Polygonal Cannonball Numbers

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1 Introduction

Recently I watched the video `https://www.youtube.com/watch?v=q6L06pyt9CA`, featuring Matt Parker. Being a huge fan of Matt and of Numperphile, and being rather full of myself, my first response was naturally one of doubt (also because of the spirit of mathematical enquiry and all that). I decided to have my own crack at the problem.

I reasoned that checking if a number is polygonal should be a roughly $\mathcal{O}(1)$ operation as we can find the nth term of the base-s polygonal numbers $P(s, n)$, which will be quadratic in n, and solve it for n with the quadratic formula, so to check if some cannonball numbers $C(s, n_c)$ is polygonal we just see if the corresponding n_p is an integer. Now 10^9 is a fairly small number. Seeing as my CPU's clockspeed is in the range of gigahertz, and we're just checking a tiny fraction of those numbers as we're just computing the cannonball numbers under this limit, it seems reasonable that this should be doable fairly fast.

I've thought about the problem of higher-dimensional stacks of cannonballs (ie the ones formed by adding up the cannonball numbers), but I've not done anything about it.

2 The Maths

Indeed, this approach does seem to work. Almost by definition we have the recurrence in polygonal numbers

$$P(s, n) = P(s, n-1) + n(s-2) - (s-3)$$

so we can use

$$P(s, n) = \sum_{r=1}^{n} P(s, r) - P(s, r-1)$$

$$
\begin{aligned}
&= \sum_{r=1}^{n} (n(s-2) - (s-3)) \\
&= \frac{1}{2} n(n+1)(s-2) - n(s-3) \\
&= \frac{n^2(s-2) - n(s-4)}{2}
\end{aligned}
$$

Fortunately this seems to agree with what Wikipedia thinks. Now, we have

$$
\begin{aligned}
0 &= (s-2)n^2 - (s-4)n - 2P(s,n) \\
\implies n &= \frac{s-4+\sqrt{(s-4)^2 + 8(s-2)P(s,n)}}{2s-4}
\end{aligned}
$$

Wikipedia still seems to think we're on track.

Another result that I don't really use is that

$$
\begin{aligned}
C(s,n) &= \sum_{r=1}^{n} P(s,n) \\
&= \frac{1}{2} \sum_{r=1}^{n} (n^2(s-2) - n(s-4)) \\
&= \frac{1}{2} \left(\frac{n(n+1)(2n+1)(s-2)}{6} - \frac{n(n+1)(s-4)}{2} \right) \\
&= \frac{1}{12} n(n+1)[(2n+1)(s-2) - 3(s-4)]
\end{aligned}
$$

In fact I've only used this in verification of the results.

Regardless, now we need only work our way up the $C(s,n)$s using the recurrence $C(s,n) = P(s,n) + C(s,n-1)$, and check for each if the quadratic formula gives an integer result. This is most easily done by checking if the discriminant is a perfect square and then checking that the denominator divides the numerator.

3 The Programming

For speeeeeeed I implemented this in C (although there is a long abandoned parallel Python implementation). I used 128-bit integers to be on the safe side, as 10^{19} is a little small for my liking. This meant I had to do a lot of messing around to get things to actually display in base 10.

I did briefly consider either implementing or importing some kind of arbitrary precision integer arithmetic functionality, but then I decided I wasn't going to run it on anything fast enough to have to worry about that, and I have better things to do.

There's also a slick little progress update that gets printed to STDERR, and a number of zsh scripts to save me typing.

4 The Ugly

Table 1 lists all the solutions that I've found, so far. The TEXsource of the table is in `../src/tab.tex`, which is derived from `../src/c/solutions/*`.

I have also plotted both the data in its entirety on a double logarithmic scale 2, and an excerpt from the data on a linear scale (1).

The obvious pattern that jumps out is the big line of points for all the sides congrent to 2 (mod 3).

