ESO201A

Lecture#17

(Class Lecture)

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By

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Example Problem

Air at 300 K and 200 kPa is heated at constant pressure to 600 K. Determine the change in internal energy of air per unit mass, using (a) data from the air table (Table A-17), (b) the functional form of the specific heat (Table A-2c), and (c) the average specific heat value (Table A-2b).

Solution

Assumptions: $P_{cr_{Air}} = 3.77$ MPa, $T_{cr_{Air}} = 132.5$ K (Table A-1)

Hence, $P\ (= 200\ kPa) << P_{cr_{Air}}$

$T\ (= 300\ K \text{ and } 600\ K) >> T_{cr_{Air}}$

Thus, air can be considered to be an ideal gas since it is at low pressure and high temperature relative to its critical point values.

(a) From Table A-17,

$$u_1 = u_{@300K} = 214.07 \text{ kJ/kg}$$

$$u_2 = u_{@600K} = 434.78 \text{ kJ/kg}$$

Thus,

$$\Delta u = u_2 - u_1$$

$$= (434.78 - 214.07)$$

~~= 230.71 kJ/kg~~

$$= 220.71 \text{ kJ/kg}$$

(b) From Table A-2c,

$$\bar{c}_p(T) = a + bT + cT^2 + dT^3$$

(T in K, $\bar{c}_p$ in kJ/kmol K)

where $a = 28.11$ kJ/kmol K

$b = 0.1967 \times 10^{-2}$ kJ/kmol K^2

$c = 0.4802 \times 10^{-5}$ kJ/kmol K^3

$d = -1.966 \times 10^{-9}$ kJ/kmol K^4

$\bar{c}_v = \bar{c}_p - R_u$ where $R_u = 8.315$ kJ/kmol K

$$= (a - R_u) + bT + cT^2 + dT^3$$

$$\Delta \bar{u} = \int_1^2 \bar{c}_v(T)\, dT = \int_1^2 \left[(a - R_u) + bT + cT^2 + dT^3\right] dT$$

$$\Delta \bar{u} = \int_1^2 \left[(a - R_u) \, dT \right.$$

$$+ \int_1^2 bT \, dT + \int_1^2 cT^2 \, dT$$

$$+ \int_1^2 dT^3 \, dT$$

$$= (a - R_u)\left[T_2 - T_1\right] + c\left[\frac{T_2^3}{3} - \frac{T_1^3}{3}\right]$$

$$+ b\left[\frac{T_2^2}{2} - \frac{T_1^2}{2}\right]$$

$$+ d\left[\frac{T_2^4}{4} - \frac{T_1^4}{4}\right]$$

$$= (a - R_u)\left[T_2 - T_1\right] + \frac{b}{2}\left[T_2^2 - T_1^2\right]$$

$$+ \frac{c}{3}\left[T_2^3 - T_1^3\right] + \frac{d}{4}\left[T_2^4 - T_1^4\right]$$

$$= (28.11 - 8.315)(600 - 300)$$

$$+ \frac{0.1967 \times 10^{-2}}{2}\left[600^2 - 300^2\right]$$

$$+ \frac{0.4802 \times 10^{-5}}{3}\left[600^3 - 300^3\right]$$

$$+ \left(-\frac{1.966 \times 10^{-9}}{4}\right)\left[600^4 - 300^4\right]$$

$= 19.795 \times 300$

$+ 0.09835 \times 10^{4} \times 10^{-2} (36 - 9)$

$+ 0.1600666666 \times 10^{6} (36 - 9) \times 10^{-5}$

$- 0.4915 \times 10^{-9} \times 10^{8} (36 - 9)$

$= 5938.5 + 9.835 (27)$

$+ 1.60066666 (27)$

$- 0.04195 \times 27$

$= 5938.5 + 265.545$

$+ 43.21799982$

$- 1.32705$

$= 6245.9 \text{ kJ/kmol}$

$$\Delta u = \frac{\Delta \bar{u}}{M_{air}} = \frac{6245.9 \text{ kJ/kmol}}{28.97 \text{ kg/kmol}}$$

