1 Solvent injection

Chemical solvents, such as DME that are soluble in both oil and water, can be dissolved in water and injected into the reservoir. When in contact with oil in the reservoir, most of the solvent goes into the oleic phase due to the higher affinity of the solvent towards oil, increasing the molar volume of oil (swelling) and decreasing its viscosity. Swelling of the oleic phase increases the oil saturation, thus make oil more mobile. This mobility is also enhanced by the lower viscosity of the oil-solvent mixture.

2 Mathematical model

The flow of the oleic and aqueous phases can be described by a material balance and the extended Darcy's law, which reads

$$\frac{\partial}{\partial t}\left(\varphi\rho_w S_w\right) + \nabla.\left(\rho_w \mathbf{u_w}\right) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\left(\varphi\rho_o S_o\right) + \nabla.\left(\rho_o \mathbf{u_o}\right) = 0, \tag{2}$$

$$\mathbf{u_w} = -\frac{k k_{rw}}{\mu_w}\left(\nabla p_w - \rho_w \mathbf{g}\right), \tag{3}$$

$$\mathbf{u_o} = -\frac{k k_{ro}}{\mu_o}\left(\nabla p_o - \rho_o \mathbf{g}\right), \tag{4}$$

$$p_c\left(S_w\right) = p_o - p_w, \tag{5}$$

$$S_w + S_o = 1. \tag{6}$$

The transport of the solvent is described by

$$\frac{\partial}{\partial t}\left[\varphi\left(c_w S_w \rho_w + c_o\left(1 - S_w\right)\rho_o\right)\right] + \nabla.\left(\mathbf{u_o} c_o \rho_o + \mathbf{u_w} c_w \rho_w\right) = 0, \tag{7}$$

$$K = \frac{c_o}{c_w}, \tag{8}$$

where c_w [-] and c_o [-] are the mass fraction of the the solvent in the aqueous and oleic phase, respectively, and K [-] is the partition coefficient of the solvent between the phases. The density (ρ_w [kg/m^3] and ρ_o [kg/m^3]) and viscosity (μ_w [Pa.s] and μ_o [Pa.s]) of each phase is a function of the solvent concentration c_w and c_o. Moreover, the K value is also a function of c_w. This makes the above system of equations nonlinear, that needs to be linearized.

Table 1: Thermodynamic and transport properties of the DME-enhanced water flooding equations

Parameter	Value	Unit	Note
φ	0.4	-	Porosity
k	0.01×10^{-12}	m^2	Permeability
ρ_w	1000	kg/m^3	Water density
ρ_o	800	kg/m^3	Oil density
μ_w	0.001	Pa.s	Water viscosity
μ_o	0.002	Pa.s	Oil viscosity
k_{ro}^0	0.8	-	Oil relative permeability endpoint
k_{rw}^0	0.2	-	Water relative permeability endpoint
n_o	2.0	-	Oil relative permeability exponent
n_w	2.0	-	Water relative permeability exponent
S_{wc}	0.08	-	Irreducible water saturation
$S_{or,max}$	0.3	-	Residual oil saturation in absence solvent
m	-0.4	-	Slope of the residual oil saturation change with solvent concentration
K	2.0	-	Distribution coefficient
u	1.0	m/day	Darcy velocity

2.1 The simplified system

It is probably possible to solve the above equations analytically. I can simplify the equations, with the following assumptions:

1. All the physical ant transport properties of oil and water are constant.
2. The distribution coefficient K is constant.
3. Gravity and capillary effects are negligible.

$$\varphi\frac{\partial S_w}{\partial t}+u\frac{\partial f_w}{\partial x}=0,$$

$$\varphi\frac{\partial}{\partial t}\left[\left(S_w\rho_w+K\rho_o\left(1-S_w\right)\right)c_w\right]+u\nabla.\left(K\rho_o\left(1-f_w\right)c_w+\rho_w f_w c_w\right)=0$$

$$f_w=\frac{\frac{k_{rw}}{\mu_w}}{\frac{k_{rw}}{\mu_w}+\frac{k_{ro}}{\mu_o}},$$

$$k_{rw}=k_{rw}^0\left(\frac{S_w-S_{wc}}{1-S_{wc}-S_{or}}\right)^{n_w},$$

$$k_{ro}=k_{ro}^0\left(\frac{1-S_w-S_{or}}{1-S_{wc}-S_{or}}\right)^{n_o},$$

$$S_{or}=mKc_w+S_{or,max}.$$

The values of the constants are reported in Table 1.

