Planetary Motion

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December 2018

1 2D Two-Body Problem

We started simply by modelling the Earth's orbit around a fixed Sun. The only force we applied to the Earth was Newton's universal gravitation (eq. 1). In this equation, G is the gravitational constant which we used with a value of 39.478 $\frac{AU^3}{(solarunits)^2 * year}$. M_E and M_S are the masses of the sun and Earth respectively. We used $3.00 * 10^6$ solar units for the mass of Earth and 1 solar unit for the mass of the Sun. r_{ES} is the distance from the Sun to the Earth which we assigned to $1AU$.

In order to compute this force we broke it up into x and y components, the equation for the x-component is given in eq. 2 and the y-component equation was analogous but with y-values in place of the x-values in the numerator.

$$F_G = \frac{GM_E M_S}{r_{ES}^2} \tag{1}$$

$$F_{G,x} = \frac{GM_E M_S (x_E - x_S)}{r_{ES}^3} \tag{2}$$

In order to produce an orbit from the attractive force we gave Earth a starting velocity of $2\pi \frac{AU}{year}$ in the y-direction and a starting position of $1AU$ in the x-direction.

We used the force equation described by eq. 2 to solve Newton's Second Law in its differential form (eq. 3). We broke this second order equation into two pairs of first order ODEs, one pair for x and another analogous pair for y (eq. 4, 5).

$$\frac{d^2x}{dt^2} = \frac{F}{m} \tag{3}$$

$$\frac{dv}{dt} = \frac{F}{m} \tag{4}$$

$$\frac{dx}{dt} = v \tag{5}$$

In order to simulate this system we used the Euler-Cromer method where we update the velocities using the previous timestep's position and velocity values and than use these updated velocities to update the positions. We simulated our orbit for 5 years with a time step of 0.002 years. Our results can be seen graphically in figure 1. We saw the expected result of Earth's stable circular orbit around the Sun.

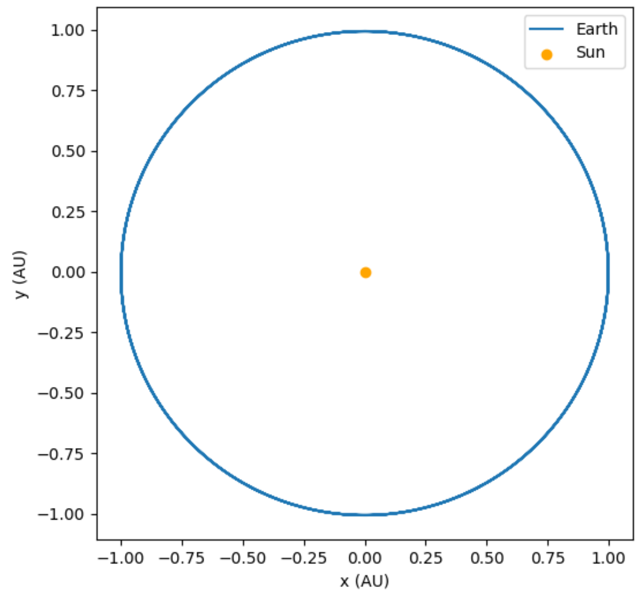

Figure 1: Earth's orbit around the sun for five years. Earth traces out a circular trajectory around the sun when subjected to gravitational attraction

2 Restricted Three-Body Problem

After we had the stable orbit working we added Jupiter to our solar system. We modelled the Sun's pull on both the Earth and Jupiter as well as Earth and Jupiter's pulls on each other. We used the same force equation (eq 2) to describe these attractions as we did for those between the Sun and the Earth but with appropriate position and mass parameters for the bodies in question.

We used a mass of $9.55 * 10^{-}4$ solar units for Jupiter with a starting x-position of $5.2AU$ and a starting y-velocity of $\frac{2\pi}{11.68}\frac{AU}{/}year$. Its starting y-positions and x-veloities were both zero.

