Solution

ESO201A

End-Sem Exam
(2022-23-I)

1. (a) We know, for a reversible process, the application of 1st law gives

$$T\,ds = du + P\,dv \qquad (1)$$

$$\Rightarrow ds = \frac{du}{T} + \frac{P}{T}dv$$

For an ideal gas, $u = f(T)$ only.

Hence, $du = c_v\,dT \qquad (2)$

Also, for an ideal gas,

$$Pv = RT \qquad (3)$$

$$\text{or } \frac{P}{T} = \frac{R}{v}$$

Therefore, from eqs. (1) - (3),

$$ds = c_v \frac{dT}{T} + R\frac{dv}{v}$$

$$\Rightarrow s_2 - s_1 = \int_1^2 c_v \frac{dT}{T} + \int_1^2 R\frac{dv}{v}$$

Since c_v is treated as constant and R is a constant,

$$s_2 - s_1 = c_v \int_1^2 \frac{dT}{T} + R\int_1^2 \frac{dv}{v} \qquad (4)$$

$$= c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

(2 points)

Since, the process in the tank is constant volume, $v_2 = v_1$ and hence, from eq. (4),

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} \qquad (\text{kJ/kg K})$$

or $$S_2 - S_1 = m c_v \ln \frac{T_2}{T_1}$$

or $$\boxed{\Delta S_{air} = m c_v \ln \frac{T_2}{T_1}} \qquad (5)$$

Thus, from eq. (5),

$$\Delta S_{air} = (5)(0.718) \ln \left(\frac{27+273}{327+273} \right)$$

$$= (5)(0.718) \ln \frac{300}{600}$$

$$= (5)(0.718) \ln (0.5)$$

$$= (5)(0.718)(-0.693)$$

$$= -2.48787$$

$$\approx \boxed{-2.488 \text{ kJ/K}}$$

(3 points)

Total = 2+3 = 5

(b) An energy balance on the tank as the system gives

$$\cancel{E_{in}}^{\to 0} - E_{out} = \Delta U = U_2 - U_1$$

$$\Rightarrow Q_{out} = m c_v (T_1 - T_2)$$

$$= (5)(0.718)(327 - 27)$$

$$= 1077 \text{ kJ}$$

(2 points)

$$\boxed{E_{out} = Q_{out}}$$

The entropy change of the surroundings is

$$\Delta S_{surr} = \frac{Q_{out}}{T_{surr}} = \frac{1077}{300}$$

$$= 3.59 \text{ KJ/K}$$ (1 point)

The total entropy change of the universe due to this process is

$$\Delta S_{Universe} = \Delta S_{air} + \Delta S_{surr}$$

$$= -2.488 + 3.59$$

$$= \boxed{1.102 \text{ KJ/K}}$$

(2 points)

Total = 2+1+2
= 5

Gross total : 5+5
= 10

2. The cylinder is taken as the system (Fig. 1). This is a closed system since no mass crosses the boundary of the system during the process. The moving boundary work, W_b, will have to be considered. Also, electrical W_e is done on the system and heat is lost from the system.

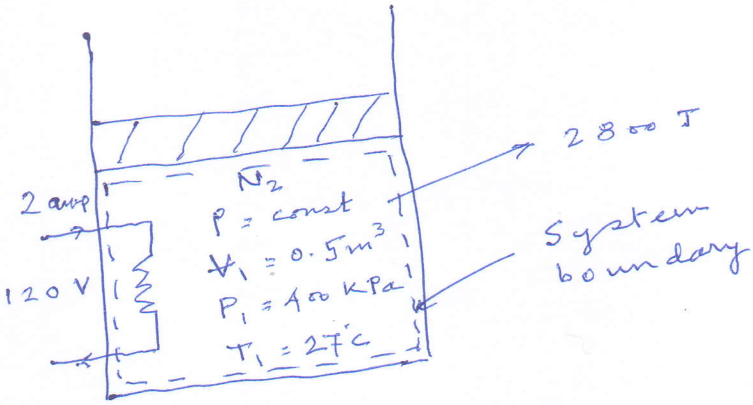


Fig. 1

The P – V diagram is shown in Fig. 2.
We assume the process to be quasi-equilibrium.

