Università degli Studi di Torino

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Laurea Magistrale
INFORMATICA
Corso di
Valutazione delle Prestazioni:
SIMULAZIONE e MODELLI
(Simulation and Modelling)

Master of Science
STOCHASTIC AND DATA SCIENCE
Course in
SIMULATION

Docenti/Instructors
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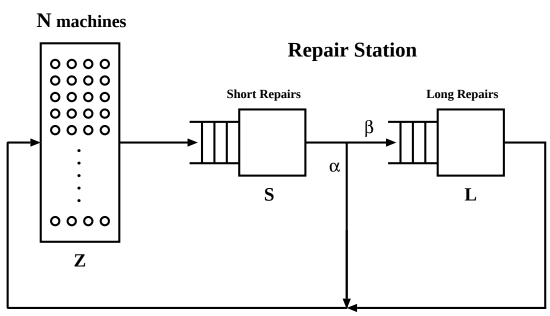
November 18-th, 2024 **Homework Nr. 7** Due: December 2-nd, 2024

Consider the Machine_Repairman system studied in Homework Nr.6 with a variation in the organization of the repair infrastructure as described here and reported in the subsequent picture.

Failures can be of different types. Often the repairs are performed locally and quickly, but in certain cases the repairs need to be performed by an external lab with the possible procurement of spare parts. The occurrence of serious failures is captured in the model by the presence of a probabilistic choice β which directs the failed machines to the external repair lab.



- The number of machines in the system is $N = 10$
- Working periods between failures are random variables (X) with negative exponential distributions with parameter $Z = E[X] = 3000\,min$.
- Short repair times are instances of a random variable Y that has a negative exponential distribution with parameter $S = E[Y] = 40\,min$
- Long repair times are instances of a random variable V that has a negative exponential distribution with parameter $S = E[V] = 960\,min$
- The probability of routing a machine from the Short_Repair Station to the Long_Repair lab is $\beta = 0.2$.

1) (20 points) Compute the interval estimate of the **average waiting times at the long repair station** using the *Regenerative Method*, stopping the simulation when you have $(1 - \alpha) = 95\%$ confidence in your result and when the confidence interval width is globally smaller than 10% ($\mp 5\%$) of the point estimate (center of the confidence interval).

Define a regeneration point that is appropriate for the result that you want to obtain from this simulation and explain the reasons of your choice.

To simplify the implementation of your simulator, make sure that when you decide to stop your simulation, the number of regeneration cycles that you are considering for the construction of the sample used as the basis for the computation of the confidence interval is larger than 40 (so that you can use the value of $z_{\alpha/2} = 1.96$ in the final formula). If the criteria that you use to end your simulation suggests to stop the run earlier, continue your simulation nevertheless until you reach this last condition.
Notice that, since all the random times considered in this model have negative exponential distributions, the choice of the regeneration point is quite simple.
Use different (independent) random number streams for the generation of the instances of the random variables considered in the model (failure time, short repair time, long repair time, and routing probability).
In order to get an initial feeling for the quality of the results provided by your simulator using the regenerative method, consider that a numerical evaluation of the model shows that

- with $\beta = 0.2$, the long-term average waiting time at the long repair station is equal to $1811.030708\,min$.

Hints: To make the implementation of the Regenerative Method straightforward, define the following functions:

- *RegPoint* implements the criteria used to decide whether the occurrence of a certain event corresponds to a regeneration point;
- *CollectRegStatistics* every time a regeneration point is found, accumulates in properly defined variables the measures collected during the regeneration cycle;
- *ResetMeasures* after storing the values collected during the regeneration cycle, resets the values of the "accumulators" to be ready for collecting measures during the next regeneration cycle;
- *ComputeConfidenceIntervals* after simulating for a sufficient number of regeneration cycles, computes the confidence intervals on the basis of the accumulated values;
- *DecideToStop* checks whether the termination criteria are satisfied.

2) (10 points) Choose a regeneration point that allows you to estimate (with the same confidence and precision used for Part 1) the average waiting time at the long repair station of the system, assuming that its repair time is now represented by a random variable with hyper-exponential distribution

$$f_X(x) = \alpha_1 * 1/\mu_1 * exp(-x/\mu 1) + \alpha_2 * 1/\mu_2 * exp(-x/\mu 2)$$

with parameters $\alpha_1 = 0.95$, $\alpha_2 = 0.05$, $\mu_1 = 10\,min$, and $\mu_2 = 19010\,min$.
Notice that if you decide to use the same regeneration point introduced for Part 1, you must explain why it is appropriate also in this case.
To make sure that this new version of your simulator works properly and that the results that it provides are reliable you can **validate** it by introducing a(n extremely) small modification in the definition/coding of the model to make it suitable to be solved with the Mean Value Analysis algorithm implemented in Homework_04.
Explain the modification that you made and comment on the results of the validation experiment.
To make the grading of this homework simpler, explain in details the actions corresponding to the occurrence of all the events envisioned for this simulator. Provide a detailed description of the regeneration point(s) you identified. Finally, comment your code to make it (easily) readable.

GOOD LUCK WITH YOUR WORK!