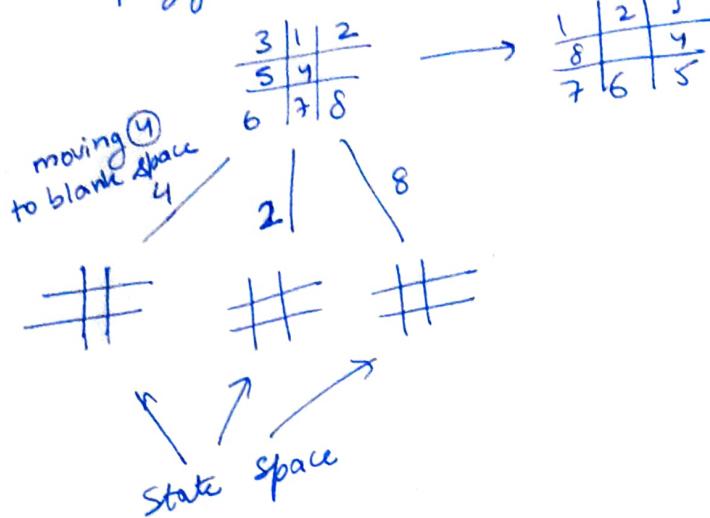


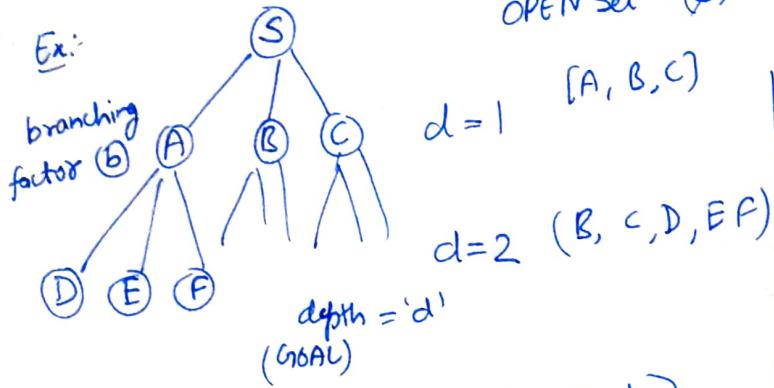
5/8/25

Artificial Intelligence

→ 8-puzzle.



Brute force = Generate & Test



Closed set → [contains nodes that have been examined]

2.) DFS } comparison points
BFS } (a) Time complexity
↳ depends on (CLOSED) SET

→ The nodes that have to be examined in future

Pseudo Code [Generate & Test]

```

OPEN ← {S}
while OPEN is not empty
{
    Pick any node n from OPEN
    OPEN ← OPEN - {n}
    Closed = Closed ∪ {n}
    if (n = GOAL) return True
    else
        OPEN ← OPEN ∪
            {neighbours} -
            {CLOSED}
}
return failure
  
```

$$\text{DFS} \rightarrow \text{Time} = \frac{1}{2} [\text{Goal at first node} + \text{GOAL at last node}]$$

$$= \frac{1}{2} [d + \frac{b^{d+1}-1}{b-1}]$$

$$\approx \frac{b^d}{2}$$

$b \rightarrow$ branching factor
↳ no. of branches each node have

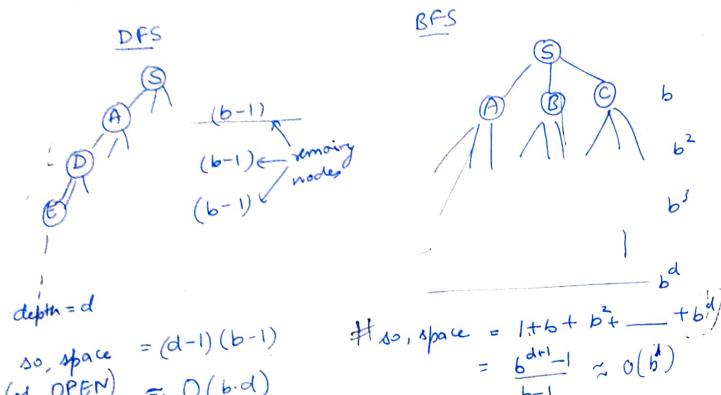
$$\rightarrow 1 + b + b^2 + \dots + b^d$$

$$\begin{aligned} \text{BFS} \rightarrow \text{Time} &= \frac{1}{2} \left[\text{all nodes at depth } d-1 + \text{goal at last node} \right] \\ &\quad + b + b^2 + \dots + b^{d-1} + b + b^2 + \dots + b^d \\ &= \frac{1}{2} \left[\frac{b^d - 1}{b-1} + \frac{b^{d+1} - 1}{b-1} \right] \\ &= \frac{b^d}{2} \left(\frac{1+b}{b} \right) \approx \frac{b^d}{2} \left(\frac{1+b}{b} \right) \end{aligned}$$

so, DFS is marginally better than BFS $\left[\frac{b^d}{2} < \frac{b^d}{2} \left(\frac{1+b}{b} \right) \right]$

$$\frac{\text{BFS}}{\text{DFS}} = \frac{1+b}{b}$$

(b) Space Comp. \rightarrow OPEN set determines space comp.



(c) Quality of set \rightarrow That is probability of finding the goal (starting cell is KTC) \rightarrow BFS \geq DFS \rightarrow BFS has better quality of set than DFS

(d) Completeness \rightarrow Both DFS & BFS are complete in nature

(can also always help to reach the goal)

6/8/25
→ MDB-DFS \rightarrow Depth Bound DFS
↳ find depth till search range algo then terminate.
↳ Not complete

(5) Depth First Iterative Deepening (DFID):-

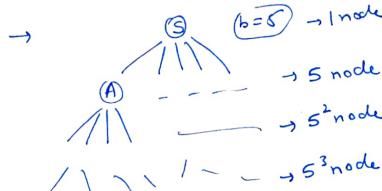
↳ Variable Depth

↳ first search till a fixed depth using DFS, if goal state not found then we inc. the depth and the whole tree will be regenerated till that depth

↳ complete

↳ Quality of set \rightarrow best

↳ Time & Space is similar to DFS complexity



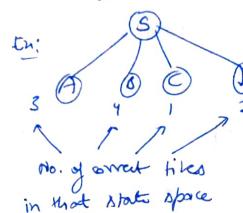
so, to generate the whole it is not heavy on the memory because we have generate for eg: $125 = 5^3$ nodes at $d=3$. for that we have to also generate the previous tree nodes. since the ratio is not significant $(\frac{31}{125})$ therefore it doesn't put stress on memory.

(Ch-3) in book

Heuristic Algorithms :-

To check how we are close to we are to goal state

let h_1 = no. of correct tiles



\rightarrow So, B is best state to continue with.

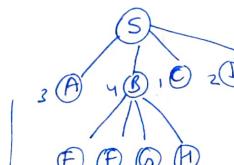
(i) Best First Search :-

Priorit^y Queue
(Min heap)
OPEN = [A, C, D, E, F, G, H]
set \rightarrow CLOSED = [S, B]

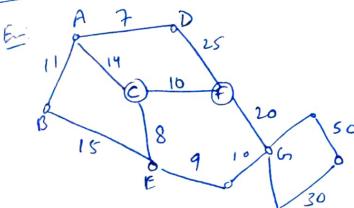
\rightarrow Best A after exploring B.

\rightarrow Time & Space depends on heuristic funcⁿ

\rightarrow Complete Algo. Not gives always optimal solⁿ

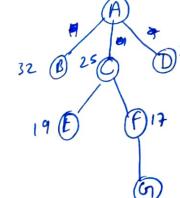


Now, we will check all the heuristic value of nodes present in OPEN set, so A will be explored next.



St. line distance

$$\begin{aligned} A \rightarrow G &= 40 \\ B \rightarrow G &= 32 \\ C \rightarrow G &= 25 \\ D \rightarrow G &= 35 \\ E \rightarrow G &= 19 \end{aligned}$$



7/08/25

#(ii) Hill Climbing

Constant Space Complexity \rightarrow since we are only examining the best node.

Linear Time Comp. \rightarrow cause we are only exploring one node at each depth.

\downarrow chosen node should be better than parent and all other nodes are deleted. If we don't find any better heuristic value, the algo. will terminate and it will return the parent node.

(ii) BEAM Search :-

(modified Hill Climbing) \rightarrow beam width $\rightarrow b$ = no. of best chosen nodes chosen at each depth

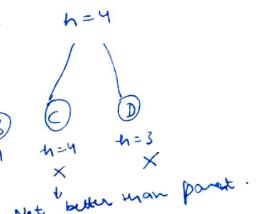
\rightarrow all best 'b' nodes should be better than parent

for $b=2$

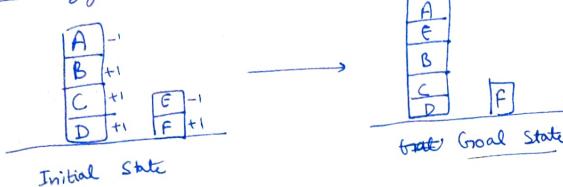
$h=3$ $h=4$

\rightarrow $\geq b$ nodes are chosen at each depth.

