Q1) X: no. of bombs ~~dro~~ ~~hatt~~ that hit the target

for destroying at least 2 bombs must hit target

⇒ let n bombs are dropped

$$P(X \geq 2) = 1 - P(X=0) - P(X=1) \geq 0.99$$

$$1 - P(X=0) - P(X=1) \geq 0.99$$

$$0.01 \geq P(X=0) + P(X=1)$$

$$ {}^nC_0\left(\frac{1}{2}\right)^n + {}^nC_1\left(\frac{1}{2}\right)^n$$

$$0.01 \geq \frac{(n+1)}{2^n}$$

$$2^n \geq 100n+100$$

$$P(X=x) = {}^nC_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{n-x}$$

$$= {}^nC_x\left(\frac{1}{2}\right)^n$$

$$\cancel{0 = 2^n - 100n - 100}$$

$$2^n \geq 100n+100$$

$$\boxed{n \geq 11}$$

Ans: Atleast 11 bombs must be dropped to completely destroy the target

Q2) (i) Atleast even chance of drawing a heart

~~X~~ X — 0 : ~~card drawn is not a heart~~
— 1 : ~~card drawn is a heart~~

X : no. of cards with hearts drawn out of the pack of 52 cards

~~P(X=1) = 1/4~~

p : probability of drawing heart $= \frac{1}{4}$

q : $1-p = \frac{3}{4}$

$$P(x=0) < \frac{1}{4}$$

let n number of cards are drawn

$$P(x=0) = {}^nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n < \frac{1}{4}$$

$$\left(\frac{3}{4}\right)^n < \frac{1}{4}$$

$$\left(\frac{3}{4}\right)^n < 0.25$$

$\Rightarrow$ ~~n=4~~ smallest n to satisfy above eq^n is $n=5$

$\Rightarrow$ Atleast 5 cards must be drawn

(ii) Probability of getting a heart is greater than $\frac{3}{4}$

$$P(x \neq 0) \geq \frac{3}{4}$$

$$1 - P(X=0) > \frac{3}{4}$$

$$P(X=0) < \frac{1}{4}$$

$\Rightarrow$ we have to atleast draw 5 cards

Q3) Assuming occurance of typographical error in a page a poisson process

X: no. of typographical errors per page in a book

$\lambda_x = \frac{390}{520}$ $n = 520$

~~P(X=5)~~ P(X=0) for a random sample of 5 pages

Z: no. of typographical errors in a set of 5 pages

$\lambda_z = 5\lambda_x = \frac{5 \times 390}{520} = \frac{195}{52} = 3.75$

$P(Z=0) = \frac{\lambda_z^{z} e^{-\lambda_z}}{z!} = \frac{(3.75)^0 e^{-3.75}}{0!}$

$P(Z=0) = e^{-3.75}$

$$\boxed{P(Z=0) = \frac{1}{e^{15/4}}}$$

Ans: Probability that random sample of 5 pages contain 0 errors is $\frac{1}{e^{15/4}}$

Q4) x: no. of lenses defective in a packet of 10 lenses

$p = \frac{1}{500}$ for a lens to be defective

$\lambda = 10 \times \frac{1}{500} = \frac{1}{50}$

$\boxed{\lambda = \frac{1}{50}}$

$P(x=0) \Rightarrow$ Probability of packet with no defective lens.

$\Rightarrow \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(\frac{1}{50})^0 e^{-\frac{1}{50}}}{0!} = \frac{1}{e^{\frac{1}{50}}}$

$\boxed{P(x=0) = \frac{1}{e^{0.02}}}$

no. of packets with no defective lens in consignment of 20,000

$= 20,000 \times \frac{1}{e^{0.02}}$

≈ 19603 packets.