We simulated this situation for 40 years with a time step of 0.01 years. Our results can be seen in figures 2-4. In figure 2 and in figure 3 we see stable circular orbits around the Sun for both Earth and Jupiter which is expected given that our starting conditions resemble the actual solar system. In figure 3 we used a mass ten-times greater than Jupiter's actual mass but this was not enough of a difference to produce a visible effect on the orbit of the Earth. In figure 4, however we see a large difference in the behavior of the Earth. We increased Jupiter's mass by a factor of 1000, making it within 5% of the mass of the sun. In order to produce the plot we had to reduce our timestep to $1 * 10^{-}5$ years. Any higher timestep we tried resulted in the Earth ejecting from the system. In this plot the Earth moves very erratically, perhaps even colliding with the sun had we allowed for that.

3 Full Three-Body Problem

Finally we allowed the Sun to leave its fixed position at the origin and feel the attractive forces of gravity like the other two bodies. We set the initial positions by finding the center of mass at $t = 0$ with the sun

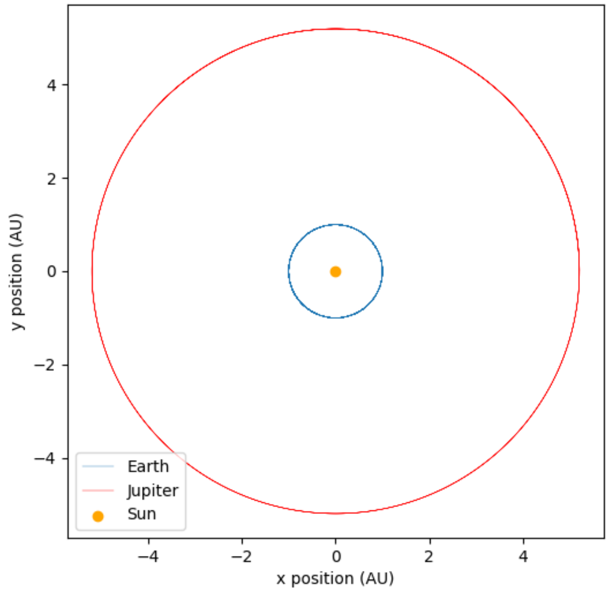

Figure 2: Orbits of the Earth and Jupiter around the sun. In this plot we can see stable circular orbits for both the Earth and Jupiter

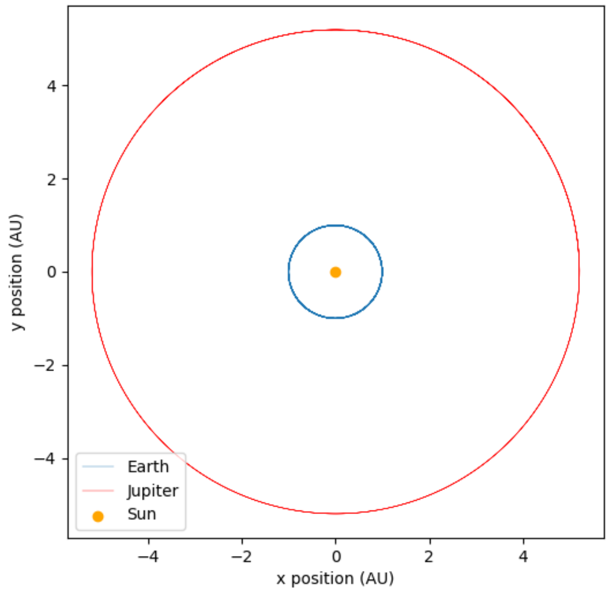

Figure 3: Orbits for Earth and Jupiter with ten-times its normal mass. Again, we see stable circular orbits for both Earth and Jupiter