\rightarrow Time & Space comp. same as Hill climbing



Block Puzzle :-



$$h = 4 - 2 = +2$$

→ Apply Hill or ~~Beam~~ Search.

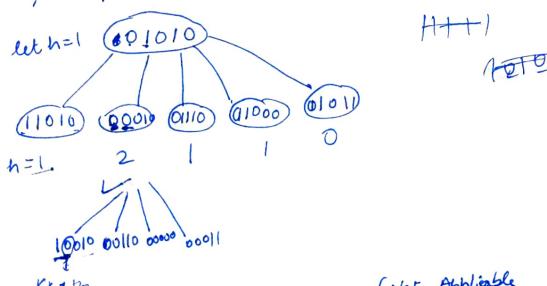
2-SAT

$$(a \vee b) \wedge (c \vee d) \wedge (d \vee e) \rightarrow \text{CNF form}$$

3 clauses
and each clause has maximum of two literals hence 2-SAT
 a, b, c, d, e

$$\# 3\text{-SAT} \quad \neg (a \vee b \vee c) \wedge (d \vee a \vee b') \wedge (e \vee d \vee c')$$

let $a=0, b=1, c=0, d=1, e=0$ initial state



(3) Variable Neighbourhood Approach :- (Not Applicable)

(variation of hill climbing)
Changing one bit at a time = 5 nodes

Changing two bits at a time = 10 nodes

L increased probability of finding both heuristic value.

for n literals,

$$\text{change one bit at a time} = nC_1 \\ \text{in two } = nC_2$$

In initial depth, start with low dense heuristic f^- , like ~~change~~ nC_1 , and then after few depths increase the density of f^- like nC_2 or nC_3 .

12/08/25

TABU Search SAT Problem :-

- We can assume multiple initial cond.
- Like H.C.

for eg: to solve ~~SAT~~ Problem

we can assume 0010, 1100, —

TABU Search SAT Problem :-

4 literals a, b, c, d

$$(a \vee b) \wedge (c \vee d) \wedge (a \vee b') \wedge (a \vee d')$$

Table Tennis
1-bit cannot be changed to certain levels that we decide (let 2)

$t(n) = 1$

$t(n) = 2$

$t(n) = 3$

$t(n) = 4$

$t(n) = 5$

$t(n) = 6$

$t(n) = 7$

$t(n) = 8$

$t(n) = 9$

$t(n) = 10$

$t(n) = 11$

$t(n) = 12$

$t(n) = 13$

$t(n) = 14$

$t(n) = 15$

$t(n) = 16$

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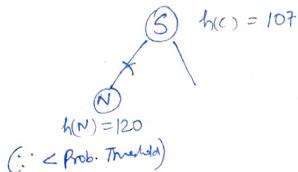
$t(n) = 219$

$t(n) = 220$

$t(n) = 221$

$t(n) = 222$

Stochastic Hill Climbing:



→ Probability threshold → A threshold value acc. to which we decide if the child node is better than parent or not.
→ If ~~the~~ it is ~~&~~ not better, we will generate another node.

and To find probability threshold,

$$\Delta E = f(N) - f(C)$$

(child) (parent)

we will sigmoid $f' \rightarrow \frac{1}{1+e^{-\Delta E/T}}$ → since it gives value \rightarrow b/w 0 & 1.

so, $\frac{1}{1+e^{-\Delta E/T}}$ → for higher value ~~of~~ ^{better} of heuristic f' .

$\frac{1}{1+e^{-\Delta E/T}}$ → for lower value in heuristic f' .
better

where, T is usually small like, $T=10$ → fixed value

$f(C) = 1087$	$h(N) = 120$	$h(N) = 150$	$h(N) = 80$
$T=1$	~ 0.8	~ 0.9	~ 0.000
$T=10$	-	-	-
$T=100000$	~ 0.5	~ 0.5	~ 0.5

so, we will use diff. values of T at diff. levels.
↳ we will start with higher ~~value~~ value of T , and decrease as we go down.
This algo. is called Simulated Annealing.

Genetic Algorithms:

- (a) Selection → Start with random n^1 and select better candidate
- (b) Crossover - Producing new n^1 from selected candidate

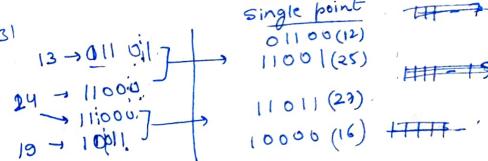
(c) Mutation :-
(Rarely used)

g:	$f''(x^2)$	Probability	Expectation
	169	$169/1340$	$\approx 0.2 \times 4 \approx 0.831$
	576	$576/1340$	$\approx 0.4 \times 4 \approx 1.67 \approx 2$
	64	$64/1340$	$\approx 0.0 \times 4 \approx 0.083 \approx 0$
	361	$361/1340$	$\approx 0.3 \times 4 \approx 1.2 \approx 1$
		<u>1340</u>	

To find max. possible value of 5 bits

(i.e. 1111)
 $= 2^5 + 2^4 + 2^3 + 2^2 + 2^1 = 31$

so, now selection



→ 1st method
→ If we have ' n ' initial candidates, * and then we generate ' n ' new candidates, then we can replace the previous weaker candidates from new better candidates.

→ 2nd method → Or we can replace entire old candidates.

→ $11001(25) \rightarrow 11001 \quad 11011 \quad 11011(27) \quad 11011$ }
we cannot change the third bit to one by any crossover.
11001 }
11001

so, now we use mutation → and we change third bit to 1.

TSP :-

$$\text{Path representation: } P_1 = 247561893 \Rightarrow 247591457$$

$$P_2 = 628391457 \Rightarrow 628361893$$

↑ repetition

To solve repetition :-

(i) Partially Mapped Crossover :-

$$P_1 = 247 | 5618 | 93$$

$$P_2 = 628 | 3914 | 57$$

$$C_1 = \frac{924}{P_2} \quad \overbrace{\begin{array}{cccc} P_1 \\ 5 & 6 & 18 & 37 \\ \hline 1 & 1 & 1 & 1 \\ P_1 & P_2 & P_2 & P_1 \end{array}}$$

$$287 \quad 3914 \quad 65$$

→ we will take some random bits from both P_1 & P_2 and then we will map them together $\begin{pmatrix} 5618 \\ 1116 \\ 3914 \end{pmatrix}$

Then, if we encounter a repetition we will use these mapping.

(ii) Ordered Crossover :-

$$C_1 = \underbrace{5 \ 6 \ 18}_{P_1} 23947 \rightarrow \text{Just put some random bits first, and discard all the repetition.}$$

$$C_2 = \underbrace{3 \ 9 \ 1 \ 4}_{P_2} 27568$$

→ ordinal representation :- Let index $i \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9$

→ write index no. of the no. in path representation

→ also cut the index you have used, then for next index, count from starting

$$P_1 = 247561893$$

↓

$$P_1 = 235331221$$

$$P_2 = 628391457$$

↓

$$P_2 = 626251111$$

$$\text{New, } P_1 = 2353 | 31221$$

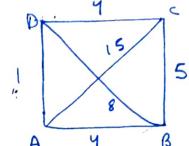
$$P_2 = 62625 | 31111$$

=>

$$14108125$$

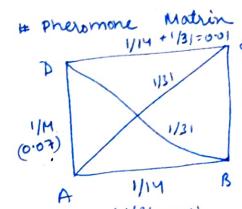
Ant Colony Optimization :-

→ Cost Matrix :-



$$\text{Path 1} \rightarrow ABCDA = 14$$

$$\text{Path 2} \rightarrow ACDBA = 31$$



→ To update the level of pheromone in the pheromone matrix, we use the path sum reciprocal.

Now to find probability of another ant going from A, based on cost & pheromone level,

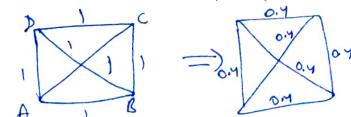
$$A \rightarrow D = \frac{(0.07) \times \frac{1}{14}}{\text{sum of all cost from A}} = \frac{0.07 \times \frac{1}{14}}{0.07 + 0.03 + 0.07} = \frac{0.07 \times \frac{1}{14}}{0.17} = \frac{0.07}{17} = 0.0041$$

$$A \rightarrow B = \frac{0.03 \times \frac{1}{15}}{0.07 \times \frac{1}{14} + 0.03 \times \frac{1}{15} + 0.07 \times \frac{1}{14}} = \frac{0.03 \times \frac{1}{15}}{0.17} = \frac{0.03}{17} = 0.0018$$

$$A \rightarrow C = \frac{0.07 \times \frac{1}{14}}{(0.07 \times \frac{1}{14}) + 0.03 \times \frac{1}{15} + 0.07 \times \frac{1}{14}} = \frac{0.07 \times \frac{1}{14}}{0.17} = \frac{0.07}{17} = 0.0041$$

→ Volatilization Effect :- $\gamma = 0.6$ (Let)
(optional)

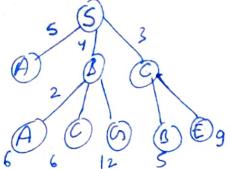
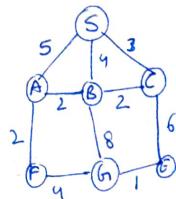
$$\text{so, } (1-\gamma) \times \text{initial level of pheromone} = (1-0.6) \times 0.07 = 0.4$$



19/08/25

Branch & Bound :-

(Solving TSP)

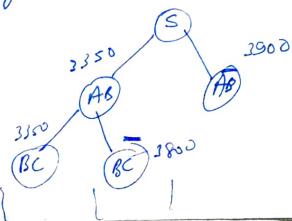


- after getting bound, removed node whose cost is less than bound
- complete
- No. of iteration we don't know.

