ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

CONFIDENTIAL

Shingirai Maresi

MARKING SCHEME

NOVEMBER 2007

MATHEMATICS 9164/1

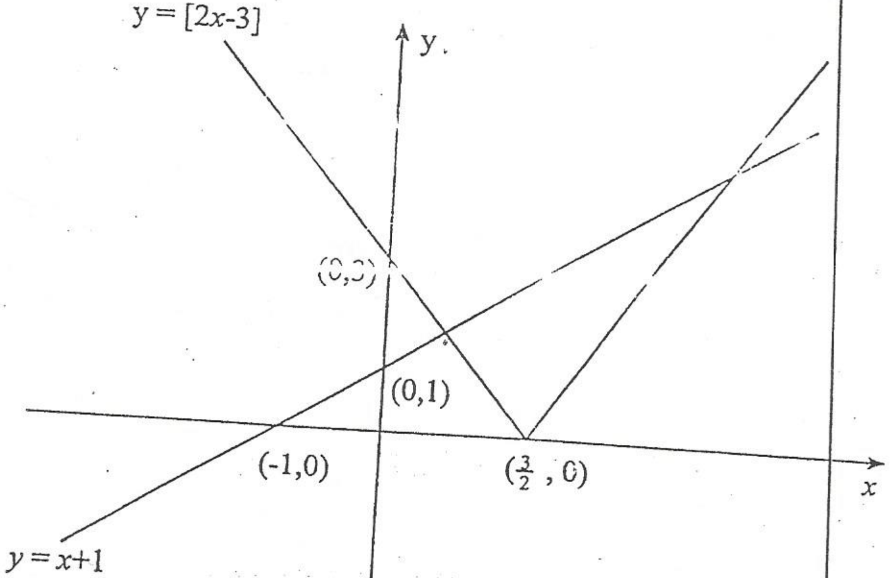
1 Complete correct method
either division or equating coefficients
obtains $a = 2$, $b = 0$ and $c = 5$ M1

	any one correct	A1
	all correct	A1
		[3]

2

$\int \frac{dy}{y} = \int \frac{dx}{dx^2}$	separating valuables and attempting to integrate	M1
$\ln y = -\frac{1}{x} + C$	correct integrals including + C	A1
$C = \ln e^2 + 1 = 3$	Use given values to find C	M1
$\ln y = -\frac{1}{x} + 3$	a.e.f	A1
		[4]

3



	Draws graph with corner at $\left(\frac{3}{2},0\right)$ and symmetric about this point, cutting – axis at (0, 3).		B1
	Draws straight graph passing through (−1, 0) and (0, 1).		B1
	Correct method for identifying both boundaries.		M1
	$\frac{2}{3} < x < 4$ correct solution		A1
			[4]
4	Attempts a correct method for rationalising denominator		M1
	obtains $\frac{6-3ai+4i+2a}{4+a^2}$ $6+2a=-3a+4$ $5a=-2$		A1
	Equates real and imaginary parts $a=-2/5$		M1
	correct equation.		A1
	$a=-\frac{2}{5}$		A1
			[5]
5	(i) $x^2+4x+1=(x+2)^2-3$		
	$(x+2)^2-3\geq-3$	correct method of obtaining range	M1
	$f(x)\geq-3$	correct range	A1
	(ii) let $(x+2)^2-3=y$		
	$x=\sqrt{y+3}-2$	making x subject of formula	M1
		correct expression with only plus	A1
	$f^{-1}: x\mapsto\sqrt{x+3}-2,\quad x\geq-3$	correct inverse	A1
		correct domain	B1
			[6]

