Department of Applied Mathematics
Delhi Technological University, Delhi

ASSIGNMENT I
2023-2024
B.Tech. V Semester
Subject Code : MC-303 **Course Title :** Stochastic Processes

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1. What is a stochastic process. Define state space and parameter space of a stochastic process. In how many ways a stochastic process may be classified, explain with examples.

9C6p^6q^3 x p

2. What is a Bernoulli process. Find the probability of getting 7 success in 10 trials. Also calculate the probability of getting 7th success in 10 trials.
3. Explain merging of two independent Bernoulli processes. Let X_t and Y_t are two independent Bernoulli processes then find the probability of an arrival in the merged process given that there is an arrival in either X_t or Y_t.
4. Define Poisson process. Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate λ. Let $T_1, T_2, ...$ be the arrival times for this process. Shoe that

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$$f_{T_1,T_2,...,T_n}(t_1, t_2, ..., t_n) = \lambda^n e^{-\lambda t_n} \; for \;\; 0 < t_1 < t_2 < ... < t_n. \qquad (1)$$

t = 2, n = 4
t = 3, n = 3

Also, find the probability that there are four arrivals in $(0, 2]$ or three arrivals in $(4, 7]$.

5. Let $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ. Let T_1 denote the time of first event and T_n denote the time between $(n-1)th$ and nth events. Find $P(T_2 > t|_{T_1=s})$.

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6. Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate λ. Show the following: Given that $N(t) = n$, the n arrival times have the same joint CDF as the order statistics of n independent Uniform $(0, t)$ random variables. To show this you can show that

$$f_{T_1,T_2,...,T_n|N(t)=n(t_1,t_2,...,t_n)} = \frac{n!}{t^n} \; for \;\; 0 < t_1 < t_2 < ... < t_n < t. \qquad (2)$$

7. What is a Renewal process. Define renewal function and renewal density function. Show that a Poisson process is a renewal process whose inter arrival times are mutually independent random variables and the random variable follows exponential distribution.
8. Suppose that in a system, a unit fails, according to a Poisson process with rate $\lambda - 3$ per day. Suppose that there are 6 spare units in an inventory and the next supply is not due in 4 days. Find the probability that the system will be out of order in next four days.

9. What do you mean by strict sense stationary and wide sence stationary process. Is the stochastic process $\{X(t); t \in T\}$ stationary, whose probability distribution under a certain condition given by

$$P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}} & \text{if n=1,2,3,...,} \\ \frac{at}{1+at} & \text{if n=0} \end{cases}$$

10. Define Gaussian process. Show that wide sense stationary and strict sense stationary are equivalent for a Gaussian process.
11. Let $X(t) = A_0 + A_1 t + A_2 t^2$, where $A_i' s$ are uncorrelated random variables with mean 0 and variance 1. Find the mean function and covariance function of $X(t)$?
12. Define standard Brownian motion and Brownian motion with variance σ^2 and drift μ. Show that a standard Brownian motion is a unique Gaussian process with $m(t) = 0$ and $a(s, t) = min\{s, t\}$, $0 \leq s \leq t$.
13. Show that Brownian motion has strict sense stationary increment property.

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B.Tech V Semester (2023-24)
MC 303: Stochastic Processes

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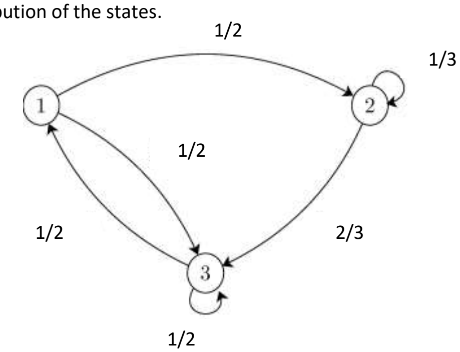
Assignment- II

