+1)$$
$$22-3x+15 = 4x+2$$
$$37-3x = 4x+2$$
$$35 = 7x$$
$$5 = x$$

Great! But the only problem is that $x = 5$ does not CHECK!!!! There is no solution. Extraneous root.
Motto: Always CHECK ANSWERS.

Sequences
Arithmetic: $a_n = a_1 + (n-1)d$

$$S_n = \frac{n(a_1+a_n)}{2}$$

Geometric: $a_n = a_1\bullet r^{n-1}$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Recursive: Example:
$a_1 = 4; \quad a_n = 2a_{n-1}$

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

Equations of Circles: $x^2+y^2=r^2$ center origin
$(x-h)^2+(y-k)^2=r^2$ center at (h,k)
$x^2+y^2+Cx+Dy+E=0$ standard form

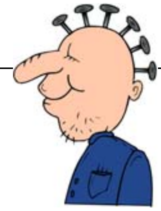
Complex Fractions:
Remember that the fraction bar means divide:
Method 1: Get common denominator top and bottom

$$\frac{\frac{2}{x^2}-\frac{4}{x}}{\frac{4}{x}-\frac{2}{x^2}} = \frac{\frac{2-4x}{x^2}}{\frac{4x-2}{x^2}} = \frac{2-4x}{x^2} \div \frac{4x-2}{x^2} = \frac{\cancel{2-4x}^{-1}}{\cancel{x^2}}\bullet\frac{\cancel{x^2}}{\cancel{4x-2}} = -1$$

Method 2: Mult. all terms by common denominator for all.

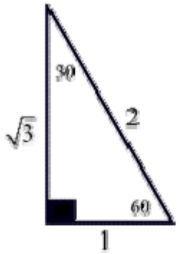
$$\frac{\frac{2}{x^2}-\frac{4}{x}}{\frac{4}{x}-\frac{2}{x^2}} = \frac{x^2\bullet\frac{2}{x^2}-x^2\bullet\frac{4}{x}}{x^2\bullet\frac{4}{x}-x^2\bullet\frac{2}{x^2}} = \frac{2-4x}{4x-2} = -1$$

Trigonometry – Things to Remember!

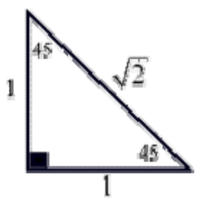


Arc Length of a Circle $= \theta r$ (in radians)

Special Right Triangles



30º-60º-90º triangle
side opposite 30º = ½ hypotenuse
side opposite 60º = ½ hypotenuse $\sqrt{3}$



45º-45º-90º triangle
hypotenuse = leg $\sqrt{2}$
leg = ½ hypotenuse $\sqrt{2}$

Law of Sines: uses 2 sides and 2 angles

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Has an ambiguous case.

Law of Cosines: uses 3 sides and 1 angle

$$c^2 = a^2 + b^2 = 2ab\cos C$$

Area of triangle: $A = \frac{1}{2}ab \sin C$
Area of parallelogram: $A = ab \sin C$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Radians and Degrees

Change to radians multiply by $\frac{\pi}{180}$

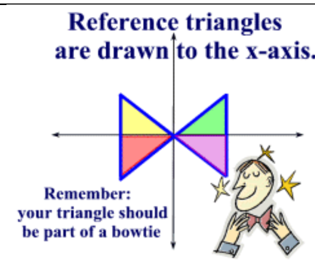
Change to degrees multiply by $\frac{180}{\pi}$

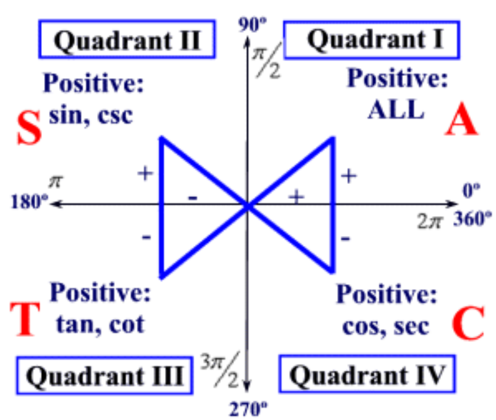
Quadrantal angles – 0, 90, 180, 270

CoFunctions: examples

$\sin\theta = \cos(90^\circ - \theta)$; $\tan\theta = \cot(90^\circ - \theta)$

Inverse notation:
$\arcsin(x) = \sin^{-1}(x)$
$\arccos(x) = \cos^{-1}(x)$
$\arctan(x) = \tan^{-1}(x)$





