An Introduction to Quantum Teleportation

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Abstract:
Quantum Teleportation is the process by which quantum information can be transmitted from one location to another. Here we discuss the theory behind quantum teleportation,its uses and limitations.

1. Introduction

Is Teleportation really possible? In this paper we'll first look at a few terms one needs to know before one can understand the process of Quantum Teleportation, then we'll look into if quantum teleportation can occur theoretically and if it has been achieved in real life, finally, we'll look into the uses,advantages and limitations of quantum teleportation.

Quantum teleportation enables one to transfer any arbitrary quantum information from one location to another, with the help of quantum mechanical phenomenon like quantum entanglement, here quantum information is the information of the state of a quantum system.

Although in theory this method can be used to transport large sets of quantum data, till date this has not been achieved with anything bigger than molecules. Currently scientists are trying to produce these teleportation results on bigger and bigger molecules, and larger and larger distances between entangled states.

2. Pre-Requisites

2.1 Qubit

A qubit or quantum bit is the basic unit of quantum information, used in quantum computing. It is the quantum analogue of the classical bit with additional dimensions associated to the quantum properties of a physical atom.
A qubit is a two-state quantum-mechanical system. Some examples are spin of the electron in which the two levels can be taken as spin up and spin down or the polarization of a single photon in which the two states can be taken to be the vertical polarization and the horizontal polarization.

Standard representation : A qubit can be represented by a linear superposition of its two orthonormal basis states or basis vectors. These vectors are usually denoted as

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

But a qubit need not just be one of these, they can be any linear combination of $|0\rangle$ and $|1\rangle$ such that,

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where α and β are probability amplitudes and are complex in nature
Probability of getting 0 is $|\alpha|^2$ and 1 is $|\beta|^2$, so

$$|\alpha|^2 + |\beta|^2 = 1.$$

2.2 Quantum Entanglement

In 1935, Albert Einstein along with 2 other scientist published a paper, in which they proposed the term Einstein–Podolsky–Rosen paradox (EPR paradox), it attempted to show that "the quantum-mechanical description of physical reality given by wave functions is not complete." Reading this paper, Erwin Schrödinger wrote a letter to Einstein, from where the term entanglement was coined from. Both, Einstein and Schrödinger were dissatisfied with the concept of entanglement, as it seemed to violate the speed limit on the transmission of information placed by the theory of relativity. Einstein later famously called Entanglement, "A spooky action at a distance".

When a group of particles are generated in a certain manner, the quantum state of each particle of the group is dependent of the other particles in the group, even if the particles are separated by a large distance. This phenomenon is known as Quantum Entanglement and the group of particles are said to be entangled with each other.

Even though this may not seem intuitive, quantum entanglement has been demonstrated practically with photons, electrons and even molecules as big as bucky balls.

Entangled particles can be created by various means, like for example, if a 0 spin particle decay's into 2 +1/2 spin particles, then if both these particles spin are measured separately on the same axis, we can see that they will always be opposite(As the decomposition must obey conservation of angular momentum). What is counter-intuitive is the fact that, that even if the particles are separated by a large distance, once 1 particle's spin is measured the other particle's spin becomes the opposite instantaneously.

2.3 Bell States

The Bell states, are the quantum states of two qubits that are the most basic examples of quantum entanglement.

In simpler terms, if two qubits in bell state were given to Alice and Bob, when Alice measure's her qubit, the probability she gets 0 or 1 is exactly the same(1/2) and the value of her qubit can be used to determine the value of bob's qubit.

Note: Suppose there are 2 qubits in state,

$$|\phi\rangle_1 = \alpha_1|0\rangle + \beta_1|1\rangle$$
$$|\phi\rangle_2 = \alpha_2|0\rangle + \beta_2|1\rangle$$

then,

$$\begin{aligned} |\phi\rangle &= |\phi\rangle_1 \otimes |\phi\rangle_2 \\ &= \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle \end{aligned}$$

Bell states or Four maximally entangled: (of two qubits A and B)

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$
$$|\Phi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B)$$
$$|\Psi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B)$$
$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$$

Let us look at what $|\Phi^+\rangle$ represents, if Alice were to measure her qubit, then the outcome she gets is perfectly random, either 0 or 1,when Bob measure's his qubit his measurement too will be perfectly random, but when they compare results, they will notice that they get the same result always. As the qubits are entangled. Corresponding results can be drawn for the other states.

