Supervisory Control of Discrete Event Systems with State-dependent Controllability and Observability

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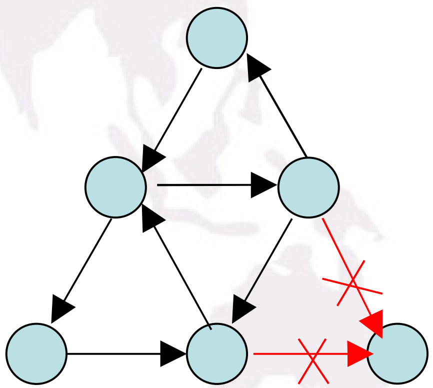
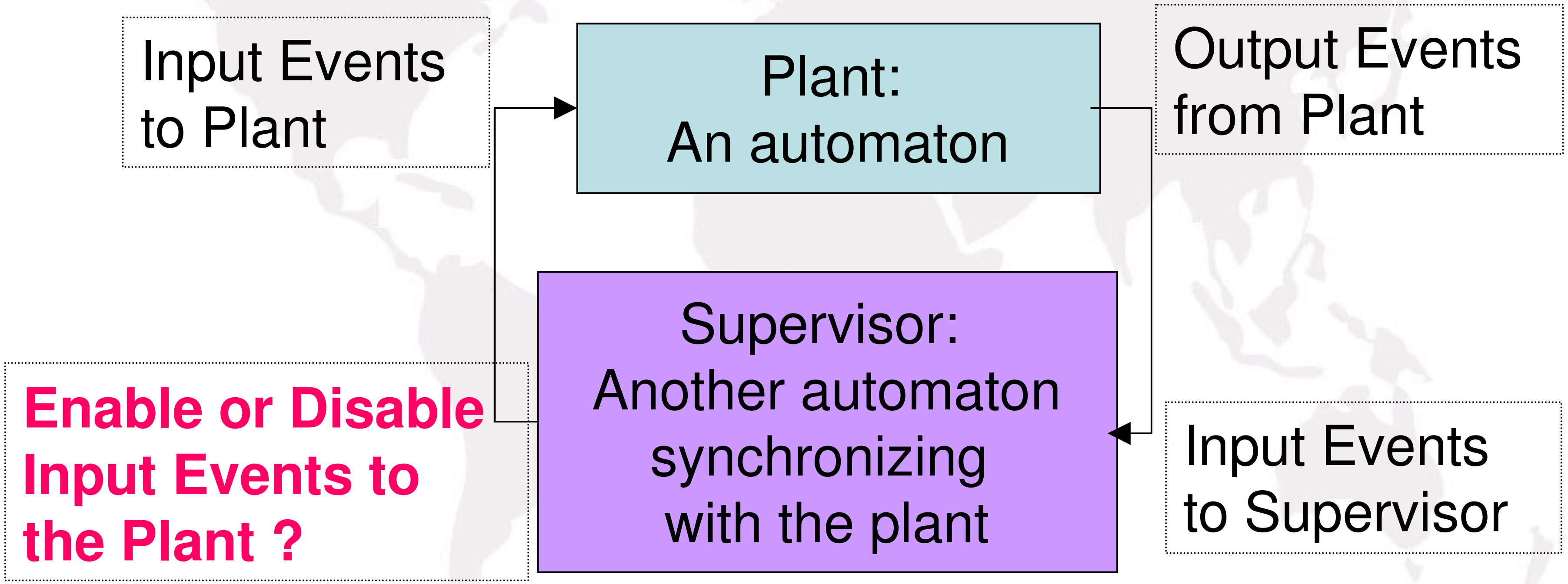
DES and SCT

- The supervisory control theory (SCT) of discrete event systems (DES) has been studied since 1980s.
- A widely-accepted framework was established by Dr. Ramadge and Dr. Wonham
  - Plant: a state transition model such as an automaton.
  - Control: Some state transitions are disabled in certain conditions.



Control and Feedback

- How is the supervisory control implemented?

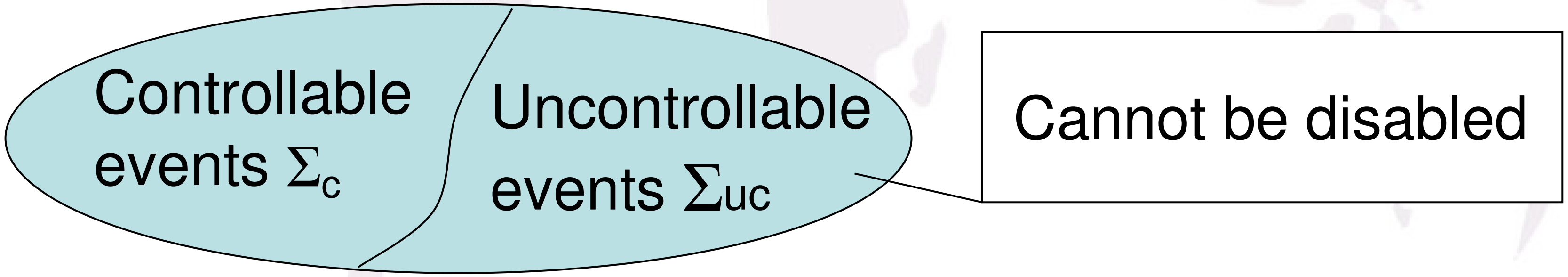


Fundamental Assumptions

- Controllability: We cannot control everything of a system. Only certain factors are controllable.
  - Some transitions can be disabled and some cannot.
- Observability: We cannot know everything about a system. Only partial information of the system can be obtained.
  - Input events are not equal to output events
  - Some information might be missing from input to output.

