Building Quantum Computers

as taught by Seth Lloyd, notes by Aaron Vontell
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These are notes on the lecture about building quantum computers, as taught by Seth Lloyd in 8.370 Quantum Computation at MIT during Fall 2016.

I. INTRODUCTION

Back in 1993, physicists began to think about electromagnetic resonance, and specifically nuclear magnetic resonance, in the context of quantum computing. By 1994, they began to think about ion trap quantum computers, as well as quantum optical devices. By the late 1990's, they also thought about the use of quantum dots and nitrogen vacancy centers in diamond. In the early 2000s, they then got into the field of superconducting qubits, which make use of optical cavities. In all of these, the qubits may be the presence of charges, the spin of an electron, etc...

A. Nuclear Magnetic Resonance (Single Qubit)

This technique involved putting atoms in a magnetic field. You zap them with microwaves, which allows you to manipulate them in specific ways.
For example, imagine we have a proton with magnetic moment μ and spin $\hbar/2$. We have a spin up state defined as $|0\rangle$, and a spin down state $|1\rangle$. We define the energies associated with these states as $E_0 = -\mu B$ and $E_1 = +\mu B$, where B is the magnetic field. Within this magnetic field, we have that the spin processes around $\vec{B}$ with a characteristic frequency where $\hbar\omega = 2\mu B$. This frequency is also known as the Larmor Frequency.
We know define the energy operator $H = -\hbar\omega/2\sigma_z$, which has two eigenstates $|0\rangle$ and $|1\rangle$, with eigenvalues $E_0 = -\hbar\omega/2$ and $E_1 = +\hbar\omega/2$.

This energy operator also defines the time evolution of a state $|\psi\rangle$, as it goes to

$$e^{-iHt/\hbar}|\psi\rangle = e^{+i\omega t\sigma_z/2}|\psi\rangle$$

where $\omega t = \theta$. The proton spin therefore processes about $-z$ axis with frequency ω. Therefore, if you can turn static magnetic fields ($\vec{B}$) on and off with high accuracy, and define the direction of this magnetic field, you can perform any rotation in $SU(2)$.

However, this is not NMR. Instead, apply an oscillating field, circulating polarized about the $-z$-axis and oscillating with frequency ω (resonance).

Let's take a look at the co-rotating frame... with respect to this frame, the static field $\vec{B}$ goes away, and the spin sits still. However, the oscillating field is now fixed, and is observed as a static field. The spin processes about the now static applied oscillatory field.

In the co-rotating frame, the evolution goes as

$$e^{+i\omega t\sigma_x/2}$$

By choosing the initial polarization of oscillatory field, one can perform any rotation $e^{-i\theta\sigma_{\hat{i}}/2}$ in the co-rotating frame. By Euler-angles, we can therefore perform any rotation in the co-rotating frame!

B. Driving Off Resonance

So what happens when you are not driving the system at resonance? Or when

$$\vec{B} = B(\hat{x}\cos\omega' t - \hat{y}\sin\omega' t), \omega' \neq \omega_0$$

The spin now sees a small oscillating field with frequency $\omega' - \omega_0$. To first order, nothing happens. To second order, the spin performs very small fast oscillations called Bloch-Siegert oscillations, with amplitude $\omega^2/(\omega' - \omega_0)^2$. For this class, we will consider the first order case.

C. Two Qubit Operations

We saw how to manipulate one qubit with NMR, but how do we manipulate two qubits? We now have two molecules with magnetic moments μ_A and μ_B. Imagine that B is a heavier atom, and as such $\mu_A > \mu_B$. We therefore have that $\hbar\omega_A = 2\mu_A B_0$ and $\hbar\omega_B = 2\mu_B B_0$.

We drive the system with an oscillating field with frequency $\omega' = \omega_A$. What happens? Spin A will be on resonance, and processes about the oscillating field in the co-rotating frame. Spin B, however, will do nothing, as we saw from the previous section on driving off resonance. If $\omega' = \omega_B$, we get the opposite.

Now what about interacting spins? Let's add an interaction of the form $\hbar\gamma/2\sigma_Z^A \otimes \sigma_z^B$. We then have

$$H = \hbar/2(-\omega_A\sigma_z^A \otimes I_B - \omega_B I_A \otimes \sigma_z^B + \gamma\sigma_z^A \otimes \sigma_z^B)$$

With this we get eigenstates $|00\rangle_{AB}$ with $E_{00} = \frac{\hbar}{2}(-\omega_A - \omega_B + \gamma)$, $|01\rangle_{AB}$ with $E_{01} = \frac{\hbar}{2}(-\omega_A + \omega_B - \gamma)$, etc...

Define $\omega_A^\uparrow = \omega_A - \gamma$ to be the Larmor frequency of A when B is up, and $\omega_A^\downarrow = \omega_A + \gamma$ to be the Larmor frequency of A when B is down.