MSO205 PRACTICE PROBLEMS SET 6

Question 1. Consider a continuous RV X with the p.d.f.

$$f_X(x) := \begin{cases} \frac{1}{2}, \text{ if } x \in (-1, 0), \\ \frac{1}{3}, \text{ if } x \in (0, \frac{3}{2}), \\ 0, \text{ otherwise.} \end{cases}$$

Consider the RV $Y = X^4$.

(a) First find the DF F_Y and then compute the p.d.f. f_Y.

(b) First find the p.d.f. f_Y and then compute the DF F_Y.

Question 2. Consider a discrete RV X with the p.m.f.

$$f_X(x) := \begin{cases} \frac{1}{4}\left(\frac{3}{4}\right)^x, \text{ if } x \in \{0, 1, 2, \cdots\}, \\ 0, \text{ otherwise.} \end{cases}$$

Compute the expectation $\mathbb{E}X$ and the variance $Var(X)$, if these exist.

Question 3. Consider a continuous RV X with the p.d.f.

$$f_X(x) := \begin{cases} \frac{1}{2}, \text{ if } x \in (-1, 0), \\ \frac{1}{3}, \text{ if } x \in (0, \frac{3}{2}), \\ 0, \text{ otherwise.} \end{cases}$$

Compute the expectation $\mathbb{E}X$ and the variance $Var(X)$, if these exist.

Question 4. Let X be a continuous RV with p.d.f. f_X. Is $|X|$ also a continuous RV?

Question 5. Let X be an RV with $\mathbb{E}|X| < \infty$.

(i) If $\mathbb{P}(X \geq 0) = 1$ and $\mathbb{E}X = 0$, show that $\mathbb{P}(X = 0) = 1$.

(ii) If $\mathbb{P}(X \geq 1) = 1$, then show that $\mathbb{E}X \geq 1$.

(iii) If X is a discrete RV such that $\mathbb{P}(X \in \{0, 1, 2, \cdots\}) = 1$ and $\mathbb{E}X < 1$, then show that $\mathbb{P}(X = 0) > 0$.

Question 6. Let X be an RV with $\mathbb{E}X^2 < \infty$. Show that $Var(X) = 0$ if and only if $\mathbb{P}(X = \mu_1') = 1$.

Question 7. Fix a positive integer n. Find examples of discrete/continuous RVs such that $\mathbb{E}X^n$ exists but $\mathbb{E}X^{n+1}$ does not exist.

Question 8. Let X be an RV with $\mathbb{E}|X-a|^n < \infty$, where $n > 1$ is some positive integer and a is some real number. Choose a positive integer m with $m \leq n$ and let b be any real number. Show that $\mathbb{E}(X-b)^m$ exists. Is it true that

$$\mathbb{E}(X-b)^m = \sum_{k=0}^{m} \binom{m}{k} (-b)^{m-k}\, \mathbb{E}X^k \ ?$$

Question 9. Let X be a discrete RV with support $S_X \subset \{0, 1, 2, \cdots\}$.

(a) If $\mathbb{E}X$ exists, evaluate the limit $\lim_{n\to\infty} n\mathbb{P}(X > n)$.
(b) If $\mathbb{E}X^2$ exists, evaluate the limit $\lim_{n\to\infty} n^2\mathbb{P}(X > n)$.

Question 10. Compute the MGF in each of the following cases and hence compute the mean and Variance.

(i) Fix $p \in (0,1)$ and let n be a positive integer. Consider a discrete RV X with the p.m.f.

$$f_X(x) := \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & \text{if } x \in \{0, 1, 2, \cdots, n\}, \\ 0, & \text{otherwise.} \end{cases}$$

(ii) Fix $\lambda > 0$. Consider a continuous RV X with the p.d.f.

$$f_X(x) := \begin{cases} \lambda e^{-\lambda x}, & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$