ASSIGNMENT - 1

MODEL - SOLUTION

(CHE221A)

1.8-3 Given; $P_A = 0.2\ MPa$, $V_A = 0.01\ m^3$

$U = 2.5PV + \text{constant}$ — (1)

→ For A → B;

$\Delta W_{AB} = -P\Delta V$

$= -(0.2\times10^6)(0.03-0.01)$

$\boxed{W_{AB} = -4000J}$

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$U_A = (2.5)(0.2\times10^6)(0.01) + C$ [Using (1)]

$\boxed{U_A = 5000 + C}$

$U_B = (2.5)(0.2\times10^6)(0.03)$

$\boxed{U_B = 15000 + C}$

$\therefore \Delta U_{AB} = U_B - U_A$ (3)

$\boxed{\Delta U_{AB} = 10000J}$

→ Using 1st Law of Thermodynamics,

$q_{AB} = \Delta U_{AB} - W_{AB}$

$\boxed{q_{AB} = 14000J}$

→ For B → C;

W_{BC} = Area under $\overline{BC}$ segment in PV diagram

$\boxed{W_{BC} = 7000J}$

$$U_C = 2.5 \times (0.5 \times 10^6) \times 0.01 + C$$

$$\boxed{U_C = 12500 + C}$$

$$\Delta U_{BC} = U_C - U_B$$

$$\boxed{\Delta U_{BC} = -2500\,J}$$ (3)

$$\Delta U_{BC} = Q_{BC} + W_{BC}$$

$$\Rightarrow \boxed{Q_{BC} = -9500\,J}$$

→ For C to A :-

$$\Delta W_{CA} = 0$$

From 1st of law of Thermodynamics (3)

$$\Delta U_{CA} = \Delta Q_{CA}$$

$$\therefore \boxed{\Delta Q_{CA} = -7500\,J}$$

→ Given; $P = 10^5 + (V-0.02)^2 \times 10^9$

$$W_{AB} = -\int P\,dV$$

$$= -\int_{V_A}^{V_B} \left[10^5 + 10^9 (V-0.02)^2\right] dV$$

$$= -\left\{10^5 [V]_{0.01}^{0.03} + \left[\frac{(V-0.02)^3}{3}\right]_{0.01}^{0.03} \times 10^9\right\}$$

$$= -\{2000 + 666.67\}$$ (3)

$$\boxed{W_{AB} = -2666.67\,J}$$

8-4 Given,

$$U = 2.5PV + \text{constant}$$

$$\therefore dU = 2.5PdV + 2.5VdP \quad - (1)$$

→ From 1st Law of Thermodynamics;

$$dU = dQ + dW$$

For adiabat,

$$dQ = 0$$

$$\therefore dU = dW$$

(5)

From (1);

$$2.5PdV + 2.5VdP = -PdV \qquad [\because dW = -PdV]$$

~~$\therefore 2.5PdV = -3.5P$~~

$$2.5VdP = -3.5PdV$$

$$\frac{dP}{P} = -\frac{7}{5}\frac{dV}{V}$$

Integrating;

$$\int_{P_1}^{P_2} \frac{dP}{P} = -\frac{7}{5}\int_{V_1}^{V_2} \frac{dV}{V}$$

$$[\ln(P)]_{P_2}^{P_1} = -7/5[\ln(V)]_{V_2}^{V_1}$$

$$\Rightarrow P_1^5 V_1^7 = P_2^5 V_2^7$$

Hence;

Equation of adiabat is as below:-

$$\boxed{P^5V^7 = k}$$

1.8-7

Given; $U = APV^2$, $N = 2$

~~Let~~ To get U in terms of P, V & N we write

$$\boxed{U = APV^2 \cdot f(N)}$$

Here $f(N) = aN^m$;

Moreover; $f(2) = a \cdot 2^m$

$$\therefore \boxed{1 = a \cdot 2^m}$$

→ Rewriting in terms of P;

(3)

$$P = \frac{U}{(AV^2)\cdot(a2^m)} \quad — (1)$$

→ If we ~~increase~~ double the system then;

$$P = \frac{2U}{A(4V^2).a4^m} \quad — (2)$$

→ Since, P does not change (1) = (2)

$$\therefore \frac{\cancel{2}\cancel{U}}{\cancel{A}(\cancel{4}\cancel{V^2})(\cancel{a}4^m)} = \frac{\cancel{U}}{\cancel{A}\cancel{V^2}(\cancel{a}2^m)}$$

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$$2^m = 2 \cdot 4^m$$

$$m\ln(2) = \ln(2) + m\ln(4)$$

$$\Rightarrow \boxed{m = -1} \text{ \& } \boxed{a = 2}$$

$$\therefore U = APV^2 \cdot a(N)^{-1}$$

$$\boxed{U = \frac{2APV^2}{N}}$$

[1-10-1]

