ESO201A

Lecture#31

(Class Lecture)

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By

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Refrigerators

We Know that heat cannot be transferred from a low-temperature body to a high-temperature body. This can be done with a refrigerator or heat pump. Figure 1 shows a vapour-compression refrigeration cycle. The working fluid is the refrigerant, such as R-134a or ammonia, which goes through a thermodynamic cycle. Heat is transferred to the refrigerant in the evaporator, where its pressure and temperature are low. Work is done on the refrigerant in the compressor, and heat is transferred from it in the condenser, where its pressure and temperature are high. The pressure drops as the refrigerant flows through the throttle or capillary tube.

Thus, in a refrigerator or heat pump, we have a device that operates in a cycle, that requires work input, and that transfers heat from a low-temperature body to a high-temperature body.

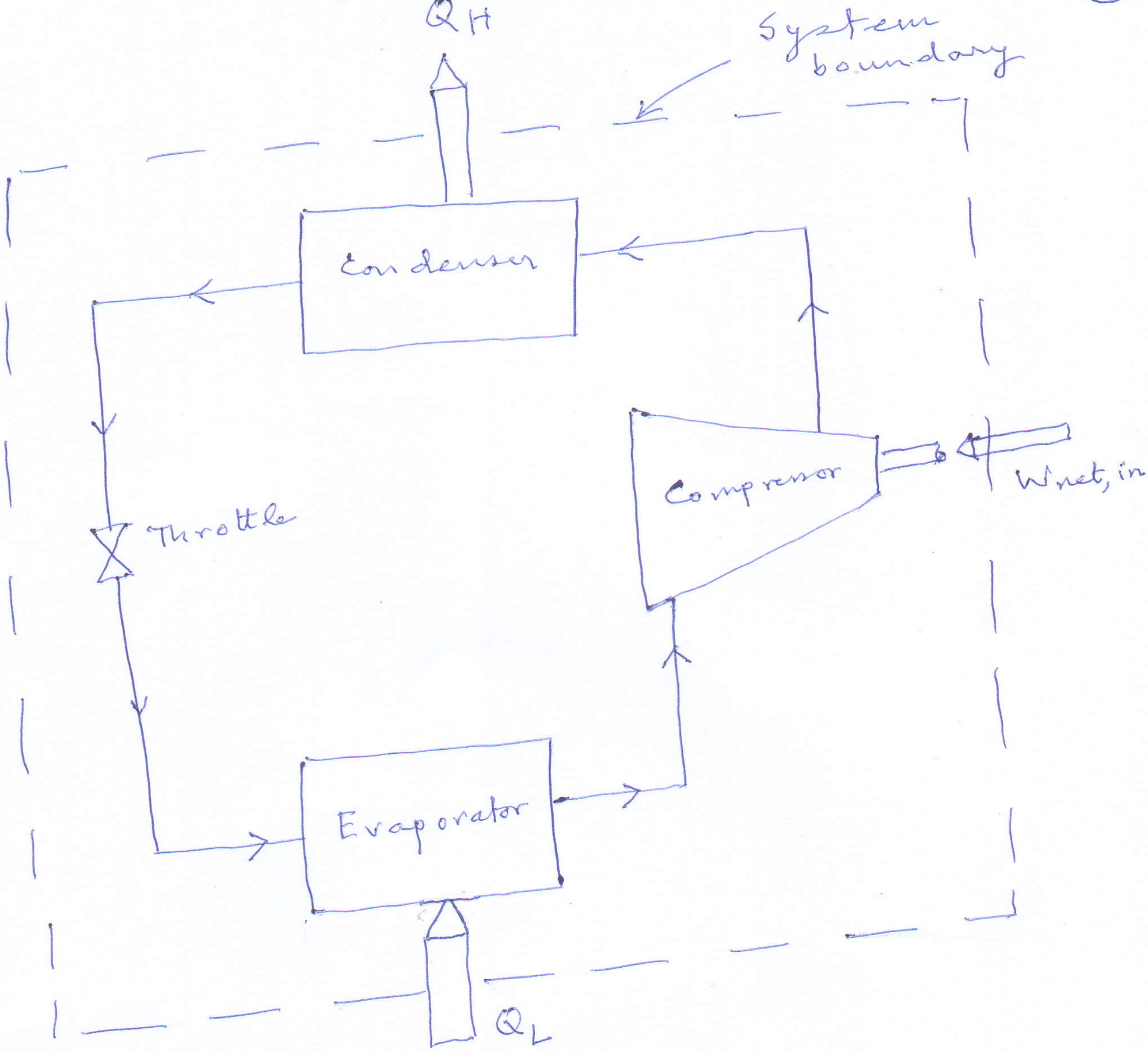


Fig. 1 Schematic diagram of a simple refrigeration cycle

In Fig. 2 Q_L is the magnitude of the heat removed from the refrigerated space at temperature T_L, Q_H is the magnitude of the heat rejected to the warm environment at temperature T_H and $W_{net, in}$ is the net work input to the refrigerator.