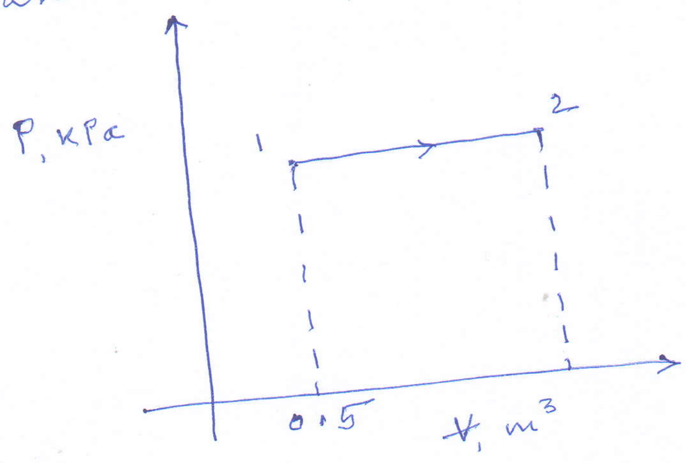


Fig. 2

The electrical work:

$$W_e = V I \Delta t$$

$$= (120)(2)(5 \times 60)/1000$$

$$= 72 \text{ kJ}$$

The mass of nitrogen is determined from the ideal gas relation:

$$m = \frac{P_1 V_1}{R T_1} = \frac{(400)(0.5)}{(0.297)(300 \text{ K})}$$

$$= 2.245 \text{ kg}$$

The energy balance on the system gives,

$$E_{in} - E_{out} = \Delta E_{sys}$$

$$W_{e,in} - Q_{out} - W_{b,out} = \Delta U$$

$$\Rightarrow W_{e,in} - Q_{out} = W_{b,out} + \Delta U$$

$$\Rightarrow W_{e,in} - Q_{out} = P(V_2 - V_1) + (U_2 - U_1)$$

$$= (P V_2 + U_2) - (P V_1 + U_1)$$

$$= H_2 - H_1$$

$$= m(h_2 - h_1)$$

$$= m c_p (T_2 - T_1)$$

$$\Rightarrow 72 - 2.8 = (2.245)(1.039)(T_2 - 27)$$

$$\Rightarrow T_2 - 27 = \frac{69.2}{2.333} = 29.7$$

$$\Rightarrow T_2 = 27 + 29.7 = \boxed{56.7^\circ C}$$

(15 points)

3. We take the tank (with a portion under the valve) as the system, which is a control volume since mass crosses the boundary (Fig. 1). We assume the direction of heat transfer is to be tank (to be verified).

