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ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

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CONFIDENTIAL

MARKING SCHEME

JUNE 2015

MATHEMATICS 9164/1

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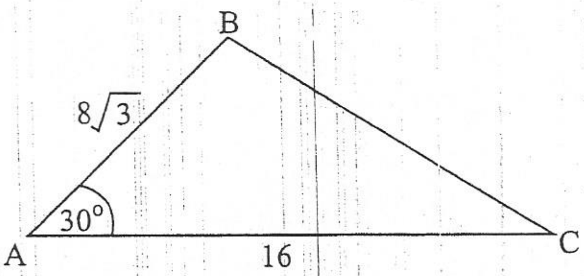


(i) $BC^2 = (8\sqrt{3})^2 + 16^2 - 2 \times 8\sqrt{3} \times 16 \cos 30°$ M1

use of cosine rule

$= 448 - 384$

$BC = 8$ (cm) A1

(ii) 1. $\dfrac{\sin A\hat{C}B}{8\sqrt{3}} = \dfrac{\sin 30}{8}$ use of sine rule or a. e. M1

$= 60°$ A1

2. Thus $B = 90°$

$\therefore$ radius of the circle $= \dfrac{16}{2} = 8\text{cm}$ B1 [5]

2 (i) $(1+ax)^n = 1 + nax + \dfrac{n(n-1)}{2}a^2x^2$ correct B1

$na = -6$

$\dfrac{n(n-1)}{2}a^2 = \dfrac{81}{4}$

comparing coefficients M1 for at least one

$a = -\dfrac{6}{n} \Rightarrow \dfrac{n(n-1)}{2} \cdot \dfrac{36}{n^2} = \dfrac{81}{4}$ solving simultaneously M1

$\dfrac{2n-2}{n} = \dfrac{9}{4}$

$n = -8$ A1

$a = \frac{-6}{-8} = \frac{3}{4}$ or equivalent A1

(ii) Valid for $\left|\frac{3}{4}x\right| < 1$

$|x| < \frac{4}{3}$ a. e. $-\frac{4}{3} < x < 4/3$ B1 [6]

3 (i) $f(x) = x^3 - e^{3\sin x}$

$y = x^3 - e^{3\sin}$

$\left.\begin{array}{l} f(2) = -7.3006036029 \\ f(2.5) = +9.603018052 \end{array}\right\}$

M1A1 for at least 1 correct

Change of sign means root between $x = 2$ and $x = 2.5$ ... A1 both correct and conclusion

(ii) $f'(x) = 3x^2 - 3\cos x e^{3\sin x}$ B1

$f'(2) = 31.10189339$

$x_2 = 2 - \frac{f(2)}{f^1(2)}$ M1 N/R Structure. values substituted.

$= 2 + \frac{7.300603629}{31.10189339}$

$= 2.23473$

$= 2.23$ A1 [6]

4 (i) $2\cos 2x + 2\sqrt{3}\sin 2x = R\cos(2x - \propto)$

$= R\cos 2x\cos\propto + R\sin 2x\sin\propto$

$R = 4$ B1

$\left.\begin{array}{l} \therefore R\cos\propto = 2 \\ R\sin\propto = 2\sqrt{3} \end{array}\right\} \Rightarrow \tan\propto = \frac{2\sqrt{3}}{2}$

$\propto = 60°$ B1

(ii) $\operatorname{Cos}2x + \sqrt{3}\sin 2x = \sqrt{2}$

$2\operatorname{Cos}2x + 2\sqrt{3}\sin 2x = 2\sqrt{2}$

$\frac{4}{4}\cos(2x-60) = \frac{2\sqrt{2}}{4}$

$\therefore\ 2x - 60 = \pm 45°;315°,405°,675°,\ldots \cos^{-1}\frac{\sqrt{2}}{2}$ M1

for $\pm 45°$ or A1

$\frac{2x}{2} = \frac{15}{2};\ \frac{105}{2};\ \frac{375}{2};\ \frac{465}{2};\ \frac{735}{2}$

$x = 7.5°;\ 52.5°;\ 187.5°;\ 232.5°$ A1A1 for a pair

[6]

