MSO205 PRACTICE PROBLEMS SET 11

Question 1. Let X and Y be i.i.d. $N(0,1)$ RVs. Identify the distribution of $\frac{X}{Y}$ and $\frac{X}{|Y|}$.

Question 2. Let $X \sim F_{m,n}$. Identify the distribution of $\frac{n}{n+mX}$.

Question 3. Let X and Y be i.i.d. $Exponential(\lambda)$ RVs, for some $\lambda > 0$. Identify the distribution of $\frac{X}{Y}$.

Question 4. Let $Y \sim N_p(b, K)$. Then for any $c \in \mathbb{R}^n$ and a $n \times p$ real matrix B, consider the n dimensional random vector $Z = c + BY$. Show that $Z \sim N_n(c + Bb, BKB^t)$.

Question 5. Let X be a p-dimensional random vector, $a \in \mathbb{R}^m$ and A be an $m \times p$ real matrix. Then show that the Characteristic function of the m-dimensional random vector $Y = a + AX$ given by

$$\Phi_Y(u) = \exp(iu^t a)\,\Phi_X(A^t u), u \in \mathbb{R}^m.$$

Question 6. Show that $\mathbb{E}|X|^\alpha < \infty, \forall \alpha \in (0,1)$ when $X \sim Cauchy(0,1)$.