

ESO201A
Lecture#22
(Class Lecture)

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By

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①

Example Problem

Refrigerant R-134a enters the capillary tube of a refrigerator as saturated liquid at 0.8 MPa and is throttled to a pressure of 0.12 MPa. Determine the quality of the refrigerant at the final state and the temperature drop during this process.

Solution

Assumptions

$$1. \dot{Q} = 0 \quad 3. \dot{W} = 0$$

$$2. \Delta KE = 0 \quad 4. \Delta PE = 0 \quad (800 \text{ kPa})$$

$$\text{At inlet: } \left. \begin{array}{l} P_1 = 0.8 \text{ MPa} \\ \text{(sat. liquid)} \end{array} \right\} \begin{array}{l} T_1 = T_{\text{sat}@0.8 \text{ MPa}} \\ = 31.31^\circ\text{C} \end{array}$$

$$h_1 = h_f @ 0.8 \text{ MPa} \\ = 95.47 \text{ kJ/kg}$$

(Table A-12)

(2)

At exit : $P_2 = 0.12 \text{ MPa}$ \nearrow (120 kPa)
 $(h_2 = h_1)$ \rightarrow $h_f = 22.49 \text{ kJ/kg}$
 $h_g = 236.97 \text{ kJ/kg}$
 $T_{\text{sat}} = -22.32^\circ\text{C}$
 (Table A-12)

Obviously, $h_f < h_2 < h_g$.

Thus, the refrigerant exists as a saturated mixture at the exit state.

The quality at this state is :

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{h_2 - h_f}{h_g - h_f}$$

$$= \frac{95.47 - 22.49}{236.97 - 22.49}$$

$$= \frac{72.98}{214.48}$$

$$= \boxed{0.34}$$

Since the exit state is a saturated mixture at 0.12 MPa, the exit temperature must be the saturation

(3)

temperature at this pressure, which is -22.32°C . Then the temperature change for this process becomes

$$\Delta T = T_2 - T_1 = (-22.32 - 31.31)^{\circ}\text{C}$$

$$= \boxed{-53.63^{\circ}\text{C}}$$

Note : The temperature of the refrigerant drops by 53.63°C during this throttling process. Also 34% ^(by mass) of the refrigerant vaporizes during this throttling process, and the energy needed to vaporize this refrigerant is absorbed from the refrigerant itself.

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The Joule - Thomson Coefficient

The temperature behaviour of a fluid during a throttling ($h = \text{constant}$) process is described by the Joule-Thomson coefficient, defined as

$$\mu_{JT} = \left(\frac{\partial T}{\partial P} \right)_h$$

(1)

Notice that if

$$\mu_{JT} \begin{cases} < 0 & \text{temperature increases} \\ = 0 & \text{temperature remains constant} \\ > 0 & \text{temperature decreases} \end{cases}$$

during the throttling process.

Consider Fig. 1 which shows the throttling process for fixed P_1, T_1 while P_2, T_2 vary.

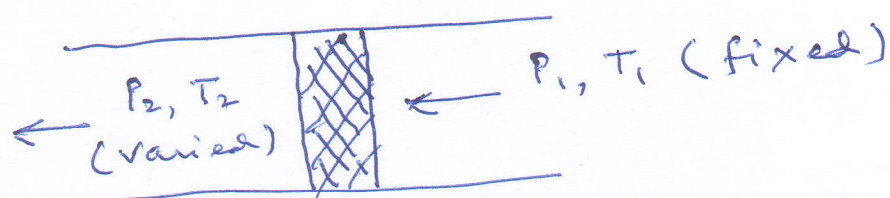


Fig. 1

A gas at a fixed T_1 and P_1 (thus fixed enthalpy) is forced through a porous plug, and T_2, P_2 are measured. The experiment is repeated for different sizes of porous plugs, each giving a different set of T_2 and P_2 . Plotting the temperatures against pressures gives us an $h = \text{constant}$ line on a $T-P$ diagram, as shown in Fig. 2.

* P_2 is set at a value lower than P_1 and T_2 is measured. The pressures P_1 and P_2 are maintained by means of a compressor. The flow is steady.