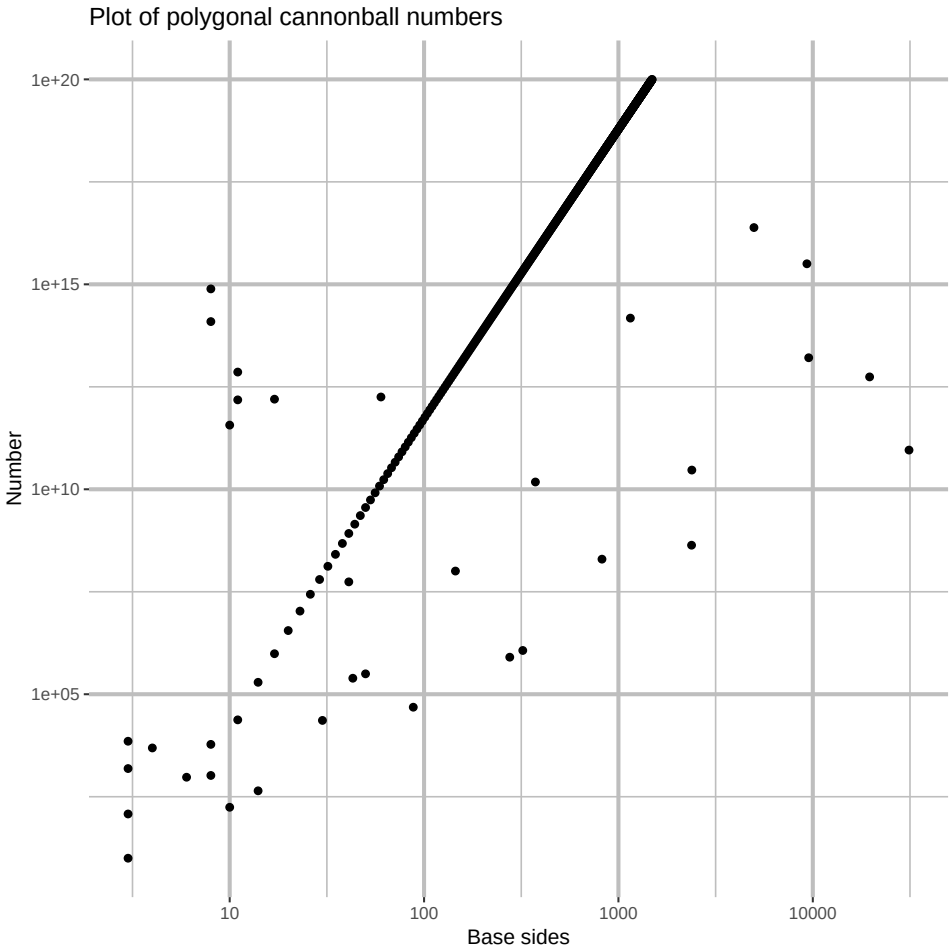

Figure 1: Log plot

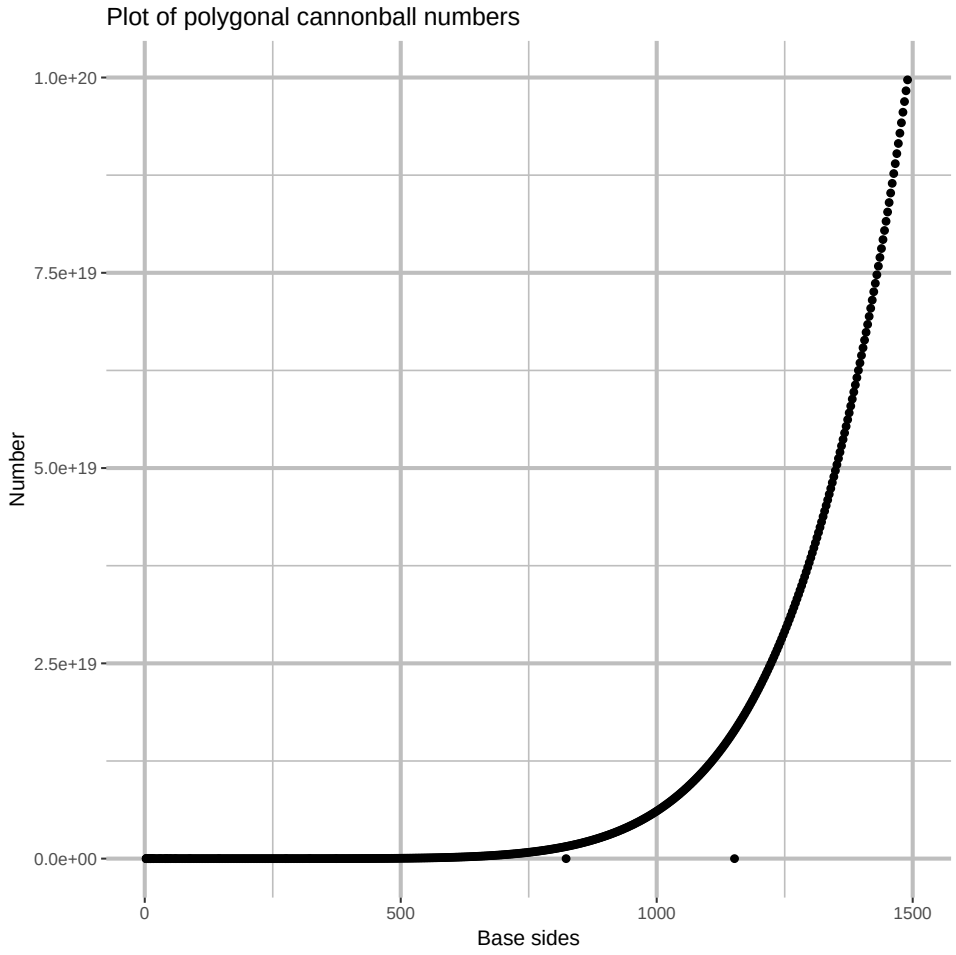

Figure 2: Linear plot

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
3	10	4	3
3	120	15	8
3	1540	55	20
3	7140	119	34
4	4900	70	24
6	946	22	11
8	1045	19	10
8	5985	45	18
8	123395663059845	6413415	49785
8	774611255177760	16068720	91839
10	175	7	5
10	368050005576	303336	6511
11	23725	73	25
11	1519937678700	581175	10044
11	7248070597636	1269127	16906
14	441	9	6
14	195661	181	46
17	975061	361	73
17	1580765544996	459096	8583
20	3578401	631	106

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
23	10680265	1009	145
26	27453385	1513	190
29	63016921	2161	241
30	23001	41	17
32	132361021	2971	298
35	258815701	3961	361
38	477132085	5149	430
41	55202400	1683	204
41	837244045	6553	505
43	245905	110	33
44	1408778281	8191	586
47	2286380881	10081	673
50	314755	115	34
50	3595928401	12241	766
53	5501691505	14689	865
56	8214519205	17443	970
59	12001111741	20521	1081
60	1785508245600	248132	5695
62	17194450141	23941	1198
65	24205450501	27721	1321
68	33535911025	31879	1450
71	45792819865	36433	1585
74	61704091801	41401	1726
77	82135801801	46801	1873
80	108110983501	52651	2026
83	140830060645	58969	2185
86	181692979525	65773	2350
88	48280	34	15
89	232323110461	73081	2521
92	294592986361	80911	2698
95	370651946401	89281	2881
98	462955752865	98209	3070
101	574298249185	107713	3265
104	707845127221	117811	3466
107	867169871821	128521	3673
110	1056291950701	139861	3886
113	1279717317685	151849	4105
116	1542481297345	164503	4330
119	1850193919081	177841	4561
122	2209087768681	191881	4798
125	2626068425401	206641	5041
128	3108767552605	222139	5290
131	3665598710005	238393	5545
134	4305815955541	255421	5806
137	5039575304941	273241	6073
140	5877999117001	291871	6346
143	6833243472625	311329	6625
145	101337426	1191	162
146	7918568615665	331633	6910
149	9148412523601	352801	7201

