ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

CONFIDENTIAL

MARKING SCHEME

NOVEMBER 2010

MATHEMATICS

9164/1

Multiplying equation by $\log_a b$ to give

$(\log_a b)^2 + 2 = 3\log_a b$ M1

Attempt to solve the quadratic equation $(\log_a b - 2)(\log_a b - 1) = 0$ M1

Using given condition to choose required factor $(\log_a b - 2) = 0$ M1

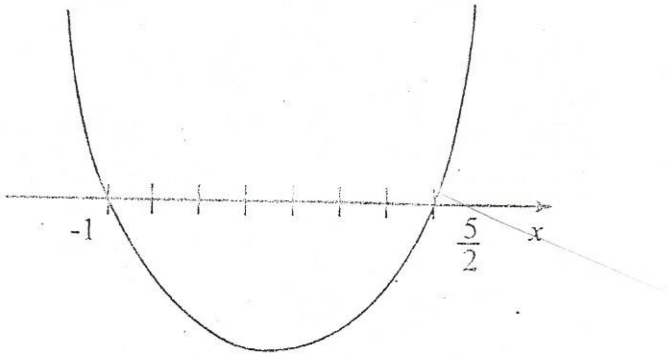
Obtaining $b = a^2$ A1

2 (a) $x^2(x^2 - 3x - 5) + 3x(x^2 - 3x - 5) - 2(x^2 - 3x - 5)$

$= x^4 - 3x^3 - 5x^2 + 3x^3 - 9x^2 - 15x - 2x^2 + 6x + 10$

$= x^4 - 16x^2 - 9x + 10$ correct method c.a.o. B1

(b) $(2x-5)(x+1) < 0$



M1

A1

$$-1 < x < \frac{5}{2}$$ A1

3 $\dfrac{(1+i)(3-4i)}{(3+4i)(3-4i)} = \dfrac{3-41+3i+4}{9+16} = \dfrac{7-i}{25}$ M1

A1

$= \dfrac{7}{25} - \dfrac{1}{25}i$ A1

$|z| = \dfrac{1}{25}\sqrt{49+1} = \dfrac{1}{25}\sqrt{50}$ M1

$= \dfrac{1}{25}.5\sqrt{2} = \dfrac{1}{5}\sqrt{2}$ A1

[5]

4 (a) $S_n = \frac{n}{2}(-10+25)$

$= \frac{15n}{2}$ seen or implied. B1

$\frac{15n}{2} > 300$ M1

$n > 40$

$n = 41$ A1

(b) n^{th} term

$25 = -10 + 40d$ M1

$d = \frac{7}{8}$ A1

[5]

5 Use correct procedures for finding $\cos\left(A\hat{O}B\right)$ M1

Obtain given answer correctly AG A1

[2]

State or imply $\left|\overrightarrow{OP}\right| = \left|\overrightarrow{OA}\right| \cos A\hat{O}B$ M1

Obtain 2N2 A1

Use $\overrightarrow{OP} = \left|\overrightarrow{OP}\right| \cdot \frac{\overrightarrow{OB}}{\left|\overrightarrow{OB}\right|}$ or equivalent M1

Obtain answer $-\frac{8}{5}i + 2j + \frac{6}{5}k$ or equivalent A1

[4]

[6]

6 $k = \dfrac{m^2+1}{m^2-1}$ or equivalent – $\dfrac{(m+1)}{m}\Big/\dfrac{m-1}{m}$ B1

$\Rightarrow m^2 = \dfrac{k+1}{k-1}$ m subject of formula M1 (for attempting to solve form)

$\Rightarrow 10^x = \left(\dfrac{k+1}{k-1}\right)^{\frac{1}{2}}$ M1 (for involving x and attempting to solve)

$\Rightarrow x = \dfrac{1}{2} lg\left(\dfrac{k+1}{k-1}\right)$ A1 (AG – validly obtained)

$\Rightarrow x = \dfrac{1}{2} lg\left(\dfrac{12}{10}\right) = \dfrac{1}{2} lg12 - \dfrac{1}{2} lg10$ M1 (substituting $k = 11$ and use of log laws)

