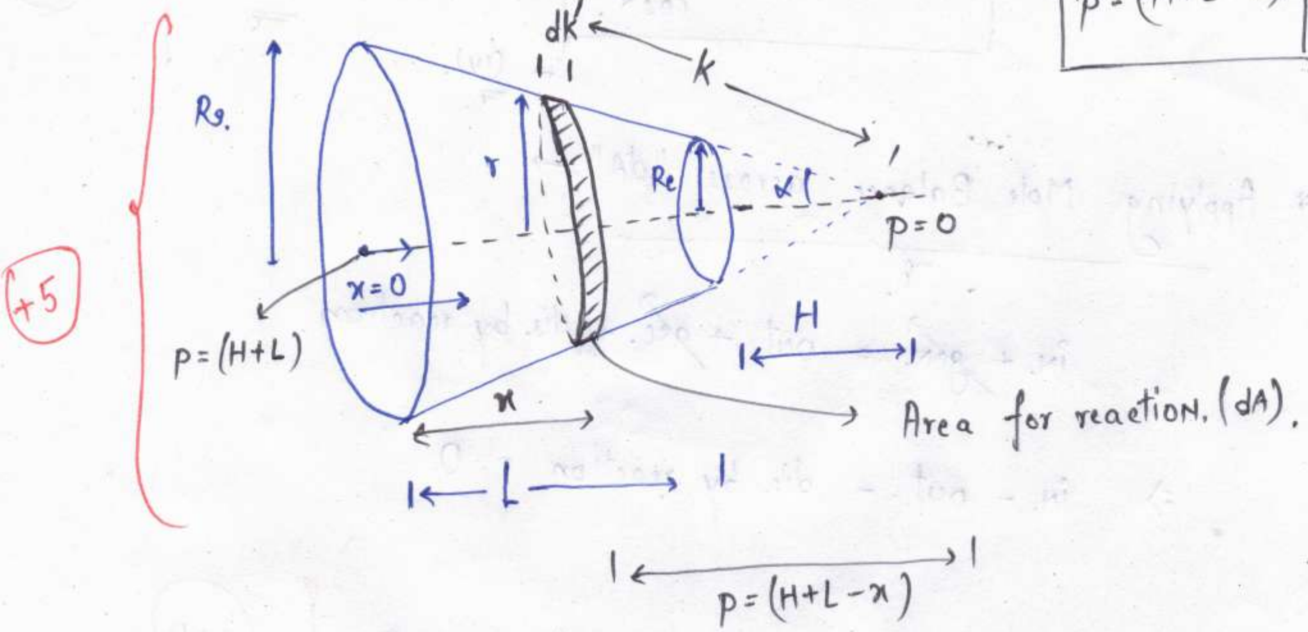
Quiz 3 : CRE - 631

Solution

$$\boxed{p = (H + L - x)}$$



From Trigonometry:

$$\tan\alpha = \frac{R_e}{H} = \frac{R_s}{(H+L)}$$

* Defining a new variable "p":

such that:

$$p = (H + L - x)$$

$$\Rightarrow \boxed{dp = -dx.} \quad \text{- (i)}$$

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also,

$$\cos\alpha = \frac{(H+L-x)}{k} = \frac{p}{k}$$

$$\Rightarrow \boxed{dp = \cos\alpha \, dk.} \quad \text{- (ii)}$$

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and,

$$\tan\alpha = r/p$$

$$\Rightarrow \boxed{r = p\tan\alpha.} \quad \text{- (iii)}$$

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(2)

*. Area Element for reaction:

$$dA = 2\pi r \Delta k$$

$$\Rightarrow \boxed{dA = 2\pi (p\tan\alpha) \frac{\Delta p}{\cos\alpha}} \quad \ldots \text{ from (ii) \& (iii)} \quad \text{— (iv).}$$

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*. Applying Mole Balance across "dA": →

in. + ~~gen.~~ = out. + ~~acc.~~ + dis. by reaction.

