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18.01 Single Variable Calculus Fall 2006

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18.01 Practice Questions for Exam 3 – Fall 2006

$$a) \quad \int_0^1 \frac{x \, dx}{\sqrt{1+3x^2}}$$

1. Evaluate
$$a$$
) $\int_0^1 \frac{x \, dx}{\sqrt{1+3x^2}}$ b) $\int_{\pi/3}^{\pi/2} \cos^3 x \, \sin 2x \, dx$

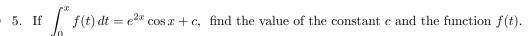
2. Evaluate $\int_{0}^{1} x \, dx$ directly from its definition as the limit of a sum.

Use upper sums (circumscribed rectangles). You can use the formula $\sum_{i=1}^{n} i = \frac{1}{2}n(n+1)$.

3. A bank gives interest at the rate r, compounded continuously, so that an amount A_0 deposited grows after t years to an amount $A(t) = A_0 e^{rt}$.

You make a daily deposit at the constant annual rate k; in other words, over the time period Δt you deposit $k\Delta t$ dollars. Set up a definite integral (give reasoning) which tells how much is in your account at the end of one year. (Do not evaluate the integral.)

- Consider the function defined by $F(x) = \int_0^x \sqrt{3 + \sin t} \, dt$. Without attempting to find an explicit formula for F(x),
 - a) (5) show that $F(1) \leq 2$;
- b) (5) determine whether F(x) is convex ("concave up") or concave ("concave down") on the interval 0 < x < 1; show work or give reasoning;
 - c) (10) give in terms of values of F(x) the value of $\int_{1}^{2} \sqrt{3 + \sin 2t} \, dt$.



- 6. A glass vase has the shape of the solid obtained by rotating about the y-axis the area in the first quadrant lying over the x-interval [0,a] and under the graph of $y=\sqrt{x}$. By slicing it horizontally, determine how much glass it contains.
- 7. A right circular cone has height 5 and base radius 1; it is over-filled with ice cream, in the usual way. Place the cone so its vertex is at the origin, and its axis lies along the positive y-axis, and take the cross-section containing the x-axis. The top of this cross-section is a piece of the parabola $y = 6 - x^2$. (The whole filled ice-cream cone is gotten by rotating this cross-section about the y-axis.)

What is the volume of the ice cream? (Suggestion: use cylindrical shells.)

- 8. Rectangles are inscribed as shown in the quarter-circle of radius a, with the point x being chosen randomly on the interval [0, a]. Find the average value of their area.
- Find the approximate value given for the integral below by the trapezoidal rule and also by Simpson's rule, taking n=2 (i.e., dividing the interval of integration into two equal subintervals):

$$\int_0^{\pi/2} \sin^6 x \, dx$$

Other possible problems: Volumes by vertical slicing 4B, Work problems (P.Set 5)