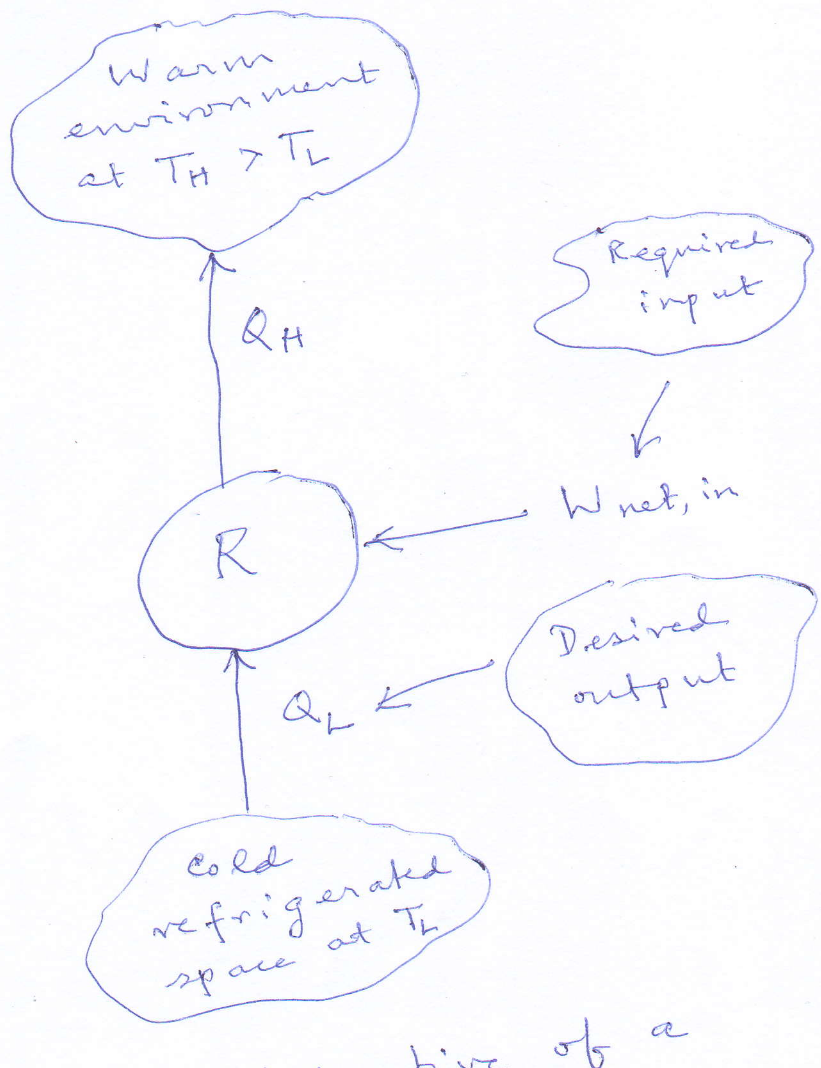


Fig. 2 The objective of a refrigerator is to remove Q_L from the cooled space

Coefficient of Performance

The efficiency of a refrigerator is expressed in terms of the coefficient of performance (COP), denoted by COP_R. The objective of a refrigerator is to remove heat (Q_L) from the refrigerated space. To accomplish this objective, it requires a work input of $W_{net,in}$. Then the COP of a refrigerator can be expressed as

$$COP_R = \frac{\text{Desired output}}{\text{Required input}}$$

$$= \frac{Q_L}{W_{net,in}} \qquad (1)$$

This relation can also be expressed as

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} \qquad (2)$$

Considering the system boundary as shown in Fig. 1, the conservation of energy principle requires that