1. What is random walk. Give real work processes which can be realized as random walk. Discuss different types of random walk and individual question of interest one can find in terms of probabilities.
2. Consider a random walk in one dimension with independent, non-identical displacements given by Cauchy's PDF,
$$p_n(x) = \frac{A_n}{a_n^2 + x^2}$$
For some positive sequence $\{a_n\}$.
   (a) Derive the characteristic function of the above and determine A_n. Show that the characteristic function is not analytic at the origin.
   (b) Derive the PDF of the position after N steps, $P_N(x)$. For $a_n = a$, how does the half-width of $P_N(x)$ scale with N?
3. Consider a random walk with two absorbing barriers $a = 5$ and $b = -4$ with probability of moving one forward step is $p = 0.3$ and probability of moving one forward step $q = 0.4$ with initial position of the particle at the origin. Find the probability of absorption at $a = 5$ and $b = -4$. Also, find the probability that absorption occurs for $N = 10$.
4. Consider a simple random walk $X_n = \sum z_i$, and suppose it starts from 0. As usual, $P(z_i = 1) = p$, $P(z_i = -1) = q = 1 - p$. Compute $E(e^{\alpha X_n})\alpha \in R$. Comment on the asymptotic behavior of X_n.
5. In ref to Question 4, show that
$$E(X_m|X_n) = \frac{m}{n}X_n \ for\ m \leq n$$
$$otherwise\ X_n\ for\ m > n$$

B.Tech V Semester (2023-24)
MC 303: Stochastic Processes

Assignment- IV

1. Consider a continuous-time Markov chain X(t) with the jump chain shown in following Figure. Assume $v_1 = 2, v_2 = 3$ and $v_3 = 4$. Find the generator matrix G for the chain and steady state distribution of the states.



2. A hospital owns two identical and independent power generators. The time to breakdown for each is exponential with parameter λ and the time for repair of a malfunctioning one is exponential with parameterμ. Let $X(t)$ be the Markov process which is the number of operational generators at time $t \geq 0$. Assume $X(0) = 2$. Prove that the probability that both generators are functional at time $t > 0$ is
$$\frac{\mu^2}{(\lambda+\mu)^2} + \frac{\lambda^2 e^{-2(\lambda+\mu)t}}{(\lambda+\mu)^2} + \frac{2\lambda\mu e^{-(\lambda+\mu)t}}{(\lambda+\mu)^2}$$
3. Let $\alpha > 0$ and consider the random walk X_n on the non-negative integers with a reflecting barrier at 0 defined by
$$p_{i,i+1} = \frac{\alpha}{1+\alpha}, p_{i,i-1} = \frac{1}{1+\alpha}\ for\ all\ i \geq 1$$
   a. Find the stationary distribution of this Markov chain for $\alpha < 1$
   b. Does it have a stationary distribution for $\alpha \geq 1$?

Further, Let $Y_0, Y_1, Y_2, \ldots$ be independent exponential random variables with parameters $\mu_0, \mu_1, \mu_2, \ldots$ respectively. Now modify the Markov chain X_n of into a continuous time Markov chain by postulating that the holding time in state j before transition to $j-1$ and $j+1$ is random according to Y_j.

   c. Explain why this is a Continuous time Markov chain.
   d. Find the infinitesimal generator.
   e. Find its stationary distribution by making reasonable assumption on μ_j and $\alpha < 1$.

4. Consider a continuous time Markov chain observed at the times of a Poisson process with rate λ. Let $X = \{X(t) : t \geq 0\}$ be a continuous time Markov chain with stationary distribution π. Let $S1, S2, \ldots$ be the event times of a Poisson process with rate λ. Define $Y_n = X(S_n)$ for $n \geq 1$. Then $Y = \{Yn : n \geq 1\}$ is a discrete time Markov chain. Show that the stationary distribution of Y is also π.

5. Consider a service system with 2 servers. Customers arrive to the system according to a Poisson process with rate λ. All service times are independent exponential random variables with rate μ. If a customer arrives to the system when there is at least one server free the customer immediately goes into service and then departs the system once service is completed. Otherwise the customer waits in a queue, which can accomodate an unlimited number of waiting customers. Customers in queue are served in a first-come first-served manner. This system is known as the M/M/2 queue. Let X(t) denote the number of customers in the system (in service or in queue) at time t.

