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Parameter Estimation

1) Let (X1, X2, ...) be random sample of size n taken from normal population with parameter mean = θ1 & variance = θ2. Find the max. likelihood estimates of these two population parameters.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

X1, X2, .. Xn } Sample of size n

$$L(X1, X2, .. X_n) = f(x_1) \cdot f(x_2) \ldots f(x_n)$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) \ldots$$

Taking ln on both side we get,

$$\ln(L) = -\frac{n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n}\left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

Taking derivative w.r.t μ

$$\frac{d\ln(L)}{d\mu} = 0 + \sum_{i=1}^{n} - \left(\frac{2(x_i-\mu)}{2\sigma^2}\right) = 0$$

$$= \sum_{i=1}^{n}(x_i - \mu) = 0$$

$$= n\bar{X} - n\mu = 0$$

$$\bar{X} = \mu$$

Hence $\theta_1 = \bar{X}$ i.e sample mean

$$\ln(L) = \frac{-n}{2} \ln(2\pi \sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i - \mu)^2}{2\sigma^2} \right)$$

Taking derivative w.r.t σ^2

$$\frac{d \ln(L)}{d\sigma^2} = \frac{-n}{\cancel{2}\,\cancel{\sigma^2}} + \sum_{i=1}^{n} - \frac{(x_i - \mu)^2}{\cancel{2}(\sigma^2)^{\cancel{2}}} = 0$$

$$-n + \sum_{i=1}^{n} - \frac{(x_i - \mu)^2}{\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

Hence $\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$

2) Let $X_1, X_2 \ldots X_n$ be random sample from $B(m, \theta)$ distribution when $\theta \in \Theta = (0,1)$ is unknown ~~&~~ m is +ve integer. Compute value of θ using M.L.E

Binomial Distribution $= {}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1}^{n} \left(\log ({}^nC_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right)$$

$$\log L = \sum_{i=1}^{n} \log({}^nC_{x_i}) + \log\theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i=1}^{n} (n-x_i)$$

Diff. w.r.t θ

$$\frac{d}{d\theta} \log (L) = 0$$

$$\Rightarrow \frac{1}{\theta} \sum x_i - \frac{1}{1-\theta} \sum (n-x_i) = 0$$

$$\Rightarrow \frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum x_i = 0$$

$$\Rightarrow \frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta}$$

$$\Rightarrow \frac{\sum x_i}{\theta} = n^2$$

$$\boxed{\theta = \frac{\sum x_i}{n^2}}$$

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