ABX	AB	A	B	C	D	E
1500	900 ←	A	0	300	600	900
1300	800 ←	B	300	0	500	800
1100	1100 ←	C	600	500	0	1100
1700	1700 ←	D	900	800	1100	0
2200	2200 ←	E	1000	1200	1400	1500
= 7800	* = 6700					

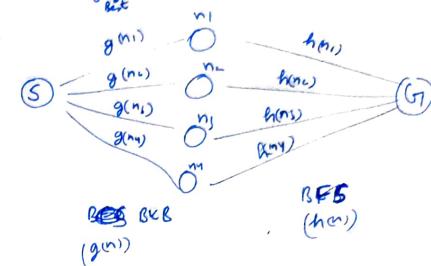
Estimated cost = $\frac{6700}{2} = 3350$, for $\bar{AB} = 2500$
1/2 bound for AB

→ Selection of min. of two and adding it the get estimated cost for every non selected edge or selected edge



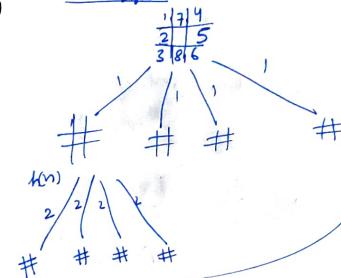
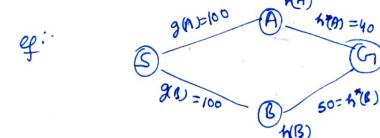
A* algo!

↳ combination of BFS & Branch & Bound



value we evaluate $f(n) = g(n) + h(n)$

8-Puzzle :-

Now for $h(n) \leq h^*(n)$ 

(Underestimation) ✓

Case II :- $h(A)=30, h(B)=20$

$$S \rightarrow (B) \rightarrow h = 120$$

but actual = 150 ($h^*(B)$)

so, we will evaluate

$$S \rightarrow A \rightarrow h = 130 \vee$$

h=140 so optimal answer is found

Time = $O(b^d)$
Space = $O(b^d)$

so, $[g^*(n) \leq g(n)]$

Case I (overestimation)
 $h(A)=80, h(B)=70$

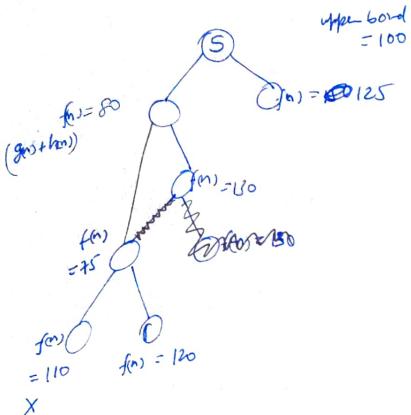
$$S \rightarrow B \rightarrow h = 150$$

but we ignore (A),
but if we $S \rightarrow A \rightarrow h = 140$
so, optimal answer gets found

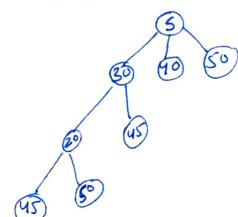
$\therefore h(n) \leq h^*(n)$

Iterative deepening A* (IDA):

like DFID but we compute
 $f(n)$



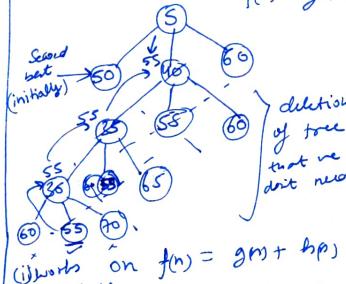
Best first Search ($h(n)$):



- (i) works on $h(n)$
- (ii) only picks the best possible value based on $h(n)$.

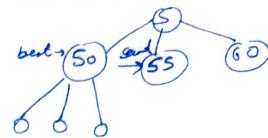
go down, when we don't have initial second best (i.e. 50) we update the best value of existing initial second best (i.e. 50) we update the best value of existing node (i.e. 55) to its top most parent, then its becomes the second best value and 50 is the best value & we will explore it.

Recursive Best first Search :-
 $f(n) = g(n) + h(n)$



- (i) works always on $f(n) = g(n) + h(n)$
- (ii) we maintain a pointer to the second best value (50)
- (iii) then we exhaust the node that is selected first (i.e. 40) as we can't find any best values other than 50

New tree



Threshing → when we generate & delete existing various paths acc. to our need.

A* Envelope = $f(n) = g(n) + h(n)$

$$f(S) = 14$$

$$S \rightarrow B \rightarrow 4 + 12 = 16$$

$$S \rightarrow C \rightarrow 3 + 11 = 14$$

$$SC \rightarrow E \rightarrow 3 + 10 + 4 = 17$$

$$SC \rightarrow D \rightarrow 3 + 7 + 6 = 16$$

~~$$SCD \rightarrow E \rightarrow 3 + 7 + 2 + 4 = 16$$~~

~~$$SCD \rightarrow F \rightarrow 3 + 7 + 2 + 6 = 16$$~~

$$SCD \rightarrow G \rightarrow 3 + 7 + 2 + 6 = 16$$

$$SB \rightarrow F = 4 + 5 + 11 = 20$$

Q1	
Q2	
Q3	
Q4	

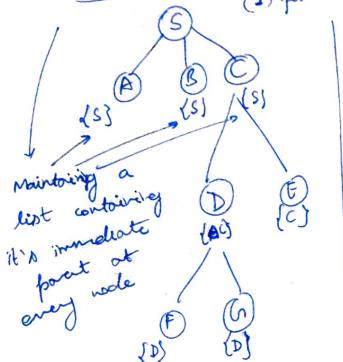
Pruning the Closed list

↳ Eliminating the unnecessary nodes present in the cloned list

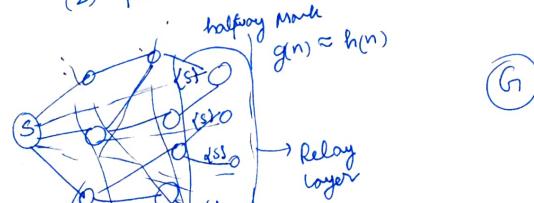
Conc → (i) Stop looking back (Not exploring the nodes we have already explored)
 Purpose of closed list (ii) Path construction
 ↳ Path from starting to goal node.

L Path from starting to goal node.

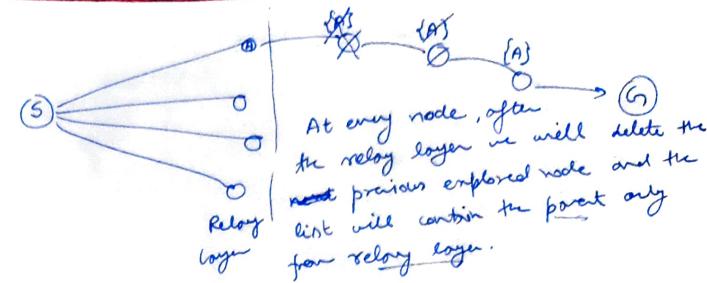
(i) DCFS (Divide & Conquer frontier Search) :-
 (1) for Stop Looking back



(2) for Path construction

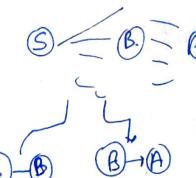
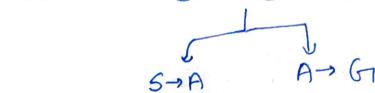


Addition: After reaching the halfway nodes, we will delete all the previous nodes, and anyone's parent will be (S) stored in the stack as $\{S\}$. And the layer of all nodes at halfway marks are called relay layer.