   (a) Write the state space and infinitesimal generator for the process $\{X(t) : t \geq 0\}$.
   (b) Compute the stationary distribution.
   (c) We say that overtaking occurs when a customer departs the system before another customer who arrived earlier. In steady state, find the probability that an arriving customer overtakes another customer (you may assume that the state of the system at each arrival instant is distributed according to the stationary distribution).

B.Tech V Semester (2023-24)
MC 303: Stochastic Processes
Assignment- V

1. A system starts working at time t = 0. Its lifetime has approximately a normal distribution with mean value $\mu = 120$ and standard deviation $\sigma = 24$ [hours]. After a failure, the system is replaced by an equivalent new one in negligible time and immediately resumes its work. How many spare systems must be available in order to be able maintain the replacement process over an interval of length 1000 hours
   (1) with probability 0.60?
   (2) with probability 0.79 ?
2. Use the Laplace transformation to find the renewal function H(t) of an ordinary renewal process whose cycle lengths have an Erlang distribution with parameters n = 3 and λ=1.
3. An ordinary renewal process has the renewal function H(t) = t /15. Determine the probability $P(N(15) \geq 2)$.
4. What is the significance of studying the Renewal process?
5. What is the relation between the renewal process and Poisson process?
6. Explain queuing behavior in respect to the stochastic process with possible scenarios?
7. A call center with one operator has an incoming call rate of three calls per hour. On the average, the operator spends 15 minutes for each call. Answer the following questions:
   a. How busy is the operator?
   b. How likely is it that a customer will have to wait on hold?
   c. How long can we expect customers to have to wait on hold?
   d. What arrival rate can a single operator reasonably handle without causing excessive waiting (less than 5 minutes).
8. For a small batch computing system the processing time per job is exponentially distributed with an average time of 3 minutes. Jobs arrive randomly at an average rate of one job every 4 minutes and are processed on a first-come-first-served basis. The manager of the installation has the following concerns.
   (a) What is the probability that an arriving job will require more than 20 minutes to be processed (the job turn-around time exceeds 20 minutes)?
   (b) A queue of jobs waiting to be processed will form, occasionally. What is the average number of jobs waiting in this queue?

9. A branch office of a large engineering firm has one on-line terminal connected to a central computer system for 16 hours each day. Engineers, who work throughout the city, drive to the branch office to use the terminal for making routine calculations. The arrival pattern of engineers is random (Poisson) with an average of 20 persons per day using the terminal. The distribution of time spent by an engineer at the terminal is exponential with an average time of 30 minutes. Thus the terminal is 5/8 utilized (20 x 1/2 = 10 hours out of 16 hours available). The branch manager receives complaints from the staff about the length of time many of them have to wait to use the terminal. It does not seem reasonable to the manager to procure another terminal when the present one is only used five-eighths of the time, on the average. How can queuing theory help this manager?

10. Traffic to a message switching center for one of the outgoing communication lines arrives in a random pattern at an average rate of 240 messages per minute. The line has a transmission rate of 800 characters per second. The message length distribution (including control characters) is approximately exponential with an average length of 176 characters. Calculate the principal statistical measures of system performance assuming that a very large number of message buffers are provided. What is the probability that 10 or more messages are waiting to be transmitted?

... three exits. What is the expected time taken for the mouse to leave the maze?

3. Let X be a discrete random variable with PGF $G_X(s) = \frac{s}{5}(2 + 3s^2)$. Find the distribution of X.

4. Let $X \sim Poisson(\lambda)$. The PGF of X is $G_X(s) = \exp^{\lambda(s-1)}$. Find $E(X)$ and $Var(X)$.

5. Let X and Y are independent random variable with $X \sim Poisson(\lambda)$ and $Y \sim Poisson(\mu)$. Find the distribution of random variable $Z = X + Y$.

6. Describe the Brownian Motion and elaborate its applications by taking some numerical illustration.

7. Describe Gaussian Process and give its applications.

2. What is time homogeneous continuous Markov chain? Give one example each of time homogeneous and time non-homogeneous Markov chain. Derive the holding time distribution in state i for a time homogeneous Markov chain.

