Linear regression

import library

```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
import matplotlib.colors as colors
from mpl_toolkits.mplot3d import Axes3D
```

```
from google.colab import drive
drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

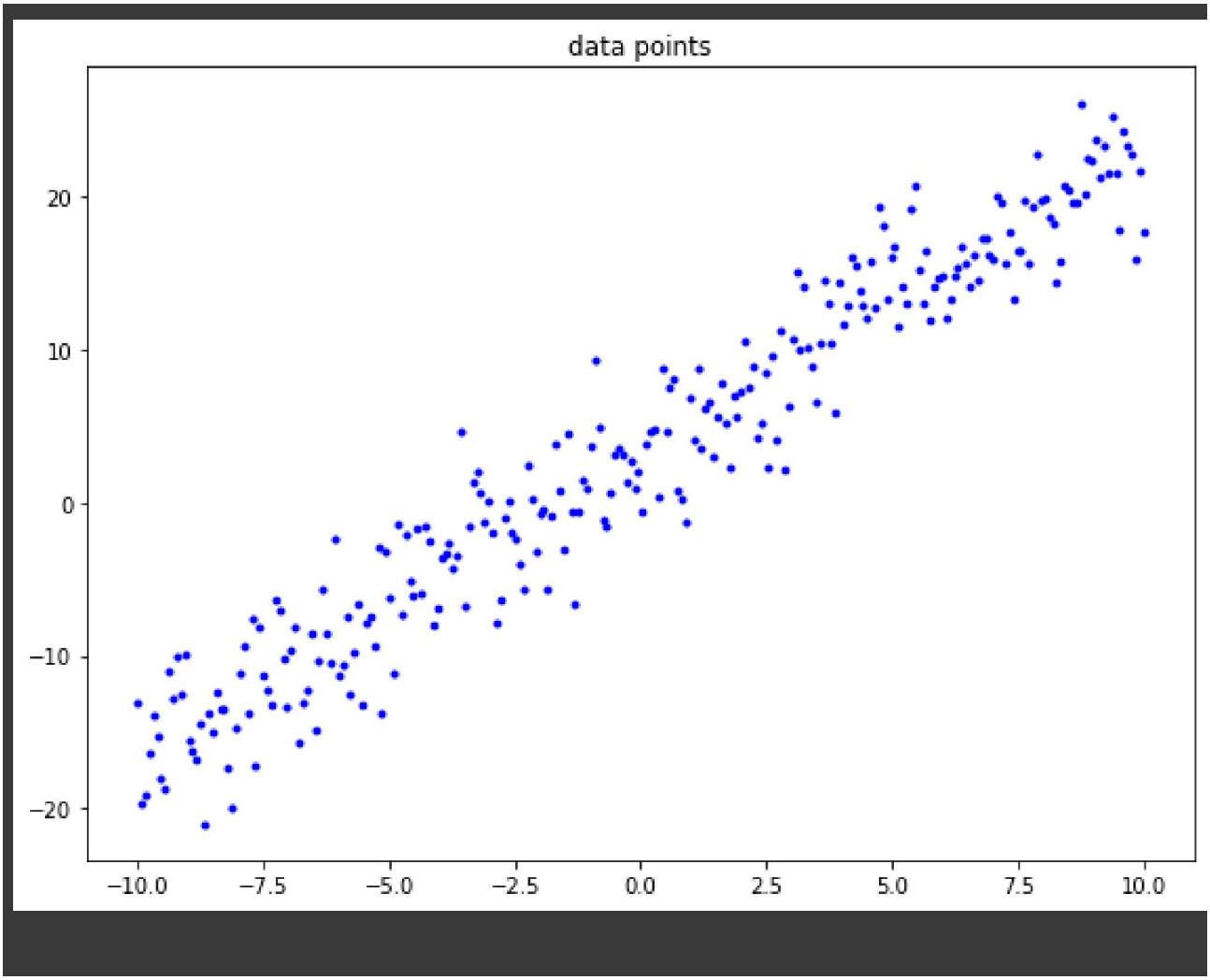
```
cd /content/drive/MyDrive/ML
```

```
/content/drive/MyDrive/ML
```

load data points

- $\{(x_i, y_i)\}_{i=1}^{n}$

