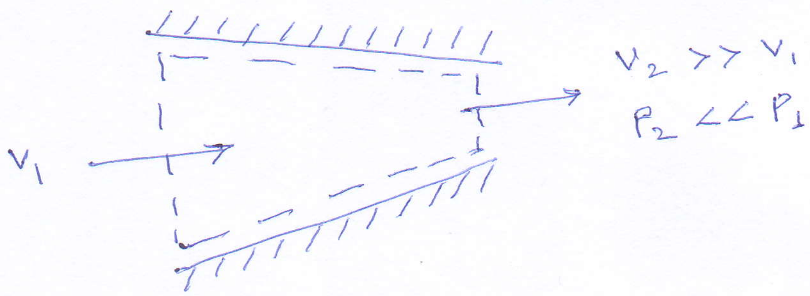
Nozzles and Diffusers

Applications : Jet engines, rockets, spacecraft and even garden hoses

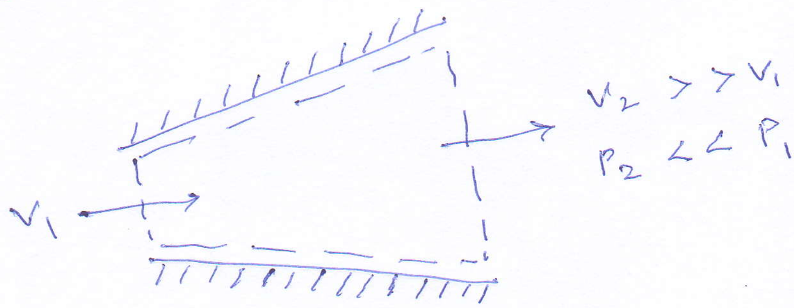
Nozzle (Mach no. < 1)

Subsonic



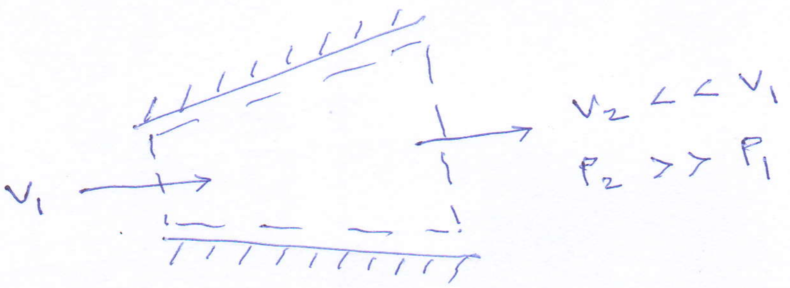
Nozzle (Mach no. > 1)

Supersonic



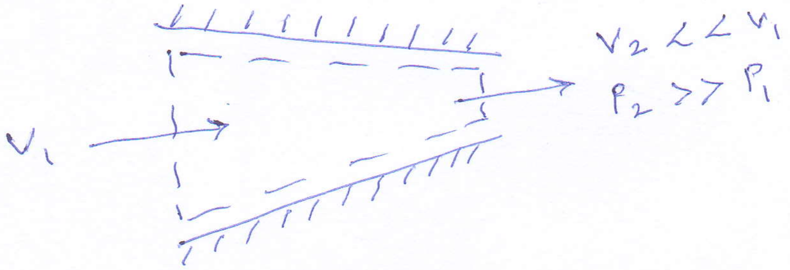
Diffuser (Mach no. < 1)

Subsonic



Diffuser (Mach no. > 1)

Supersonic



Nozzles and Diffusers

Salient Features

1. $\dot{Q} = 0$ since the fluid has high velocities and thus it does not spend enough time in the device for any significant heat transfer.

2. $\dot{W} = 0$ ($\dot{W}_{boundary} = 0$, $\dot{W}_{shear} = 0$)

3. $\Delta Pe = 0$

4. $\Delta Ke \neq 0$

Effect of Area Variation on properties in reversible adiabatic (isentropic) flow

The differential momentum equation for 1D, steady, frictionless flow with no heat transfer is :

$$\frac{dP}{\rho} + d\left(\frac{V^2}{2}\right) = 0 \qquad (1)$$

$$\text{or} \quad dP = -\rho V dV$$

Dividing by ρV^2, we obtain

$$\frac{dP}{\rho V^2} = -\frac{dV}{V} \qquad (2)$$

Now, for 1D, steady flow, the mass balance equation is

$$\rho A V = \text{constant} \qquad (3)$$

Taking the natural logarithm of both sides yields

$$\ln \rho + \ln A + \ln V = \ln C \qquad (4)$$

Differentiating,

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad (5)$$

Solving eq. (5) for $\frac{dA}{A}$ gives

$$\frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho}$$

Substituting from eq. (2),

$$\frac{dA}{A} = \frac{dp}{\rho V^2} - \frac{d\rho}{\rho}$$

or $$\frac{dA}{A} = \frac{dp}{\rho V^2}\left[1 - \frac{V^2}{dp/d\rho}\right]$$

For an isentropic process,

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial \rho}\right)_s = c^2$$

Hence,

$$\frac{dA}{A} = \frac{dp}{\rho V^2}\left[1 - \frac{V^2}{c^2}\right]$$

$$= \frac{dp}{\rho V^2}\left[1 - M^2\right] \qquad (6)$$

where $M = \frac{V}{c} = \frac{\text{Flow speed}}{\text{speed of sound}}$ = Mach. no.