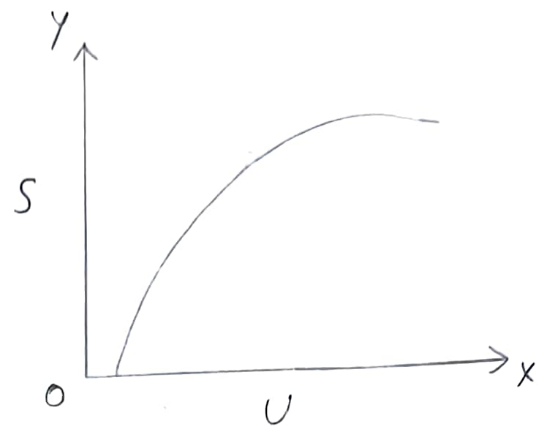
(f) $S = NR \ln(UV/N^2R\theta v_0)$

Let: $NR = A$ & $\frac{V}{N^2R\theta v_0} = B$

$\therefore S = A \ln(UB)$

→ Rewriting in terms of U we get;

$$U = \frac{1}{B} e^{S/A}$$



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$$\rightarrow \left(\frac{\partial U}{\partial S}\right)_{V,N} = \frac{1}{B} e^{S/A} \cdot \frac{1}{A}$$

$$\left(\frac{\partial U}{\partial S}\right)_{V,N} = \frac{1}{AB} e^{S/A}$$

Moreover,

$$\left(\frac{\partial U}{\partial S}\right)_{V,N} = T$$

$$\therefore \quad \frac{1}{AB} e^{S/A} = T$$

→ As LHS term can become zero only when $S \rightarrow -\infty$, but this violates 4^{th} postulate of Thermodynamics which says at zero absolute temp. entropy of system is also zero.

6-3

Given

$$\frac{1}{T^{(1)}} = \frac{3}{2} R \frac{N^{(1)}}{U^{(1)}} , \quad N^{(1)} = 2$$

$$\frac{1}{T^{(2)}} = \frac{5}{2} R \frac{N^{(2)}}{U^{(2)}} , \quad N^{(2)} = 3$$

$$U_{Total} = 2.5 \times 10^{3} J$$

$\because$ The wall is diathermal, the system will achieve thermal eqb when

$$\frac{1}{T^{(1)}} = \frac{1}{T^{(2)}}$$

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$$\frac{3}{\cancel{2}} \cancel{R} \frac{N^{(1)}}{U^{(1)}} = \frac{5}{\cancel{2}} \cancel{R} \frac{N^{(2)}}{U^{(2)}}$$

putting the value of $N^{(1)}$ & $N^{(2)}$

$$\Rightarrow \frac{6}{U^{(1)}} = \frac{15}{U^{(2)}}$$

$$\boxed{15 U^{(1)} - 6 U^{(2)} = 0} \quad — (1)$$

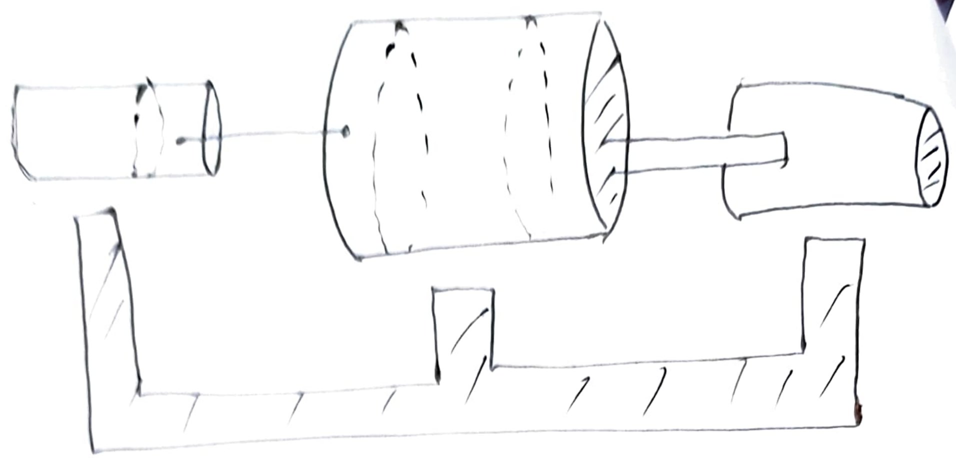
we also have

$$U_{Total} = U^{(1)} + U^{(2)} = 2500 J \quad — (2)$$

on solving equation (1) & (2)

$$\boxed{\begin{aligned} U^{(1)} &= 714.29 J \\ U^{(2)} &= 1785.714 J \end{aligned}}$$

