Polygonal Cannonball Numbers

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1 Introduction

Recently I watched the video `https://www.youtube.com/watch?v=q6L06pyt9CA`, featuring Matt Parker. Being a huge fan of Matt and of Numperphile, and being rather full of myself, my first response was naturally one of doubt (also because of the spirit of mathematical enquiry and all that). I decided to have my own crack at the problem.

I reasoned that checking if a number is polygonal should be a roughly $\mathcal{O}(1)$ operation as we can find the nth term of the base-s polygonal numbers $P(s, n)$, which will be quadratic in n, and solve it for n with the quadratic formula, so to check if some cannonball numbers $C(s, n_c)$ is polygonal we just see if the corresponding n_p is an integer. Now 10^9 is a fairly small number. Seeing as my CPU's clockspeed is in the range of gigahertz, and we're just checking a tiny fraction of those numbers as we're just computing the cannonball numbers under this limit, it seems reasonable that this should be doable fairly fast.

I've thought about the problem of higher-dimensional stacks of cannonballs (ie the ones formed by adding up the cannonball numbers), but I've not done anything about it.

2 The Maths

Indeed, this approach does seem to work. Almost by definition we have the recurrence in polygonal numbers

$$P(s, n) = P(s, n-1) + n(s-2) - (s-3)$$

so we can use

$$P(s, n) = \sum_{r=1}^{n} P(s, r) - P(s, r-1)$$

$$
\begin{aligned}
&= \sum_{r=1}^{n}(n(s-2)-(s-3)) \\
&= \frac{1}{2}n(n+1)(s-2)-n(s-3) \\
&= \frac{n^2(s-2)-n(s-4)}{2}
\end{aligned}
$$

Fortunately this seems to agree with what Wikipedia thinks. Now, we have

$$
\begin{aligned}
0 &= (s-2)n^2-(s-4)n-2P(s,n) \\
\implies n &= \frac{s-4+\sqrt{(s-4)^2+8(s-2)P(s,n)}}{2s-4}
\end{aligned}
$$

Wikipedia still seems to think we're on track.

Another result that I don't really use is that

$$
\begin{aligned}
C(s,n) &= \sum_{r=1}^{n}P(s,n) \\
&= \frac{1}{2}\sum_{r=1}^{n}(n^2(s-2)-n(s-4)) \\
&= \frac{1}{2}\left(\frac{n(n+1)(2n+1)(s-2)}{6}-\frac{n(n+1)(s-4)}{2}\right) \\
&= \frac{1}{12}n(n+1)[(2n+1)(s-2)-3(s-4)]
\end{aligned}
$$

In fact I've only used this in verification of the results.

Regardless, now we need only work our way up the $C(s,n)$s using the recurrence $C(s,n) = P(s,n)+C(s,n-1)$, and check for each if the quadratic formula gives an integer result. This is most easily done by checking if the discriminant is a perfect square and then checking that the denominator divides the numerator.

3 The Programming

For speeeeeeed I implemented this in C (although there is a long abandoned parallel Python implementation). I used 128-bit integers to be on the safe side, as 10^{19} is a little small for my liking. This meant I had to do a lot of messing around to get things to actually display in base 10. This program is shown in Listing 1.

I did briefly consider either implementing or importing some kind of arbitrary precision integer arithmetic functionality, but then I decided I wasn't going to run it on anything fast enough to have to worry about that, and I have better things to do.

There's also a slick little progress update that gets printed to STDERR, and a number of zsh scripts to save me typing.

I also have a program that verifies results, removes duplicates and formats them into a LaTeX table (spoilers for table 1), shown in listing 2.

```
1 // Finding cannonball numbers that are equal to a polygonal number of the same
2 // base. See https://www.youtube.com/watch?v=q6LO6pyt9CA
3
```

```c
#include <stdio.h>
#include <math.h>
#include <stdlib.h>

// Macro to calculate the n-th polygonal number of side s. It's a macro so I
// don't have to keep typing it but it stays efficient.
// There also also some other macros with the nth term of a cannonball number
#define POLYGONAL(s, n) ((n * n * (s - 2) - n * (s - 4)) >> 1)
#define CANNON(s, n) n * (n + 1) * ((s - 2) * (2 * n + 1) - 6 * (s - 4)) / 12
// Symbolic constants for the default values of the parameters
#define MAX_CHECK_DEFAULT ipow(10, 11)
#define MAX_BASE_DEFAULT 31265
// How many numbers to check before giving an update
#define UPDATE_CYCLES ipow(10, 6) * 5

// integer type being used to represent cannonball numbers
typedef __int128_t cannonball_int;
// maximum possible amount of memory needing to be allocated to represent a
// cannonball_int in base 10 (in an ASCII-encoded string)
#define CANNON_INT_STR_LEN (int)(sizeof(cannonball_int) * log10(0xff) + 2)

// custom function to format a cannonball int into a base 10 string, as printf
// doesn't know how.
void fmt_c(cannonball_int n, char *target) {
    ssize_t i = 0;
    ssize_t size;
    cannonball_int tmp;
    while (n != 0) {
        target[i++] = '0' + (n % 10);
        n = n / 10;
    }
    size = i;
    target[size--] = '\0';
    // reverse it because we built the string back to front
    for (i--; i > size - i; i--) {
        tmp = target[i];
        target[i] = target[size - i];
        target[size - i] = tmp;
    }
}

// Integer exponentiation by squaring - basically just so I can write integers
// in standard form.
cannonball_int ipow(cannonball_int base, cannonball_int exp) {
    cannonball_int result = 1;
    while (exp) {
        if (exp & 1)
            result *= base;
        exp >>= 1;
        base *= base;
    }
```

```
55      return result;
56  }
57
58  // Find the integer square root, with the bit-shifting algorithm. This is used
59  // when applying the quadratic formula to see if there are rational solutions.
60  cannonball_int isqrt(cannonball_int n) {
61      cannonball_int small, large;
62      if (n < 2) {
63          return n;
64      } else {
65          small = isqrt(n >> 2) << 1;
66          large = small + 1;
67          if (large * large > n) {
68              return small;
69          } else {
70              return large;
71          }
72      }
73  }
74
75  // Routine to check all cannonball numbers of side `base` up to `max` to see if
76  // they are also a polyhedral number of side `base`.
77  void check_base(cannonball_int base, cannonball_int max_check,
78                  cannonball_int max_base) {
79      char *c_1 = malloc(CANNON_INT_STR_LEN),
80           *c_2 = malloc(CANNON_INT_STR_LEN),
81           *c_3 = malloc(CANNON_INT_STR_LEN),
82           *c_4 = malloc(CANNON_INT_STR_LEN);
83      cannonball_int i, cannonballs;
84      cannonball_int discriminant, discriminant_sqrt, numerator, denominator;
85      denominator = 2 * base - 4;
86      for (  i = 2, cannonballs = 1 + POLYGONAL(base, 2);
87             cannonballs <= max_check;
88             i++, cannonballs += POLYGONAL(base, i)) {
89          if (i % UPDATE_CYCLES == 0 || (i == 2 && base % UPDATE_CYCLES == 0)) {
90              fmt_c(base, c_1);
91              fprintf(stderr, "\r%3.0f%% %3.0f%% %s",
92                      100.0 * base / max_base,
93
                        ↪  // As cannonballs grows roughly cubically, take a cube root
94                      // to linearise the progress
95                      100.0 * pow(1.0 * cannonballs / max_check, 1.0 / 3),
96                      c_1);
97              fflush(stderr);
98          }
99          discriminant = (base - 4) * (base - 4) + 8 * (base - 2) * cannonballs;
100         discriminant_sqrt = isqrt(discriminant);
101         if (discriminant_sqrt * discriminant_sqrt == discriminant) {
102             numerator = base - 4 + discriminant_sqrt;
103             if (numerator % denominator == 0) {
```

```
104
                    ↪  // not using %n$ syntax but just passing the same argument twice
105                 // because of something something ISO C
106                 fmt_c(cannonballs, c_1);
107                 fmt_c(base, c_2);
108                 fmt_c(numerator / denominator, c_3);
109                 fmt_c(i, c_4);
110                 fprintf(stderr, "\r");
111                 printf(">%s == P(%s, %s) == C(%s, %s)\n",
112                        c_1, c_2, c_3, c_2, c_4);
113             }
114         }
115     }
116     free(c_1); free(c_2); free(c_3); free(c_4);
117 }
118
119 int main(int argc, char **argv) {
120     cannonball_int base,
121                    max_check = MAX_CHECK_DEFAULT,
122                    max_base = MAX_BASE_DEFAULT;
123     char *c_1 = malloc(CANNON_INT_STR_LEN),
124          *c_2 = malloc(CANNON_INT_STR_LEN);
125     if (argc >= 2) {
126         max_check = (cannonball_int)strtold(argv[1], NULL);
127     }
128     if (argc >= 3) {
129         max_base = (cannonball_int)strtold(argv[2], NULL);
130     }
131     fmt_c(max_check, c_1);
132     fmt_c(max_base, c_2);
133     printf("Finding polygonal cannonball numbers <= %s, with base <= %s\n",
134            c_1, c_2);
135     printf("Using integers of width %zu bytes, which go up to about %.5e\n",
136            sizeof(cannonball_int), exp(log(0xff) * sizeof(cannonball_int)));
137     for (base = 3; base <= max_base && base <= max_check; base++) {
138         check_base(base, max_check, max_base);
139     }
140     free(c_1); free(c_2);
141     return 0;
142 }
```

