Solution

Mid-Sem Exam

(ESO201A)

2022-23-I

1. (i) Work done, W

$$= 2\int_1^2 P\,d\bar{v}$$

$$= 2\int_1^2 \left[\frac{R_u T}{\bar{v}-b} - \frac{a}{\bar{v}^2}\right] d\bar{v}$$

$$= 2\left[\int_1^2 \frac{R_u T}{\bar{v}-b}\,d\bar{v} - a\int_1^2 \frac{d\bar{v}}{\bar{v}^2}\right]$$

$$= 2\left[R_u T\int_1^2 \frac{d\bar{v}}{\bar{v}-b} - a\int_1^2 \frac{d\bar{v}}{\bar{v}^2}\right]$$

$$= 2\left[R_u T \left| \ln(\bar{v}-b) \right|_1^2 - a\left| \frac{v^{-2+1}}{-2+1} \right|_1^2\right]$$

$$= 2\left[R_u T \ln\left(\frac{\bar{v}_2-b}{\bar{v}_1-b}\right) + a\left(\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1}\right)\right]$$

$$= 2\left[(8.315)(300)\ln\right.$$

$$= 2\left[(8.315)(300)\ln\left(\frac{5-0.0373}{30-0.0373}\right) + 423.3\left(\frac{1}{5}-\frac{1}{30}\right)\right]$$

$$= 2\left[(8.315)(300)\ln(0.1656) + 423.3(0.2-0.0333)\right]$$

$$= 2\left[(8.315)(300)(-1.7982) + 423.3(0.1667)\right]$$

$$= 2\left[-4485.61 + 70.56411\right]$$

$$= 2(-4415.04589)$$

$$= -8830.09178$$

$$= \boxed{-8830.09\ \text{KJ}}$$

(5 points)

(ii) Given :

$$d\bar{u} = \bar{c}_v\, dT + \frac{a}{\bar{v}^2} d\bar{v}$$

Since the process is isothermal,

$dT = 0$.

Hence, $d\bar{u} = \frac{a}{\bar{v}^2} d\bar{v}$

$$\Rightarrow \int_1^2 d\bar{u} = a \int_1^2 \frac{d\bar{v}}{\bar{v}^2}$$

$$\Rightarrow \bar{u}_2 - \bar{u}_1 = a \left| \frac{\bar{v}^{-2+1}}{-2+1} \right|_1^2$$

$$\Rightarrow \bar{u}_2 - \bar{u}_1 = a \left| -\frac{1}{\bar{v}} \right|_1^2$$

$$= -a \left[\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1} \right]$$

$$= a \left[\frac{1}{\bar{v}_1} - \frac{1}{\bar{v}_2} \right]$$

$$\Rightarrow \bar{u}_2 - \bar{u}_1 = (423.3) \left[\frac{1}{30} - \frac{1}{5} \right)$$

$$= 423.3\,(0.0333 - 0.2)$$

$$= -70.56411 \text{ kJ/kmol}$$

$$\Rightarrow U_2 - U_1 = 2(-70.56411)$$

$$= -141.12822 = \boxed{-141.13 \text{ kJ}}$$

(3 points)

(iii) The gas is taken as the system (closed system).

Applying the 1st law of thermodynamics assuming $\Delta KE = 0$ and $\Delta PE = 0$,

$$Q - W = \Delta U$$

$$\Rightarrow Q = \Delta U + W$$

$$= -141.13 + (-8830.09)$$

$$= \boxed{-8971.22 \text{ KJ}}$$

(2 points)

Total : 5 + 3 + 2
= 10 points

2. $P = 500$ kPa $P_{cr} = 3.77$ MPa

$T = 27°C = 300$ K $T_{cr} = 132.5$ K

Since $P << P_{cr}$ $(P/P_{cr} << 1)$

$T >> T_{cr}$ $(T/T_{cr} > 2)$

ideal gas assumption can be made. (1 point)

We take the cylinder containing the air as the system (Fig. 1).

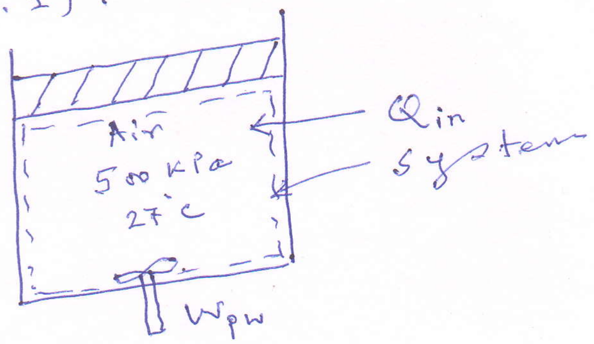


Fig. 1

We assume $\Delta KE = 0$, $\Delta PE = 0$.