From eq. (6), we see that for $M < 1$ (subsonic), an area change causes a pressure change of the same sign (positive dA means positive dp for $M < 1$); for $M > 1$, an area change causes a pressure change of the opposite sign.

Substituting eq. (2) into eq. (6),

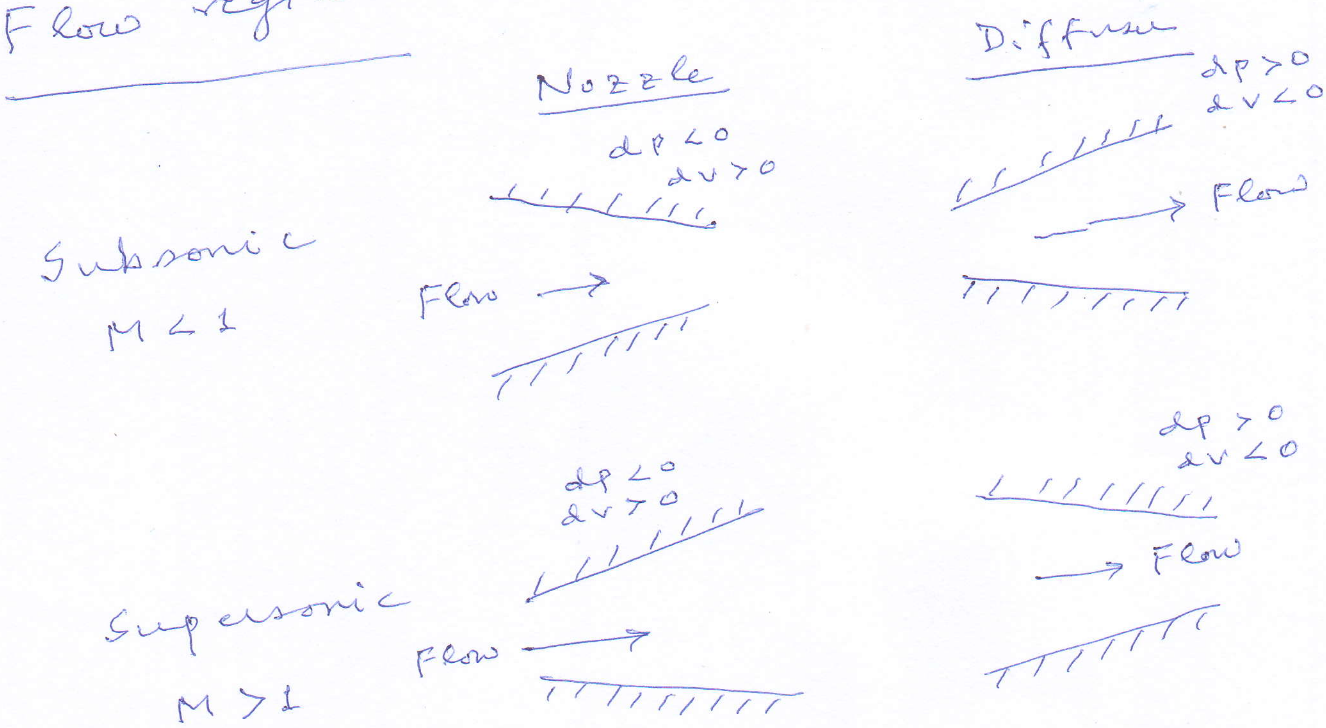
$$\frac{dA}{A} = -\frac{dV}{V}(1 - M^2) \quad (7)$$

From eq. (7), we see that for $M < 1$ an area change causes a velocity change of the opposite sign (positive dA means negative dV); for $M > 1$ an area change causes a velocity change of the same sign.

Note that for $M < 0.3$, the flow is considered as incompressible. Thus, eqs. (6) and (7) are also applicable for incompressible flow.

Summary

Nozzle and Diffuser shapes as a function of inlet Mach number.



Mach number (M) should be calculated based on the inlet velocity. Thus, v in eqs. (6) and (7) is the inlet velocity. Similarly, 'ρ' in eq. (6) is the density of the fluid at the inlet.

Example Problem

Air at 10°c and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m². The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine (a) the mass flow rate of the air and (b) the temperature of the air leaving the diffuser.

$P_{cr} = 3.77$ MPa

$T_{cr} = 132.5$ K

Since $P << P_{cr}$ and $T >> T_{cr}$ ideal gas assumption can be made.

Solution

Assumptions

1. This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{cv} = 0$ and $\Delta E_{cv} = 0$.

2. Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values.