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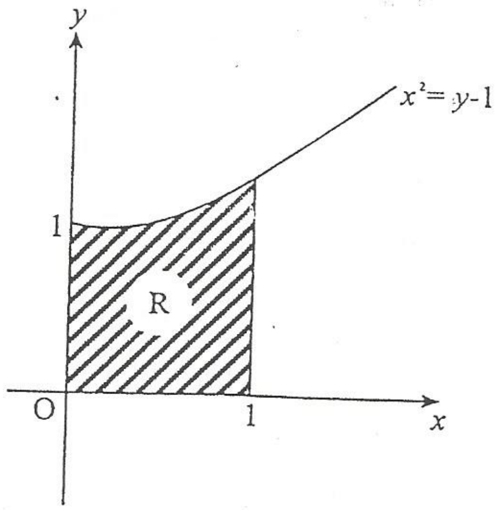
6



$$V_x = \pi\int_0^1 y^2\,dx = \pi\int_0^1 \left(x^2+1\right)^2 dx$$ B1 B1

$$= \pi\int_0^1 \left(x^4 + 2x^2 + 1\right)dx$$

$$= \pi\left[\frac{x^5}{5} + \frac{2x^3}{3} + x\right]_0^1$$ M1A1

$$\pi\left[\frac{1}{5} + \frac{2}{3} + 1\right]$$ M1

$$= \frac{28\pi}{15}$$ A1

[6]

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7 $V = \frac{\pi x^2 l}{4}$	correct formula	B1
$\delta V = \frac{dv}{dx}\delta x$		B1
$\delta V = \frac{\pi x l \delta x}{2}$	attempting to obtain δV in terms of x	M1
	correct expression	A1
$\frac{\delta V}{V} = \frac{\pi x l}{2} / \frac{\pi x^2 l}{4}$	obtaining expression for relative error	M1
$= 2\frac{\delta x}{x}$	correct solution validiy obtained	A1 [6]
8 (a) (i) $t = 0$, $\overrightarrow{OP} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	correct $\overrightarrow{PQ}$	B1
$\overrightarrow{PQ} = \begin{pmatrix} -5 \\ +2 \\ 2 \end{pmatrix}$		
Distance PQ = $\lvert\overrightarrow{PQ}\rvert = \sqrt{(-5)^2 + (+2)^2 + 2^2} = \sqrt{33}$	c.a. use of formula	M1A1
(ii) Mid point $\frac{1}{2}\left[\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}\right] = \begin{pmatrix} \frac{1}{2} \\ 0 \\ 1 \end{pmatrix} = \left(\frac{1}{2}\underline{i} + \underline{k}\right)$		B1
(b) $\begin{pmatrix} 2t+3 \\ t-1 \\ 3t \end{pmatrix} \cdot \begin{pmatrix} t-2 \\ 3t+1 \\ t+2 \end{pmatrix} = 0$	Use of scalar product = 0 solve	M1
$2t^2 - t - 6 + 3t^2 - 2t - 1 + 3t^2 + 6t = 0$		
$8t^2 + 3t - 7 = 0$	correct equation	A1
$t = -\frac{3 \pm \sqrt{9 + 224}}{\cancel{8}\ 16}$		
$= 1.5(33)$ OR – ve rejected only.	c.a.	A1

0.77

9	(i)	Obtains $y^2 - 4y - 1 = 0$ and attempts to solve it		M1
		obtains $2 \pm \sqrt{5}$		A1
		selects $2^x > 0$ and attempts solution		M1
		obtains $x = ln\frac{(2+\sqrt{5})}{ln2}$ only		A1
	(ii)	rationalises the denominator		M1
		obtains correct rationalisation		A1
		obtains given answer validly		A1 [7]
10	(a)	$\sin A = \frac{13\sin 45}{10}$	Use sine rule to find A	M1
		A = 66.815° OR 113.184°	Correct p.v	A1
		B = 180 – (45 + A)	Identify ambiguous A and apply to finding B	M1
		B = 68° OR 22°	each correct value	A1 A1 [5]
	(b)	straight Line AB = 2(5sin 1°)		
		Arc AB = 5 x 2	Use trig and arc formula at least one correct Compound	M1
		Total distance = 10(1 + sin 1°C)		A1
		= 18(4) m		A1 [3]

