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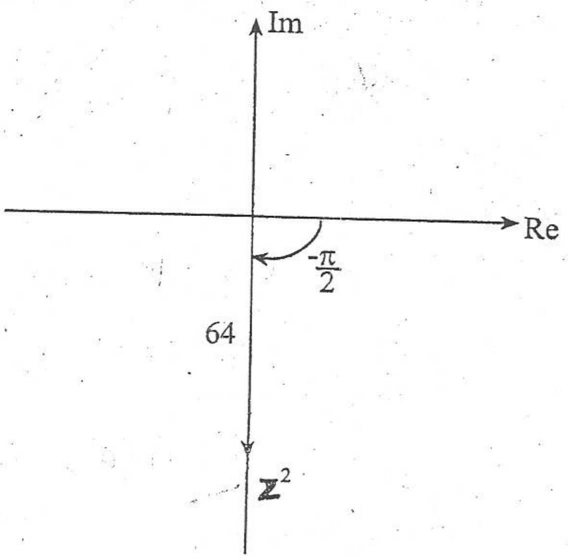
N 2008 9164/1

1 $|z^2| = 8^2 = 64$ B1

$\arg(z^2) = 2 \times \frac{3\pi}{4} = \frac{3\pi}{2}$

$m = -\frac{\pi}{2}$ (p.v.) B1



$z^2 = 0 - 64i$ correct answer (condone omission of 0) B1 [4]

2 (i) $y = 2$ Equation of perpendicular bisector B1

$y = 2$ and $2x + y = 4$, solving simultaneously M1

$x = 1$, substitutes in given equation correctly A1

(ii) $r = \sqrt{(1+1)^2 + (-2\times 1)^2}$ or equiv. correct method M1

$r = 2\sqrt{2}\ (\sqrt{8})$ obtain correct answer A1

7 (a) $1+\left(-\frac{1}{4}\right)(-4x)+\frac{\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)}{2.1}(-4x)^2+\frac{\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)\left(-\frac{9}{4}\right)}{3.2.1}(-4x)^3$

$= 1+x+\frac{5x^2}{2}+\frac{15}{2}x^3$ Use Binomial theorem/first three terms correct M1 A1

correct simplified answer A1

Valid for $-\frac{1}{4}<x<\frac{1}{4}$ B1

(b) $\ln 5+\ln\left(1-\frac{3x}{5}\right)$ Use log laws to obtain expandable form and apply standard series M1

$\ln 5-\left(\frac{3x}{5}\right)-\frac{\left(\frac{3x}{5}\right)^2}{2}-\frac{\left(\frac{3x}{5}\right)^3}{3}$ correct expansion of ln A1

$\ln 5-\frac{3x}{5}-\frac{9}{50}x^2-\frac{9x^3}{125}$ c.a. A1

Valid for $-\frac{5}{3}\le x<\frac{5}{3}$ B1 [4]

8 $\ln\left(\frac{x^3}{\sqrt{1+x^2}}\right)=3\ln x-\frac{1}{2}\ln\left(1+x^2\right)$ B1

(i) $\frac{d}{dx}(\ln y)=\frac{1}{y}\frac{dy}{dx}$ B1

(ii) $\frac{d}{dx}\ln\left(\frac{x^3}{\sqrt{1+x^2}}\right)=\frac{3}{x}-\frac{x}{1+x^2}$ Use chain rule M1

a.e.f. A1

$\frac{1}{y}\frac{dy}{dx}=\frac{3}{x}-\frac{x}{1+x^2}$ equating (i) + (ii) and making $\frac{dy}{dx}$ subject M1

$\frac{dy}{dx}=y\left(\frac{3}{x}-\frac{x}{1+x^2}\right)$ correct $\frac{dy}{dx}$ including y A1

$= \dfrac{x^2(3+2x^2)}{x\sqrt{(1+x^2)^3}}$ a.e.f. in x only A1

$\dfrac{dy}{dx} = 0$ for $x = 0$ (twice) A1

9 For $x = -1$ $\dfrac{dy}{dx} = \dfrac{3}{4} + k$

For $x = 1$ $\dfrac{dy}{dx} = \dfrac{3}{4} - k$ for both correct B1

$\left(\dfrac{3}{4} + k\right)\left(\dfrac{3}{4} - k\right) = -1$ M1

$\dfrac{9}{16} - k^2 + 1 = 0$ A1

$k^2 = \dfrac{25}{16}$

$k = \dfrac{5}{4}$ c.a.o A1

$\int_0^y dy = \int_4^x \left(\dfrac{3}{4} - \dfrac{5}{4}x\right)dx$

$y\Big]_0^y = \dfrac{3}{4}x - \dfrac{5}{8}x^2\Big]_4^x$ Attempting to integrate
and getting correct integrals M1A1

