SCI 1.102 : Classical Mechanics

Quiz : 21 Apr 2022

Time : 45 minutes

Max. marks = 25

1. For the figures below:

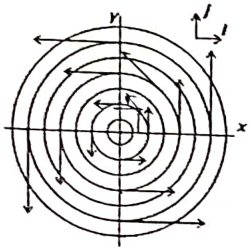

Fig. 1

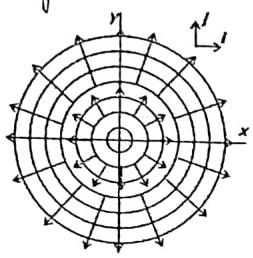

Fig. 2

(a) Find suitable values of values of a and b for the vectors depicted, $\vec{V} = a\hat{i} + b\hat{j}$;

(b) Obtain the magnitudes of the gradient and the curl for each case. 2+3

2. For a particle rotating in a 2-D space

(a) Derive the expression for kinetic energy in polar coordinates;

(b) Construct the Lagrangian, and write Hamilton's equations of motion for generalised coordinates for the system.

(c) Using Hamilton's equations, show that angular momentum for the problem is a conserved quantity. 2+2+2

3. Show that the Hamiltonian of a system is a Legendre transform of its Lagrangian. 4

4. A particle is subjected to the potential $V(x) = -C_1 x$, where C_1 is a constant. The particle travels from $x = 0$ to $x = a$ in a time interval t_0. Assume that the motion can be expressed as $x(t) = A + Bt + Ct^2$, Find A, B and C, such that the action is a minimum. 5

5. An object of 100 N suspended from the end of a vertical spring of negligible mass stretches it by 0.16m. ($mg = 100N$) $g = 10\ m/s^2$

(a) Write an equation of motion for the variable z [Hint : remember gravity and use Hooke's law] .

(b) Determine the position of the object at any time if it is pulled down 0.05m and then released.

(c) Find the amplitude, period and frequency of the motion.

(d) If in part (b) above, while releasing the object, it is given an initial velocity of 0.05m/s, how will the results in (b) and (c) change? $1+1+2\times 1\frac{1}{2}$