Introduction to Quantum Mechanics (from QC and QI)

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1 Linear Algebra

1.1 Overview of Dirac Notation

Notation	Description
z^*	Complex conjugate of the complex number z
$\|\psi\rangle$	Column vector, also known as a ket
$\langle\psi\|$	Row vector, also known as a bra
$\langle\varphi\|\psi\rangle$	Inner product between vectors $\|\varphi\rangle$ and $\|\psi\rangle$
$\|\varphi\rangle \otimes \|\psi\rangle$	Tensor product of $\|\varphi\rangle$ and $\|\psi\rangle$
$\|\varphi\rangle\|\psi\rangle$	Tensor product of $\|\varphi\rangle$ and $\|\psi\rangle$
A^*	Complex conjugate of the A matrix
A^T	Transposed of the A matrix
$A\dagger$	Hermitian conjugate or adjoint of the A matrix $= (A^T)^*$

1.2 Bases and Linear Independence

Exercise 2.1 Linear Dependence Show that $(1, -1), (1, 2)$ and $(2, 1)$ are linearly dependent. This is equivalent to finding the set of complex numbers $a_1, ..., a_n$ that satisfy the equation $a_1|v_1\rangle + a_2|v_2\rangle + ... + a_n|v_n\rangle = 0$. Therefore, we have

$$a_1(1, -1) + a_2(1, 2) + a_3(2, 1) = 0$$

One set that satisfies this equality (while maintaining $a_i \neq 0$ for at least one value of i) is

$$a_1 = 1, a_2 = 1, and a_3 = -1$$

Therefore, this set of vectors is linearly dependent.

1.3 Linear Operators and Matrices

An m by n matrix A with entries A_{ij} is a linear operator sending vectors in the vector space $\mathbb{C}^n$ to the vector space $\mathbb{C}^m$. The operation can be written as (for all i)

$$A\left(\Sigma a_i|v_i\rangle\right) = \Sigma a_i A|v_i\rangle$$

Four important matrices are the **Pauli matrices**:

$$\sigma_0 = \sigma_I = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \sigma_1 = \sigma_x = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_2 = \sigma_y = Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_3 = \sigma_z = Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

1.4 Inner Products

$$((y_1, ..., y_n), (z_1, ..., z_n)) = \Sigma y_i^* z_i$$

Two vectors $|v\rangle$ and $|w\rangle$ are orthogonal if $\langle v|w\rangle = 0$

Also note that in the context of this material, an inner product space is exactly the same thing as a Hilbert space.

The norm of a vector $|v\rangle$ is given by $||\ |v\rangle|| = \sqrt{\langle v|v\rangle}$, where a norm of 1 indicates that v is a unit vector. If not a unit vector, we can normalize $|v\rangle$ by dividing it by its norm.