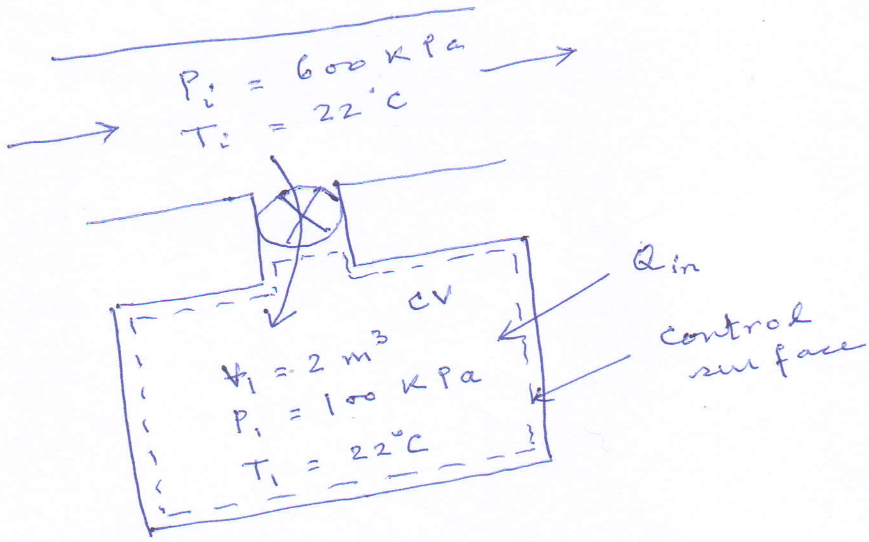


Fig. 1

(a) Mass balance:

$$m_{in} - \cancel{m_{out}}^{\,0} = \Delta m_{sys} \qquad (1)$$

$$\Rightarrow \quad m_i = m_2 - m_1$$

Energy balance:

$$E_{in} - \cancel{E_{out}}^{\,0} = \Delta E_{sys}$$

$$\Rightarrow \quad Q_{in} + m_i h_i = m_2 u_2 - m_1 u_1$$

(since W, Δke, Δpe are zero)

$$\Rightarrow \quad Q_{in} = -m_i h_i + m_2 u_2 - m_1 u_1 \qquad (2)$$

From the ideal-gas relation,

$$m_1 = \frac{P_1 \forall}{RT_1} = \frac{(100)(2)}{(0.287)(295)}$$

$$= 2.362 \text{ kg}$$

$$m_2 = \frac{P_2 \forall}{RT_2} = \frac{(600)(2)}{(0.287)(350)}$$

$$= 11.946 \text{ kg}$$

From eq. (1),

$$m_i = m_2 - m_1 = 11.946 - 2.362$$

$$= 9.584 \text{ kg}$$

(6 points)

(b) From eq. (2),

$$Q_{in} = -m_i h_i + m_2 u_2 - m_1 u_1$$

$$= -(9.584)(295.17) + (11.946)(250.07) - (2.362)(210.49)$$

$$= -2828.91 + 2987.34 - 497.18$$

$$= -338.75 \text{ kJ}$$

(9 points)

Total: 6+9 = 15

$$\Rightarrow \boxed{Q_{out} = 338.75 \text{ kJ}}$$

The negative sign for heat transfer indicates that the assumed direction is wrong. Therefore, we reversed the direction.

4. The combustion chamber is taken as a system which in this case is a control volume since mass is crossing the system (Fig. 1). We assume that the combustion is complete except that there is free O_2 in the products.

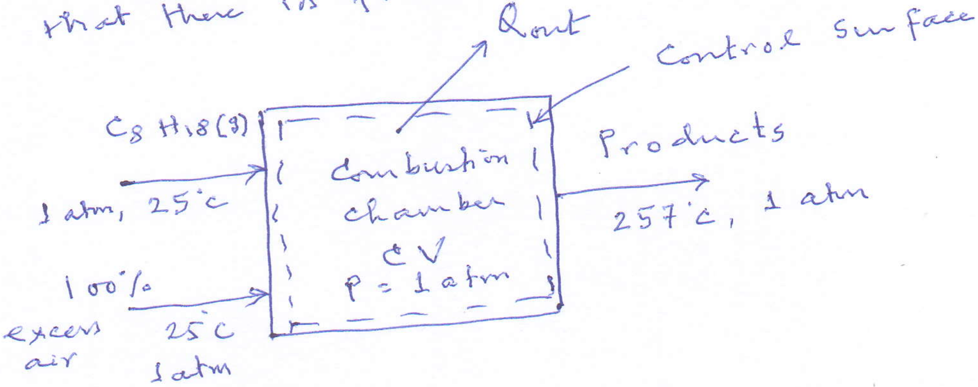


Fig. 1

The combustion reaction for stoichiometric air is

$$C_8H_{18} + 12.5\left[O_2 + 3.76 N_2\right] \rightarrow 8CO_2 + 9H_2O + (12.5 \times 3.76) N_2$$