```
filename    = 'assignment_06_data.csv'
data        = np.loadtxt(filename, delimiter = ',')

x           = data[0, :]    # independent variable
y           = data[1, :]    # dependent variable

plt.figure(figsize=(8,6))

plt.plot(x, y, '.', color = 'blue')
plt.title('data points')

plt.tight_layout()
plt.show()
```



compute the residual

```
def compute_residual(x, y, theta0, theta1):

    n           = len(x)
    residual    = np.zeros(n)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    residual = theta0 + theta1*x - y

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return residual
```

compute the loss

- useful functions: `np.inner`

```
def compute_loss(x, y, theta0, theta1):

    n           = len(x)
    loss        = 0

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    r = compute_residual(x, y, theta0, theta1)
    loss = np.sum(1/2/n*(np.inner(r , r)))

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return loss
```

compute the gradient with respect to θ_0

- useful functions: `np.inner`

```
def compute_gradient_theta0(x, y, theta0, theta1):

    n           = len(x)
    derivative  = 0

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    temp = np.ones(n)
    derivative = 1/n*(np.inner(compute_residual(x,y,theta0,theta1), temp))

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return derivative
```

compute the gradient with respect to θ_1

- useful functions: `np.inner`

```
def compute_gradient_theta1(x, y, theta0, theta1):

    n           = len(x)
    derivative  = 0

    # +++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    derivative = 1/n*(np.inner(compute_residual(x,y,theta0,theta1),x))

    #
    # +++++++++++++++++++++++++++++++++++++++++++++++++++

    return derivative
```

gradient descent for the optimization

```
number_iteration    = 500
learning_rate       = 0.01

theta0              = 0
theta1              = 0

list_theta0         = np.zeros(number_iteration)
list_theta1         = np.zeros(number_iteration)
list_loss           = np.zeros(number_iteration)

for i in range(number_iteration):

    # +++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    theta0  = theta0-learning_rate*compute_gradient_theta0(x,y,theta0, theta1)
    theta1  = theta1-learning_rate*compute_gradient_theta1(x,y,theta0, theta1)
    loss    = compute_loss(x,y,theta0, theta1)
    #
    # +++++++++++++++++++++++++++++++++++++++++++++++++++

    list_theta0[i]  = theta0
```

```
    list_theta1[i]  = theta1
```

---

functions for presenting the results

```
def function_result_01():

    plt.figure(figsize=(8,6))
    plt.plot(x, y, '.', color='blue')
    plt.title('data points')
    plt.show()
```

```
def function_result_02():

    plt.figure(figsize=(8,6))
    ax = plt.gca()
    plt.plot(list_theta0, '-', color='blue', label=r'$Wtheta_0$')
    plt.plot(list_theta1, '-', color='red', label=r'$Wtheta_1$')
    plt.title('model parameters')
    ax.legend()
    plt.show()
```

```
def function_result_03():

    plt.figure(figsize=(8,6))
    plt.plot(list_loss, '-', color='blue')
    plt.title('loss curve')
    plt.show()
```

```
def function_result_04():

