Chapter 2: Introduction to Quantum Mechanics Examples

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1 Intro to Quantum Mechanics

1.1 Linear Dependence

Show that $(1, -1), (1, 2)$ and $(2, 1)$ are linearly dependent.

Solution

The solution to the linear system is $x = y = -z$. So let $x = y = 2, z = -2$ then $2(1, -1) + 2(1, 2) - 2(2, 1) = (0, 0)$■

1.2 Matrix Representations

Suppose V is a vector space with basis vectors $|0\rangle$ and $|1\rangle$, and A is a linear operator from V to V such that $A|0\rangle = |1\rangle$ and $A|1\rangle = |0\rangle$. Give a matrix representation for A, with respect to the input basis $(|0\rangle, |1\rangle)$ and the output basis $(|0\rangle, |1\rangle)$.

Solution

$\sigma_x \equiv X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ will flip a qubit in the standard basis. Let $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and let $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ then $X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ■

1.3 Matrix Representation for Operator Products

Suppose A is a linear operator from vector space V to vector space W, and B is a linear operator from vector space W to vector space X. Let $|v_i\rangle, |w_j\rangle$ and $|x_k\rangle$ be bases for the vector spaces V, W, and X, respectively. Show that the matrix representation for the linear transformation BA is the matrix product of the matrix representation for B and A, with respect to the appropriate bases.

Solution ■