The combustion reaction with 100% excess air is

$$C_8H_{18} + 25\left[O_2 + 3.76N_2\right] \rightarrow 8CO_2 + 9H_2O + 94 N_2 + 12.5 O_2$$

On energy balance applied to the CV,

$$E_{in} - E_{out} = \Delta E_{sys}$$

Since it is a steady-state, steady-flow process, we can write

$$E_{in} = E_{out}$$

$$\bar{Q}_{in} + \sum_R N_i (\bar{h}_f^\circ + \Delta\bar{h})_i = \bar{Q}_{out} + \sum_P N_e (\bar{h}_f^\circ + \Delta\bar{h})_e$$

Note that $W = 0$ since the boundary is rigid.

Since $\bar{Q}_{in} = 0$, we get

$$-\bar{Q}_{out} = \sum_P N_e (\bar{h}_f^\circ + \Delta\bar{h})_e - \sum_R N_i (\bar{h}_f^\circ + \Delta\bar{h})_i \qquad (1)$$

where $\Delta\bar{h} = \bar{h} - \bar{h}_o$.

$\Delta\bar{h}$ is also called sensible enthalpy relative to 25°C, 1 atm.

Assuming the air and the combustion products to be ideal gases, we have $h = h(T)$.

From Table 1,

$$\bar{h}^{\circ}_{f,\,O_2} = 0, \quad \bar{h}^{\circ}_{f,\,N_2} = 0.$$

$$\sum_R N_i (\bar{h}^{\circ}_f + \Delta \bar{h})_i = \bar{h}^{\circ}_{f,\,C_8H_{18}(g)}$$

$$= -208,450 \text{ kJ/kmol fuel}$$

$$\sum_P N_e (\bar{h}^{\circ}_f + \Delta \bar{h})_e = N_{CO_2} (\bar{h}^{\circ}_f + \Delta \bar{h})_{CO_2}$$

$$+ N_{H_2O} (\bar{h}^{\circ}_f + \Delta \bar{h})_{H_2O}$$

$$+ N_{O_2} (\Delta \bar{h})_{O_2} + N_{N_2} (\Delta \bar{h})_{N_2}$$

$$= 8(-393,520 + \bar{h}_{530K,\,CO_2} - \bar{h}_{298K,\,CO_2})$$

$$+ 9(-241,820 + \bar{h}_{530K,\,H_2O} - \bar{h}_{298K,\,H_2O})$$

$$+ 12.5(\bar{h}_{530K,\,O_2} - \bar{h}_{298K,\,O_2})$$

$$+ 94(\bar{h}_{530K,\,N_2} - \bar{h}_{298K,\,N_2})$$

$$= 8(-393{,}520 + 19{,}029 - 9364)$$
$$+ 9(-241{,}820 + 17{,}889 - 9904)$$
$$+ 12.5(15{,}708 - 8682)$$
$$+ 94(15{,}469 - 8669)$$

$$= 8(-383{,}855) + 9(-233{,}835)$$
$$+ 12.5(7026) + 94(6800)$$

$$= -3{,}070{,}840 - 2{,}104{,}515$$
$$+ 87{,}825 + 639{,}200$$

$$= -4{,}448{,}330 \text{ KJ/Kmol fuel}$$

Hence, from eq. (1),

$$-\bar{Q}_{out} = -4{,}448{,}330 - (-208{,}450)$$

$$= -4{,}239{,}880$$

$$\Rightarrow \bar{Q}_{out} = 4{,}239{,}880 \text{ KJ/Kmol fuel}$$

(13 points)