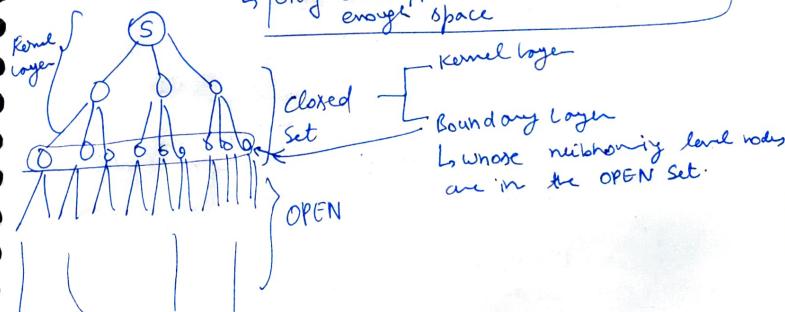
Path reconstruction

↳ we have $S \rightarrow A \rightarrow G$

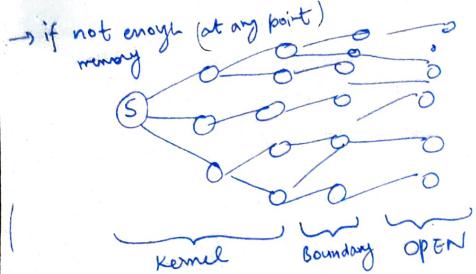


(ii) SMGS (Sparse Memory Graph Search):

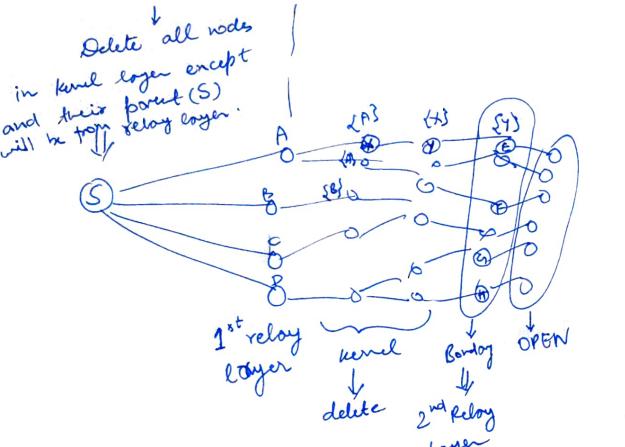
↳ Only delete/prune when you don't have enough space



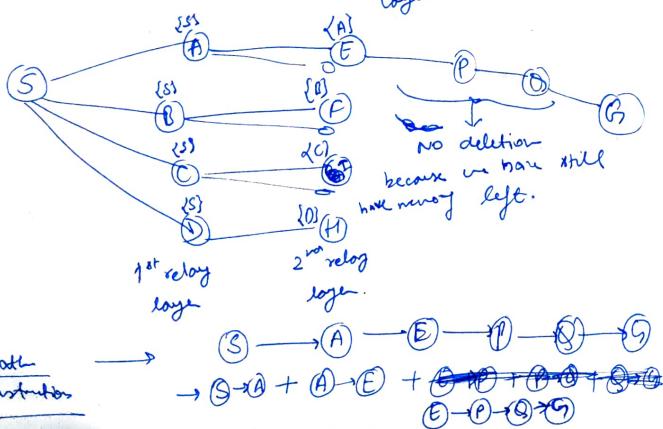
→ Acc. to SMGS :- If you have enough memory than avoid pruning.



(G)



(G)



28/8/25

Pruning the OPEN List :-

(i) → Hill Climbing

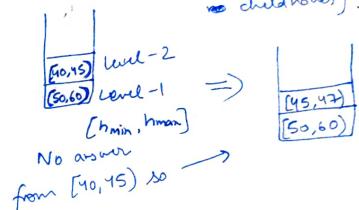
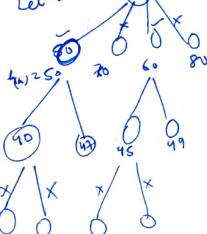
(ii) → BEAM Search

(iii) → BFHS (Breadth First Heuristic Search) → Not complete

↳ We set an upper bound depending on a heuristic f . and if the heuristic value is higher than the bound then we discard that node.

(iv) → BEAM Stack Search :- [complete] because we also search discarded nodes if we didn't find any better \Rightarrow childnodes).

Let $b=2$



like in (40, 45)

If we didn't find any answer better than parent, then we will take the best value from hmax and make it the new hmax and the hmax will become new hmin

$$\begin{array}{l} [40, 45] \Rightarrow [45, 47] \Rightarrow [47, 49] \\ \text{hmin hmax} \quad \text{hmin hmax} \quad \text{hmin hmax} \\ \text{if no answer} \quad \text{if no answer} \end{array}$$

Merging pruning of OPEN & CLOSED list Algo:-

(No pruning) (Closed) (Open)

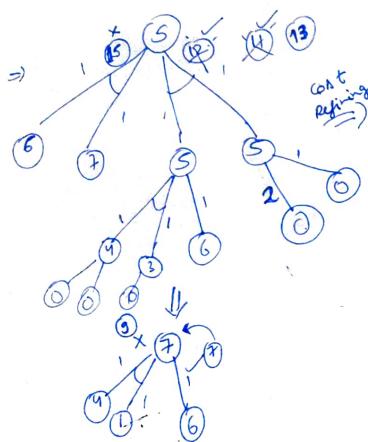
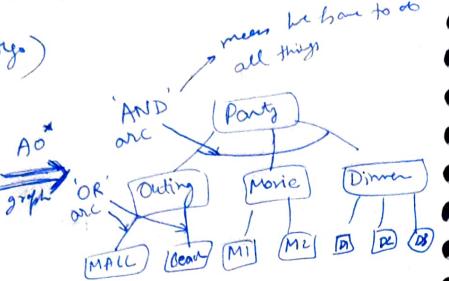
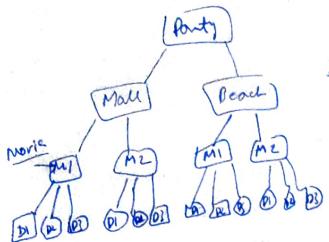
(i) DCBSS → Divide & Conquer Beam Stack Search. (DCFS + BSS)

(ii) DCBFHS → Divide & Conquer Beam First Heuristic Search.

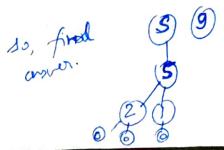
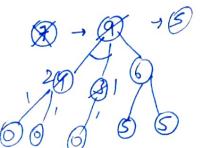
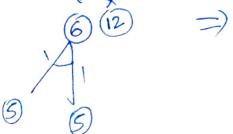
(DCFS + BFHS)

(Closed) (Open)

AO* Algo. (AND OR Algo.)



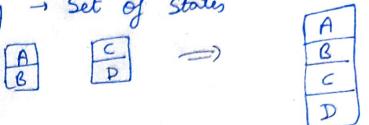
Only generate the level at each node, then compose and update next after refining at each node.



Note: It may or may not give the optimal sol.

02/09/25

Planning → Set of States



Initial State

on Table (B)

on Table (D)

on (A, B)

on (C, D)

Arm Empty

Clean (A)

Clean (C)

(Represent States)
Domain Predicate (S)

on(X, Y) → X is on Y

on Table → on Table

(T(X))

Clean(X) → No one is above 'X'.

Holding(X) → Robert holds X

ArmEmpty → Robert not holding anything

Action (Y)

Pickup(X) → X on Table
No one is above

Put Down (X) → Putting block on table

Unstack (X, Y) → Pick X from Y

Stack (X, Y) → Putting X on Y

After Action Effect

Holding(X)

A.E., onTable, clean(X)

Holding(X), Clean(X)

A.E. clean X, on(X, Y)

Pre Cond:-

on T(X) & clean(X) & A.E.

Holding(X)

On (X, Y) & clean(X) & A.E.

Clean(Y), & holding(Y)

← For Pickup

← for putdown(X)

← Unstack (X, Y)

← stack (X, Y)

Action
unstack(A, B)
Holding(A)
Clean(B)
on(C, D)
Clean C

domain predicate
unstack (C, D)
Holding (A)
Clean (B)
on (C, D)
Clean C

FSSP

↓
forward State Space Planning

of all the Algo's we have learnt that come under this

In a stepwise action A = program(S, a) -> S' = [S - effect(a)] ∪ [effect + (a)]

BSSP (Backward State Space Planning) :-

$$G \rightarrow G'$$

$$G' = [G - \text{effect}^+(a)] \cup \text{pre cond}^-(a)$$

$$[\text{on}(A, B) \wedge \text{on}(B, D)]$$

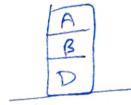
Stack
(B, D)
holding(B)

04/09/25

Goal Stack Planning :-



=>



Order doesn't matter

Goal state predicate

On(A, B) & On(B, D)
↳ On(A, B)
↳ On(B, D) pop
sm → Stack (B, D)
Holding(B) & Clean(D)
↳ Holding(B) pop
↳ Clean(D) pop
Unstack(C, D) AP
A, E, On(C, D) & Clean(C)

compose it from start state if not true then pop and write the action what needs to be done

Not True
Action →
precond →
Top
(Reciprocal for representation)
and along with action also push its precond
1st action →
when an action gets popped that means that will be the first action of our Plan
Also now, the state will change, acc. to action
Robot