$P(x=1)$ = Probability of packet with 1 defective

$\Rightarrow \frac{\frac{1}{50} \times e^{-\frac{1}{50}}}{1} \approx 0.0196$

no. of packets with 1 defective in consignment = $20,000 \times 0.0196$

≈ 392 packets

$P(x=2) \Rightarrow \frac{(\frac{1}{50})^2 \times e^{-\frac{1}{50}}}{2!} = \frac{0.9801}{2500 \times 2} = 0.00019602$

no. of packet with 2 defective in consignment $\approx$ 4 packets

Q5) $p = \frac{5}{100}$

Atleast 4 items are to be examined to get 2 defective

$\Rightarrow$ there is 1 defective item in first 3 items

$$P(x=3) \Rightarrow {}^3C_1 \left(\frac{5}{100}\right)^1 \left(\frac{95}{100}\right)^2$$

$\Rightarrow$ either 4th, 5th, 6th .... be the 2nd defective item

$$P = {}^3C_1 \left(\frac{5}{100}\right)^1 \left(\frac{95}{100}\right)^2 \left[\frac{5}{100} + \frac{5}{100} \times \frac{95}{100} + \frac{5}{100} \times \frac{95}{100} \times \frac{95}{100} + \dots\dots\right]$$

$$= {}^3C_1 \left(\frac{5}{100}\right)^2 \left(\frac{95}{100}\right)^2 \left[1 + \frac{95}{100} + \left(\frac{95}{100}\right)^2 + \left(\frac{95}{100}\right)^3 + \dots\right]$$

$$= 3 \times \left(\frac{5}{100}\right)^{\cancel{2}} \left(\frac{95}{100}\right)^2 \times \frac{1}{\frac{5}{\cancel{100}}}$$

$$= \frac{3 \times 5 \times 95 \times 95}{1000000}$$

$$\boxed{P = 0.135}$$

Ans = Probability of Atleast 4 items are to be examined to get 2 defective is 0.135.

Q6 Probability that a drug ~~recover~~ gives relief : 80% = 0.8

X: no. of people who ~~are~~ got relieved by the drug given in a given week.

Acc. to problem

⇒ A : 5^{th} person to ~~recover~~ get relief is 7^{th} person to get ~~relief~~ drug

$$P(X=4) = {}^6C_4 (0.8)^4 (0.2)^2$$

$$P(A) = {}^6C_4 (0.8)^4 (0.2)^2 \times (0.8)$$

$$= {}^6C_4 (0.8)^5 (0.2)^2$$

$$\boxed{P(A) = 0.016}$$

Q7) P (that the target is destroyed by a shot) $= 0.5 = p$

$q = 1-p$

X: no. of shots fired ~~tit~~ till the target is destroyed.

$$P = p\,q^{x-1}$$

~~P(target is de~~

$$P(X=6) = (0.5)\,(0.5)^5$$

$$= (0.5)^6$$

$$= \frac{1}{2^6} = \frac{1}{64}$$

$$\boxed{P = 0.015}$$

Ans: Probability that target was destroyed on the 6th attempt to shoot $= 0.015$.

Q8)

P(that switch fails) = 0.001 = p

q = 0.999

P(switch fail after 1200 times) = $(0.999)^{1200} \times [0.001 + (0.001)(0.999) + (0.001)(0.999)(0.999) + \ldots]$

$P = (0.999)^{1200} \times \cancel{(0.001)} \times \frac{1}{\cancel{(0.001)}}$

$P = (0.999)^{1200}$

$\boxed{P = 0.301}$

Ans: Probability ~~of~~ that switch fails after 1200 times of use = 0.301.

8

89) numbers on tickets : 00, 01, 10, 11

sum of the tickets drawn : 23
5 times

Total no. of ways of drawing ticket 5 times : 4^5

$= 1024$ way

no. of ways when sum is 23 is

coefficient of x^{23} in $(x^0 + x^1 + x^{11} + x^{10})^5$

$= (1+x)^5 (1+x^{10})^5$

$= (1 + 5x + 10x^2 + 10x^3 + \dots)$
$(1 + 5x^{10} + 10x^{20} + \dots)$

coefficient of x^{23} is : 100

Required Probability : $\frac{100}{1024} = \frac{25}{256}$

Q10) ~~Ans~~ avg life = μ = 1000 hrs

Standard deviation = σ = 200 hrs

σ^2 = 40000

X : Burning hours of a street lamp

given $X \sim N(1000, 40000)$

$$Z = \frac{X - \mu}{\sigma} = \frac{X - 1000}{200}$$

(i) fail in first 800 hrs of burning.

$X = 800$

$$P(X < 800) = P\left(Z < \frac{800 - 1000}{200}\right) = P(Z < -1)$$

$$\boxed{P(Z < -1) = 0.5 - 0.3413}$$

$$\boxed{P(Z < -1) = 0.1587}$$

(ii) Between 800 to 1200 burning hours

$$P(800 < X < 1200) = P(-1 < Z < 1) = 2 \times 0.3413$$

$$\boxed{P = 0.6826}$$

Q11) X: no. of items are defective in a batch of 100

mean = $\mu = (100)\,0.4 = 40$

Standard deviation = $SD = \sigma = \sqrt{100 \times (0.4)(0.6)}$

$= \sqrt{24} \approx 4.9$

$$Z = \frac{X-\mu}{SD} = \frac{X-40}{4.9} \sim N(0,1)$$

i) At least 44 are defective

$$P(X \geq 44) = P(43.5 \leq X \leq 100.5)$$

$$= P\left(\frac{3.5}{4.9} \leq Z \leq \frac{60.5}{4.9}\right)$$

$$= P(0.71 \leq Z \leq 12.35)$$

$$= 0.5 - 0.2611$$

$\boxed{\cancel{P = 0.2420}}$

$\boxed{P = 0.2389}$

(ii) Exactly 44 defective

$$P(X=44) = P(43.5 < X < 44.5)$$

$$= P(0.71 < Z < 0.91)$$

$= \cancel{0.5 - 0.2611 - 0.3186}$ $0.3186 - 0.2611$

$= \cancel{0.5} \quad \cancel{97} = 0.0575$

$\boxed{P = 0.575}$

Q12) on avg 3 trucks arrive at a warehouse in a hour

$\boxed{\beta = \frac{1}{3} \text{ hr}}$ or $= \frac{60}{3}$ min $=$ 20 min

$\boxed{\beta = 20 \text{ min}}$

i) time taken between arrival of successive truck is 10 min:

using exponential distribution:

x: time taken by the truck to arrive the warehouse in minute

$$P(X<10) = \int_0^{10} \frac{1}{\beta} e^{-\frac{x}{\beta}} dx$$

$$= \frac{1}{20} \int_0^{10} e^{-\frac{x}{20}} dx$$

$$= \frac{1}{20} \times (-20) \left[e^{-\frac{x}{20}} \right]_0^{10}$$

$$= -\left[e^{-\frac{1}{2}} - 1 \right]$$

$$= 1 - \frac{1}{\sqrt{e}} = \frac{\sqrt{e}-1}{\sqrt{e}}$$

$$\boxed{P(x<10) = \frac{\sqrt{e}-1}{\sqrt{e}}}$$

Q17) x: [illegible]

(ii) Atleast 50 minutes

$$P(X \geq 50) = \int_{50}^{\infty} \frac{1}{\beta} e^{-\frac{x}{\beta}} dx$$

$$= \frac{1}{20} \int_{50}^{\infty} e^{\frac{-x}{20}} dx$$

$$= -\left[e^{-\frac{x}{10}} \right]_{50}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-\frac{5}{2}} \right]$$

$$= e^{-\frac{5}{2}}$$

$$\boxed{P = \frac{1}{e^{2.5}}}$$

Q13)

X: time in minutes for which the shower will last

$a = 2$

considering X to be distributed exponentially:

$f(x) = a e^{-ax}$ for $x > 0$

=) Probability that shower last more than 3 minute..

