PRACTICE PROBLEM SET 2

(These problems have been sourced from various sources)

1. (a) A liquid film of thickness δ flows down an inclined flat plate of width W. If the velocity profile of the falling film is given by

$$v_z(x) = \left(\frac{\rho g \delta^2 \cos\beta}{2\mu}\right)\left\{1 - \left(\frac{x}{\delta}\right)^2\right\}, \quad 0 \leq x \leq \delta, 0 \leq y \leq W,$$

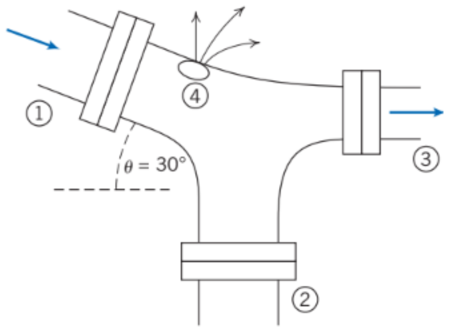
where μ is the fluid viscosity and β (constant) is the angle of inclination of the plate with the vertically downward direction, determine the maximum velocity $v_{z,\max}$, the average velocity $\overline{v_z}$, and the mass flow rate $\dot{m}$. Also, determine the shear stress profile τ_{xz}.

   (b) Repeat experiment 1a for a flow through a circular pipe of radius R and length L, whose velocity profile is given by

$$v_z(r) = \left(\frac{(P_0 - P_L)R^2}{4\mu L}\right)\left\{1 - \left(\frac{r}{R}\right)^2\right\}, \quad 0 \leq r \leq R, 0 \leq z \leq L.$$

2. A horizontal conveyor belt moving at 0.5 m/s receives sand from a hopper. The sand falls vertically from the hopper to the belt at a speed of 1.5 m/s and a flow rate of 250 kg/s (the bulk density of sand is approximately 1600 kg/m^3). The conveyor belt is initially empty but begins to fill with sand. If friction in the drive system and rollers is negligible, find the tension required to pull the belt while the conveyor is filling.

3. Consider the steady flow in a water pipe joint shown in the diagram. The areas are $A_1 = 0.2$ m^2, $A_2 = 0.2$ m^2, and $A_3 = 0.15$ m^2. In addition, fluid is lost out of a hole at 4, estimated at a rate of 0.1 m^3/s. The average speeds at sections 1 and 3 are $V_1 = 5$ m/s and $V_3 = 12$ m/s, respectively. Find the velocity at section 2.

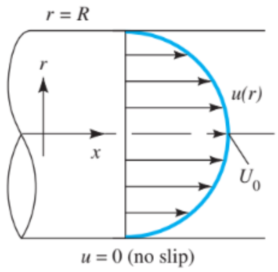


4. You are trying to pump storm water out of your basement during a storm. The pump can extract 27.5 gpm. The water level in the basement is now sinking by about 4 in./hr. What is the flow rate (gpm) from the storm into the basement? The basement is 30 ft × 20 ft.

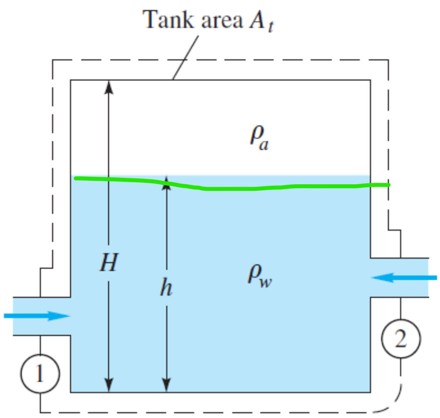
5. For steady viscous flow through a circular tube, the axial velocity profile is given by

$$u = U_0\left(1 - \frac{r}{R}\right)^m$$

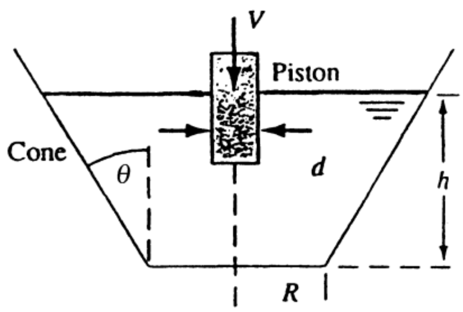
such that u varies from zero at the wall (r = R), or no slip, to a maximum $u = U_0$ at the centerline $r = 0$. For highly viscous (laminar) flow $m \approx \frac{1}{2}$, while for less viscous (turbulent) flow $m \approx \frac{1}{7}$. Compute the average velocity if the density is constant.



6. The tank, shown below, is being filled with water by two one-dimensional inlets. Air is trapped at the top of the tank. The water height is h.
   (a) Find an expression for the change in water height dh/dt.
   (b) Compute dh/dt if $D_1 = 1$ in, $D_2 = 3$ in, $V_1 = 3$ ft/s, $V_2 = 2$ ft/s, and $A_t = 2$ ft^2, assuming water at 20 °C.



7. The cone frustum in the figure contains incompressible liquid to depth h. A solid piston of diameter d penetrates the surface at velocity V. Derive an analytic expression for the rate of rise dh/dt of the liquid surface.



8. A tank of volume 0.05 m^3 contains air at 800 kPa (absolute) and 15 °C. At t= 0, air begins escaping from the tank through a valve with a flow area of 65 mm^2. The air passing through the valve has a speed of 300 m/s and a density of 6 kg/m^3. Determine the instantaneous rate of change of density in the tank at t= 0.