Listing 1: The main C source code

```
1 #!/usr/bin/env python3
2
3 """
4 Program to verify polygonal cannonball numbers and then do a little
5 post-processing.
6 """
7
8 import argparse
```

```
9
10 from cannonball import polygonal
11 from re import findall
12 from itertools import chain
13 from math import log10, inf
14
15 def cannonball(s, n):
16     """
17     Derived cubic nth term of cannonball numbers.
18     """
19     return n * (n + 1) * ((2 * n + 1) * (s - 2) - 3 * (s - 4)) // 12
20
21 def check_line(line):
22     """
23     Parse and check one line, just by extracting all present integers with some
24     regex.
25     """
26     C, s, n_P, _, n_C = map(int, findall(r"\d+", line))
27     if not (C == cannonball(s, n_C) == polygonal(s, n_P)):
28         raise ValueError("line {!r} incorrect".format(line))
29     return s, C, n_P, n_C
30
31 def check_files(files):
32     """
33     Parse and check all the solutions in each file
34     """
35     solutions = set()
36     for line in chain.from_iterable(files):
37         if line.startswith(">"):
38             solutions.add(check_line(line))
39     output_solutions(solutions)
40
41 def solutions_key(sol):
42     """
43     The idea here is to push all the boring solutions to the end
44     """
45     s, C, n_P, n_C = sol
46     if (s > 100 and
47         s % 3 == 2 and
48         log10(C) > -3 + 7 * log10(s) and
49         log10(C) < -2.5 + 7.5 * log10(s)):
50         return (inf, *sol)
51     return sol
52
53 def output_solutions(solutions_):
54     """
55     Write solutions to a LaTeX table
56     """
57     solutions = list(sorted(solutions_, key=solutions_key))
58     # write the output as LaTeX. We're not here to be pretty, so might as well
59     # play a few rounds of code golf.
```

```
    for solution in solutions:
        print((" {} ".join("&" * 5)[2:-1] + r"\\").format(*solution))

def get_args():
    """
    Get arguments from command line
    """
    parser = argparse.ArgumentParser(description=__doc__)
    parser.add_argument("--files", type=argparse.FileType("r"),
                        nargs="*", help="list of files to read")
    return parser.parse_args()

if __name__ == "__main__":
    args = get_args()
    check_files(args.files)
```

Listing 2: Python verification program

4 The Ugly

Table 1 lists all the solutions that I've found, so far. The TEX source of the table is in `../src/tab.tex`, which is derived from `../src/c/solutions/*`. It has been ordered so that all the solutions along the trendy line are grouped together at the end, so they can be viewed separately from the more flavourful, stylish and individualistic solutions.

I have also plotted both the data in its entirety on a double logarithmic scale 2, and an excerpt from the data on a linear scale (1).

The obvious pattern that jumps out is the big line of points for all the sides congrent to 2 (mod 3). Particularly because it looks like such a straight line on the log-log plot, we would expect it to be modelled well as a constant multiple of some power of s. I drew two lines that seemed to roughly bound it, and used those to extract the points on the line and then do some linear regression on that (figure 3). I obtained the formula

$$C = 0.0051274 \cdot s^{7.023781}$$

which seems to be accurate to within probably about 1%, I've not really checked properly.

The R code I used to achieve all this is in Listing 3.

```
library(ggplot2)

soln_df <- read.table("solutions.tsv")
colnames(soln_df) <- c("s", "C", "n_P", "n_C")

trend_df <- subset(soln_df, log10(C) > -3 + 7 * log10(s) &
                            log10(C) < -2.5 + 7.5 * log10(s) &
                            s > 100)

model <- lm(log(C) ~ log(s), data=trend_df)
intercept <- coef(summary(model))["(Intercept)", "Estimate"]
grad <- coef(summary(model))["log(s)", "Estimate"]
```

```
cat("\\begin{equation*}\n")
cat("C =", exp(intercept), "\\cdot s ^ {", grad, "}\n")
cat("\\end{equation*}\n")

ggplot(soln_df, aes(s, C)) +
    geom_point() +
    ggtitle("Log plot of polygonal cannonball numbers") +
    labs(x="s - sides of base polygon", y="C - number of cannonballs") +
    theme(panel.grid.minor = element_line(colour="gray", size=0.4),
          panel.grid.major = element_line(colour="gray", size=1),
          panel.background = element_blank()) +
    scale_x_log10() +
    scale_y_log10() +
    geom_abline(intercept = -3, slope = 7, linetype="dotted") +
    geom_abline(intercept = -2.5, slope = 7.5, linetype="dotted")

ggplot(soln_df, aes(s, C)) +
    geom_point() +
    ggtitle("Linear plot of subrange") +
    labs(x="s - sides of base polygon", y="C - number of cannonballs") +
    theme(panel.grid.minor = element_line(colour="gray", size=0.4),
          panel.grid.major = element_line(colour="gray", size=1),
          panel.background = element_blank()) +
    xlim(0, tail(trend_df$s, n = 1))

ggplot(trend_df, aes(s, C)) +
    geom_point() +
    ggtitle("Log plot of the subset") +
    labs(x="s - sides of base polygon", y="C - number of cannonballs") +
    theme(panel.grid.minor = element_line(colour="gray", size=0.4),
          panel.grid.major = element_line(colour="gray", size=1),
          panel.background = element_blank()) +
    scale_x_log10() +
    scale_y_log10() +
    geom_smooth(method = "lm", linetype="dashed")
```