- PLAN**
- 1) Unstack (C, D)
 - 2) Put down (C)
 - 3) Unstack (A, B)
 - 4) Put down (A)
 - 5) Pickup (B)
 - 6) Stack (B, D)
 - 7) Pickup (A)
 - 8) Stack (A, B)

initial state will change
sm Action
→

3rd Action →

2nd Action →

4th Action →

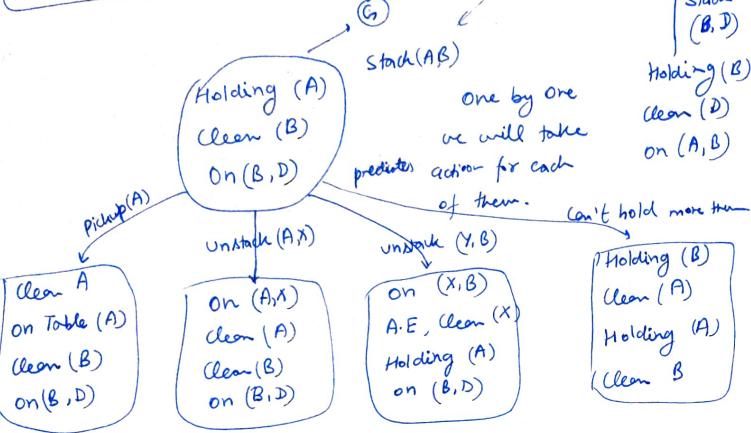
8th Action →

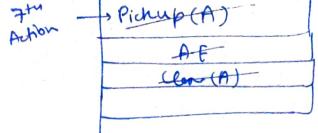
New stack after 3rd action

[B | D] C

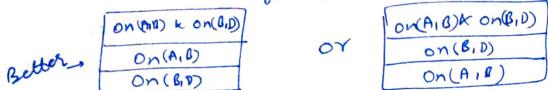
Drawback FSSS

↳ Large branching factor



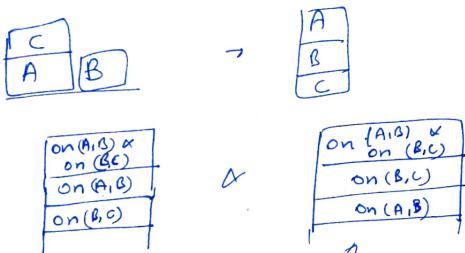


→ Goals are serializable → that means a specific order of goals will be better than other



→ Both will give correct answer
But we have to do test work in fixture.

→ Susanman Anomaly → we have to rethink

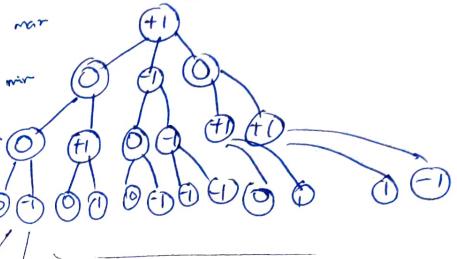


In both cases we have to do one action twice so there is no better serial ∴ this is an Susanman Anomaly

Wolfram
Game Planning

↳ back-up rule

Tic-Tac-Toe software → more all game

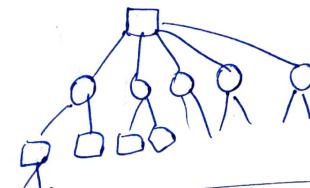


II.

Chess avg. branching factor ≈ 35

no. of moves to end game = 50 (both players)

35^{50} → Back up rule failed.



Chess → we go upto K-level

material value
positional value

(intermediate board pos) & generate legal



30/09/25

(en. Email spam or not spam)

Naive Bayes' classifier \rightarrow generative supervised learning algo.

\rightarrow categorical data

The prob. of new example, belonging to class $y \in Y$ can be calculated using Bayes' Thm,

$$P(y|x_{\text{new}}) = \frac{P(x_{\text{new}}|y) \cdot P(y)}{\sum P(x_{\text{new}}|y) \cdot P(y)}$$

↓
class label

Example	words	label (Class)
Algo, Tree, Graph		CS
Tree, life, Gene, Algo		Bio
Graphs, NP, Algo, Tree		CS
Protein, Assay, Cell		Bio

$$\text{Now, } P(y=CS) = 2/4 = 0.5$$

$$P(y=Bio) = 2/4 = 0.5$$

\rightarrow class-word prob.

$$P(\text{Algo} | y=CS) = \frac{P(\text{Algo}, y=CS)}{\sum P_w(w, y=CS)}$$

$$= \frac{\# \text{Algo. in 'CS' doc}}{\# (\text{Total no. of words in CS doc})} = \frac{2}{7}$$

$$\approx 0.3$$

$$\text{Similar, } P(\text{true} | y=CS) = 2/7$$

$$P(\text{Algo}, y=Bio) = 1/7$$

$$P(y=CS|x_{\text{new}}) \propto P(y=CS) \prod_w P(w|y=CS)$$

Prob. of doc x_{new}
 contain the words
 (Algo, Tree)

$$\propto P(y=CS) * P(\text{Algo}|y=CS) * P(\text{true}|y=CS)$$

$$\propto \frac{1}{2} * \frac{2}{7} * \frac{2}{7} = \frac{4}{98}$$

$$P(y=CS|x_{\text{new}}) \approx \frac{4}{98} = 0.04$$

Similarly,

$$P(y=Bio|x_{\text{new}}) = P(y=Bio) * P(\text{not } x_{\text{new}} | Bio)$$

$$= (1/2) * (2/7 * 1/7)$$

$$= 1/98$$

Now

$$P(y=CS|x_{\text{new}}) = \frac{P(y=CS) * P(x_{\text{new}}|y=CS)}{P(x_{\text{new}})}$$

$$= \frac{4/98}{2/49 + 1/98} = 4/5 = 0.2$$

$$P(y=Bio|x_{\text{new}}) = \frac{P(y=Bio) * P(x_{\text{new}}|y=Bio)}{P(x_{\text{new}})}$$

$$= \frac{1/98}{2/49 + 1/98} = 1/5 = 0.2$$

So, $P(y='CS'|x_{\text{new}}) > P(y='Bio'|x_{\text{new}})$, so we assign the 'CS' to x_{new} (ie new doc) containing the words (Algo, Tree)

→ Numerical data

Study	Sleep	Result
2	10	fail
4	12	fail
3	11	fail
8	8	Pass
6	9	Pass
7	7	Pass

→ In numerical data if Naive Bayes' is applied then it is called Gaussian Naive Bayes'

$$P(y|x_{\text{new}}) = \frac{P(x_{\text{new}}|y) \cdot P(y)}{\sum_j P(x_{\text{new}}|y_j) \cdot P(y_j)}$$

↓
Study = 4
Sleep = 6

Now, $P(\text{Fail}) = P(\text{Pass}) = 0.5$ ($= \frac{3}{6}$)

Mean = ? Variance = ?

For Fail class

$$\begin{aligned} \text{Study (mean)} &= 3 \\ \text{Sleep (mean)} &= 11 \\ \text{Study variance} &= \frac{(2-3)^2 + (4-3)^2 + (3-3)^2}{3} = 2/3 \\ \text{Sleep variance} &= \frac{(10-11)^2 + (12-11)^2 + (11-11)^2}{3} \\ &= 2/3 \end{aligned}$$

For Pass class

$$\begin{aligned} \text{Study mean} &= 7 \\ \text{Sleep mean} &= 8 \\ \text{Study variance} &= \frac{(8-7)^2 + (6-7)^2 + (7-7)^2}{3} \\ &= 2/3 \\ \text{Sleep variance} &= \frac{(8-8)^2 + (9-8)^2 + (7-8)^2}{3} \\ &= 2/3 \end{aligned}$$

Gaussian NB

$$P(x_{\text{new}} | \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left[\frac{-(x_{\text{new}} - \mu)^2}{2\sigma^2}\right]$$

for x_{new}
Study = 4
Sleep = 6

for Fail class

$$P(\text{Study} = 4 | \mu = 3, \sigma^2 = 2/3)$$

$$= \frac{1}{\sqrt{2\pi(2/3)}} \times e^{-\frac{(4-3)^2}{2 \times 2/3}}$$

$$= 0.33 \times 0.275$$

$$= 0.09 \quad (\approx \text{let } 0.08)$$

$$P(\text{Sleep} = 6 | \mu = 3, \sigma^2 = 2/3)$$

$$= \frac{1}{\sqrt{2\pi(2/3)}} \times e^{-\frac{(6-3)^2}{2 \times 2/3}}$$

$$= 0.33 \times 0.001$$

$$\text{let } \approx 0.04$$

for Pass class

$$P(\text{Study} = 4 | \mu = 7, \sigma^2 = 2/3)$$

$$= \frac{1}{\sqrt{2\pi(2/3)}} \times e^{-\frac{(4-7)^2}{2 \times 2/3}}$$

$$(\approx \text{let } 0.01)$$

$$P(\text{Sleep} = 6 | \mu = 7, \sigma^2 = 2/3)$$

$$= \frac{1}{\sqrt{2\pi(2/3)}} \times e^{-\frac{(6-7)^2}{2 \times 2/3}}$$

$$\text{let } \approx 0.01$$

Now,

$$0.03 \times 0.04 \times P(\text{Fail})$$

$$0.03 \times 0.04 \times 0.5$$

$$\Rightarrow \text{Let } (0.7)$$

$$\rightarrow (0.04)(0.01) + P(\text{Pass})$$

$$\rightarrow 0.04 \times 0.01 \approx 0.5$$

$$\Rightarrow \text{let } = (0.8)$$

Their sum for now ($\text{Study} = 4$) \Rightarrow
 $\text{Sleep} = 6$

$$P(\text{Fail}|x_{\text{new}}) < P(\text{Pass}|x_{\text{new}})$$

\Rightarrow Their now will pass.

