$$V^{\mathrm{Swpt}}(t_0) = N \cdot \sum_{k=i+1}^{m} c_k V_p^{\mathrm{ZCB}}(t_0, T_k),$$

Payer

$$V^{\mathrm{Swpt}}(t_0) = N \cdot P(t_0, T_i)$$
$$\times \mathbb{E}^{T_i}\left[\max\left(\sum_{k=i+1}^{m} \tau_k P(T_i, T_k)\Big(\ell(T_i; T_{k-1}, T_k) - K\Big), 0\right) \Big| \mathcal{F}(t_0)\right].$$

$$\boxed{\begin{aligned} \left(1+\tau_k \ell_k(T_m, T_{k-1}, T_k)\right) &= \frac{P(T_m, T_{k-1})}{P(T_m, T_k)} \\ \ell_k(T_m, T_{k-1}, T_k) &= \frac{P(T_m, T_{k-1}) - P(T_m, T_k)}{\tau_k P(T_m, T_k)} \end{aligned}}$$

Receiver

$$\frac{V_0}{P(0, T_m)} = E^{T_m}\left(\max\left(\sum_{k=m+1}^{n} \tau_k P(T_m, T_k)\left(Y - \ell(T_m; T_{k-1}, T_k)\right), 0\right)\right)$$
$$= E^{T_m}\left(\max\left(Y \sum_{k=m+1}^{n} \tau_k P(T_m, T_k) - (1 - P(T_m, T_n)), 0\right)\right)$$
$$= E^{T_m}\left(\max\left(\sum_{k=m+1}^{n} c_k P(T_m, T_k) - 1, 0\right)\right)$$

where $c_k = Y\tau_k$ for $k = m+1, \dots, n-1$

$c_n = 1 + Y\tau_k$ for $k = n$

$$= E^{T_m}\left(\max\left(\sum_{k=m+1}^{n} c_k e^{A_k + B_k r_{T_m}} - \sum_{k=m+1}^{n} c_k e^{A_k + B_k r^*}, 0\right)\right)$$
$$= \sum_{k=m+1}^{n} c_k E^{T_m}\left(\max\left(e^{A_k + B_k r_{T_m}} - \hat{K}, 0\right)\right)$$

$$V_0^{Rec} = \sum_{k=m+1}^{n} c_k V_{ZCB}^{Call}\left(0, P(0; T_m, T_k), \hat{K}\right)$$

$$V_0^{Rec} - V_0^{Pay} = \sum_{k=m+1}^{n} c_k \left(V_{ZCB}^{Call} - V_{ZCB}^{Put} \right)$$

$$= \sum_{k=m+1}^{n} c_k \left(P(0,T_k) - \hat{K} P(0,T_m) \right)$$

$$\boxed{C - P = P(0,T_1) \frac{P(0,T_2)}{P(0,T_1)} - k P(0,T_1) = P(0,T_2) - k P(0,T_1)}$$

$$= \sum_{k=m+1}^{n} c_k P(0,T_k) - \sum_{k=m+1}^{n} c_k e^{A_k + B_k r^*} P(0,T_m)$$

$$= \sum_{k=m+1}^{n} c_k P(0,T_k) - P(0,T_m)$$

$$= \sum_{k=m+1}^{n} Y \tau_k P(0,T_k) + P(0,T_n) - P(0,T_m)$$

Alternative

$$V_0^{Rec} = P(0,T_m) E^T\left((Fix - Float)_+ \right)$$

$$V_0^{Payer} = P(0,T_m) E^T\left((Float - Fix)_+ \right)$$

$$V_0^{Rec.} - V_0^{Payer} = P(0,T_m) E^T\left(\max(Fix - Float, 0) - \max(Float - Fix, 0) \right)$$

$$= P(0,T_m) E^T\left(\max(Fix, Float) - Float - \left(\max(Float, Fix) - Fix \right) \right)$$

$$= P(0,T_m) E^T\left(Fix - Float \right)$$

$\min(X,Y) + \max(X,Y) = X+Y$

$\max(Y-X,0) + X + \min(X-Y,0) + Y = X+Y$

$\max(Y-X,0) + \min(X-Y,0) = 0$

$\max(Y-X,0) = -\min((Y-X),0)$

$$V_0^{Rec} = V_0^{Payer} + \sum_{k=m+1}^{n} Y \tau_k P(0,T_k) + P(0,T_n) - P(0,T_m)$$