Listing 3: R graphical analysis

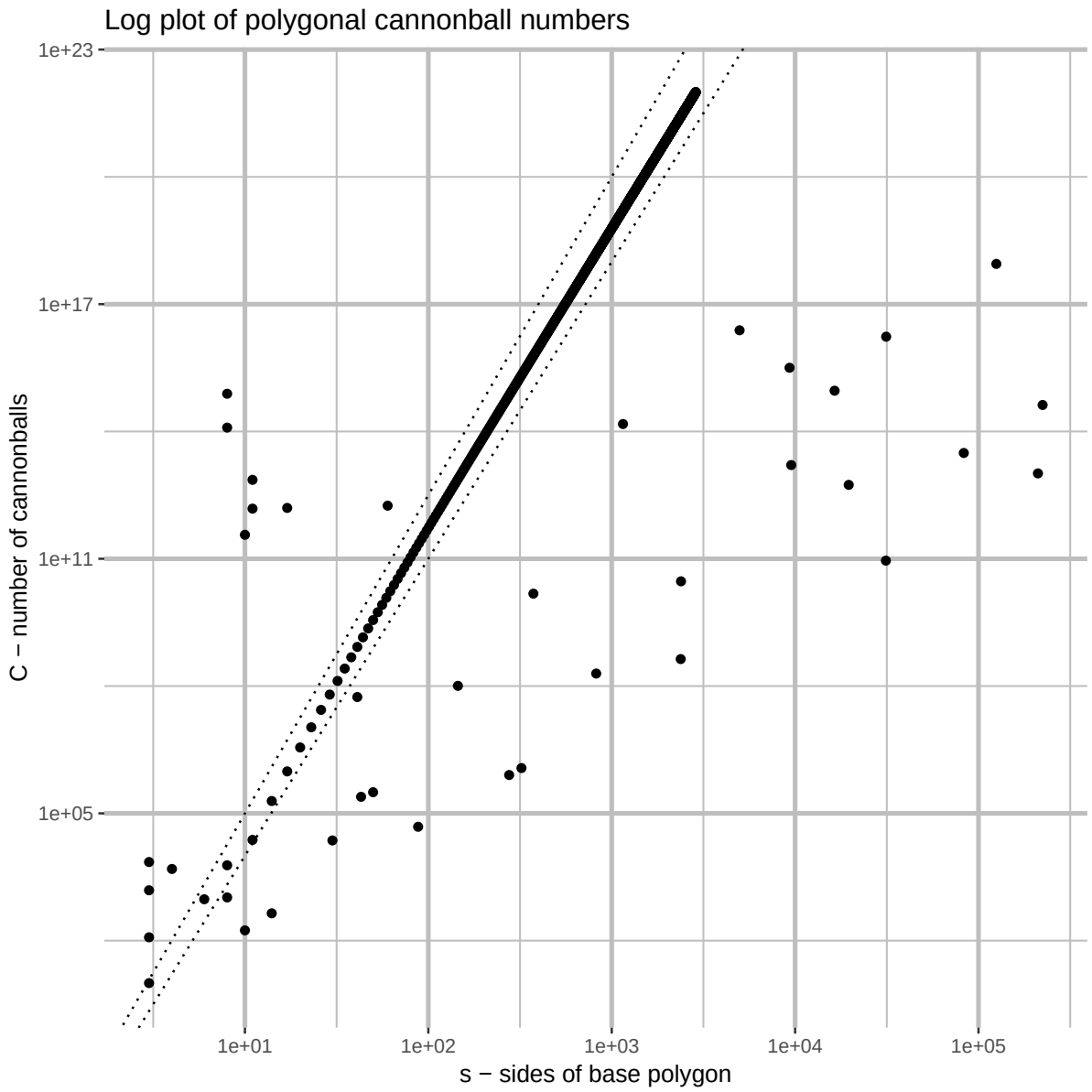

Figure 1: Log plot

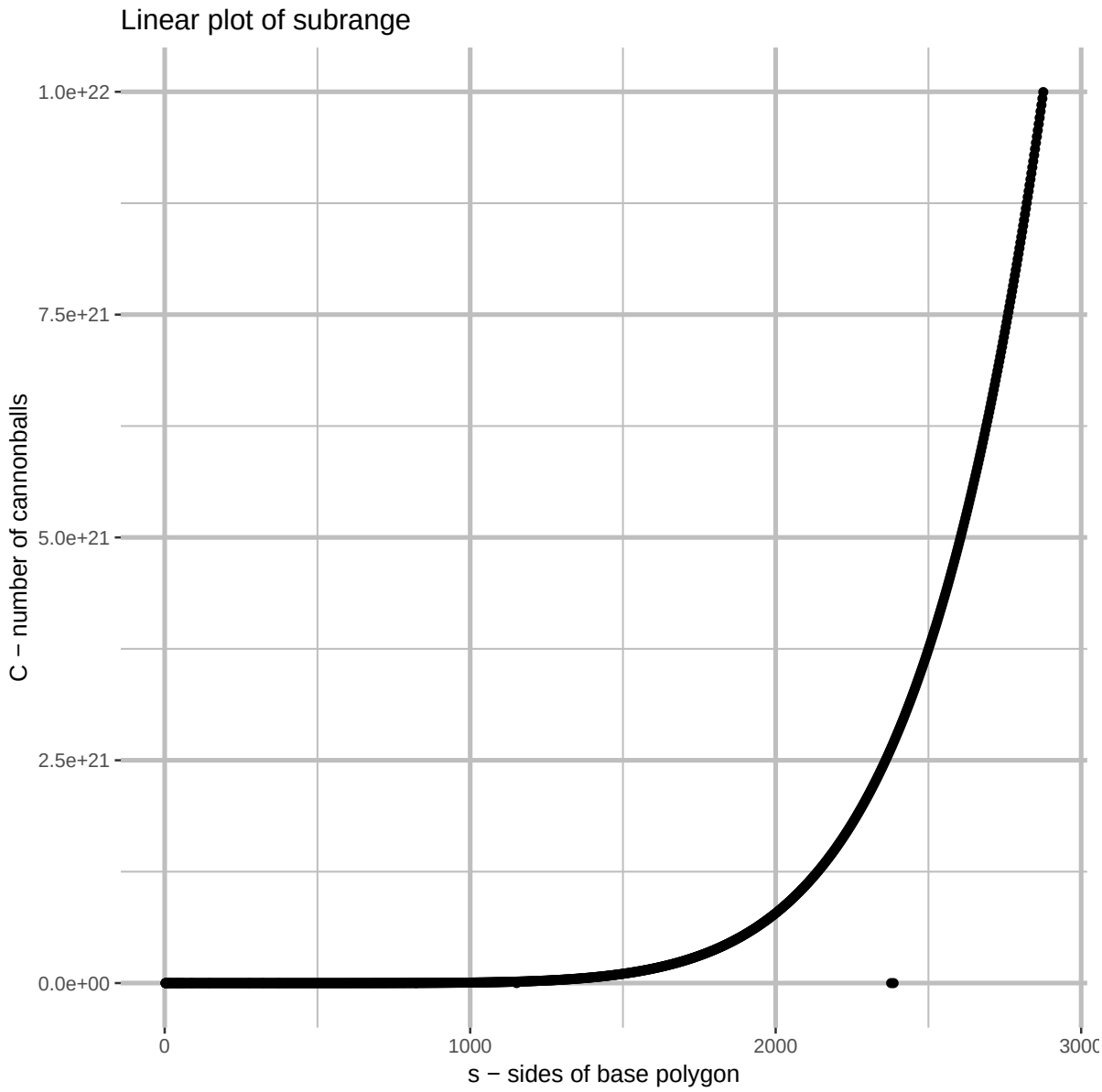

Figure 2: Linear plot

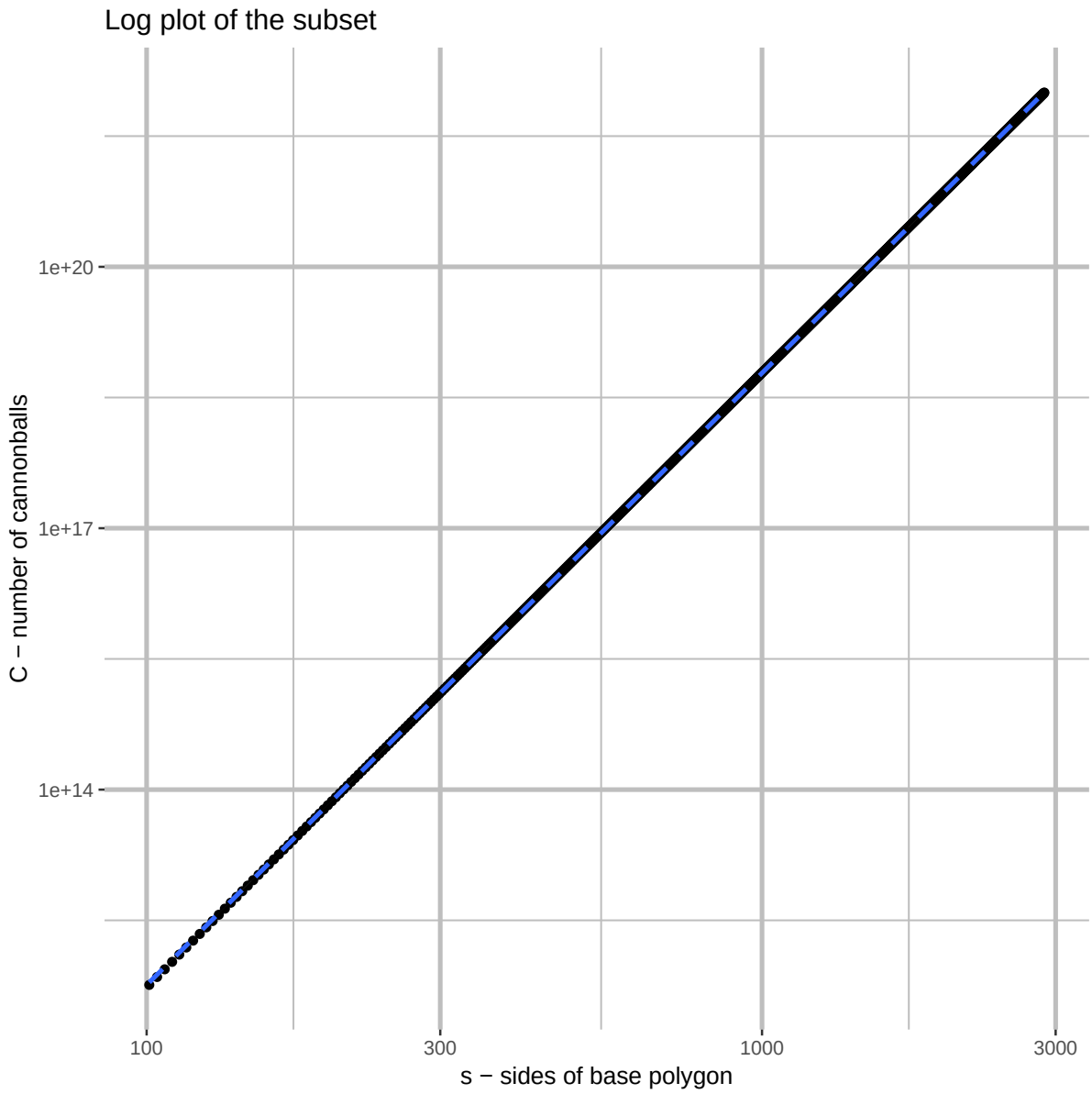

Figure 3: Log plot of the interesting bit

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
3	10	4	3
3	120	15	8
3	1540	55	20
3	7140	119	34
4	4900	70	24
6	946	22	11
8	1045	19	10
8	5985	45	18
8	123395663059845	6413415	49785
8	774611255177760	16068720	91839
10	175	7	5
10	368050005576	303336	6511
11	23725	73	25

