Isotropic smoothing of image via Heat equation

In [8]:

```
from google.colab import drive
drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

In [9]:

```
cd /content/drive/MyDrive/ML
```

```
/content/drive/MyDrive/ML
```

import library

In [10]:

```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
from matplotlib import cm
import matplotlib.colors as colors
from skimage import color
from skimage import io
```

load input image

- filename for the input image is 'barbara_color.jpeg'

In [11]:

```
I0 = io.imread('barbara_color.jpeg')
```

check the size of the input image

In [12]:

```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
num_row     = I0.shape[0]
num_column  = I0.shape[1]
num_channel = I0.shape[2]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('number of rows of I0 = ', num_row)
print('number of columns of I0 = ', num_column)
print('number of channels of I0 = ', num_channel)
```

```
number of rows of I0 =  512
number of columns of I0 =  512
number of channels of I0 =  3
```

convert the color image into a grey image

In [13]:

```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
```

```
#
I = color.rgb2gray(I0)

num_row     = I0.shape[0]
num_column  = I0.shape[1]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('number of rows of I = ', num_row)
print('number of columns of I = ', num_column)
```

```
number of rows of I =  512
number of columns of I =  512
```

normalize the converted image

- normalize the converted grey scale image so that its maximum value is 1 and its minimum value is 0

In [14]:

```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks

I = (I - np.min(I))/(np.max(I) - np.min(I))

#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('maximum value of I = ', np.max(I))
print('minimum value of I = ', np.min(I))
```

```
maximum value of I =  1.0
minimum value of I =  0.0
```

define a function to compute the derivative of input matrix in x(row)-direction

- forward difference : $I[x+1,y] - I[x,y]$

In [15]:

```
def compute_derivative_x_forward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    i0 = np.roll(I, shift = -1, axis = 0)
    for j in range(0 , num_column):
      i0[num_row-1][j] = i0[num_row-2][j]
    D = i0 - I

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

- backward difference : $I[x,y] - I[x-1,y]$

In [26]:

```
def compute_derivative_x_backward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    i0 = np.roll(I, shift = 1, axis = 0)
    for j in range(0 , num_column):
      i0[num_row-1][j] = i0[num_row-2][j]
    D = I - i0




    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

define a function to compute the derivative of input matrix in y(column)-direction

- forward difference : $I[x, y+1] - I[x, y]$

In [2]:

```
def compute_derivative_y_forward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    i0 = np.roll(I, shift = -1, axis = 1)
    for i in range(0 , num_column):
      i0[i][num_row-1] = i0[i][num_row-2]
    D = i0 - I


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

- backward difference : $I[x, y] - I[x, y-1]$

In [1]:

```
def compute_derivative_y_backward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    i0 = np.roll(I, shift = 1, axis = 1)
    for i in range(0 , num_column):
      i0[i][num_row-1] = i0[i][num_row-2]
    D = I - i0
```

```
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

define a function to compute the laplacian of input matrix

- $\Delta I = \nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$
- $\Delta I = I[x+1, y] + I[x-1, y] + I[x, y+1] + I[x, y-1] - 4 * I[x, y]$
- ΔI = derivative_x_forward - derivative_x_backward + derivative_y_forward - derivative_y_backward

In [7]:

```
def compute_laplace(I):

    laplace = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    laplace = (compute_derivative_x_forward(I) - compute_derivative_x_backward(I) + c

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return laplace
```

define a function to compute the heat equation of data I with a time step

- $I = I + \delta t * \Delta I$

In [28]:

```
def heat_equation(I, time_step):

    I_update = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    delta_t = time_step

    I_update = I + time_step*compute_laplace(I)

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return I_update
```

run the heat eq`uation over iterations

In [34]:

```
def run_heat_equation(I, time_step, number_iteration):

    I_update = np.zeros(I.shape)
```

```
    #I_update = heat_equation(I, time_step)
    for t in range(number_iteration):
        # ++++++++++++++++++++++++++++++++++++++++++++++++++
        # complete the blanks
        #
        if t == 0 :
          I_update = heat_equation(I, time_step)

        else:
          I_update = heat_equation(I_update, time_step)


        #
        # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return I_update
```

---

functions for presenting the results

In [4]:

```
def function_result_01():

    plt.figure(figsize=(8,6))
    plt.imshow(I0)
    plt.show()
```

In [16]:

```
def function_result_02():

    plt.figure(figsize=(8,6))
    plt.imshow(I, cmap='gray', vmin=0, vmax=1, interpolation='none')
    plt.show()
```

In [17]:

```
def function_result_03():

    L = compute_laplace(I)

    plt.figure(figsize=(8,6))
    plt.imshow(L, cmap='gray')
    plt.show()
```

In [18]:

```
def function_result_04():

    time_step   = 0.25
    I_update    = heat_equation(I, time_step)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [19]:

```
def function_result_05():
```

```python
    time_step           = 0.25
    number_iteration    = 128

    I_update = run_heat_equation(I, time_step, number_iteration)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [20]:

```python
def function_result_06():

    time_step           = 0.25
    number_iteration    = 512

    I_update = run_heat_equation(I, time_step, number_iteration)

    plt.figure(figsize=(8,6))
    plt.imshow(I_update, vmin=0, vmax=1, cmap='gray')
    plt.show()
```

In [21]:

```python
def function_result_07():

    L = compute_laplace(I)

    value1 = L[0, 0]
    value2 = L[-1, -1]
    value3 = L[100, 100]
    value4 = L[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [22]:

```python
def function_result_08():

    time_step   = 0.25
    I_update    = heat_equation(I, time_step)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [23]:

```python
def function_result_09():

    time_step           = 0.25
    number_iteration    = 128

    I_update = run_heat_equation(I, time_step, number_iteration)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
```

```
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [24]:

```
def function_result_10():

    time_step           = 0.25
    number_iteration    = 512

    I_update = run_heat_equation(I, time_step, number_iteration)

    value1 = I_update[0, 0]
    value2 = I_update[-1, -1]
    value3 = I_update[100, 100]
    value4 = I_update[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

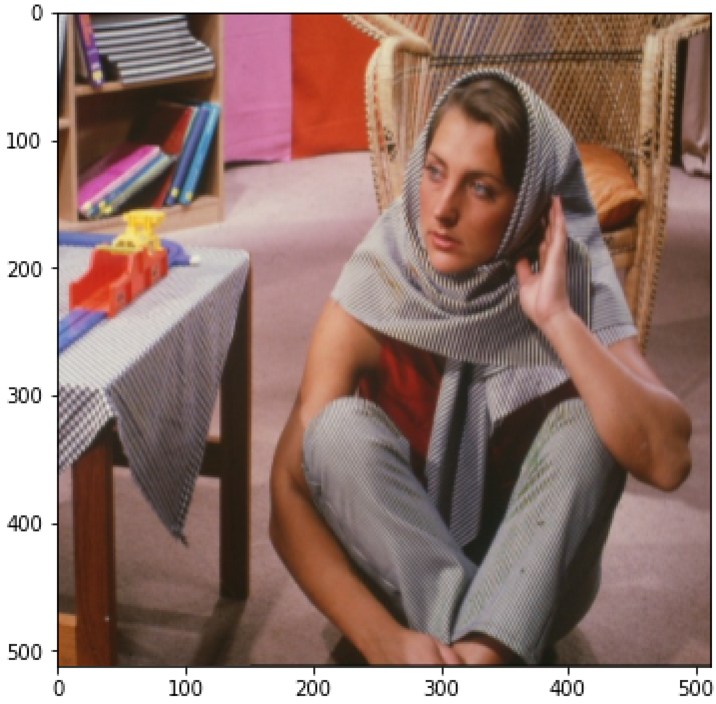
---

results

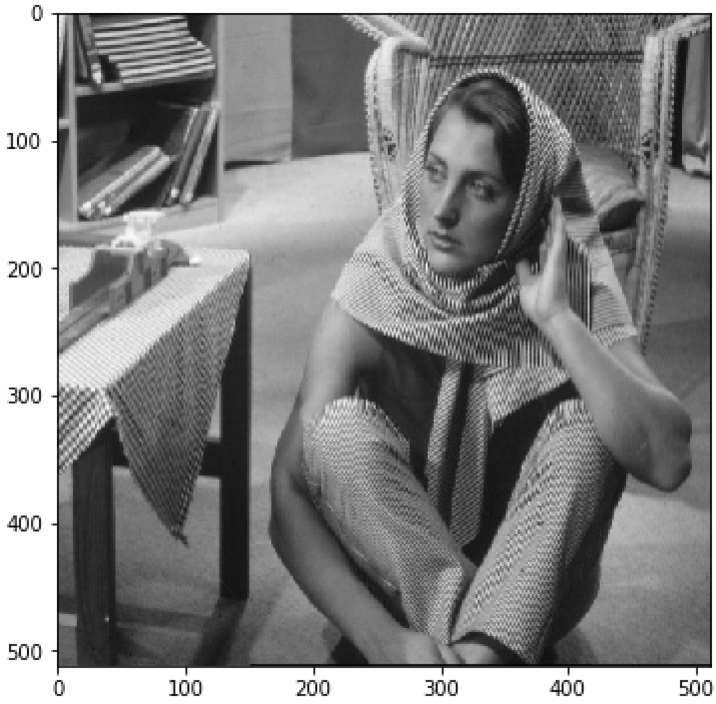
In [35]:

```
number_result = 10

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('**************************************************')
    print(title)
    print('**************************************************')
    eval(name_function)
```

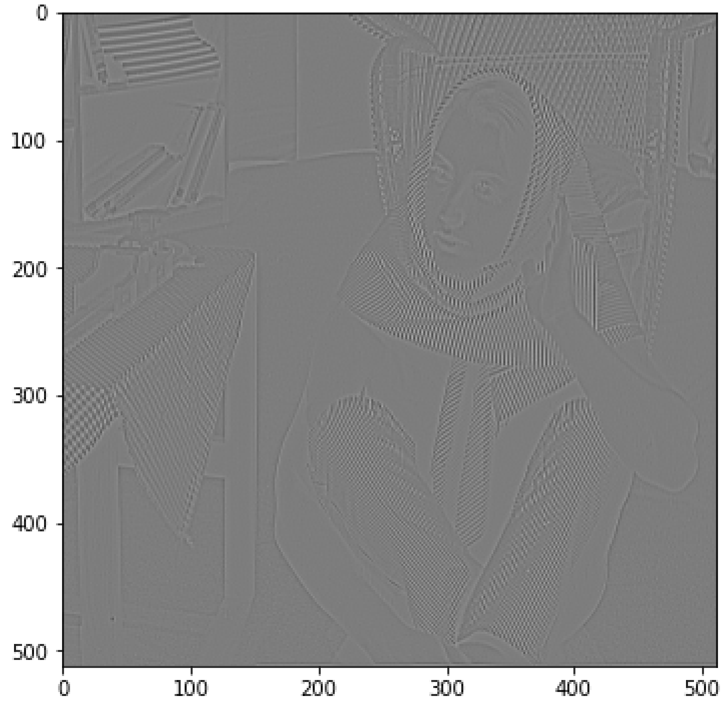
```
**************************************************
## [RESULT 01]
**************************************************
```



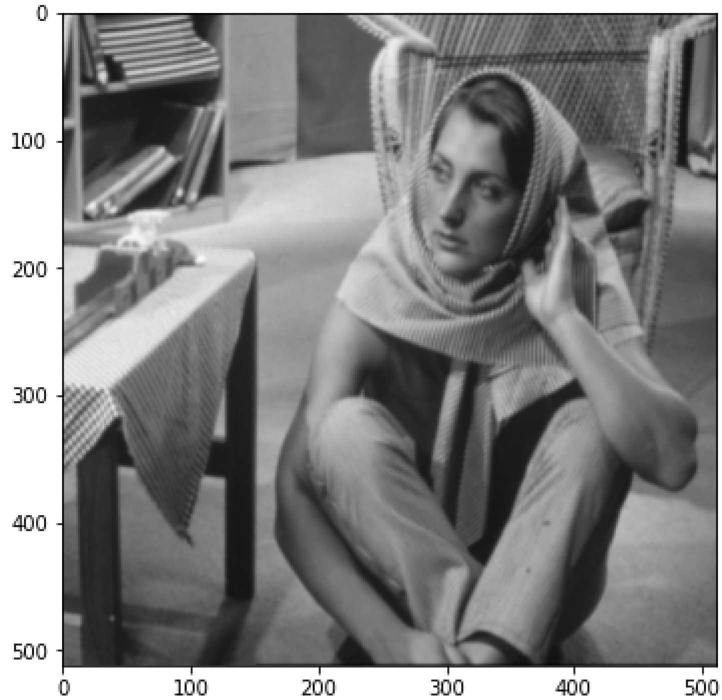
****
[RESULT 02]
****



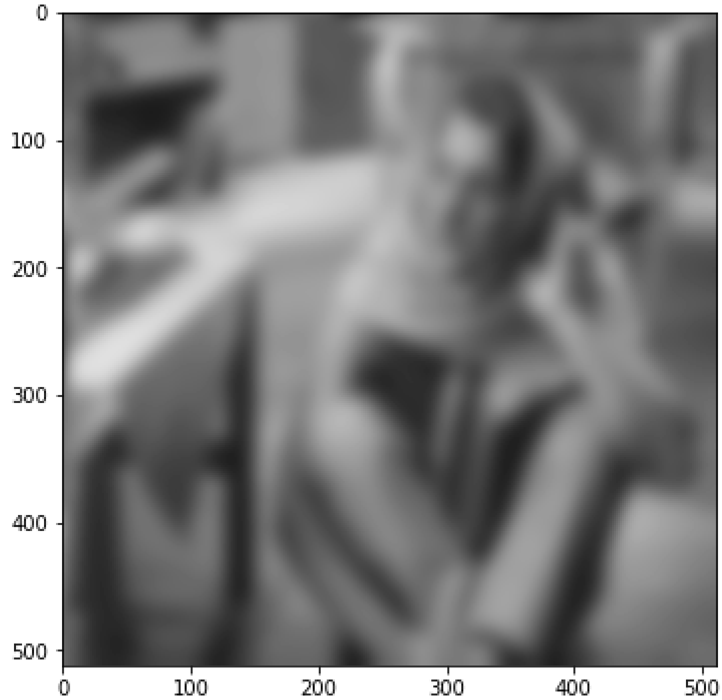
****
[RESULT 03]
****



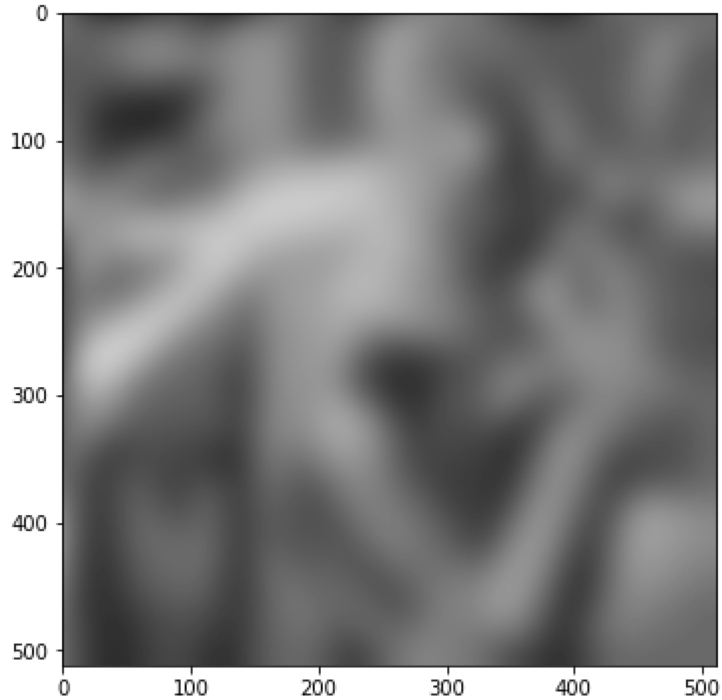
****
[RESULT 04]
****



****
[RESULT 05]
****



****
[RESULT 06]
****



****
[RESULT 07]
****
value1 = 1.214975715366264
value2 = 0.36458415157353663
value3 = -0.11096969959074021
value4 = -0.057368751329410106
****
[RESULT 08]
****
value1 = 0.32823288359174274
value2 = 0.16498050345050977
value3 = 0.5126456173363071
value4 = 0.5929656202312226
****
[RESULT 09]

```
**************************************************
value1 =  0.4167787629810381
value2 =  0.4198011975571742
value3 =  0.3678389043817923
value4 =  0.6014962322332662
**************************************************
## [RESULT 10]
**************************************************
value1 =  0.41029130348867193
value2 =  0.4174457523498733
value3 =  0.3512767875894531
value4 =  0.6310803621724141
```

In [ ]: