06/06/24 Assignment-4 CSA0672 - DAA T. Godwin Abilash 192321012

1) a) $x(n) = x(n-1) + 5 \quad n > 1 \quad x(1) = 0$

$x(n) = x(n-1) + 5 \rightarrow ①$

$x(n-1) = x(n-1-1) + 5$
$= x(n-2) + 5 \rightarrow ②$

$x(n-2) = x(n-2-1) + 5$
$= x(n-3) + 5 \rightarrow ③$

Sub eq ③ in ②

$x(n-1) = x(n-3) + 5 + 5$
$= x(n-3) + 10 \rightarrow ④$

Sub eq ④ in eq ①

$x(n) = x(n-3) + 10 + 5$
$= x(n-3) + 15$

for some k,

$x(n) = x(n-k) + 5k \rightarrow ⑤$

$n - k = 1$
$n - 1 = k$

eq ⑤ $\Rightarrow x(n) = x(1) + 5(n-1)$

$x(n) = 0 + 5n - 5$

$O(n)$

b) $x(n) = 3x(n-1) \quad x(1) = 4$

$x(n) = 3x(n-1) \rightarrow ①$

$x(n-1) = 3x(n-1-1) = 3x(n-2) \rightarrow ②$

$x(n-2) = 3x(n-2-1) = 3x(n-3) \rightarrow ③$

Sub eq ③ in ②,

$x(n-1) = 3[3x(n-3)]$
$= 9x(n-3) \rightarrow ④$

Sub eq ④ in eq ①

$x(n) = 3[9x(n-3)]$
$= 27x(n-3)$

for some k,

$x(n) = 3^k x(n-k) \rightarrow ⑤$

$n - k = 1 \Rightarrow k = n - 1$

eq ⑤ $\Rightarrow x(n) = 3^{n-1} x(1)$
$= 3^{n-1} 4 = 3^n 3^{-1} 4$

$O(3^n)$

d) $x(n) = x(n/3) + 1 \quad x(1) = 1$

$x(n) = x(n/3) + 1 \rightarrow ①$

$x(n-1) \quad x(n/3) = x(n/3/3) + 1 \rightarrow ②$
$= x(\frac{n}{3^2}) + 1$

$x(n-2) \quad x(n/9) = x(n/9/3) + 1.$
$= x(\frac{n}{3^3}) + 1 \rightarrow ③$

Sub eq ③ in eq ②

$x(n/3) = x(\frac{n}{3^3}) + ① ④$

Sub eq ④ in eq ①,

$x(n) = x(\frac{n}{3^3}) + 3$

Some k,

$x(n) = x(\frac{n}{3^k}) + k$

$\frac{n}{3^k} = 1 \Rightarrow n = 3^k$

$k = \log_3 n$

eq ⑤

$x(n) = x(1) + \log n$
$= 1 + \log_3 n$

$O(\log_3 n)$

i) $x(n) = x(n/2) + n \quad n > 1 \quad x(1) = 1 \quad n = 2^k$

$$x(n) = x\left(\frac{n}{2}\right) + n \rightarrow ①$$

$$x(n) = \left[x\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$$

$$x(n) = x\left(\frac{n}{2^2}\right) + \frac{n}{2} + n \rightarrow ②$$

$$x(n) = x\left(\frac{n}{2^3}\right) + \frac{n}{2^2} + \frac{n}{2} + n$$

$$\vdots$$

$$x(n) = x\left(\frac{n}{2^k}\right) + \frac{n}{2^{k-1}} + \frac{n}{2^{k-2}} + \dots \frac{n}{2} + n$$

$$x(n) = x\left(\frac{n}{2^k}\right) + \frac{n}{2^{k-1}} + \dots \frac{n}{2^2} + \frac{n}{2} + n.$$

Assume $\frac{n}{2^k} = 1$

$n = 2^k$ and $k = \log n$

$$x(n) = x(1) + n\left[\frac{1}{2^{k-1}} + \frac{1}{2^{k-2}} + \dots \frac{1}{2} + 1\right]$$

$$x(n) = 1 + n[1 + 1]$$

$$x(n) = 1 + 2n$$

$O(n)$

2) i) $T(n) = T(n/2) + 1$ when $n = 2^k$ for all $k \geq 0$

by using substitution method

$$T(n) = T(n/2) + 1 \rightarrow ①$$

$$T(n/2) = T(n/2^2) + 1 \rightarrow ②$$

Substitute ② in ①

$$T(n) = (T(n/2^2) + 1] + 1$$

$$T(n) = T(n/2^2) + 2 \rightarrow ③$$

$$T(n) = T(n/2^3) + 3 \rightarrow ④$$

$$T(n) = T(n/2^k) + k \rightarrow ⑤$$

Assume $\frac{n}{2^k} = 1 \quad , n = 2^k$

$k = \log n$

$$T(n) = T(1) + \log n$$

$$T(n) = 1 + \log n$$

$O(\log n)$

3) b) $T(n) = T(n-1) + 1$ when $n > 1$

(one comparison at every step)

$T(1) = 0$ (No comparison)

$$T(n) = T(1) + T(n-1)$$

$$= 0 + n - 1$$

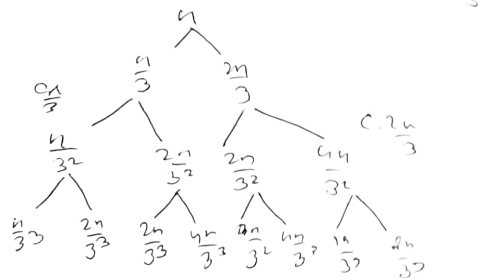
$$= n - 1$$

$O(n)$

ii) $T(n) = T(n/3) + T(2n/3) + cn$

$$T(n/3) = T\left(\frac{n}{3^2}\right) + T\left(\frac{2n}{3^2}\right) + \frac{c.n}{3}$$

$$T(2n/3) = T(2n/3^2) + T(4n/3^2) + \frac{c2n}{3}$$



$$\frac{n}{3^2} + \frac{2n}{3^2} + \frac{2n}{3^2} + \frac{4n}{3^2} = \frac{9n}{3^2}$$

$\frac{n}{3^k} = 1 \quad , k = \log_3 n$ ($\log_3 n$ with base)

$c.n \log_3 n$

$O(n \log n)$

3) a) the algorithm finds the minimum value in the array, efficiency breaking down the problem into smaller sub problem.

$n = 1 \Rightarrow$ There is only one element

4) Analyse the order of growth

if $f(n) = 2n^2 + 5$ $\quad g(n) = 7n$

	$f(n)$	$g(n)$
n	$2n^2+5$	$7n$
$n=1$	7	7
$n=2$	13	14
$n=3$	23	21

$n \geq 3$ $\quad f(n) \geq g(n).c$

$f(n)$ is always greater than or equal to $g(n)$ when $n_0 \geq 3$

$$f(n) = \Omega(g(n))$$