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
11	1519937678700	581175	10044
11	7248070597636	1269127	16906
14	441	9	6
14	195661	181	46
17	975061	361	73
17	1580765544996	459096	8583
20	3578401	631	106
23	10680265	1009	145
26	27453385	1513	190
29	63016921	2161	241
30	23001	41	17
32	132361021	2971	298
35	258815701	3961	361
38	477132085	5149	430
41	55202400	1683	204
41	837244045	6553	505
43	245905	110	33
44	1408778281	8191	586
47	2286380881	10081	673
50	314755	115	34
50	3595928401	12241	766
53	5501691505	14689	865
56	8214519205	17443	970
59	12001111741	20521	1081
60	1785508245600	248132	5695
62	17194450141	23941	1198
65	24205450501	27721	1321
68	33535911025	31879	1450
71	45792819865	36433	1585
74	61704091801	41401	1726
77	82135801801	46801	1873
80	108110983501	52651	2026
83	140830060645	58969	2185
86	181692979525	65773	2350
88	48280	34	15
89	232323110461	73081	2521
92	294592986361	80911	2698
95	370651946401	89281	2881
98	462955752865	98209	3070
145	101337426	1191	162
276	801801	77	26
322	1169686	86	28
374	15064335000	9000	624
823	197427385	694	113
1152	149979784926720	510720	9215
2378	432684460	604	103
2386	29437553530	4970	420
4980	24264913354964425	3122317	30810
9325	3176083959788026	825436	12691
9525	16195753597485	58322	2169

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
16420	913053565546276	333506	6936
19605	5519583702676	23731	1191
31265	90525801730	2407	259
31368	17147031694579605	1045635	14858
83135	31148407558500	27375	1310
125070	890348736143873526	3773306	34956
210903	10290361955160	9879	664
223613	421687634347915	61414	2245
101	574298249185	107713	3265
104	707845127221	117811	3466
107	867169871821	128521	3673
110	1056291950701	139861	3886
113	1279717317685	151849	4105
116	1542481297345	164503	4330
119	1850193919081	177841	4561
122	2209087768681	191881	4798
125	2626068425401	206641	5041
128	3108767552605	222139	5290
131	3665598710005	238393	5545
134	4305815955541	255421	5806
137	5039575304941	273241	6073
140	5877999117001	291871	6346
143	6833243472625	311329	6625
146	7918568615665	331633	6910
149	9148412523601	352801	7201
152	10538467676101	374851	7498
155	12105761089501	397801	7801
158	13868737685245	421669	8110
161	15847347060325	446473	8425
164	18063133727761	472231	8746
167	20539330895161	498961	9073
170	23300957849401	526681	9406
173	26374921015465	555409	9745
176	29790118757485	585163	10090
179	33577549990021	615961	10441
182	37770426667621	647821	10798
185	42404290220701	680761	11161
188	47517132005785	714799	11530
191	53149517838145	749953	11905
194	59344716674881	786241	12286
197	66148833516481	823681	12673
200	73610946594901	862291	13066
203	81783248916205	902089	13465
206	90721194225805	943093	13870
209	100483647464341	985321	14281
212	111133039782241	1028791	14698
215	122735528181001	1073521	15121
218	135361159849225	1119529	15550
221	149084041261465	1166833	15985
224	163982512107901	1215451	16426

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
227	180139324122901	1265401	16873
230	197641824880501	1316701	17326
233	216582146624845	1369369	17785
236	237057400203625	1423423	18250
239	259169874172561	1478881	18721
242	283027239138961	1535761	19198
245	308742757412401	1594081	19681
248	336435498030565	1653859	20170
251	366230557228285	1715113	20665
254	398259284417821	1777861	21166
257	432659513748421	1842121	21673
260	469575801313201	1907911	22186
263	509159668071385	1975249	22705
266	551569848553945	2044153	23230
269	596972545420681	2114641	23761
272	645541689936781	2186731	24298
275	697459208436901	2260441	24841
278	752915294844805	2335789	25390
281	812108689316605	2412793	25945
284	875246963075641	2491471	26506
287	942546809507041	2571841	27073
290	1014234341580001	2653921	27646
293	1090545395665825	2737729	28225
296	1171725841819765	2823283	28810
299	1258031900594701	2910601	29401
302	1349730466454701	2999701	29998
305	1447099437856501	3090601	30601
308	1550428054066945	3183319	31210
311	1660017238784425	3277873	31825
314	1776179950632361	3374281	32446
317	1899241540592761	3472561	33073
320	2029540116447901	3572731	33706
323	2167426914298165	3674809	34345
326	2313266677224085	3778813	34990
329	2467438041160621	3884761	35641
332	2630333928051721	3992671	36298
335	2802361946353201	4102561	36961
338	2983944798951985	4214449	37630
341	3175520698569745	4328353	38305
344	3377543790718981	4444291	38986
347	3590484584279581	4562281	39673
350	3814830389763901	4682341	40366
353	4051085765338405	4804489	41065
356	4299772970669905	4928743	41770
359	4561432428664441	5055121	42481
362	4836623195166841	5183641	43198
365	5125923436689001	5314321	43921
368	5429930916234925	5447179	44650
371	5749263487290565	5582233	45385
374	6084559596046501	5719501	46126

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
377	6436478791921501	5859001	46873
380	6805702246455001	6000751	47626
383	7192933280636545	6144769	48385
386	7598897900740225	6291073	49150
389	8024345342732161	6439681	49921
392	8470048625319061	6590611	50698
395	8936805111705901	6743881	51481
398	9425437080130765	6899509	52270
401	9936792303244885	7057513	53065
404	10471744636405921	7217911	53866
407	11031194614952521	7380721	54673
410	11616070060528201	7545961	55486
413	12227326696522585	7713649	56305
416	12865948772698045	7883803	57130
419	13532949699069781	8056441	57961
422	14229372689107381	8231581	58798
425	14956291412325901	8409241	59641
428	15714810656334505	8589439	60490
431	16506066998410705	8772193	61345
434	17331229486668241	8957521	62206
437	18191500330886641	9145441	63073
440	19088115603070501	9335971	63946
443	20022345947806525	9529129	64825
446	20995497302486365	9724933	65710
449	22008911627463301	9923401	66601
452	23063967646210801	10124551	67498
455	24162081595551001	10328401	68401
458	25304707986021145	10534969	69310
461	26493340372446025	10744273	70225
464	27729512134784461	10956331	71146
467	29014797269317861	11171161	72073
470	30350811190248901	11388781	73006
473	31739211541778365	11609209	73945
476	33181699020728185	11832463	74890
479	34680018209778721	12058561	75841
482	36235958421388321	12287521	76798
485	37851354552463201	12519361	77761
488	39528087949845685	12754099	78730
491	41268087286688845	12991753	79705
494	43073329449785581	13232341	80686
497	44945840437920181	13475881	81673
500	46887696271310401	13722391	82666
503	48901023912208105	13971889	83665
506	50988002196726505	14224393	84670
509	53150862777962041	14479921	85681
512	55391891080478941	14738491	86698
515	57713427266224501	15000121	87721
518	60117867211943125	15264829	88750
521	62607663498157165	15532633	89785
524	65185326409782601	15803551	90826

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
527	67853424948447601	16077601	91873
530	70614587856582001	16354801	92926
533	73471504653345745	16635169	93985
536	76426926682464325	16918723	95050
539	79483668172039261	17205481	96121
542	82644607306401661	17495461	97198
545	85912687310076901	17788681	98281
548	89290917543928465	18085159	99370
551	92782374613548985	18384913	100465
554	96390203489966521	18687961	101566
557	100117618642734121	18994321	102673
560	103967905185470701	19304011	103786
563	107944420033921285	19617049	104905
566	112050593076604645	19933453	106030
569	116289928358116381	20253241	107161
572	120666005275155481	20576431	108298
575	125182479785342401	20903041	109441
578	129843085628896705	21233089	110590
581	134651635563242305	21566593	111745
584	139612022610608341	21903571	112906
587	144728221318693741	22244041	114073
590	150004289034463501	22588021	115246
593	155444367191144725	22935529	116425
596	161052682608490465	23286583	117610
599	166833548806379401	23641201	118801
602	172791367331819401	23999401	119998
605	178930629099423001	24361201	121201
608	185255915745422845	24726619	122410
611	191771900995295125	25095673	123625
614	198483352045059061	25468381	124846
617	205395130956320461	25844761	126073
620	212512196065127401	26224831	127306
623	219839603404706065	26608609	128545
626	227382508142144785	26996113	129790
629	235146166029094321	27387361	131041
632	243135934866552421	27782371	132298
635	251357275983800701	28181161	133561
638	259815755731561885	28583749	134830
641	268517046989445445	28990153	136105
644	277466930687749681	29400391	137386
647	286671297343688281	29814481	138673
650	296136148612109401	30232441	139966
653	305867598850775305	30654289	141265
656	315871876700270605	31080043	142570
659	326155326678607141	31509721	143881
662	336724410790593541	31943341	145198
665	347585710152037501	32380921	146521
668	358745926628848825	32822479	147850
671	370211884491111265	33268033	149185
674	381990532082191201	33717601	150526