    f = theta0 + theta1 * x
```

```
    plt.figure(figsize=(8,6))
    ax = plt.gca()
    plt.plot(x, y, '.', color='blue', label='data point')
    plt.plot(x, f, '-', color='red', label='regression')
    plt.title('regression')
    ax.legend()
    plt.show()
```

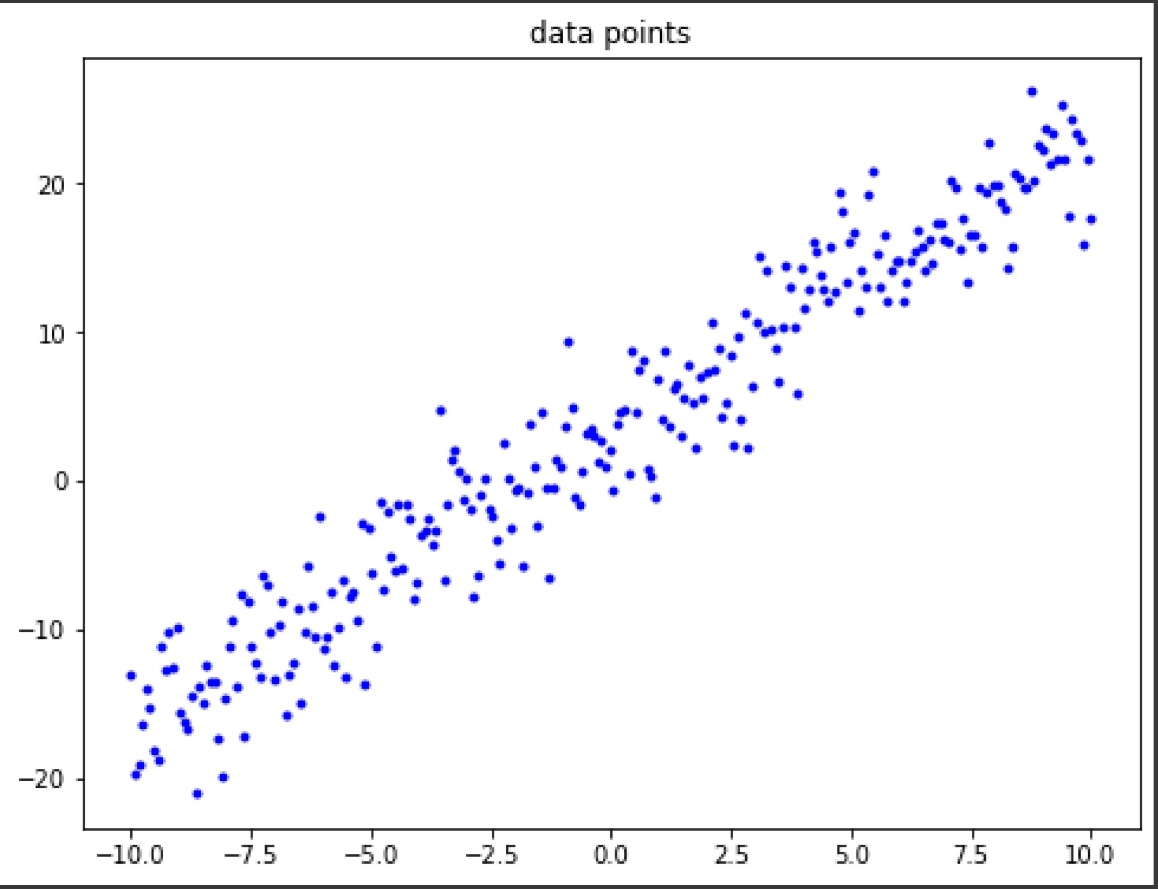
```
def function_result_05():

    X0  = np.arange(-10, 10, 0.1)
    X1  = np.arange(-10, 10, 0.1)

    grid_theta0, grid_theta1 = np.meshgrid(X0, X1)

    grid_loss   = np.zeros(grid_theta0.shape)

    for i, t0 in enumerate(X0):
        for j, t1 in enumerate(X1):

            grid_loss[j, i] = compute_loss(x, y, t0, t1)

    fig = plt.figure(figsize=(8,6))

    ax  = fig.add_subplot(111, projection='3d')
    plt.title('loss surface')

    ax  = plt.axes(projection='3d')

    ax.set_xlabel(r'$Wtheta_0$')
    ax.set_ylabel(r'$Wtheta_1$')
    ax.set_zlabel('loss')