$P(X>3 \mid X>2) = P(X>1)$ as exponential distribution lack memory

$\Rightarrow P(X > s+t \mid P(X>t)) = P(X>s)$

$$P(X>1) = \int_1^\infty 2 e^{-2x}\,dx = \cancel{2}\left[\frac{e^{-2x}}{-\cancel{2}}\right]_1^\infty$$

$$= -\left[e^{-\infty} - e^{-2}\right]$$

$$\boxed{P = \frac{1}{e^2}}$$

Q17) x: proportion of defectives in a lot

Q14) x: survival time of mice in weeks after a certain dose of toxicant

X follows gamma distribution

$f(x) = \frac{1}{\beta \Gamma\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}$ $x > 0$

ATQ

$\beta = 5$: avg time for survival of mice in weeks

α = 60 weeks

$$P(X > 60) = \int_0^{60} \frac{1}{5\Gamma 60} x^{59} e^{-\frac{x}{5}} dx$$

proportion of ...

Q16) X: lifetime of a back battery in hrs

X having weibull distribution

$\alpha = 0.1$ $\beta = 0.5$

a) mean lifetime

$$\mu = \alpha^{-\frac{1}{\beta}} \Gamma\left(1+\frac{1}{\beta}\right)$$

$$= (0.1)^{-2} \Gamma 1+2$$

$$= \Gamma 3 \; (0.1)^{-2}$$

$$= 100 \times 2 \times 1$$

$$\boxed{\mu = 200 \text{ hrs}}$$

b) $P(X>300) = \int_{300}^{\infty} 0.1 \times 0.5 \; x^{0.5-1} e^{-0.1 x^{0.5}} dx$

$$= \frac{5}{100} \int_{300}^{\infty} x^{-0.5} e^{-\frac{x^{0.5}}{10}} dx$$

$x^{0.5} = y$

$$= 0.1 \int_{\sqrt{300}}^{\infty} e^{-\frac{y}{10}} dy$$

$\boxed{P \approx \frac{1}{e^{1.73}}}$ $= 0.1 \left[e^{-\frac{y}{10}} \times -10 \right]_{\sqrt{300}}^{\infty} = e^{-\frac{\sqrt{300}}{10}}$

Q17) x: proportion of defectives in a lot

⇒ X follows beta distribution

$\alpha = 2, \beta = 3$

a) avg of defectives

μ = mean = $\frac{\alpha}{\alpha+\beta} = \frac{2}{5}$

$\boxed{\mu = 0.4}$ avg proportion of defectives

b) $P(X > 0.3)$

$= \int_{0.3}^{1} \frac{\Gamma 2+3}{\Gamma 2 \Gamma 3} x^1 (1-x)^2 \, dx$

$\frac{4!}{2!} \int_{0.3}^{1} x(1+x^2-2x) \, dx$

$= 12 \int_{0.3}^{1} x^3 + x - 2x^2 \, dx$

$= 12 \left[\frac{x^4}{4} + \frac{x^2}{2} - \frac{2}{3}x^3 \right]_{0.3}^{1}$

$\boxed{P = 0.6517}$

$= 12 \left[\frac{1}{4} + \frac{1}{2} - \frac{2}{3} - \frac{81}{40000} - \frac{9}{200} + \frac{18}{1000} \right]$

12(

$= 12 \left[\frac{1}{12} - 0.002025 - 0.045 + 0.018 \right]$

Q18) X : 65 66 67 67 68 69 70 72

Y: 61 68 65 68 72 72 69 71

correlation coefficient : $r_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y}$

$cov(x,y) = E(xy) - E(x)E(y)$

$E(x) = 68$ $E(y) = 68.25$

$E(xy) = \frac{\sum xy}{n}$

$= 4646.25$

$\boxed{cov(x,y) = 5.25}$

$\sigma_x = 4.5$ $\sigma_y = 12.43$

$r_{xy} = \frac{5.25}{4.5 \times 12.43} = 0.093$

$\boxed{r_{xy} = 0.093}$