Physics of Lumen Formation and Interaction

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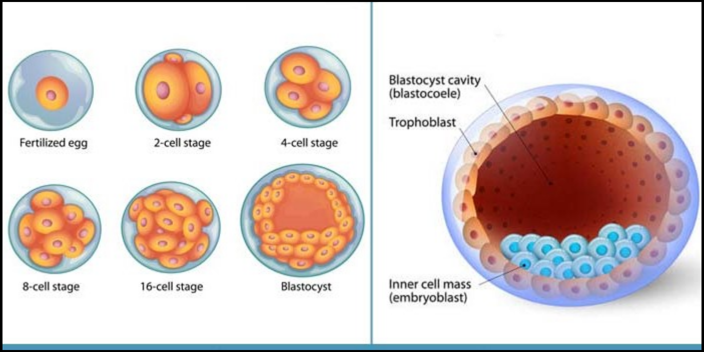
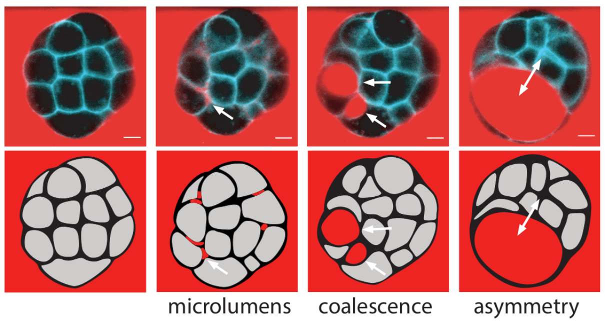
How do living beings form tubes and cavities?



What are the possible mechanisms?

- Coarsening

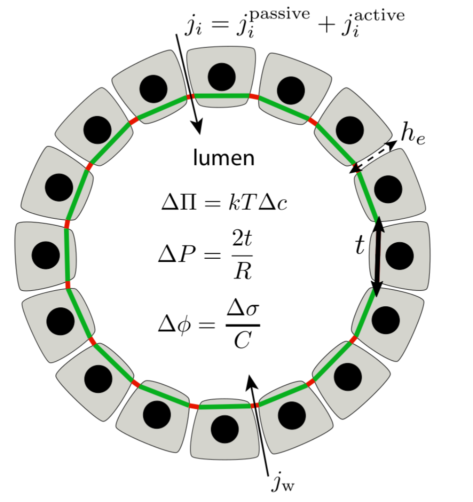


- Fluid Pumping

- Electrostatic Interactions

$$\nabla^2 \phi = \frac{1}{\ell_{DH}^2} \phi$$

Fluxes of water and solutes



Fluxes of water and solutes

Say there are N ionic solutes and the i^{th} solute has a charge q_i

$$j_w = \lambda_w(\Delta\Pi - \Delta P) - \sum_{i=1}^{N} \lambda_{w,i} \left(\Delta c_i + \frac{q_i \overline{c_i}}{k_B T} \Delta\phi \right) + j_w^{active} \quad (1)$$

$$j_i = -\lambda_i \left(\Delta c_i + \frac{q_i \overline{c_i}}{k_B T} \Delta\phi \right) + \lambda_{i,w}(\Delta\Pi - \Delta P) - \sum_{j \neq i} \lambda_{i,j} \left(\Delta c_j + \frac{q_j \overline{c_j}}{k_B T} \Delta\phi \right) + j_i^{active}$$