5 (a) (i) $f(x) = -x^3 + 2x^2 + 3x - 6$

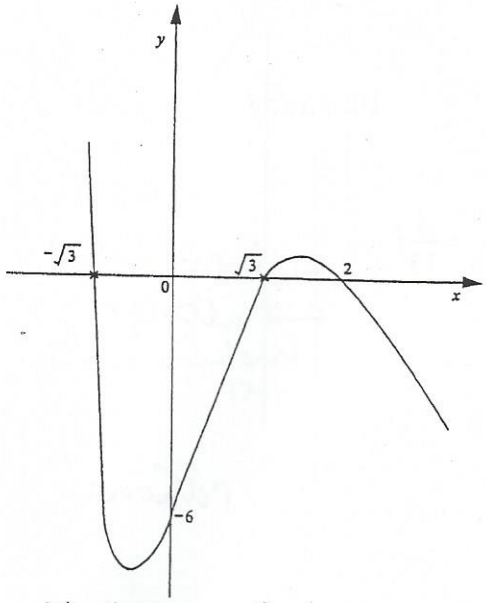
$f(2) = -8 + 8 + 6 - 6 = 0 \Rightarrow x - 2$ is a factor or a.e. B1

$$\begin{array}{r|l} & -x^2 + 3 \\ \hline x-2 & -x^3 + 2x^2 + 3x - 6 \\ & -x^3 + 2x^2 \\ & \qquad 3x - 6 \\ & \qquad -(3x - 6) \end{array}$$

M1A1

$f(x) = (x-2)(3-x^2) = (x-2)(\sqrt{3}-x)(\sqrt{3}+x)$ A1

(ii)



B1 shape

B1 intercept

(b) $f(x)>0 \ \{(x<-\sqrt{3}) \cup (\sqrt{3}<x<2)\}$ B1B1 [8]

6 (i) 1. $\overrightarrow{AB}=5i+2j-k-(3i-pj-k)=2i+(2+p)j$

$\overrightarrow{BC}=7i+(2+\sqrt{5});-k-(5i+2j-k)=2i+\sqrt{5}j$ B1 either

If $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ $\quad 2+p=\sqrt{5}$ M1

$p=\sqrt{5}-2$ A1

2. If $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{BC}$ $\quad \{2i+(2+p)\}j\bullet(2i+\sqrt{5}j)=0$ M1

$4+\sqrt{5}(2+p)=0$

$\sqrt{5}(2+p)=-4 \Rightarrow 2+p=-\frac{4}{\sqrt{5}}$

$\therefore \quad p=\frac{-4}{\sqrt{5}}-2 \ \ or \ \ \frac{-4\sqrt{5}}{5}-2$ or $a.e$ A1

(ii) 1. $|\overrightarrow{BC}|=|2i+\sqrt{5}j|=\sqrt{4+5}=3$ M1 ~~B1~~ his $\frac{B2}{|BC|}$

$\therefore \ \hat{r}=\frac{1}{3}(2i+\sqrt{5})$ is the unit vector A1 ~~B1~~

2. $V = \frac{15}{3}\left(2i+\sqrt{5}j\right) = 5\left(2i+\sqrt{5}j\right)$

$= 10i + 5\sqrt{5}j$ B1 [8]

7 (i) $\frac{w}{u} = \frac{3-4i}{u} = \frac{2}{13}+\frac{3}{13}i$ subst into two and attempting to make u subject M1

$\therefore = \frac{13(3-4i)}{2+3i}$ or a. e. B1

$= 13\frac{(3-4i)(2-3i)(2-3i)}{(2+3i)(2-3i)}$ rationalising M1

$= 13\frac{(6-9i-8i-12)}{4+9}$ *for* $i^2 = -1$ use of i^2 M1

$= -6-17i$ A1

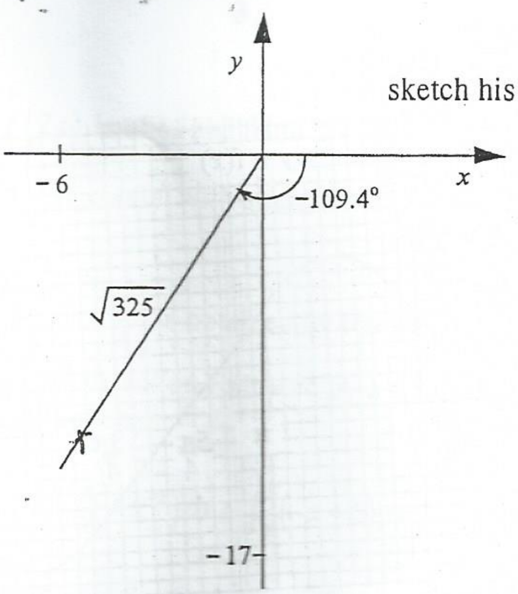
(ii) 1. $|u| = \sqrt{36+289} = \sqrt{325}$ or $5\sqrt{13}$ or a.e. B1

2. $\arg u = -180+\tan^{-1}\frac{17}{6}$ M1

$= -180+70.6$ or $-\pi+1.2315$

$= -109.4°$ or -1.91 A1

(iii)



sketch his u B1

For $|u|$ and argu on sketch B1 [9]