3. Explain matrix transition probability function in reference to a continuous time Markov chain. What's the concept parallel to this in case discrete time Markov chain? If $P(t)$ is the matrix transition probability function, then show that $P(t + s) = P(t) + P(s)$.

... Any attempt to cheat/copy ... net or book will lead to the cancellation of the exam of concern students.

1. In a classical loss model M/M/c/c, if the call arrival rate is 2 per minute and on the average if a subscriber occupies a line for 3 minutes, then in case of a 7 lines telephone exchange find
   (i) probability of customer loss,
   (ii) mean number of busy channels.
2. Find the distribution of the waiting time in the queue in case of M/M/1 model. In case arrival and service rate is equal find the mean and variance time in the queue in steady state.
3. In case of M/M/1/N model, where N is the maximum capacity of the system if arrival rate is 2 units per minutes, the average service time is 2 minutes and N = 5, then find
   (i) the probability that a customer is not allowed to join the system,
   (ii) mean number of customers in the system,
   (iii) if the mean service time reduces to 0.5 minutes, then what's the probability distribution of the customers in the system ?
4. Design a problem of which results in a specific queuing model. Check for the existence of its steady state solutions. If the system is in working mode from 9 AM to 5 PM, find
   (i) the expected number of idle periods and the mean duration of each idle period,
   (ii) the expected number of customers served between 9 AM to 5 PM.

1. Design a problem which models a Markov chain. Modified this so that Markov property no more holds. Show how this modified problem can be viewed as a Markov chain.
2. Give example of two Markov chains in which
   (i) Every state communicate with every other state.
   (ii) No state communicate with any other state.

   Give their state diagrams. Further prove that relation "*communication*" is transitive.
3. A system is assumed to be Markov chain with transition matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 0.6 & 0.4 \\ 0.8 & 0 & 0.2 \end{bmatrix} \end{array}$$

   The profit that the system makes per unit time in State 1 and 2 are Rs. 1000 and Rs. 600 respectively, whereas, when in State 3, the system generates a loss of Rs. 100 per unit time. Find the expected profit per unit time after sufficiently long time.
4. Define absorbing Markov chain and fundamental matrix. Illustrate the application of fundamental matrix by considering a suitable example.

Class Test 3 MC 303 Stochastic Processes Marks Distribution(15)

1. In a classical loss model M/M/c/c, if the call arrival rate is 2 per minute and on the average if a subscriber occupies a line for 3 minutes, then in case of a 7 lines telephone exchange find
   3 (i) probability of customer loss, $c=7,\ \lambda=2,\ \mu=\frac{1}{3}\quad \lambda/\mu=6$
   2 (ii) mean number of busy channels. $P_c=\dfrac{(6)^7/7!}{1+6+\frac{(6)^2}{2!}+\cdots\frac{(6)^7}{7!}}=0.185$

   $\rho(1-P_7)=6(1-0.185)=4.89$

2. Find the distribution of the waiting time in the queue in case of M/M/1 model. In case arrival and service rate is equal find the mean and variance time in the queue in steady state. $\lambda=\mu=1$

   3 $P_r\{w_q=0\}=1-\rho.\quad f_q(w_q)=\rho\mu(1-\rho)e^{-(1-\rho)\mu w_q}$

   2 Steady state solution does not exist.

3. In case of M/M/1/N model, where N is the maximum capacity of the system if arrival rate is 2 units per minutes, the average service time is 2 minutes and N = 5, then find $\lambda=2\ \ \mu=1/2\ \ N=5\quad \lambda/\mu=4$
   2 (i) the probability that a customer is not allowed to join the system, $P_5=(1-4)(4)^5/[1-(4)^6]=(-3)(1024)/-(4095)=0.75$
   2 (ii) mean number of customers in the system, $\sum_{n=1}^{5} nP_n=4.46$
   1 (iii) if the mean service time reduces to 0.5 minutes, then what's the probability distribution of the customers in the system ?