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
152	10538467676101	374851	7498
155	12105761089501	397801	7801
158	13868737685245	421669	8110
161	15847347060325	446473	8425
164	18063133727761	472231	8746
167	20539330895161	498961	9073
170	23300957849401	526681	9406
173	26374921015465	555409	9745
176	29790118757485	585163	10090
179	33577549990021	615961	10441
182	37770426667621	647821	10798
185	42404290220701	680761	11161
188	47517132005785	714799	11530
191	53149517838145	749953	11905
194	59344716674881	786241	12286
197	66148833516481	823681	12673
200	73610946594901	862291	13066
203	81783248916205	902089	13465
206	90721194225805	943093	13870
209	100483647464341	985321	14281
212	111133039782241	1028791	14698
215	122735528181001	1073521	15121
218	135361159849225	1119529	15550
221	149084041261465	1166833	15985
224	163982512107901	1215451	16426
227	180139324122901	1265401	16873
230	197641824880501	1316701	17326
233	216582146624845	1369369	17785
236	237057400203625	1423423	18250
239	259169874172561	1478881	18721
242	283027239138961	1535761	19198
245	308742757412401	1594081	19681
248	336435498030565	1653859	20170
251	366230557228285	1715113	20665
254	398259284417821	1777861	21166
257	432659513748421	1842121	21673
260	469575801313201	1907911	22186
263	509159668071385	1975249	22705
266	551569848553945	2044153	23230
269	596972545420681	2114641	23761
272	645541689936781	2186731	24298
275	697459208436901	2260441	24841
276	801801	77	26
278	752915294844805	2335789	25390
281	812108689316605	2412793	25945
284	875246963075641	2491471	26506
287	942546809507041	2571841	27073
290	1014234341580001	2653921	27646
293	1090545395665825	2737729	28225
296	1171725841819765	2823283	28810

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
299	1258031900594701	2910601	29401
302	1349730466454701	2999701	29998
305	1447099437856501	3090601	30601
308	1550428054066945	3183319	31210
311	1660017238784425	3277873	31825
314	1776179950632361	3374281	32446
317	1899241540592761	3472561	33073
320	2029540116447901	3572731	33706
322	1169686	86	28
323	2167426914298165	3674809	34345
326	2313266677224085	3778813	34990
329	2467438041160621	3884761	35641
332	2630333928051721	3992671	36298
335	2802361946353201	4102561	36961
338	2983944798951985	4214449	37630
341	3175520698569745	4328353	38305
344	3377543790718981	4444291	38986
347	3590484584279581	4562281	39673
350	3814830389763901	4682341	40366
353	4051085765338405	4804489	41065
356	4299772970669905	4928743	41770
359	4561432428664441	5055121	42481
362	4836623195166841	5183641	43198
365	5125923436689001	5314321	43921
368	5429930916234925	5447179	44650
371	5749263487290565	5582233	45385
374	15064335000	9000	624
374	6084559596046501	5719501	46126
377	6436478791921501	5859001	46873
380	6805702246455001	6000751	47626
383	7192933280636545	6144769	48385
386	7598897900740225	6291073	49150
389	8024345342732161	6439681	49921
392	8470048625319061	6590611	50698
395	8936805111705901	6743881	51481
398	9425437080130765	6899509	52270
401	9936792303244885	7057513	53065
404	10471744636405921	7217911	53866
407	11031194614952521	7380721	54673
410	11616070060528201	7545961	55486
413	12227326696522585	7713649	56305
416	12865948772698045	7883803	57130
419	13532949699069781	8056441	57961
422	14229372689107381	8231581	58798
425	14956291412325901	8409241	59641
428	15714810656334505	8589439	60490
431	16506066998410705	8772193	61345
434	17331229486668241	8957521	62206
437	18191500330886641	9145441	63073
440	19088115603070501	9335971	63946