2.2 Initial and boundary conditions

The porous medium is initially saturated with oil and water, with a water saturation of $S_{w0} = S_{wc}$. Initially, the concentration of solvent in the system is zero, $c_{w0} = 0$. Water in injected from the left boundary with a saturation of one ($S_{w,b} = 1.0$) and with a solvent concentration (mass fraction) of $c_w = 0.2$.

2.3 Equations and unknowns

We can solve the above equations for three primary unknowns, i.e., p_w, c_w, and S_w. We first do some algebraic operations to simplify the above equations. First, by adding Eqs. (1 and 2) and then replacing Eqs. (6 and 3 and 4 and 5) gives

$$\frac{\partial}{\partial t}\left(\varphi\left(\rho_w S_w + \rho_o\left(1 - S_w\right)\right)\right) + \nabla.\left(-\rho_w \frac{kk_{rw}}{\mu_w}\left(\nabla p_w - \rho_w \mathbf{g}\right) - \rho_o \frac{kk_{ro}}{\mu_o}\left(\nabla p_w + \nabla p_c - \rho_o \mathbf{g}\right)\right) = 0.$$

Let's work with the original mass balance equation that are shorter and easier to linearize:

$$\frac{\partial}{\partial t}\left(\varphi \rho_w S_w\right) + \nabla.\left(-\rho_w \frac{kk_{rw}}{\mu_w}\left(\nabla p_w - \rho_w \mathbf{g}\right)\right) = 0, \tag{9}$$

$$\frac{\partial}{\partial t}\left(\varphi \rho_o\left(1 - S_w\right)\right) + \nabla.\left(-\rho_o \frac{kk_{ro}}{\mu_o}\left(\nabla p_w + \nabla p_c - \rho_o \mathbf{g}\right)\right) = 0, \tag{10}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left[\varphi\left(c_w S_w \rho_w + K c_w\left(1 - S_w\right)\rho_o\right)\right] + \nabla.\left(-\rho_w c_w \frac{kk_{rw}}{\mu_w}\left(\nabla p_w - \rho_w \mathbf{g}\right)\right) + \\ + \nabla.\left(-\rho_o K c_w \frac{kk_{ro}}{\mu_o}\left(\nabla p_w + \nabla p_c - \rho_o \mathbf{g}\right)\right) = 0\end{aligned} \tag{11}$$

3 Linearization

The linearization can be done by using the Taylor expansion. For example, for the following term, we have:

$$f\left(s,c\right)\nabla p = f\left(s_0,c_0\right)\nabla p + \left(\frac{\partial f}{\partial s}\right)_{s_0,c_0}\nabla p_0\left(s - s_0\right) + \left(\frac{\partial f}{\partial c}\right)_{s_0,c_0}\nabla p_0\left(c - c_0\right) + h.o.t$$

For other terms, we can use the Taylor expansion as

$$f\left(s,c\right) = f\left(s_0,c_0\right) + \left(\frac{\partial f}{\partial s}\right)_{s_0,c_0}\left(s - s_0\right) + \left(\frac{\partial f}{\partial c}\right)_{s_0,c_0}\left(c - c_0\right) + h.o.t$$

For convenience we define

$$\left(p,S,c\right) = \left(p_w,S_w,c_w\right)$$

Let's do the linearization for each term separately.

For Eq. (9)

$$\frac{\partial}{\partial t}\left(\varphi \rho_w S_w\right) + \nabla . \left(-\rho_w \frac{k k_{rw}}{\mu_w}\left(\nabla p_w - \rho_w \mathbf{g}\right)\right) = 0$$

$$\rho_w S_w = \rho_w\left(c_0\right) S_0 + S_0 \frac{\partial \rho_w}{\partial c}\left(c - c_0\right) + \left(\rho_w\right)_{c_0}\left(S - S_0\right) + h.o.t =$$
$$\left(\rho_w\right)_{c_0} S + S_0\left(\frac{\partial \rho_w}{\partial c}\right)_{c_0}\left(c - c_0\right) + h.o.t$$

$$\frac{\partial}{\partial t}\left(\varphi \rho_w S_w\right) = \varphi\left(\rho_w\right)_{c_0} \frac{\partial\left(S - S_0\right)}{\partial t} + S_0 \varphi\left(\frac{\partial \rho_w}{\partial c}\right)_{c_0} \frac{\partial\left(c - c_0\right)}{\partial t}$$

