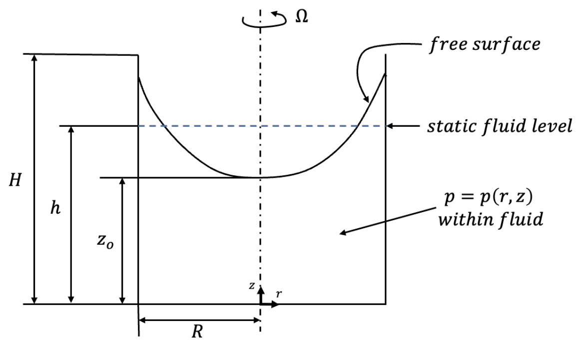
PRACTICE PROBLEM SET 3

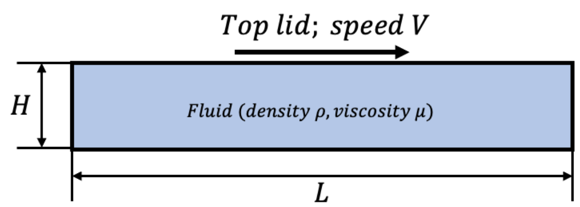
(These problems have been sourced from various sources)

1. (a) A liquid of constant density and viscosity is in a cylindrical container of radius R, as shown in the below figure. The container is caused to rotate about its axis at an angular velocity Ω. Show that the shape of the free surface of the liquid when a steady state has been established can be expressed by the relation: $z - z_0 = (\Omega^2/2g)r^2$.



   (b) For the same setup, if H is the height of the cylindrical container and liquid is filled up to height h, deduce an expression for the maximum angular velocity $\Omega_{\max}$ at which the cylinder can be rotated such that no liquid spills out of the container (Hint: Express the free surface in terms of H).

2. A cuboid container of length L, width W, and height H, where $L, W \gg H$, is shown below. The container is covered with a sufficiently long flat lid at the top. If the lid is slid horizontally with a speed V:



A) Roughly sketch the horizontal velocity profile far from the ends.

B) Deduce the governing equations and specify the boundary conditions.

C) How does this problem differ from the simple Couette flow problem solved in the lectures? What are the additional restrictions imposed on the current system?

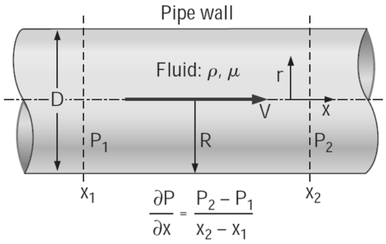
D) Using the information from sections B and C, deduce the expression for the velocity profile far from the ends.

E) Will there be a pressure drop? If yes, deduce the expression for the same.

F) Calculate the shear stress on the moving lid.

3. Consider steady, incompressible, laminar flow of a Newtonian fluid in an infinitely long round pipe of diameter D or radius $R = \frac{D}{2}$. We ignore the effects of gravity. A constant pressure gradient $\frac{\partial P}{\partial x}$ is applied in the x-direction,

$$\frac{\partial P}{\partial x} = \frac{P_2 - P_1}{x_2 - x_1} \text{ constant} \tag{1}$$



where x_1 and x_2 are two arbitrary locations along the x-axis, and P_1 and P_2 are the pressures at those two locations. Note that we adopt a modified cylindrical coordinate system here with x instead of z for the axial component, namely, (r, u, x) and (u_r, u_θ, u). Derive an expression for the velocity field inside the pipe and estimate the viscous shear force per unit surface area acting on the pipe wall.

4. The viscous oil is set into steady motion by a concentric inner cylinder moving axially at velocity U inside a fixed outer cylinder. Assuming constant pressure and density and a purely axial fluid motion, solve for the fluid velocity distribution $v_z(r)$. What are the proper boundary conditions? (Refer figure 1)

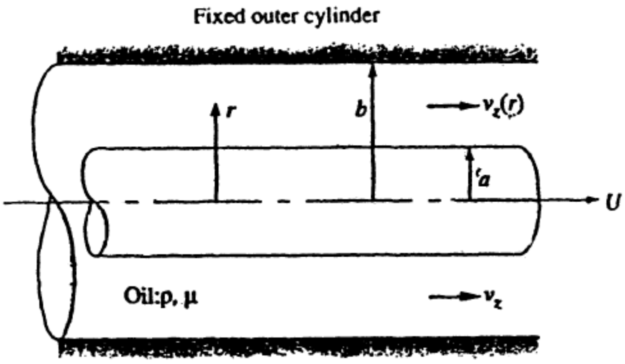
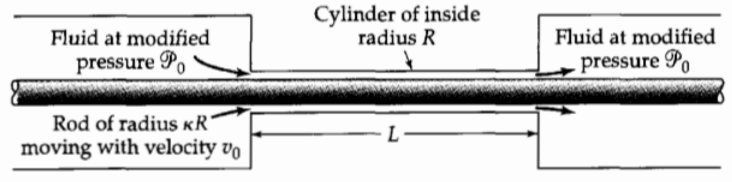


Figure 1: Schematics for question 6

5. The space between two coaxial cylinders is filled with an incompressible fluid at constant temperature. The radii of the inner and outer wetted surfaces are κR and R, respectively. The angular velocities of rotation of the inner and outer cylinders are Ω_i and Ω_o. Determine the velocity distribution in the fluid and the torques exerted by the fluid on the two cylinders needed to maintain the motion.

6. Fluid A has a viscosity twice that of fluid B, while Fluid A has a density half that of fluid B. A plane-Couette experiment is conducted. Which fluid would you expect to exert a larger stress on the moving plate? Which fluid is expected to reach steady earlier? Why?

7. A cylindrical rod of radius kR moves axially with velocity $v_z = v_o$ along the axis of a cylindrical cavity of radius R as seen in the figure. The pressure at both ends of the cavity is the same, so that the fluid moves through the annular region solely because of the rod motion.



A) Find the velocity distribution in the narrow annular region.

B) Find the mass rate of flow through the annular region.

C) Obtain the viscous force acting on the rod over the length L.

8. Show that the two-dimensional flow field of $u = Kx$, $v = -Ky$, $w = 0$ is an exact solution to the incompressible Navier-Stokes equation. Neglecting gravity, compute the pressure field $p(x, y)$ and relate it to the absolute velocity $V = \sqrt{u^2 + v^2}$. Interpret the result.

9. A viscous liquid of constant density and viscosity falls due to gravity between two parallel plates a distance $2h$ apart, as in the figure. The flow is fully developed, that is, $w = w(x)$ only. There are no pressure gradients, only gravity. Set up and solve the Navier-Stokes equation for the velocity profile $w(x)$.

10. An oil film drains steadily down the side of a vertical wall, as shown. After an initial development at the top of the wall, the film becomes independent of z and of constant thickness. Assume that $w = w(x)$ only that the atmosphere offers no shear resistance to the film. Solve Navier-Stokes for $w(x)$.