Question 1: Please refer to the class notes for more details.

Assumption: No parallel edges in the graph.

$$c_f(u \to v) = \begin{cases} c(u\to v) - f(u \to v) & \text{if } u \to v \in E, \\ f(v \to u) & \text{if } v \to u \in E, \\ 0 & \text{otherwise.} \end{cases}$$

2 marks for this defn.

Residual graph: $G_{res} = (V, E_{res})$ // Vertex set remains the same.

$$u \to v \in E_{res} \quad \text{iff} \quad c_f(u \to v) > 0.$$

1 mark for this.

Question 2:

Case: Ordering matters

Recursive defn: $f(n) = f(n-1) + f(n-2)$

Base cases: $f(1) = 1$, $f(2) = 2$.

Case: Ordering does not matter.

Equivalently, we are looking for no. of integral solutions to $2x + y = n$.

$$N(n, x, y) = \sum_{i=1}^{n/2} N(n, x, i)$$

No. of integral slns to $2x+y=n$

# of integral solutions to $2x + i = n$

Give partial marking as you desire.

Question 3: Follow the hint. 1 mark for the set up.

For $i \in [0,n]$ and $j \in [0,m]$

$M_{i,j}$ entry is 1 if $S_3[1, i+j]$ is formed by interleavi-ing of $S_1[1,i]$ and $S_2[1,j]$ in some order.

Consider $M_{i+1,j}$. This is true if $M_{i,j}$ is true and if $S_1[i+1] = S_3[i+j+1]$.

$\Big\}$ 2 marks for this

Similarly, $M_{i,j+1}$ is true if $M_{i,j}$ is true and $S_2[j+1] = S_3[i+j+1]$. 2 marks for this

Base cases: $M_{0,j}$ = true iff $S_3[1,j] = S_2[1,j]$ $\forall j \in [1,m]$

2 marks for base case. $M_{i,0}$ = true iff $S_3[1,i] = S_1[1,i]$ $\forall i \in [1,n]$

$M_{0,0}$ = true.