Question 4. (3 + 1 marks) Fix $p \in (0,1)$. Let X_1, X_2, X_3 be a random sample of size 3 from the $Bernoulli(p)$ distribution. Find the joint distribution of $X_{(2)}$ and $X_{(3)}$. Are $X_{(2)}$ and $X_{(3)}$ independent? Justify your answer.
Answer:

MSO205: Introduction to Probability Theory
Quiz 3 (October 29, 2024)

6:20 pm - 7:20 pm (+20 minutes for DAP students)
Maximum Marks: 15

Instructions:

1. Electronic devices (mobiles, calculators etc.) are prohibited.
2. DO NOT do any rough work on this sheet. Additional sheets for rough work shall be provided. DO NOT attach any additional sheet to this page.
3. If necessary, you may write your answer using a simple fraction.

Name:

Roll No.:

Question 1. (3 marks) Let X be a continuous RV with DF F_X and p.d.f. f_X such that $\mathbb{P}(0 < X \leq 2) = 1$. Is it necessary that the variance $Var(X) \leq 1$? Justify.
Answer:

Please turn over

Question 2. (2 + 2 marks) A box contains 8 electric bulbs, out of which 4 are defective. All the bulbs look identical. Suppose 3 bulbs drawn at random and without replacement, and then tested. Let X denote the number of defective bulbs in this random experiment. Identify the distribution of X and compute $\mathbb{E}[X(X-1)(X-2)]$.
Answer:

Question 3. (3 + 1 marks) Let $Z = (X, Y)$ be a 2-dimensional continuous random vector with the joint p.d.f.

$$f_Z(x, y) = \begin{cases} 8xy, & \text{if } 0 < x < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional p.d.f. $f_{X|Y}(x \mid \frac{1}{3}), \forall x \in \mathbb{R}$ and compute $\mathbb{E}[X \mid Y = \frac{1}{3}]$.
Answer:

Please turn over

Question 4. (3 + 1 marks) Fix $p \in (0,1)$. Let X_1, X_2, X_3 be a random sample of size 3 from the $Bernoulli(p)$ distribution. Find the joint distribution of $X_{(2)}$ and $X_{(3)}$. Are $X_{(2)}$ and $X_{(3)}$ independent? Justify your answer.
Answer:

MSO205: Introduction to Probability Theory
Quiz 3 (October 29, 2024)

6:20 pm - 7:20 pm (+20 minutes for DAP students)
Maximum Marks: 15

Instructions:

1. Electronic devices (mobiles, calculators etc.) are prohibited.
2. DO NOT do any rough work on this sheet. Additional sheets for rough work shall be provided. DO NOT attach any additional sheet to this page.
3. If necessary, you may write your answer using a simple fraction.

Name:

Roll No.:

Question 1. (3 marks) Let X be a continuous RV with DF F_X and p.d.f. f_X such that $\mathbb{P}(0 < X \leq 2) = 1$. Is it necessary that the variance $Var(X) \leq 1$? Justify.
Answer:

Please turn over

Question 2. $(2 + 2$ marks) A box contains 8 electric bulbs, out of which 4 are defective. All the bulbs look identical. Suppose 3 bulbs drawn at random and without replacement, and then tested. Let X denote the number of defective bulbs in this random experiment. Identify the distribution of X and compute $\mathbb{E}[X(X-1)(X-2)]$.
Answer:

Question 3. $(3 + 1$ marks) Let $Z = (X, Y)$ be a 2-dimensional continuous random vector with the joint p.d.f.

$$f_Z(x, y) = \begin{cases} 8xy, & \text{if } 0 < x < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional p.d.f. $f_{X|Y}(x \mid \frac{1}{4}), \forall x \in \mathbb{R}$ and compute $\mathbb{E}[X \mid Y = \frac{1}{4}]$.
Answer:

Please turn over

Question 4. (3 + 1 marks) Fix $p \in (0, 1)$. Let X_1, X_2, X_3 be a random sample of size 3 from the $Bernoulli(p)$ distribution. Find the joint distribution of $X_{(2)}$ and $X_{(3)}$. Are $X_{(2)}$ and $X_{(3)}$ independent? Justify your answer.

Answer:

MSO205: Introduction to Probability Theory
Quiz 3 (October 29, 2024)

6:20 pm - 7:20 pm (+20 minutes for DAP students)
Maximum Marks: 15

Instructions:

1. Electronic devices (mobiles, calculators etc.) are prohibited.
2. DO NOT do any rough work on this sheet. Additional sheets for rough work shall be provided. DO NOT attach any additional sheet to this page.
3. If necessary, you may write your answer using a simple fraction.

Name:

Roll No.:

Question 1. (3 marks) Let X be a continuous RV with DF F_X and p.d.f. f_X such that $\mathbb{P}(0 < X \leq 2) = 1$. Is it necessary that the variance $Var(X) \leq 1$? Justify.

Answer:

Please turn over

Question 2. ($2 + 2$ marks) A box contains 8 electric bulbs, out of which 4 are defective. All the bulbs look identical. Suppose 3 bulbs drawn at random and without replacement, and then tested. Let X denote the number of defective bulbs in this random experiment. Identify the distribution of X and compute $\mathbb{E}[X(X-1)(X-2)]$.
Answer:

Question 3. ($3 + 1$ marks) Let $Z = (X, Y)$ be a 2-dimensional continuous random vector with the joint p.d.f.

$$f_Z(x, y) = \begin{cases} 8xy, & \text{if } 0 < y < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional p.d.f. $f_{Y|X}(y \mid \frac{1}{3}), \forall y \in \mathbb{R}$ and compute $\mathbb{E}[Y \mid X = \frac{1}{3}]$.
Answer:

Please turn over

Question 4. (3 + 1 marks) Fix $p \in (0, 1)$. Let X_1, X_2, X_3 be a random sample of size 3 from the $Bernoulli(p)$ distribution. Find the joint distribution of $X_{(2)}$ and $X_{(3)}$. Are $X_{(2)}$ and $X_{(3)}$ independent? Justify your answer.
Answer:

MSO205: Introduction to Probability Theory
Quiz 3 (October 29, 2024)

6:20 pm - 7:20 pm (+20 minutes for DAP students)
Maximum Marks: 15

Instructions:

1. Electronic devices (mobiles, calculators etc.) are prohibited.
2. DO NOT do any rough work on this sheet. Additional sheets for rough work shall be provided. DO NOT attach any additional sheet to this page.
3. If necessary, you may write your answer using a simple fraction.

Name:

Roll No.:

Question 1. (3 marks) Let X be a continuous RV with DF F_X and p.d.f. f_X such that $\mathbb{P}(0 < X \leq 2) = 1$. Is it necessary that the variance $Var(X) \leq 1$? Justify.
Answer:

Please turn over

Question 2. (2 + 2 marks) A box contains 8 electric bulbs, out of which 4 are defective. All the bulbs look identical. Suppose 3 bulbs drawn at random and without replacement, and then tested. Let X denote the number of defective bulbs in this random experiment. Identify the distribution of X and compute $\mathbb{E}[X(X-1)(X-2)]$.
Answer:

Question 3. (3 + 1 marks) Let $Z = (X, Y)$ be a 2-dimensional continuous random vector with the joint p.d.f.

$$f_Z(x, y) = \begin{cases} 8xy, & \text{if } 0 < y < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional p.d.f. $f_{Y|X}(y \mid \frac{1}{4}), \forall y \in \mathbb{R}$ and compute $\mathbb{E}[Y \mid X = \frac{1}{4}]$.
Answer:

Please turn over