Computing in Sciences: Spring 2022
ROLL NUMBER: 2021113008

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1. Let $z(t)$ be a closed curve in complex plane parametrized by t such that $z(t) = z(t+T)$. Assume that the function can be represented by series as:

$$z(t) = \sum_{n=0}^{n=\infty} c_n e^{(i2\pi nt/T)} \tag{1}$$

find c_n; note that constant c_n can be complex.

$$\cancel{z(t) = \sum_{n=0}^{\infty} (a+ib)\, e^{\frac{i2\pi nt}{T}}}$$

$$\cancel{= \sum_{n=0}^{\infty} a e^{\frac{i2\pi nt}{T}} + i \sum_{n=0}^{\infty} b e}$$

$$\cancel{= \sum_{n=0}^{\infty} (a+ib)\left(\cos\frac{2\pi nt}{T} + i\sin\frac{2\pi nt}{T}\right)}$$

$$\cancel{= \sum_{n=0}^{\infty}\left(a\cos\frac{2\pi nt}{T} - b\sin\frac{2\pi nt}{T}\right) + i\sum_{n=0}^{\infty}\left(a\sin\frac{2\pi nt}{T} + b\cos\frac{2\pi nt}{T}\right)}$$

$$z(t) = \sum c_n e^{\frac{i2\pi nt}{T}}$$

$$z(t+T) = \sum c_n e^{\frac{i2\pi nt}{T} + i} = \sum c_n e^{\frac{i2\pi n(t+T)}{T}} \cancel{\times e^{i}}$$

$$= \cancel{z(t) \times e^{i}}$$

$$= \sum c_n e^{\frac{i2\pi n}{T} + i2\pi n}$$

$$\int_0^T dt\, z(t)\, e^{-\frac{i2\pi nt}{T}}.$$

$$z(t+T) = \sum z(t) \times e^{i2\pi n}.$$

$$e^{i2\pi n} = 1$$

Let $f(t)$ is a continuous differentiable function with a period T, i.e. $f(t+T) = f(t)$ for any real t. If the function is approximated as

$$f(t) = \sum_{n=0}^{n=\infty} \left(a_n \cos\left(2\pi n \frac{t}{T}\right) + b_n \sin\left(2\pi n \frac{t}{T}\right)\right) \tag{2}$$

determine the constants a_n and b_n