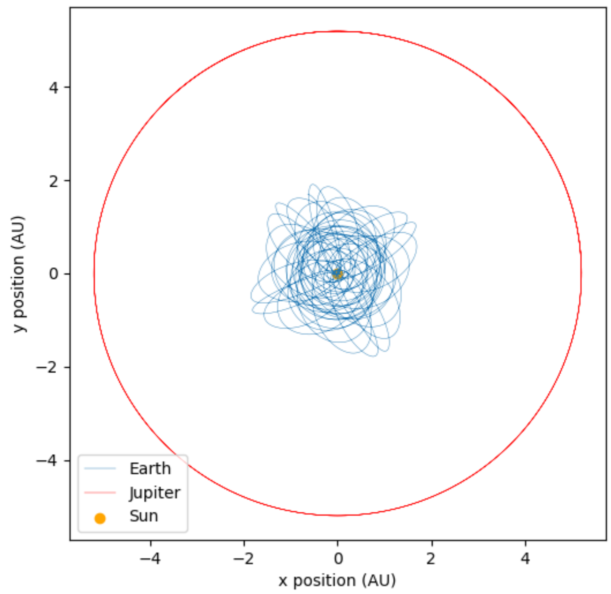

Figure 4: Orbits for Earth anf Jupiter with 1000-times its normal mass. Here we see that Jupiter maintains a stable circular orbit around the Sun while Earth moves very erratically. We used a timestep of 1e-5 for this plot to maintain resolution for the Earth's movement.

initially placed at the origin and adjusting all of their positions. We used equation 6 to find the center of mass and the following equation to make the adjustments.

$$x_{CM} = \frac{x_S M_S + x_E M_E + x_J M_J}{M_S + M_E + M_J} \tag{6}$$

$$x_{adj} = x_{start} - x_{CM}$$

We ran our simulation over the same time period as in the restricted three-body problem and with the same time step. We also used the same increasing masses of Jupiter. Our results can be seen in figures 5-8. In figure 5, we see similar results to the one's seen in figure 2. Our planetary orbits look almost identical but the sun actually travels in a tight circle rather than staying in place. In figure 6 we started to see deviations from the restricted model. The Earth's orbit remained circular overall but there were some irregularities in it. For both of these plots Jupiter's orbit did not appear to change. We see Jupiter's behavior change dramatically in figures 7 and 8 where its influence on the Sun becomes very apparent. The two appear to diverge but actually enter into a binary orbit which can be seen in 9. In figure 7 we see the Earth ejected from its proximity to the Sun and perhaps transferring to Jupiter or leaving the system but when we decreased our timestep from $1*10^-3$ years to $1*10^-4$ years we can see it actually stay mostly under the sun's influence.

To elucidate what was happening with the apparent divergence between Jupiter and the Sun in figures 7 and 8 we let the simulation run for 4000 simulated years. After increasing our duration we can actually see

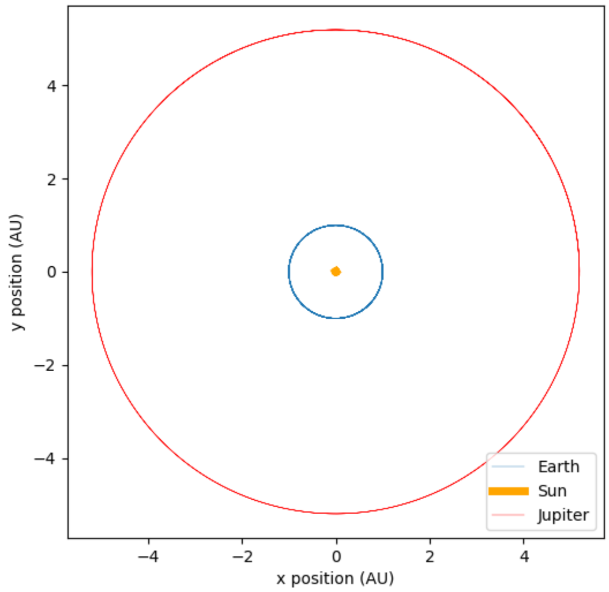

Figure 5: Full three-body problem. Its hard to make out in the figure but the Sun moves in a very tight circle while the other two bodies retain their stable circular orbits seen before.