Osmotic Pressure Electric Potential

Osmotic pressure can form lumens

$$j_w = \lambda_w(\Delta\Pi - \Delta P)$$

For MDCK cells,

$$\lambda_w \sim (0.1 - 1) \times 10^{-7} \mu m s^{-1} Pa^{-1}$$

During growth of the zebrafish inner ear

$$j_w \sim 1 - 8\mu m/h$$

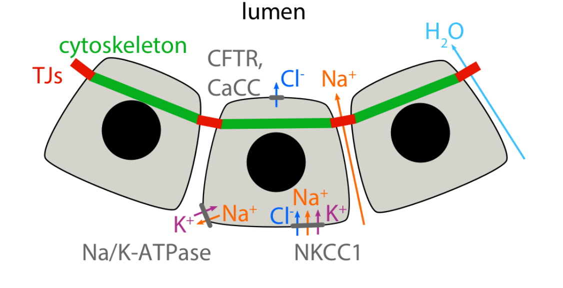
This gives

$$\Delta\Pi - \Delta P \sim 3 - 200 kPa$$

Direct measurements $\Longrightarrow$

$$\Delta P \sim 100 - 300 Pa$$

Active pumps are crucial in lumen formation
No pumps, no lumen



Inhibition of Na/K-ATPase pump blocks lumen formation.
Doesn't end there. Primary channels are followed by secondary channels. Therefore, a massive contribution.

Spherical ~~Cow~~ Lumen

$$\dot{R} = \lambda_w(\Delta\Pi - \Delta P)$$

$$\frac{R}{3}\dot{c_i^\ell} = -\dot{R}c_i^\ell - \lambda_i[\Delta c_i + \frac{q_i\overline{c_i}}{k_B T}\Delta\phi] + j_i^{active}$$

Where

$$\Delta\phi = \frac{R}{6C}\sum_i q_i \Delta c_i$$

$$\Delta\Pi = k_B T \sum_i \Delta c_i$$

$$\Delta P = \frac{2t}{R}$$

Pump-leak mechanism for Spherical Lumen

-e j^{active}

e *passive*

$$c_+^o = c_-^o = c_0$$
$$\lambda_+ = \lambda_- = \lambda$$

Steady state:

at low capacitance and constant lumen radius R

$$\Delta c_- = \Delta c_+ = \frac{j^{active}}{2\lambda}$$
$$\Delta\phi = -\frac{\Delta c_+}{\overline{c}}\left(\frac{k_B T}{e} \sim \; 27mV\right)$$

For constant R,

$$\Delta P = \Delta\Pi$$
$$\implies R = \frac{2t\lambda}{k_B T j^{active}}$$

Is $R = \frac{2t\lambda}{k_B T j^{active}}$ stable? Depends!

- constitutive law of monolayer tension
- active flux density j^{active}

Considering $t = t_0 + 2K\frac{\delta R}{R_0}$,

$$\delta P = 2[2K - t_0]\frac{\delta R}{R_0}$$

$\therefore$ stabilisation for $K > t_0/2$

How do smaller lumens form bigger lumens?

Mouse blastocyst

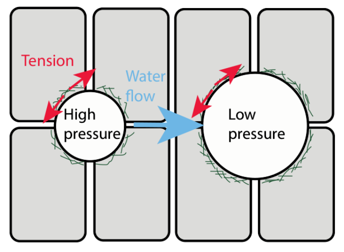
Maybe the same as in bubbles?

Then, we need a “Surface tension”

But there is an INSTABILITY

Can we predict the time scale of coarsening?

$$\mathbf{v}^f = \mathbf{v}^c + \frac{\nabla P^f}{\kappa}$$



Hydraulic resistance

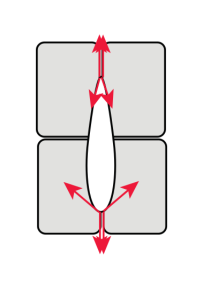
Poiseuille $\implies \kappa \sim \frac{32\eta_w}{\varrho\delta^2}$

Then, typical pressure equilibration time scale

$$\tau \sim \frac{R^2\ell\kappa}{2t} \sim \frac{16R^2\ell\eta_w}{t\varrho\delta^2}$$

Nonspherical ~~cows~~ lumens

Differences in interfacial tension



Inhomogeneous epithelial thinning

So, what have we learnt?

Several mechanisms of lumen formation

Fluxes as possible explanations.

Found time scales for coarsening