Quantum Mechanics

aka

Magic!

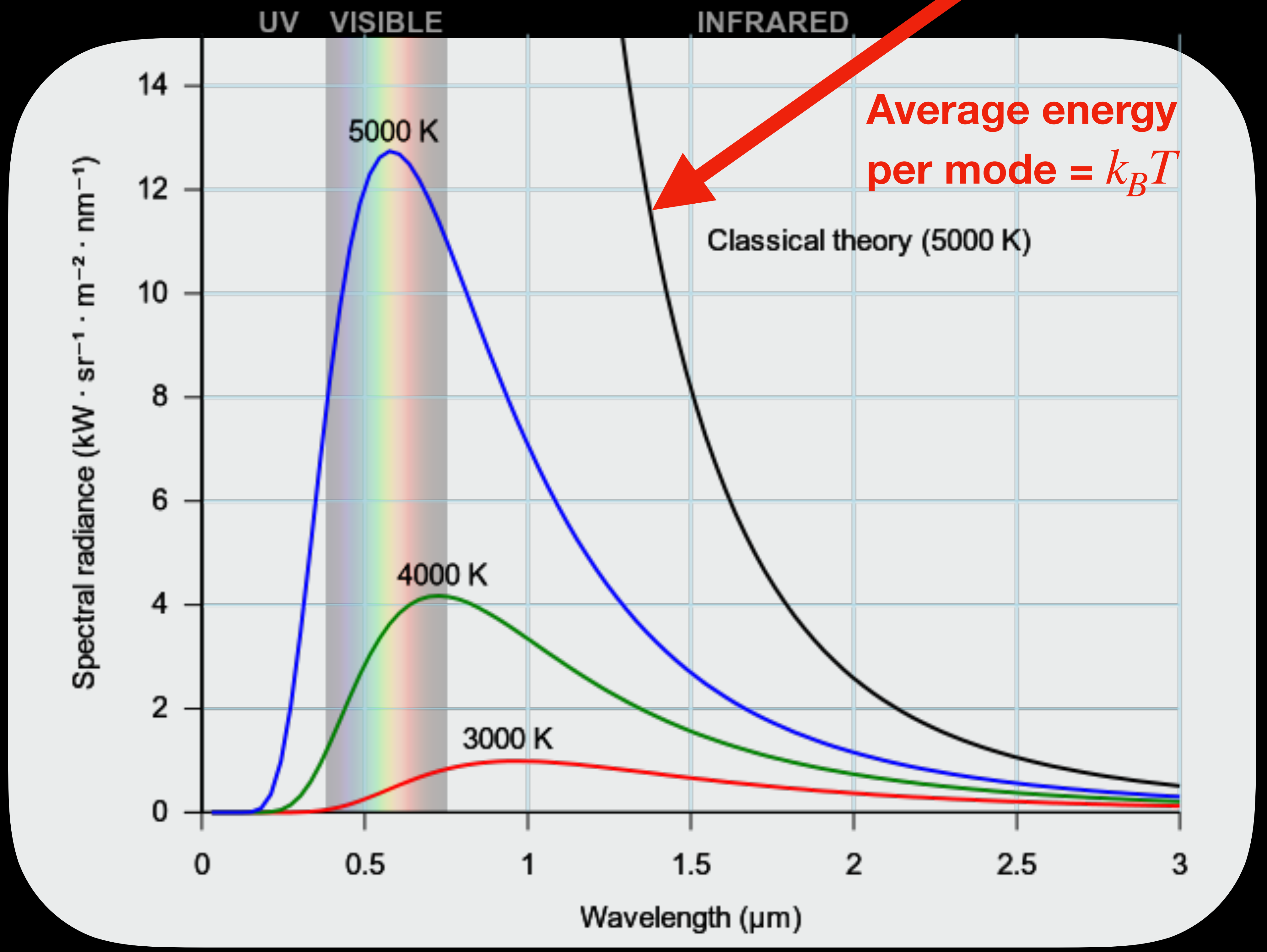
Imaging Bell-type nonlocal behavior, Moreau et al., Science Advances, Vol. 5, no. 7, eaaw2563 (2019)
https://advances.sciencemag.org/content/5/7/eaaw2563

Why Quantum?