For example, let us add $|\Phi^+\rangle$ and $|\Phi^-\rangle$,

$$\begin{aligned} |\Phi^+\rangle + |\Phi^-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle) + \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle - |1\rangle \otimes |1\rangle) \\ &= \frac{2}{\sqrt{2}}(|0\rangle \otimes |0\rangle) \\ &= \sqrt{2}(|0\rangle \otimes |0\rangle) \end{aligned}$$

$$\implies |0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle)$$

Similarly,

$$|0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle)$$

$$|0\rangle \otimes |1\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle + |\Psi^-\rangle)$$
$$|1\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle - |\Psi^-\rangle)$$
$$|1\rangle \otimes |1\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle - |\Phi^-\rangle)$$

Bell State Measurements : It is when we measure a particular quantum system such that it can be broken down into the four bell operator basis vectors.

Let us look at an example of Bell State measurement in the realm of Quantum computing. If there exists a pair of entangled qubits A and B.
To do a Bell-State Measurement on this system we first apply a CNOT gate to the qubits which un-entangles the qubits, we then pass one qubit through a Hadamard gate. Finally the Quantum Information can now be interpreted into Classical Information.

3. How do you Teleport a Quantum State?

Let us assume Alice has photon or a particle with quantum state

$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$$

which she would like to teleport to Bob. Here it is not necessary to know the value of α and β.

For Alice to teleport her quantum state to Bob, both of them should pre-arrange sharing a pair of entangled particles, for our convenience let us say this pair of particles are qubits in one of the maximally entangled bell states, $|\Psi^-\rangle$, and let us name these entangled qubits B and C, and the photon or particle with quantum state $|\phi\rangle$ as A.
Now Alice has particle A and qubit B, wheras Bob has only qubit C which is entangled to B.

$$|\phi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A$$
$$|\Psi^-\rangle_{BC} = \frac{1}{\sqrt{2}}(|0\rangle_B \otimes |1\rangle_C - |1\rangle_B \otimes |0\rangle_C)$$

Now,If we look at the complete system of the 3 particles,then,

$$\begin{aligned}
|\Psi\rangle_{ABC} &= |\phi\rangle_A \otimes |\Psi^-\rangle_{BC} \\
&= (\alpha|0\rangle_A + \beta|1\rangle_A) \otimes (\frac{1}{\sqrt{2}}(|0\rangle_B \otimes |1\rangle_C - |1\rangle_B \otimes |0\rangle_C)) \\
&= \frac{\alpha}{\sqrt{2}}(|0\rangle_A|0\rangle_B|1\rangle_C - |0\rangle_A|1\rangle_B|0\rangle_C) + \frac{\beta}{\sqrt{2}}(|1\rangle_A|0\rangle_B|1\rangle_C - |1\rangle_A|1\rangle_B|0\rangle_C)
\end{aligned}$$

Now,we can see that all direct products of type $|\rangle_A|\rangle_B$ can be written in terms of Bell operator basis vectors $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$ and $|\Psi^-\rangle$.

We know this from what was discussed above,(Bell States in pre-requisites)

$$
\begin{aligned}
|0\rangle|0\rangle &= \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle) \\
|0\rangle|1\rangle &= \frac{1}{\sqrt{2}}(|\Psi^+\rangle + |\Psi^-\rangle) \\
|1\rangle|0\rangle &= \frac{1}{\sqrt{2}}(|\Psi^+\rangle - |\Psi^-\rangle) \\
|1\rangle|1\rangle &= \frac{1}{\sqrt{2}}(|\Phi^+\rangle - |\Phi^-\rangle)
\end{aligned}
$$