$\Rightarrow x = \dfrac{1}{2} lg3 + lg2 - \dfrac{1}{2}$ AG A1(validly obtained)[6]

7. (a) Mirror line or line of reflection (symmetry line) B1 CAO

(b) A(3,0) and $B\left(-1\dfrac{1}{2},0\right)$ B1B1 (both correct)

(c) Equation of BC : $y = 2x + 3$ / ~~Grad BC~~ grad of any one of AC $y = \frac{1}{2}x - \frac{3}{2}$ ~~B1~~M1 A1 for each
Correct method to solve for x and y M1
Obtain $x = -3$ and $y = -3$ A1 B2 for C(-3,-3)
[7]

8 (a) Use change of variable method correctly to obtain $\int_0^1 u^{\frac{1}{2}} du$ or equivalent M1 with correct limits or incorrect limits

Obtain correct integral $\left[\dfrac{2u^{\frac{3}{2}}}{3}\right]_0^1$ A1 even with wrong limits or missing

Obtain correct answer $\dfrac{2}{3}$ A1

(b) $\cos 3y = -\frac{1}{\sqrt{2}}$

$3y = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ with at least one angle correct or wrong M1

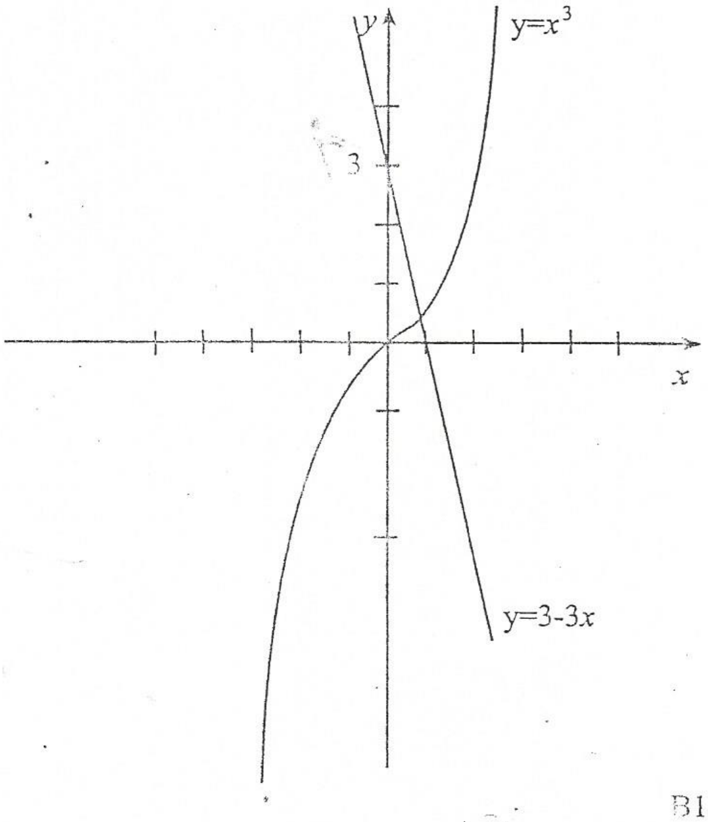
at least one correct ∠

3y = 135, 225°, 495° 585° for 1[st] two A1

y = 45°, 75°, 165°, 195° for any pair correct A1A1 [7]

for each pair

9 (a)



for any one

B1 both correct

For showing they met once B1

(b) for substituting in $x^3 + 3x - 3$ and showing change of sign
$x = 0{,}8; \quad x = 1$ change of sign must be seen B1

(c) (i) $x_1 = 0{,}8$

$$x_2 = 0{,}8 - \left[\frac{(0{,}8)^3 + 3(0{,}8) - 3}{3(0{,}8)^2 + 3}\right]$$ M1

$$= \quad 0{,}818$$ → 3 s.f A1

(ii) $x_{n+1} = \frac{3 - x_n^{\ 3}}{3}$

$$x_1 = \frac{3-(0,8)^3}{3}$$

$= \quad 0,82933$ or 0,83 B1

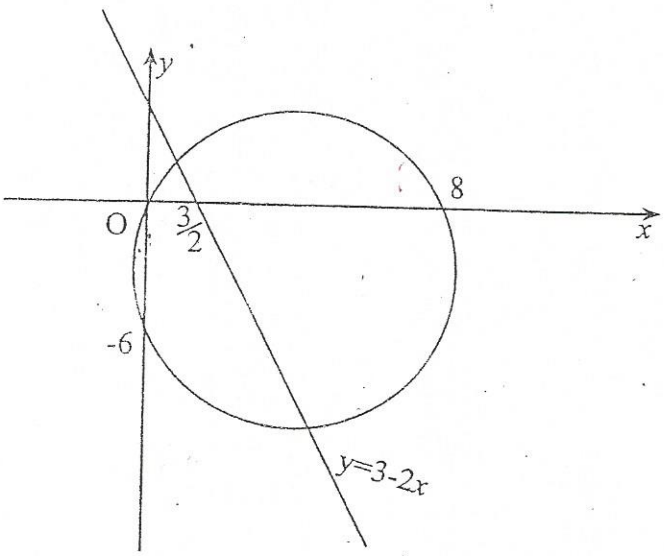
$$x_2 = \frac{3-(0,8293)^3}{3}$$

$= \quad 0,810$ B1 [7]

10 (i) Equation of circle $(x-4)^2 + (y+3)^2 = 5^2$ or equiv. B1

(ii) Pt (x, y) satisfies the inequality $x^2 + y^2 - 8x + 6y < 0$ or equiv B1

(iii)



circle with correct intercepts B1

line intersecting circle at 2 points B1

(iv) Putting $y = 3-2x$ in inequality to give

$$x^2 + (3-2x)^2 - 8x + 6(3-2x) < 0$$ including solving simultaneously M1

Method of finding critical values $(5x-27)(x-1) < 0$ ~~M1~~ A1 for correct critical values 1 & 27/5

$$\begin{array}{c|c|c} ++ & --- & ++ \\ \hline 1 & & \frac{27}{5} \end{array}$$

obtain soln $1 < x < \frac{27}{5}$ for correct solution A1 [7]

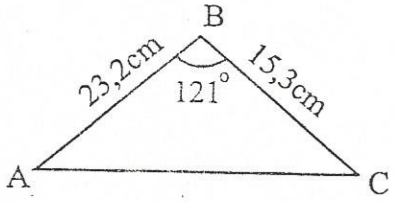
11 (i) $\frac{dx}{dt} = \frac{2}{3+2t}$ B1

		$\frac{dy}{dt} = 6te^{3t^2}$		B1	
		$\frac{dy}{dx} = 6te^{3t2} \times \frac{3+2t}{2}$		M1	
		$= \quad 3t(3+2t)e^{3t^2}$		A1	
	(ii)	When $\frac{dy}{dx} = 0, \quad 3te^{3t^2}(3+2t) = 0$		M1	
		$\Rightarrow t = 0$ or $\frac{-3}{2}, \quad e^{3t^2} \neq 0$.		A1	
		Coordinates of turning point ($ln\ 3$, 1) since x is undefined for $t = \frac{-3}{2}$.		B1	[7]
12	(a)	$(2)^4 + (2)^3a + (2)^2b + 16(2) - 12 = 0$			
		$(-2)^4 + (-2)^3a + (-2)^2b + 16(-2) - 12 = 0$	for using factor theorem	M1	
		$8a + 4b = -36$ $-8a + 4b = 28$	for obtaining correct 3–term equations in a and b.	A1	
		$8b = -8$	for an attempt to solve the two equations simultaneously	M1	
		$a = -4$ and $b = -1$	for both a and b correct	A1	
					[4]
	(ii)	$$\begin{array}{r} x^2 - 4x + 3 \\ x^2 - 4 \overline{)\, x^4 - 4x^3 - x^2 + 16x - 12} \\ x^4 \quad\quad - 4x^2 \quad\quad\quad\quad\quad \\ \hline -4x^3 + 3x^2 + 16x \quad\quad \\ -4x^3 \quad\quad\quad + 16x \quad\quad \\ \hline 3x^2 \quad\quad - 12 \\ 3x^2 \quad\quad - 12 \\ \hline \end{array}$$	for valid method of finding the second quadratic factor $x^2 - 4x + 3$	M1	
			for obtaining correct $x^2 - 4x + 3$	A1	
		$x^2 - 4x + 3 = (x-3)(x-1) = 0$	for factorising quadratic polynomial	M1	