Trig Functions

$$\sin\theta = \frac{o}{h};\quad \cos\theta = \frac{a}{h};\quad \tan\theta = \frac{o}{a}$$

$$\csc\theta = \frac{h}{o};\quad \sec\theta = \frac{h}{a};\quad \cot\theta = \frac{a}{o}$$

Reciprocal Functions

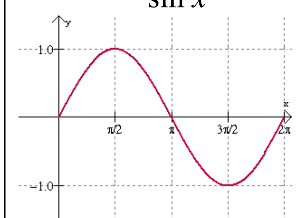
$$\sin\theta = \frac{1}{\csc\theta};\quad \cos\theta = \frac{1}{\sec\theta};\quad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta};\quad \sec\theta = \frac{1}{\cos\theta};\quad \cot\theta = \frac{1}{\tan\theta}$$

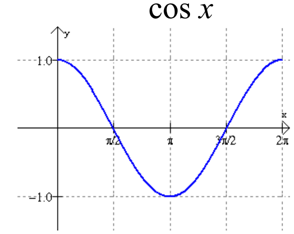
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trig Graphs

sin x



cos x



amplitude = ½ | max – min| *(think height)*

period = horizontal length of 1 complete cycle

frequency = number of cycles in 2π

sinusoidal curve = any curve expressed as
$y = A\sin(B(x - C)) + D$

phase shift = measure of horizontal shifting

Statistics and Probability – Things to Remember!

Statistics:

$mean = \bar{x} = \dfrac{x_1 + x_2 + \ldots + x_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$

median = middle number in ordered data
mode = value occurring most often

range = difference between largest and smallest

mean absolute deviation (MAD):

$population\ MAD = \dfrac{1}{n}\sum_{i=1}^{n}\left|x_i - \bar{x}\right|$

variance:

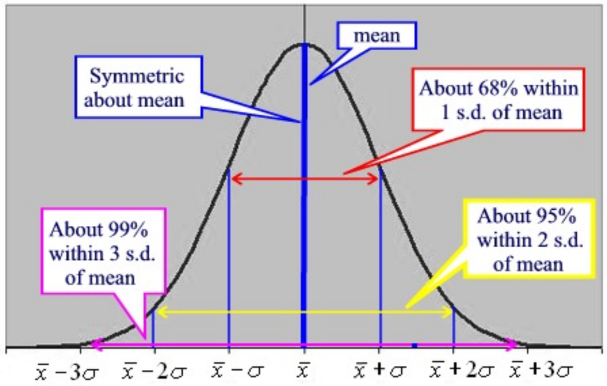
$population\ \text{variance} = (\sigma x)^2 = \dfrac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$

standard deviation:
population standard deviation =

$\sigma x = \sqrt{\dfrac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$

Sx = sample standard deviation
σ_x = population standard deviation

Normal Distribution and Standard Deviation



Binomial Probability

${}_nC_r \bullet p^r \bullet q^{n-r}$ **"exactly"** r times

or $\dbinom{n}{r} \bullet p^r \bullet (1-p)^{n-r}$

[TI Calculator: binompdf(*n, p, r*)]

When computing **"at least"** and **"at most"** probabilities, it is necessary to consider, in addition to the given probability,

• all probabilities larger than the given probability (**"at least"**)
[TI Calculator: 1 – binomcdf(*n, p, r*-1)]

• all probabilities smaller than the given probability (**"at most"**)
[TI Calculator: binomcdf(*n, p, r*)]

Probability

Permutation: without replacement and order matters

${}_nP_r = \dfrac{n!}{(n-r)!}$

Combination: without replacement and order does not matter

${}_nC_r = \dbinom{n}{r} = \dfrac{{}_nP_r}{r!} = \dfrac{n!}{r!(n-r)!}$

Empirical Probability

$P(E) = \dfrac{\#\text{ of times event } E \text{ occurs}}{\text{total }\#\text{ of observed occurrences}}$

Theoretical Probability

$P(E) = \dfrac{n(E)}{n(S)} = \dfrac{\#\text{ of outcomes in } E}{\text{total }\#\text{ of outcomes in } S}$

P(A and B) = P(A)•P(B)
for independent events
P(A and B) = P(A)•P(B| A)
for dependent events

P(A') = 1 – P(A)

P(A or B) = P(A) + P(B) – P(A and B)
for not mutually exclusive

P(A or B) = P(A) + P(B)
for mutually exclusive

$P(B\,|\,A) = \dfrac{P(A \text{ and } B)}{P(A)}$ (conditional)

General Decay/Growth

- $V = V_0(1 + r)^t$
- $V = V_0(1 - r)^t$
- V= value
- $V_0 = initial\ value$
- r=rate % to grow or decay
- t= time(years)

Compound Interest Formula

- $A = P\left[1+\frac{r}{h}\right]^{nt}$
- a=investment value
- p=principal (initial investment)
- r=rate %
- n=time compound per year
- t=years