Applying this to $|\Psi\rangle_{ABC}$,

$$
\begin{aligned}
|\Psi\rangle_{ABC} = \frac{\alpha}{\sqrt{2}}((\frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle))|1\rangle_C - (\frac{1}{\sqrt{2}}(|\Psi^+\rangle + |\Psi^-\rangle))|0\rangle_C) \\
+ \frac{\beta}{\sqrt{2}}((\frac{1}{\sqrt{2}}(|\Psi^+\rangle - |\Psi^-\rangle))|1\rangle_C - (\frac{1}{\sqrt{2}}(|\Phi^+\rangle - |\Phi^-\rangle))|0\rangle_C)
\end{aligned}
$$

On separating the terms and writing it back in terms of Bell basis vectors,

$$
\begin{aligned}
|\Psi\rangle_{ABC} = \frac{1}{2}[\ & |\Psi^-\rangle_{AB}(-\alpha|0\rangle_C - \beta|1\rangle_C) \\
&+ |\Psi^+\rangle_{AB}(-\alpha|0\rangle_C + \beta|1\rangle_C) \\
&+ |\Phi^-\rangle_{AB}(+\alpha|1\rangle_C + \beta|0\rangle_C) \\
&+ |\Phi^+\rangle_{AB}(+\alpha|1\rangle_C - \beta|0\rangle_C)\ ]
\end{aligned}
$$

We can see that,we now have a 4 term superposition. Till now all we have done is change of basis on Alice's part of the system. We have not done any operation or measurement on the system yet and the 3 particles A, B and C are still in the same quantum state.

Note that even though we didn't know the initial quantum state of A, the four measurement outcomes which we got are equally likely.(i.e, the probability of getting any 1 outcome from the superimposed outcomes is $\frac{1}{4}$)

Now, This is where the actual teleportation of state occurs, Alice now makes a Bell State Measurement on her particles A,B.

The 3 state system now collapses into one of the four superposition terms we got above with equal probability.
After Bell State Measurement, particles B,C are no longer entangled whereas particles A,B are now entangled. Bob's particle C takes one of the four superposition terms we got above.