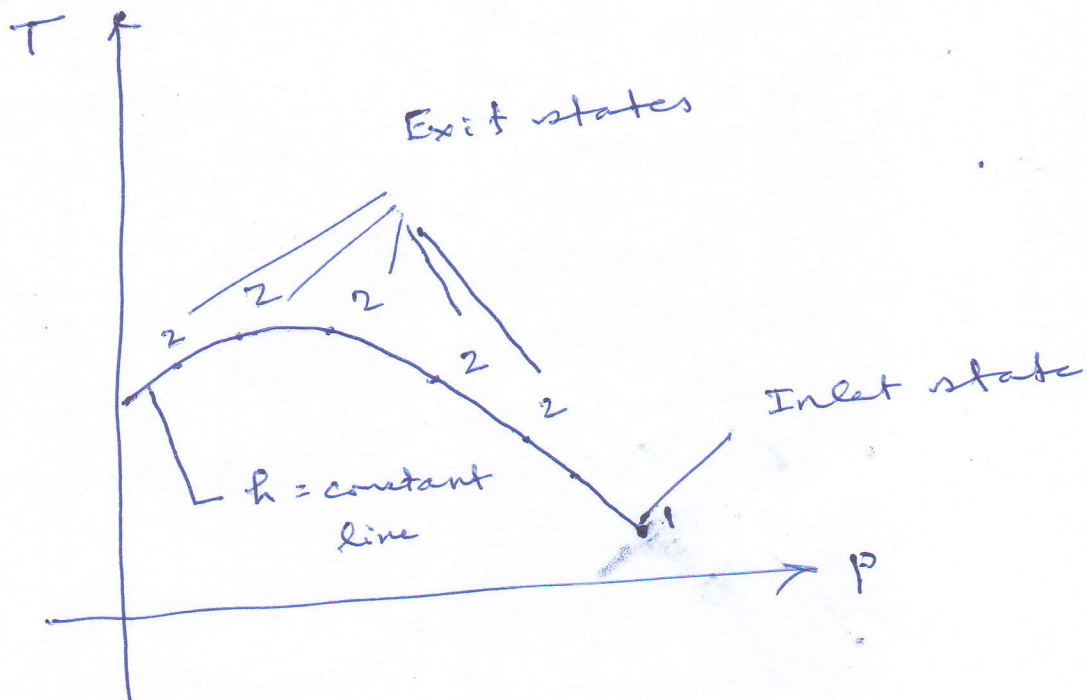


Fig. 2

Repeating the experiment for different sets of inlet pressure and temperature and plotting the results, we can construct a T - P diagram for a substance with several $h = \text{constant}$ lines, as shown in Fig. 3.

Note that an isenthalpic curve is not the graph of a throttling process. No such graph can be drawn, because in any throttling process the intermediate irreversible states traversed by a gas cannot be described by means of thermodynamic coordinates. An isenthalpic curve is the locus of all points representing equilibrium initial and final states of the same enthalpy.