   $\lambda=\mu=2\quad \rho=1$ uniform $P_n=\frac{1}{6},\ n=0,1,2,3,4,5,6$

4. Design a problem ~~of~~ which results in a specific queuing model. Check for the existence of its steady state solutions. If the system is in working mode from 9 AM to 5 PM, find
   2+1 (i) the expected number of idle periods and the mean duration of each idle period,
   (ii) the expected number of customers served between 9 AM to 5 PM.

   $T\lambda(1-\rho)\qquad \dfrac{T(1-\rho)}{T\lambda(1-\rho)}=1/\lambda$

   $\dfrac{1}{(1-\rho)}\times T\rho.$

   $\dfrac{\lambda}{\mu}.\mu\left(1-\dfrac{\lambda}{\mu}\right)$

   $\lambda\left(\dfrac{\mu-\lambda}{\mu}\right)$

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V SEMESTER (Summer Semester)

B.Tech.

END TERM EXAMINATION **July-2023**

MC 303 Stochastic Process

Time: 3 Hours **Max. Marks: 40**

Note: Answer any five questions. (Assume suitable missing data, if any.)

1) Solve the following:
   a) Describe a random walk with two absorbing barriers. Give a suitable example of any such real world process. Derive its limiting distribution. (4) [CO2]
   b) Discuss Brownian motion and renewal process. Give a suitable example of each. (4) [CO1]

2)
   a) Derive the n-step transition probability of a Markov chain. Discuss an application of this. (4)[CO1, CO3]
   b) How one can identify that a specific state is transient recurrent. Explain with example. (4) [CO3]

3)
   a) Derive the steady state solution for the Birth and death process. (4) [CO2, CO3]
   b) Give an example of a distribution function which has memoryless property with proof. (4) [CO2, CO3]

4)
   a) Discuss Kendall's notation for representing a queuing model. Derive the performance parameters such as average waiting time in queue and average queue length in M|M|1 model. (4) [CO1, CO3]

   b) Assume a suitable distribution for the recurrence time between two successive renewals, find the distribution for the waiting time until the nth renewal and expected number of renewal till time t. (4) CO3,CO4]

5)
   a) How does a renewal process differ from a Poisson process? Discuss renewal equation and solved it using Laplace transform. (4) [CO5]
   b) What is Erlang loss model in queues? Explain with a suitable example. (4) [CO4, CO5]

6)
   a) The reliability assurance given by a company for its fuses is as per Bernoulli distribution with the probability of a specific fuse being defective as 0.05, independent of others. Fuses are being sold in a packing of 20 each with the money back guarantee that no more than 3 fuses were defective in a packet of 20.
      i. Find the expected number of packets for which money is to be return out of 1000 such packets sold.
      ii. In case packets are sold to customers in a specific order, find the probability that 20th customer will be the 2nd one for which the money will be refunded.
      iii. In case fuse in packets are placed in a specific order, find the probability that 12th fuse will be the 2nd defective fuse. (6) [CO5]
   b) Consider a transistor battery having exponential lifetime with mean as 2 months. In case six such spares batteries are available and if the time to replace a battery is negligible, find the probability that transistor will work for at least one year. (2) [CO5]

******All the Best******

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FIFTH SEMESTER **B.Tech. Mathematics & Computing**

Mid Semester Exam, *Sept. 2019*

Code & Title: MC 303 Stochastic Processes

Time: One and half hrs. *Max. Marks : 30*

Note : Answer all questions. All questions carry equal marks. Assume suitable missing data, if any. You can ask for statistical tables.

1. Explain Bernoulli process. Give one example each of homogeneous and non homogeneous Bernoulli processes. In case of a Bernoulli process find the distribution of the number of succeeding trials before the next success. Is it memory less or with memory?

2. Explain birth and death process. Derive the distribution of the number of departures at time t in case of a pure death process with departure rate $\mu > 0$ and initial inventories N.

3. What is a renewal process? Give examples. How does it differ from a Poission process? In case of a renewal process if inter renewal process is exponentially distributed with parameter $\mu > 0$, then find the renewal function and renewal distribution.

