ESO201A

Lecture#25

(Class Lecture)

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By

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Modelling of Unsteady-flow Processes

Mass Balance

The overall process occurs during time Δt. At any instant of time during the process, the continuity (that is, conservation of mass) equation is

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{cv}}{dt}$$

$$\text{or} \quad \sum \dot{m}_i - \sum \dot{m}_e = \frac{dm_{cv}}{dt}$$

$$\text{or} \quad \frac{dm_{cv}}{dt} + \sum \dot{m}_e - \sum \dot{m}_i = 0$$

where i = inlet, e = exit.

Integrating over time Δt gives the change of mass in the control volume during the overall process, that is, in the time interval Δt.

Note : Summation is applicable when there are multiple inlets and outlets.

or $$\int_0^{\Delta t} \frac{dm_{cv}}{dt}\,dt + \int_0^{\Delta t} \sum \dot{m}_e\,dt - \int_0^{\Delta t} \sum \dot{m}_i\,dt = 0$$

1st term
$$\int_0^{\Delta t} \left(\frac{dm_{cv}}{dt} \right) dt = (m_2 - m_1)_{cv}$$

where 1 = initial state and 2 = final state of the control volume.

The total mass leaving the control volume during time Δt is

2nd Term
$$\int_0^{\Delta t} \left(\sum \dot{m}_e \right) dt = \sum m_e$$

and the total mass entering the control volume during time t is

3rd term
$$\int_0^{\Delta t} \left(\sum \dot{m}_i \right) dt = \sum m_i$$

Therefore, for this period of time Δt, we can write the continuity equation for the transient process as

$$(m_2 - m_1)_{cv} + \sum m_e - \sum m_i = 0$$

or $$\boxed{\sum m_i - \sum m_e = (m_2 - m_1)_{cv}} \quad (1)$$

Often one or more terms in eq. (1) are zero. For example, $\sum \dot{m}_i$ is zero if no mass enters the control volume during the process, $\sum m_e = 0$ if no mass leaves, and $m_1 = 0$ if the control volume is initially evacuated.

Energy Balance

$$\left(\dot{Q}_{in} + \dot{W}_{in} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right) \right)$$

$$- \left(\dot{Q}_{out} + \dot{W}_{out} + \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e \right) \right)$$

$$= \frac{d E_{cv}}{dt}$$

$$\Rightarrow \left(\dot{Q}_{in} - \dot{Q}_{out} \right) - \left(\dot{W}_{out} - \dot{W}_{in} \right)$$

$$+ \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right)$$

$$- \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e \right)$$

$$= \frac{d E_{cv}}{dt}$$

$$\Rightarrow \quad \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right) = \frac{dE_{cv}}{dt}$$

where $\dot{Q}_{cv} = \dot{Q}_{in} - \dot{Q}_{out}$

$\dot{W}_{cv} = \dot{W}_{out} - \dot{W}_{in}$

$$\Rightarrow \quad \dot{Q}_{cv} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) = \frac{dE_{cv}}{dt} + \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right) + \dot{W}_{cv}$$

Since at any instant of time the state within the control volume is uniform, the first law for the transient process becomes

$$\dot{Q}_{cv} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) = \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right) + \frac{d}{dt}\left[m\left(u + \frac{V^2}{2} + gz\right)\right]_{cv} + \dot{W}_{cv}$$

Let us now integrate this equation over time interval Δt, during which time we have

$$\int_0^{\Delta t} \dot{Q}_{cv}\, dt = Q_{cv}$$

$$\int_0^{\Delta t} \left[\sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right) \right] dt = \sum m_i \left(h_i + \frac{V_i^2}{2} + g z_i \right)$$

$$\int_0^{\Delta t} \left[\sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e \right) \right] dt = \sum m_e \left(h_e + \frac{V_e^2}{2} + g z_e \right)$$

$$\int_0^{\Delta t} \dot{W}_{cv}\, dt = W_{cv}$$

$$\int_0^{\Delta t} \frac{d}{dt} \left[m \left(u + \frac{V^2}{2} + g z \right) \right]_{cv} dt = \left[m_2 \left(u_2 + \frac{V_2^2}{2} + g z_2 \right) - m_1 \left(u_1 + \frac{V_1^2}{2} + g z_1 \right) \right]_{cv}$$

Therefore, for this period of time Δt, we can write the first law for the transient process as

$$Q_{cv} + \sum m_i \left(h_i + \frac{V_i^2}{2} + g z_i \right)$$
$$= \sum m_e \left(h_e + \frac{V_e^2}{2} + g z_e \right)$$
$$+ \left[m_2 \left(u_2 + \frac{V_2^2}{2} + g z_2 \right) - m_1 \left(u_1 + \frac{V_1^2}{2} + g z_1 \right) \right]_{cv}$$
$$+ W_{cv}$$

or

$$\boxed{Q_{cv} - W_{cv} + \sum m_i \theta_i = \sum m_e \theta_e + \left[m_2 e_2 - m_1 e_1 \right]_{cv}} \quad (2)$$

where $\theta = h + ke + pe$
$e = u + ke + pe$

When $ke \approx 0$, $pe \approx 0$ or $\Delta KE \approx 0$, $\Delta PE \approx 0$ in the CV and fluid streams, eq. (2) simplifies to

$$Q_{cv} - W_{cv} + \sum m_i h_i = \sum m_e h_e + \left[m_2 u_2 - m_1 u_1 \right]_{cv} \quad (3)$$

Example Problem

Steam at a pressure of 1.4 MPa and a temperature of 300°C is flowing in a pipe (Fig. 1). Connected to this pipe through a valve is an evacuated tank. The valve is opened and the tank fills with steam until the pressure is 1.4 MPa, and then the valve is closed. The process takes place adiabatically, and kinetic energies and potential energies are negligible. Determine the final temperature of steam.