$\boxed{\text{Mair from Table A-1}}$

$= 215.6 \text{ kJ/kg}$

Absolute % difference $= \left| \dfrac{\Delta u_{integration} - \Delta u_{Table}}{\Delta u_{Table}} \right|$

$$= \left| \frac{215.6 - 220.71}{220.71} \right|$$

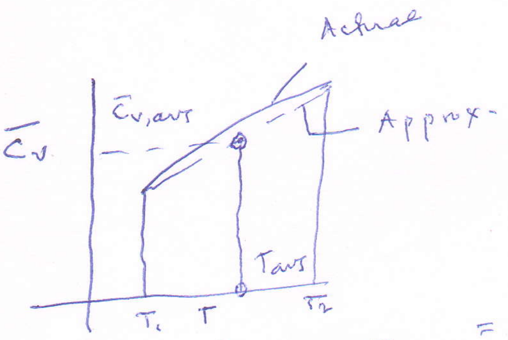
$$= 2.315\%$$

$$\approx 2.3\%$$

(c) $$T_{avg} = \frac{T_1 + T_2}{2}$$

$$= \frac{300 + 600}{2}$$

$$= 450\ K$$



$$c_{v,avg} = c_v @ 450\ K = 0.733\ kJ/kg\ K$$

(Table A-26)

Thus, $$\Delta u = c_{v,avg}(T_2 - T_1)$$

$$= 0.733\ (600 - 300)$$

$$= 219.9\ kJ/kg\ K$$

$$\text{Absolute \% Difference} = \left| \frac{\Delta u_{c_v,avg} - \Delta u_{Table}}{\Delta u_{Table}} \right|$$

$$= \left| \frac{219.9 - 220.71}{220.71} \right|$$

$$= 0.367\% \approx 0.4\%$$

Energy Analysis of Control Volumes

Flow Work

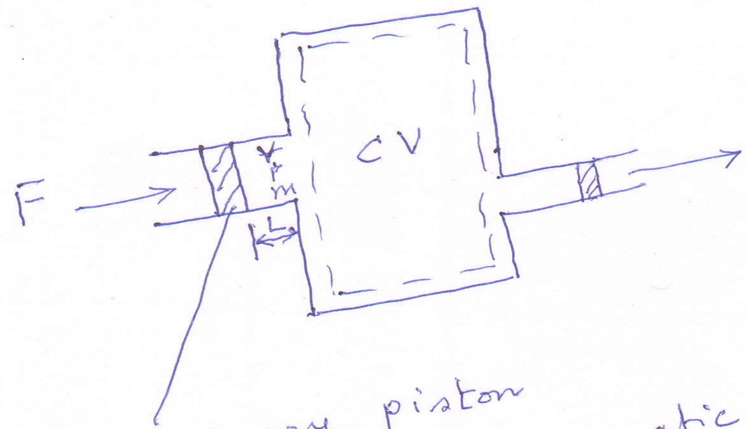


Fig. 1 schematic for flow work

- Unlike closed systems, control volumes involve mass flow across their boundaries.
- Some work is required to push the mass into or out of the control volume
- This work is known as flow work, or flow energy, and is necessary for maintaining a continuous flow through a control volume.

Flow Work (contd.)

Consider a fluid element of volume $\forall$ as shown in Fig. 1. The fluid immediately upstream forces the fluid element to enter the CV. Thus it can be regarded as an imaginary piston. The fluid element can be chosen to be sufficiently small so that it has uniform properties throughout.

If the fluid pressure is P and the cross-sectional area of the fluid element is A (Fig. 2), the force applied on the imaginary piston by the fluid element is

$$F = PA \qquad (1)$$

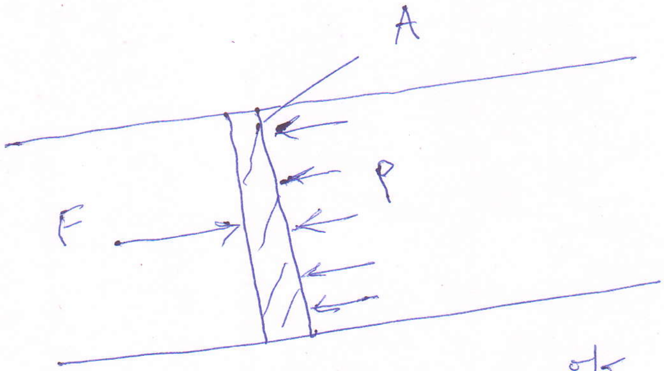


Fig. 2 In the absence of acceleration, $F = PA$.

