```
(*
equation17.nb

 produced by D.F.Gochberg for paper:
  D.F.Gochberg, M.D.Does, Z.Zu, C.L.Lankford. Towards
  an analytic solution for pulsed CEST.NMR in Biomedicine 2017

  You are free to use this code for non-commercial purposes,
but please cite the above manuscript if you use the code, or parts thereof,
to help produce a manuscript or presentation figure, as appropriate. Thanks.

  Based on: analytic_CEST_zb_ss and lab notebook 27,
p.29 (Dan's reminder to himself)
*)

(* equivalent formulations of steady state: *)

(* form used in some matlab calculations: *)
ss = Inverse[{{r2b + kba, -dwb, 0}, {dwb, r2b + kba, -w1}, {0, w1, r1b + kba}}].
    {kba * r1a * dwa * w1 / (R1rho * (w1^2 + dwa^2)), 0, kba * dwa^2 * r1a / (R1rho * (w1^2 + dwa^2)) + r1b};

(* form used to derive eqn 17: *)

ss3 = Inverse[{{r2b + kba, -dwb, 0}, {dwb, r2b + kba, -w1}, {0, w1, r1b + kba}}].{{kba * zaSS * w1 / dwa}, {0}, {kba * zaSS + r1b}};
(* The ratio between zaSS and xaSS is sinA/cosA = w1a/dwa *)
```

```
(* equation 17 comes from combining the 2 numerator and denominator terms: *)
ss3Zb = Collect[ Simplify[ss3[[3, 1]]], {zaSS} ]
ss3ZbTerm1Num = Collect[ Numerator[ss3Zb[[1]]], r1b, Simplify]
ss3ZbTerm1Den = Collect[ Denominator[ss3Zb[[1]]], w1, Simplify]
ss3ZbTerm2Num = Collect[ Numerator[ss3Zb[[2]]] / dwa, zaSS, Simplify]
ss3ZbTerm2Den = Collect[ Denominator[ss3Zb[[2]]] / dwa, w1, Simplify]
```

$$\frac{r1b\left(dwb^2+(kba+r2b)^2\right)}{dwb^2\,(kba+r1b)+(kba+r2b)\left(kba^2+r1b\,r2b+kba\,(r1b+r2b)+w1^2\right)}+\frac{\left(dwa\,kba\left(dwb^2+(kba+r2b)^2\right)+dwb\,kba\,w1^2\right)zaSS}{dwa\left(dwb^2\,(kba+r1b)+(kba+r2b)\left(kba^2+r1b\,r2b+kba\,(r1b+r2b)+w1^2\right)\right)}$$

$$r1b\left(dwb^2+(kba+r2b)^2\right)$$

$$(kba+r1b)\left(dwb^2+(kba+r2b)^2\right)+(kba+r2b)\,w1^2$$

$$kba\left(dwb^2+(kba+r2b)^2+\frac{dwb\,w1^2}{dwa}\right)zaSS$$

$$(kba+r1b)\left(dwb^2+(kba+r2b)^2\right)+(kba+r2b)\,w1^2$$

```
(* unitless form of Zb steady state value is
 used to derive eqn 28 in equations3and4and28.nb *)

normTransforms = {
    r1b → alpha * w1,
    r2b → beta * w1,
    kba → k * w1,
    dwb → delta * w1,
    dwa → deltaA * w1
   };

ss3ZbNorm = Simplify[ss3Zb //. normTransforms]
```

$$\left(alpha\,deltaA\left(beta^2+delta^2+2\,beta\,k+k^2\right)+k\left(delta+delta^2\,deltaA+deltaA\,(beta+k)^2\right)zaSS\right)\Big/\left(deltaA\left(beta^2\,(alpha+k)+alpha\left(delta^2+k^2\right)+k\left(1+delta^2+k^2\right)+beta\left(1+2\,alpha\,k+2\,k^2\right)\right)\right)$$