4. A barber shop serves one customer at a time and provides three seats for waiting customers. If the place is full, customers go elsewhere. Arrivals occur according to a Poisson distribution with mean of 4 per hour. The time to get a haircut is exponential with mean 15 minutes. Determine, (i) steady-state probabilities, (ii) expected number of customers in the shop, (iii) probability that customers will go elsewhere because the shop is full.

5. Define Markov chain. Give an example each of a Markov and a non-Markov chain. What are Chapman-Kolmogorov equations ? Design a suitable problem of your choice to illustrate its application.

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FIFTH SEMESTER **B.Tech. Mathematics & Computing**

SUPPLEMENTARY EXAMINATION, *Feb 2019*

Code & Title: MC 303 Stochastic Processes

Time: 3:00 Hours *Max. Marks : 40*

Note : Answer all question by selecting any two parts from each questions. All questions carry equal marks. Assume suitable missing data, if any.

1[a] Differentiate between a random variable and a random process. Classify a stochastic process based on its state and parameter with an example of each type and graphical representation.

[b] What is a Poisson process? Give example. State its important properties. Show that it is a Markov process.

[c] Describe birth and death process and find its steady state solution.

2[a] Describe random walk with two absorbing barriers. Show that the probability that the particle continues to move indefinitely between the two such barriers is zero.

[b] Show that in case of an unrestricted simple random walk, if the probability of a jump upward is greater than the probability of a jump downward, then the particle will drift to ∞ with probability one.

[c]Describe a random walk of your choice with finite number of states with one absorbing barrier and one reflecting barrier. Consider suitable values of the different parameters and find the probability of absorption.

3[a] Explain Bernoulli process? Give examples, both of homogeneous and non homogeneous.

[b] Explain ergodic Markov chain. Consider an ergodic Markov chain of your choice and find the steady state probability distribution for that. What is its significance ?

[c] Two gamblers having an equal probability of loss or gain of Rs 1 at a time, start a game with a fortune of Rs 3 each. If at any stage a specific gambler is having fortune of Rs 4 then find the probability of losing all his fortune by the next 8 trials.

4[a] What's a renewal process. Give example. Consider a renewal counting process with a renewal function of your choice. Find the probability distribution of the number of renewals by a specific time of your choice.

[b] A service centre opens at 9 AM. From 9 AM until 3 PM customers arrive at a Poisson rate of four per hr. and from 3 PM until 9 PM arrival is at a Poisson rate of 6 per hr. Find the probability distribution of the number of customers entering the store on a given day. Also the mean and variance for the same.

[c] Define a Markov chain. Give example. How do you find n-step transition probability matrix of a Markov chain; explain by considering a suitable example.

5[a] Describe M/M/1 queuing system. Find the expected numbers in the queue, and in the system.

[b] Find the probability of losing a customer in M/M/c/c queue model by considering suitable values of the various parameters.

[c] In a railway yard goods trains arrive at the rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes then find the (i) mean queue size, (ii) probability that the queue size exceeds 10, (iii) mean number of trains departed per busy period.

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FIFTH SEMESTER **B.Tech. Mathematics & Computing**

END SEMESTER EXAMINATION, *Nov 2019*

Code & Title: MC 303 Stochastic Processes

Time: 3:00 Hours *Max. Marks : 40*

Note : Answer all five questions selecting any two parts from each question. Questions carry equal marks. Assume suitable missing data, if any.

1[a] Define stochastic process. Give the classification based on state and parameter of a stochastic process. Give an example of each type and their graphical representations.

[b] Describe Poisson process. Give example of homogeneous and non homogeneous Poisson processes. Find the distribution of the number of events in any time interval t in a Poisson process with parameter λ. What is the distribution of the inter-arrival times between two successive events ?

[c] Describe a stationary process. Show that two state Markov chain in continuous time is a stationary process.

2[a] Describe random walk with two absorbing barriers. Give example. Find the probability of absorption at a specific barrier.

