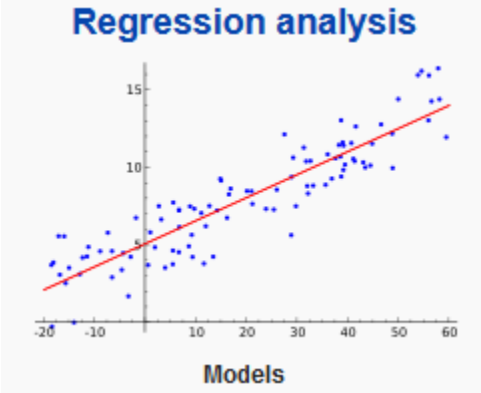
1. **Explain the linear regression algorithm in detail.**

In statistics, linear regression is a linear approach to modeling the relationship between a scalar response (or dependent variable) and one or more explanatory variables (or independent variables). The case of one explanatory variable is called simple linear regression. For more than one explanatory variable, the process is called multiple linear regression. This term is distinct from multivariate linear regression, where multiple correlated dependent variables are predicted, rather than a single scalar variable.
In linear regression, the relationships are modeled using linear predictor functions whose unknown model parameters are estimated from the data. Such models are called linear models. Most commonly, the conditional mean of the response given the values of the explanatory variables (or predictors) is assumed to be an affine function of those values; less commonly, the conditional median or some other quantile is used. Like all forms of regression analysis, linear regression focuses on the conditional probability distribution of the response given the values of the predictors, rather than on the joint probability distribution of all these variables, which is the domain of multivariate analysis.
Linear regression was the first type of regression analysis to be studied rigorously, and to be used extensively in practical applications. This is because models which depend linearly on their unknown parameters are easier to fit than models which are non-linearly related to their parameters and because the statistical properties of the resulting estimators are easier to determine.
Linear regression has many practical uses. Most applications fall into one of the following two broad categories:

- If the goal is prediction, or forecasting, or error reduction, linear regression can be used to fit a predictive model to an observed data set of values of the response and explanatory variables. After developing such a model, if additional values of the explanatory variables are collected without an accompanying response value, the fitted model can be used to make a prediction of the response.
- If the goal is to explain variation in the response variable that can be attributed to variation in the explanatory variables, linear regression analysis can be applied to quantify the strength of the relationship between the response and the explanatory variables, and in particular to determine whether some explanatory variables may have no linear relationship with the response at all, or to identify which subsets of explanatory variables may contain redundant information about the response.

Linear regression models are often fitted using the least squares approach, but they may also be fitted in other ways, such as by minimizing the "lack of fit" in some other norm (as with least absolute deviations regression), or by minimizing a penalized version of the least squares cost function as in ridge regression (L2-norm penalty) and lasso (L1-norm penalty). Conversely, the least squares approach can be used to fit models that are not linear models. Thus, although the terms "least squares" and "linear model" are closely linked, they are not synonymous.



2. What are the assumptions of linear regression regarding residuals?

Linear regression is an analysis that assesses whether one or more predictor variables explain the dependent (criterion) variable. The regression has below key assumptions:

a. Linear relationship
b. No or little multicollinearity
c. Homoscedasticity of residuals or equal variance
d. Mean of residuals is zero
e. Normality of residuals
f. No auto-correlation of residuals

3. What is the coefficient of correlation and the coefficient of determination?

Correlation Coefficient (r):
The quantity r, called the linear correlation coefficient, measures the strength and the direction of a linear relationship between two variables. The linear correlation coefficient is sometimes referred to as the Pearson product moment correlation coefficient in honor of its developer Karl Pearson.

