For Performance Measurement

# ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

**General Certificate of Education Advanced Level**

## MATHEMATICS

**PAPER 1**

**9164/1**

**NOVEMBER 2008 SESSION**

3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae

**TIME** 3 hours

## INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

**This question paper consists of 7 printed pages and 1 blank page.**

1 A complex number z has modulus 8 and argument $\frac{3\pi}{4}$.

State the modulus and argument of $z^2$. [2]

Using these values show the number $z^2$ on an Argand diagram, and hence express $z^2$ in the form $a + bi$. [2]

2 The centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ lies on the line $2x + y = 4$ and the circle passes through the points $(-1; 0)$ and $(-1; 4)$.

(i) Show that the coordinates of the centre of the circle are $(1; 2)$. [3]

(ii) Find the radius of the circle, giving your answer in exact form. [2]

3 Given that $(x-1)$ and $(x-b)$ are factors of the polynomial $f(x) = x^3 + 2ax^2 + bx - 1$, where $a > 0$ and $b > 0$, find the values of $a$ and $b$. [5]

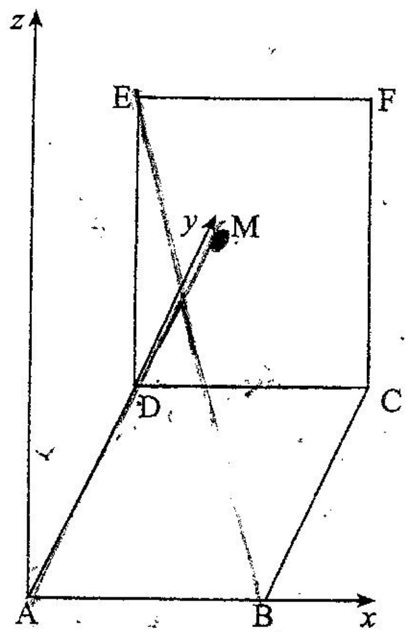
4 (a) On the same axes, sketch the graphs of $y = |x-1| - 2$ and $y = -|x-1|$. [2]

(b) Solve the equation $|x-1| - 2 = -|x-1|$. [2]

Hence solve the inequality

$$|x-1| - 2 > -|x-1|.$$ [2]

5 The sides of a square ABCD are each of length 4 cm. The rectangle CDEF lies in a plane perpendicular to the plane ABCD and DE = CF = 8 cm. M is the centre of the rectangle CDEF. (See diagram).




Taking the point A as the origin and unit vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ in the directions $\overrightarrow{AB}, \overrightarrow{AD}$ and $\overrightarrow{DE}$, calculate the angle between the line AM and the line BE. [6]

6 Given that $y = \dfrac{e^{2x}}{\sin x}$, find an expression for $\dfrac{dy}{dx}$. [2]

Hence find the $x$-coordinate of the turning point in the range $0 < x < \dfrac{\pi}{2}$.
Determine whether this turning point is a maximum or a minimum. [5]

7 For each of the following expressions, obtain a series of simplified terms, in ascending powers of $x$, up to and including the term in $x^3$ stating the set of values for which each expansion is valid.

**(a)** $(1-4x)^{-\frac{1}{4}}$. [4]

**(b)** $ln(5-3x)$. [4]



[Turn over

8 Express $ln\left(\frac{x^3}{\sqrt{1+x^2}}\right)$ in the form

$$a ln x + b ln(1+x^2),$$

where $a$ and $b$ are constants to be found. [1]

Differentiate with respect to $x$:

**(i)** $lny$, [1]

**(ii)** $ln\left(\frac{x^3}{\sqrt{1+x^2}}\right)$. [2]

Using the above results, or otherwise, find $\frac{dy}{dx}$ in terms of $x$, given that $y = \frac{x^3}{\sqrt{1+x^2}}$. [3]

Hence, find values of $x$ for which $\frac{dy}{dx} = 0$. [1]

9 A curve is such that $\frac{dy}{dx} = \frac{3}{4} - kx$, where $k$ is a positive constant. Given that the tangents to the curve at the points where $x = -1$ and 1 are perpendicular, find the value of $k$. [4]

Given that the curve passes through the point (4; 0), find its equation. [4]

10 **(a)** Find the sum of all the integers between 100 and 600 which are exactly divisible by 7. [5]

**(b)** The sum to infinity of a geometrical progression is $\frac{11}{3}$. Given that the first term is 6, find the common ratio.

