1

Z - TRANSFORM

Find the z-transform and the ROC of the signal

$$x(n) = -b^n u(-n-1)$$

The given signal is of infinite duration and anticausal

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$u(-n-1) = 0$ for $n \geq 0$
$= 1$ for $n \leq 1$

$$= -\sum_{n=-\infty}^{-1} b^n z^{-n} = -\sum_{n=1}^{\infty} (b^{-1} z)^{-n}$$

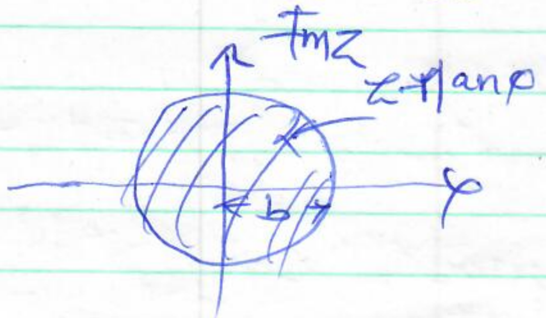
$$= -\left[\sum_{n=0}^{\infty} (b^{-1} z)^{n} - 1\right]$$

The above series coverage for $|b^{-1} z| < 1$

$|z| < b$

$$X(z) = -\left[\frac{1}{1-b^{-1}z} - 1\right]$$

$$= \frac{z}{z-b} \quad \text{ROC } |z| < b$$



(2)

Poles and zero of a system function

We know the system function

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} = \frac{1}{z^M} \sum_{k=0}^{M} b_k z^{M-k}.$$

System function reduces to

$$H(z) = \frac{b_0}{1 + \sum_{k=1}^{N} a_k z^{-k}}.$$

Ex) Determine the pole-zero plot for the system described by difference equation.

$$y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n) - x(n-1)$$

Taking z-transform on both sides.

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = X(z) - z^{-1} X(z)$$

$$Y(z)\left[1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right] = X(z)\left[1 - z^{-1}\right]$$

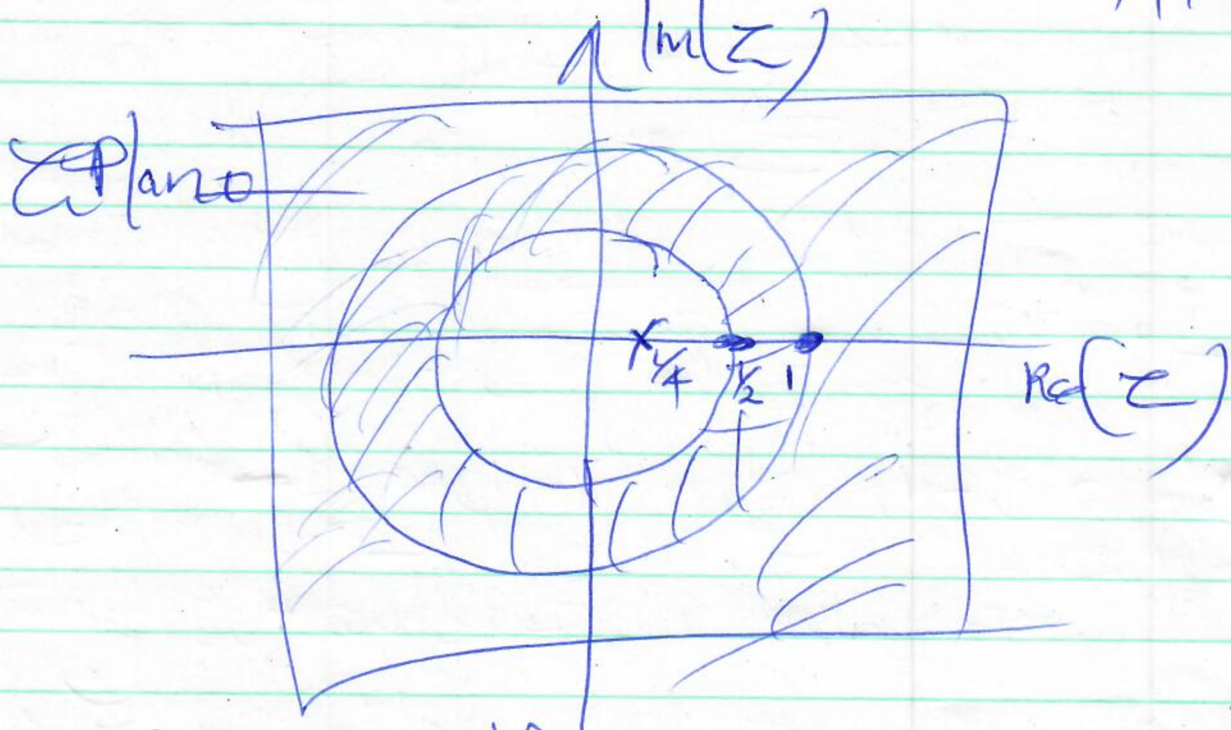
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

(3)

$$= \frac{1-z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)} \quad \text{ROC } |z| > \frac{1}{2}$$

$$\frac{z(z-1)}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{2}\right)}$$

Zeros at $z = 0, 1$ Poles at $z = \frac{1}{4}, \frac{1}{2}$



→ ROC of the system function includes unit circle
→ ROC of the system function cannot contain any poles

Find the poles-zeros plot for the system described by difference eq

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n) - x(n-1)$$

4

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}.$$

=> Find the system function and impulse response of the system described by the difference equation $y(n) = \frac{1}{5} y(n-1) + x(n)$

Taking z transform on both side we get

$$Y(z) = \frac{1}{5} z^{-1} Y(z) + X(z)$$

$$Y(z)\left[1 - \frac{1}{5} z^{-1}\right] = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{5} z^{-1}}$$

Find the inverse -z transform

$$h(n) = \left(\frac{1}{5}\right)^n u(n)$$

=> Find the system function and the impulse response of the system described by difference equation ~~$y(n) = x(n)$~~

$$y(n) = x(n) + 2x(n-1) - 4x(n-2) + x(n-3)$$

5

Z - TRANSFORM

1) Find the z-transform of the sequence

$$x(n) = \left(\frac{1}{3}\right)^{n-1} u(n-1)$$

We know the z-transform of the sequence $\left(\frac{1}{3}\right)^n u(n)$ as

$$Z\left[\left(\frac{1}{3}\right)^n u(n)\right] = \frac{z}{z-\frac{1}{3}}$$

using shift property we have

$$Z[x(n-1)] = z^{-1} X(z)$$

$$\therefore Z\left[\left(\frac{1}{3}\right)^{n} u(n-1)\right] = z^{-1} \frac{z}{z-\frac{1}{3}}$$

$$= \frac{1}{z-\frac{1}{3}}$$

F) The System function

The system is described by a linear constant coefficient difference equation of the form

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

Taking z transform on both sides, applying shifting property

$$Y(z) = -\sum_{k=1}^{N} a_k Y(z) z^{-k} + \sum_{k=0}^{M} b_k X(z) z^{-k}$$

$$Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = \sum_{k=0}^{M} b_k z^{-k} X(z)$$