11) $\frac{6x^2(3x-4)^2 - 2x^3 \times 6(3x-4)}{(3x-4)^4}$

11	$\frac{dy}{dx} = \frac{(3x-4)^2 . 6x^2 - 12x^3(3x-4)}{(3x-4)^4}$	correct method	M1
	$\frac{\cancel{(3x-4)}\{(3x-4)6x^2 - 12x^3\}}{(3x-4)^{\cancel{4}\,3}}$	correct derivative	A1
	$\frac{6x^2[x-4]}{(3x-4)^3} = 0$ $= \frac{6x^2(3x-4-2x)}{(3x-4)^3}$	equating to zero and attempt to solve	M1
	$6x^2(x-4) = 0$		
	$x = 0$ *or* $x = 4$	correct x values	A1
	when $x = 0$ $y = 0$		
	when $x = 4$ $y = 2$	correct y – values	A1
	when $x = 0$, $\frac{d^2y}{dx^2} = 0$	applying nature test to at least one point	M1
	at $x = -1$ $\frac{dy}{dx} = 0.087$ at $x = 1$ $\frac{dy}{dx} = 18$	not a minimum (for identifying a minimum at $x = 4$ and not a minimum at $x = 0$)	A1
	minimum point (4, 2)	correct conclusion	A1 [8]
12	$4\sin^3\theta + 3\cos\theta\sin^2\theta = 2\sin^2\theta$	using relevant identities	M1
	$\sin^2\theta[4\sin\theta + 3\cos\theta - 2] = 0$	obtaining given answer validly	A1
	$\sin^2\theta = 0$ *or* $4\sin\theta + 3\cos\theta - 2 = 0$	either factor equal to zero	M1
	$\sin^2\theta = 0$		
	$\theta = 0°, \ 180°, \ 360°$	for all correct	A1
	$4\sin\theta + 3\cos\theta = R\sin(\theta + \alpha)$		M1
	$R = \sqrt{3^2 + 4^2} = 5$	for using $R\cos(\theta - \alpha)$ or $R\sin(\theta - \alpha)$	
	$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$	for both R and α	A1
	$5\sin\left(\theta + \tan^{-1}\left(\frac{3}{4}\right)\right) - 2 = 0$		

$\theta + \tan^{-1}\left(\frac{3}{4}\right) = 23.578, 156.422$		M1
$= 383.578$	for using principle value	A1
	correct principle value
$\theta = 119.55, 346.708$	
$= 120°, 347°$	both correct	A1 [9]

13 (a) (i) Award for
substitutions $n = 1, 2, 3$ into $n^2 - 7n$ — B1
subtraction of successive terms — M1
Ans $-6, -4, -2$ — A1
For the first correct term for all correct terms — A1

(ii) identifying series as AP and attempting expression for rth term — M1

Obtains $-8 + 2r$ — A1 [6]

(b) Distances are in arithmetic progression first ~~tension~~ term $= 30$
common difference $= 12$ — B1

Total distance $= \frac{10}{2}\left[2 \times 30 + (10-1) \times 12\right]$ applying Sn formula — M1A1

correct numerical substitution

$= 840$ m — A1 [4]

14 $\int \frac{dm}{m} = -\frac{k}{5}\int dt$ — attempting to separate and integrate both sides — M1

$\ln m = -\frac{k}{5}t + C$ — For correct integrals including C — A1

$\ln Mo = 0 + C$
$C = \ln Mo$ — to determining constant of integration M1

$\ln M = -\frac{k}{5}t + \ln Mo$ — correct general solution — A1

$\ln\left(\frac{M}{Mo}\right) = -\frac{k}{5}t$

$$\frac{M}{Mo} = e^{-\frac{k}{5}t}$$ for exponentiation M1

$$M = Moe^{-\frac{k}{5}t} \text{ (AG)}$$ for correct answer validly obtained A1

$$\frac{9}{10}Mo = Moe^{-\frac{k}{5}(1)}$$

$$\ln\frac{9}{10} = -\frac{k}{5}t$$ using 0.9 Mo to find k M1

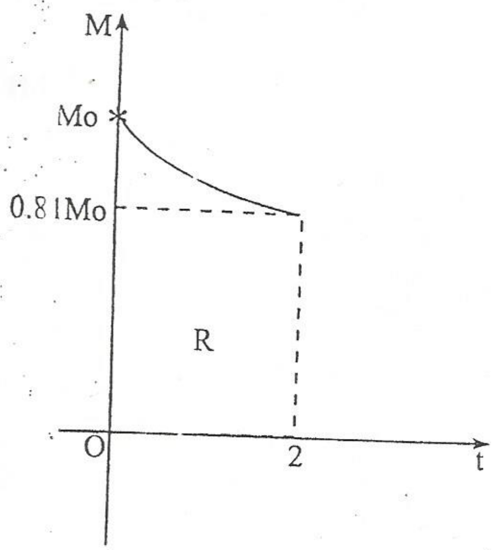
$$k = -5\ln\frac{9}{10} \text{ or } k = 5\ln\frac{10}{9}$$ A1

