Chapter 5: The Quantum Fourier Transform and its Applications

John Martinez
john.r.martinez14@gmail.com

1 The Quantum Fourier Transform and its Applications

1.1 The Quantum Fourier Transform

The *discrete fourier transform* takes as input a vector of complex numbers $x_0, \ldots, X_{N-1}$ where the length N of the vector is a fixed parameter. It outputs the transformed data, a vector of complex numbers $y_0, \ldots, y_{N-1}$, defined by

$$y_N \equiv \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{2\pi ijk/N}.$$

The *quantum fourier transform* is exactly the same transformation, although the conventional notation for the quantum fourier transform is somewhat different. The quantum fourier transform on an orthonormal basis $|0\rangle, \ldots, |N-1\rangle$ is defined to be linear operator with the followin action on the basis states:

$$|j\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ijk/N} |k\rangle.$$

The action on an arbitrary state maybe written

$$\sum_{j=0}^{N-1} x_j |j\rangle \rightarrow \sum_{k=0}^{N-1} y_k |k\rangle.$$

It is not ovbious, but this transformation is a unitary transformation.