$$Q_H = Q_L + W_{net,in}$$

or $$W_{net,in} = Q_H - Q_L \qquad (3)$$

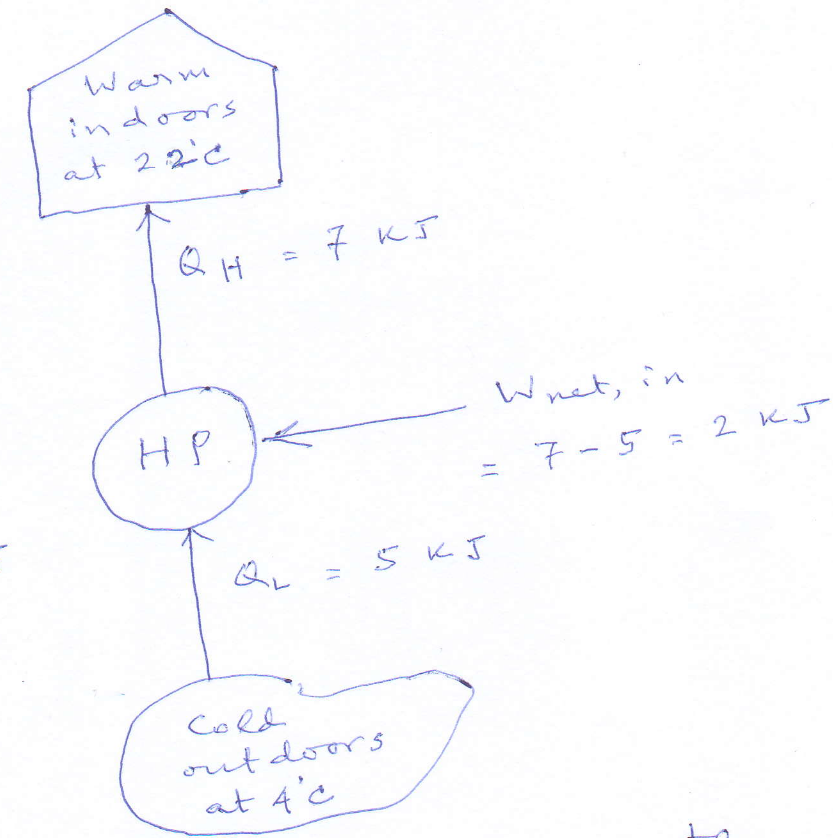
Then from eq. (1),

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1} \qquad (4)$$

It may be noted that the value of COP_R can be greater than unity. That is, the amount of heat removed from the refrigerated space can be greater than the amount of work input. This is in contrast with the thermal efficiency of engines, which can never be 100%. This is the reason why the efficiency of a refrigerator is denoted by another term — the coefficient of performance.

Heat Pumps

Another device that transfers heat from a low-temperature medium to a high-temperature one is the heat pump, shown schematically in Fig. 3.



$$COP_{HP} = \frac{Q_H}{W_{net,in}} = \frac{7}{2} = 3.5$$

Fig. 3 The work supplied to a heat pump is used to extract energy from the cold outdoors and carry it into the warm indoors.

The objective of a heat pump, however, is to maintain a heated space at a high temperature (comfort temperature). This is accomplished by absorbing heat from a low-temperature source such as cold outside air in winter, and supplying the heat to the high-temperature medium such as a house (Fig. 3 on p. 6).

The measure of performance of a heat pump is also expressed in terms of the coefficient of performance (COP_{HP}), as defined as

$$COP_{HP} = \frac{\text{Desired output}}{\text{Required input}}$$

$$= \frac{Q_H}{W_{net, in}} \qquad (5)$$

Equation (5) can also be expressed as

$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}} \qquad (6)$$

Note that

$$COP_R = \frac{1}{\frac{Q_H}{Q_L} - 1} \qquad (7)$$

$$\Rightarrow \frac{Q_H}{Q_L} - 1 = \frac{1}{COP_R}$$

$$\Rightarrow \frac{Q_H}{Q_L} = 1 + \frac{1}{COP_R}$$

$$= \frac{1 + COP_R}{COP_R}$$

$$\Rightarrow \frac{Q_L}{Q_H} = \frac{COP_R}{1 + COP_R} \qquad (8)$$

Putting eq. (8) into eq. (6),

$$COP_{HP} = \frac{1}{1 - \frac{COP_R}{1 + COP_R}}$$

$$= \frac{1}{\frac{1 + COP_R - COP_R}{1 + COP_R}}$$

$$= 1 + COP_R$$

Thus, $$\boxed{COP_{HP} = 1 + COP_R} \quad (9)$$

for fixed values of Q_L and Q_H.