Assumption of Controllability

- The state transitions are directly caused by input events
- Only certain input events are possibly disabled
- The events imported to the automaton are partitioned into two disjoint subsets
  - Controllable subset and uncontrollable subset.
  - C: {*Input Event*} → {*Controllable, Uncontrollable*}

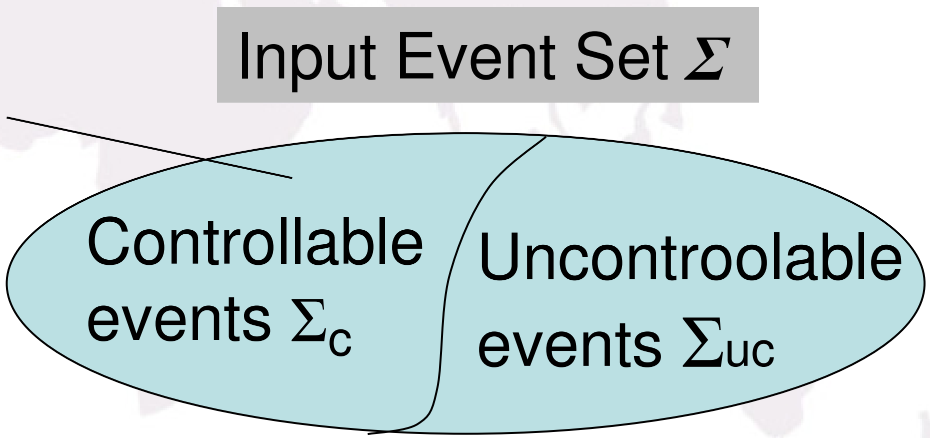


Assumption of Controllability

- The system is controlled by disabling certain controllable events that cause undesired state transitions.
- The uncontrollable events cannot be disabled.

Controllable events Σ_c

Some of them are disabled in certain states



Assumption of Observability

- Only partial information can be obtained
  - The input events are projected to output events
  - An observation function

$$M: \{Input\ Event\} \rightarrow \{Output\ Event\} \cup \{\varepsilon\}$$

  - Some information is missing from the input to the output of a system: *Using Empty String* $\boldsymbol{\varepsilon}$

Assumption of Observability

- A widely-used projection is that the input events are partitioned by two disjoint subsets, i.e., an observable subset and unobservable subset.

Σ

Unobservable events Σ_{uo}

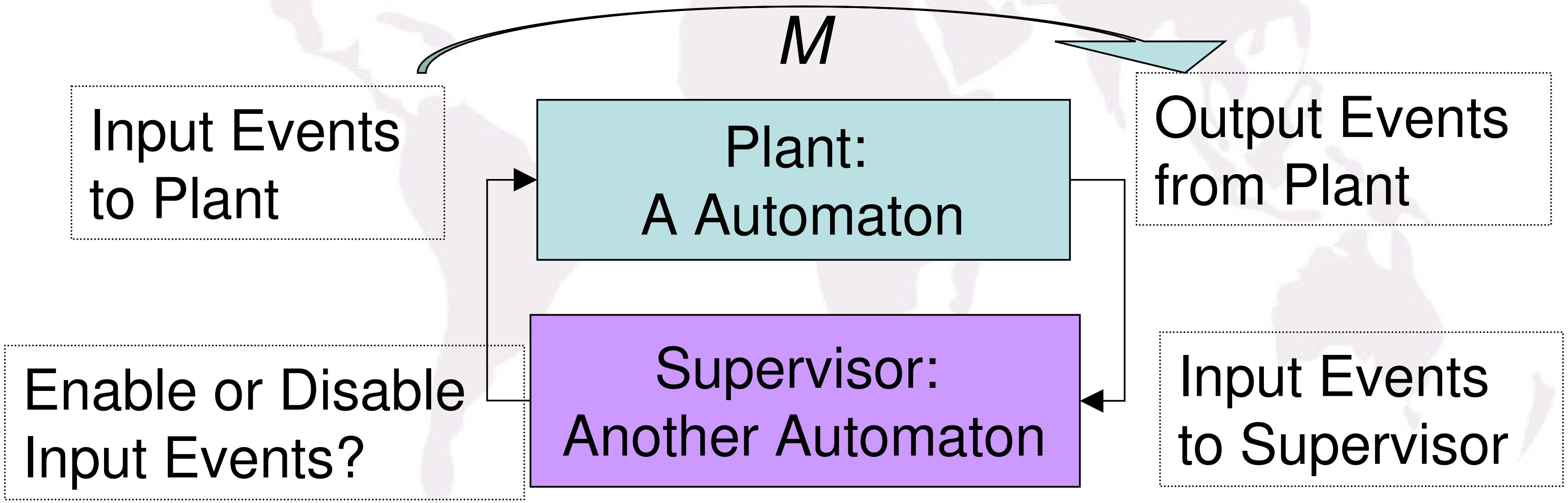
Observable events Σ_o

Mapped to empty string ε: Unobservalbe events imply that information is missing

Mapped to itself: Observalbe events imply that information is preserved in the output