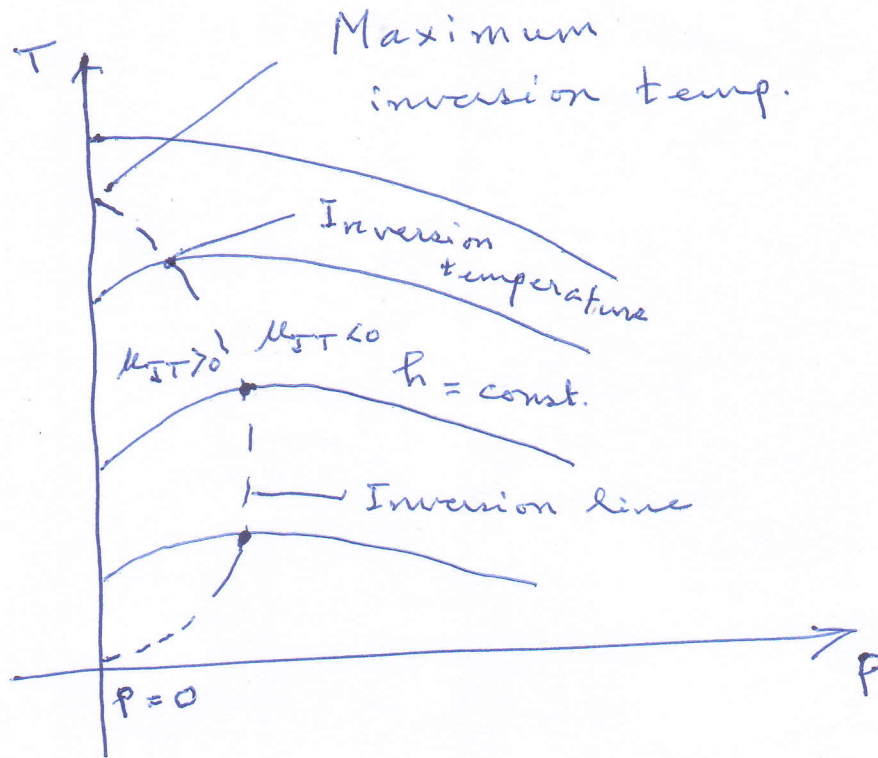


Fig. 3

Some $h = \text{const}$ lines pass through a point of zero slope or $\mu_{ST} = 0$. The line that passes through these points is called the inversion line, and the temperature at a point where a $h = \text{const}$ line intersects the inversion line is called the inversion temperature. The temperature at the intersection of the $p = 0$ line (ordinate) and the upper part of the inversion line is called the maximum inversion temperature.

Notice that the slopes of the $h = \text{const}$ lines are negative ($\mu_{JT} < 0$) at states to the right of the inversion line and positive ($\mu_{JT} > 0$) to the left of the inversion line.

A throttling process proceeds along a constant-enthalpy line in the direction of decreasing pressure, that is from right to left. It is clear from the diagram that a cooling effect cannot be achieved by throttling unless the fluid is below its maximum inversion temperature.

This presents problems for substances whose maximum inversion temperature is well below room temperature. For hydrogen, for example, the maximum inversion temperature is -68°C . Thus hydrogen must be cooled below this temperature if any further cooling is to be achieved by throttling.