$y = \dfrac{3}{4}x - \dfrac{5}{8}x^2 - 3 + 10$ Use of limits to find constant M1

$y = \dfrac{3}{4}x - \dfrac{5}{8}x^2 + 7$ A1 [8]

10	(a)	$a = 105$	correct u_1	B1
		$Un = 105 + (n-1)7 < 600$		
		$n < 71.7$	Use term formula to find no of terms	M1
		71 terms	correct no of terms	A1
		$S_{71} = \frac{71}{2}[2 \times 105 + (71-1)7]$	use Sn formula	M1
		$= 24\,850$	c.a.	A1
	(b)	$\frac{11}{3} = \frac{6}{1-r}$	use S_∞ formula M1	
		$r = -\frac{7}{11}$	c.a.	A1
		$6\left\|\left(-\frac{7}{11}\right)\right\|^{n-1} < 10^{-5}$	set up relevant inequality using GP term	M1
		$n-1 > \frac{\ln\left(\frac{10^{-5}}{6}\right)}{\ln\left(\frac{7}{11}\right)} = 29.4$	Use logs to solve	M1
		$n > 30.4$	correct solution	A1
		$n = 31$	c.a.o.	A1
11	(a)	$X\max = 5(\text{when } \theta = \pi)$		B1
		$y = 2$ (5; 2)		B1
		$x_{\min} = 1\,(\text{when } \theta = 0)$		B1
		$y = 2$ (1; 2)		B1

(b). $\cos\theta = \dfrac{3-x}{2}$ substitute for $\cos\theta$ & $\sin\theta$ M1

$\sin\theta = \dfrac{y-2}{3}$

$\left(\dfrac{3-x}{2}\right)^2 + \left(\dfrac{y-2}{3}\right)^2 = 1$ a.e.f A1

Coefficients of x^2 and y^2 are NOT equal and so equation is NOT of circle A1

(c) $\dfrac{dy}{dx} = 3\cos\theta \; \dfrac{dx}{d\theta} = 2\sin\theta$ using chain rule M1

$\dfrac{dy}{dx} = \dfrac{3\cos\theta}{2\sin\theta}$ a.e.f A1

where $\theta = \dfrac{\pi}{3}$ $x = 2$

$y = 2 + \dfrac{3\sqrt{3}}{2}$ use of values for $\theta = \dfrac{\pi}{3}$ in tanget equation M1

$\dfrac{dy}{dx} = \dfrac{3}{2}.\dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{2}$ all correct values A1

$y - \left(2 + 3\dfrac{\sqrt{3}}{2}\right) = \dfrac{\sqrt{3}}{2}(x-2)$

$2y = \sqrt{3}x + 4 + \sqrt{3}$ a.e.f A1

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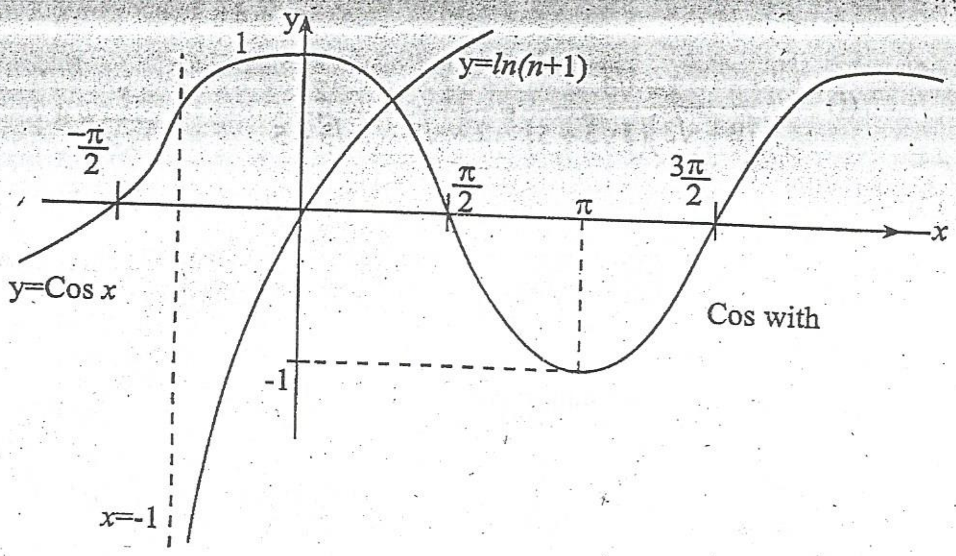