Assumption of Observability

- Control of a system is implemented by using partial information provided by observable events.
- The observation function characterizes how much information the system provides to the outside, especially to the supervisor to be synthesized.

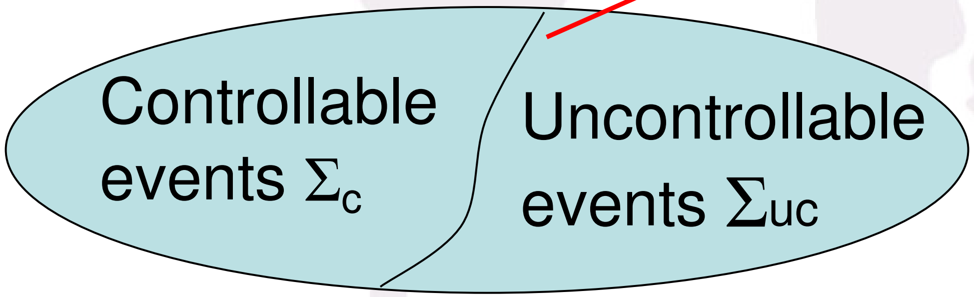


Assumptions in SCT

- In sum, the aforementioned study was established based on two important assumptions:
  - (1) We cannot control everything of a system. Only certain events are controllable.
  - (2) We cannot know everything about a system. Only certain events are observable.
- So far the assumptions have been followed up in most of branches in the supervisory control theory
- The controllability and observability of an event are **static properties:** the properties of an event do not change in the system evolvement

State-Dependency

- Our study will follow the assumptions, but will extend the original assumptions by making the controllability and observability of an event **<u>dynamically changing</u>** in the system evolvement
- The new model is consistent with the existing framework, and it is an extention of the traditional model.



The partition is NOT static, but dynamically changeable

State-Dependent Controllability

- The new model characterizes the situation that an event is controllable in one state while uncontrollable in another state.
- To depict the state-dependent controllability, a controllability function is introduced.

C: {*State*} × {*Input Event*} → {*Controllable, Uncontrollable*}

- The partition of controllable events and uncontrollable events is dynamically changeable as the system state transits from one to another.

State-Dependent Observability

- The state-dependent observability assumes that the output of observation does not only depend on the input events, but also the state where the system stays.
- To depict the state-dependent observability, the observation function is renewed.

$$\boldsymbol{M}\textbf{:}\ \boxed{\{\boldsymbol{State}\}} \times \{\boldsymbol{Input\ Event}\} \rightarrow \{\boldsymbol{Output\ Event}\} \cup \{\varepsilon\}$$

- In other words, the observability of an event is changeable as the system state transits from one to another.

Comparison

- Traditional Controllability and Observability
  - **C: {*Input Event*} → {*Controllable, Uncontrollable*}**
  - ***M*: {*Input Event*} → {*Output Event*}** ∪ {ε}
- State-Dependent Controllability and Observability
  - **C: {*State*} × {*Input Event*} → {*Controllable, Uncontrollable*}**
  - ***M*: {*State*} × {*Input Event*} → {*Output Event*}** ∪ {ε}

State-Dependent
Controllability and Observability

Solution

- Supervisor Existence Problem
  - A necessary and sufficient condition is derived for existence of a supervisor based on $(C,L(G))$-invariability and $(M, \Sigma_{uo}, L(G))$ of a language.
- Supervisor Synthesis Problem
  - Focus on how to synthesize a required supervisor to meet a given specification.
  - The solvability of the problem is first discussed. Given the specification as a regular language, algorithms are developed to synthesize a required supervisor if it exists.

Solution

- State-dependent controllability and observability are extension of the classic model in the tradititional framework, and the solution is also an extension of the classic results.
- The idea is obtained from some practial problems in computer science and manufacturing systems.
  - Coordination of Process and Interruptions Sevice Routine (ISR)
  - Flexible Manufacturing Systems: Transpot Lines with Load and Unload Machines

Solution

- Detailed mathematical solutions
  - P. Wang and K. Cai, Supervisory Control of Discrete Event Systems with State-Dependent Controllability (2009), International Journal of System Science. 40(4), 357-366.
  - P. Wang, X. Wang and K. Cai, Supervisory Control of Discrete Event Systems with State-Dependent Observability.

Thank You