[b] Show that in case of an unrestricted simple random walk, if the probability of upward jump is greater than the probability of downward jump, then the particle will drift to ∞ with probability one. Discuss the case when both probabilities are equal.

[c] Discuss random walk with two reflecting barriers. Find its steady state solution. How can you modify this model as with one reflecting barrier and one absorbing barrier?

3[a] Describe a Markov chain by giving a suitable example. Modify this example so that the chain becomes non-Markovian. Show that the relation '*communication*' in a Markov chain is an equivalence relation.

[b] Define transient and recurrent states of Markov chain. Give examples. Find the condition for a state i to be recurrent/transient. Verify same by considering suitable example.

[c] A communication source can generate one of the three possible messages 0, 1, 2. The transmission is described by the t.p.m. $\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$. If the initial state probability distribution is $p^{(0)} = (0.3, 0.3, 0.4)$. Find $p^{(3)}$ and limiting distribution.

4[a] Define a renewal process. Derive and solve renewal equation.

[b] Consider a renewal counting process with renewal function proportional to time t. Derive the n-fold convolution of the distribution function of the number of renewals and hence show that the number of renewal in time t is a Poisson process.

[c] A particle performs a random walk with absorbing barriers at 0 and 4 . Considering the upward and downward jumps to be equally probable, write the t.p.m. and classify the various states. In case the particle starts from state 2, find the probability that it gets absorbed at the 10th step.

5[a] Describe M/M/1 queue model. In steady state find the distribution of the waiting time in the system and in the queue. Hence find the mean waiting time in (i) system, (ii) queue.

[b] Describe M/M/c/c queue model. If c = 10 and arrival rate equals the departure rate then find the probability of customer loss.

[c] In a railway yard goods trains arrive at the rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes then find the (i) mean queue size, (ii) probability that the queue size exceeds 10, (iii) mean number of trains departed per busy period.

on machine B. The profit ...

per doll. What should be daily production of each of the two dolls?

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V SEMESTER

MID TERM EXAMINATION **B.Tech.**

September-2024

MC 303 Stochastic Process

Time: 1 Hour 30 Minutes **Max. Marks: 20**

Note: Answer all questions. (Assume suitable missing data, if any.)

1) How can you determine that a process is a stochastic process? Stochastic processes can be categorized based on nature of index parameter (Discrete/Continuous) and state space (Discrete/Continuous). Categorize Bernoulli process, Poisson process, Renewal process, counting process, Gaussian Process, Brownian Motion based on state space and index set with proper justification. (5) [CO1, CO2]

2) Explain the counting process and give definition of Poisson process accordingly. A random variable X has a Poisson distribution with parameter $\lambda > 0$ and for positive integer $k \geq 0, P[X = k] = \frac{e^{-\lambda}\lambda^k}{k!}$. Prove that the probability generating function for random variable X, $G_X(t) = e^{-\lambda(1-t)}$. Let two independent random variable X and Y follows Poisson(λ_1) and Poisson(λ_2), respectively where λ_1 and λ_2 are parameter representing the arrival rates. Then derive the distribution of the random variable X+Y. (5) [CO3]

3) Discuss random walk problem using any real world scenario of your choice and explain the advantages of such random walk simulations on those real world scenario. Describe unrestricted random walk. Derive the probability of position of particle performing the random walk $j \leq X_n \leq k$ at time point n, $P(j \leq X_n \leq k)$ using approximation by central limit theorem. (5) [CO3, CO4]

4) Let $f_{ja}^{(n)}$ be the probability distribution of absorption at barrier a for possible time points $n \in \{0,1,2,3,...\}$ given the particle started the random walk with arbitrary initial position $X_0 = j$. The generating function can be defined as $F_{ja}(s) = \sum_{n=0}^{\infty} f_{ja}^{(n)} s^n$. Derive the second order difference equation in generating function and discuss the boundary condition on this difference equation in detail. (5) [CO3, CO4]

**********All The Best******

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V SEMESTER

B.Tech.

END TERM EXAMINATION December-2024

MC 303 Stochastic Processes

Time: 3 Hours Max. Marks: 40

Note: Answer any four questions. (Assume suitable missing data, if any.)