Then heat transfer per kg of fuel is

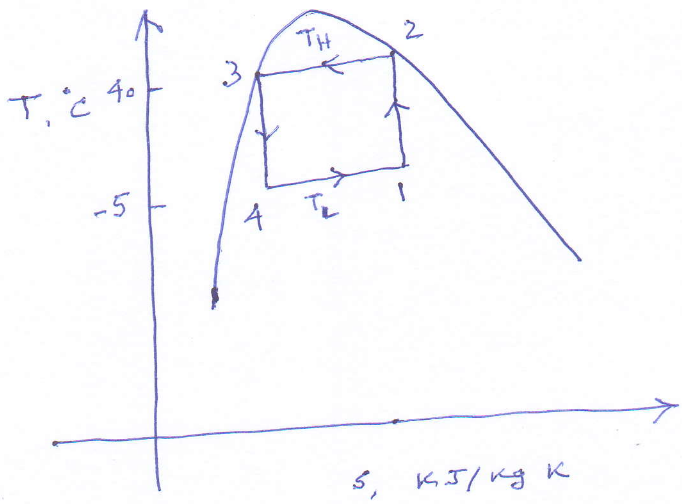
$$Q_{out} = \frac{\bar{Q}_{out}}{M_{C_8H_{18}}} = \frac{4,239,880 \text{ KJ/Kmol fuel}}{114 \text{ Kg/Kmol fuel}}$$

$$= \boxed{37191.9 \text{ KJ/Kg fuel}}$$

(2 points)

Gross total : 13+2
= 15

5. (a) The T-s diagram of the cycle is given below (Fig. 1).



(3 points)

Fig. 1

(b) State 3 is saturated liquid.

$$s_4 = s_3 = 0.4473 = s_{f@40°C}$$

$$= s_{f@-5°C} + x_4 \, s_{fg@-5°C}$$

$$= 0.1989 + x_4 (0.8477)$$

$$\Rightarrow x_4 (0.8477) = 0.4473 - 0.1989$$

$$\Rightarrow x_4 = \frac{0.4473 - 0.1989}{0.8477}$$

$$= \frac{0.2484}{0.8477} = \boxed{0.293}$$

(2.5 points)

State 2 is saturated vapour.

$s_1 = s_2 = s_{g@40°C} = 0.9552 = s_{f@-5°C} + x_1\, s_{fg@-5°C}$

$= 0.1989 + x_1 (0.8477)$

$\Rightarrow x_1 (0.8477) = 0.9552 - 0.1989$

$\Rightarrow x_1 = \frac{0.9552 - 0.1989}{0.8477}$

$= \frac{0.7563}{0.8477} \simeq \boxed{0.892}$

(2.5 points)
Total = 2.5 + 2.5
= 5

(c) $(COP)_{\text{Carnot HP}} = \frac{q_H}{w_{in}} = \frac{T_H}{T_H - T_L}$

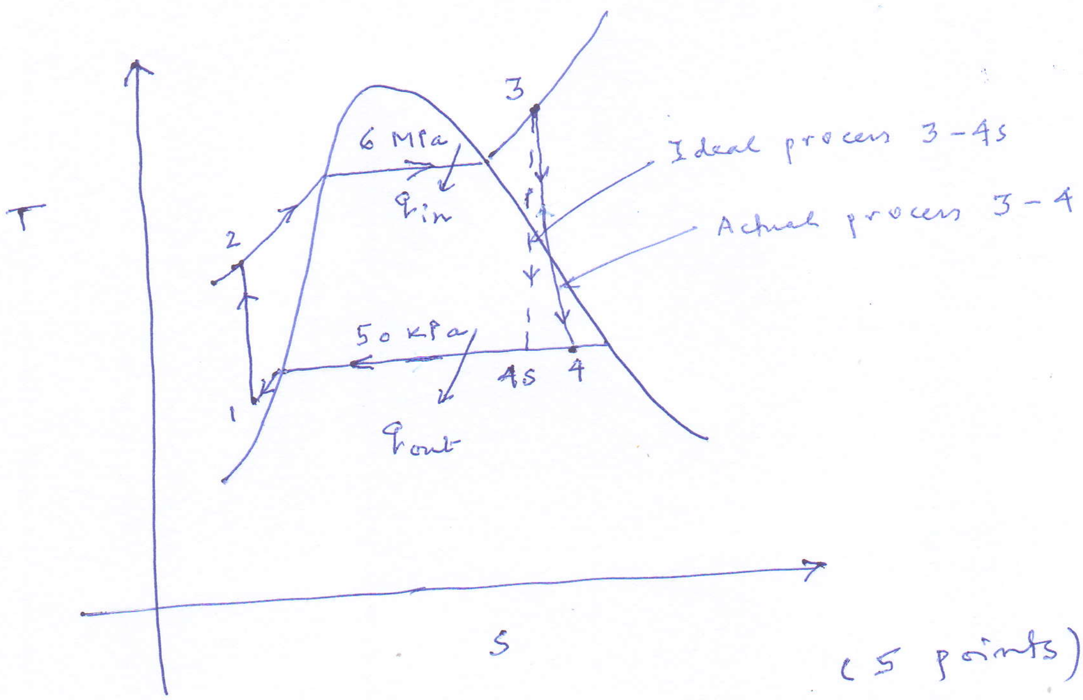
$= \frac{40 + 273}{(40 + 273) - (-5 + 273)}$