The Second Law of Thermodynamics: The Clausius Statement

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a cooler body to hotter body.

See Fig. 4.

This statement is related to the refrigerator or heat pump. In effect, it states that it is impossible to construct a refrigerator that operates without an input of work. This also implies that the COP is always less than infinity since $W_{net,in}$ can never be zero.

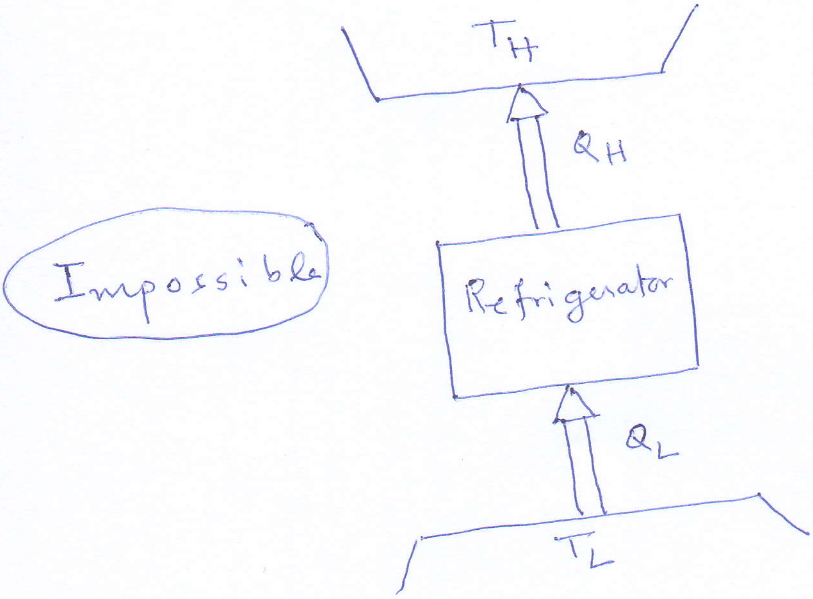


Fig. 4 The Clausius Statement

Observations about the two statements

Two observations can be made about these two statements.

1. Both are negative statements. It is, of course, impossible to prove these negative statements. However, we can say that the second law of thermodynamics (like every other law of nature) rests on experimental evidence. No experiment has ever been conducted that contradicts the second law. The basis of the second law is therefore experimental evidence.

2. A second observation is that these two statements of the second law are equivalent. Two statements are equivalent if the truth of either statement implies the truth of the other or if the violation of either statement implies the violation of the other.

Equivalence of the Two Statements

This exercise proves that the violation of the Clausius statement results in a violation of the Kelvin-Planck statement.

The device at the left in Fig. 5(a) is a refrigerator that requires no work input and thus violates the Clausius statement. Let an amount of heat Q_L be transferred from the low-temperature reservoir to the refrigerator, and let the same amount of heat Q_L be transferred to the high temperature reservoir.

Let an amount of heat Q_H that is greater than Q_L be transferred from the high-temperature reservoir to the heat engine (at the right in Fig. 5(a)), and let the engine reject the amount of heat Q_L as it does an amount of work, W, that equals $Q_H - Q_L$.