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
443	20022345947806525	9529129	64825
446	20995497302486365	9724933	65710
449	22008911627463301	9923401	66601
452	23063967646210801	10124551	67498
455	24162081595551001	10328401	68401
458	25304707986021145	10534969	69310
461	26493340372446025	10744273	70225
464	27729512134784461	10956331	71146
467	29014797269317861	11171161	72073
470	30350811190248901	11388781	73006
473	31739211541778365	11609209	73945
476	33181699020728185	11832463	74890
479	34680018209778721	12058561	75841
482	36235958421388321	12287521	76798
485	37851354552463201	12519361	77761
488	39528087949845685	12754099	78730
491	41268087286688845	12991753	79705
494	43073329449785581	13232341	80686
497	44945840437920181	13475881	81673
500	46887696271310401	13722391	82666
503	48901023912208105	13971889	83665
506	50988002196726505	14224393	84670
509	53150862777962041	14479921	85681
512	55391891080478941	14738491	86698
515	57713427266224501	15000121	87721
518	60117867211943125	15264829	88750
521	62607663498157165	15532633	89785
524	65185326409782601	15803551	90826
527	67853424948447601	16077601	91873
530	70614587856582001	16354801	92926
533	73471504653345745	16635169	93985
536	76426926682464325	16918723	95050
539	79483668172039261	17205481	96121
542	82644607306401661	17495461	97198
545	85912687310076901	17788681	98281
548	89290917543928465	18085159	99370
551	92782374613548985	18384913	100465
554	96390203489966521	18687961	101566
557	100117618642734121	18994321	102673
560	103967905185470701	19304011	103786
563	107944420033921285	19617049	104905
566	112050593076604645	19933453	106030
569	116289928358116381	20253241	107161
572	120666005275155481	20576431	108298
575	125182479785342401	20903041	109441
578	129843085628896705	21233089	110590
581	134651635563242305	21566593	111745
584	139612022610608341	21903571	112906
587	144728221318693741	22244041	114073
590	150004289034463501	22588021	115246

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
593	155444367191144725	22935529	116425
596	161052682608490465	23286583	117610
599	166833548806379401	23641201	118801
602	172791367331819401	23999401	119998
605	178930629099423001	24361201	121201
608	185255915745422845	24726619	122410
611	191771900995295125	25095673	123625
614	198483352045059061	25468381	124846
617	205395130956320461	25844761	126073
620	212512196065127401	26224831	127306
623	219839603404706065	26608609	128545
626	227382508142144785	26996113	129790
629	235146166029094321	27387361	131041
632	243135934866552421	27782371	132298
635	251357275983800701	28181161	133561
638	259815755731561885	28583749	134830
641	268517046989445445	28990153	136105
644	277466930687749681	29400391	137386
647	286671297343688281	29814481	138673
650	296136148612109401	30232441	139966
653	305867598850775305	30654289	141265
656	315871876700270605	31080043	142570
659	326155326678607141	31509721	143881
662	336724410790593541	31943341	145198
665	347585710152037501	32380921	146521
668	358745926628848825	32822479	147850
671	370211884491111265	33268033	149185
674	381990532082191201	33717601	150526
677	394088943502951201	34171201	151873
680	406514320311136501	34628851	153226
683	419273993236002445	35090569	154585
686	432375423908250925	35556373	155950
689	445826206605343861	36026281	157321
692	459634070012261761	36500311	158698
695	473806878997775401	36978481	160081
698	488352636406298665	37460809	161470
701	503279484865390585	37947313	162865
704	518595708608974621	38438011	164266
707	534309735316343221	38932921	165673
710	550430137967015701	39432061	167086
713	566965636711517485	39935449	168505
716	583925100758148745	40443103	169930
719	601317550275810481	40955041	171361
722	619152158312956081	41471281	172798
725	637438252732736401	41991841	174241
728	656185318164406405	42516739	175690
731	675402997971061405	43045993	177145
734	695101096233770941	43579621	178606
737	715289579752178341	44117641	180073
740	735978580061634001	44660071	181546