    ax.plot_surface(grid_theta0, grid_theta1, grid_loss, rstride=1, cstride=1, cmap=

    plt.tight_layout()
    plt.show()
```

---

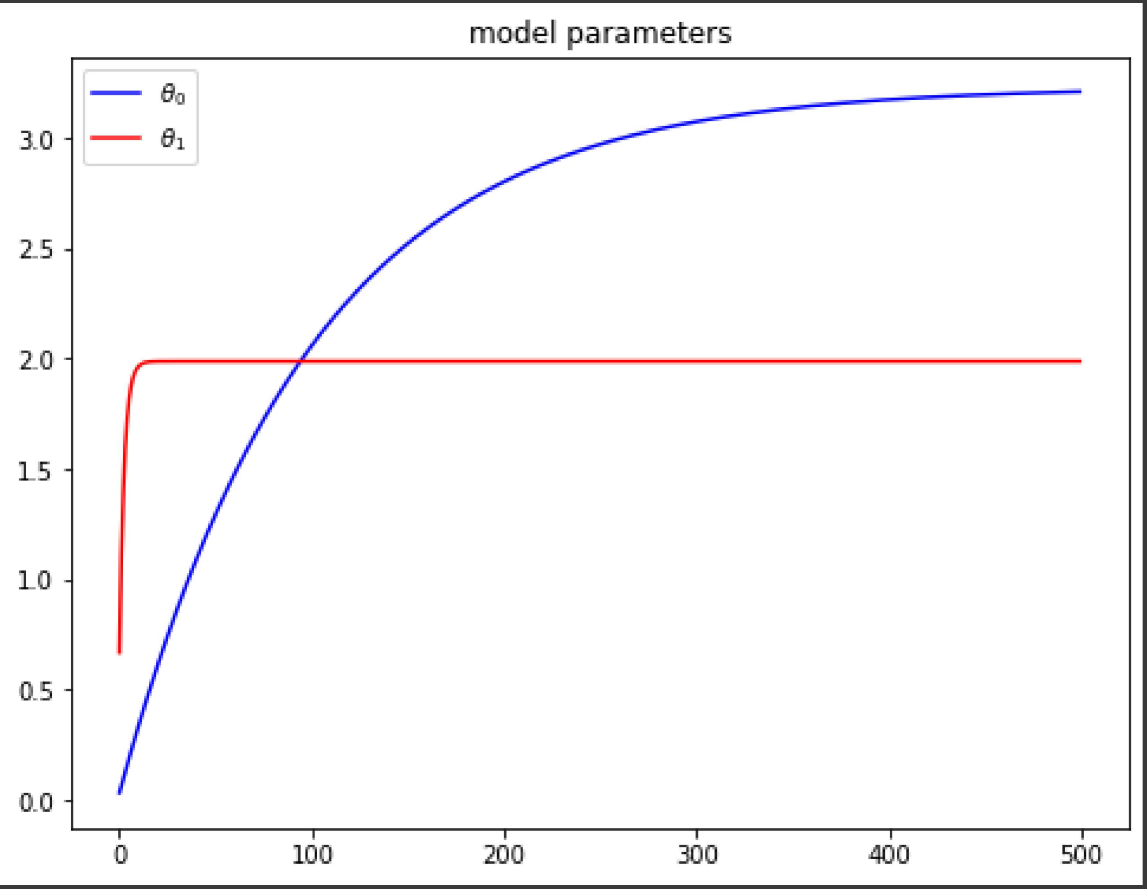
results

```
number_result = 5

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('************************************************')
    print(title)
    print('************************************************')
    eval(name_function)
```

```
**************************************************
## [RESULT 01]
**************************************************
```

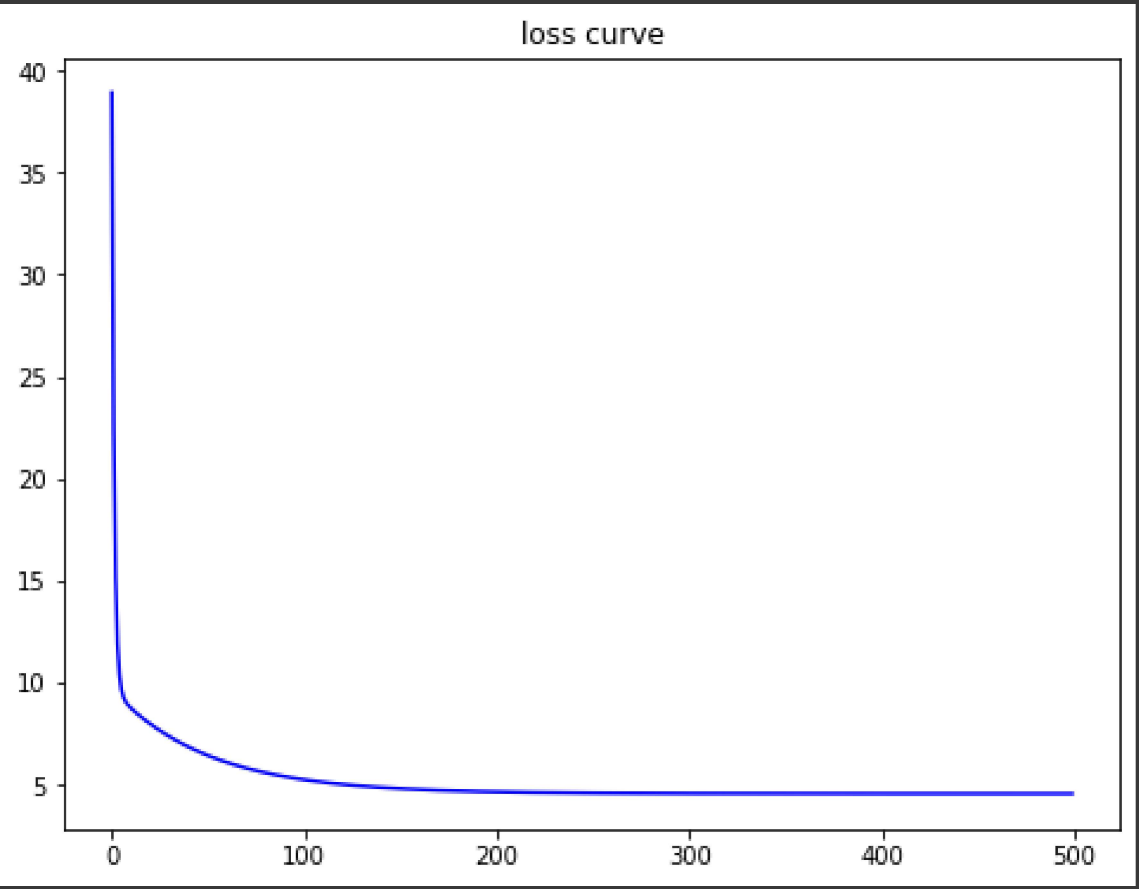