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
677	394088943502951201	34171201	151873
680	406514320311136501	34628851	153226
683	419273993236002445	35090569	154585
686	432375423908250925	35556373	155950
689	445826206605343861	36026281	157321
692	459634070012261761	36500311	158698
695	473806878997775401	36978481	160081
698	488352636406298665	37460809	161470
701	503279484865390585	37947313	162865
704	518595708608974621	38438011	164266
707	534309735316343221	38932921	165673
710	550430137967015701	39432061	167086
713	566965636711517485	39935449	168505
716	583925100758148745	40443103	169930
719	601317550275810481	40955041	171361
722	619152158312956081	41471281	172798
725	637438252732736401	41991841	174241
728	656185318164406405	42516739	175690
731	675402997971061405	43045993	177145
734	695101096233770941	43579621	178606
737	715289579752178341	44117641	180073
740	735978580061634001	44660071	181546
743	757178395466930425	45206929	183025
746	778899493092707065	45758233	184510
749	801152510950593001	46314001	186001
752	823948260023155501	46874251	187498
755	847297726364722501	47439001	189001
758	871212073219147045	48008269	190510
761	895702643154581725	48582073	192025
764	920780960215331161	49160431	193546
767	946458732090850561	49743361	195073
770	972747852301958401	50330881	196606
773	999660402404331265	50923009	198145
776	1027208654209348885	51519763	199690
779	1055405072022357421	52121161	201241
782	1084262314898419021	52727221	202798
785	1113793238915615701	53337961	204361
788	1144010899465975585	53953399	205930
791	1174928553564089545	54573553	207505
794	1206559662173486281	55198441	209086
797	1238917892550833881	55828081	210673
800	1272017120608035901	56462491	212266
803	1305871433292290005	57101689	213865
806	1340495130984177205	57745693	215470
809	1375902729913849741	58394521	217081
812	1412108964595385641	59048191	218698
815	1449128790279378001	59706721	220321
818	1486977385423827025	60370129	221950
821	1525670154183402865	61038433	223585
824	1565222728917147301	61711651	225226

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
827	1605650972714682301	62389801	226873
830	1646970981940993501	63072901	228526
833	1689199088799856645	63760969	230185
836	1732351863915975025	64454023	231850
839	1776446118935895961	65152081	233521
842	1821498909147774361	65855161	235198
845	1867527536120051401	66563281	236881
848	1914549550359116365	67276459	238570
851	1962582753986019685	67994713	240265
854	2011645203432305221	68718061	241966
857	2061755212155029821	69446521	243673
860	2112931353371038201	70180111	245386
863	2165192462810561185	70918849	247105
866	2218557641490205345	71662753	248830
869	2273046258505402081	72411841	250561
872	2328677953842384181	73166131	252298
875	2385472641209757901	73925641	254041
878	2443450510889738605	74690389	255790
881	2502632032609118005	75460393	257545
884	2563037958430031041	76235671	259306
887	2624689325660590441	77016241	261073
890	2687607459785457001	77802121	262846
893	2751813977416413625	78593329	264625
896	2817330789263011165	79389883	266410
899	2884180103123354101	80191801	268201
902	2952384426895094101	80999101	269998
905	3021966571606699501	81811801	271801
908	3092949654469068745	82629919	273610
911	3165357101947555825	83453473	275425
914	3239212652854475761	84282481	277246
917	3314540361462158161	85116961	279073
920	3391364600636616901	85956931	280906
923	3469710064991903965	86802409	282745
926	3549601774065215485	87653413	284590
929	3631065075512818021	88509961	286441
932	3714125648326863121	89372071	288298
935	3798809506073158201	90239761	290161
938	3885143000149961785	91113049	292030
941	3973152823067871145	91991953	293905
944	4062866011750870381	92876491	295786
947	4154309950858606981	93766681	297673
950	4247512376129964901	94662541	299566
953	4342501377748002205	95564089	301465
956	4439305403726321305	96471343	303370
959	4537953263316939841	97384321	305281
962	4638474130439730241	98303041	307198
965	4740897547133496001	99227521	309121
968	4845253427028752725	100157779	311050
971	4951572058842281965	101093833	312985
974	5059884109893525901	102035701	314926

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
977	5170220629642890901	102983401	316873
980	5282613053252028001	103936951	318826
983	5397093205166158345	104896369	320785
986	5513693302718511625	105861673	322750
989	5632445959756945561	106832881	324721
992	5753384190292814461	107810011	326698
995	5876541412172154901	108793081	328681
998	6001951450769256565	109782109	330670
1001	6129648542702686285	110777113	332665
1004	6259667339573833321	111778111	334666
1007	6392042911728043921	112785121	336673
1010	6526810752038413201	113798161	338686
1013	6664006779712302385	114817249	340705
1016	6803667344120649445	115842403	342730
1019	6945829228650141181	116873641	344761
1022	7090529654578314781	117910981	346798
1025	7237806284971656901	118954441	348841
1028	7387697228606768305	120004039	350890
1031	7540241043914662105	121059793	352945
1034	7695476742948263641	122121721	355006
1037	7853443795373180041	123189841	357073
1040	8014182132481807501	124264171	359146
1043	8177732151230844325	125344729	361225
1046	8344134718302277765	126431533	363310
1049	8513431174187912701	127524601	365401
1052	8685663337297510201	128623951	367498
1055	8860873508090604001	129729601	369601
1058	9039104473232062945	130841569	371710
1061	9220399509771467425	131959873	373825
1064	9404802389346367861	133084531	375946
1067	9592357382409493261	134215561	378073
1070	9783109262479977901	135352981	380206
1073	9977103310418674165	136496809	382345
1076	10174385318727619585	137647063	384490
1079	10375001595873726121	138803761	386641
1082	10578998970636759721	139966921	388798
1085	10786424796481678201	141136561	390961
1088	10997326955955395485	142312699	393130
1091	11211753865108040245	143495353	395305
1094	11429754477938776981	144684541	397486
1097	11651378290866257581	145880281	399673
1100	11876675347223771401	147082591	401866
1103	12105696241779161905	148291489	404065
1106	12338492125279577905	149506993	406270
1109	12575114709021127441	150729121	408481
1112	12815616269443502341	151957891	410698
1115	13060049652749641501	153193321	412921
1118	13308468279550500925	154435429	415150
1121	13560926149534998565	155684233	417385
1124	13817477846165202001	156939751	419626

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1127	14078178541396827001	158202001	421873
1130	14343084000425115001	159471001	424126
1133	14612250586456157545	160746769	426385
1136	14885735265503735725	162029323	428650
1139	15163595611211742661	163318681	430921
1142	15445889809702257061	164614861	433198
1145	15732676664449335901	165917881	435481
1148	16024015601178594265	167227759	437770
1151	16319966672792640385	168544513	440065
1154	16620590564322433921	169868161	442366
1157	16925948597904635521	171198721	444673
1160	17236102737785015701	172536211	446986
1163	17551115595347991085	173880649	449305
1166	17871050434172356045	175232053	451630
1169	18195971175113277781	176590441	453961
1172	18525942401410622881	177955831	456298
1175	18861029363823683401	179328241	458641
1178	19201297985792370505	180707689	460990
1181	19546814868624943705	182094193	463345
1184	19897647296712343741	183487771	465706
1187	20253863242769197141	184888441	468073
1190	20615531373101560501	186296221	470446
1193	20982721052901472525	187711129	472825
1196	21355502351568381865	189133183	475210
1199	21733946048057518801	190562401	477601
1202	22118123636255278801	191998801	479998
1205	22508107330381686001	193442401	482401
1208	22903970070420004645	194893219	484810
1211	23305785527573566525	196351273	487225
1214	23713628109749882461	197816581	489646
1217	24127572967072105861	199289161	492073
1220	24547695997417916401	200769031	494506
1223	24974073851985891865	202256209	496945
1226	25406783940889436185	203750713	499390
1229	25845904438778331721	205252561	501841
1232	26291514290487983821	206761771	504298
1235	26743693216716425701	208278361	506761
1238	27202521719729151685	209802349	509230
1241	27668081089091846845	211333753	511705
1244	28140453407431081081	212872591	514186
1247	28619721556223035681	214418881	516673
1250	29105969221610330401	215972641	519166
1253	29599280900247019105	217533889	521665
1256	30099741905171822005	219102643	524170
1259	30607438371709662541	220678921	526681
1262	31122457263401576941	222262741	529198
1265	31644886377963064501	223854121	531721
1268	32174814353270946625	225453079	534250
1271	32712330673378802665	227059633	536785
1274	33257525674561050601	228673801	539326

