Assignment-2

Probability and Statistics (MC-205)

Department of Applied Mathematics

1. (a) Find the expectation of the number on a die when thrown. CO2
(b) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.

Ans: 7/2 , 7

2. Let the random variable X have the p.m.f. CO2

$$f(x) = \frac{(|x|+1)^2}{9}, \quad x = -1,0,1.$$

Compute $E(X), E(X^2)$ and $E(3X^2 - 2X + 4)$.

Ans: $E(X) = 0, E(X^2) = 8/9$, $E(3X^2 - 2X + 4) = 60/9$

3. Let the random variable X has the distribution CO2

$$P(X = 0) = P(X = 2) = p,$$

$$P(X = 1) = 1 - 2p\,; \qquad for\ 0 \le p \le 1/2$$

For what p is the $Var(X)$ maximum.

Ans: $p = \frac{1}{2}$

4. Urn A contain 5 cards numbered from 1 to 5 and urn B contains 4 cards numbered from 1 to 4. One card is drawn from each of these urns. Find the probability function of the number which appears on the cards drawn and its mathematical expectation. CO2

Ans: 11/2

5. In a lottery 8000 tickets are to be sold at Rs. 5 each. The price is a Rs. 12,000 T.V. If two tickets are purchased what is the expected gain? CO2

Ans: Rs.-7.0

6. A Rs. 5000 item can be insured for its total value by paying an yearly premium of Rs. N. If the probability of theft in a year is estimated to be .01, what premium should the insurance company change if it wants the expected gain to equal Rs. 1000? CO2

Ans: Rs.1050

7. In a lottery m tickets are drawn at a time out of n tickets numbered from 1 to n. Find the expected value of the sum of the number on the tickets drawn.

Ans: $\frac{m(n+1)}{2}$

8. Starting from the origin, unit steps are taken to the right with probability p and to the left with probability $q = 1 - p$. Assuming independent movements, find the mean and variance of the distance moved from origin after n steps.

Ans: $E(X) = 2p - 1, Var(X) = 4p(1 - p)$

9. For the two random variables X and Y with same probability distribution, show that $Cov(X - Y, X + Y) = 0$ CO2

10. Show that in 40,000 tosses of a balanced coin, the probability is at least 0.99 that proportion of head will fall between 0.475and 0.525. CO1

11. A random variable, X has mean μ and variance σ^2. Find the mean and the variance of $(X - \mu)/\sigma$.

Ans: $Mean = 0, Var = 1$

12. Let the random variable X assume the value r with the probability law

$$P(X = r) = q^{r-1}p\,; \quad r = 1,2,3\,...$$

Find the m.g.f. of X and hence its mean and variance.

Ans: $M_X(t) = p * \frac{e^t}{1-qe^t}$, $\mu = \frac{1}{p}$, $\sigma^2 = \frac{q^2}{p}$,

13. A random variable X has the density function CO2

$$f(x) = \begin{cases} \frac{1}{2\sqrt{x}} & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

Obtain the m.g.f. and hence the mean and variance.

Hint: expand in infinite series

14. For a distribution $f(x) = 2^{-x}$, $x = 1,2,3\,...$ prove that Chebychev's inequality gives CO1

$$P[|X - 2| \leq 2] > 1/2$$

While the actual probability is $15/16$.

15. Two unbiased dice are thrown. If X is the sum of the numbers showing up, prove that

$$P[|X - 7| \geq 3] \leq \frac{35}{54}$$