$$\frac{\rho_w}{\mu_w} k_{rw} \nabla p_w = \left(\frac{\rho_w}{\mu_w}\right)_{c_0}\left(k_{rw}\right)_{S_0} \nabla p + \left(\frac{\rho_w}{\mu_w}\right)_{c_0}\left(\frac{\partial k_{rw}}{\partial S}\right)_{S_0} \nabla p_0\left(S - S_0\right) + \left(\frac{\partial \frac{\rho_w}{\mu_w}}{\partial c}\right)_{c_0}\left(k_{rw}\right)_{S_0} \nabla p_0\left(c - c_0\right) + h.o.t$$

$$\frac{\rho_w^2}{\mu_w} k_{rw} \mathbf{g} = \left(\frac{\rho_w^2}{\mu_w}\right)_{c_0}\left(k_{rw}\right)_{S_0} \mathbf{g} + \left(\frac{\rho_w^2}{\mu_w}\right)_{c_0}\left(\frac{\mathrm{d} k_{rw}}{\mathrm{d} S}\right)_{S_0} \mathbf{g}\left(S - S_0\right) + \left(\frac{\mathrm{d} \frac{\rho_w^2}{\mu_w}}{\mathrm{d} c}\right)_{c_0}\left(k_{rw}\right)_{S_0} \mathbf{g}\left(c - c_0\right) + h.o.t$$

For Eq. (10):

$$\frac{\partial}{\partial t}\left(\varphi \rho_o\left(1 - S_w\right)\right) + \nabla . \left(-\rho_o \frac{k k_{ro}}{\mu_o}\left(\nabla p_w + \nabla p_c - \rho_o \mathbf{g}\right)\right) = 0$$

$$\rho_o\left(1 - S_w\right) = \left(\rho_o\right)_{c_0}(1 - S) + \left(1 - S_0\right)\left(\frac{\mathrm{d} \rho_o}{\mathrm{d} c}\right)_{c_0}\left(c - c_0\right) + h.o.t$$

$$\left(\rho_o(K c)\right)' = (K c)' \rho_o'(K c)$$

$$\frac{\rho_o}{\mu_o} k_{ro} \nabla p_w = \left(\frac{\rho_o}{\mu_o}\right)_{c_0}\left(k_{ro}\right)_{S_0} \nabla p + \left(\frac{\partial \frac{\rho_o}{\mu_o}}{\partial c}\right)_{c_0}\left(k_{ro}\right)_{S_0} \nabla p_0\left(c - c_0\right) + \left(\frac{\rho_o}{\mu_o}\right)_{c_0}\left(\frac{\partial k_{ro}}{\partial S}\right)_{S_0} \nabla p_0\left(S - S_0\right) + h.o.t$$

$$\frac{\rho_o}{\mu_o} k_{ro} \nabla p_c = \frac{\rho_o}{\mu_o} k_{ro} \frac{\mathrm{d} p_c}{\mathrm{d} S} \nabla S = \left(\frac{\rho_o}{\mu_o}\right)_{c_0}\left(k_{ro} \frac{\mathrm{d} p_c}{\mathrm{d} S}\right)_{S_0} \nabla S +$$
$$+\left(\frac{\mathrm{d} \frac{\rho_o}{\mu_o}}{\mathrm{d} c}\right)_{c_0}\left(k_{ro} \frac{\mathrm{d} p_c}{\mathrm{d} S}\right)_{S_0} \nabla S_0\left(c - c_0\right) + \left(\frac{\rho_o}{\mu_o}\right)_{c_0}\left(\frac{\mathrm{d}\left(k_{ro} \frac{\mathrm{d} p_c}{\mathrm{d} S}\right)}{\mathrm{d} S}\right)_{S_0} \nabla S_0\left(S - S_0\right)$$

$$\frac{\rho_o^2}{\mu_o} k_{ro} \mathbf{g} = \left(\frac{\rho_o^2}{\mu_o}\right)_{c_0}\left(k_{ro}\right)_{S_0} \mathbf{g} + \left(\frac{\rho_o^2}{\mu_o}\right)_{c_0}\left(\frac{\partial k_{ro}}{\partial S}\right)_{S_0} \mathbf{g}\left(S - S_0\right) + \left(\frac{\partial \frac{\rho_o^2}{\mu_o}}{\partial c}\right)_{c_0}\left(k_{ro}\right)_{S_0} \mathbf{g}\left(c - c_0\right) + h.o.t$$