Mixing Chambers

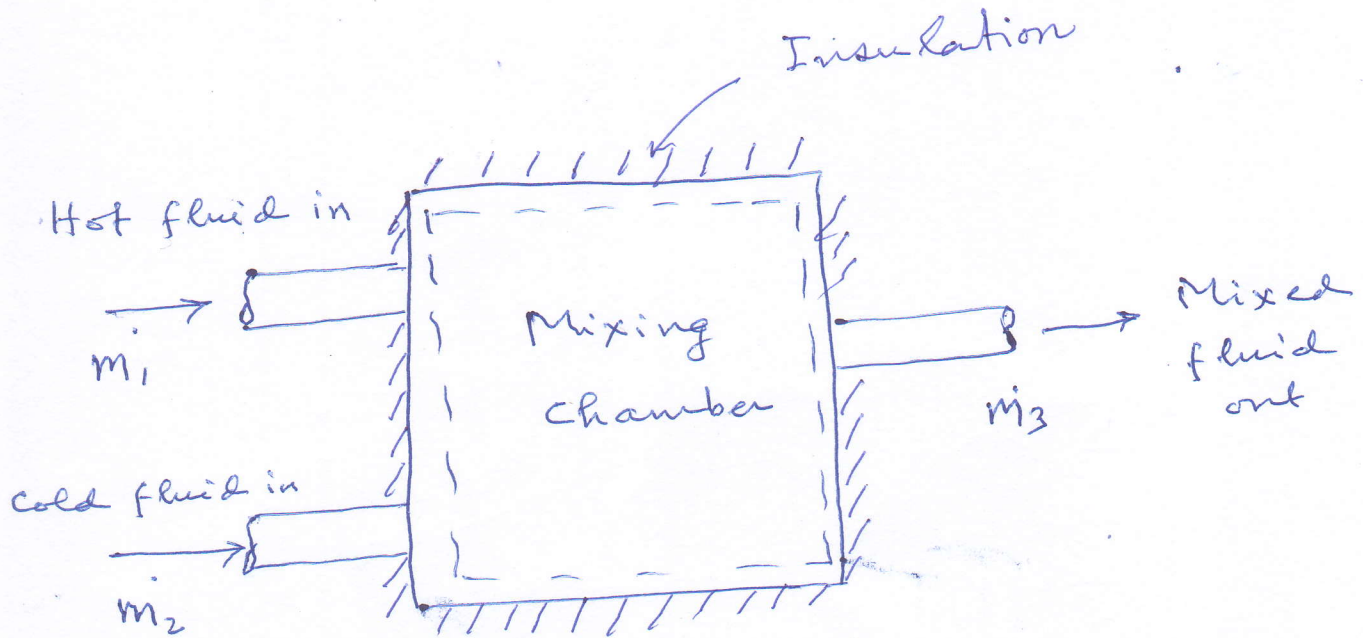


Fig.1 Mixing Chamber

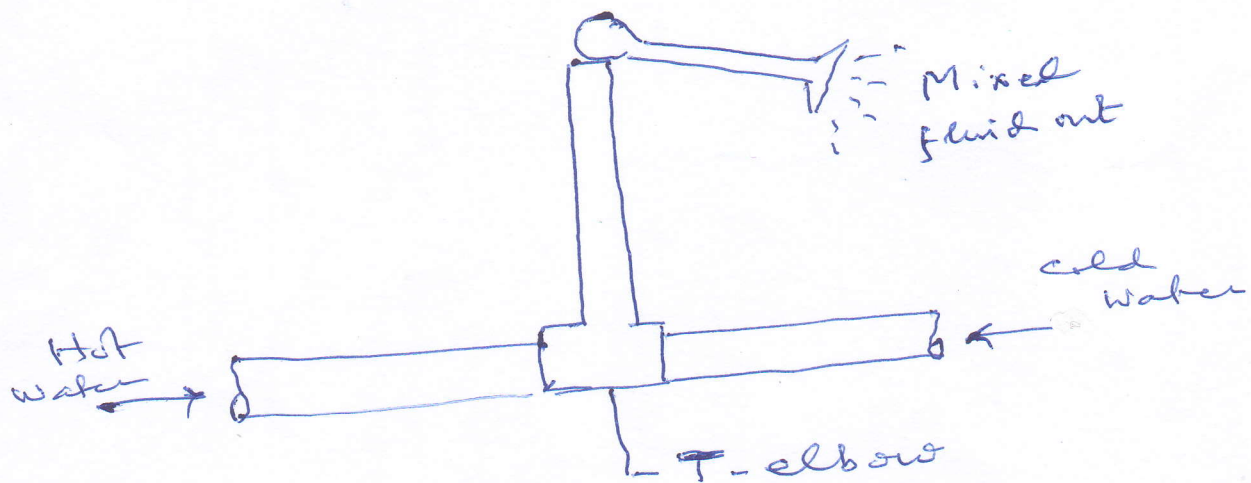


Fig.2 An ordinary shower

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Conservation of Mass

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\Rightarrow \boxed{\dot{m}_1 + \dot{m}_2 = \dot{m}_3}$$

Conservation of Energy

$$\dot{Q} = 0 \text{ (Well insulated)}$$

$$\dot{W} = 0 \text{ (Does not involve any kind of work)}$$

$$\Delta K_e = 0$$

$$\Delta P_e = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\Rightarrow \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$\Rightarrow \boxed{\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3}$$

Example Problem

Consider a mixing chamber where hot water 60°C is mixed with cold water at 10°C . If it is desired that a steady stream of warm water at 45°C be supplied, determine the ratio of mass flow rates of the hot to cold water. Assume the heat losses from the mixing chamber to be negligible and the mixing to take place at a pressure of 150 kPa . See Fig. 1

Solution :

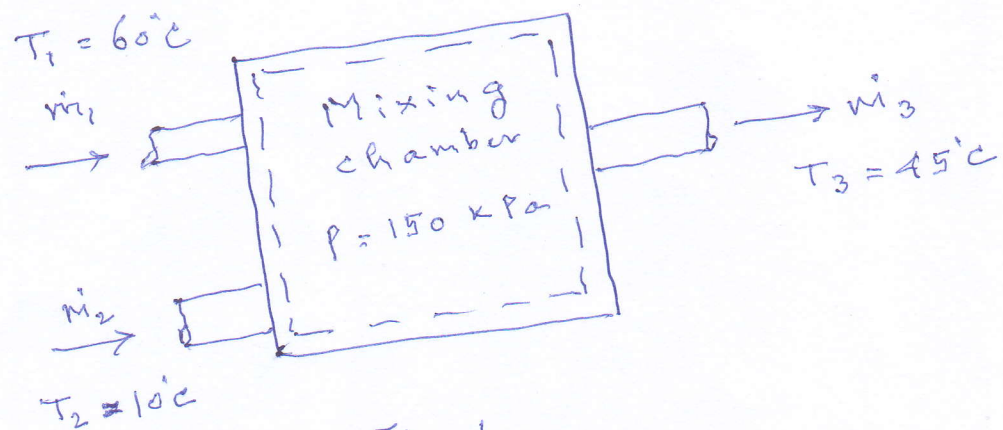


Fig. 1

(2)

