Solution (Quiz-1)

QN-1 (i) Given, $T = \frac{3as^2}{NV}$ —— (a)

& $P = \frac{as^3}{NV^2}$ —— (b)

from the Gibbs-Duhem equation

$$N d\mu = V dp - S dT \quad \text{—— (1)}$$

2

$$N d\mu = V d\left(\frac{as^3}{NV^2}\right) - S d\left(\frac{3as^2}{NV}\right)$$

$$= \frac{Va}{N} d\left(\frac{s^3}{V^2}\right) - \frac{3as}{N} d\left(\frac{s^2}{V}\right)$$

$$= \frac{Va}{N}\left\{ s^3(-2V^{-3})dV + \frac{1}{V^2}\times(3s^2)ds \right\}$$

$$- \frac{3as}{N}\left\{ s^2(-1\times V^{-2})dV + \frac{1}{V}\times 2s\, ds \right\}$$

2

$$= \frac{Va}{N}\left\{ -\frac{2s^3}{V^3}dV + \frac{3s^2}{V^2}ds \right\}$$

$$- \frac{3as}{N}\left\{ -\frac{s^2}{V^2}dV + \frac{2s}{V}ds \right\}$$

$$= \left(\frac{-2as^3}{NV^2} + \frac{3as^3}{NV^2}\right)dV +$$

$$\left(\frac{3as^2}{NV} - \frac{6as^2}{NV}\right)ds$$

$$N\,d\mu = \frac{as^3}{NV^2}\,dV - \frac{3as^2}{NV}\,ds \quad (1)$$

$$\frac{N^2}{a}\,d\mu = -\frac{3s^2}{V}\,ds + \frac{s^3}{V^2}\,dV$$

$$\frac{N^2}{a}\,d\mu = \left(-\frac{1}{V}\right)d(s^3) + s^3\,d\left(-\frac{1}{V}\right) \quad (2)$$

$$\frac{N^2}{a}\,d\mu = d\left(-\frac{1}{V}s^3\right) \quad (2)$$

$$d\mu = d\left(-\frac{as^3}{N^2V}\right)$$

$$\int d\mu = \int d\left(-\frac{as^3}{N^2V}\right) + \text{constant}$$

$$\mu = -\frac{as^3}{N^2V} + \text{constant}$$

$$\Rightarrow \boxed{\mu = -\frac{as^3}{N^2V} + \mu_0} \quad (2) \quad (1)$$

1) From Euler equation

$$U = TS - PV + \mu N \quad \text{--- (3)} \qquad (2)$$

After substituting T, P & μ from (a), (b), (2) to (3), we get

$$U = \left(\frac{3as^2}{NV}\right)S - \left(\frac{as^3}{NV^2}\right)V + \left(\mu_0 - \frac{as^3}{N^2V}\right)N$$

$$U = \frac{3as^3}{NV} - \frac{as^3}{NV} + \mu_0 N - \frac{as^3}{NV} \qquad (2)$$

Fundamental Equation in Enthalpy Representation

$$\boxed{U = \frac{as^3}{NV} + \mu_0 N} \qquad (1)$$

(2) $H = 2\left(\frac{CS^3P}{N}\right)^{1/2}$

→ Using:

$$\left(\frac{\partial H}{\partial P}\right)_{S,N} = V \quad 4$$

$$\therefore V = 2\left(\frac{CS^3}{N}\right)^{1/2}\cdot\frac{1}{2}P^{-1/2} \quad 2$$

$$V = \left(\frac{CS^3}{N}\right)^{1/2}\frac{1}{P^{1/2}} \Rightarrow \boxed{P = \frac{CS^3}{NV^2}} \quad -(1) \quad 1$$

→ $H = PV + U$ 4

$$2\cdot\left(\frac{CS^3}{N}\cdot\frac{CS^3}{NV^2}\right)^{1/2} = \frac{CS^3}{N\cancel{V^2}}\cdot\cancel{V} + U \quad 3 \quad [\text{using } (1)]$$

$$\frac{2CS^3}{NV} = \frac{CS^3}{NV} + U$$

$$\Rightarrow \boxed{U = \frac{CS^3}{NV}} \quad 1$$

Fundament eqn in
Internal energy representation