Quantum Walks and Photosynthesis

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These are notes on the lecture about more of the physics behind quantum walks and photosynthesis, including quantum cryptography, as taught by Seth Lloyd in 8.370 Quantum Computation at MIT during Fall 2016.

I. QUANTUM WALKS

Suppose we have a bunch of sites, and we have a particle that can move along these sites. If this were a classical particle, and had equal probability of moving to the left and right, then it basically takes a random walk along the path, and gradually diffuse outward.

Quantum mechanically, however, something different happens. If we have a rate J at which this particle moves, then we have

$$H = J \sum_j |j+1\rangle\langle j| + |j\rangle\langle j+1|$$

If we have an initial wave packet, then the wave corresponding to the position of the particle propagates with a speed $v = 2J$. In the quantum mechanical case, there is a ballistic-type movement, and rather than spreading out in square root of time (as in the classical case), it spreads out linearly in time.

This means that quantum walks can do some cool things that classical walks can't. For example, take two branching trees that are facing each other and connected in the middle. If we have a classical walk to get from one root to the other root, then when it encounters a branch that splits, it is twice as likely to move forward rather than backward. However, if it is on the other side and encounters a joint where two branches meet, then it is likely to go back. Therefore, the particle would get stuck in the middle. Not good!

With a quantum walk, however, we have the Hamiltonian at each node / junction which tells us how to propagate. At each step, we have that the wave is symmetrically located at each node on that level, and therefore the walk looks as if it is walking along a line, since it only considers the fully symmetrized state. The wave propagates through ballistically with velocity $2J$, and goes straight from the origin to the destination, all due to the symmetry of the situation.

For a non-random walk we could just define a rule for the classical case, such as always taking a certain direction on the fork. However, it is provable that for a randomized tree, it is possible that you will always get stuck at a certain point. The quantum walk, however, has a feature that allows it to pretty much ignore the scrambling. For example, say that we add energy disorder to the walk over some range Δw. Then instead we have the Hamiltonian (this of which is called **Anderson Localization**)

$$H = J \sum_j |j+1\rangle\langle j| + |j\rangle\langle j+1| + \Delta w_j |j\rangle\langle j|$$

As you can see, there is some localization, as there is the chance that the walker gets stuck in one state. The localization length which characterizes this is

$$l = 2\pi^2 \frac{J^2}{\Delta w^2}$$

This destructive interference is so powerful that it causes the walker to get stuck. Therefore, if we change the branching factor, or add some energy fluctuations, we get this localization and the symmetry of the walk is destroyed.

II. WHAT'S THIS ABOUT PHOTOSYNTHESIS?

As an exciton makes its way to a reaction center for photosynthesis, it was thought that the exciton takes a random walk through the photosynthesis structure. However, researches found that the exciton actually takes a quantum walk through the structure to find the reaction center.

Now what happens if Anderson localization occurs? At zero temperature, the exciton gets completely stuck, but as you raise the temperature, it gets unstuck and makes its way to the reaction center. It turns that at 290 kelvin, it makes it through with the best efficiency, and also gets stuck if you increase the temperature too much. At low temperature, the particle takes a combined quantum and classical walk. The net effect is a random walk with step size equal to the localization l and step time equal to τ, or the decoherence / environmental interaction time. The square root of the expectation value is therefore $\sqrt{\langle x^2(t)\rangle} = l\sqrt{t/\tau}$.

If we keep increasing the temperature, then we just propagate for the decoherence time, and therefore spend less time being stuck. Therefore, when τ is greater than $\tau_{LOC} = l/\bar{v}$, where $\bar{v}$ is the average velocity, we get the above expectation value.

Now what happens when $\tau < \tau_{LOC}$? Instead we just jitter around and have shorter and shorter step sizes. We propagate inherently, and only perform a quantum walk up to the decoherence time.

Now let's take a classical random walk with step time = τ, but a step size = $v\tau$ (where the velocity is commonly $2J$). This says that the expected thread, or standard deviation, of where this particle is goes as

$$sqrt\langle x^2(t)\rangle = v\tau\sqrt{t/\tau} = v\sqrt{t\tau}$$

which tells us that as a function of temperature (since $\frac{1}{\tau} \propto T$) the diffusion rate increases and tapers off as T increases, and then drops off and asymptotes to 0 after $1/\tau_{loc}$, where $t_{loc} = \pi^2 J/\Delta w^2$. It turns out that $1/\tau_{loc}$ is proportional to temperature and coupling to the environment, and inversely proportional to the decay rate of environmental correlations, all of which you can measure. Therefore, we can optimize this to create the best diffusion rate, which happens to be $T\lambda/\gamma$.

III. QUANTUM CRYPTOGRAPHY

We would like to use quantum mechanics to distribute secure keys, through Quantum Key Distribution protocols. We want each to have a random number k, which is a secret key, of which no one else possesses. With this, Alice can take any message, add it in a bit-wise fashion to the random number, and transmits it. Bob can then add k to the encoded message to get the original message. However, this is a one time pad, and can therefore only be used once, and Eve cannot interpret what the original message was. We therefore use a protocol such as BB84 (omitted from these notes)