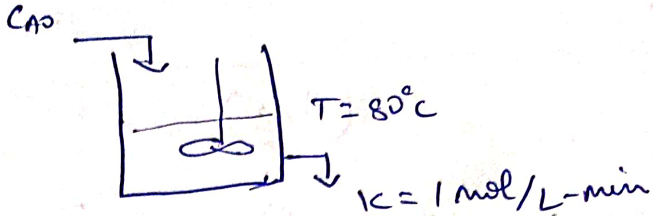
Quiz-5 Solutions

1) $C_{A0} = 75 \text{ mol/L}$

$k = 1 \text{ mol/L-min}$



$$E(t) = \frac{\exp[-t/100]}{75}$$

For a zero order reaction: $C_{A0}\dfrac{dx}{dt} = k$

$$\frac{dx}{dt} = k/C_{A0}$$

$$\int_0^x dx = \frac{k}{C_{A0}} \int_0^t dt$$

$$\boxed{x = \frac{kt}{C_{A0}}} \Rightarrow t = 0 \text{ to } 75 \text{ min}$$

$x = 0$ at $t = 0$

$x = 1$ at $t \geq 75$

$$\bar{x} = \frac{\int_0^\infty x(t)\, E(t)\, dt}{\int_0^\infty E(t)\, dt} \qquad \text{[Refer Fogler]}$$

$$\int_0^\infty E(t)\, dt = \int_0^\infty \frac{\exp[-t/100]}{75} = \frac{4}{3}$$

$$\bar{x} = \frac{\int_0^{75} \frac{kt}{C_{A0}} \frac{\exp[-t/100]}{75} dt + \int_{75}^\infty \frac{\exp(-t/100)}{75} dt}{(4/3)}$$

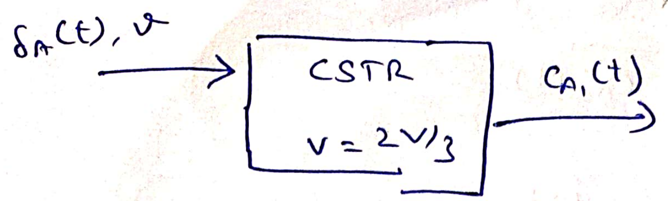
$$= \frac{\frac{1}{(75)^2}\int_0^{75} t \cdot \exp(-t/100)\, dt + \int_{75}^{\infty} \frac{\exp(-t/100)}{75}\, dt}{(4/3)}$$

$$= \frac{\frac{1}{(75)^2}\left[\frac{1-(75/100+1)e^{-75/100}}{(1/100)^2}\right] + \left(\frac{4}{3}\right)\exp(-3/4)}{(4/3)}$$

$$= \frac{\left(\frac{4}{3}\right)^2 [0.17336) + (4/3)(0.4723)}{(4/3)}$$

$$\boxed{\bar{X} = 0.7035}$$

2.



Assuming initially A moles of pulse.

In - out = Accumulation.

$$A\,\delta_A(t) - v\,C_{A1}(t) = \frac{2V}{3}\frac{dC_{A1}}{dt}$$

$$\frac{A}{v}\delta_A(t) - C_{A1}(t) = \frac{2\tau}{3}\frac{dC_{A1}}{dt}$$

(If we define, $\tau = \frac{V}{v}$, τ is not the residence time of the reactor]

$$\frac{2\tau}{3}\frac{dC_{A1}}{dt} + C_{A1}(t) = \frac{A}{v}\delta_A(t)$$

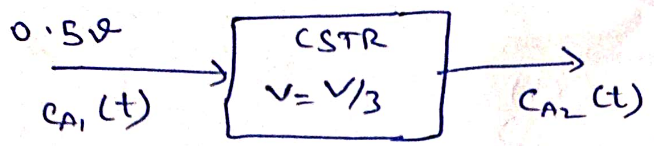
$$\frac{d}{dt}\left[e^{3t/2\tau}C_{A1}(t)\right] = \frac{3A}{2v\tau}e^{(3t/2\tau)}\delta_A(t)$$

$$\int_0^t d\left[e^{3t/2\tau}C_{A1}(t)\right] = \frac{3A}{2v\tau}\int_0^t \delta_A(t)\,e^{(3t/2\tau)}\,dt$$