Only one intersection so ONLY ONE ROOT B1

$f(x) = \ln(x+1) - \cos x$

$f(0.8) = -0.108$ Use interval test M1

$f(0.9) = 0.02$ Correct numerical values A1

Change of sign $\Rightarrow$ root between 0.8, 0.9 valid conclusion A1

$f'(x) = \dfrac{1}{x+1} + \sin x$ correct $f'(x)$ B1

$x_2 = 0.9 - \dfrac{f(0.9)}{f'(0.9)}$ correct NR structure M1

$=$ 0.884542 correct numerical expression c.a.o A1

$x_2 = 0.8845(4SF)$

f(0.88445) = −0.000079

f(0.88455) = +0.00005

change of sign interval test or further interaction

$\Rightarrow$ root lies between o.88455 and 0.88455 M1

which is 0.8845 to 4SF as given by x_2 valid conclusion A1

13 (a) $\dfrac{1+x}{(2+x)(3x+5)} = \dfrac{1}{2+x} - \dfrac{2}{3x+5}$ — correct method — M1

c.a. — A1

-80
-64
-144 -18
8

(i) $\dfrac{1+\sqrt{3}}{(2+\sqrt{3})(3\sqrt{3}+5)} = \dfrac{1}{2+\sqrt{3}} - \dfrac{2}{3\sqrt{3}+5}$

$= \dfrac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} - \dfrac{2(3\sqrt{3}-5)}{(3\sqrt{3}+5)(3\sqrt{3}-5)}$ — Rationalise — M1

$= \dfrac{2-\sqrt{3}}{1} - 2\dfrac{(3\sqrt{3}-5)}{2}$ — one correct term — A1

$= 7-4\sqrt{3}$ — c.a.o — A1

(ii) $\int_{-1}^{1}\left(\dfrac{1}{2+x} - \dfrac{2}{3x+5}\right)dx$

$= \left[\ln(2+x) - \dfrac{2}{3}\ln(3x+5)\right]_{-1}^{1}$ — attempting integral — M1

correct integral — A1

$= \ln 3 - \dfrac{2}{3}\ln 8 - \left(\ln 1 - \dfrac{2}{3}\ln 2\right)$ — correct use of $\int$ limits — M1

$= \ln 3 - \dfrac{4}{3}\ln 2$ — correctly obtained — A1

(b) $V = \pi\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \operatorname{cosec}^2 x\,dx$ — use correct form of $\int$ with limits — M1

$= \pi[-\cot x]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$ — correct integral — A1

$= \pi\left[-\left(\dfrac{-1}{\sqrt{3}}\right) - \left(-\dfrac{1}{\sqrt{3}}\right)\right]$ — correct use of limits — M1

$= \dfrac{2\pi}{\sqrt{3}}$ — single term c.a. — A1

14 (a) $v = \frac{1}{3} \times 4x^2 \times h$

$h = \frac{3v}{4x^2}$ B1

Height of triangular face, H is given by

$H^2 = h^2 + x^2$

$= \frac{9V^2}{16x^4} + x^2$ M1

$= \frac{9V^2 + 16x^2}{16x^4}$ A1

Area of triangular face, A

$A = \frac{1}{2}.2x.H = xH$ s.o.i B1

squaring expression for area

$A^2 = x^2H^2$ M1

$= \frac{x^2(9V^2 + 16x^6)}{16x^4}$ substituting M1

$= \frac{9V^2}{16x^2} + x^4$ as required AG A1

Differentiating with respect to x

$2A\frac{dA}{dx} = \frac{-32x.9v^2}{256x^4} + 4x^3$ M1A1

A is least when $\frac{dA}{dx} = 0$

i.e. $4x^3.256x^4 = 32x.9v^2$ M1

$x = \sqrt[6]{\frac{9V^2}{32}}$ A1 [11]

(b) Finds volume of solid as $\frac{2}{3}\pi r^3 + 20\pi r^2$ B1

Uses $\frac{dv}{dt} = \frac{dv}{dr}.\frac{dv}{dt}$ is chain rule M1

Obtains $(2\pi r^2 + 40\pi)2$ or equiv. A1

$19\ 200\ \pi$ Ans A1 [4]