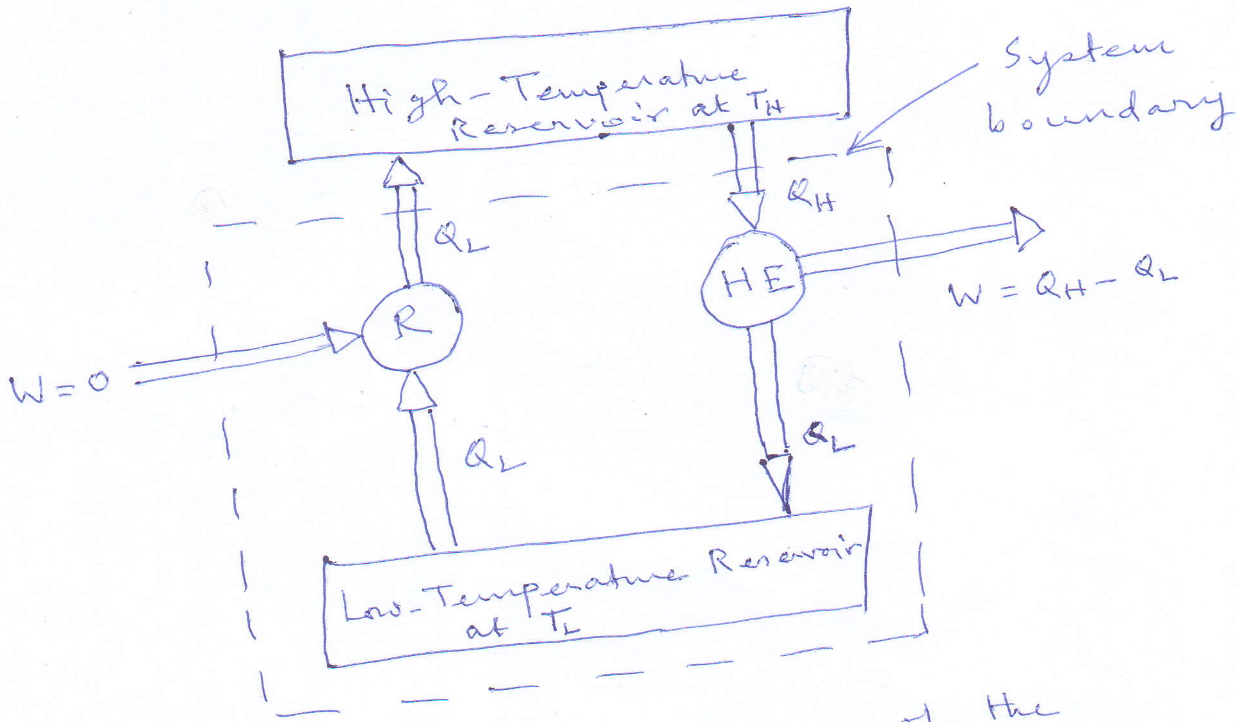


Fig. 5 (a) Demonstration of the equivalence of the two statements of the second law

Because there is no net heat transfer to the low-temperature reservoir, the low-temperature reservoir, along with the heat engine and the refrigerator, can be considered together as a device that operates in a cycle and produce no effect other than raising of a weight (work) and the exchange of heat with a single reservoir (see Fig. 5(b))

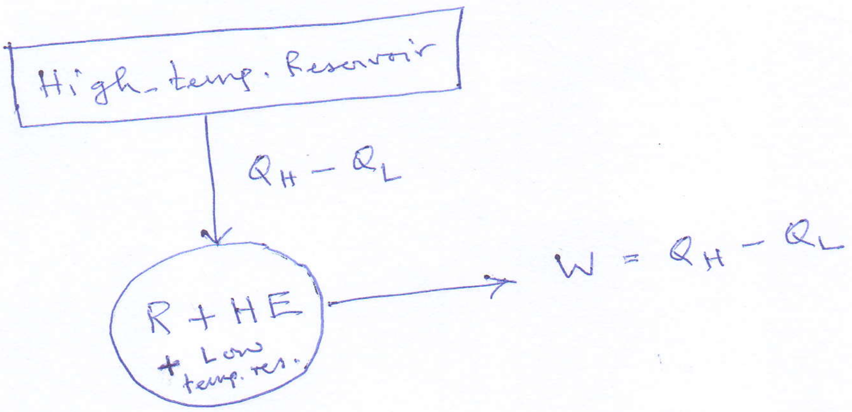


Fig. 5(b) The equivalent heat engine

Thus, a violation of the clausius statement implies a violation of the Kelvin-planck statement. The complete equivalence of these two statements is established when it is shown that a violation of the Kelvin-planck statement implies a violation of the clausius statement. This can also be proved.

Violation of Kelvin-Planck Statement Leads to Violation of Clausius Statement

Consider the heat-engine-refrigerator combination shown in Fig. 6(a). The heat engine is assumed to have, in violation of the Kelvin-Planck statement, a thermal efficiency of 100%, and therefore it converts all the Q_H it receives to work W. This work is now supplied to a refrigerator that removes heat in the amount of Q_L from the low-temperature reservoir and rejects heat in the amount of $Q_L + Q_H$ to the high-temperature reservoir. During this process, the high-temperature reservoir receives a net amount of heat, $(Q_L + Q_H) - Q_H = Q_L$. Thus, the combination of these two devices can be viewed as an equivalent refrigerator (Fig. 6(b)), that transfers heat in an amount of Q_L from a cooler body to a warmer one without requiring any work input from outside. This is clearly a violation of the Clausius statement. Therefore, a violation of Kelvin-Planck statement results in the violation of the Clausius statement.

