B.Tech V Semester (2025-26)
MC 303: Stochastic Processes
Assignment- II

Task 1: Simulation of Customer Arrivals at a Bank

Objective:

To simulate a real-world arrival system using a homogeneous Poisson process and analyze service load and waiting times.

Problem Description:

Customers arrive at a bank according to a Poisson process with rate λ customers per minute. Each customer requires a random service time (for example, exponentially distributed with mean $1/\mu$ minutes). The bank has a single service counter.

Tasks:

1. Simulate arrivals and departures for a working period (say, 4 hours).
2. Track queue length, waiting times, and server utilization.
3. Compare analytical vs. simulated mean waiting time.

Expected Outcomes:

- Plot: number of customers in system vs. time
- Average waiting time, queue length, and server utilization

Implementation Hints:

- Generate inter-arrival times using exponential(λ)
- Generate service times using exponential(μ)
- Use a time-driven or event-driven simulation
- Libraries: numpy, matplotlib (Python) or random number utilities in C++/Java

Task 2: Modelling Call Traffic from Multiple Sources

Objective:

To model the aggregation of independent Poisson processes and simulate selective filtering of events.

Problem Description:

Consider three call centers (A, B, C) receiving calls as independent Poisson processes with rates $\lambda_1 = 2$/min, $\lambda_2 = 1.5$/min, $\lambda_3 = 0.5$/min.

1. Superpose these processes to obtain a combined arrival stream.
2. Thin the combined stream to model a situation where only priority calls (with probability $p = 0.3$) are passed to a central operator.
3. Simulate for 2 hours and estimate:

  - Mean inter-arrival time of priority calls
  - Distribution of time between successive priority calls
  - Empirical verification that thinning yields a Poisson process with rate $p(\lambda_1 + \lambda_2 + \lambda_3)$

Expected Outcomes:

- Histogram of inter-arrival times (should match exponential distribution)
- Comparison between simulated and theoretical rates
- Insight into load balancing among call centers

Implementation Hints:

- Use random exponential inter-arrivals for each source
- Merge and sort all event times (superposition)
- Retain events with uniform random filter (thinning)

Task 3: Machine Replacement Policy

Objective:

To model equipment life cycles and maintenance schedules using a renewal process and study the long-term replacement rate.

Problem Description:

A factory operates a machine whose lifetime (in months) follows a Weibull distribution with parameters $\alpha = 2$, $\beta = 10$. When a machine fails, it is replaced immediately with a new one of the same type.

1. Simulate the renewal process for 10 years (120 months).
2. Estimate:
   - The number of renewals up to time t
   - The long-run renewal rate $N(t)/t$
3. Compare the empirical renewal rate with the theoretical value $1/E[X]$.
4. Extend: Introduce preventive replacement after 8 months if failure hasn't occurred.

Compute expected cost per month if
  - Replacement on failure costs = ₹5000
  - Preventive replacement costs = ₹3000

Expected Outcomes:

- Plot of cumulative renewals vs. time
- Comparison of empirical vs. theoretical renewal rates
- Optimal replacement interval minimizing cost

Implementation Hints:

- Use random generation from Weibull distribution (numpy.random.weibull in Python)
- Maintain total operational time and count renewals
- Run multiple simulation trials for stability