$= \frac{313}{45} = 6.955 \approx \boxed{6.96}$

(2 points)

Grand total
= 3 + 5 + 2
= 10

6. The T-s diagram of the cycle is given below. See Fig. 1.



(5 points)

Fig. 1

From the given data,

$P_1 = 50 \text{ kPa}$

$T_1 = T_{sat@50 \text{ kPa}} - 6.3$

$= 81.3 - 6.3$

$= 75°C$

$h_1 \cong h_{f@75°C}$

$= 314.03 \text{ kJ/kg}$

$v_1 = v_{f@75°C}$

$= 0.001026 \text{ m}^3/\text{kg}$

$w_{p,in} = v_1 (P_2 - P_1)$

$= (0.001026)(6000 - 50)$

$= 6.1 \text{ kJ/kg}$

$h_2 = h_1 + w_{p,in}$

$= 314.03 + 6.1$

$= 320.13$ kJ/kg

$P_3 = 6000$ kPa, $T_3 = 450^\circ C$ } $h_3 = 3302.9$ kJ/kg, $s_3 = 6.7219$ kJ/kg K

$P_4 = 50$ kPa, $s_{4s} = s_3$ } $x_{4s} = \dfrac{s_{4s} - s_f}{s_{fg}}$

$= \dfrac{6.7219 - 1.0912}{6.5019}$

$= 0.866$

$h_{4s} = h_f + x_{4s} h_{fg}$

$= 340.54 + (0.866)(2304.7)$

$= 2336.4$ kJ/kg

$\eta_T = \dfrac{h_3 - h_4}{h_3 - h_{4s}}$

$\Rightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s})$

$= 3302.9 - (0.94)(3302.9 - 2336.4)$

$= 2394.4$ kJ/kg

Thus,

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2)$$

$$= (20)(3302.9 - 320.13)$$

$$= \boxed{59655.4 \text{ kW}}$$ (4 points)

$$\dot{W}_{T\,out} = \dot{m}(h_3 - h_4)$$

$$= (20)(3302.9 - 2394.4)$$

$$= 18,170 \text{ kW}$$

$$\dot{W}_{P,\,in} = \dot{m}\, w_{P,\,in} = (20)(6.1)$$

$$= \boxed{122 \text{ kW}}$$ (4 points)

