Assignment - 2

Q.1 Define the convergence of a sequence. Prove the following:

a) $\lim \left(\frac{1}{n^2+1}\right) = 0$,

b) $\lim \left(\frac{3n+2}{n+1}\right) = 3$.

Q.2 Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Then the sequence $(\sqrt{x_n})$ of positive square roots converges and $\lim(\sqrt{x_n}) = \sqrt{x}$.

Q.3 Let (x_n) be a sequence of positive real numbers such that $L = \lim \left(\frac{x_{n+1}}{x_n}\right)$ exists. If $L < 1$, then (x_n) converges and $\lim(x_n) = 0$.

Q.4 Find the limits of the following sequences:

a) $\lim((2 + 1/n)^2)$

b) $\lim \left(\frac{(-1)^n}{n+2}\right)$

c) $\lim \left(\frac{\sqrt{n}-1}{\sqrt{n}+1}\right)$

d) $\lim \left(\frac{n+1}{n\sqrt{n}}\right)$

Q.5 Give an example of two divergent sequences X and Y such that:

a) Their sum $X + Y$ converges,

b) Their product XY converges.

Q.6 Define Monotone sequences. State and prove Monotone Convergence theorem.

Q.7 Define subsequences and nested intervals. Prove that a bounded sequence of real numbers has a convergent subsequence.

Q.8 Let $y_n = \sqrt{n+1} - \sqrt{n}$ for $n \in \mathbb{N}$. Show that $(\sqrt{n}y_n)$ converges. Find the limit.

Q.9 Define Cauchy sequences. Show from the definition that the following are Cauchy sequences.

a) $\left(\frac{n+1}{n}\right)$,

b) $\left(1 + \frac{1}{2!} + \cdots + \frac{1}{n!}\right)$.

Q.10 State and prove Cauchy Convergence Criterion.