ZIMSEC

For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MATHEMATICS 9164/1

PAPER 1

NOVEMBER 2007 SESSION 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

[Turn over

1 Find the values of a, b and c such that

$$2x^4 + 6x^3 + 7x^2 + 15x + 5 = (x^2 + 3x + 1)(ax^2 + bx + c)$$

for all values of x. [3]

2 Solve the differential equation $x^2 \dfrac{dy}{dx} = y$, given that $y = e^2$ when $x = 1$. [4]

3 Sketch, on the same axes, the graphs of $y = |2x - 3|$ and $y = x + 1$. Hence or otherwise, solve the inequality $|2x - 3| < x + 1$. [4]

4 The complex number $\dfrac{3 + 2i}{2 + ai}$ can be expressed in the form $x + iy$, where x and y are real. Find the value of a given that $x = y$. [5]

5 A function is defined by

$$f : x \mapsto x^2 + 4x + 1, \text{ for } x \geq -2.$$

Find

(i) the range of the function, [2]

(ii) an expression for $f^{-1}(x)$, stating its domain. [4]

6 The region R is bounded by the graph of $x^2 = y - 1$; $x = 0$; $y = 0$ and $x = 1$. Sketch the region R. [1]

R is rotated completely about the x–axis. Find the volume of the solid of revolution generated giving your answer in terms of π. [5]

7 A student in a physics class measured the diameter x, of a cylindrical wire of fixed length l. He used this to calculate the volume V of the wire. If an error δx, in the measurement of the diameter, results in an error of δv in the calculated volume, write down a relationship between δx and δv. Hence show that the relative error in the calculated volume is approximately twice the relative error in the diameter. [6]

8 Two birds, P and Q fly such that their position vectors with respect to an origin O are given by

$$\overrightarrow{OP} = (2t+3)\boldsymbol{i} + (t-1)\boldsymbol{j} + 3t\boldsymbol{k} \text{ and}$$

$$\overrightarrow{OQ} = (t-2)\boldsymbol{i} + (3t+1)\boldsymbol{j} + (t+2)\boldsymbol{k}$$

for $0 \leq t \leq 10$, where $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are unit vectors of magnitude 1 metre in the x, y and z directions respectively.

(a) For the time $t = 0$,

(i) calculate the distance between the two birds, [3]

(ii) find the position vector of the point mid-way between the two birds. [1]

(b) Find the value of t for which $P\hat{O}Q = 90°$, giving your answer to 2 significant figures. [3]

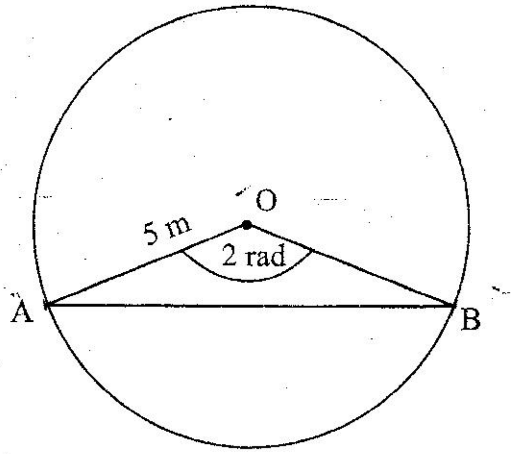
9 Given that $2^x - 2^{-x} = 4$,

(i) solve the equation for x, [4]

(ii) show that $\left|2^x + 2^{-x}\right| = 2\sqrt{5}$. [3]

10 **(a)** In triangle ABC, $c = 10$ cm, $a = 13$ cm and $C = 45°$. Find the possible values of B. [5]

(b)



The diagram shows the cross-section of a circular pond centre O and radius 5 metres. The angle between radii OA and OB is 2 radians. A frog swims in a straight line from A to B and then hops back to A along the minor arc AB. Calculate the total distance travelled by the frog. [3]

11 Find the coordinates of each of the stationary points on the curve $y = \dfrac{2x^3}{(3x-4)^2}$. [5]

Show that there is only one minimum point and state its coordinates. [3]