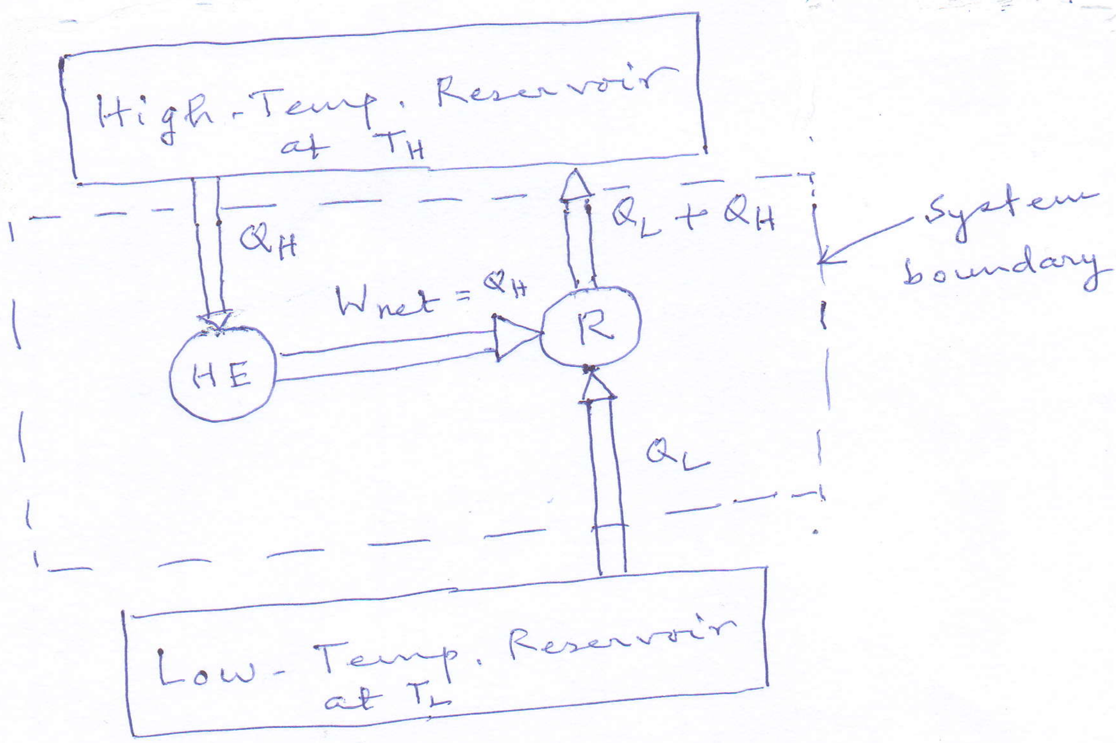


Fig. 6(a) Demonstration of the equivalence of the two statements of the second law

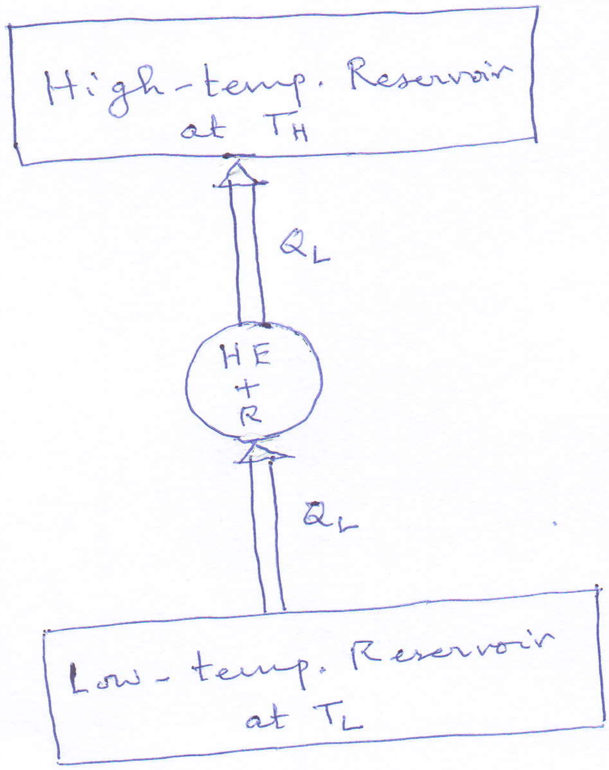


Fig. 6 (b) The equivalent refrigerator