Blackbody Radiation: Ultraviolet Catastrophe

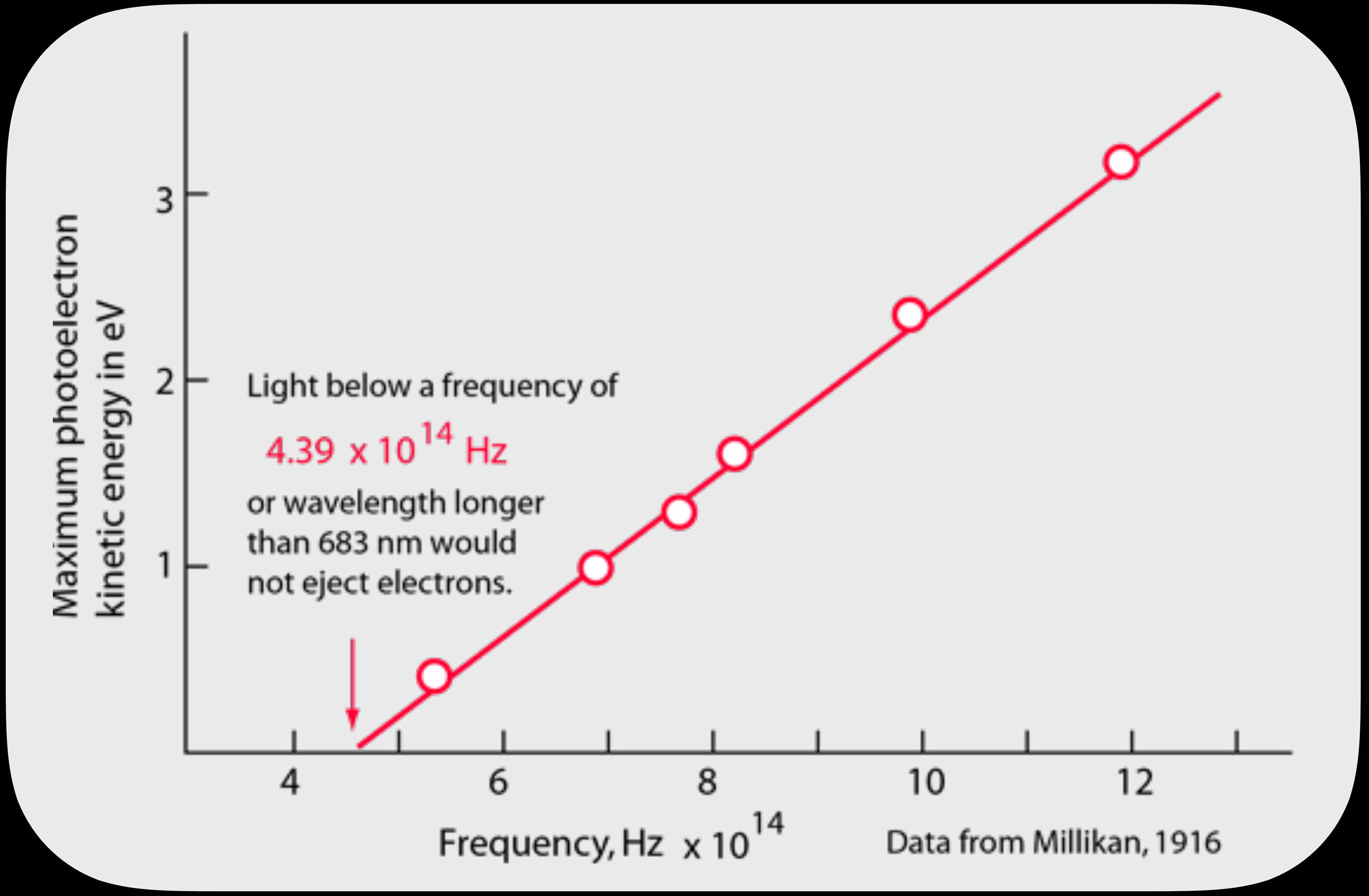
The number of independent standing waves inside a cavity

$$g(\nu)d\nu = \frac{8\pi\nu^2}{c^3}d\nu \Rightarrow \text{Rayleigh–Jeans Formula}$$