8 (i) $y \; = \; 1+\cos\left(\dfrac{\pi}{3}e^{3\vartheta}\right)$ and $x = 2-\sin\left(\dfrac{\pi}{3}e^{3\vartheta}\right)$

$$\frac{dy}{d\vartheta} = -\pi e^{3\vartheta}\sin\left(\frac{\pi}{3}e^{3\vartheta}\right); \quad \frac{dx}{d\vartheta} = -\pi e^{3\vartheta}\cos\left(\frac{\pi}{3}e^{3\vartheta}\right)$$ B1 for at least 1 correct

$$\frac{dy}{dx} = \frac{-\pi e^{3\vartheta}\sin\left(\frac{\pi}{3}e^{3\vartheta}\right)}{-\pi e^{3\vartheta}\cos\left(\frac{\pi}{3}e^{3\vartheta}\right)} = \tan\left(\frac{\pi}{3}e^{3\vartheta}\right)$$ M1A1

$$\vartheta = 0 \Rightarrow \frac{dy}{dx} = \tan\frac{\pi}{3} = \sqrt{3}$$ ~~B1~~ A1

(ii) $(y-1)^2 \; = \; \cos\left(\dfrac{\pi}{3}e^{3\vartheta}\right)$ (1) Sq

M1 for either (1) or (2)

$(x-2)^2 \; = \; \left(-\sin\left(\dfrac{\pi}{3}e^{3\vartheta}\right)\right)^2$ (2)

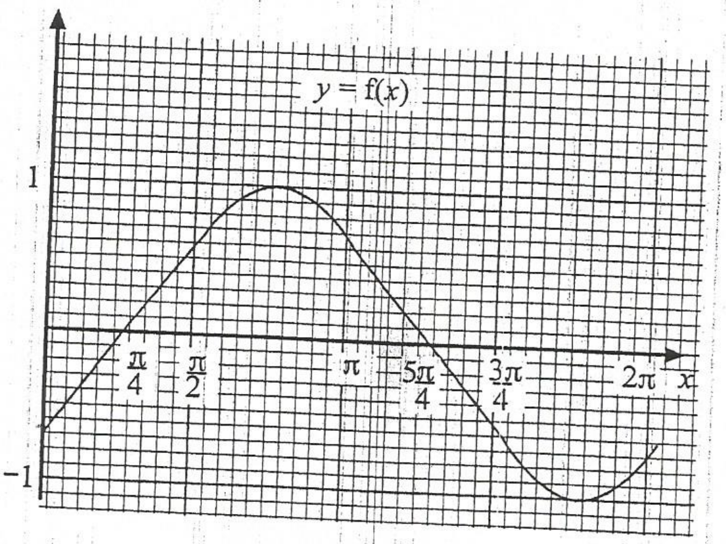
(1) + (2) $(x-2)^2+(y-1)^2 = 1$ add M1A1

This is a circle of radius 1 with centre at (2,1)

B1 for ~~both~~ circle

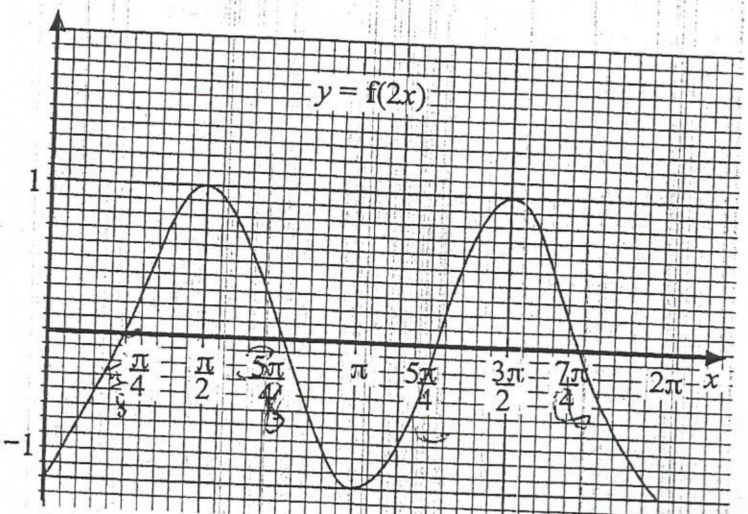
B1 for ~~both~~ centre and radius [9]

9 (i)