$\Rightarrow \quad x = 3$ or 1 — for obtaining the other correct roots (both) — A1 [8]

or equivalent

13 (a)



By Cosine rule

$AC^2 = 15,3^2 + 23,2^2 - 2(15,3)(23,2)\cos 121°$ — M1 using cos rule

$= 1137,96583$ — ~~A1~~

$AC = \sqrt{1137,96583}$

$= 33,73374913$ — A1

By sine Rule

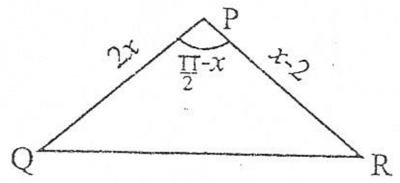
$\dfrac{\text{Sin}A}{15,3} = \dfrac{\text{Sin}121°}{33,73374913}$ — OR equivalent — M1

$\text{Sin}A = \dfrac{15,3\,\text{Sin}121°}{33,73374913}$

$A = 22,877968°$

Angle BAC = 22,9° to the nearest 0,1° — A1 after simplifying

(b)



$QR^2 = (2x)^2 + (x-2)^2 - 2(2x)(x-2)\cos\left(\dfrac{\pi}{2} - x\right)$ — M1

$QR^2 = 4x^2 + 5x^2 - 4x + 4 - 2(2x)(2x)(\sin x)$ — seen or implied

Since $\cos\left(\dfrac{\pi}{2} - x\right) = \sin x$ — B1

$= 4x^2 + x^2 - 4x + 4 - 4x^3 + 8x^2$ — A1

since $\sin x \approx x$ when x is small.

$= -4x^3 + 13x^2 - 4x + 4$ — A1 [8]

	(i)	$(2x-1)(x^2+4)$		M1 for attempting to expand or equivalent
		$2x^3-x^2+8x-4$		A1 obtaining AG validly
	(ii)	$\dfrac{A}{2x-1}+\dfrac{Bx+C}{x^2+4}=\dfrac{x^2-2x+20}{(2x-1)(x^2+4)}$		$\dfrac{A}{2x-1}+\dfrac{Bx+C}{x^2+4}$
		$\Rightarrow A(x^2+4)+(Bx+C)(2x-1)\equiv x^2+2x+20$		M1 (for complete correct method) correct fraction
		$A=5$		B1 (for correct value of A)
		$B=-2,\ C=0$		A1
		$\dfrac{x^2+2x+20}{2x^3-x^2+8x-4}=\dfrac{5}{2x-1}-\dfrac{2x}{x^2+4}$		~~A1~~
	(iii)	$\int_1^3\dfrac{5}{2x-1}dx-\int_1^3\dfrac{2x}{x^2+4}dx$		
		$=\quad \dfrac{5}{2}ln(2x-1)\Big\|_1^3-ln(x^2+4)\Big\|_1^3$		M1 (for obtaining 2 long terms)
				A1 (for correct integrals)
		$=\quad \dfrac{5}{2}\ln5-ln13+ln5$		M1 (for using limits correctly)
		$=\quad$ 3,068 (083336) Ω 3,07	C A O	A1 [10]
15	(i)	$-4\left[\left(x-\dfrac{3}{2}\right)^2-\dfrac{9}{4}\right]$		Method of completing the square or any valid method ie identifies M1
		$=\quad 9-(2x-3)^2$	c.a.o	A1
		or equivalent		

(ii) when $y = 8 \Rightarrow \quad 4x^2 - 12x + 8 = 0$ — and attempt to solve the quadratic eqn

