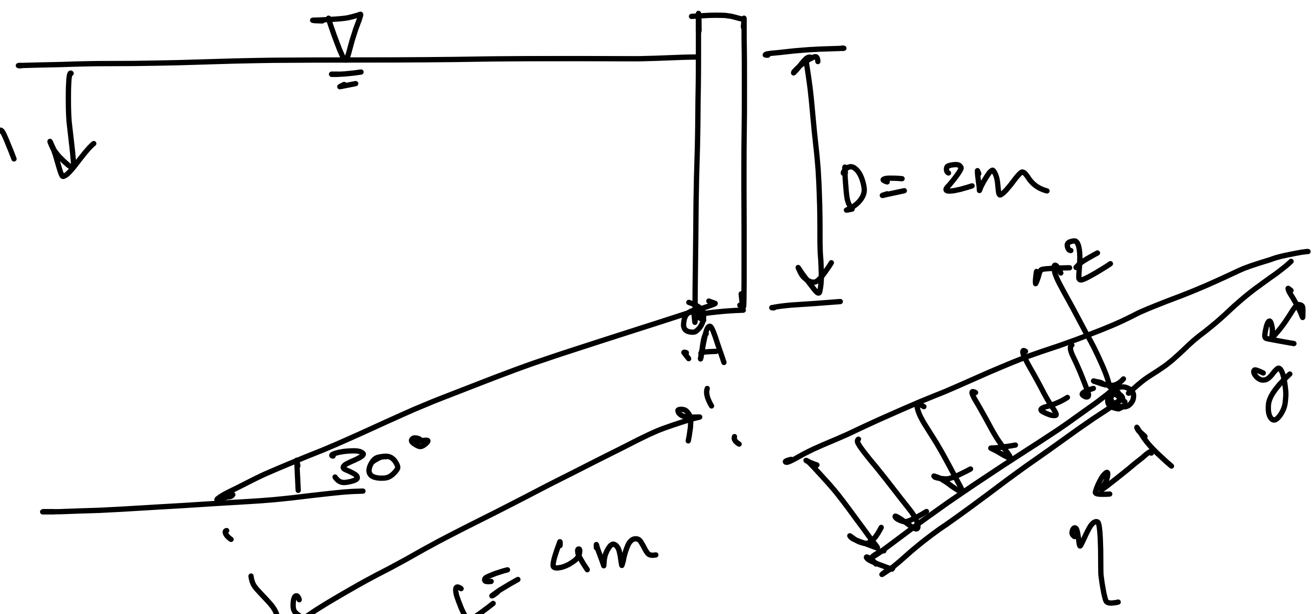
1.



Resultant force (F_R) of the water & the air on the gate is

$$F_R = \int_A p \, dA \qquad (1)$$

$$p = p_0 + \rho g h$$

$$h = D + \eta \sin 30^\circ$$

$$dA = w \, d\eta$$

Let $p = \rho g (D + \eta \sin 30^\circ)$, ignore p_0 as it is on both sides of the gate

Substituting —

$$F_R = \int_0^L \rho g (D + \eta \sin 30^\circ) \, w \, d\eta$$

$$= \rho g w \int_0^L [D + \eta \sin 30^\circ] \, d\eta$$

$$= \rho g w \left[D\eta + \frac{\eta^2}{2} \sin 30^\circ \right]_0^L$$

$$= \rho g w \left[DL + \frac{L^2}{2} \sin 30^\circ \right]$$

$g = 9.8 \ ms^{-2}$

$w = 5m$ $\quad D = 2m$

$L = 4m$

$\rho = 1000 \ kg/m^3$

$$\therefore \boxed{F_R = 588.5 \ kN} \ \text{Ans}$$

Dir^n of force is $\perp$ to the surface

2. $v = 50\ ms^{-1}$

$$T = \frac{-3}{500} z + 15$$

calculate material derivative

$$\frac{DT}{Dt} = \cancel{\frac{\partial T}{\partial t}} + \underline{v} \cdot \nabla T$$

steady state

$$\frac{DT}{Dt} = \underline{v} \cdot \nabla T \qquad \underline{v} = -50\,\hat{k}\ ms^{-1}$$