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1277	33810490551385740601	230295601	541873
1280	34371317362816129501	231925051	544426
1283	34940099038341104245	233562169	546985
1286	35516929384134522325	235206973	549550
1289	36101903089243537261	236859481	552121
1292	36695115731805977161	238519711	554698
1295	37296663785296844401	240187681	557281
1298	37906644624804004465	241863409	559870
1301	38525156533333131985	243546913	562465
1304	39152298708141982021	245238211	565066
1307	39788171267104054621	246937321	567673
1310	40432875255101720701	248644261	570286
1313	41086512650448877285	250359049	572905
1316	41749186371343200145	252081703	575530
1319	42421000282348061881	253812241	578161
1322	43102059200904183481	255550681	580798
1325	43792468903871087401	257297041	583441
1328	44492336134098420205	259051339	586090
1331	45201768607027212805	260813593	588745
1334	45920875017321146341	262583821	591406
1337	46649765045527891741	264362041	594073
1340	47388549364770591001	266148271	596746
1343	48137339647469548225	267942529	599425
1346	48896248572094198465	269744833	602110
1349	49665389829945422401	271555201	604801
1352	50444878131968274901	273373651	607498
1355	51234829215595195501	275200201	610201
1358	52035359851619768845	277034869	612910
1361	52846587851101103125	278877673	615625
1364	53668632072298894561	280728631	618346
1367	54501612427639245961	282587761	621073
1370	55345649890711307401	284455081	623806
1373	56200866503294807065	286330609	626545
1376	57067385382418540285	288214363	629290
1379	57945330727449884821	290106361	632041
1382	58834827827215410421	292006621	634798
1385	59736003067152650701	293915161	637561
1388	60648983936493105385	295831999	640330
1391	61573899035476540945	297757153	643105
1394	62510878082596657681	299690641	645886
1397	63460051921878191281	301632481	648673
1400	64421552530185516901	303582691	651466
1403	65395513024562823805	305541289	654265
1406	66382067669605928605	307508293	657070
1409	67381351884865795141	309483721	659881
1412	68393502252283829041	311467591	662698
1415	69418656523659015001	313459921	665521
1418	70456953628146964825	315460729	668350
1421	71508533679790944265	317470033	671185
1424	72573537985084946701	319487851	674026

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1427	73652109050568881701	321514201	676873
1430	74744390590455946501	323549101	679726
1433	75850527534292248445	325592569	682585
1436	76970666034648746425	327644623	685450
1439	78104953474845579361	329705281	688321
1442	79253538476708849761	331774561	691198
1445	80416570908359930401	333852481	694081
1448	81594201892037362165	335939059	696970
1451	82786583811951411085	338034313	699865
1454	83993870322171352621	340138261	702766
1457	85216216354545551221	342250921	705673
1460	86453778126654403201	344372311	708586
1463	87706713149796210985	346502449	711505
1466	88975180237006056745	348641353	714430
1469	90259339511107743481	350789041	717361
1472	91559352412798871581	352945531	720298
1475	92875381708769118901	355110841	723241
1478	94207591499851792405	357284989	726190
1481	95556147229208719405	359467993	729145
1484	96921215690548546441	361659871	732106
1487	98302965036378513841	363860641	735073
1490	99701564786289774001	366070321	738046
1493	101117185835276321425	368288929	741025
1496	102550000462087602565	370516483	744010
1499	104000182337614873501	372753001	747001
1502	105467906533311373501	374998501	749998
1505	106953349529646382501	377253001	753001
1508	108456689224593230545	379516519	756010
1511	109978104942151327225	381789073	759025
1514	111517777440902279161	384070681	762046
1517	113075888922600163561	386361361	765073
1520	114652623040796025901	388661131	768106
1523	116248164909496669765	390970009	771145
1526	117862701111857806885	393288013	774190
1529	119496419708911635421	395615161	777241
1532	121149510248328914521	397951471	780298
1535	122822163773215603201	400296961	783361
1538	124514572830944131585	402651649	786430
1541	126226931482019372545	405015553	789505
1544	127959435308979381781	407388691	792586
1547	129712281425330974381	409771081	795673
1550	131485668484520205901	412162741	798766
1553	133279796688937826005	414563689	801865
1556	135094867798959772705	416973943	804970
1559	136931085142022775241	419393521	808081
1562	138788653621735133641	421822441	811198
1565	140667779727022743001	424260721	814321
1568	142568671541310430525	426708379	817450
1571	144491538751738673365	429165433	820585
1574	146436592658415765301	431631901	823726

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1577	148404046183705500301	434107801	826873
1580	150394113881550441001	436593151	830026
1583	152407011946830840145	439087969	833185
1586	154442958224759283025	441592273	836350
1589	156502172220311118961	444106081	839521
1592	158584875107690749861	446629411	842698
1595	160691289739833843901	449162281	845881
1598	162821640657945542365	451704709	849070
1601	164976154101074727685	454256713	852265
1604	167155058015724420721	456818311	855466
1607	169358582065498375321	459389521	858673
1610	171586957640783938201	461970361	861886
1613	173840417868471242185	464560849	865105
1616	176119197621708800845	467161003	868330
1619	178423533529695572581	469770841	871561
1622	180753663987509562181	472390381	874798
1625	183109829165973027901	475019641	878041
1628	185492271021554362105	477658639	881290
1631	187901233306306713505	480307393	884545
1634	190336961577843419041	482965921	887806
1637	192799703209350313441	485634241	891073
1640	195289707399634984501	488312371	894346
1643	197807225183213042125	491000329	897625
1646	200352509440431469165	493698133	900910
1649	202925814907629122101	496405801	904201
1652	205527398187334449601	499123351	907498
1655	208157517758500497001	501850801	910801
1658	210816433986777264745	504588169	914110
1661	213504409134821488825	507335473	917425
1664	216221707372643911261	510092731	920746
1667	218968594787994108661	512859961	924073
1670	221745339396782946901	515637181	927406
1673	224552211153542729965	518424409	930745
1676	227389481961925110985	521221663	934090
1679	230257425685236833521	524028961	937441
1682	233156318157013371121	526846321	940798
1685	236086437191630533201	529673761	944161
1688	239048062594954105285	532511299	947530
1691	242041476175027591645	535358953	950905
1694	245066961752798128381	538216741	954286
1697	248124805172880634981	541084681	957673
1700	251215294314360272401	543962791	961066
1703	254338719101633275705	546851089	964465
1706	257495371515286229305	549749593	967870
1709	260685545603013852841	552658321	971281
1712	263909537490575365741	555577291	974698
1715	267167645392789498501	558506521	978121
1718	270460169624568218725	561446029	981550
1721	273787412611989239965	564395833	984985
1724	277149678903407381401	567355951	988426

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1727	280547275180604846401	570326401	991873
1730	283980510269980488001	573307201	995326
1733	287449695153778129345	576298369	998785
1736	290955142981354007125	579299923	1002250
1739	294497169080483406061	582311881	1005721
1742	298076090968706552461	585334261	1009198
1745	301692228364713834901	588367081	1012681
1748	305345903199770420065	591410359	1016170
1751	309037439629180331785	594464113	1019665
1754	312767164043790061321	597528361	1023166
1757	316535405081531776921	600603121	1026673
1760	320342493639006200701	603688411	1030186
1763	324188762883105220885	606784249	1033705
1766	328074548262674307445	609890653	1037230
1769	332000187520214799181	613007641	1040761
1772	335966020703626130281	616135231	1044298
1775	339972390177988064401	619273441	1047841
1778	344019640637383004305	622422289	1051390
1781	348108119116758445105	625581793	1054945
1784	352238175003829639141	628751971	1058506
1787	356410160051022540541	631932841	1062073
1790	360624428387457097501	635124421	1065646
1793	364881336530970960325	638326729	1069225
1796	369181243400183673265	641539783	1072810
1799	373524510326601418201	644763601	1076401
1802	377911501066762378201	647998201	1079998
1805	382342581814422789001	651243601	1083601
1808	386818121212783746445	654499819	1087210
1811	391338490366758837925	657766873	1090825
1814	395904062855282665861	661044781	1094446
1817	400515214743660331261	664333561	1098073
1820	405172324595957945401	667633231	1101706
1823	409875773487434237665	670943809	1105345
1826	414625945017013327585	674265313	1108990
1829	419423225319798729121	677597761	1112641
1832	424268003079628655221	680941171	1116298
1835	429160669541672690701	684295561	1119961
1838	434101618525069901485	687660949	1123630
1841	439091246435608448245	691037353	1127305
1844	444129952278446772481	694424791	1130986
1847	449218137670876423081	697823281	1134673
1850	454356206855126591401	701232841	1138366
1853	459544566711210422905	704653489	1142065
1856	464783626769813173405	708085243	1145770
1859	470073799225222277941	711528121	1149481
1862	475415498948299400341	714982141	1153198
1865	480809143499494531501	718447321	1156921
1868	486255153141902204425	721923679	1160650
1871	491753950854359894065	725411233	1164385
1874	497305962344588670001	728910001	1168126

