

## MSO205 PRACTICE PROBLEMS SET 1

Question 1. Write down the sample spaces of the following random experiments.

- (a) Shuffle a standard deck of cards and draw the first card.
- (b) A box contains 3 identical red balls and 2 identical green balls. Draw a ball from the box blindfolded and then check (and note down) the colour of the ball. Put the ball back into the box.
- (c) Throw a standard six-sided die two times and add up the two numbers obtained.

Question 2. Let  $(\Omega, \mathcal{F}, \mathbb{P})$  be a probability space associated with a random experiment  $\mathcal{E}$ .

- (a) Let  $B \in \mathcal{F}$  be such that  $\mathbb{P}(B) = 1$ . For any event  $A \in \mathcal{F}$ , show that  $\mathbb{P}(A) = \mathbb{P}(A \cap B)$ .  
(Hint: What is  $\mathbb{P}(A \cap B^c)$ ?)
- (b) (Boole's inequality) Let  $n \geq 2$  be any integer and let  $E_1, E_2, \dots, E_n$  be events in  $\mathcal{F}$ . Prove that

$$\mathbb{P}\left(\bigcup_{i=1}^n E_i\right) \leq \sum_{i=1}^n \mathbb{P}(E_i).$$

- (c) Let  $\{E_n\}_n$  be a sequence of events in  $\mathcal{F}$ . Show that

$$\mathbb{P}\left(\bigcup_{n=1}^{\infty} E_n\right) \leq \sum_{n=1}^{\infty} \mathbb{P}(E_n).$$

(Hint: Take  $A_1 := E_1$  and for  $n \geq 2$ , define  $A_n := E_n \cap (E_1 \cup E_2 \cup \dots \cup E_{n-1})^c$ . Prove that  $\bigcup_n A_n = \bigcup_n E_n$ . Use the  $A_n$ 's.)

- (d) Let  $n \geq 2$  be any integer and let  $E_1, E_2, \dots, E_n$  be events in  $\mathcal{F}$ . Prove that

$$\mathbb{P}\left(\bigcap_{i=1}^n E_i\right) \geq \sum_{i=1}^n \mathbb{P}(E_i) - (n-1).$$

Question 3. Let  $(\Omega, \mathcal{F}, \mathbb{P})$  be the probability space associated with a random experiment  $\mathcal{E}$ . Let  $A, B, C$  be events such that  $\mathbb{P}(A) = 0.3, \mathbb{P}(B) = 0.4, \mathbb{P}(A \cap B) = 0.2, \mathbb{P}(C) = 0.1$ . Further assume that  $A, C$  are mutually exclusive and  $B, C$  are mutually exclusive.

Find the probabilities that

- (a) exactly one of the events  $A$  or  $B$  occurs
- (b) at least one of the events  $A, B$  or  $C$  occurs
- (c) none of  $A$  and  $B$  will occur
- (d)  $A$  occurs, but  $C$  does not occur.