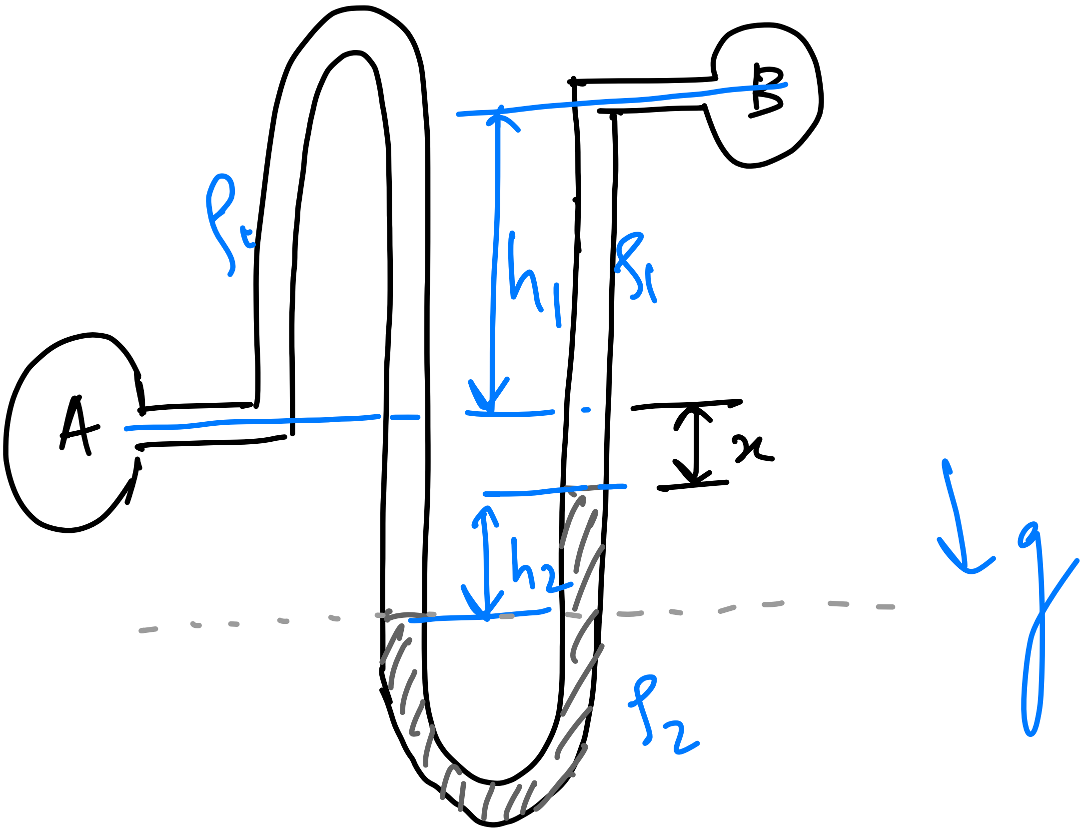
$$\frac{DT}{Dt} = (\underbrace{\cancel{v_x}}_{0}\,\hat{i} + \underbrace{\cancel{v_y}}_{0}\,\hat{j} + v_z\,\hat{k}) \cdot \left(\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}\right)$$

only in z-dirn

$$= (-50\,\hat{k}) \cdot \left(\frac{-3}{500}\,\hat{k}\right)$$

$$= \boxed{0.3\ ^{\circ}C\ s^{-1}} \text{ Ans}$$

3.



$$P_A + \rho_1 (x + h_2) g = P_B + \rho_1 (h_1 + x) g + \rho_2 h_2 g$$

$$P_A + \cancel{\rho_1 x g} + \rho_1 h_2 g = P_B + \rho_1 h_1 g + \cancel{\rho_1 x g} + \rho_2 h_2 g$$

$$\boxed{P_A - P_B = \rho_1 (h_1 - h_2) g + \rho_2 h_2 g}$$

Ans.

For freely falling manometer,

$$P_A - P_B = 0$$