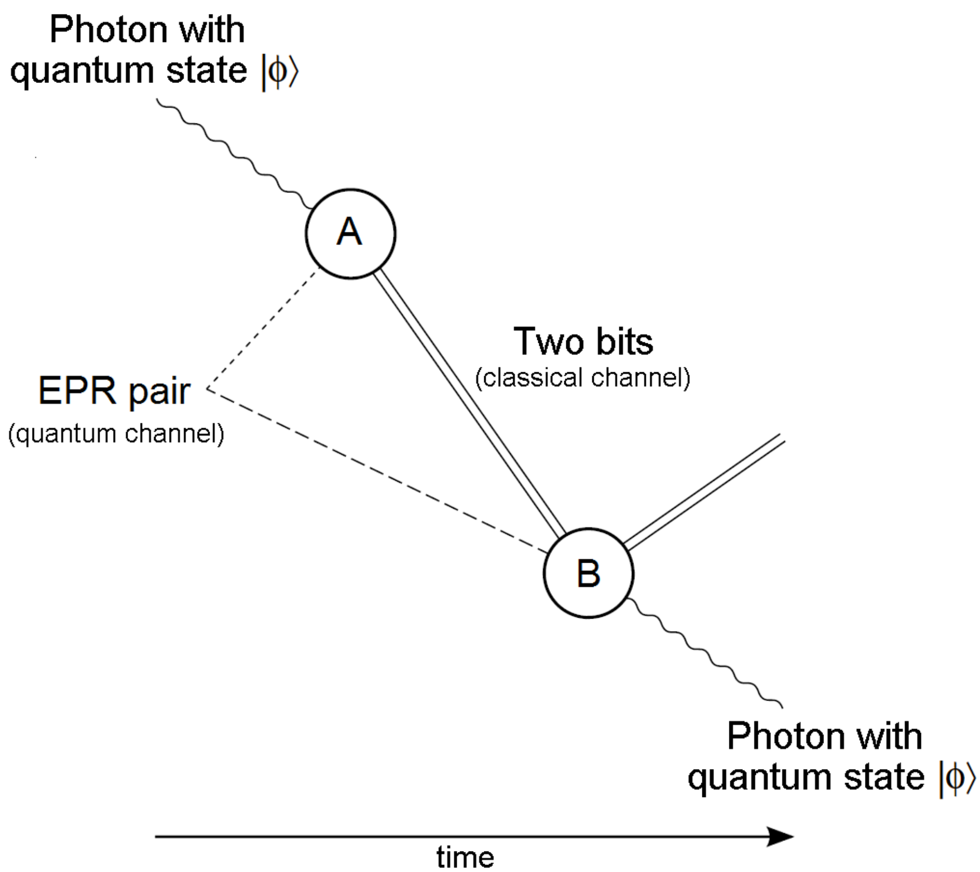

Figure 1: Schematic Representation of Quantum Teleportation

Alice now sends the result of her Bell's measurement to Bob via a classical channel.(i.e two classical bits can used to transmit which one of the 4 states Alice got.)

Bob now using the information he got from Alice can determine which state his paricle C is in,

$$-\alpha|0\rangle_C - \beta|1\rangle_C$$

$$-\alpha|0\rangle_C + \beta|1\rangle_C$$

$$+\alpha|1\rangle_C + \beta|0\rangle_C$$

$$+\alpha|1\rangle_C - \beta|0\rangle_C$$

We can see that all the four states are pretty similar to the Initial state of particle A, $|\phi\rangle$.That is, Bob only needs to do a unitary transformation particle C to change it to $|\phi\rangle$. Now using the information sent by Alice, Bob can now pass the qubit C through the necessary unitary quantum gate to change its state to $|\phi\rangle$.

After all is said and done,finally
$\rightarrow$ Bob has qubit C whose state is $|\phi\rangle_C = \alpha|0\rangle_C + \beta|1\rangle_C$
$\rightarrow$ Alice's qubit B becomes an (undefined) part of an entangled state with particle A.

$\Rightarrow$ Hence, We have Successfully transported Quantum state of A to Qubit C !