3. The potential energy change is zero, $\Delta pe = 0$.

4. Heat transfer is negligible.

5. Kinetic energy at the diffuser exit is negligible.

6. There are no work interactions.

We take the diffuser as the control volume (or system). See Fig. 1. For steady flow,

$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$

$P_1 = 80\ kPa$

$T_1 = 10^\circ C = 283\ K$

$V_1 = 200\ m/s$

$A_1 = 0.4\ m^2$

$R_{air} = 0.287\ \frac{kPa \cdot m^3}{kg\,K}$

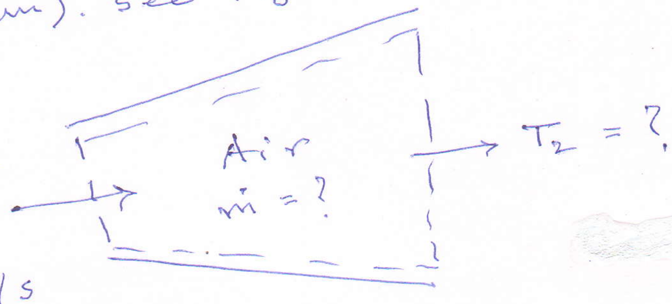


Fig. 1

(a) $v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(283)}{80}$

$= 1.015\ m^3/kg$

$\rho_1 = \frac{1}{v_1}$

Then, $\dot{m} = \frac{1}{v_1} V_1 A_1$

$= \frac{1}{1.015} (200)(0.4)$

$= 78.8\ kg/s$

(b) Energy balance equation:

$\dot{E}_{in} - \dot{E}_{out} = \cancelto{0\ (\text{Steady})}{\frac{dE_{system}}{dt}} = 0$

$\Rightarrow \dot{E}_{in} = \dot{E}_{out}$

$$\Rightarrow \quad \dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

(since $\dot{Q} = 0$, $\dot{W} = 0$ and $\Delta pe = 0$)

$$\Rightarrow \quad h_2 = h_1 - \frac{V_2^2 - V_1^2}{2}$$

Since $V_2 << V_1$,

$$h_2 = h_1 + \frac{V_1^2}{2}$$

The enthalpy of air at the diffuser inlet is determined from the air table (Table A-17) to be

$h_1 = h_{@283K} = 283.14$ kJ/kg (by linear interpolation)

$h_{@280K} = 280.13$ kJ/kg

$h_{@285K} = 285.14$ kJ/kg

Substituting, we get

$$h_2 = 283.14 + \frac{(200)^2}{2} \frac{1}{1000}$$

$$= \boxed{303.14 \text{ kJ/kg}} \; *$$

(since the unit m^2/s^2 is equal to 1 J/kg, we are dividing it by 1000 to convert it to kJ/kg.)

From Table A-17, the temperature corresponding to this enthalpy value is:

$$\boxed{T_2 = 303.1 K}^{*}$$ by ~~linear~~ linear interpolation

$h = 300.19$ kJ/kg corresponds to $T = 300$ K

$h = 305.22$ kJ/kg corresponds to $T = 305$ K

Conclusions:

This result shows that the temperature of the air increases by about 20.1°C as it is slowed down in the diffuser. The temperature rise of the air is mainly due to the conversion of kinetic energy to internal energy.

* See the linear interpolations on the next page (p.12).

Linear Interpolation for h_2

$$y = y_0 + m(x - x_0)$$

$$m = \frac{285.14 - 280.13}{285 - 280}$$

$$= \frac{5.01}{5} = 1.002$$

$$y_{283\,K} = 280.13 + 1.002(283 - 280)$$

$$= 280.13 + 1.002(3)$$

$$= 280.13 + 3.006$$

$$= 283.136$$

$$\simeq 283.14 \text{ kJ/kg}$$

$$\boxed{\begin{array}{l} y \rightarrow h \\ x \rightarrow T \end{array}}$$

Linear Interpolation for T_2

$$y = y_0 + m(x - x_0)$$

$$\boxed{\begin{array}{l} y \rightarrow T \\ x \rightarrow h \end{array}}$$

$$m = \frac{305 - 300}{305.22 - 300.14} = \frac{5}{5.08}$$

$$= 0.984$$

$$y_{303.14\text{ kJ/kg}} = 300 + 0.984(303.14 - 300)$$

$$= 300 + 0.984(3.14)$$

$$= 300 + 3.08976$$

$$= 303.08976$$

$$\simeq 303.1 \text{ K}$$

Important points to Note

In evaluating 'h' as a function of 'T' or 'T' as a function of 'h' we assume that the motion in no way alters the thermodynamic equation of state. In microscopic terms, even though the fluid is accelerating or decelerating, the molecules behave locally as if there were no bulk motion. Satisfactory results are obtained with this idealization, and this experimental support is sufficient justification for its use. In other words, thermodynamic property tables and charts can be used for both non-flow and flow processes, assuming local thermodynamic equilibrium in the latter.