Doubling-Time Growth Formula

- $N = N_0 \cdot 2^{\frac{t}{d}}$
- N=population at time
- $N_0 = initial\ population$
- d=doubling time (years,days,hours)
- t=given time

Half like Decay formula

- $N = N_0 \cdot (\frac{1}{2})^{t/h}$
- N=amount remaining
- N_0=initial amount
- h=half life
- t=time

Determinant

a real number created from square matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - cb$$

3x3 diagonal

- $\begin{matrix} a & b & c\,| & a & b \\ d & e & f\,| & d & e \\ g & h & i\,| & g & h \end{matrix}$

= aei+bfg+cdh-gec+hfa+idb

Matrix

- row x column

Change of Base Formula

- $\log_a x = \frac{\log_b x}{\log_b a}$

Examples:

- is $x + i\sqrt{3}$ a factor of

$x^3 - 4x^2 + 3x - 12$?

$$(x + i\sqrt{3})(\,x + i\sqrt{3})=$$
$$x^2 + i\sqrt{3x} - i\sqrt{3x} - i^2\sqrt{9}$$
$$= x^2 + 3$$
$$\begin{array}{r} x - 4 \\ x^2 + 3\overline{\big)\,x^3 - 4x^2 + 3x - 12} \\ \underline{-x^3 + 0x^2 + 3x} \\ 0 - 4x^2 + 0 - 12 \\ \underline{-4x^2 \quad\quad - 12} \\ 0 \end{array}$$

yes it is a factor

- $\sqrt{-3} = i\sqrt{3}$
- $\sqrt{-36} = i\sqrt{36} = 6i$

- $\begin{matrix} 7 & 3 \\ 2 & 5 \\ 6 & 8 \end{matrix} \quad \begin{matrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{matrix}$

3x2 2x3 final product 3x3

7(7)=3(8)

7(4)+3(1)

7(9)+3(5)

.....etc

- $\log_3 5 = \frac{\log_{10} 5}{\log_{10} 3}$

ALGEBRA EQUATIONS FOR MULTIPLYING BINOMIALS

In algebra, multiplying binomials is easier if you recognize their patterns. You multiply the sum and difference of binomials and multiply by squaring and cubing to find some of the special products in algebra. See if you can spot the patterns in these equations:

- Sum and difference: $(a + b)(a - b) = a^2 - b^2$
- Binomial squared: $(a + b)^2 = a^2 + 2ab + b^2$
- Binomial cubed: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

STANDARD EQUATIONS OF ALGEBRAIC CONICS

Conics are curved algebraic forms that come from slicing a cone with a plane. Use these equations to graph algebraic conics, such as circles, ellipses, parabolas, and hyperbolas:

Parabolas: $y - k = a(x - h)^2$

$$x - h = a(y - k)^2$$

Circle: $(x - h)^2 + (y - k)^2 = r^2$

Ellipse: $$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Hyperbola: $$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

CRAMER'S RULE FOR LINEAR ALGEBRA

Named for Gabriel Cramer, Cramer's Rule provides a solution for a system of two linear algebraic equations in terms of determinants — the numbers associated with a specific, square matrix.

The solution of the linear system $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$ is $x = \dfrac{ce - bf}{ae - bd}, \quad y = \dfrac{af - cd}{ae - bd}$