Decision Tree (Discrimination Tree)

Experience	Education	Hand-on	Salary (Classified)
Low	B.Tech	No	Low
Low	P.H.D.	No	Medium
Medium	P.H.D.	No	Medium
High	M.Tech	No	High
High	M.Tech	Yes	Very High

Possible Output:

Exp → Low, medium, high

Education → B.Tech, M.Tech, P.H.D.

Hand-on → Yes, No

Salary → Low, medium, high, very high

→ Let Total rows = 48

Salary class label

$$\text{Low} = 10, \text{ Medium} = 15, \text{ High} = 15, \text{ Very High} = 8$$

Entropy → Uncertainty with the dataset

n	w	Gender
6	70	M
5	60	M
5.5	40	M

$$\text{Entropy} = \sum (-P_i \log_2(P_i))$$

$$\text{For salary class label} = \left(\frac{-10}{48} \times \log\left(\frac{10}{48}\right) \right) + \left(\frac{-15}{48} \times \log\left(\frac{15}{48}\right) \right) + \left(\frac{-15}{48} \times \log\left(\frac{15}{48}\right) \right) + \left(\frac{8}{48} \times \log\left(\frac{8}{48}\right) \right) \approx 0.48$$

→ Education

B.Tech	M.Tech	P.H.D.	Salary
7	4	5	Low
3	7	1	Medium
8	8	18	High
$\Sigma = 25$			Very High

→ Entropy

$$\begin{aligned} & \text{B.Tech} \quad \text{M.Tech} \quad \text{P.H.D.} \\ & \left(-\frac{7}{15} \times \log \frac{7}{15} \right) + \left(-\frac{4}{15} \times \log \frac{4}{15} \right) + \left(-\frac{5}{18} \times \log \frac{5}{18} \right) + \\ & \left(-\frac{8}{15} \times \log \frac{8}{15} \right) + \left(-\frac{8}{15} \times \log \frac{8}{15} \right) + \left(-\frac{5}{18} \times \log \frac{5}{18} \right) + \\ & \approx 0.02 \quad \approx 0.04 \quad \approx 0.01 \end{aligned}$$

Info. Gain [Education]

$$\text{Gain}(S, A) = \text{Entropy}(S) - \sum_{\text{values}(A)} \frac{1}{|S|} \times \text{Entropy}(S_i)$$

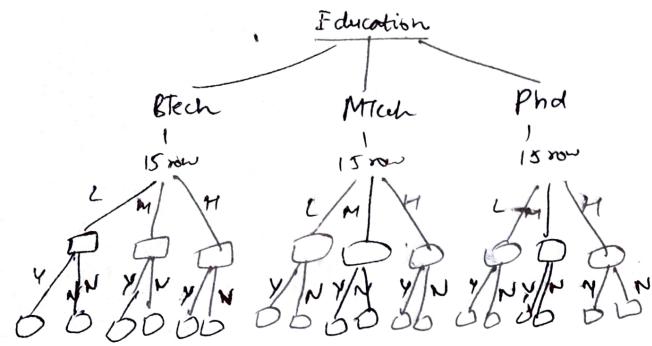
$$= 1.8 - \left[\frac{15}{48} \times 0.02 + \frac{15}{48} \times 0.04 + \frac{18}{48} \times 0.01 \right]$$

$$= 1.8 - \left(\frac{0.3}{48} + \frac{0.6}{48} + \frac{0.18}{48} \right) \approx 0.51$$

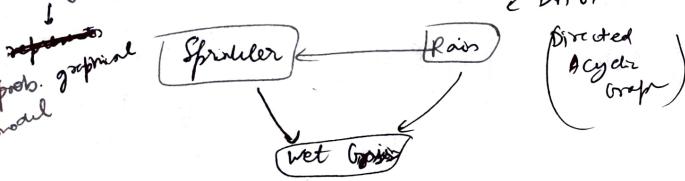
$$\text{Similarly, I.Gain (Exp)} \approx 0.3$$

$$\text{I.Gain (Hand-on)} \approx 0.2$$

→ Better I.Gain → Better distinguishable factor.



Bayesian Network :-



Given, $P(R) = 0.6$, so $P(\sim R) = 0.4$
 $P(S|R)$

Prob. of sprinkler depending on Rain,

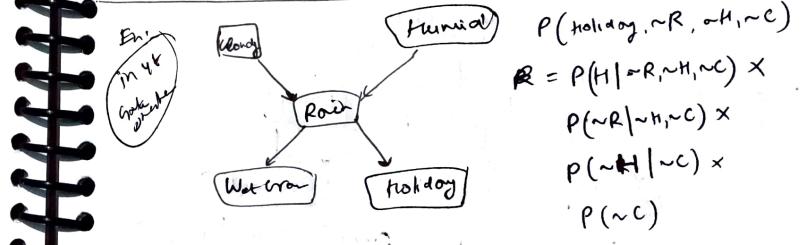
Rain	Sprinkler	No Sprinkler
R	0.6	0.4
$\sim R$	0.3	0.7

Prob. of wet grass depending upon Sprinkler & Rain,

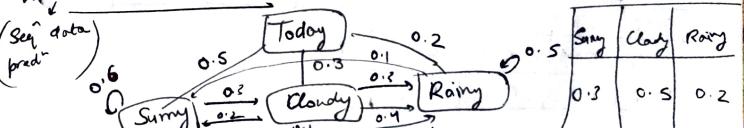
Rain	Sprinkler	wet grass	No wet grass
R	$y(s)$	0.99	0.01
$\sim R$	$N(s)$	0.7	0.3
R	y	0.6	0.4
$\sim R$	N	0.2	0.8

Q1) $P(\text{Wet Grass, Sprinkler, No Rain})$
 $= P(W, S, \sim R) = P(W \cap S \cap \sim R)$
 $= P(W | S, \sim R) \times P(S, \sim R) \times P(\sim R)$
 $= 0.6 \times 0.3 \times 0.4$
 $= 0.072$

Q1) $P(\sim W \cap S \cap \sim R) = P(\sim W | S, \sim R) \times P(S | \sim R) \times P(\sim R)$
 $= 0.4 \times 0.3 \times 0.4$
 $= 0.048$



Markov Model :- (only depends on immediate past)



Given → Next Day Table (Transition Table)

	Sunny	Cloudy	Rainy
Sunny	0.6	0.3	0.1
Cloudy	0.2	0.5	0.3
Rainy	0.1	0.4	0.5

Ex:- (i) Today 1st) $\xrightarrow{\text{Rainy}} \text{Sunny}$ $\xrightarrow{\text{P}(S|R)}$ $\xrightarrow{\text{Cloudy}}$ $\xrightarrow{\text{P}(C|S)}$ $\xrightarrow{\text{Rainy}}$ $\xrightarrow{\text{P}(R|C)}$

$P(R) = 0.2$ $P(S|R) = 0.1$ $P(C|S) = 0.3$ $P(R|C) = 0.3$

$= 0.0012$

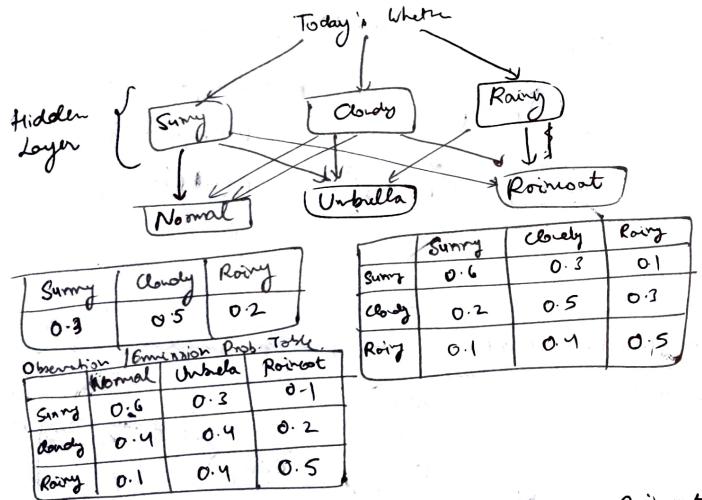
(ii) $S \rightarrow C \rightarrow R \rightarrow S$

$$\Rightarrow P(S) \times P(C|S) \times P(R|C) \times P(S|R)$$

$$\Rightarrow 0.3 \times 0.3 \times 0.3 \times 0.1$$

$$\Rightarrow 0.0027$$

Hidden Markov Model :-



Ex:- Raincoat $\xrightarrow{C|R/S} \text{Normal} \xrightarrow{C|R/S} \text{Umbrella} \xrightarrow{C|R/S} \text{Raincoat}$

$3 \times 3 \times 3 \times 3 = 81$ cases

Now out of 81 cases, we have given the same day's weather

Day \rightarrow Rainy \downarrow $\xrightarrow{\text{Sunny}}$ \downarrow $\xrightarrow{\text{Cloudy}}$ \downarrow $\xrightarrow{\text{Rainy}}$

$\xrightarrow{\text{Rainy}}$ $\xrightarrow{\text{Normal}}$ $\xrightarrow{\text{Umbrella}}$ $\xrightarrow{\text{Raincoat}}$

