Assignment 2: Due on Thu, 3 Sep 2020

Q1) The Cartesian components L_i; $i = 1, 2, 3$ of the angular momentum operator are related to the position and momentum operators by $\hat{L}_i = \epsilon_{ijk}\hat{x}_j\hat{p}_k$.

[summation convention : sum over the repeated indices j and k .

If you find it difficult, write it as $L_i = x_j p_k - x_k p_j$, where the indices i, j and k are different and each is one from the set $\{x, y, z\}$ or $\{q_1, q_2, q_3\}$ in a cyclic order as shown. If the order is reversed then a negative sign is required. For convenience, we will not be using below the hat ($\hat{}$) that denotes an operator. You have to remember that these are operators].

in detail, $L_x = yp_z - zp_y = -(zp_y - yp_z)$, etc.

(a) Show: $[L_i, L_j] = i\hbar\epsilon_{ijk}L_k$, i.e., $[L_x, L_y] = i\hbar L_z$

(b) Show: $\left[L_i, L^2\right] = \left[L_i, r^2\right] = \left[L_i, p^2\right] = 0$

Q2) Let $\vec{A}$ be a vector operator s.t. $[L_i, A_j] = i\hbar\epsilon_{ijk}L_k$ [i.e., $[L_x, A_y] = i\hbar L_z$, etc.]

Show: $\vec{L}.\vec{A} = \vec{A}.\vec{L}$

Q3) Using first order perturbation theory, calculate the energy of the n-th state for a particle of mass m moving in a 1-dim infinite potential well of length $2L$,

(i) with walls at $x = 0$ and $x = 2L$, which is modified at the bottom by the perturbation $\lambda V_0 \sin\left(\frac{\pi x}{2L}\right)$, where $\lambda \ll 1$.

(ii) with walls at $x = -L$ and $x = L$, which is modified at the bottom by the following perturbation with $V_0 \ll 1$.

$$H' = \begin{cases} -V_0 & -L \le x \le L \\ 0 \text{ elsewhere} & \end{cases} \qquad H' = \begin{cases} -V_0 & -\frac{L}{2} \le x \le \frac{L}{2} \\ 0 \text{ elsewhere} & \end{cases}$$

[Note that the boundary of the 'box' on the left in (ii) is not at $x = 0$]

Q4) For a 1-dim harmonic oscillator, the spring constant changes from k to $k(1 + \epsilon)$, where ϵ is small. Calculate the first order perturbation in the energy.