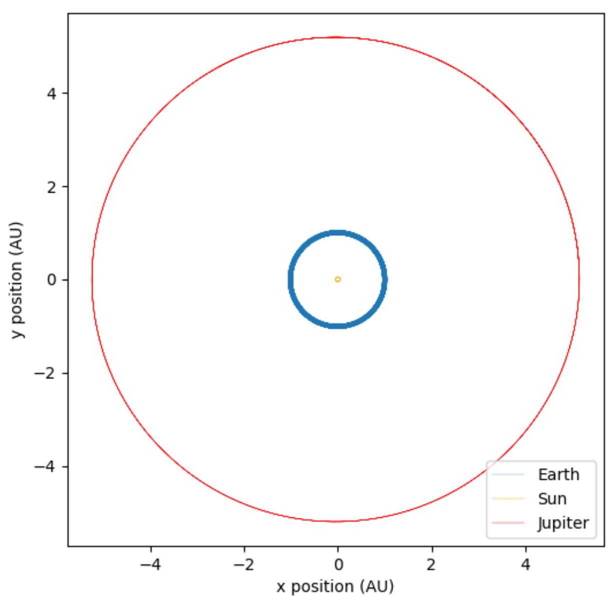

Figure 6: Full three-body problem with Jupiters mass increased ten-times. This plot again shows the circular orbits of the three bodies. The thickness of the line for Earth's trajectory suggest a slight irregularity in its orbit.

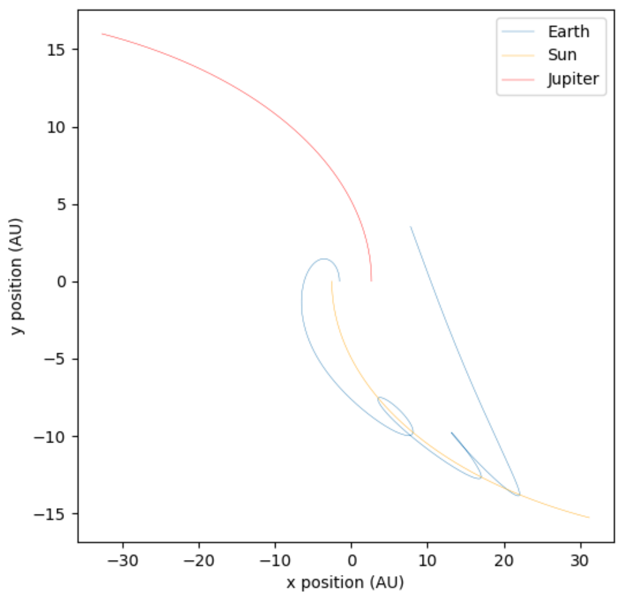

Figure 7: Full three-body problem with Jupiter's mass increased 1000 times. In this plot we see different behavior than before. The Earth bounces around erratically as the Sun and Jupiter form a completely distinct binary orbit.

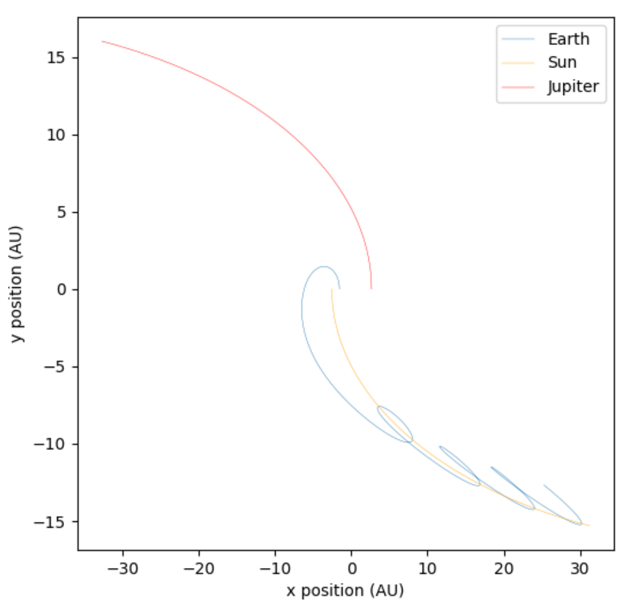

Figure 8: Full three-body problem with 1000-times larger Jupiter and a smaller time step. For this plot we decreased the timestep to 1e-4 years. We can see more reasonable behavior from Earth as it does not get ejected from Solar influence.