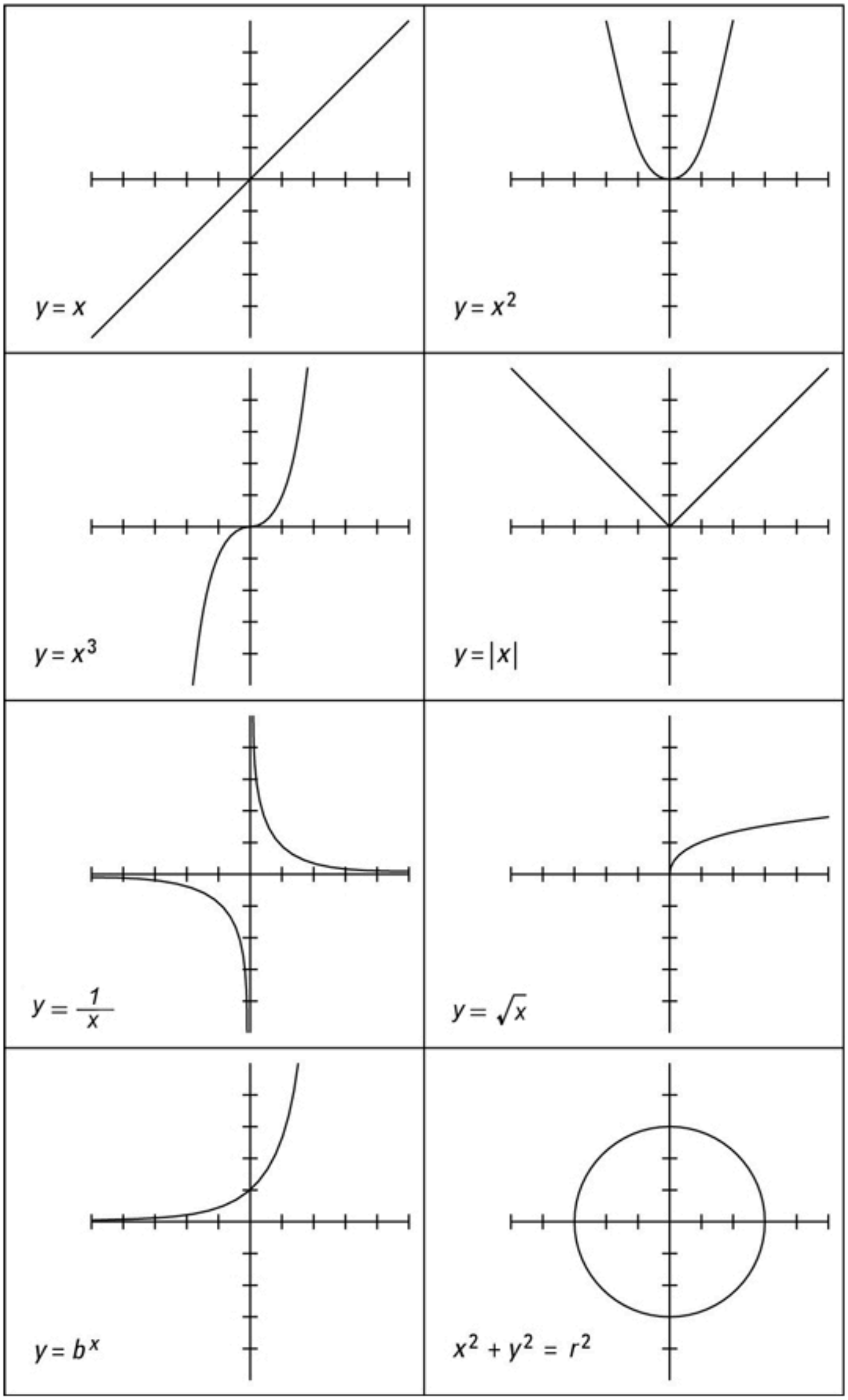
USING ALGEBRA TO FIND THE SUMS OF SEQUENCES

Algebra can help you add a series of numbers (the sum of sequences) more quickly than you would be able to with straight addition. Adding integers, squares, cubes, and terms in an arithmetic or geometric sequence is simple with these algebraic formulas:

Sum of the first n positive integers:	$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
Sum of the first n squares:	$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
Sum of the first n cubes:	$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$
Sum of the first n terms of an arithmetic sequence:	$S_n = \frac{n}{2}[2a_1 + (n-1)d] = \frac{n}{2}(a_1 + a_n)$
Sum of the first n terms of a geometric sequence:	$S_n = \frac{g_1(1-r^n)}{1-r}$
Sum of all of the terms of a geometric sequence with $\lvert r \rvert < 1$:	$S_n \to \frac{g_1}{1-r}$

EIGHT BASIC ALGEBRAIC CURVES

Algebra is all about graphing relationships, and the curve is one of the most basic shapes used. Here's a look at eight of the most frequently used graphs.



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$$= x^2 + 3$$
$$x - 4$$
$$x^2 + 3\sqrt{x^3 - 4x^2 + 3x - 12}$$
$$\underline{-x^3 + 0x^2 + 3x}$$
$$0 - 4x^2 + 0 - 12$$
$$\underline{-4x^2 \quad - 12}$$
$$0$$

yes it is a factor

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Word Problem Formulas

Distance	*D*istance = *r*ate X *t*ime
Interest	*I*nterest = *p*rincipal X *r*ate X *t*ime
Compound Interest	$A = P(1 + \frac{r}{n})^{nt}$
Work accomplished	work *a*ccomplished = (*r*ate of work) X (*t*ime worked)

Graphing Formulas

Distance (for graphs)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_2 \neq x_1$
Slope-Intercept Form	$y = mx + b$

Geometry Formulas	
Perimeter	P = sum of sides
Area of a triangle	$A = ½bh$
Area of a rectangle	$A = lw$
Area of a parallelogram	$A = bh$
Area of a trapezoid	$A = ½\cdot(b_1 + b_2)\cdot h$
Area of a circle	$A = \pi r_2$