**MSO205 PRACTICE PROBLEMS SET 13**

*Question* 1. Refer to Question 3 of problem set 12. Show by an example that the continuous mapping theorem does not hold for converge in $r$-th mean/moment.

*Question* 2. Suppose $X, X_1, X_2, \cdots, Y, Y_1, Y_2, \cdots$ be RVs defined on the same probability space. Fix $\alpha, \beta \in \mathbb{R}$.

(i) Suppose $X_n \xrightarrow{n\to\infty} X$ and $Y_n \xrightarrow{n\to\infty} Y$ in $r$-th mean for some $r \geq 1$. Show that $\alpha X_n + \beta Y_n \xrightarrow{n\to\infty} \alpha X + \beta Y$ in $r$-th mean.

(ii) Suppose $X_n \xrightarrow[n\to\infty]{\text{a.s.}} X$ and $Y_n \xrightarrow[n\to\infty]{\text{a.s.}} Y$. Show that $\alpha X_n + \beta Y_n \xrightarrow[n\to\infty]{\text{a.s.}} \alpha X + \beta Y$.

*Question* 3. Construct an example of a sequence of RVs $\{X_n\}_n$ converging in law/distribution, but not in probability.

*Question* 4. Consider a sequence $\{X_n\}_n$ of RVs with $X_n \sim N(\frac{1}{n}, 1 - \frac{1}{n}), \forall n$. Does this sequence converge in law/distribution?

*Question* 5. Suppose that a continuous RV $X$ has a quantile of order $\frac{1}{3}$ at 5. Consider a random sample of size 100 from the distribution of $X$. What is the probability (approximately) that more than 40 sample values are more than 5? Express the approximate value in terms of $\Phi$, the DF of $N(0, 1)$ distribution.

*Question* 6. Fix $\lambda > 0$. Let $X_1, X_2, \cdots$ be a sequence of i.i.d. RVs with $Exponential(\lambda)$ distribution. Consider the sample mean $\bar{X}_n := \frac{1}{n}\sum_{j=1}^{n} X_j, \forall n$. Show that

$$\sqrt{n}\left(\frac{1}{\bar{X}_n} - \frac{1}{\lambda}\right) \xrightarrow[n\to\infty]{d} N\left(0, \frac{1}{\lambda^2}\right).$$