Since air is treated as an ideal gas and since $u = f(T)$ only for an ideal gas and the process is isothermal, $\Delta u = 0$ or $\Delta U = 0$.

This is a closed system since no mass crosses the boundaries of the system.

The energy balance for this system can be expressed as

$$E_{in} - E_{out} = \Delta E_{system}$$
$$= \cancel{\Delta U}^{0} + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0}$$

$$\Rightarrow (Q_{in} + W_{in}) - (Q_{out} + W_{out}) = 0$$

$$\Rightarrow (Q_{in} - \cancel{Q_{out}}^{0}) - (W_{out} - W_{in}) = 0$$

$$\Rightarrow Q_{in} - W_{out} + W_{in} = 0$$

$$\Rightarrow Q_{in} = -W_{in} + W_{out}$$ or $q_{in} = -W_{in} + W_{out}$

or $\boxed{q_{in} = W_{out} - W_{in}}$ (1)

Note $W_{in} = W_{pw}$

and $W_{out} = \int P\, dV$

There fore, $W_{in} = W_{pw} = 50\,(1) = 50$ KJ

$$W_{out} = \int_1^2 P\, dV = mRT \int_1^2 \frac{dV}{V}$$

$$= mRT [\ln V]_1^2 = mRT \ln \frac{V_2}{V_1}$$

$$= (1)(0.287)(300) \ln (3)$$

$$= (0.287)(300)(1.0986)$$

$$= 94.58946$$

$$= 94.59 \text{ KJ}$$

$$= 94.6 \text{ KJ}$$

Thus, $w_{out} = 94.6$ kJ/kg

Also, $w_{in} = 50$ kJ/kg

From eq. (1), we can write

$$q_{in} = w_{out} - w_{in}$$

$$= 94.6 - 50$$

$$= \boxed{44.6 \text{ kJ/kg}}$$

(7 points)

Total : 1 + 7 = 8 points

3. Given :

$$R = 0.5182 \text{ kPa}\cdot\text{m}^3/\text{kg K}$$

$$T_{cr} = 191.1 \text{ K}$$

$$P_{cr} = 4.64 \text{ MPa}$$

The system is shown in Fig. 1.

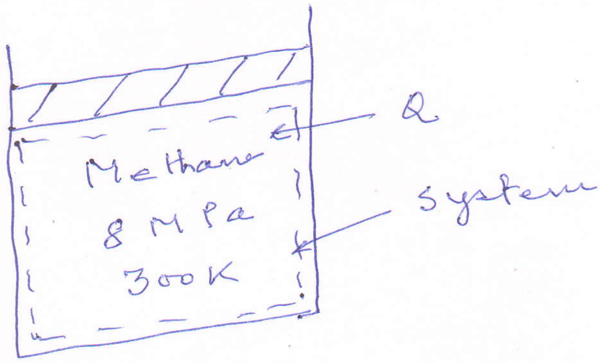


Fig. 1

<u>At the initial state</u>

$$T_{R_1} = \frac{T_1}{T_{cr}} = \frac{300}{191.1} = 1.570$$

$$P_{R_1} = \frac{P_1}{P_{cr}} = \frac{8}{4.64} = 1.724$$

From the compressibility factor chart we get (see Fig. 2),

$$Z_1 = 0.88$$

$$v_{R_1} = 0.80$$

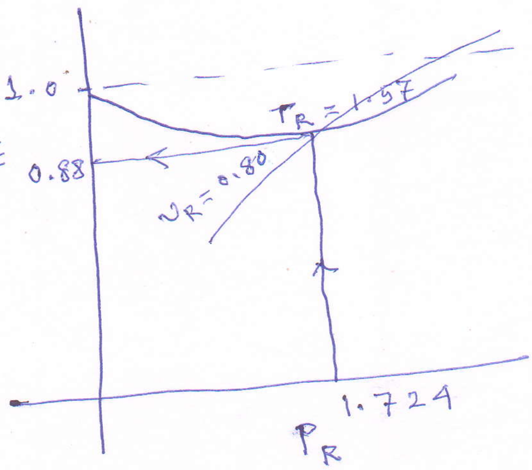


Fig. 2

At the final state,

$$P_{R_2} = P_{R_1} = 1.724$$ (since the process is isobaric)

