

**MSO205: Introduction to Probability Theory**  
**Quiz 1 (August 31, 2024)**

**Duration: 2:45 pm - 3:30 pm (+15 minutes for DAP students)**

**Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. Other than the student ID card and writing materials, no other materials shall be allowed. Electronic devices (mobiles, calculators etc.) are prohibited. If such a device is found on person during the quiz, appropriate action shall be taken.
3. DO NOT do any rough work on this sheet. Additional sheets for rough work shall be provided. DO NOT attach any additional sheet to this page.
4. Write your answers in the designated place. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit.

**Name:**

**Roll No.:**

Question 1. Is the following statement ‘True’ or ‘False’? Underline/Tick ‘True’ or ‘False’, as appropriate, in the space designated below. [1 mark]

Statement: The probability mass function of a discrete RV can be uniquely determined from the distribution function.

Answer:

Question 2. Consider the random experiment of drawing a card at random from a standard deck of cards. Write down the sample space. [1 mark]

Answer:

Question 3. Suppose a standard six-sided die is rolled at random. If an even integer is observed, we choose box A containing 2 white balls and 4 black balls. Otherwise, we choose box B containing 3 white balls and 4 red balls. All balls of the same colour are identical. Next, a ball is drawn at random from the chosen box.

- (a) What is the probability that the ball drawn is white? Justify your answer. [2 marks]
- (b) If the ball drawn is white, what is the probability that box A was chosen? Justify your answer. [2 marks]

You may write your answer as a simple fraction.

Question 4. Let  $X$  be an RV with the following distribution function  $F_X : \mathbb{R} \rightarrow [0, 1]$  given by

$$F_X(x) = \begin{cases} 0, & \text{if } x < 0, \\ \frac{1}{5} + \frac{x}{4}, & \text{if } 0 \leq x \leq 2, \\ \frac{1}{2} + \frac{x-1}{5}, & \text{if } 2 < x < \frac{7}{2}, \\ 1, & \text{if } x \geq \frac{7}{2}. \end{cases}$$

- (a) Identify the points of discontinuity of  $F_X$ , if any. Also compute the jumps at these points, if any. [4 marks]
- (b) Is  $X$  discrete or continuous? Justify your answer. [2 marks]
- (c) Compute  $\mathbb{P}(X > 0)$  and  $\mathbb{P}(X > 1 \mid X \leq \frac{5}{2})$ . [1 + 2 marks]

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