$$\dot{W}_{net} = \dot{W}_{T\,out} - \dot{W}_{P,\,in}$$

$$= 18,170 - 122$$

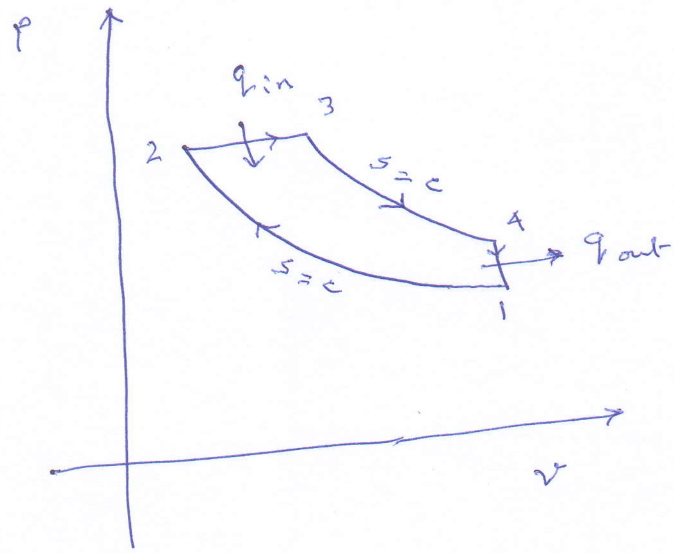
$$= \boxed{18,048 \text{ kW}}$$ (4 points)

and

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{18,048}{59655.4} = \boxed{0.3025}$$ (3 points)

Total : 5 + 4 + 4 + 4 + 3 = 20

7. The p–v diagram of the cycle is given below (Fig. 1).



(3 points)

Fig. 1

(a) Process 1 – 2 : isentropic compression.

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$= (293)(20)^{1.4-1}$$

$$= (293)(20)^{0.4}$$

$$= (293)(3.31445)$$

$$= 971.1 \text{ K}$$

Process 2-3 : P = constant heat addition.

$$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2}$$

$$\Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2200}{971.1} = 2.265$$

Process 3-4 : isentropic expansion.

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{k-1}$$

$$= T_3 \left(\frac{2.265\, V_2}{V_4}\right)^{k-1}$$

$$= T_3 \left(\frac{2.265}{V_4/V_2}\right)^{k-1}$$

Since $\frac{V_4}{V_2} = \frac{V_1}{V_2} = r = 20$,

$$T_4 = T_3 \left(\frac{2.265}{r}\right)^{k-1}$$

$$= (2200)\left(\frac{2.265}{20}\right)^{1.4-1}$$

$$= (2200)(0.11325)^{0.4}$$

$$= (2200)(0.4184)$$

$$= 920.48$$

$$\approx 920.5\ K$$

$$q_{in} = h_3 - h_2 = c_p (T_3 - T_2)$$

$$= (1.005)(2200 - 971.1)$$

$$= 1235.04 \text{ kJ/kg}$$

$$q_{out} = u_4 - u_1 = c_v (T_4 - T_1)$$

$$= (0.718)(920.5 - 293)$$

$$= 450.5 \text{ kJ/kg}$$

$$w_{net, out} = q_{in} - q_{out}$$

$$= 1235.04 - 450.5$$

$$= 784.54 \text{ kJ/kg}$$

$$\eta_{th} = \frac{w_{net, out}}{q_{in}} = \frac{784.54}{1235.04} = 0.635 = \boxed{63.5\%}$$

(7 points)

(b) $$v_1 = \frac{R T_1}{P_1} = \frac{(0.287)(293)}{95}$$

$$= 0.885 \text{ m}^3/\text{kg} = v_{max}$$

$$v_{min} = v_2 = \frac{v_{max}}{r}$$

$$MEP = \frac{w_{net,\,out}}{v_1 - v_2} = \frac{w_{net,\,out}}{v_1\left(1 - \frac{v_2}{v_1}\right)}$$

$$= \frac{w_{net,\,out}}{v_1\left(1 - \frac{1}{r}\right)} = \frac{784.54}{(0.885)\left(1 - \frac{1}{20}\right)}$$

$$= \frac{784.54}{(0.885)(0.95)} = \frac{784.54}{0.84075}$$

$$= \boxed{933.14 \text{ kPa}}$$

(5 points)

Total : 3 + 7 + 5
= 15