$$M = Moe^{t\ln\frac{9}{10}}$$

$$M = \left(\frac{9}{10}\right)^t Mo$$

$$M = \left(\frac{9}{10}\right)^2 Mo$$ subst. t = 2 M1

= 0.81 Mo (81% of Mo) C.a. A1



clear indication of end points B1

correct shape B1

[12]

15	(a)		applies a relevant algebraic manipulation obtains given answer validly	M1 A1
	(b)	(i)	obtains $-x-\frac{x^2}{2}-\frac{x^3}{3}-\ldots$	B1
		(ii)	obtains $\ln\left(1-\frac{1}{n^2}\right)$ as $-\frac{1}{n^2}-\frac{1}{2n^4}-\frac{1}{3n^6}-\ldots$	B1
			for $\ln\left(1-\frac{1}{n^2}\right)^{-1}$ Apples correct log law	M1
			obtains $+\frac{1}{n^2}+\frac{1}{2n^4}+\frac{1}{3n^6}+\ldots$	A1
		(iii)	Uses log laws	M1
			obtains $2\ln(n)-\ln(n+1)-\ln(n-1)$	A1
		(vi)	$n=10$ soi	B1
			Equates $2\ln 10-\ln(11)-\ln 9=\frac{1}{10^2}+\frac{1}{2\times 10^4}+\frac{1}{3\times 10^6}+\ldots$	B1
			Makes $\ln 11$ subject of formula	M1
			Correct numerical value for $\ln 11$	
			Ans 2.397895(27)	A1
				[12]

16	(i)	$4a+8b-c=20$	obtain any	M1
		$6a+6b+c=-10$	equation by substitution	
		$4a-c=4$	correct set of equations	A1
		$8b=16$		
		$b=2$		
		$10a=-10$	solving equations using correct method	M1
		$a=-1$		
		$c=4(-1)-4$	for obtaining one unknown	A1
		$=-8$		
		The equation is $x^2+y^2-2x+4b-8=0$	A1 correct equation	
		$(x-1)^2-1+(y+2)^2-4-8=0$	completing square	M1
		$(x-1)^2+(y+2)^2=13$	correct answer obtained validly	A1
	(ii)	$\dfrac{y-(0)}{x-(-2)}=\dfrac{-2-0}{1-(-2)}$	correct centre	B1M1
			correct equation unsimplified	
		$2x+3y+4=0$	correct equation	A1
	(i i)	$\dfrac{dy}{dx}=-\dfrac{(x-1)}{y+2}$	method of finding the gradient of tangent	M1
		at point (3,1) $\dfrac{dy}{dx}=-\dfrac{2}{3}$	correct gradient	A1
		from part (ii) gradient of the diameter is $-\dfrac{2}{3}$ which is equation to the gradient of the tangent hence the tangent is parallel to the diameter.		A1

correct conclusion using $-\dfrac{2}{3}$ from part (iii) and equation in part (ii).

MARKING INSTRUCTIONS

MATHEMATICS 9164

1. Mark in red. Do not cross out or obliterate any work. Errors which determine marks should be indicated by ringing or a cross or underlining, and omission by λ. Each page must have some indication that it has been seen, e.g. a tick in the margin. Correct answers should be ticked. In cases of particular difficulty, comments written on the script may be helpful should the script be reviewed at a later stage. Blank pages should be struck through to indicate that they have been seen.

2. For a partially correct part of a question, exhibit the detailed marks, e.g. M1 A0 *in the margin* at the point where the marks have been first earned. Please give sufficient detail to allow your marking to be understood. For a completely correct **part**, only the total mark for that part need to be given, *in the margin.* Do NOT use subtotals (underlined or otherwise). The question total should be ringed and placed in the margin at the end of the question. *This total MUST equal the sum of all the marks in the margin for that question* and should be entered against the question number in the question grid on the front of the script. (Note: To facilitate administrative checks, *please use the left hand margin of left hand pages).*

   If a candidate's answer is in two instalments, indicate the carried forward total at the end of the first part by, for example, 3 and the brought forward total at the start of the second instalment by, for example, 3.