2.7-1



$dV^{(1)} = A_1 dl_1$, $dV^{(2)} = A dl_2$, $dV^{(3)} = A_3 dl_3$

$$\boxed{dU = dU^{(1)} + dU^{(2)} + dU^{(3)} = 0}$$

$$dV^{(3)} = -3\,dV^{(1)} - \frac{3}{2}\,dV^{(2)} \quad (\because A_1 : A_3 : A_2 = 1:3:2)$$

Now,

$$dS = \frac{1}{T_1}\,dU^{(1)} + \frac{P^{(1)}}{T^{(1)}}\,dV^{(1)} + \frac{1}{T^2}\,dU^{(2)} + \frac{P^{(2)}}{T^{(2)}}\,dV^{(2)} + \frac{1}{T^{(3)}}\,dU^{(3)} + \frac{P^{(3)}}{T^{(3)}}\,dV^3$$

at eq^{b} $dS = 0$

$$\left(\frac{1}{T^{(1)}} - \frac{1}{T^{(3)}}\right)dU^{(1)} + \left(\frac{1}{T^{(2)}} - \frac{1}{T^{(3)}}\right)dU^{(2)} + \left(\frac{P^{(1)}}{T^{(1)}} - \frac{3P^{(3)}}{T^{(3)}}\right)dV^{1} + \left(\frac{P^{(2)}}{T^{(2)}} - \frac{3}{2}\frac{P^{(3)}}{T^{(3)}}\right)dV^{(2)} = 0$$

So,
$$\boxed{\frac{1}{T^{(1)}} = \frac{1}{T^{(2)}} = \frac{1}{T^{(3)}}}$$

$$P^{(1)} = 3P^{(3)}$$
$$P^{(2)} = \frac{3}{2}P^{(3)}$$

$\therefore P = $ Force × Area
$P^{(1)} = F \times A^{(1)}$
$\therefore A_1 : A_3 : A_2 = 1:3:2$

$$\boxed{\text{Ratio of pressure} = 1 : \frac{1}{2} : \frac{1}{3}}$$

As we know

$$U = U(S, V, x_1, x_2, \ldots)$$

where $x_i = \frac{N_i}{N}$

Taking total differential of equation we get,

$$du = \left(\frac{\partial U}{\partial S}\right)_{V, x_1, x_2 \ldots} ds + \left(\frac{\partial U}{\partial V}\right)_{S, x_1, x_2 \ldots} dV + \sum_i \left(\frac{\partial U}{\partial x_i}\right)_{S, V, x_{i \neq j}} dx_i$$

Also, we know

$$\left(\frac{\partial U}{\partial V}\right)_{S, x_1, x_2 \ldots} = -P \quad \Bigg| \quad \left(\frac{\partial U}{\partial S}\right)_{V, x_1, x_2 \ldots} = T \quad \Bigg| \quad \left(\frac{\partial U}{\partial x_i}\right)_{S, V, x_{i \neq j}} = \mu_i$$

So,

$$\boxed{du = Tds - PdV + \sum_j^r \mu_j dx_j} \quad \text{--- (1)}$$

(5)

$$du = Tds - PdV + \sum_{J=1}^{r-1} \mu_j dx_j + \mu_r dx_r$$

$\therefore \quad x_1 + x_2 + x_3 + \text{----} \; x_r = 1$

$$\Rightarrow \underbrace{dx_1 + dx_2 + dx_3 + \text{---}}_{\sum_{J=1}^{r-1} dx_j} + dx_r = 0$$

$$\Rightarrow \quad dx_r = -\sum_{J=1}^{r-1} dx_j \quad \text{--- (2)}$$

putting dx_r in eqn (2)

$$du = Tds - PdV + \sum_{J=1}^{r-1} \mu_j dx_j + \mu_r \sum_{J=1}^{r-1} -dx_j$$

$$\boxed{dU = Tds - PdV + \sum_{J=1}^{r-1} (\mu_j - \mu_r) dx_j}$$

proved

2.2-1

Q-8-1

Ai, we have

$$U = \left(\frac{V_0 \theta}{R^2}\right) \frac{S^3}{NV}$$

So, Equation of state in term of temprature (we know):-

$$\left(\frac{\partial U}{\partial S}\right)_{N,V} = T$$

$$\Rightarrow \boxed{T = \frac{3S^2 V_0 \theta}{R^2 NV} = \left(\frac{V_0 \theta}{R^2}\right) \frac{3S^2}{NV}}$$

