Assignment 1: Due on Mon, 24 Aug 2020

Q1) A particle of mass m is in the state, $\psi(x,t) = A.\exp(-a[(\frac{m}{\hbar}x^2) + it])$, where A and a are real positive constants.

(a) Find A.

(b) For what potential energy function $V(x)$ does $\psi(x,t)$ satisfy the Schrödinger equation ?

(c) Calculate the expectation values of $\hat{x}$, $\hat{x}^2$, $\hat{p}$, $\hat{p}^2$.

Q2) Normalise the wavefunction,

$\psi(x,t) = A.\sin(2\pi x/a)e^{(-\frac{iEt}{\hbar})}; \; \forall -\frac{a}{2} < x < \frac{a}{2}$

$= 0; \; \forall x < -\frac{a}{2} \; or \; x > +\frac{a}{2}$,

such that the total probability of finding the particle in the region of length a is 1.

Q3) Why does the probability density function have to be everywhere real, non-negative, and of finite and definite.

Q4) Define an operator $\hat{\mathcal{O}} = \left[\hat{A},\, \hat{B}\right]$

s.t. $\hat{\mathcal{O}}f == \hat{A}\hat{B}f - \hat{B}\hat{A}f$, where f is a function on which the operators $\hat{A}$ and $\hat{B}$ act.

[$\therefore$we can write an operator identity, $\left[\hat{A},\, \hat{B}\right] = \hat{A}\hat{B} - \hat{B}\hat{A}$; (commutator of $\hat{A}$ and $\hat{B}$)]

Show : (i) $[\hat{x},\, \hat{p}] = i\hbar\hat{1}$, where $\hat{1}$ is the identity operator

(ii) $\left[\hat{A},\, \hat{B}\hat{C}\right] = \left[\hat{A},\, \hat{B}\right]\hat{C} + \hat{B}\left[\hat{A},\, \hat{C}\right]$

(iii) $\left[\hat{A}\hat{B},\, \hat{C}\right] = \hat{A}\left[\hat{B},\, \hat{C}\right] + \left[\hat{A},\, \hat{C}\right]\hat{B}$