$P(R|R) = 0.6 \times P(N|S) \times P(U|C) \times P(R|R)$

$= 0.5$

$\times \frac{\text{Today Rainy}}{0.2} \times \frac{\text{Next sunny day}}{0.1} \times \frac{\text{Next cloudy day}}{0.3} \times \frac{\text{Next rainy day}}{0.3}$

$\Rightarrow 0.5 \times 0.6 \times 0.4 \times 0.5 \times 0.2 \times 0.1 \times 0.3 \times 0.3$

$\Rightarrow 0.000072$

Prolog

→ logical programming

e.g.: cakes are delicious
chocolates are delicious
pickles are delicious

→ Priya likes food if food is delicious

Prolog:- delicious(cakes).
" (chocolates).
" (pickles). } facts (end with full stop)

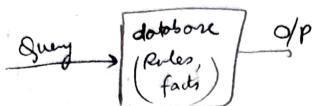
likes(Priya, food) :- delicious(food),
} from statement } If statement

Variable - with capital letter (starts with Y)
constants → small lowercase

In above example,
→ like, priya, cakes, chocolates, pickles } constants - small
→ food } variable, can be cake, pickle
→ cake, chocolate, pickles } constants - small
+ capital letter

ex:- cakes are sweet } adjective noun → Sweet(cakes)
chocolates are sweet } adjective noun → Sweet(chocolate)

ex:- Prokash likes food if food is delicious and tastes sweet.
likes(Prokash, food) :- delicious(food), sweet(food).
if and used
separates by comma(,)



ex:- if (OR used in if cond)
↳ separate by semicolon (;)
likes(prokash, food); delicious(food); sweet(food).

for Queries :-

ex:- Priya likes which food?

Query => ? likes(Priya, food) } gives only first matched result

O/P => X = cakes

for all output : press ;

Ex:- give all person with like food?

⇒ ? likes(Y, X).

Y = priya

X = cakes

→ & press;

Ex:- car(maruti, white). } if we want only the
car(maruti, black). } colour irrespective of the
car(suzuki, grey). } company for query.
car(" ", yellow). } ? car(-, color).
O/P :- color = white
& o/p press;

9/10/28

miss backtracking

Prologue (extension .pl)

; → and
; → orTest: ML &
Prologue

the likes (Protosh, X) :- sweet (X), delicious (X).] rule
 it in outer sweet (chocolate). "If this is true [fact
 甜 in outer sweet (cakes).]
 delicious (cakes).

(i) query ?- likes (Protosh, Food)
 Food = cakes; { ∵ it satisfies both cond }
 NO
 sweet & delicious

Now, to check if there are more available answer to the query then we add ';' after the first output (like Food = cakes;), and if there is no answer possible then it says NO.

(ii) ?- likes(protosh, cakes) ← If this is true then answer will be yes otherwise no.
 Yes

Prologue :- likes (Protosh, X) :- sweet (X), delicious (X), wants (X),
 sweet (chocolate)
 sweet (cakes)
 delicious (cakes)
 delicious (chocolate)
 sweet (party)
 delicious (party)

query:- ?- likes (protosh, food)
 output → Food = chocolates
 Food = cakes
 Food = party
 NO.

→ To get output without using semicolon again & again.
 → fail (x) means that the work you have done till now is null / failed,
 so it produces it again.

program to find max/min of two no.

max (X, Y, X) :- X ≥ Y

query: ?- max (10, 8, X)
 X = 10

[← clearly if (10 > 8)]

? - max (8, 10, X)

[← clearly if (8 > 10)]

no

So, for this case we will add another rule

max (y, y) :- y >= y

therefore final rules,

max (x, y, z) :- x >= y

max (x, y, z) :- y >= z

program to find factorial.

fact (0, 1).

fact (N, result) :- N > 0, NI = N - 1,

fact (NI, RI)

Result = N * RI

working

query:- fact (3, X)

N = 3

NI = 2

fact (2, R)

Result = 3 * R

fact (2, R1)

N = 2

NI = 1

fact (1, R2)

R2 = 1 * R1

fact (1, R2)

R1 = 2 * R2

fact (1, R2)
 N = 1
 NI = 0
 fact (0, R3 = 1)
 R2 = 1 * R3
 R2 = 1 * 1
 R1 = 2 * R2
 R1 = 2 * 1
 R1 = 2

List = $[1, 2, 3, 4] = \underbrace{[1]}_{\substack{\text{Head} \\ (\text{First element})}} / \underbrace{[2, 3, 4]}_{(\text{Remaining elements})} = [H | T]$

Program to find sum of elements of the list.

sum ($[]$, 0)

sum ($[H | T]$, Result) :- sum ($[T]$, RI),
Result = H + RI.

query
sum $[1, 2, 3], X$

X = 6

Given any list if any ^{given} no. belongs to the list or not.

member ($[x] -$, x)

member ($- | T$, x) :- member (T , x)

member ($[1, 2, 3]$, 3)

yes

- → represents
anonymous variable

Determine the length of list

Append / Concatenation of two lists

Reversing a list

finding last element of a list

14/10/25

CUTS (!) It prevents backtracking

lines (prokash, food) :- sweet (food), delicious (food),
 !, healthy (food).

sweet (cakes)
 " (chocolate)
 delicious (cakes)
 " (chocolate)
 healthy (chocolate)

no backtracking for other variable

? lines (Prokash, X),
 no { because there is no. healthy food (cake)
 It will not do backtrace but if one
 variable satisfies all it's condn then it
 will search for new variable after pressing ;,
 ff (then we can press semi colon)

Predicate Logic

- ex:-
- ① marcus was a man → man (marcus)
 - ② " " christian → christian (marcus)
 - ③ all christian are roman → $\forall n : \text{christian}(X) \rightarrow \text{roman}(X)$
 - ④ all romans were either loyal to caesar or hated him → $\forall n : \text{roman}(X) \rightarrow \text{loyal}(n, \text{caesar}) \vee \text{hate}(n, \text{caesar})$
 - ⑤ everyone is loyal to someone → $\forall x, \exists y : \text{loyal}(x, y)$
 - ⑥ everyone is loyal to someone → $\forall x, \exists y : \text{loyal}(x, y)$
 - people only try to assassinate ruler they
 are not loyal to → $\forall x, \forall y : \text{person}(x) \wedge \text{ruler}(y) \wedge \neg \text{loyal}(x, y) \rightarrow \text{try_assassinate}(x, y) \rightarrow \neg \text{loyal}(x, y)$
- Marcus try to assassinate Caesar
- (8) → try to assassinate (marcus, caesar)

(9) $\forall n : \text{man}(n) \rightarrow \text{person}(n)$

Prove that

Marcus is not loyal to Caesar

using 7th
 $\text{person}(m) \wedge \text{ruler}(c) \wedge \neg \text{try_assassinate}(M, C)$

↓⑧

$\text{person}(m) \wedge \text{ruler}(c)$

↓⑨

$\text{person}(m)$

↓⑩

$\text{man}(m)$

↓⑪

true

Using resolutions

(i) $P \vee Q \rightarrow P \vee Q$

(ii) $P \rightarrow R \rightarrow \neg P \vee R$

(iii) $Q \rightarrow R \rightarrow \neg Q \vee R$

↓⑫

⑫ CNF :- $(A \vee B) \wedge (C \vee D) \wedge (E \vee F) \Rightarrow$ prove is True
How to solve :- Starts with negation & assume it to be true &
 reach the contradiction point

Say :- Assume $\neg R$ is True

using ① & ②

(iv) $\neg Q \vee R \leftarrow$ (because at least one of Q or R is True)

using ④ & ⑤,

(vi) $\neg P \rightarrow Q$

using ② & ③,

(vii) $R \rightarrow$

It contradicts the fact that $\neg R$ is True,
 ∴ our assumption is wrong, hence R is True

6/10/28

Solve using resolution. Prove R.

$$1) \sim(P \rightarrow Q) \rightarrow Q,$$

$$2) (P \rightarrow P) \rightarrow R$$

$$3) (R \rightarrow S) \rightarrow \sim(S \rightarrow Q)$$

$$\text{eq: } 1) \sim(P \rightarrow Q) \vee Q$$

$$\rightarrow (\sim P \vee Q) \vee Q$$

$$= (P \wedge Q) \vee Q$$

$$= (P \vee Q) \wedge (Q \vee Q)$$

$$= P \vee Q - (i)$$

$$(2) \sim(r \sim P \vee P) \vee R$$

$$(P \wedge \sim P) \vee R = (P \vee R) \wedge \frac{(\sim P \vee R)}{(iii)}$$

$$(3) (R \rightarrow S) \rightarrow \sim(S \rightarrow Q)$$

$$\Rightarrow \sim(\sim R \vee S) \vee \sim(\sim S \vee Q)$$

$$\Rightarrow (R \wedge \sim S) \vee (S \wedge \sim Q)$$

$$= \frac{(R \vee S)}{(iv)} \wedge \frac{(R \wedge \sim Q)}{(v)} \wedge \frac{\sim(S \vee \sim Q)}{(vi)}$$

$$\text{Let } \sim R \quad (7)$$

$$\text{using } S \wedge (7) \rightarrow \sim Q$$

$$\text{using } I \rightarrow P$$

$$\text{using } 3 \rightarrow R \text{ is True}$$

(contradiction)

$$\therefore \sim R \text{ is wrong assumption}$$

$$\therefore R \text{ is true}$$

gt → greater than

lt → less than

gt(10,1) → 10 is greater than 1

lt(1,10) → 1 is smaller than 10.

born(marcus, 40)

all number mortal → man(x) → mortal

all christians died due to volcanic
eruption,

$\forall x \Rightarrow \text{christian}(x) \wedge \text{eruption}(79)$

→ died(x, 79)

No, mortal lives

$\forall x \text{ mortal}(x) \wedge \text{born}(x, t_1) \wedge \text{gt}$
 $(t_2 - t_1, 150)$

→ died(x, t_2)

✓ alive means not dead

alive(x) → → dead(x)

✓ If someone dies then, he is
dead all future times.