Wikipedia

Photoelectric Effect



hyperphysics.phy-astr.gsu.edu

Davisson–Germer electron diffraction experiment

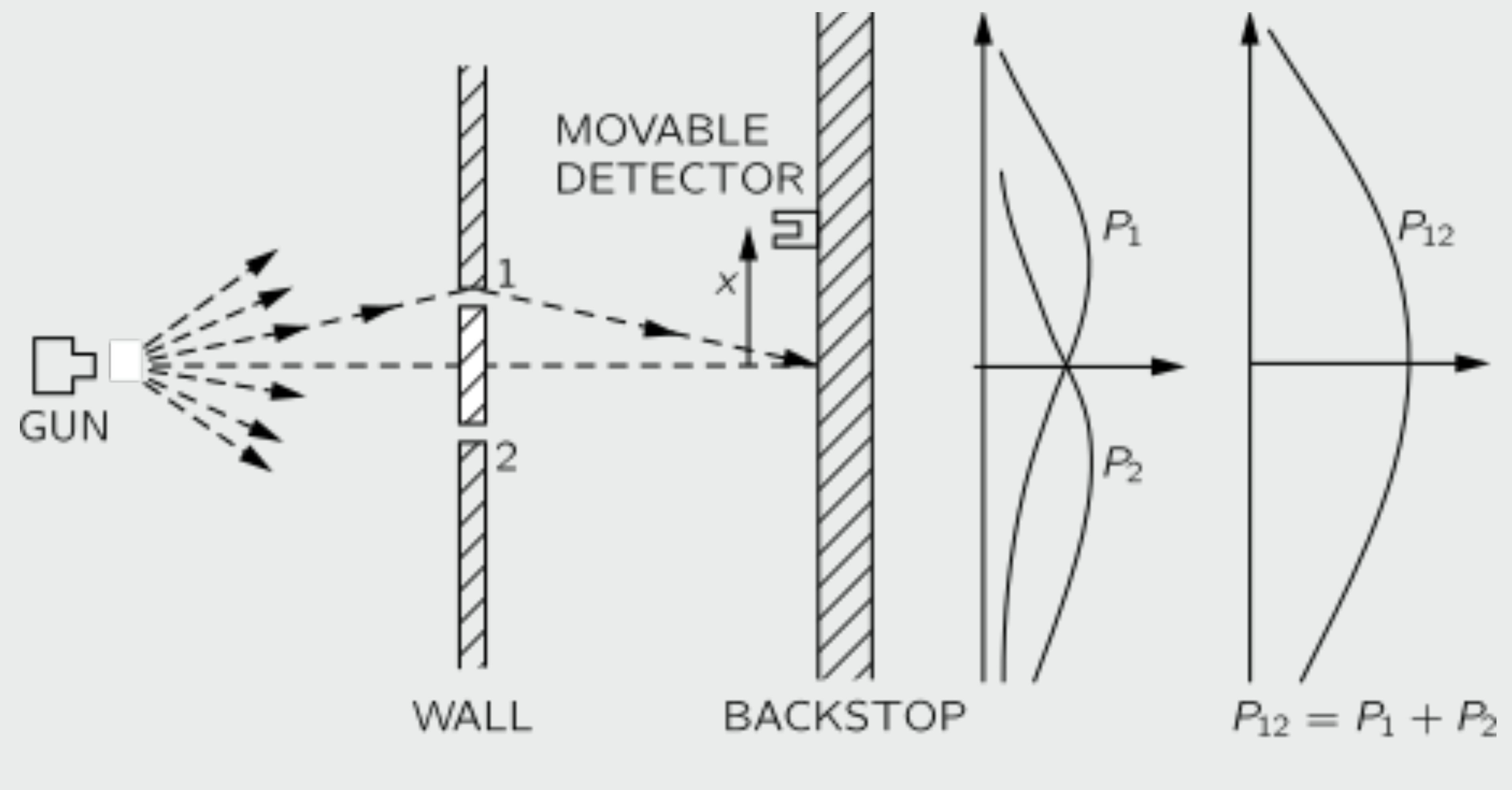
Stability of atomic orbitals

…

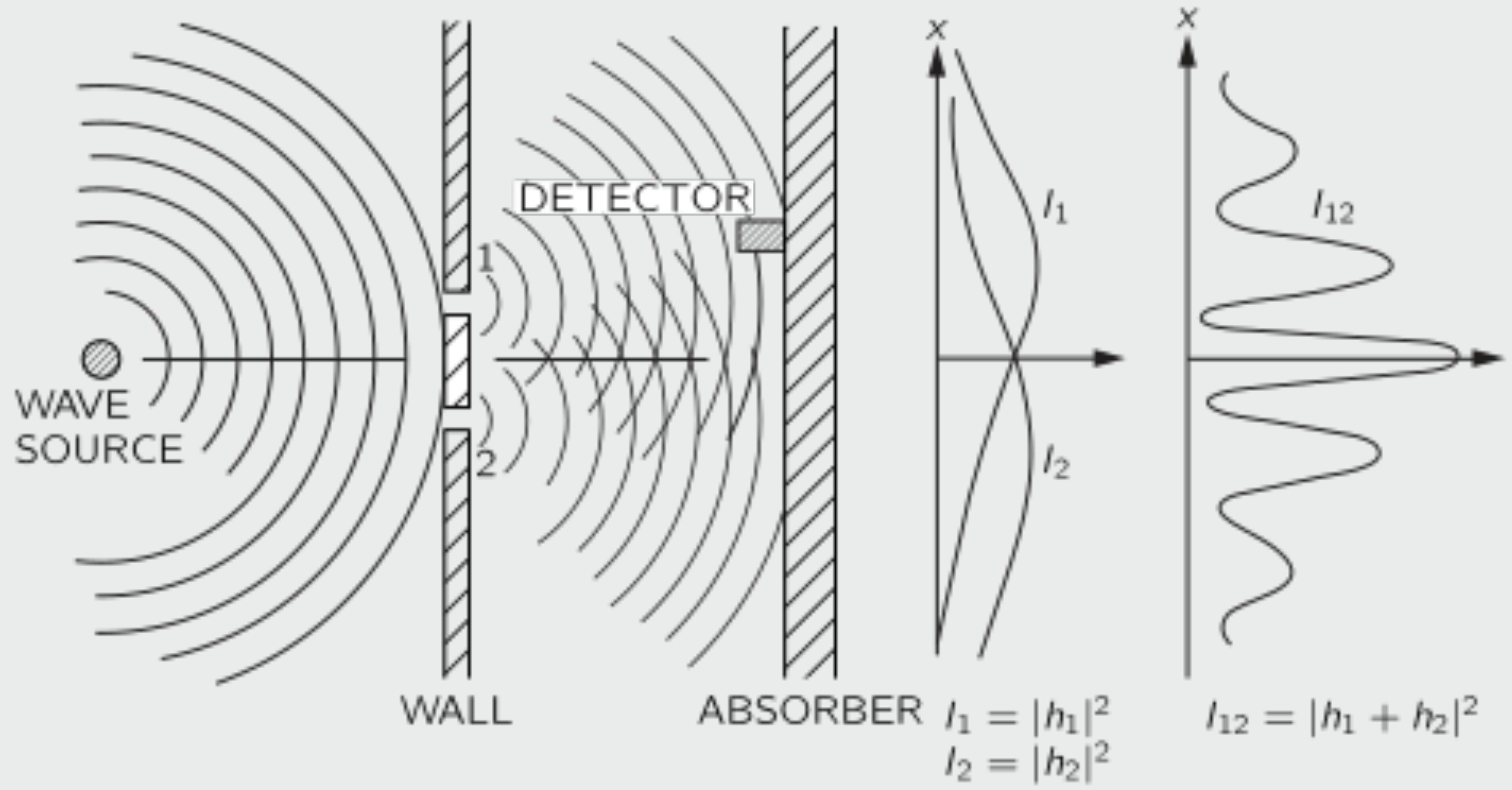
Bullets vs Waves

Feynman Lectures

An experiment with bullets



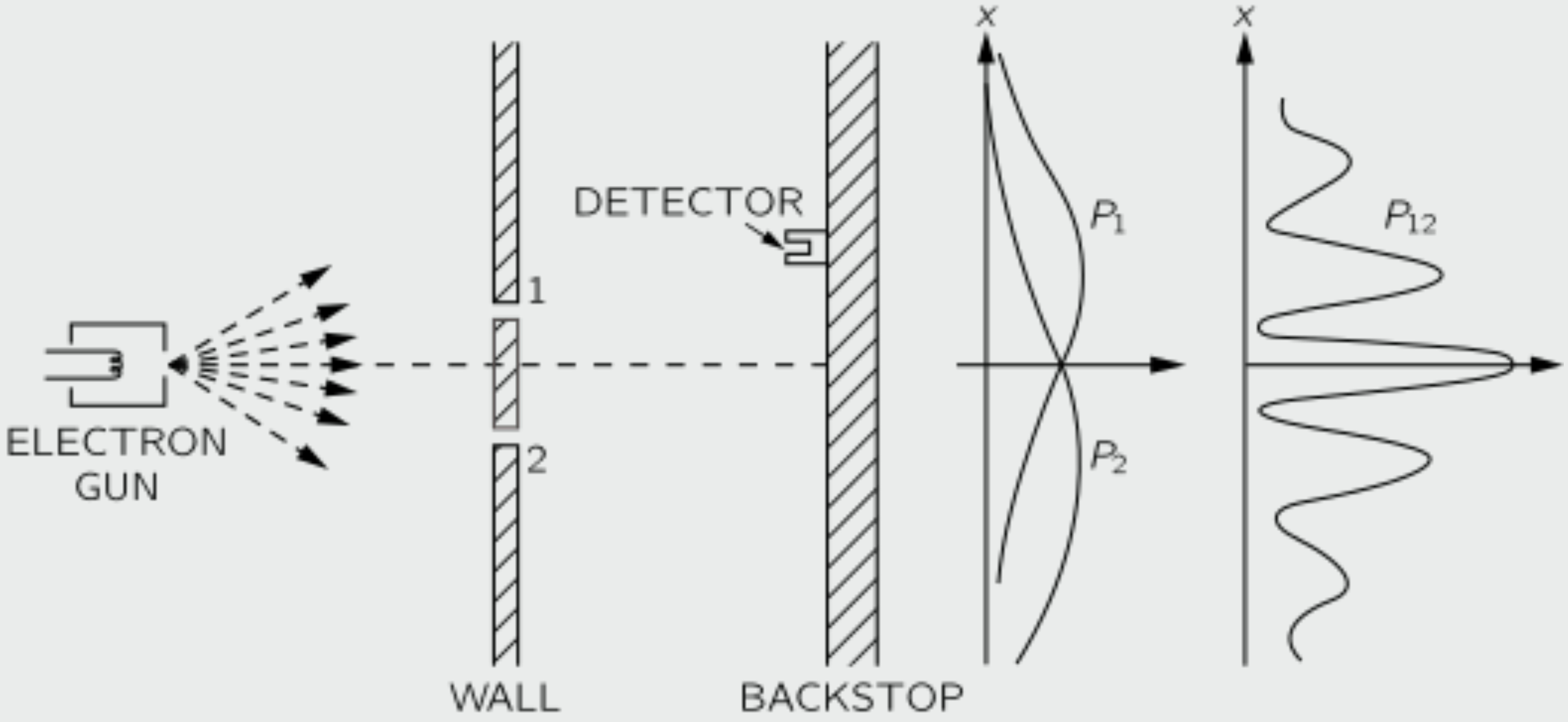
The same experiment with waves



Enter Electrons

Feynman Lectures

The same experiment with electrons



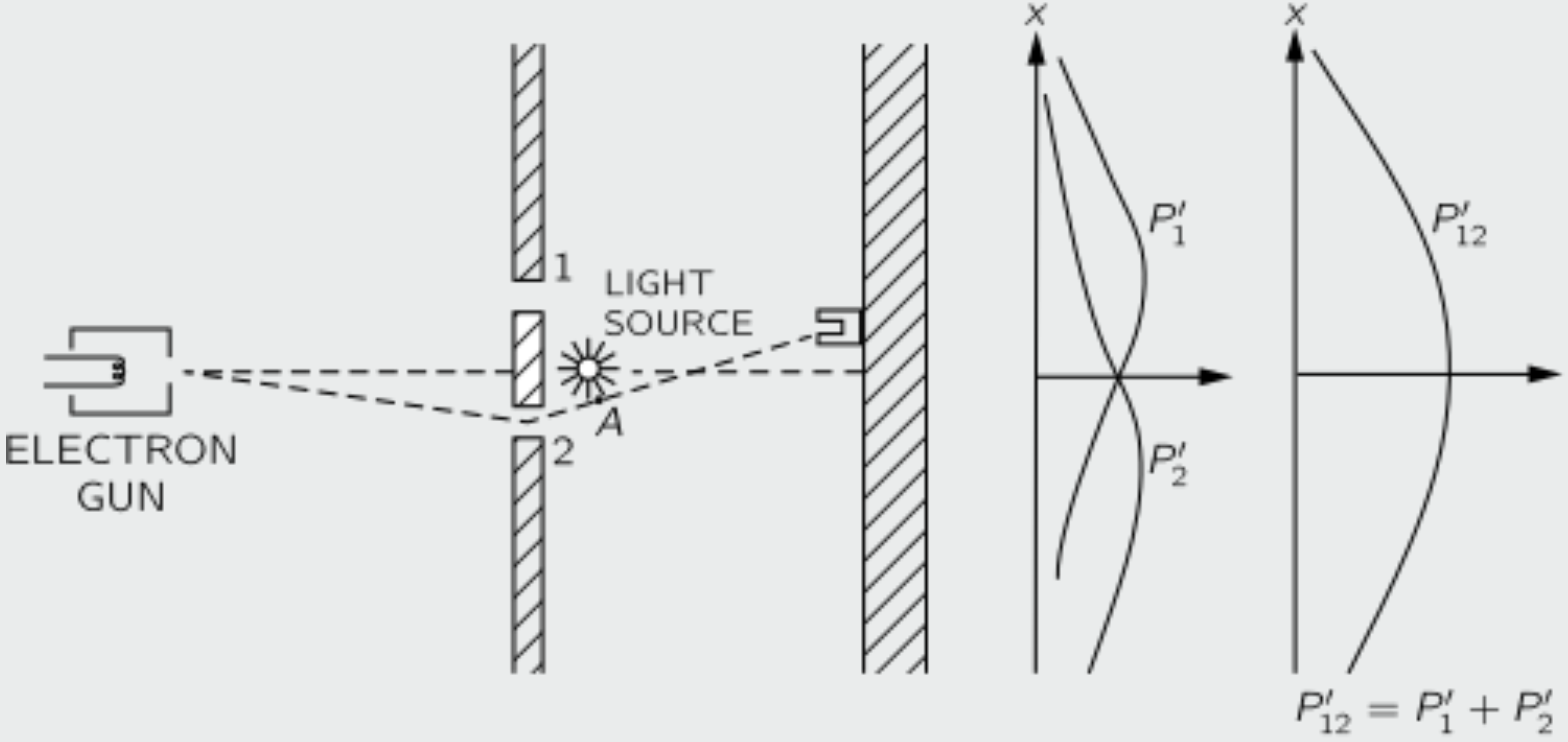
The definition of probability is not changed, but the method to calculate it is completely different

$P_{12} \neq P_1 + P_2$. **The two probabilities, P_1 and P_2 are related to two complex numbers**

The Ways of Electrons

Feynman Lectures

Which hole?



When we watch them $P_{12} = P_1 + P_2$

Curiouser

Feynman Lectures / Feynman, Hibbs

We are changing the pattern on the screen just by watching the electrons. How is this possible?

To watch them we used light. When an electron collides with a photon its chance of arrival at the detector is possibly altered.

Can we use weaker light and thus expect a weaker effect? A negligible disturbance certainly cannot be presumed to produce the finite change in the distribution.

Light comes in photons of energy $\hbar\nu$ or of momentum $\hbar/\lambda$. Weakening the light just means using fewer photons so that we may miss seeing an electron.

The electrons we miss are distributed according to the interference law, while those we do see (and which therefore have scattered a photon) arrive at the detector with the probability $P_{12} = P_1 + P_2$

It might still be suggested that weaker effects could be produced by using light of longer wavelength. But there is a limit to this. If light of too long a wavelength is used, we will not be able to tell whether it was scattered from hole 1 or hole 2.