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
743	757178395466930425	45206929	183025
746	778899493092707065	45758233	184510
749	801152510950593001	46314001	186001
752	823948260023155501	46874251	187498
755	847297726364722501	47439001	189001
758	871212073219147045	48008269	190510
761	895702643154581725	48582073	192025
764	920780960215331161	49160431	193546
767	946458732090850561	49743361	195073
770	972747852301958401	50330881	196606
773	999660402404331265	50923009	198145
776	1027208654209348885	51519763	199690
779	1055405072022357421	52121161	201241
782	1084262314898419021	52727221	202798
785	1113793238915615701	53337961	204361
788	1144010899465975585	53953399	205930
791	1174928553564089545	54573553	207505
794	1206559662173486281	55198441	209086
797	1238917892550833881	55828081	210673
800	1272017120608035901	56462491	212266
803	1305871433292290005	57101689	213865
806	1340495130984177205	57745693	215470
809	1375902729913849741	58394521	217081
812	1412108964595385641	59048191	218698
815	1449128790279378001	59706721	220321
818	1486977385423827025	60370129	221950
821	1525670154183402865	61038433	223585
823	197427385	694	113
824	1565222728917147301	61711651	225226
827	1605650972714682301	62389801	226873
830	1646970981940993501	63072901	228526
833	1689199088799856645	63760969	230185
836	1732351863915975025	64454023	231850
839	1776446118935895961	65152081	233521
842	1821498909147774361	65855161	235198
845	1867527536120051401	66563281	236881
848	1914549550359116365	67276459	238570
851	1962582753986019685	67994713	240265
854	2011645203432305221	68718061	241966
857	2061755212155029821	69446521	243673
860	2112931353371038201	70180111	245386
863	2165192462810561185	70918849	247105
866	2218557641490205345	71662753	248830
869	2273046258505402081	72411841	250561
872	2328677953842384181	73166131	252298
875	2385472641209757901	73925641	254041
878	2443450510889738605	74690389	255790
881	2502632032609118005	75460393	257545
884	2563037958430031041	76235671	259306
887	2624689325660590441	77016241	261073

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
890	2687607459785457001	77802121	262846
893	2751813977416413625	78593329	264625
896	2817330789263011165	79389883	266410
899	2884180103123354101	80191801	268201
902	2952384426895094101	80999101	269998
905	3021966571606699501	81811801	271801
908	3092949654469068745	82629919	273610
911	3165357101947555825	83453473	275425
914	3239212652854475761	84282481	277246
917	3314540361462158161	85116961	279073
920	3391364600636616901	85956931	280906
923	3469710064991903965	86802409	282745
926	3549601774065215485	87653413	284590
929	3631065075512818021	88509961	286441
932	3714125648326863121	89372071	288298
935	3798809506073158201	90239761	290161
938	3885143000149961785	91113049	292030
941	3973152823067871145	91991953	293905
944	4062866011750870381	92876491	295786
947	4154309950858606981	93766681	297673
950	4247512376129964901	94662541	299566
953	4342501377748002205	95564089	301465
956	4439305403726321305	96471343	303370
959	4537953263316939841	97384321	305281
962	4638474130439730241	98303041	307198
965	4740897547133496001	99227521	309121
968	4845253427028752725	100157779	311050
971	4951572058842281965	101093833	312985
974	5059884109893525901	102035701	314926
977	5170220629642890901	102983401	316873
980	5282613053252028001	103936951	318826
983	5397093205166158345	104896369	320785
986	5513693302718511625	105861673	322750
989	5632445959756945561	106832881	324721
992	5753384190292814461	107810011	326698
995	5876541412172154901	108793081	328681
998	6001951450769256565	109782109	330670
1001	6129648542702686285	110777113	332665
1004	6259667339573833321	111778111	334666
1007	6392042911728043921	112785121	336673
1010	6526810752038413201	113798161	338686
1013	6664006779712302385	114817249	340705
1016	6803667344120649445	115842403	342730
1019	6945829228650141181	116873641	344761
1022	7090529654578314781	117910981	346798
1025	7237806284971656901	118954441	348841
1028	7387697228606768305	120004039	350890
1031	7540241043914662105	121059793	352945
1034	7695476742948263641	122121721	355006
1037	7853443795373180041	123189841	357073