1)

a) Discuss generalized Poisson queuing model and write the formula for calculating the steady state probability of n customers in the systems. (2) [CO1] [BTL1]

b) Explain different steady state measures of performance in queuing systems. (2) [CO2] [BTL2]

c) Differential among transient state, recurrent state and absorbing state in context of markov chain with a suitable example. (3) [CO5] [BTL2]

d) Explain the recursive formulation of mean passage time. Discuss its significance in the analysis of Markov process. (3) [CO3] [BTL4]

2)

a) Describe a random walk with two absorbing barriers. Give a suitable example of any such real world process. Derive the probability of absorption at lower barrier. (6) [CO2, CO3] [BTL2]

b) Consider a simple random walk $X_n = \sum Z_i$, and suppose it starts from 0. As usual, $P(Z_i = 1) = p$, $P(Z_i = -1) = q = 1 - p$. Compute $E(e^{\alpha X_n})$ $\alpha \in R$. Comment on the asymptotic behavior of X_n. (2) [CO2] [BTL3]

c) Can we analyze the random walk problem with Markov chain for obtaining the steady state probability? If so, what should be your assumptions on random walk problem? (2) [CO4] [BTL3]

3)

a) Assume that a computer system is in one of three states: busy, idle, or undergoing repair, respectively denoted by states 0, 1, and 2. Observing its state at 2 P.M. each day, we believe that the system approximately behaves like a homogeneous Markov chain with the transition probability matrix:

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$

Prove that the chain is irreducible, and determine the steady-state probabilities. (4) [CO4] [BTL3]

b) Prove the Chapman-Kolmogorov Equations $P_{ij}(t+s) = \sum_{k \in S} P_{ik}(t) P_{kj}(s)$ for continuous-time Markov chain with state space S. (4) [CO5] [BTL4]

c) Define continuous time Markov chain (CTMC) along with two real world example. (2) [CO5] [BTL2]

4)

a) Discuss the consequences of higher expected waiting time in a queuing system. Discuss some suggestions to solve the issues raised due to higher expected waiting time. (2) [CO3] [BTL3]

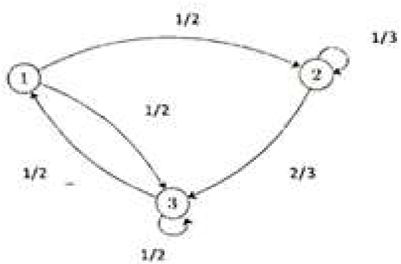
b) For a small batch computing system, the processing time per job is exponentially distributed with an average time of 3 minutes. Jobs arrive randomly at an average rate of one job every 4 minutes and are processed on a first-come-first-served basis. The manager of the installation has the following concerns. (4) [CO2] [BTL3]

i) What is the probability that an arriving job will require more than 20 minutes to be processed (the job turn-around time exceeds 20 minutes)?

ii) A queue of jobs waiting to be processed will form, occasionally. What is the average number of jobs waiting in this queue?

c) Consider a continuous-time Markov chain X(t) with the jump chain shown in following Figure. Assume $v_1 = 2, v_2 = 3$ *and* $v_3 = 4$. Find the generator matrix G for the chain and steady state distribution of the states.

(4) [CO2] [BTL3]



$\frac{4}{15}$ 3 8

5)

a) Define Renewal process. Give two real world examples of renewal processes. Let $N(t)$ represents number of renewals upto time t and S_n represent time of n^{th} renewal. Comment on the distribution of S_n if $N(t)$ follows the poission distribution. (3) [CO1] [BTL4]

b) Define the renewal function $H(t)$ and renewal density function $h(t)$ Derive the renewal equation $h(t) = f(t) + \int_0^t h(t-u)f(u)du \quad t \geq 0$.
(5) [CO3] [BTL5]

c) An ordinary renewal process has the renewal function $H(t) = \frac{t}{5}$. Determine the probability $Prob[N(15) < 2]$. (2) [CO2] [BTL4] $4e^{-3}$

**************All The Best*************