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Equation of state in term of pressure is as follow:-

$$P = -\left(\frac{\partial U}{\partial V}\right)_{S,N} = -\frac{V_0 \theta}{R^2}\left(-\frac{S^3}{NV^2}\right)$$

$$\boxed{P = +\left(\frac{V_0 \theta}{R^2}\right) \frac{S^3}{NV^2}}$$

Equation of state in term of chemical potential is as follow:-

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V} = \frac{V_0 \theta}{R^2}\left(\frac{-S^3}{N^2 V}\right)$$

$$\boxed{\mu = -\left(\frac{V_0 \theta}{R^2}\right) \frac{S^3}{N^2 V}}$$

For Equation of state to be homogenous zero order

we replace (S, V, N): $S \to KS$, $N \to KN$, $V \to KV$

where K is arbitary constant

Let's take $T' = \left(\frac{V_0 \theta}{R^2}\right) \frac{3K^2 S^2}{(KN)\cdot(KV)}$

$$\boxed{T' = \left(\frac{V_0 \theta}{R^2}\right) \frac{3S^2}{NV} = T}$$

Similarly, we get $P' = P$ & $\mu'_i = \mu$ proved.

2.8-1

$$S = NA + NR\ln\left(\frac{U^{3/2}V}{N^{5/2}}\right) - N_1R\ln\left(\frac{N_1}{N}\right) - N_2R\ln\left(N_2/N\right)$$

$N = N_1 + N_2$

$$\boxed{\frac{\partial S}{\partial U} = \frac{3}{2}\frac{NR}{U}}$$

Given data

$V_T = 10L$

$N_1^{(1)} = 0.5$, $N_2^{(1)} = 0.75$, $V^1 = 5L$, $T^{(1)} = 300k$

$N_1^{(2)} = 1$, $N_2^{(2)} = 0.5$, $V^2 = 5L$ & $T^{(2)} = 250k$

For eq

$$\boxed{\frac{\mu_1^{(1)}}{T^{(1)}} = \frac{\mu_1^{(2)}}{T^{(2)}}}, \quad \boxed{\frac{1}{T^{(1)}} = \frac{1}{T^{(2)}}}$$

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$$\frac{\partial S}{\partial U} = \frac{1}{T} = \frac{3}{2}\frac{NR}{U}$$

$$\frac{1}{T^{(1)}} = \frac{3}{2}\frac{N^{(1)}R}{U^{(1)}}, \quad \frac{P^{(1)}}{T^{(1)}} = R\frac{N^{(1)}}{V^{(1)}}$$

$$\frac{\mu^1}{T^1} = A + R\ln\left[\frac{U^{(1)3/2}V^{(1)}}{N^{(1)5/2}}\right] - \frac{5}{2}R - R\ln\left(\frac{N_1'}{N'}\right)$$

$$\frac{1}{300} = \frac{3}{2}\times 8.3\times\frac{1.25}{U^{(1)}}, \quad \frac{1}{250} = \frac{3}{2}\times 8.3\times\frac{1.5}{U^{(2)}}$$

$\therefore$ $U^{(1)} = 4668.75\,J$ & $U^{(2)} = 4668.75\,J$

$$U_T = U^{(1)} + U^{(2)} = 9337.5\,J$$

for eq^n:

$$\frac{1}{T^{(1)}} = \frac{1}{T^{(2)}}, \quad \frac{\mu_1^{(1)}}{T^{(1)}} = \frac{\mu_1^{(2)}}{T^{(2)}}$$

so we have,

$$\frac{N_1^{(1)} + 0.75}{U^{(1)}} = \frac{N_1^{(2)} + 0.5}{U^{(2)}}$$

$$\frac{[U^{(1)}]^{3/2}}{(N_1^{(1)} + 0.75)^{3/2} N_1^{(1)}} = \frac{(U^{(2)})^{3/2}}{(N_1^{(2)} + 0.5)^{3/2} N_1^{(2)}}$$

$\therefore$ we have

$$\frac{1}{N^{(1)}} = \frac{1}{N^{(2)}}$$

or $N^{(1)} = N^{(2)}$

$\therefore N = N^{(1)} + N^{(2)} = 1.5$

So, $N^{(1)} = N^{(2)} = 0.75$

The final internal energy can be calculated using,

$$U^{(1)} = \left(\frac{0.75 + 0.75}{0.75 + 0.75}\right) U^{(2)}$$

$$U^{(1)} + U^{(2)} = 9337.5$$

So,

$$U^{(1)} = 5093.18, \quad U^{(2)} = 4244.32$$

Thus the final eq^b temperature is,

$$\frac{1}{T} = \frac{3}{2} R \times \frac{N^{(1)}}{U^{(1)}} = \frac{3}{2} \times 8.3 \times \frac{1.5}{5093.18}$$

giving $T = 272.73$, the pressure can then be calculated by using

$$p^{(1)} = \frac{N^{(1)} R T^{(1)}}{V} = \frac{1.5 \times 8.31 \times 272.3}{5}$$

$$= 679.1 \text{ pa}$$

$$p^{(2)} = \frac{N^{(2)} R T^{(2)}}{V} = 565.9 \text{ pa}$$