```
**************************************************
## [RESULT 02]
**************************************************
```

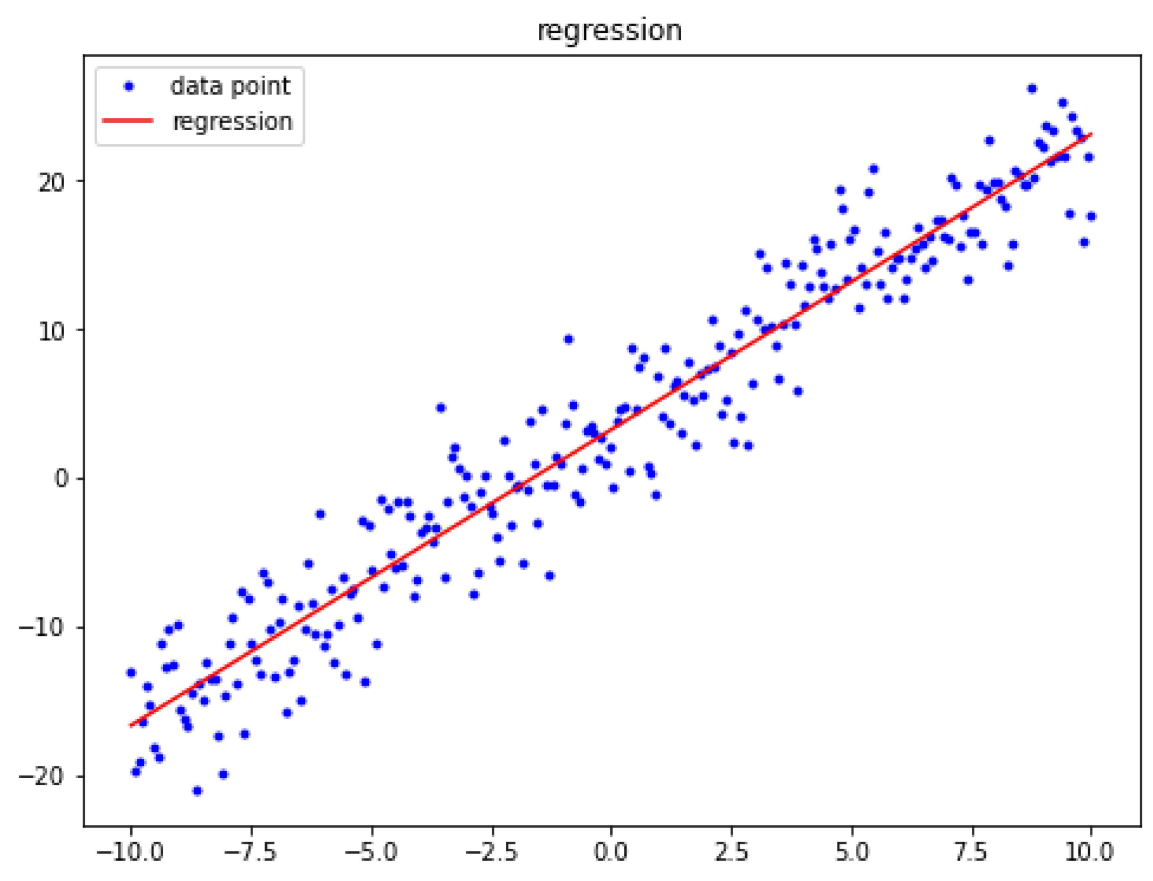



