Chern classes: characteristic classes of complex vector bundles

Charlie Godfrey

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Abstract

Associated to a complex vector bundle $E \xrightarrow{\pi} X$ over a CW complex X are a sequence of cohomology classes $c_i(E) \in H^{2i}(X;\mathbb{Z})$, called the **Chern classes** of E. They tell us things about the vector bundle - for instance, they can be viewed as obstructions to the existence of pointwise linearly independent global sections.

In this talk, I'll (briefly!) sketch a pleasant construction of the Chern classes, based on what's known as the "splitting principle," and write down a short list of axioms that describe them completely. Then we'll have some Yoneda-style fun with universal vector bundles and their universal Chern classes. Time permitting, I'll introduce the **Chern numbers** of a compact complex manifold and try to explain what they have to do with "complex cobordism." Along the way we'll compute the Chern classes of several interesting vector bundles, such as $\mathscr{O}(1)$ on $\mathbb{P}^n_{\mathbb{C}}$.