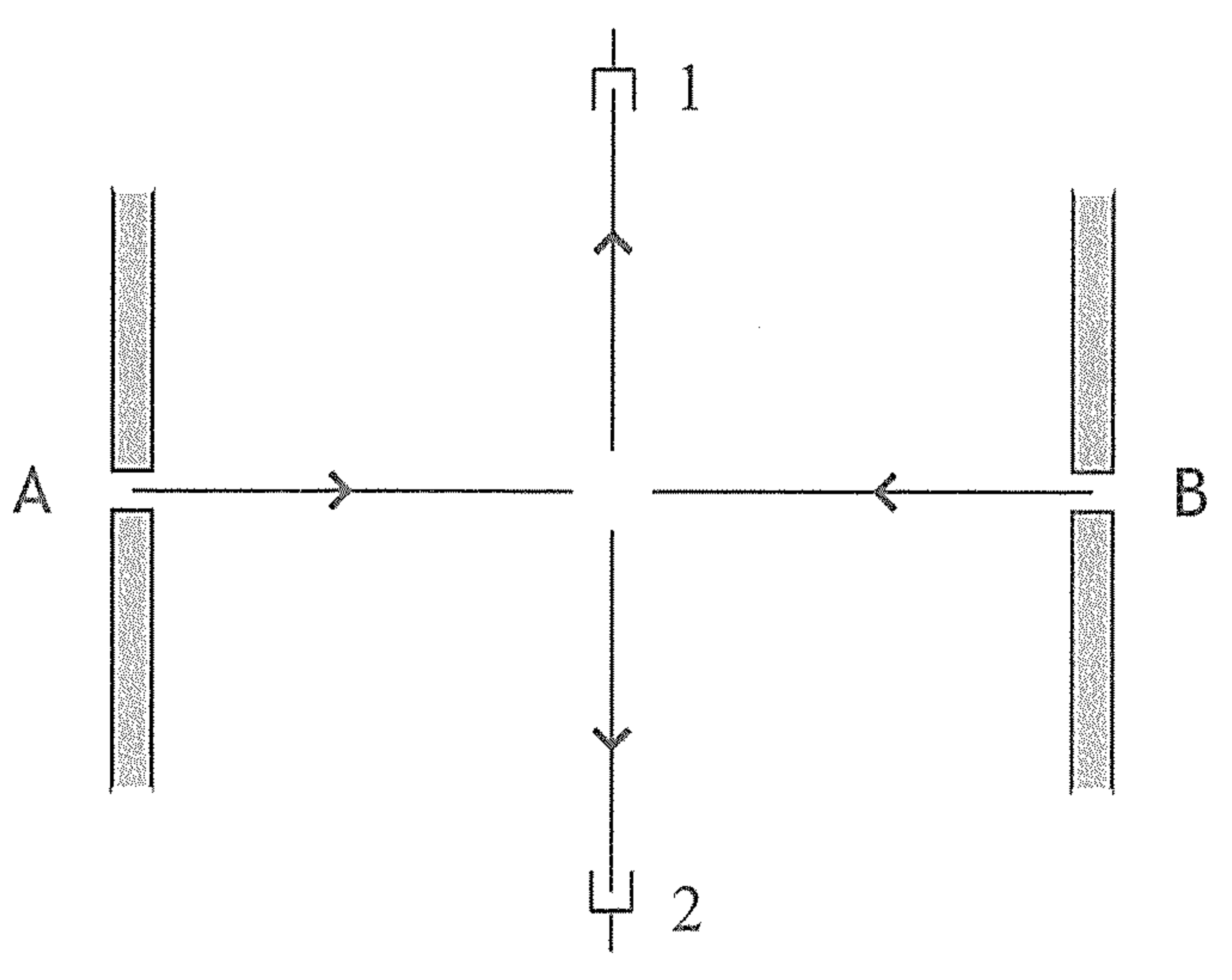
… and Curiouser!

Feynman, Hibbs

The uncertainty principle

Any determination of the alternative taken by a process capable of taking more than one alternatives destroys the interference between alternatives.

- Feynman's qualitative formulation of the uncertainty principle



Let the amplitude of scattering for the particle to start from A and end up at 1 (and the one to start from B and end up at 2) be $\alpha\,(1,A)$ so that the probability is $p = |\alpha\,(1,A)|^2$. This is also equal to $|\alpha\,(2,A)|^2$ as the scattering is by 90^o.

If the particles are different (like two different nuclei or two electrons with different spins — assuming the scattering is soft, i.e., can not flip the spins)

$$p(1, A \text{ or } B) = |\alpha(1,A)|^2 + |\alpha(1,B)|^2 = 2p$$

If we wish, we can distinguish the two cases by measuring.

If the particles are alpha particles (i.e., no way to tell them apart)

$$p(1, A \text{ or } B) = |\alpha(1,A) + \alpha(1,B)|^2 = 4p$$

If the particles are electrons with the same spin orientation, i.e., both up or both down

$$p(1, A \text{ or } B) = |\alpha(1,A) - \alpha(1,B)|^2 = 0$$

90^o scattering not possible.

The Schrödinger Equation

Unlike classical physics, in Quantum Mechanics we solve the Schrödinger equation to get the "wave function" of a particle

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

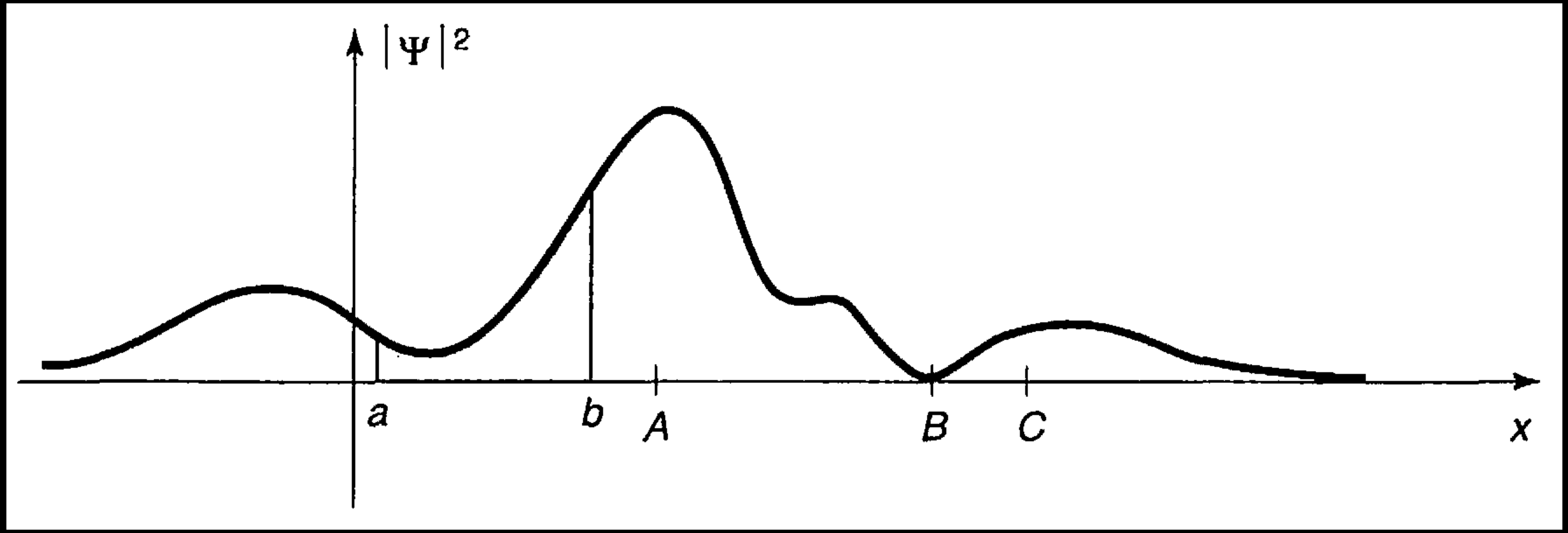
$$\hbar = \frac{h}{2\pi} = 1.054572 \times 10^{-34}\ \text{J s}$$