For Eq. (11):

$$\frac{\partial}{\partial t}\left[\varphi\left(c_w S_w \rho_w + K c_w \left(1 - S_w\right)\rho_o\right)\right] + \nabla.\left(-\rho_w c_w \frac{k k_{rw}}{\mu_w}\left(\nabla p_w - \rho_w \mathbf{g}\right)\right) + \\ + \nabla.\left(-\rho_o K c_w \frac{k k_{ro}}{\mu_o}\left(\nabla p_w + \nabla p_c - \rho_o \mathbf{g}\right)\right) = 0$$

$$c_w S_w \rho_w = c_0 \left(\rho_w\right)_{c_0} S + S_0 \left(\frac{\partial\left(c\rho_w\right)}{\partial c}\right)_{c_0}\left(c - c_0\right) + h.o.t$$

$$K c_w \left(1 - S_w\right)\rho_o = c_0 \left(K\rho_o\right)_{c_0}\left(1 - S\right) + \left(1 - S_0\right)\left(\frac{\partial\left(K c\rho_o\right)}{\partial c}\right)_{c_0}\left(c - c_0\right) + h.o.t$$

$$\frac{c_w \rho_w}{\mu_w} k_{rw} \nabla p_w = \left(\frac{c\rho_w}{\mu_w}\right)_{c_0}\left(k_{rw}\right)_{S_0}\nabla p + \left(\frac{c\rho_w}{\mu_w}\right)_{c_0}\left(\frac{\partial k_{rw}}{\partial S}\right)_{S_0}\nabla p_0 \left(S - S_0\right) + \left(\frac{\partial \frac{c\rho_w}{\mu_w}}{\partial c}\right)_{c_0}\left(k_{rw}\right)_{S_0}\nabla p_0\left(c - c_0\right) + h.$$

$$\frac{c_w \rho_w^2}{\mu_w} k_{rw} \mathbf{g} = \left(\frac{c\rho_w^2}{\mu_w}\right)_{c_0}\left(k_{rw}\right)_{S_0}\mathbf{g} + \left(\frac{c\rho_w^2}{\mu_w}\right)_{c_0}\left(\frac{\partial k_{rw}}{\partial S}\right)_{S_0}\mathbf{g}\left(S - S_0\right) + \left(\frac{\partial \frac{c\rho_w^2}{\mu_w}}{\partial c}\right)_{c_0}\left(k_{rw}\right)_{S_0}\mathbf{g}\left(c - c_0\right) + h.o.t$$

$$\frac{K c_w \rho_o}{\mu_o} k_{ro} \nabla p_w = \left(\frac{K c\rho_o}{\mu_o}\right)_{c_0}\left(k_{ro}\right)_{S_0}\nabla p + \left(\frac{K c\rho_o}{\mu_o}\right)_{c_0}\left(\frac{\mathrm{d} k_{ro}}{\mathrm{d} S}\right)_{S_0}\nabla p_0 \left(S - S_0\right) + \left(\frac{\partial \frac{K c\rho_o}{\mu_o}}{\partial c}\right)_{c_0}\left(k_{ro}\right)_{S_0}\nabla p_0\left(c - c_0\right.$$

$$\frac{K c_w \rho_o}{\mu_o} k_{ro} \nabla p_c = \frac{K c_w \rho_o}{\mu_o} k_{ro} \frac{\mathrm{d} p_c}{\mathrm{d} S}\nabla S = \left(\frac{K c\rho_o}{\mu_o}\right)_{c_0}\left(k_{ro}\frac{\mathrm{d} p_c}{\mathrm{d} S}\right)_{S_0}\nabla S + \\ + \left(\frac{\mathrm{d}\frac{K c\rho_o}{\mu_o}}{\mathrm{d} c}\right)_{c_0}\left(k_{ro}\frac{\mathrm{d} p_c}{\mathrm{d} S}\right)_{S_0}\nabla S_0 \left(c - c_0\right) + \left(\frac{K c\rho_o}{\mu_o}\right)_{c_0}\left(\frac{\mathrm{d}\left(k_{ro}\frac{\mathrm{d} p_c}{\mathrm{d} S}\right)}{\mathrm{d} S}\right)_{S_0}\nabla S_0\left(S - S_0\right)$$

$$\frac{K c_w \rho_o^2}{\mu_o} k_{ro} \mathbf{g} = \left(\frac{K c\rho_o^2}{\mu_o}\right)_{c_0}\left(k_{ro}\right)_{S_0}\mathbf{g} + \left(\frac{K c\rho_o^2}{\mu_o}\right)_{c_0}\left(\frac{\partial k_{ro}}{\partial S}\right)_{S_0}\mathbf{g}\left(S - S_0\right) + \left(\frac{\partial \frac{K c\rho_o^2}{\mu_o}}{\partial c}\right)_{c_0}\left(k_{ro}\right)_{S_0}\mathbf{g}\left(c - c_0\right) + h.o.t$$