Flow Work (Contd.)

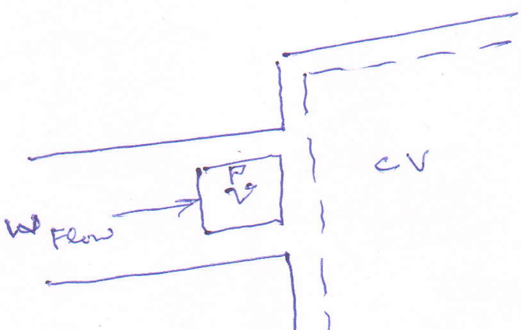
To push the entire fluid element into the CV, this force must act through a distance L. Thus, the work done in pushing the fluid element across the boundary is :

$$W_{Flow} = FL = PAL = PV \qquad (2)$$

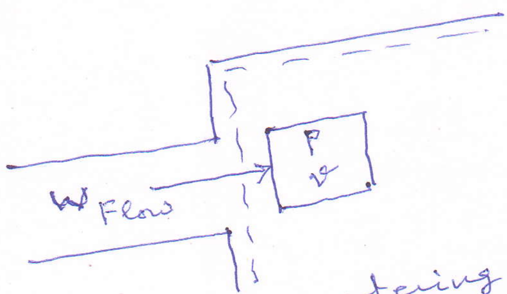
or

$$w_{Flow} = Pv \qquad (3)$$

The flow work relation is the same whether the fluid is pushed into or out of the CV (Fig. 3).



(a) Before entering the CV



(b) After entering the CV

Fig. 3 Flow work to push a fluid into or out of CV = Pv ~~= Flow work~~

Total Energy of a Flowing Fluid

Total energy of a substance per unit mass is:

$$e = u + ke + pe \tag{4}$$

$$= u + \frac{V^2}{2} + gz$$

where V is the velocity of the fluid and z is the elevation of the system relative to some external reference plane.

The fluid entering or leaving a CV possesses an additional form of energy, Pv. Then the total energy of a flowing fluid on a unit-mass basis (denoted by θ) becomes

$$\theta = Pv + e$$

$$= Pv + (u + ke + pe)$$

$$= (u + Pv) + \frac{V^2}{2} + gz$$

$$= h + \frac{V^2}{2} + gz \tag{5}$$

Thus, the flow work is embedded in the expression for enthalpy, h.

Energy Transport by Mass

Amount of energy transport:

$$E_{mass} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \qquad (6)$$

Rate of energy transport:

$$\dot{E}_{mass} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \qquad (7)$$

When ~~KE~~ ke and pe of a fluid stream are negligible, as is often the case,

$$E_{mass} = mh \qquad (8)$$

$$\dot{E}_{mass} = \dot{m}h \qquad (9)$$

In general, the total energy transported by mass into or out of the CV is not easy to determine since the properties of the mass at each inlet or exit may be changing with time as well as over cross-section. Thus, the only way to determine the energy transport through an opening as a result of mass flow is to consider sufficiently small differential masses δm that have uniform properties and to add their total energies during the flow.

Energy Transport by Mass (contd.)

At an inlet, for example,

it becomes

$$E_{in,\, mass} = \int_{m_i} \theta_i \, \delta m_i$$

$$= \int_{m_i} \left(h_i + \frac{V_i^2}{2} + g z_i \right) \delta m_i \qquad (10)$$

Most flows encountered in practice can be approximated as being steady and one-dimensional, and thus the simple relations in eqs. (6) and (7) can be used to represent the energy transported by a fluid stream.