$$v_{R_2} = 1.5\, v_{R_1} = 1.5(0.80) = 1.2$$

From the compressibility factor chart (see Fig. 3) we get,

$$Z_2 = 0.975$$

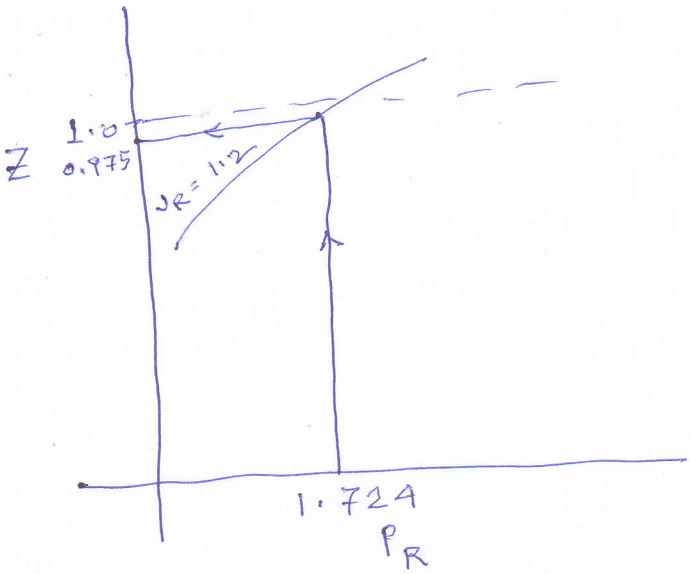


Fig. 3

Thus,

$$T_2 = \frac{P_2 v_2}{Z_2 R} = \frac{P_2 v_{R_2} R T_{cr}}{P_{cr} Z_2 R}$$

$$= \frac{P_2}{Z_2} \frac{v_{R_2} T_{cr}}{P_{cr}}$$

$$\left(\because \frac{v_2}{v_{R_2}} = \frac{R T_{cr}}{P_{cr}} \right)$$

$$\Rightarrow T_2 = \frac{8000 \text{ kPa} (1.2)(191.1\text{K})}{0.975 \ (4640 \text{ kPa})}$$

$$= 405.517 \text{ K}$$

$$= \boxed{405.5 \text{ K}}$$

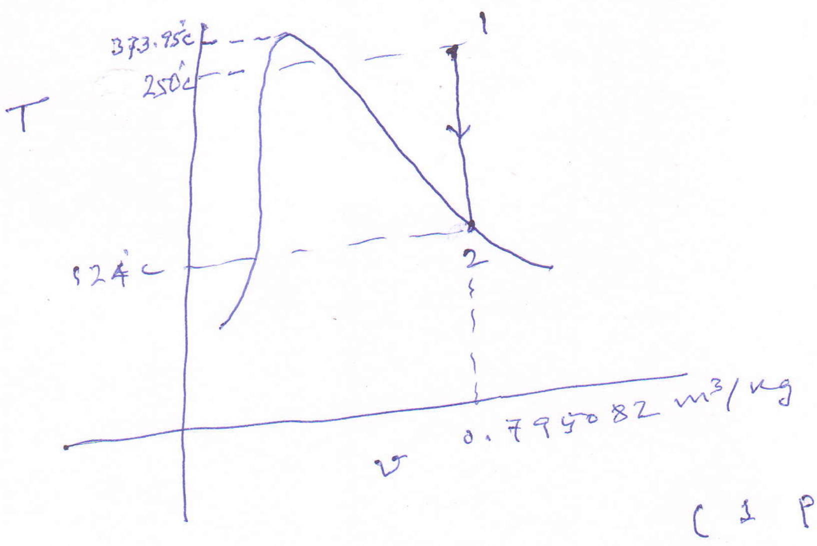
(6 points)

4. Since the container is rigid it is a constant volume process ($v = \frac{V}{m}$ = constant). Hence, the initial specific volume is equal to the final specific volume, that is,

$$v_1 = v_2 = v_{g@124°C} = 0.795082 \text{ m}^3/\text{kg}$$

(by linear interpolation, from Table A-4)
See P. 12.

since the vapour starts condensing at 124°C. See Fig. 1.



(1 point for T-v diagram along with the liq-vap mixture dome)

Then from Table A-6,

$$\left.\begin{aligned} T_1 &= 250^\circ C \\ v_1 &= 0.795082\ m^3/kg \end{aligned}\right\} \boxed{P_1 = 0.3\ MPa}$$

At $P = 0.30$ MPa, $250^\circ C$,
$v = 0.79645\ m^3/kg$.
Hence, the closest value is taken.

Linear interpolation to obtain $v_{g@124^\circ C}$

From Table A-4

x (T)	y (v_g)
$120^\circ C$	$0.89133\ m^3/kg$
$125^\circ C$	$0.77012\ m^3/kg$

$$y = y_0 + m(x - x_0)$$

where $m = \dfrac{0.77012 - 0.89133}{125 - 120}$

$= -0.024062\ m^3/kg^\circ C$

$$y_{@124^\circ C} = 0.89133 - 0.024062(124 - 120)$$

$= 0.795082\ m^3/kg$ (5 points)

Total: 1+5 = 6 points