Assignment - 1

Q.1 **a)** If $a \in \mathbb{R}$ satisfies $a.a = a$, prove that either $a = 0$ or $a = 1$.

b) If $a \neq 0$ and $b \neq 0$, show that $\frac{1}{ab} = \left(\frac{1}{a}\right)\left(\frac{1}{b}\right)$

Q.2 Use Mathematical Induction to show that if $a \in \mathbb{R}$ and $m, n \in \mathbb{N}$, then $a^{m+n} = a^m a^n$ and $(a^m)^n = a^{mn}$.

Q.3 If $a, b \in \mathbb{R}$, then show that

a) $||a| - |b|| \leq |a - b|$,

b) $|a - b| \leq |a| + |b|$.

Q.4 Define ε-neighborhood of a point x. Let $a \in \mathbb{R}$, if x belongs to the neighborhood of a ($V_\varepsilon(a) = (a - \varepsilon, a + \varepsilon)$ for every $\epsilon > 0$, then $x = a$.

Q.5 Define completeness property of $\mathbb{R}$. Find the Supremum and Infimum of the following:

a) $S_1 = \{1 - (-1)^n/n : n \in \mathbb{N}\}$,

b) $S_2 = \{1/n - 1/m : n, m \in \mathbb{N}\}$,

c) $S_3 = \{x \in \mathbb{R} : x > 0\}$.

Q.6 State and prove Archimedean Property.

Q.7 **a)** Prove that there exists a positive real number x such that $x^2 = 2$.

b) Prove that there does not exist a rational number r such that $r^2 = 2$

Q.8 State and prove Bernoulli's Inequality.

Q.9 Define countable set. Prove that set of real numbers $\mathbb{R}$ is not countable.

Q.10 If x and y are real numbers with $x < y$, then there exists an irrational number z such that $x < x < y$.