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1040	8014182132481807501	124264171	359146
1043	8177732151230844325	125344729	361225
1046	8344134718302277765	126431533	363310
1049	8513431174187912701	127524601	365401
1052	8685663337297510201	128623951	367498
1055	8860873508090604001	129729601	369601
1058	9039104473232062945	130841569	371710
1061	9220399509771467425	131959873	373825
1064	9404802389346367861	133084531	375946
1067	9592357382409493261	134215561	378073
1070	9783109262479977901	135352981	380206
1073	9977103310418674165	136496809	382345
1076	10174385318727619585	137647063	384490
1079	10375001595873726121	138803761	386641
1082	10578998970636759721	139966921	388798
1085	10786424796481678201	141136561	390961
1088	10997326955955395485	142312699	393130
1091	11211753865108040245	143495353	395305
1094	11429754477938776981	144684541	397486
1097	11651378290866257581	145880281	399673
1100	11876675347223771401	147082591	401866
1103	12105696241779161905	148291489	404065
1106	12338492125279577905	149506993	406270
1109	12575114709021127441	150729121	408481
1112	12815616269443502341	151957891	410698
1115	13060049652749641501	153193321	412921
1118	13308468279550500925	154435429	415150
1121	13560926149534998565	155684233	417385
1124	13817477846165202001	156939751	419626
1127	14078178541396827001	158202001	421873
1130	14343084000425115001	159471001	424126
1133	14612250586456157545	160746769	426385
1136	14885735265503735725	162029323	428650
1139	15163595611211742661	163318681	430921
1142	15445889809702257061	164614861	433198
1145	15732676664449335901	165917881	435481
1148	16024015601178594265	167227759	437770
1151	16319966672792640385	168544513	440065
1152	149979784926720	510720	9215
1154	16620590564322433921	169868161	442366
1157	16925948597904635521	171198721	444673
1160	17236102737785015701	172536211	446986
1163	17551115595347991085	173880649	449305
1166	17871050434172356045	175232053	451630
1169	18195971175113277781	176590441	453961
1172	18525942401410622881	177955831	456298
1175	18861029363823683401	179328241	458641
1178	19201297985792370505	180707689	460990
1181	19546814868624943705	182094193	463345
1184	19897647296712343741	183487771	465706