Solution:

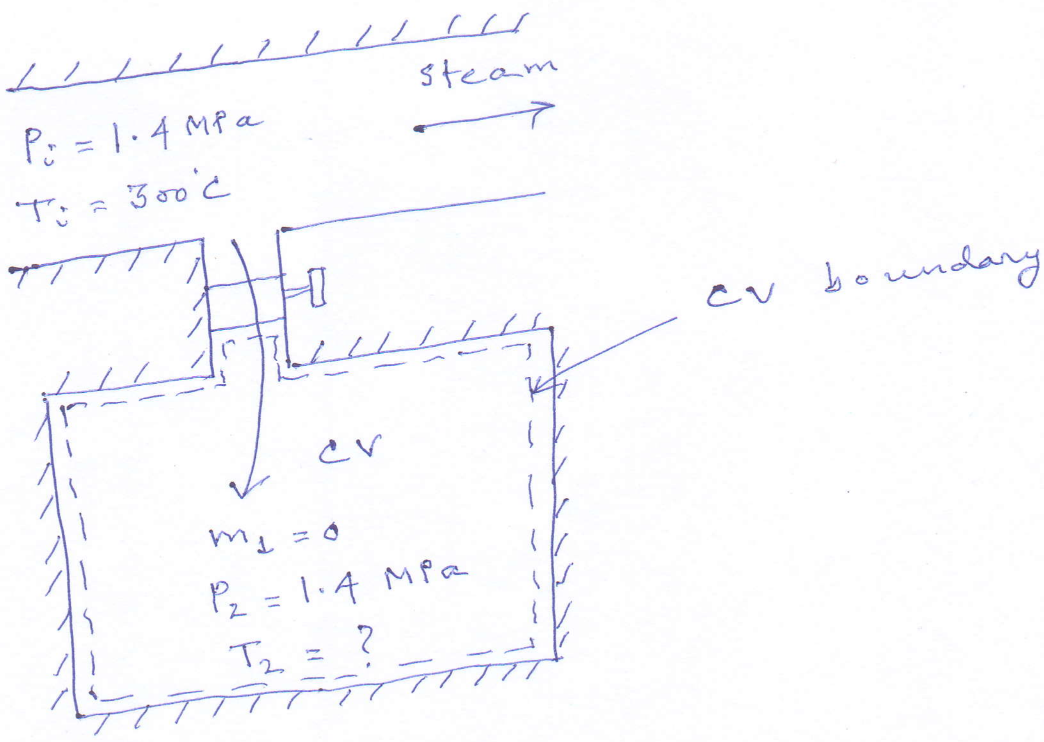

Fig. 1

From the 1st law, we have

$$Q_{cv} + \sum m_i \left(h_i + \frac{V_i^2}{2} + gZ_i\right)$$

$$= \sum m_e \left(h_e + \frac{V_e^2}{2} + gZ_e\right)$$

$$+ \left[m_2 \left(u_2 + \frac{V_2^2}{2} + gZ_2\right) - m_1 \left(u_1 + \frac{V_1^2}{2} + gZ_1\right)\right]_{cv}$$

$$+ W_{cv} \qquad (1)$$

We note that

$Q_{cv} = 0$ (Adiabatic)

$W_{cv} = 0$ (Rigid tank)

$m_e = 0$ (No exit)

$(m_1)_{cv} = 0$ (evacuated)

Also, $ke \approx 0$, $pe \approx 0$.

Therefore, from eq. (1) we can write

$$m_i h_i = m_2 u_2 \qquad (1a)$$

The continuity equation is

$$\sum m_i - \sum m_e = (m_2 - m_1)_{cv} \qquad (2)$$

From eq. (2),

$$m_2 = m_i \qquad (2a)$$

Therefore, combining eqs. (1a) and (2a) we get

$$\boxed{h_i = u_2} \qquad (3)$$

That is, the final internal energy in the tank is equal to the enthalpy of the steam entering the tank.

The enthalpy of the steam at the inlet state is

$$\left.\begin{aligned} P_i &= 1.4 \text{ MPa} \\ T_i &= 300^\circ\text{C} \end{aligned}\right\} \quad h_i = 3040.9 \text{ kJ/kg} \quad (\text{Table A-6})$$

Thus, $u_2 = h_i = 3040.9$ kJ/kg

Note that the inlet steam is superheated since $T_{sat@1.4\,MPa} = 195.04^\circ$c (Table A-5) which is lower than $T_i = 300^\circ$c.

Since the final pressure is given as 1.4 MPa, we know two properties at the final state and therefore, the final state is determined.

$$\left.\begin{aligned} P_2 &= 1.4 \text{ MPa} \\ u_2 &= 3040.9 \text{ MPa} \end{aligned}\right\} \quad \boxed{T_2 = 452.04^\circ\text{C}}$$

(by interpolation)
(see Table A-6)

Note that the temperature of the steam in the tank has increased by 152.04°C.

The temperature rise occurs because the flow energy is converted to internal energy once the flow ceases to exist in the c.v.