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1877	502911616062376170001	732420001	1171873
1880	508571343212801962501	735941251	1175626
1883	514285577769505366045	739473769	1179385
1886	520054756487995793725	743017573	1183150
1889	525879318919005690661	746572681	1186921
1892	531759707421886132561	750139111	1190698
1895	537696367178045153401	753716881	1194481
1898	543689746204428870265	757306009	1198270
1901	549740295367045473385	760906513	1202065
1904	555848468394532149421	764518411	1205866
1907	562014721891765006021	768141721	1209673
1910	568239515353512065701	771776461	1213486
1913	574523311178129397085	775422649	1217305
1916	580866574681300451545	779080303	1221130
1919	587269774109818673281	782749441	1224961
1922	593733380655413450881	786430081	1228798
1925	600257868468619478401	790122241	1232641
1928	606843714672689594005	793825939	1236490
1931	613491399377551164205	797541193	1240345
1934	620201405693806081741	801268021	1244206
1937	626974219746774445141	805006441	1248073
1940	633810330690581988001	808756471	1251946
1943	640710230722291326025	812518129	1255825
1946	647674415096077089865	816291433	1259710
1949	654703382137445011801	820076401	1263601
1952	661797633257495034301	823873051	1267498
1955	668957672967228508501	827681401	1271401
1958	676184008891899550645	831501469	1275310
1961	683477151785410624525	835333273	1279225
1964	690837615544752417961	839176831	1283146
1967	698265917224488081361	843032161	1287073
1970	705762577051281896401	846899281	1291006
1973	713328118438472442865	850778209	1294945
1976	720963068000690331685	854668963	1298890
1979	728667955568520572221	858571561	1302841
1982	736443314203209641821	862486021	1306798
1985	744289680211417325701	866412361	1310761
1988	752207593160013395185	870350599	1314730
1991	760197595890919192345	874300753	1318705
1994	768260234535994189081	878262841	1322686
1997	776396058531967588681	882236881	1326673
2000	784605620635415037901	886222891	1330666
2003	792889476937780517605	890220889	1334665
2006	801248186880443480005	894230893	1338670
2009	809682313269831300541	898252921	1342681
2012	818192422292577112441	902286991	1346698
2015	826779083530723092001	906333121	1350721
2018	835442869976969262625	910391329	1354750
2021	844184358049967885665	914461633	1358785
2024	853004127609663506101	918544051	1362826

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
2027	861902761972678721101	922638601	1366873
2030	870880847927745739501	926745301	1370926
2033	879938975751183800245	930864169	1374985
2036	889077739222422517825	934995223	1379050
2039	898297735639571222761	939138481	1383121
2042	907599565835034365161	943293961	1387198
2045	916983834191173049401	947461681	1391281
2048	926451148656012767965	951641659	1395370
2051	936002120758997402485	955833913	1399465
2054	945637365626789560021	960038461	1403566
2057	955357501999117312621	964255321	1407673
2060	965163152244667408201	968484511	1411786
2063	975054942377025020785	972726049	1415905
2066	985033502070660108145	976979953	1420030
2069	995099464676960444881	981246241	1424161
2072	1005253467240311398981	985524931	1428298
2075	1015496150514222519901	989816041	1432441
2078	1025828158977501006205	994119589	1436590
2081	1036250140850472120805	998435593	1440745
2084	1046762748111246621841	1002764071	1444906
2087	1057366636512035277241	1007105041	1449073
2090	1068062465595510531001	1011458521	1453246
2093	1078850898711215389225	1015824529	1457425
2096	1089732603032019593965	1020203083	1461610
2099	1100708249570623152901	1024594201	1465801
2102	1111778513196107292901	1028997901	1469998
2105	1122944072650532905501	1033414201	1474201
2108	1134205610565586552345	1037843119	1478410
2111	1145563813479274098625	1042284673	1482625
2114	1157019371852662042561	1046738881	1486846
2117	1168572980086666608961	1051205761	1491073
2120	1180225336538890674901	1055685331	1495306
2123	1191977143540508595565	1060177609	1499545
2126	1203829107413198998285	1064682613	1503790
2129	1215781938486125612821	1069200361	1508041
2132	1227836351112966205921	1073730871	1512298
2135	1239993063688989688201	1078274161	1516561
2138	1252252798668181461385	1082830249	1520830
2141	1264616282580417073945	1087399153	1525105
2144	1277084246048684253181	1091980891	1529386
2147	1289657423806353381781	1096575481	1533673
2150	1302336554714496486901	1101182941	1537966
2153	1315122381779254809805	1105803289	1542265
2156	1328015652169255024105	1110436543	1546570
2159	1341017117233074170641	1115082721	1550881
2162	1354127532516753377041	1119741841	1555198
2165	1367347657781360430001	1124413921	1559521
2168	1380678257020601268325	1129098979	1563850
2171	1394120098478480464765	1133797033	1568185
2174	1407673954667010764701	1138508101	1572526

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
2177	1421340602383971749701	1143232201	1576873
2180	1435120822730717694001	1147969351	1581226
2183	1449015401130034681945	1152719569	1585585
2186	1463025127344047054425	1157482873	1589950
2189	1477150795492173252361	1162259281	1594321
2192	1491393204069131125261	1167048811	1598698
2195	1505753155962992772901	1171851481	1603081
2198	1520231458473288988165	1176667309	1607470
2201	1534828923329163369085	1181496313	1611865
2204	1549546366707576168121	1186338511	1616266
2207	1564384609251557946721	1191193921	1620673
2210	1579344476088513103201	1196062561	1625086
2213	1594426796848573341985	1200944449	1629505
2216	1609632405683001152245	1205839603	1633930
2219	1624962141282643363981	1210748041	1638361
2222	1640416846896434849581	1215669781	1642798
2225	1655997370349952438901	1220604841	1647241
2228	1671704564064019115905	1225553239	1651690
2231	1687539285073358564905	1230514993	1656145
2234	1703502395045300134441	1235490121	1660606
2237	1719594760298534286841	1240478641	1665073
2240	1735817251821918601501	1245480571	1669546
2243	1752170745293334399925	1250495929	1674025
2246	1768656121098594060565	1255524733	1678510
2249	1785274264350399091501	1260567001	1683001
2252	1802026064907349029001	1265622751	1687498
2255	1818912417393001230001	1270692001	1692001
2258	1835934221214981626545	1275774769	1696510
2261	1853092380584146510225	1280871073	1701025
2264	1870387804533795414661	1285980931	1705546
2267	1887821406938935164061	1291104361	1710073
2270	1905394106535595155901	1296241381	1714606
2273	1923106826940193945765	1301392009	1719145
2276	1940960496668957202385	1306556263	1723690
2279	1958956049157387100921	1311734161	1728241
2282	1977094422779783222521	1316925721	1732798
2285	1995376560868815028201	1322130961	1737361
2288	2013803411735145975085	1327349899	1741930
2291	2032375928687109343045	1332582553	1746505
2294	2051095070050435839781	1337828941	1751086
2297	2069961799188033052381	1343089081	1755673
2300	2088977084519816813401	1348362991	1760266
2303	2108141899542594549505	1353650689	1764865
2306	2127457222850000680705	1358952193	1769470
2309	2146924038152484138241	1364267521	1774081
2312	2166543334297348069141	1369596691	1778698
2315	2186316105288841795501	1374939721	1783321
2318	2206243350308305096525	1380296629	1787950
2321	2226326073734364881365	1385667433	1792585
2324	2246565285163184320801	1391052151	1797226