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1187	20253863242769197141	184888441	468073
1190	20615531373101560501	186296221	470446
1193	20982721052901472525	187711129	472825
1196	21355502351568381865	189133183	475210
1199	21733946048057518801	190562401	477601
1202	22118123636255278801	191998801	479998
1205	22508107330381686001	193442401	482401
1208	22903970070420004645	194893219	484810
1211	23305785527573566525	196351273	487225
1214	23713628109749882461	197816581	489646
1217	24127572967072105861	199289161	492073
1220	24547695997417916401	200769031	494506
1223	24974073851985891865	202256209	496945
1226	25406783940889436185	203750713	499390
1229	25845904438778331721	205252561	501841
1232	26291514290487983821	206761771	504298
1235	26743693216716425701	208278361	506761
1238	27202521719729151685	209802349	509230
1241	27668081089091846845	211333753	511705
1244	28140453407431081081	212872591	514186
1247	28619721556223035681	214418881	516673
1250	29105969221610330401	215972641	519166
1253	29599280900247019105	217533889	521665
1256	30099741905171822005	219102643	524170
1259	30607438371709662541	220678921	526681
1262	31122457263401576941	222262741	529198
1265	31644886377963064501	223854121	531721
1268	32174814353270946625	225453079	534250
1271	32712330673378802665	227059633	536785
1274	33257525674561050601	228673801	539326
1277	33810490551385740601	230295601	541873
1280	34371317362816129501	231925051	544426
1283	34940099038341104245	233562169	546985
1286	35516929384134522325	235206973	549550
1289	36101903089243537261	236859481	552121
1292	36695115731805977161	238519711	554698
1295	37296663785296844401	240187681	557281
1298	37906644624804004465	241863409	559870
1301	38525156533333131985	243546913	562465
1304	39152298708141982021	245238211	565066
1307	39788171267104054621	246937321	567673
1310	40432875255101720701	248644261	570286
1313	41086512650448877285	250359049	572905
1316	41749186371343200145	252081703	575530
1319	42421000282348061881	253812241	578161
1322	43102059200904183481	255550681	580798
1325	43792468903871087401	257297041	583441
1328	44492336134098420205	259051339	586090
1331	45201768607027212805	260813593	588745
1334	45920875017321146341	262583821	591406

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1337	46649765045527891741	264362041	594073
1340	47388549364770591001	266148271	596746
1343	48137339647469548225	267942529	599425
1346	48896248572094198465	269744833	602110
1349	49665389829945422401	271555201	604801
1352	50444878131968274901	273373651	607498
1355	51234829215595195501	275200201	610201
1358	52035359851619768845	277034869	612910
1361	52846587851101103125	278877673	615625
1364	53668632072298894561	280728631	618346
1367	54501612427639245961	282587761	621073
1370	55345649890711307401	284455081	623806
1373	56200866503294807065	286330609	626545
1376	57067385382418540285	288214363	629290
1379	57945330727449884821	290106361	632041
1382	58834827827215410421	292006621	634798
1385	59736003067152650701	293915161	637561
1388	60648983936493105385	295831999	640330
1391	61573899035476540945	297757153	643105
1394	62510878082596657681	299690641	645886
1397	63460051921878191281	301632481	648673
1400	64421552530185516901	303582691	651466
1403	65395513024562823805	305541289	654265
1406	66382067669605928605	307508293	657070
1409	67381351884865795141	309483721	659881
1412	68393502252283829041	311467591	662698
1415	69418656523659015001	313459921	665521
1418	70456953628146964825	315460729	668350
1421	71508533679790944265	317470033	671185
1424	72573537985084946701	319487851	674026
1427	73652109050568881701	321514201	676873
1430	74744390590455946501	323549101	679726
1433	75850527534292248445	325592569	682585
1436	76970666034648746425	327644623	685450
1439	78104953474845579361	329705281	688321
1442	79253538476708849761	331774561	691198
1445	80416570908359930401	333852481	694081
1448	81594201892037362165	335939059	696970
1451	82786583811951411085	338034313	699865
1454	83993870322171352621	340138261	702766
1457	85216216354545551221	342250921	705673
1460	86453778126654403201	344372311	708586
1463	87706713149796210985	346502449	711505
1466	88975180237006056745	348641353	714430
1469	90259339511107743481	350789041	717361
1472	91559352412798871581	352945531	720298
1475	92875381708769118901	355110841	723241
1478	94207591499851792405	357284989	726190
1481	95556147229208719405	359467993	729145
1484	96921215690548546441	361659871	732106

s	$C(s, n_c) = P(s, n_p)$	n_p	n_c
1487	98302965036378513841	363860641	735073
1490	99701564786289774001	366070321	738046
2378	432684460	604	103
2386	29437553530	4970	420
4980	24264913354964425	3122317	30810
9325	3176083959788026	825436	12691
9525	16195753597485	58322	2169
19605	5519583702676	23731	1191
31265	90525801730	2407	259

Table 1: Polygonal Cannonball Numbers