MODERN ALGEBRA (MC-207)

ASSIGNMENT – 1

2024 (ODD SEMESTER) B.Tech III rd SEMESTER

Q1 Prove that if $(ab)^2 = a^2b^2$ in a group $G, \forall a,b \in G$ then G is Abelian.

Q2 If $(ab)^n = a^nb^n$ holds for 3 consecutive integer value of n. Show that G is Abelian.

Q3 Show that $(Z,\oplus)$ where $a \oplus b = a+b+2, \forall a,b \in Z$ is a group.

Q4 Let G be the set of all real numbers except -1 and $a * b = a+b+ab, \forall a,b \in G$ then show that G is a group.

Q5 Let G be an Abelian group and let $H = \{x \in G : x^2 = e\}$. Show $H < G$.

Q6 Let G be an Abelian group and let $H = \{x^2 : x \in G\}$. Show $H < G$.

Q7 Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$. Find $|A|, |B|, |AB|$ in SL(2,R), i.e., Group of 2X2 matrices under matrix multiplication, which has Determinant not equal to zero.

Q8 Prove that group of prime order are cyclic.

Q9 Show that U(9) is a cyclic group. What are all its generators?

Q10 Prove that subgroup of an Abelian group is Abelian.

Q11 Find the generators of $G = < a >,\ a^{12} = e$.

Q12 A non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H, \forall\, a,b \in H$.

Q13 If $|a| = 15$ find orders of the following

(i) a^3, a^6, a^9, a^{12}

(ii) a^5, a^{10}

Q14 Show that the group $(\mathbb{Q},+)$ is not a cyclic group.

Q15 Show that Center of group G ,i.e., $Z(G)$ is a subgroup of G.

Q16 If a and b are two elements of a group G , then show that

i) $o(a)=o(xax^{-1})=o(x^{-1}ax)$

ii) $o(ab)=o(ba)$

iii) $o(a)=o(a^{-1})$

iv) If $a^m=e$, then $o(a)$ divides m.

Q17 The set $G=\{1,5,7,11\}$ is a group w.r.t multiplication modulo 12. Find the order of all elements of G.