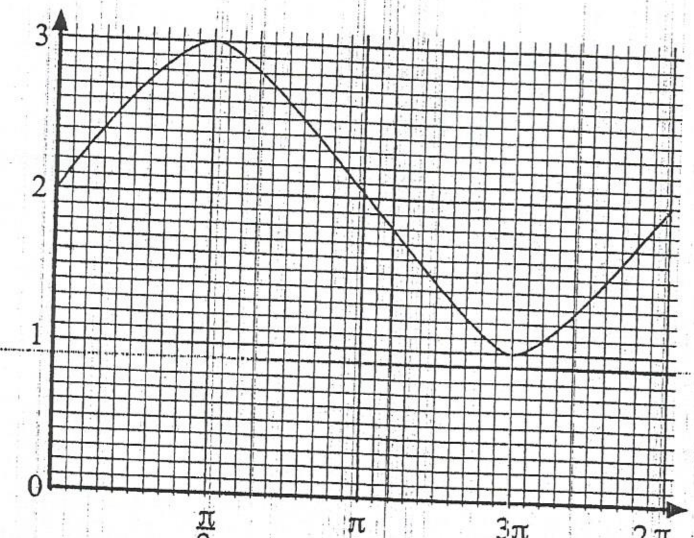
Shape B1
Intercept B1

(ii)



Shape B1
Intercept B1

(iii)



Shape B1

(ii) The graph $y = f(2x)$ is obtained from that of $y = f(x)$ by a stretch parallel to the x-axis, with stretch factor $\frac{1}{2}$ — B1, B1

The graph $y = 2 + f\left(x + \frac{\pi}{4}\right)$ is obtained by translation of $\frac{\pi}{4}$ units in the negative x-axis direction followed by a translation of 2 units in the positive y-direction. — B1, B1 [9].

10 (i) $R_1 = 4\int_1^2 (1 - x^{-2})\,dx + \int_2^5 (-x+5)\,dx$ — M1 ~~B1~~ or equivalent

$4\left[x + \frac{1}{x}\right]_1^2 + \left[-\frac{1}{2}x^2 + 5x\right]_2^5$ — A1 ~~M1~~ at least one correct integral; A1 all correct

$4\left(2 + \frac{1}{2} - (1+1)\right) + \left(-\frac{25}{2} + 25\right) - (-2+10)$ — M1 use of limits

$4 \times \frac{1}{2} + \frac{25}{2} - 8 = \frac{25}{2} - 6 = \frac{13}{2}$ a. e. — A1

(ii) $V = \pi\int_0^3 x^2\,dy,$ $\quad y = 4 - \frac{4}{x^2}$

$\frac{4}{x^2} = 4 - y$

$x^2 = \frac{4}{4-y}$

$V = \pi\int_0^3 \frac{4}{4-y}\,dy$ — B1

$[4\pi \bullet (-\ln(4-y))$

$[4\pi \bullet (-\ln(4-y))]_0^3$ — M1, A1

$-4\pi\{\ln 1 - \ln 4\}$ — M1

$4\pi \ln 4$ ✓ — a.e. A1 [10]

Sum 2

11 (i) $\frac{dm}{dt} \propto \frac{1}{t+3} \Rightarrow \frac{dm}{dt} = \frac{k}{t+3}$ B1

(ii) $m = \int \frac{k}{t+3} dt \Rightarrow m = k\ln(t+3) + c$ M1A1

$\ln 9 = k\ln 3 + c$ .... (1)

and $3\ln 9 = k\ln 27 + c = k\ln 3^3 + c$ ... (2) at least one M1 substituting the values G/sch.

$(2) - (1)$ $2\ln 9 = 2k\ln 3$ solving — M1

$k = \frac{\ln 9}{\ln 3} = 2$, and $c = 0$ A1

$\therefore M = 2\ln(t+3)$ all correct

(iii) (a) when $t = 100$, $m = 2\ln 103$ M1

$= 9.269$ A1

(b) when m = 10

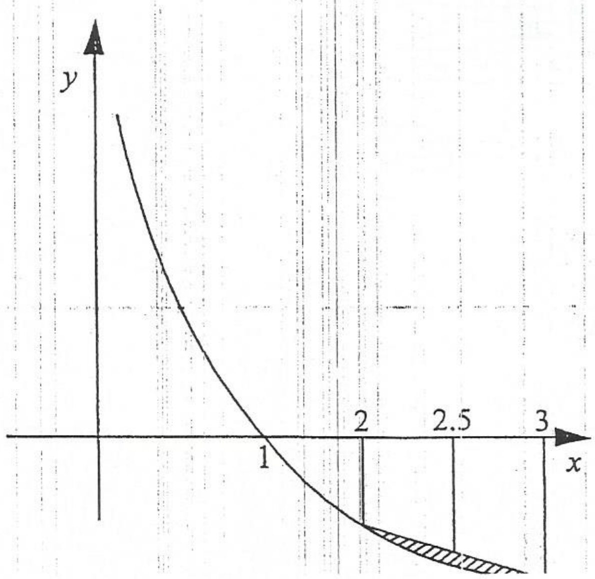
$2\ln(t+3)$

$5 = \ln(t+3)$

$t + 3 = e^5$ M1

$t = e^5 - 3$ or 145.4 years A1 $\boxed{10}$

12 (a)