```
**************************************************
```

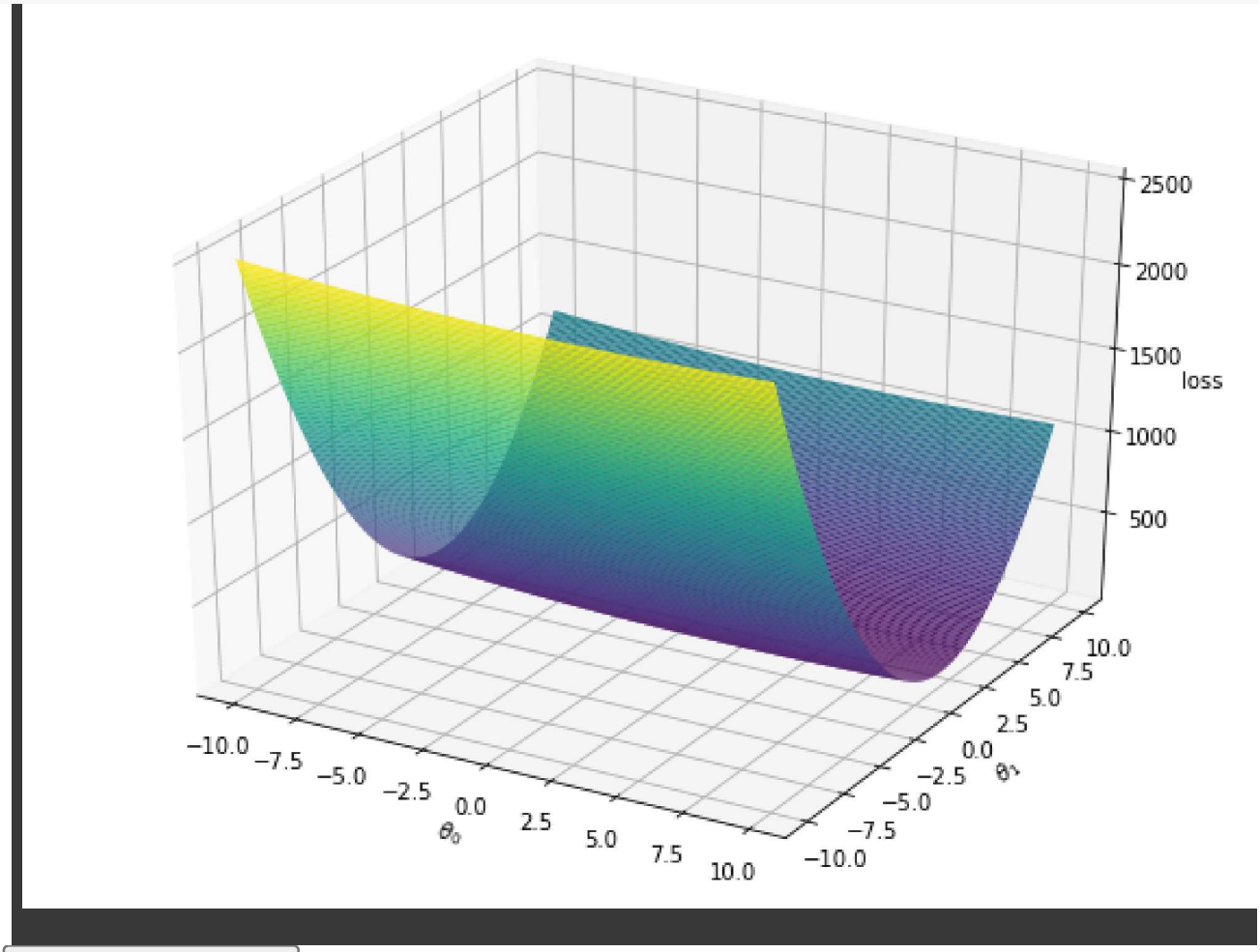
```
## [RESULT 03]
**************************************************
```




```
**************************************************
## [RESULT 04]
**************************************************
```




```
**************************************************
```



check
6초

오후 9:52에 완료됨