MODERN ALGEBRA (MC-207)
ASSIGNMENT – 2
2024 (ODD SEMESTER) B.Tech III rd SEMESTER

Q 1 Show that the set of all permutations on $s = \{1, 2, 3\}$ forms a group under the composition of functions.

Q 2 Prove that the symmetric group S_n is non abelian for $n \geq 3$.

Q 3 State and prove Lagrange's Theorem for finite groups.

Q 4
- (i) Define left cosets and right cosets of a subgroup in a group.
- (ii) Show that the number of left cosets is equal to the number of right cosets for a subgroup of a finite group.

Q5 Prove that A_n is a normal subgroup of S_n.

Q 6
- (i) Define a normal subgroup.
- (ii) Give an example of a normal subgroup of a group that is not the center of the group

Q 7 Show that every cyclic group is abelian.

Q 8 Construct the Cayley's table for Q_8.

Q 9 Find all of the left cosets of $\{1, 11\}$ in $U(30)$.

Q 10 Let G be a group of order 60. What are the possible order for the subgroup of G.

Q 11 Klein four-group K_4
- (i) Prove that K_4 is an abelian group.
- (ii) Explain why K_4 is not a cyclic group.
- (iii) Are all the subgroup of K_4 normal?

Q 12 Let $H =< (12) >$. Is H normal in S_3.

Q 13 Find all the cosets of all subgroup of Q_8.

Q 14 Prove that if $N \lhd G$ and $M \lhd G$, then $N \cap M \lhd G$.

Q 15 If H has index 2 in G. Prove that $H \lhd G$.

Q 16 What is the order of each of the following permutations.

(i) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 1 & 2 & 3 & 4 & 5 \end{bmatrix}$

(iii) $(124)(3578)$

Q 17 If $\dfrac{G}{Z(G)}$ is cyclic. Show $\dfrac{G}{Z(G)}$ is trivial.

Q 18 Let G be the set of all 2 X 2 matrices $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ where $ad \neq 0$ under Matrix multiplication. Let $N = \left\{ \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \right\}$. Prove that

a) N is a normal subgroup of G.
b) G / N is Abelian.

Q 19 Draw the following Cayley's Digraph:

i) $Cay(\{1\} : Z_6)$

ii) $Cay(\{2,3\} : Z_6)$

Q 20 Find $a^{-1}ba$ given $a = (1 \;\; 3 \;\; 5)(1 \;\; 2)$ and $b = (1 \;\; 5 \;\; 7 \;\; 9)$ in S_9.

Q 21 Determine which of the following are even/odd permutation.

$(i)\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$

$(ii)(1 \;\; 2 \;\; 3 \;\; 4 \;\; 5)(1 \;\; 2 \;\; 3)$

Q 22 Prove that quotient group of an abelian group is abelian