Hence find $n$, such that $U_n$ is the term of largest magnitude for which $|U_n| < 10^{-5}$. [6]

**11** The parametric equations of a curve are

$$x = 3 - 2\cos\theta,$$
$$y = 2 + 3\sin\theta.$$

**(a)** State the coordinates of the point on the curve for which $x$ takes its maximum and minimum values. [4]

**(b)** Use the identity $\cos^2\theta + \sin^2\theta = 1$ to derive the Cartesian equation of the curve.

Briefly explain why this is not the equation of a circle. [3]

**(c)** Find the equation of the tangent to the curve at the point where

$$\theta = \frac{\pi}{3}.$$ [5]

**12** Sketch on the same axes, the graphs of $y = ln(x+1)$ and $y = \cos x$, where $x$ is in radians. [2]

State the number of roots of the equation $ln(x+1) - \cos x = 0$. [1]

Show that one root of this equation lies between $x = 0.8$ and $x = 0.9$. [3]

Taking 0.9 as the initial approximation to this root, use the Newton-Raphson method once to obtain a second approximation, giving your answer to 6 significant figures. [4]

Show that your answer gives the root correct to 4 significant figures. [2]

**13** (a) Express $\dfrac{1+x}{(2+x)(3x+5)}$ in partial fractions. [2]

**(i)** By putting $x = \sqrt{3}$, or otherwise, obtain an expression for

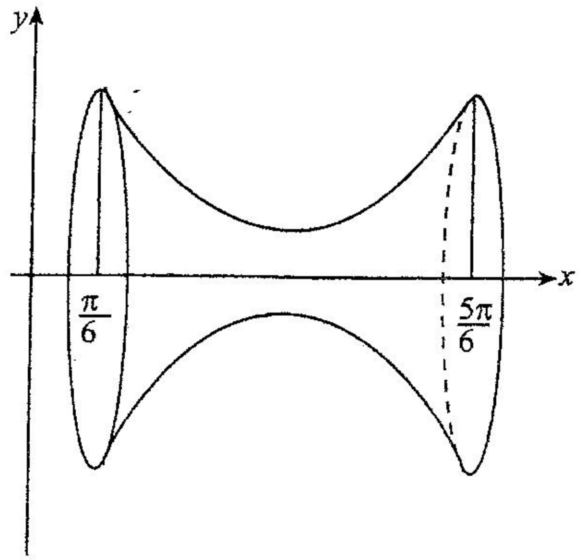
$$\frac{1+\sqrt{3}}{\left(2+\sqrt{3}\right)\left(3\sqrt{3}+5\right)}$$

in the form $a + b\sqrt{3}$, where $a$ and $b$ are integers. [3]

**(ii)** Show that

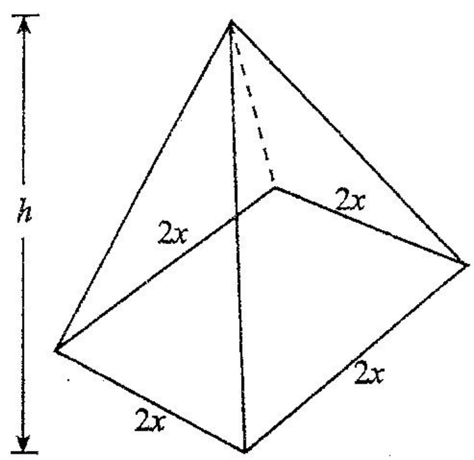
$$\int_{-1}^{1} \frac{1+x}{(2+x)(3x+5)}\,dx = ln 3 - \frac{4}{3} ln 2.$$ [4]

**(b)** The area bounded by the $x$-axis, the curve $y = \operatorname{cosec} x$ and the lines $x = \dfrac{\pi}{6}$ and $x = \dfrac{5\pi}{6}$ is rotated through 4 right angles about the $x$-axis. See diagram.




Find the exact value of the volume of the solid generated. [4]

**14** **(a)**



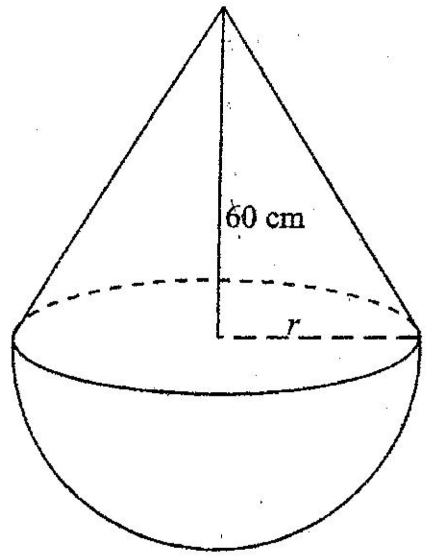

The diagram above shows a pyramid of constant volume with a horizontal square base of edge $2x$ units. Its vertex is $h$ units vertically above the centre of the base. Show that the area A, of a sloping triangular face is given by

$$A^2 = \frac{9V^2}{16x^2} + x^4.$$ [7]

Find an expression of $x$ in terms of $V$ for which A is least, leaving your answer in exact form.

(Volume of a pyramid $= \dfrac{1}{3}$ base area $\times$ height). [4]

**(b)** A solid consists of a hemisphere of radius $r$ joined to a cone of constant height 60 cm. The base of the cone is the plane face of the hemisphere. (See diagram).




If $r$ increases at a rate of 2 cm per minute, calculate the rate of increase of the volume of the solid when $r = 60$ cm. Leave your answer in terms of $\pi$. [4]

$$\left[\text{Volume of sphere} = \frac{4}{3}r^3; \text{Volume of cone} = \frac{1}{3}\pi^2 h\right]$$