   The total mark for the paper should be obtained (a) by adding all the unringed marks through the script (checking at the same time that all pages have been marked) and (b) by adding the question marks in the grid in reverse order. The two totals must, of course, tally.

3. Types of marks.

M — Method of marks for a valid method, applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However it is usually not sufficient for a candidate just to indicate an intention of using some method or just to quote the formula, the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula.

A — Accuracy Mark, which cannot be given unless the associated M mark is earned (or implied).

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B Mark for a correct result or statement independent of M marks.

Marks may not be subdivided, unless a special ruling has been made at the meeting. Unless otherwise indicated, marks once gained cannot subsequently be lost, i.e. ignore subsequent working. If in genuine doubt, give candidate benefit of doubt.

Note that dep* B1 means B1 awarded conditional on a previous *B1 having been awarded.

4. Answers with wrong or missing units, but which are otherwise correct, are not penalized.

5 If work is deleted and replaced, mark the replacement. If work is deleted without replacement, mark the deleted work provided that it is legible. For two solutions offered, count what appears the more serious attempts or the more complete attempt at the question. If attempts are indistinguishable in these respects, count the better.

6 The symbol ✓ implies that the A or B may be allowed for work correctly following on from previous incorrect results. Otherwise CAO [Correct Answer (or results) Only] –differences in notation are of course permitted. A and B marks cannot be earned by "correct" answers (or results) obtained from incorrect working. When the mark is awarded from an intermediate result, it will be agreed at the co-ordination meeting exactly what is acceptable. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise. Of course, in practice it may often happen that when a candidate has once gone wrong, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are run together by the candidate, the earlier marks are implied and full credit must be given.

7. For a *genuine* misreading, (of number or symbols) which is such that the object and the difficulty of the question remained unaltered, mark according to the scheme but following the working. (A miscopy of the candidate's own working is not a misread but an accuracy error). All M marks are available. In assessing whether to give A or B marks the candidate's own working must be 'followed through'. Then deduct MR-0, 1, 2 according as the number of 'misread' A and B marks earned is 0, 1-4, >4. If the misreading makes the question easier a further deduction E-? may be made at your discretion – this can deduct from M marks.

8. Remember that the mark scheme is designed for incorrect solutions. Correct solutions get full marks. Be alert for correct but unfamiliar or unexpected methods – often signalled by a correct result following an *apparently* incorrect

method. Such work must be properly assessed. On the other hand, work must not be judged on the answer alone, and AG answers, especially, must be validly obtained. Key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. If a method is not catered for in the scheme, mark at discretion, imitating the scheme as closely as possible. If this involves a number of candidates or you are unsure what to do, consult your Team Leader.

9. If in any case the scheme operates with considerable unfairness, mark at discretion but please give a brief reason and initial the mark. This discretion should only be used very rarely.

10. If there is any suspicion of cheating or copying, mark according to the scheme and enter the marks on the marksheet, and send the script to your Team leader, as per council instructions.

11. Notes on illness should be sent with the marksheets. Scripts should be marked as per the scheme.

12. Accuracy. In most cases the accuracy of required results will be specified in the scheme, in some cases by a range of acceptable answers. Do not penalize overspecified answers.

13. Additional abbreviations.

AG	Answer Given on the paper – watch out for "fiddles"! Answer must be validly obtained.
SR	Special Ruling – agreed scheme for specified incorrect solutions.
AEF	Any Equivalent Form.
SOS	See Other Solutions – candidate starts again later.
BOD	Benefit Of Doubt.
ISW	Ignore Subsequent Working – marks already earned (see paragraph 3 above).
CWO	Correct Working Only – often written by a 'fortuitous' answer.

14. Use of graphical (and programmable) calculators. Allow full marks for correct answers. Little working may be seen, but this should not be penalized. If mistakes are made and there is some evidence of method then allow appropriate method marks. If there is no evidence of methods we can only give zero. If in doubt consult your Team leader.