To prove, died(x, t_1) ∧ gt(t_2, t_1)

→ died(x, t_2)

To prove,

Marcus is now dead,

x now (1991)

→ died(marcus, 1991)

→ alive false.

29/10/25

CSPConstraint Satisfaction Prob.

Def. by

 $\{X\}$ = set of variables $\{D\}$ = Domain, $\{C\}$ = constraint $\{X\} = \{x_1, x_2, \dots, x_n\}$

$$\{D\} = \{1, 2, 3, 2\}$$

y

$$\begin{matrix} \text{Constant} \\ \text{on } x_1, x_2, x_3 \end{matrix} \downarrow \quad \begin{matrix} \text{rows or} \\ \text{cols} \end{matrix} \quad \begin{matrix} \text{which} \\ \text{are allowed} \end{matrix}$$

$$C_{12} = \{(1, 3), (1, 4), (2, 4), (3, 1), (4, 1), (4, 2)\}$$

$$C_{13} = \{ \quad \}$$

$$C_{14} = \{ \quad \}$$

\rightarrow Binary constraints \rightarrow constraints on two variables

How to solve

Let $a = \{x_1 = 1, x_2 = 2\}$ \leftarrow now check in C_{12}
 assignment if $(1, 2)$ is allowed
 \therefore it's not allowed.

$$\therefore a = \{x_1 = 1, x_2 = 3\}$$

$$\text{Let } \rightarrow x_3 = 2$$

Now check in C_{13} & C_{23} if $(1, 2) \& (3, 2)$ are allowed

\therefore it's not allowed,

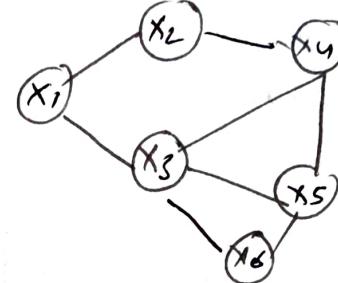
$$\therefore a = \{x_1 = 1, x_2 = 3, x_3 = 3, 1, 4 \rightarrow \text{all are not allowed}$$

so, we backtrack to x_1, x_2 & change their values.
 $\rightarrow a = \{x_1 = 1, x_2 = 4\}$

repeat steps to get the answer.

Graph colouring problem from CSPVariables = Nodes = $\{x_1, x_2, \dots, x_6\}$ Domains = $\{R, G, B\}$ Constraints = $\{x_1, x_2\} = \{(R, G), (R, B), (G, R), (G, B), (B, R), (B, G)\}$

that are allowed.

# Criptarithmetic problem

$$\begin{array}{r} \text{Message} \xrightarrow{x_1, x_2, x_3, x_4} \text{EAT} \\ + \text{THAT} \\ \hline \text{APPLE} \end{array}$$

$$\begin{array}{r} A = 1 \quad x_1 = 1 \\ P = 0 \quad x_2 = 0 \\ T = 9 \quad x_3 = 1 \\ E = 8 \quad x_4 = 2 \\ L = 3. \end{array}$$

$$\{X\} = \{A, E, H, L, P, T, x_1, x_2, x_3, x_4\}$$

$$\{D\} = \{0, 1, 2, \dots, 9\}$$

$$\{C\} = [T + T = E + 10 \cdot x_1,$$

$$A + A = L + 10 \cdot x_2,$$

$$+ x_1$$

$$E + H = P + 10 \cdot x_3,$$

$$+ x_2$$

$$T + K_3 = P + 10 \cdot x_4.$$

$$A = x_4.$$

$$\underline{\text{Ex: SEND}}$$

$$+ \text{MORE}$$

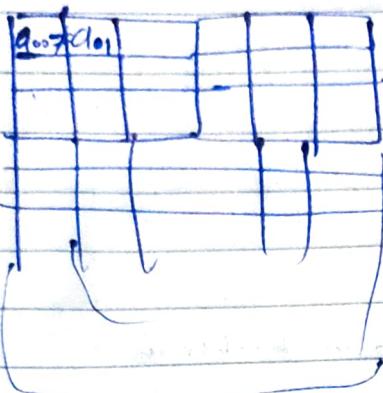
$$\hline \text{MONEY}$$

4th Nov., 2025

Page No.

AI

Sudoku as CSP



Domain = 1 to 9

Variable = cells.

Constraints = that in a ~~cell~~ column, row or a box no two cells can have same value.

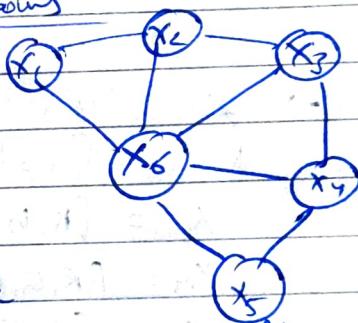
Three approaches to improve backtracking

1. forward checking / constraint propagation;

↳ checks for constraints in its neighbours

e.g. Graph colouring

(i) $x_1 = \text{[Red]}$ & let one colour



(ii) Now, check the constraints where

x_1 is present, and remove the colour you selected for x_1 in its neighbours.

$$\text{eg: } \{x_1, x_2\} = \underbrace{\{\text{RG}, \text{RB}, \text{GR}, \text{GB}, \text{BR}, \text{BG}\}}$$

only possible
so left (since $x_1 = \text{[Red]}$) $\Rightarrow x_2 \rightarrow \text{G or B}$

(iii) Now, $\therefore x_2 = \text{[G, B]}$

(iv) Now, check for x_3 ,

$$\{x_1, x_3\} = \{\text{RR}, \text{BB}, \text{GL}, \text{ }$$

→ all allowed since x_1 & x_2 are not counted.

Limitation :- If

↓
It delays in
identifying failure.

$$x_1 = \{Green\}$$

$$x_2 = \{Red\}$$

$$x_3 = \{Blue\}$$

$$x_4 = \{R, B\}$$

$$x_5 = \{B\}$$

$$x_6 = \{G\}$$

$$x_7 = \{R, G, B\}$$

Date: / /
Page No.

→ It fails when any variable is left with no domain, so we backtrack.

e.g:- $x_1 = \{Red\}$

$x_2 = \{Blue\}$

$x_3 = \{Green\}$

$x_4 = \{B\}$

$x_5 = \{\} \leftarrow \text{Non in backtrace}$

$x_6 = \{R, G, B\}$

(2)

Arc Consistency :-

[in presence of forward checking]

$$\rightarrow \begin{cases} x_1 = \{Red\} \\ x_2 = \{G, B\} \\ x_3 = \{R, B\} \\ x_4 = \{R, G, B\} \\ x_5 = \{G, B\} \\ x_6 = \{R, G, B\} \end{cases}$$

Arcs are,

$x_2 \rightarrow x_1$

$x_1 \rightarrow x_2$

Def →

If $A \rightarrow B$
 $\forall x \text{ in } A \Rightarrow \exists y \text{ in } B$
 which doesn't violate the constraint.

In arc $x_1 \rightarrow x_2$

Red Blue

since there is atleast one value in x_2 that satisfies the constraint for x_1 , arc is consistent

→ Do that for every arc

arc is consistent
otherwise backtrack.

Now, Let $x_3 = G$

$$x_1 = [R]$$

$$x_2 = [B]$$

$$x_3 = [G]$$

$$x_4 = (R, B)$$

$$x_5 = (B)$$

$$x_6 = (R, G, B)$$

Now when we check
the arc,
 $x_2 \rightarrow x_6$, we see
that the arc is
not consistent

So, we backtrack, and change the constraint
of x_3

Similarly, $x_4 \rightarrow x_6$ is also not consistent.

2nd approach in Arc consistency,

we can try to make the edge consistst,

$$\text{for eg:- } x_4 = (R, B)$$

$$x_5 = (B)$$

So we remove ~~B~~ from x_4 to make
arc $x_4 \rightarrow x_6$ consistst.

③

Ordering:-

Order in variable (forward checks)

3.(a) Variable with least domain values

↳ Assign the variable first which has the
least domain values.

3(b) when there is two or more variable with large
values then take its degree into consideration.

Ordering in domain values

~~$x_3 = [K, G, B]$~~ current
Pick that value from domain such that it removes fewest no. of values from other domains.

If we pick $x_3 = G$ or B , the domains of x_2, x_4, x_6 will be changed.

but if we pick $x_3 = R$, only the domain of x_4 will be changed.

So, $\boxed{x_3 = R} \rightarrow$ is better.