$\left[\begin{array}{l} t=0, \quad C_{A1}(t)=0 \\ t=t, \quad C_{A1}(t)=C_{A1}(t) \end{array}\right]$

$$e^{\frac{3t}{2\tau}}C_{A1}(t) = \frac{3A}{2v\tau}$$

$$\boxed{C_{A1}(t) = \frac{3A}{2v\tau}e^{-\frac{3t}{2\tau}}}$$



In − Out = Accumulation

$$0.5v\, C_{A_1}(t) - 0.5v\, C_{A_2}(t) = \frac{V}{3}\frac{dC_{A_2}}{dt}$$

$$C_{A_1}(t) - C_{A_2}(t) = \frac{2\tau}{3}\frac{dC_{A_2}}{dt}$$

$$\frac{2\tau}{3}\frac{dC_{A_2}}{dt} + C_{A_2}(t) = C_{A_1}(t) = \frac{3A}{2v\tau}e^{-3t/2\tau}$$

$$\int_0^t d\left[e^{3t/2\tau}C_{A_2}(t)\right] = \int_0^t \frac{3}{2\tau}\cdot\frac{3A}{2v\tau}e^{(-3t/2\tau)}\cdot e^{(3t/2\tau)}\,dt$$

$$e^{3t/2\tau}C_{A_2}(t) = \frac{9A}{4v\tau^2}(t-0) \qquad \left[\begin{matrix} t=0, & C_{A_2}=0 \\ t=t, & C_{A_2}=C_{A_2}(t)\end{matrix}\right]$$

$$\boxed{C_{A_2}(t) = \frac{9A}{4v\tau^2}(t)\cdot e^{-3t/2\tau}}$$

$$C_A(t) = \frac{0.5\cancel{v}\,C_{A_1}(t) + 0.5\cancel{v}\,C_{A_2}(t)}{\cancel{v}}$$

$$= \frac{1}{2}\left[C_{A_1}(t) + C_{A_2}(t)\right]$$

$$C_A(t) = \frac{1}{2}\left[\frac{3A}{2v\tau}e^{-3t/2\tau} + \frac{9At}{4v\tau^2}e^{-3t/2\tau}\right]$$

$$\boxed{C_A(t) = \frac{1}{2}\times\frac{3A}{2v\tau}\left[e^{-3t/2\tau} + \frac{3t}{2\tau}e^{-3t/2\tau}\right]}$$

$$E(t) = \frac{C_A(t)}{\int_0^\infty C_A(t)\,dt}$$

$$\int_0^\infty C_A(t)\,dt = \frac{3A}{4v\tau}\left[\int_0^\infty e^{-3t/2\tau}\,dt + \int_0^\infty \frac{3t}{2\tau}\,e^{-3t/2\tau}\,dt\right]$$

$$= \frac{3A}{4v\tau}\left[\frac{e^{-3t/2\tau}}{\left(\frac{-3}{2\tau}\right)}\right]_0^\infty + \frac{3A}{4v\tau}\cdot\frac{3}{2\tau}\int_0^\infty t\cdot e^{-\frac{3t}{2\tau}}\,dt$$

$$= \frac{3A}{4v\tau}\left[2\tau/3\right] + \frac{3A}{4v\tau}\left(\cancel{\frac{3}{2\tau}}\right)\frac{1}{\left(\frac{3}{2\tau}\right)^{\cancel{2}}}$$

$$= \frac{3A}{4v\tau}\left[\frac{2\tau}{3} + \frac{2\tau}{3}\right]$$

$$= \frac{\cancel{3}A}{\cancel{4}v\cancel{\tau}}\left[\frac{\cancel{4}\cancel{\tau}}{\cancel{3}}\right] = \frac{A}{v}$$

$$E(t) = \frac{\frac{3\cancel{A}}{4\cancel{v}\tau}\left[e^{-3t/2\tau} + \frac{3t}{2\tau}e^{-3t/2\tau}\right]}{\left(\cancel{A}/\cancel{v}\right)}$$

$$E(t) = \frac{3}{4\tau}\left[e^{-3t/2\tau} + \frac{3t}{2\tau}e^{-3t/2\tau}\right]$$