Born's statistical interpretation

The probability of finding the particle between point a and b at time t is given by

$$\int_a^b \boxed{\left|\Psi(x,t)\right|^2} dx$$

PDF



Normalisation

Since $|\Psi(x,t)|^2$ is a PDF, $\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx = 1$

The Schrödinger equation keeps it normalised.

$$\frac{d}{dt}\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx = \int_{-\infty}^{+\infty} \frac{\partial}{\partial t}|\Psi(x,t)|^2 dx = \int_{-\infty}^{+\infty} \left\{ \Psi^* \frac{\partial \Psi}{\partial t} + \frac{\partial \Psi^*}{\partial t}\Psi \right\} dx$$

$$\frac{\partial \Psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2} - \frac{i}{\hbar}V\Psi$$ and $$\frac{\partial \Psi^*}{\partial t} = -\frac{i\hbar}{2m}\frac{\partial^2 \Psi^*}{\partial x^2} + \frac{i}{\hbar}V\Psi^*$$

$$\frac{d}{dt}\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx = \frac{i\hbar}{2m}\left\{ \Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\Psi \right\}\Bigg|_{-\infty}^{+\infty}$$

Prove this!

However, since $\Psi(x,t) \to 0$ as $x \to \pm\infty$,

$$\boxed{\frac{d}{dt}\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx = 0}$$

Where was the Particle?

If, suppose, we locate a particle at point c, what will happen if we measure it immediately again?

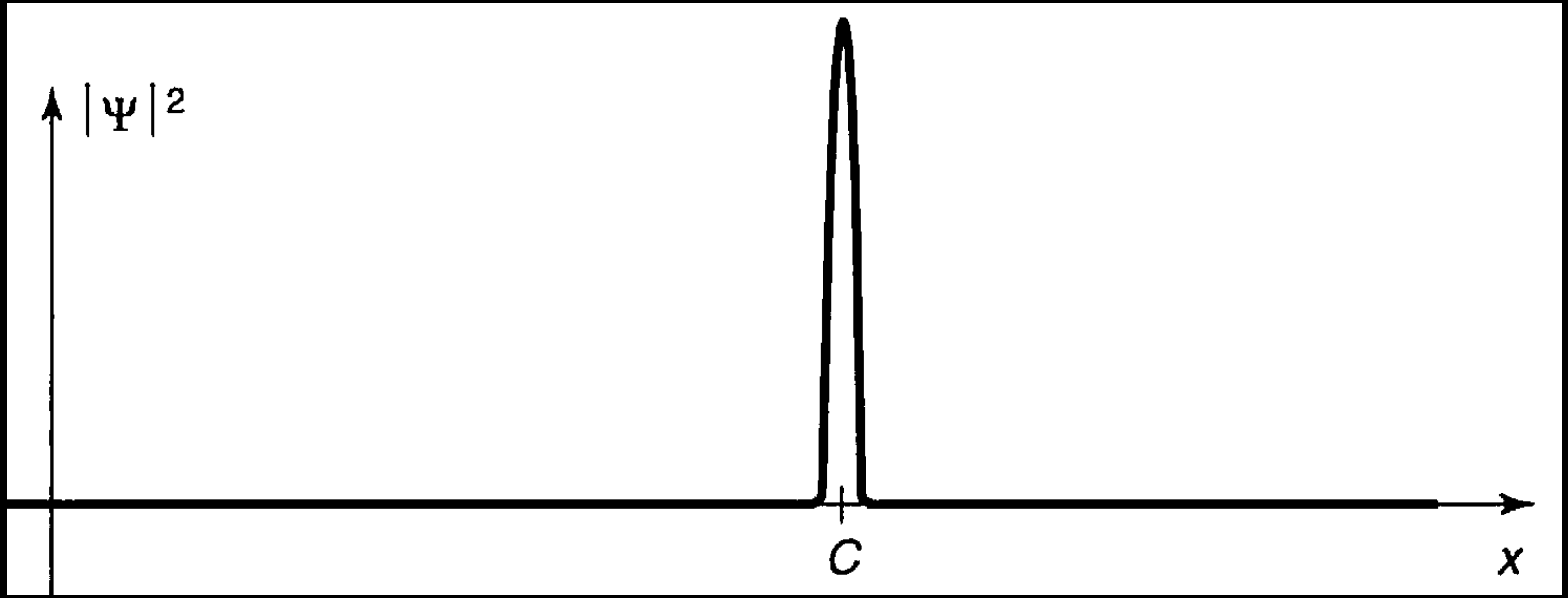
We will find it at c

But where was it before we located it for the first time?