$\Rightarrow$ in. − out. − dis. by reaction = 0

$$\Rightarrow (\pi r^2) W_A|_x - (\pi r^2) W_A|_{x+\Delta x} - k'' C_A\, dA = 0$$

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Using eq^N (iii) and (iv).

$$\left|\pi p^2 \tan^2\alpha \left(-D_{AB}\cdot\frac{dC_A}{dx}\right)\right|_x - \left|\pi p^2 \tan^2\alpha \left(-D_{AB}\cdot\frac{dC_A}{dx}\right)\right|_{x+\Delta x} - \frac{k'' C_A (2\pi) p \tan\alpha . \Delta p}{\cos\alpha} = 0$$

$$\Rightarrow \left|\cancel{\pi} p^2 . \tan^{\cancel{2}}\alpha . \left(D_{AB}\cdot\frac{dC_A}{dp}\right)\right|_p - \left|\cancel{\pi} p^2 \tan^{\cancel{2}}\alpha \left(D_{AB}\cdot\frac{dC_A}{dp}\right)\right|_{p-\Delta p} - \frac{k'' C_A (2\cancel{\pi}) p . \cancel{\tan\alpha} \Delta p}{\cos\alpha} =$$

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*. DIVIDING by Δp throughout and taking $\lim \Delta p \to 0$

$$\lim_{\Delta p \to 0} \left(\frac{\left| p^2 \left(D_{AB} \cdot \frac{dC_A}{dp} \right) \right|_{p} - \left| p^2 \left(D_{AB} \cdot \frac{dC_A}{dp} \right) \right|_{p-\Delta p}}{\Delta p} \right) = \lim_{\Delta p \to 0} \left[\frac{2k'' C_A \cdot p \cdot \cancel{\Delta p}}{\cos\alpha \cdot \tan\alpha \cdot \cancel{\Delta p}} \right]$$

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$$\Rightarrow \frac{d}{dp} \left(p^2 \left(D_{AB} \cdot \frac{dC_A}{dp} \right) \right) = \frac{2k'' C_A \cdot p}{\sin\alpha}$$

$$\Rightarrow \boxed{\frac{d}{dp} \left(p^2 \frac{dC_A}{dp} \right) = \frac{2pk'' C_A}{D_{AB} \cdot \sin\alpha}} \quad \text{— (v)}$$

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*. Non - Dimensionalizing :→

Let : $\chi = \frac{p}{H+L}$ and, $\psi = \frac{C_A}{C_{As}}$

Thus, $(H+L)\, d\chi = dp$ and, $C_{As} \cdot d\psi = dC_A$.

Plugging This in eq^n (v)

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$$\Rightarrow \frac{1}{\cancel{(H+L)}} \cdot \frac{d}{d\chi} \left[\chi^2 \frac{\cancel{(H+L)^2}}{\cancel{(H+L)}} \cdot \frac{\cancel{C_{As}} \cdot d\psi}{d\chi} \right] = \frac{2(H+L) \cdot \chi k'' \cdot \psi \cdot \cancel{C_{As}}}{D_{AB} \cdot \sin\alpha}$$

4.

$$\boxed{\frac{d}{dX}\left[X^2 \cdot \frac{d\varphi}{dX}\right] = \frac{2k''(H+L) \cdot X \cdot \varphi}{D_{AB} \cdot \sin\alpha}}$$

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Now, for Boundary conditions:

We know: at $x=0$: $C_A = C_{As}$.

at $x=L$: $\frac{dC_A}{dx} = 0$

$\Downarrow$

at $p=(H+L)$: $C_A = C_{As}$

$p=H$: $\frac{dC_A}{dp} = 0$

$\Downarrow$

at $X=1$: $\varphi = 1$

and, $X = \frac{H}{(H+L)}$: $\frac{d\varphi}{dX} = 0$

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