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V SEMESTER

B.Tech.

END TERM EXAMINATION **December-2024**

MC 303 Stochastic Processes

Time: 3 Hours **Max. Marks: 40**

Note: Answer any four questions. (Assume suitable missing data, if any.)

1)

a) Discuss generalized Poisson queuing model and write the formula for calculating the steady state probability of n customers in the systems. (2) [CO1] [BTL1]

b) Explain different steady state measures of performance in queuing systems. (2) [CO2] [BTL2]

c) Differential among trannsient state, recurrent state and absorbing state in context of markov chain with a suitable example. (3) [CO5] [BTL2]

d) Explain the recursive formulation of mean passage time. Discuss its significance in the analysis of Markov process. (3) [CO3] [BTL4]

2)

a) Describe a random walk with two absorbing barriers. Give a suitable example of any such real world process. Derive the probability of absorption at lower barrier. (6) [CO2, CO3] [BTL2]

b) Consider a simple random walk $X_n = \sum z_i$, and suppose it starts from 0. As usual, $P(z_i = 1) = p$, $P(z_i = -1) = q = 1 - p$. Compute $E(e^{\alpha X_n})$ $\alpha \in R$. Comment on the asymptotic behavior of X_n. (2) [CO2] [BTL3]

c) Can we analyze the random walk problem with Markov chain for obtaining the steady state probability? If so, what should be your assumptions on random walk problem? (2) [CO4] [BTL3]

3)

(a) Assume that a computer system is in one of three states: busy, idle, or undergoing repair, respectively denoted by states 0, 1, and 2. Observing its state at 2 P.M. each day, we believe that the system approximately behaves like a homogeneous Markov chain with the transition probability matrix:

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$

Prove that the chain is irreducible, and determine the steady-state probabilities. (4) [CO4] [BTL3]

b) Prove the Chapman-Kolmogorov Equations $P_{ij}(t+s) = \sum_{k \in S} P_{ik}(t) P_{kj}(s)$ for continuous-time Markov chain with state space S. (4) [CO5] [BTL4]

c) Define continuous time Markov chain (CTMC) along with two real world example. (2) [CO5] [BTL2]

4)

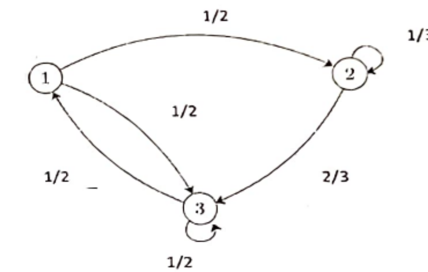
a) Discuss the consequences of higher expected waiting time in a queuing system. Discuss some suggestions to solve the issues raised due to higher expected waiting time. (2) [CO3] [BTL3]

b) For a small batch computing system, the processing time per job is exponentially distributed with an average time of 3 minutes. Jobs arrive randomly at an average rate of one job every 4 minutes and are processed on a first-come-first-served basis. The manager of the installation has the following concerns. (4) [CO2] [BTL3]

i) What is the probability that an arriving job will require more than 20 minutes to be processed (the job turn-around time exceeds 20 minutes)?

ii) A queue of jobs waiting to be processed will form, occasionally. What is the average number of jobs waiting in this queue?

c) Consider a continuous-time Markov chain X(t) with the jump chain shown in following Figure. Assume $v_1 = 2, v_2 = 3$ and $v_3 = 4$. Find the generator matrix G for the chain and steady state distribution of the states. (4) [CO2] [BTL3]



5)

a) Define Renewal process. Give two real world examples of renewal processes. Let $N(t)$ represents number of renewals upto time t and S_n represent time of n^{th} renewal. Comment on the distribution of S_n if $N(t)$ follows the poission distribution. (3) [CO1] [BTL4]

b) Define the renewal function $H(t)$ and renewal density function $h(t)$ Derive the renewal equation $h(t) = f(t) + \int_0^t h(t-u) f(u) du \;\; t \geq 0$. (5) [CO3] [BTL5]

c) An ordinary renewal process has the renewal function $H(t) = \frac{t}{5}$. Determine the probability $Prob[N(15) < 2]$. (2) [CO2] [BTL4]

**************All The Best*************