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
2327	2266961999428764505801	1396450801	1801873
2330	2287517236623298701001	1401863401	1806526
2333	2308232022117579261145	1407289969	1811185
2336	2329107386581457278525	1412730523	1815850
2339	2350144366004355029461	1418185081	1820521
2342	2371344001715831287861	1423653661	1825198
2345	2392707340406199573901	1429136281	1829881
2348	2414235434147199405865	1434632959	1834570
2351	2435929340412720623185	1440143713	1839265
2354	2457790122099580848721	1445668561	1843966
2357	2479818847548356158321	1451207521	1848673
2360	2502016590564265025701	1456760611	1853386
2363	2524384430438105610685	1462327849	1858105
2366	2546923451967246458845	1467909253	1862830
2369	2569634745476670680581	1473504841	1867561
2372	2592519406840073677681	1479114631	1872298
2375	2615578537501014485401	1484738641	1877041
2378	2638813244494120798105	1490376889	1881790
2381	2662224640466347746505	1496029393	1886545
2384	2685813843698290494541	1501696171	1891306
2387	2709581978125550723941	1507377241	1896073
2390	2733530173360157074501	1513072621	1900846
2393	2757659564712039608125	1518782329	1905625
2396	2781971293210558364665	1524506383	1910410
2399	2806466505626086077601	1530244801	1915201
2402	2831146354491645117601	1535997601	1919998
2405	2856011998124598732001	1541764801	1924801
2408	2881064600648396648245	1547546419	1929610
2411	2906305332014375109325	1553342473	1934425
2414	2931735368023611409261	1559152981	1939246
2417	2957355890348832996661	1564977961	1944073
2420	2983168086556381214401	1570817431	1948906
2423	3009173150128229743465	1576671409	1953745
2426	3035372280484057818985	1582539913	1958590
2429	3061766683003378286521	1588422961	1963441
2432	3088357569047720566621	1594320571	1968298
2435	3115146155982868595701	1600232761	1973161
2438	3142133667201153811285	1606159549	1978030
2441	3169321332143803249645	1612100953	1982905
2444	3196710386323342823881	1618056991	1987786
2447	3224302071346055850481	1624027681	1992673
2450	3252097634934496892401	1630013041	1997566
2453	3280098330950060986705	1636013089	2002465
2456	3308305419415608324805	1642027843	2007370
2459	3336720166538144453341	1648057321	2012281
2462	3365343844731556063741	1654101541	2017198
2465	3394177732639402438501	1660160521	2022121
2468	3423223115157762622225	1666234279	2027050
2471	3452481283458138385465	1672322833	2031985
2474	3481953535010413049401	1678426201	2036926

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
2477	3511641173605866239401	1684544401	2041873
2480	3541545509380244635501	1690677451	2046826
2483	3571667858836888787845	1696825369	2051785
2486	3602009544869916065125	1702988173	2056750
2489	3632571896787459804061	1709165881	2061721
2492	3663356250334964727961	1715358511	2066698
2495	3694363947718538702401	1721566081	2071681
2498	3725596337628360896065	1727788609	2076670
2501	3757054775262146414785	1734026113	2081665
2504	3788740622348667476821	1740278611	2086666
2507	3820655247171331197421	1746546121	2091673
2510	3852800024591814050701	1752828661	2096686
2513	3885176336073753076885	1759126249	2101705
2516	3917785569706493902945	1765438903	2106730
2519	3950629120228895644681	1771766641	2111761
2522	3983708389053192758281	1778109481	2116798
2525	4017024784288913909401	1784467441	2121841
2528	4050579720766857927805	1790840539	2126890
2531	4084374620063126915605	1797228793	2131945
2534	4118410910523216577141	1803632221	2137006
2537	4152690027286163838541	1810050841	2142073
2540	4187213412308751825001	1816484671	2147146
2543	4221982514389772263825	1822933729	2152225
2546	4256998789194345381265	1829398033	2157310
2549	4292263699278297361201	1835877601	2162401
2552	4327778714112595433701	1842372451	2167498
2555	4363545310107840661501	1848882601	2172601
2558	4399564970638818492445	1855408069	2177710
2561	4435839186069107145925	1861948873	2182825
2564	4472369453775743901361	1868505031	2187946
2567	4509157278173949356761	1875076561	2193073
2570	4546204170741909725401	1881663481	2198206
2573	4583511650045617238665	1888265809	2203345
2576	4621081241763768723085	1894883563	2208490
2579	4658914478712722419621	1901516761	2213641
2582	4697012900871513113221	1908165421	2218798
2585	4735378055406925640701	1914829561	2223961
2588	4774011496698626844985	1921509199	2229130
2591	4812914786364356043745	1928204353	2234305
2594	4852089493285174080481	1934915041	2239486
2597	4891537193630771026081	1941641281	2244673
2600	4931259470884832598901	1948383091	2249866
2603	4971257915870465371405	1955140489	2255065
2606	5011534126775680831405	1961913493	2260270
2609	5052089709178938365941	1968702121	2265481
2612	5092926276074747235841	1975506391	2270698
2615	5134045447899327609001	1982326321	2275921
2618	5175448852556330720425	1989161929	2281150
2621	5217138125442618227065	1996013233	2286385
2624	5259114909474100825501	2002880251	2291626

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
2627	5301380855111636200501	2009763001	2296873
2630	5343937620386986372501	2016661501	2302126
2633	5386786870928834512045	2023575769	2307385
2636	5429930279988861289225	2030505823	2312650
2639	5473369528467880826161	2037451681	2317921
2642	5517106304942036320561	2044413361	2323198
2645	5561142305689055408401	2051390881	2328481
2648	5605479234714565333765	2058384259	2333770
2651	5650118803778467993885	2065393513	2339065
2654	5695062732421374927421	2072418661	2344366
2657	5740312747991102314021	2079459721	2349673
2660	5785870585669226053201	2086516711	2354986
2663	5831737988497696990585	2093589649	2360305
2666	5877916707405516359545	2100678553	2365630
2669	5924408501235471506281	2107783441	2370961
2672	5971215136770931966381	2114904331	2376298
2675	6018338388762705960901	2122041241	2381641
2678	6065780039955957380005	2129194189	2386990
2681	6113541881117183322205	2136363193	2392345
2684	6161625711061252257241	2143548271	2397706
2687	6210033336678502880641	2150749441	2403073
2690	6258766572961903728001	2157966721	2408446
2693	6307827243034273617025	2165200129	2413825
2696	6357217178175562985365	2172449683	2419210
2699	6406938217850196192301	2179715401	2424601
2702	6456992209734474852301	2186997301	2429998
2705	6507381009744042268501	2194295401	2435401
2708	6558106482061409034145	2201609719	2440810
2711	6609170499163539870025	2208940273	2446225
2714	6660574941849501765961	2216287081	2451646
2717	6712321699268173494361	2223650161	2457073
2720	6764412668946016563901	2231029531	2462506
2723	6816849756814907681365	2238425209	2467945
2726	6869634877240032789685	2245837213	2473390
2729	6922769953047842750221	2253265561	2478841
2732	6976256915554070737321	2260710271	2484298
2735	7030097704591811413201	2268171361	2489761
2738	7084294268539661951185	2275648849	2495230
2741	7138848564349924975345	2283142753	2500705
2744	7193762557576873484581	2290653091	2506186
2747	7249038222405077829181	2298179881	2511673
2750	7304677541677794807901	2305723141	2517166
2753	7360682506925418953605	2313282889	2522665
2756	7417055118393996075505	2320859143	2528170
2759	7473797385073799126041	2328451921	2533681
2762	7530911324727966460441	2336061241	2539198
2765	7588398963921202557001	2343687121	2544721
2768	7646262338048541266125	2351329579	2550250
2771	7704503491364171656165	2358988633	2555785
2774	7763124477010326524101	2366664301	2561326

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
2777	7822127357046233639101	2374356601	2566873
2780	7881514202477129787001	2382065551	2572426
2783	7941287093283337683745	2389791169	2577985
2786	8001448118449405825825	2397533473	2583550
2789	8061999375993311345761	2405292481	2589121
2792	8122942972995725940661	2413068211	2594698
2795	8184281025629344941901	2420860681	2600281
2798	8246015659188279593965	2428669909	2605870
2801	8308149008117512610485	2436495913	2611465
2804	8370683216042417075521	2444338711	2617066
2807	8433620435798338758121	2452198321	2622673
2810	8496962829460241908201	2460074761	2628286
2813	8560712568372418601785	2467968049	2633905
2816	8624871833178261703645	2475878203	2639530
2819	8689442813850101515381	2483805241	2645161
2822	8754427709719106176981	2491749181	2650798
2825	8819828729505245889901	2499710041	2656441
2828	8885648091347321029705	2507687839	2662090
2831	8951888022833054216305	2515682593	2667745
2834	9018550761029246409841	2523694321	2673406
2837	9085638552511997100241	2531723041	2679073
2840	9153153653396988658501	2539768771	2684746
2843	9221098329369834917725	2547831529	2690425
2846	9289474855716494051965	2555911333	2696110
2849	9358285517353745820901	2564008201	2701801
2852	9427532608859733248401	2572122151	2707498
2855	9497218434504568803001	2580253201	2713201
2858	9567345308281005148345	2588401369	2718910
2861	9637915553935170531625	2596566673	2724625
2864	9708931504997368878061	2604749131	2730346
2867	9780395504812944659461	2612948761	2736073
2870	9852309906573212604901	2621165581	2741806
2873	9924677073346452321565	2629399609	2747545
2876	9997499378108967893785	2637650863	2753290

Table 1: Polygonal Cannonball Numbers