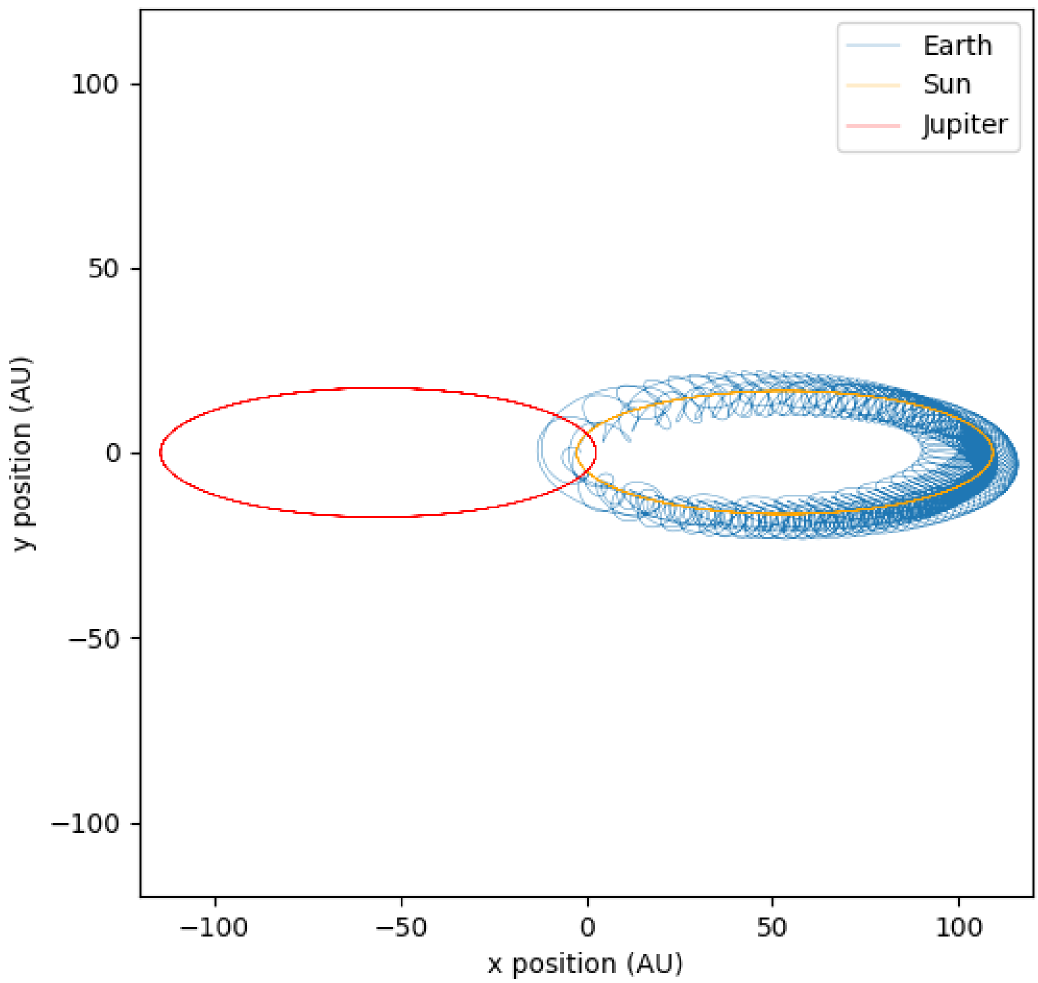

Figure 9: Full three body problem. This plot was produced after simulating our system for 4000 years. We set Jupiter's mass 1000-times larger than normal and used the smaller timestep of 1e-4 years. Earth retains its solar orbit though it becomes erratic. The Sun and Jupiter settle into a stable binary orbit.

a complete binary orbit around the center of mass between Jupiter and the Sun. We also see that the Earth mostly stays close to the sun, following it through its orbit.