12 Show that the equation

$$\frac{4\sin^2\theta}{\operatorname{cosec}\theta} + \frac{3}{\operatorname{cosec}^2\theta\,\sec\theta} = 2\sin^2\theta$$

may be written as

$$\sin^2\theta\left[4\sin\theta + 3\cos\theta - 2\right] = 0$$

and hence solve the equation for $0° \le \theta \le 360°$. [7]

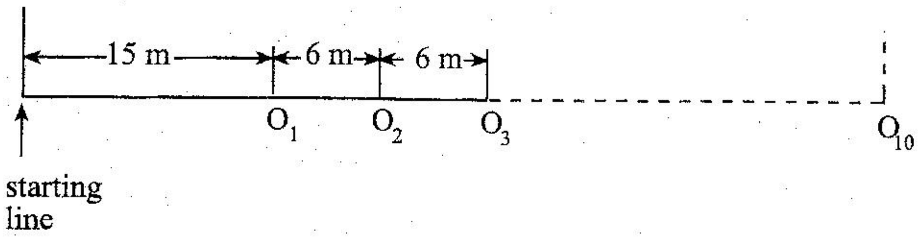
13 **(a)** It is given that $\sum_{r=1}^{n} u_r = n^2 - 7n$.

Find

(i) u_1, u_2, u_3, [4]

(ii) an expression for u_r in terms of r. [2]

(b) The diagram shows 10 oranges $O_1, O_2, O_3, \ldots O_{10}$ placed in a straight line 6 m apart with the first orange 15 m from the starting line.



In a race, each competitor runs and collects the oranges one at a time starting with O_1 and returns it to a box at the starting line. Find the total distance covered by a contestant who manages to collect all the oranges. [4]

14 A girl returning from a milling point is carrying mealie-meal in a cylindrical container. The container has a hole at its base and the mealie-meal trickles out through this hole. It is estimated that the rate of reduction of mealie-meal is proportional to the mass m of mealie-meal remaining in the container, so that this situation can be modelled by differential equation,

$\frac{dm}{dt} = -\frac{k}{5}m$, where k is a constant.

Find the general solution of this differential equation and show that it reduces to

$m = m_0 e^{-\frac{k}{5}t}$, where m_0 is the initial mass of the mealie-meal. [6]

The girl takes 2 hours to walk from the milling point to her home. Given that after one hour, ten percent of the mealie-meal is lost,

(i) calculate the percentage of mealie-meal in the container when she arrives home, [4]

(ii) sketch a graph showing the variation of the mass of the mealie-meal during the two hour journey. [2]

15 **(a)** Show that $\left(1-\frac{1}{n^2}\right)^{-1}=\left(\frac{n^2}{n^2-1}\right)$. [2]

(b) **(i)** Write down the first three terms of the series expansion of $ln(1-x)$ in ascending powers of x, up to and including the term in x^3. [1]

(ii) Hence write down the first three terms for $ln\left(1-\frac{1}{n^2}\right)$, in terms of n,

and deduce the first three terms in the expansion of $ln\left(1-\frac{1}{n^2}\right)^{-1}$. [3]

(iii) Show that $ln\left(\frac{n^2}{n^2-1}\right)=2ln(n)-ln(n+1)-ln(n-1)$. [2]

(iv) Given that $ln10=2.302\,585\,1$, $ln3=1.098\,612\,3$, and $n=10$, and using your results in **(ii)** and **(iii)** to calculate $ln11$, to six decimal places. [4]

16 **(i)** Show that the equation of the circle passing through the points (–2; –4), (3; 1) and (–2; 0) is $(x-1)^2+(y+2)^2=13$. [7]

(ii) Deduce the equation of the diameter of the circle in **(i)** in the form $ax+by+c=0$ given that *t* passes through the point (– 2; 0). [3]

(iii) Show that the tangent at the point (3; 1) is parallel to the diameter of the circle, whose equation was found in **(ii)**. [3]