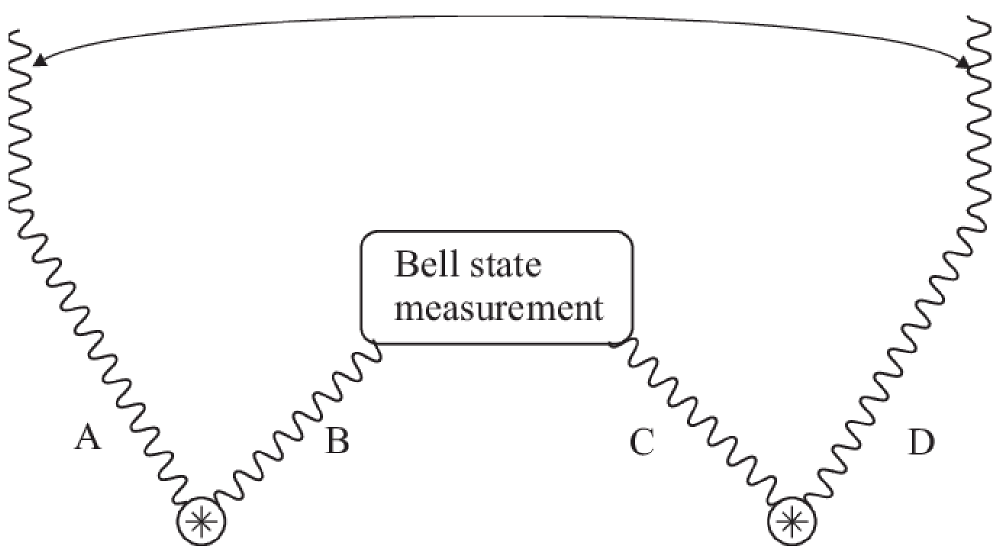

Figure 2: Quantum Entanglement Swapping

4. Uses and Limitations of Quantum Teleportation

4.1 Uses of Quantum Teleportation

- Let us assume one has a particle with quantum state $|\phi\rangle$, If he wishes to send this quantum state to someone else, he would have to transport the particle itself to that person as any attempt at measuring and copying it, could modify the state of the particle. Here with the help of Quantum Teleportation he could teleport the state of particle however far the receiver is, as long as they share a pair of entangled particles and can communicate classically.
- **Quantum Entanglement Swapping:** Let us assume assume Alice and Bob share a entangled pair of particles A and B, and Bob and Carol share a entangled pair of particles C and D.
  Now if Quantum Teleportation is used to teleport the state of particle B to particle D, then it is interesting to observe that the entanglement of particles A and B has swapped to A and D.(i.e, The particle D which now has the same state as particle B is now entangled to particle A).
- Using Quantum Teleportation one can duplicate a quantum state to make many copies, This can be achieved by taking a group of particles which follow multipartite-entanglement, instead of just 2 way entanglement. This overcomes some of the restrictions placed by the No-Cloning Theorem on cloning Quantum States.
- Using Quantum teleportation we can try to overcome Heisenberg's Uncertainty Principle, we overcome this by cloning one quantum state into many duplicates using Quantum Teleportation, Then from these clones we can measure different observable from different clones.'

4.2 Limitation of Quantum Teleportation

- Quantum Teleportation still requires a classical channel to complete the process, I.e, When the sender performs Bell State Measurement on his particles, he still needs to be able to send the results of the measurement to the receiver, so even if a quantum state can be teleported by huge distances, a classic channel of communication must always exist for Quantum Teleportation to work.
- During Quantum Teleportation, even though the state of a particle is teleported, the original particle loses it quantum state.(I.e, When particle A's state is telported to Particle C using Particle B, Finally after the Bell state measurement, particle A loses its original Quantum state, So the sender no longer has the original Quantum State anymore)

5. Quantum Teleportation in Real life

Quantum teleportation has been proved experimentally and a significant amount of effort is being put into the field to teleport larger states, longer distances,Without going into details let us look at a few quantum teleportation experiments.

- **Description:** We are going to discuss, without going into much detail, one of the first quantum teleportation experiment. The Experiment was conducted in University of Innsbruck, Austria and the results were pulished in a paper "Experimental quantum teleportation" in 1998.
  Aim: They wanted to successfully present the first demonstration of Quantum Teleportation.
  Process: They produced polarization entangled photons which they used as their entangled particles for teleportation. They realized that the bell-state analysis would be the most crucial and difficult part as it would make or break their experiment.They used an interferometric approach which requires specific timing conditions for two independent incoming photons at Alice's side.
  Result and Conclusions: In this paper, they successfully demonstrated the possibility of transferring the polarization state from one photon onto another. They effectively made the first practical demonstration of Quantum Teleportation. This experiment allowed further physicists and researchers to work on quantum communication experiments like entanglement purification and also showed that in the future Quantum teleportation could be used to link quantum computers.

- **Description:** This Quantum teleportation experiment was one of the most recent and innovative experiment to take place, Physicists successfully managed to teleport quantum information between silicon chips. They later published their results as "Chip-to-chip quantum teleportation and multi-photon entanglement in silicon",in 2019.
  Aim: The Aim of this experiment was to demonstrate chip-to-chip quantum teleportation and genuine multipartite entanglement, which had never been fully successfully done in the past.
  Process: They first created 4 single photons with high purity and indistinguishablity in an array of microresonator sources, this method gave them low levels of interference and high levels of accuracy in making their quantum states.To achieve this feat, they used both Silicon chips as well as their own fabricated by using complementary metal-oxide-semiconductor (CMOS) techniques. By making precise measurements to these photons, they managed to simultaneously break entanglement between the particles and also teleport the state to 2 quantum chips.
  Result and Conclusions: The physicists were able to successfully teleport one quantum state of a particle to 2 different chips.They were able to run these experiments with about 91% fidelity, i.e approximately 91% of the quantum information was transmitted and logged properly. This experiment opens the door for larger scale experiments that one day might lead to creation of quantum computers with super-powerful processing power.

6. Conclusion

In this paper, we discussed Quantum mechanical terms and phenomenon like Qubits, Quantum Entanglement and Bell states. We used this to learn how quantum teleportation occurs theoretically, it's uses and it's limitations. We finally looked into a few quantum teleportation experiments and how they take place. The process of quantum teleportation is being explored more and more, and is gaining traction in both the Quantum Information and

Quantum Computing field.

In the begin of the paper, we asked if Teleportation is possible. Even though at this time, teleportation may seem fictitious with the rapid development of technology and our understanding of the universe. You never know what the future might hold!

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