It was at c. This implies QM is an incomplete theory. There are hidden variables. If we know them, we get back the deterministic picture.

Is the moon really there when nobody looks? Reality and the quantum theory
N. David Mermin, **Physics Today,** April 1985

It was everywhere. The act of measurement is a physical process that forced it to take a definite position. After the first measurement the wave function collapses to a spike at c.



Bell's experiment has (almost) ruled out the first option.

$|\Psi(x,t)|^2$ is a PDF

Since $|\Psi(x,t)|^2$ is a PDF of x, if we measure the position of a particle repeatedly a large number of times, what would be the expected outcome?

$$\langle x \rangle = \int_{-\infty}^{+\infty} x\,|\Psi(x,t)|^2 dx = \int_{-\infty}^{+\infty} \Psi^*(x,t)\, x\, \Psi(x,t) dx$$

But, **there is collapse!** The wave function collapses after the first measurement. Hence, the repeated measurement would not be given by the expectation value.

Instead, we should think that if we start with an ensemble of identically prepared systems [all with the same wave function $\Psi(x,t)$—we say, all in the same state $\Psi(x,t)$] and perform the measurement on all of them at time t, we would get the expected value.

But what is the use of this?

The classical objects are made up of lots of quantum objects. So, roughly, we can think the classical objects as ensemble of quantum states. Then, if we measure the location of a classical object we expect to get the average or the "expected value", i.e., $\langle x \rangle$. This could work provided the average quantities obey the classical laws.

Ehrenfest's Theorem

How to compute $\langle p \rangle$?

We need to compute dynamical quantities like $\langle p \rangle$ or $\langle \vec{L} \rangle$ etc.

$$\langle p \rangle = m\langle v \rangle = m\frac{d\langle x \rangle}{dt} = m\int x\frac{\partial}{\partial t}|\Psi(x,t)|^2 dx = \frac{i\hbar}{2}\int x\frac{\partial}{\partial x}\left\{\Psi^*\frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\Psi\right\}dx$$

$$= -\frac{i\hbar}{2}\int\left\{\Psi^*\frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\Psi\right\}dx = -i\hbar\int\Psi^*\frac{\partial \Psi}{\partial x}dx$$

Every dynamical quantity has its operator. We can compute their expectation values by operating with the corresponding operators.

$\hat{x}$ $\hat{p}$

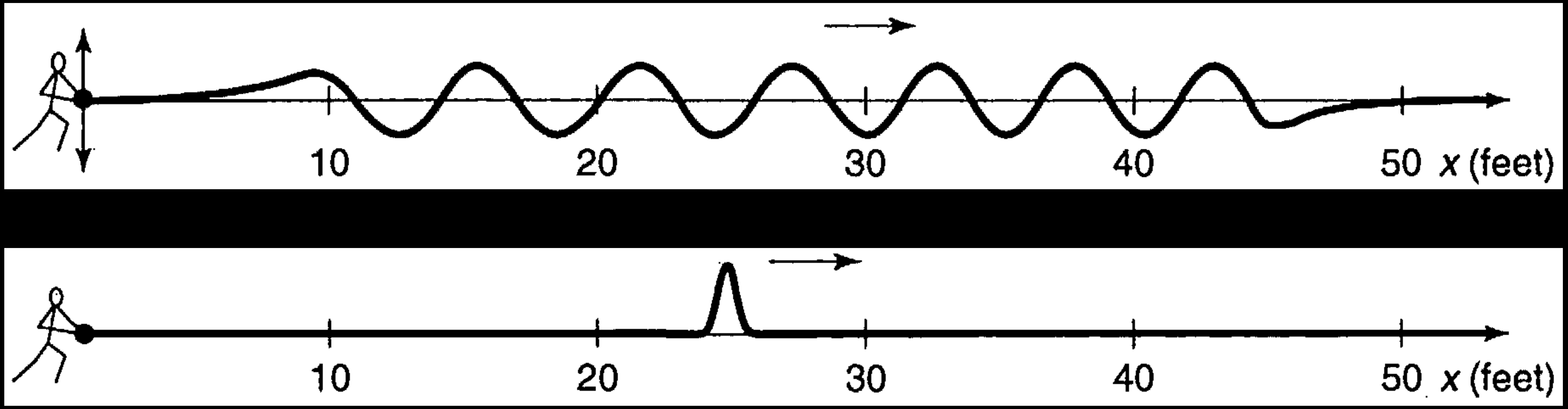
$$\langle x \rangle = \int \Psi^* \boxed{x} \Psi\, dx \qquad \langle p \rangle = \int \Psi^* \boxed{\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)} \Psi\, dx \qquad \langle T \rangle = \frac{\langle p \rangle^2}{2m} = \int \Psi^* \boxed{\frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2} \Psi\, dx$$

$\hat{T}$

$$\langle Q(x,p) \rangle = \int \Psi^* \boxed{Q\left(x, \frac{\hbar}{i}\frac{\partial}{\partial x}\right)} \Psi\, dx$$

$\hat{Q}(x,t)$

The Uncertainty Principle



Every measurement on a state yields some definite answer. However, measurements over identically prepared states vary.

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

Here, σ_q is the standard deviation, i.e., $\sigma_q = \sqrt{\langle q^2 \rangle - \langle q \rangle^2}$