B1

(b) A $\approx \frac{1}{2}\cdot\frac{1}{2}\{-ln2+2(-ln2.5)+(-ln3)\}$ $h=\frac{1}{2}$ B1 M1 ~~A1~~ with 3 ordinates.

$= -\frac{1}{4}ln(2\times 6.25\times 3) = (-)0.9061$ A1

(c) A $= -\int_2^3 1.lnx\,dx = -\left[xlnx+\int x.\frac{1}{x}dx\right]_2^3$ integration by parts M1

$= -[xlnx+x]_2^3$ A1

$= -(3ln3+3)+(2ln2+2$ M1

$= (-)0.9095$ A1

$\therefore$ percentage error $= \frac{0.9095-0.9061}{0.9095}\times 100\%$ M1

$= 0.3738 \approx 0.37\%$ A1

The trapezium rule does not include the shaded area on the graph. B1

[11]

13 (a) (i) $a+3d=42$ … (1)

$\frac{3}{2}(2a+2d)=12$

$a+d=4$ (2)

B1 for at least (1) or (2)

(1) − (2) $2d = 38$ Solving M1

$d = 19$ and $a = -15$ A1A1

(ii) $S_{20} = \frac{20}{2}(-30+19\times 19)$ M1

$= 3310$ A1

(b) (i) $ar^2 = 36$ or $ar^4 = 16$ — B1

$$\frac{ar^4}{ar^2} = \frac{16}{36}$$ — M1

$$\therefore r^2 = \frac{16}{36} \Rightarrow r = \frac{-4}{6} = \frac{-2}{3}$$ c.a.o — A1

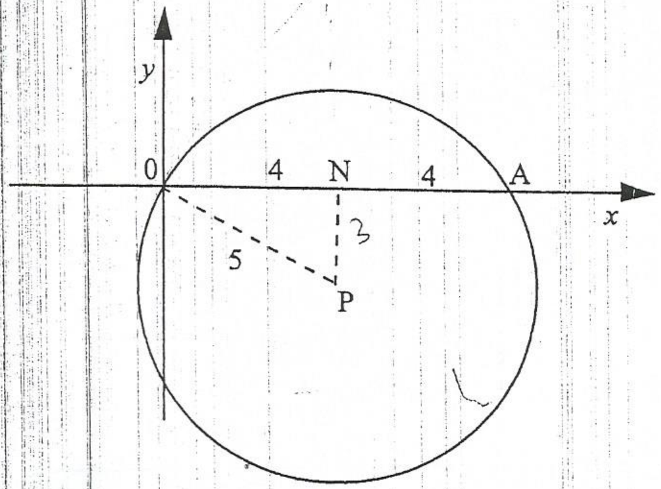
$$a = \frac{36}{\left(\pm\frac{2}{3}\right)^2} = 36 \times \frac{9}{4} = 81$$ — B1√

(ii) $$S\infty = \frac{81}{1+\frac{2}{3}} = \frac{81\times 3}{5}$$

$$= \frac{243}{5}$$ or 48.6 — B1√

[11]

14 (a)



(i) Using triangle OPN

$PN^2 = 5^2 - 4^2$ — M1

$PN = 3$ (cm)

$x \quad \therefore C\ (4, -3)$ — A1

(ii) Equation of circle is

$(x-4)^2 + (y+3)^2 = 5^2$ — M1

$x^2 + y^2 - 8x + 6y = 0$ — A1

(iii) Grad of OP $= \dfrac{3}{-4} = \dfrac{-3}{4}$ — B1

Equation of tangent is y $= \frac{4}{3}x$ M1 A1

(b) At Q, $x = 4$

Or Alt. E m B1

$y = \frac{4}{3}\times 4 = \frac{16}{3}$ M1

$OQ^2 = 4^2 + \left(\frac{16}{3}\right)^2 = \frac{400}{9} \Rightarrow OQ = \frac{20}{3}$ A1

Area of triangle OPQ $= \frac{1}{2}\times 5\times\frac{20}{3}$ M1

$= \frac{50}{3}$ or $16\frac{2}{3}$ a.e A1

[12]