Assumptions

1. Steady-state ($\Delta m_{cv} = 0$, $\Delta E_{cv} = 0$).
2. $\Delta ke = 0$, $\Delta pe = 0$.
3. $\dot{Q} = 0$.
4. $\dot{W} = 0$.

Mass balance

$$\dot{m}_{in} = \dot{m}_{out}$$

(1)

$$\Rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Energy balance

$$\dot{E}_{in} = \dot{E}_{out}$$

(2)

$$\Rightarrow \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

(since $\dot{Q} = 0$, $\dot{W} = 0$,
 $\Delta ke = 0$, $\Delta pe = 0$)

From eq. (1), $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$

Substituting eq. (1) into eq. (2),

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3 \quad (3)$$

(3)

Dividing eq. (3) by \dot{m}_2 ,

$$\frac{\dot{m}_1}{\dot{m}_2} h_1 + h_2 = \left(\frac{\dot{m}_1}{\dot{m}_2} + 1 \right) h_3 \quad (4)$$

$$\Rightarrow y h_1 + h_2 = (y + 1) h_3 \quad (5)$$

where $y = \frac{\dot{m}_1}{\dot{m}_2}$

$$\Rightarrow y = \frac{h_3 - h_2}{h_1 - h_3}$$

From Table A-5, we see that

$$T_{\text{sat@150 kPa}} = 121.35^\circ\text{C}$$

Since $T < T_{\text{sat}}$ for inlet and exit streams, the water exists as a compressed or subcooled liquid in all three streams. The data for this problem is not available in Table A-7. We know, for a compressed liquid,

$$h = h_{f@T} + v_{f@T} (P - P_{\text{sat@T}})$$

For stream 1 (Table A-4)

$$\begin{aligned} h_1 &= h_{f@60^\circ\text{C}} + v_{f@60^\circ\text{C}} (P - P_{\text{sat@60}^\circ\text{C}}) \\ &= 251.18 + 0.001017 (150 - 19.947) \\ &= 251.18 + 0.001017 (130.053) \\ &= 251.18 + 0.13274901 \\ &= 251.312749 \\ &= \boxed{251.31 \text{ kJ/kg}} \end{aligned}$$

(4)

For stream 2 (Table A-4)

$$\begin{aligned}
 h_2 &= h_{f@10^\circ\text{C}} + v_{f@10^\circ\text{C}} (P - P_{\text{sat}@10^\circ\text{C}}) \\
 &= 42.022 + 0.001000 (150 - 1.2281) \\
 &= 42.022 + 0.001000 (148.7719) \\
 &= 42.022 + 0.1487719 \\
 &= 42.1707719 \\
 &= \boxed{42.17 \text{ kJ/kg}}
 \end{aligned}$$

For stream 2 (Table A-4)

$$\begin{aligned}
 h_3 &= h_{f@45^\circ\text{C}} + v_{f@45^\circ\text{C}} (P - P_{\text{sat}@45^\circ\text{C}}) \\
 &= 188.44 + 0.001010 (150 - 9.5953) \\
 &= 188.44 + 0.001010 (140.4047) \\
 &= 188.44 + 0.14181177 \\
 &= 188.5818118 \\
 &= \boxed{188.58 \text{ kJ/kg}}
 \end{aligned}$$

From eq. (5),

$$\begin{aligned}
 y &= \frac{h_3 - h_2}{h_1 - h_3} \\
 &= \frac{188.58 - 42.17}{251.31 - 188.58} \\
 &= \frac{146.41}{62.73} \\
 &= \boxed{2.33}
 \end{aligned}$$

That is, $\frac{\dot{m}_1}{\dot{m}_2} = 2.33$

$$\text{or } \dot{m}_1 = 2.33 \dot{m}_2$$

In other words, the mass flow rate of the hot water must be 2.33 times the mass flow rate of the cold water for the mixture to leave at 45°C .