$(x-2)(x-1) = 0$ M1

$x = 2$ or 1

since $x < \frac{3}{2} \quad \Rightarrow x = 1$ A1

$\frac{dy}{dx} = 12 - 8x$ with $x = 1$ M1

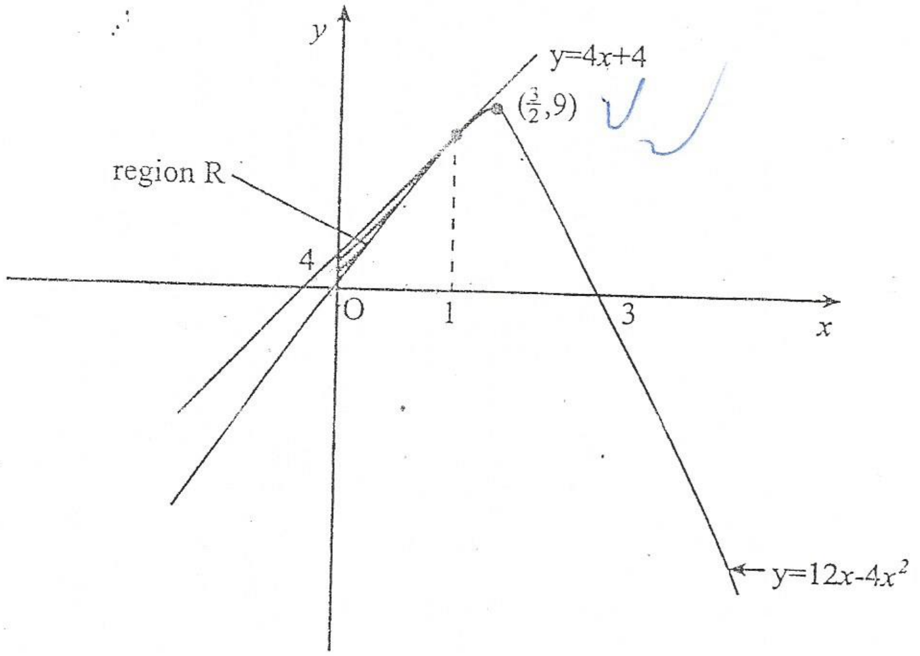
for differentiating & substituting x and simplifying

$\left.\frac{dy}{dx}\right|_{x=1} = 4$ A1

$\frac{y-8}{x-1} = 4$ M1

$y = 4x + 4$ A1

(iii)



both correct tangent & curve ~~with points, 1 and 3 labelled~~ and the shaded region B1

correct region identified B1

(iv) Volume = $\pi \int_o^b y^2 dx$

$$= \pi\left[\int_0^1 (4x+4)^2 dx - \int_0^1 (12x-4x^2)^2 dx\right]$$ correct form of volume and integration M1

$$= \pi\left[16\left(\frac{x^3}{3}+x^2+x\right)_0^1 - \left(\frac{144}{3}x^3 - \frac{96x^4}{4} + \frac{16x^5}{5}\right)_0^1\right]$$ M1

correct integral A1

$$= \frac{152}{15}\pi$$

[13

16 (a) (i) $\frac{dv}{dt} = 100 - 2{,}5h$ B1

$V = \pi r^2 h = 25\pi h$ and $\frac{dv}{dh} = 25\pi$ B1

$$\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt} = \frac{1}{25\pi}(100 - 2.5h)$$ M1

$$\therefore = \frac{1}{10\pi}(40-h)$$ AGA1

(ii) $\int \frac{1}{40-h} dh = \frac{1}{10\pi}\int dt$ and attempt to integrate M1

$-ln(40-h) = \frac{1}{10\pi}t + A$ or equivalent constant A1

$t = 0,\ h = 0 \Rightarrow A = -ln40$ M1

$ln\left(\frac{40}{40-h}\right) = \frac{1}{10\pi}t$ or equiv. A1

$\frac{40}{40-h} = e^{\frac{t}{10\pi}}$ for exponentiating even wrong M1

$h = 40 - 40e^{-\frac{t}{10\pi}} = 40\left(1 - e^{-\frac{t}{10\pi}}\right)$ or equivalent A1

The maximum value of h is 40 cm. It cannot be exceeded B1 stating maximum
since at that height the rate at which juice enters equals the rate at which it drains out of the cylinder. B1 explanation

as $t \to \infty$ $e^{-t/10\pi} \to 0$