# MSO205 Mid Semester Examination (September 20, 2024)

**Duration: 8:00 am - 10:00 am (+40 minutes for DAP students), Maximum Marks: 28**

**Question 1.** **(3 marks) A secretary types 4 letters and prepares 4 corresponding envelopes. In a hurry, she places one letter in each envelope at random. What is the probability that at least one letter is in the correct envelope? You may write your answer as a fraction.**

**Question 2.** **(2 marks) Show by an example that pairwise independence of events does not imply mutual independence of the same events.**

**Question 3.** **(2 marks) Let** $(\Omega, \mathcal{F}, \mathbb{P})$ **be a probability space such that there exist events** $A$ **and** $B$ **with** $\mathbb{P}(A) = 0.9, \mathbb{P}(B) = 0.2$. **Is it true that** $\mathbb{P}(A|B) \geq 0.3$? **Justify your answer.**

**Question 4.** **(3 + 1 marks) Let** $X$ **be an RV with the distribution function** $F_X$. **Consider the following two functions** $G_1, G_2 : \mathbb{R} \to \mathbb{R}$ **defined by**

$$G_1(x) := (F_X(x))^2, \quad G_2(x) := G_1(x) + \frac{1}{2}, \forall x \in \mathbb{R}.$$

**Can** $G_1$ **and** $G_2$ **be distribution functions? Justify your answer.**

**Question 5.** **(4 + (1.5 + 1.5) + 3 + (2 + 1) marks) Let** $X$ **be a continuous RV with the following p.d.f.** $f_X : \mathbb{R} \to [0, \infty)$ **defined by**

$$f_X(x) := \begin{cases} \exp(-x), & \textbf{if } x > 0, \\ 0, & \textbf{otherwise} \end{cases}$$

**Consider the RV** $Y := |X - 1|$. **Answer the following sub-questions with full justification.**

(a) **Compute the distribution function of** $Y$.

(b) **Show that** $Y$ **is a continuous RV and compute its p.d.f.**

(c) **Identify the support of** $Y$.

(d) **Compute a quantile of order** $0.9$ **for** $Y$. **Is it unique? You may use the fact that** $\exp(-2) \approx 0.1353$. **You may write your answer in terms of a natural logarithm, i.e.** $\log_e$ **or** $\ln$.

**Question 6.** **(2 + (1 + 1) marks) Consider a discrete RV** $X$ **with the p.m.f.**

$$f_X(x) := \begin{cases} \frac{1}{3}\left(\frac{2}{3}\right)^x, & \textbf{if } x \in \{0, 1, 2, \cdots\}, \\ 0, & \textbf{otherwise}. \end{cases}$$

(a) **Justify the existence of MGF of** $X$ **and compute it.**

(b) **Using the MGF obtained in part (a), compute the mean and variance of** $X$.