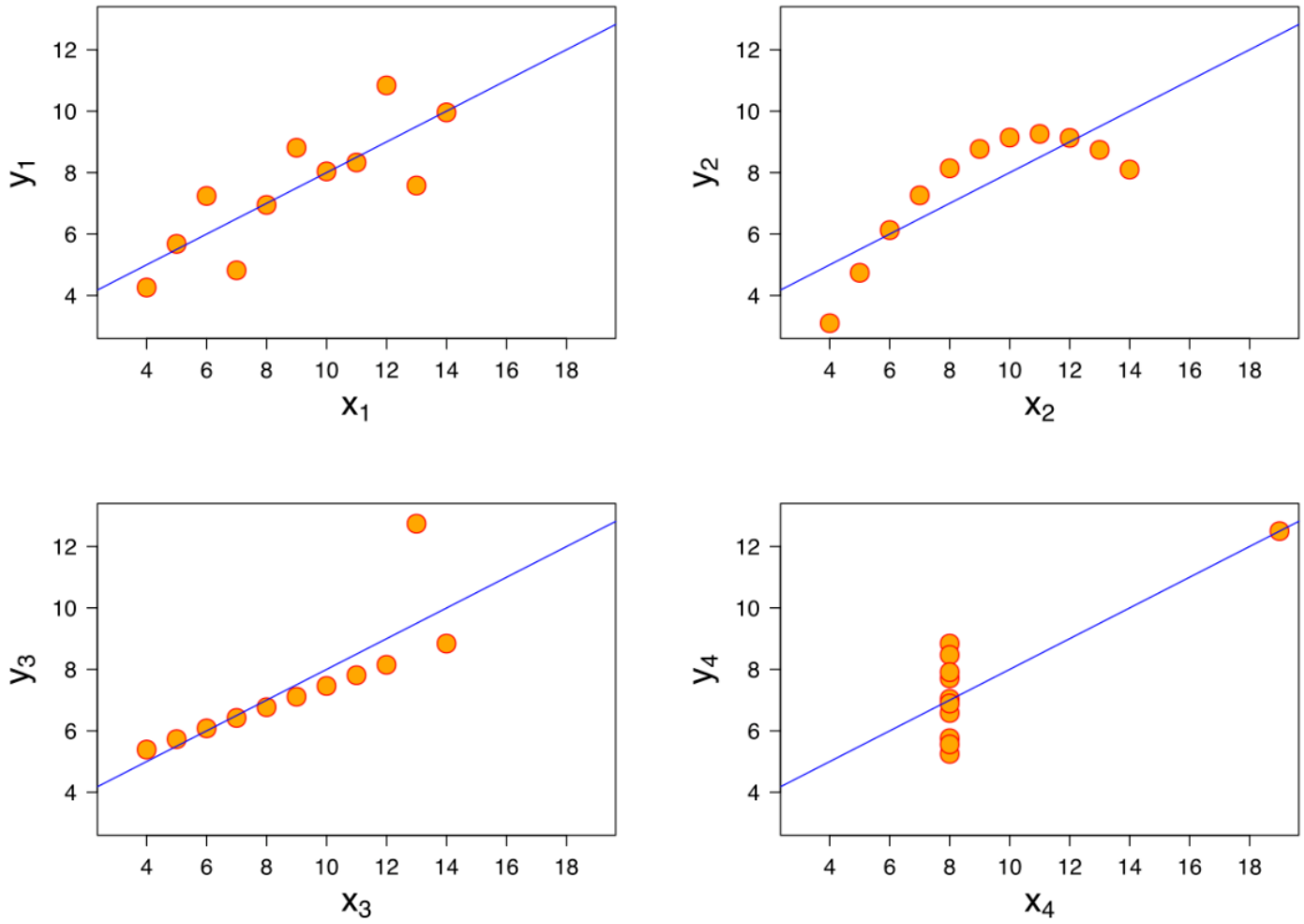
- The value of r is such that $-1 < r < +1$. The + and – signs are used for positive linear correlations and negative linear correlations, respectively.
- No correlation: If there is no linear correlation or a weak linear correlation, r is close to 0. A value near zero means that there is a random, nonlinear relationship between the two variables.
- A perfect correlation of ± 1 occurs only when the data points all lie exactly on a straight line. If r = +1, the slope of this line is positive. If r = -1, the slope of this line is negative.
- A correlation greater than 0.8 is generally described as strong, whereas a correlation less than 0.5 is generally described as weak. These values can vary based upon the "type" of data being examined. A study utilizing scientific data may require a stronger correlation than a study using social science data.

Coefficient of Determination (r^2):
The coefficient of determination, r^2, is useful because it gives the proportion of the variance (fluctuation) of one variable that is predictable from the other variable. It is a measure that allows us to determine how certain one can be in making predictions from a certain model/graph.

- The coefficient of determination is the ratio of the explained variation to the total variation.
- The coefficient of determination is such that $0 < r^2 < 1$, and denotes the strength of the linear association between x and y.
- The coefficient of determination represents the percent of the data that is the closest to the line of best fit.
- The coefficient of determination is a measure of how well the regression line represents the data. If the regression line passes exactly through every point on the scatter plot, it would be able to explain all of the variation. The further the line is away from the points, the less it is able to explain.

4. Explain the Anscombe's quartet in detail.

Anscombe's quartet comprises four data sets that have nearly identical simple descriptive statistics, have very different distributions and appear very different when graphed. Each dataset consists of eleven (x, y) points. They were constructed in 1973 by the statistician Francis Anscombe to demonstrate both the importance of graphing data before analyzing it and the effect of outliers and other influential observations on statistical properties. He described the article as being intended to counter the impression among statisticians that "numerical calculations are exact, but graphs are rough.



For all four datasets:

Property	Value	Accuracy
Mean of *x*	9	exact
Sample variance of *x*	11	exact
Mean of *y*	7.50	to 2 decimal places
Sample variance of *y*	4.125	±0.003
Correlation between *x* and *y*	0.816	to 3 decimal places
Linear regression line	*y* = 3.00 + 0.500*x*	to 2 and 3 decimal places, respectively
Coefficient of determination of the linear regression	0.67	to 2 decimal places

- The first scatter plot (top left) appears to be a simple linear relationship, corresponding to two variables correlated and following the assumption of normality.
- The second graph (top right) is not distributed normally; while a relationship between the two variables is obvious, it is not linear, and the Pearson correlation coefficient is not relevant. A more general regression and the corresponding coefficient of determination would be more appropriate.
- In the third graph (bottom left), the distribution is linear, but should have a different regression line (a robust regression would have been called for). The calculated regression is offset by the one outlier which exerts enough influence to lower the correlation coefficient from 1 to 0.816.
- Finally, the fourth graph (bottom right) shows an example when one high-leverage point is enough to produce a high correlation coefficient, even though the other data points do not indicate any relationship between the variables.

The quartet is still often used to illustrate the importance of looking at a set of data graphically before starting to analyze according to a particular type of relationship, and the inadequacy of basic statistic properties for describing realistic datasets.

The datasets are as follows. The x values are the same for the first three datasets.

Anscombe's quartet

I		II		III		IV	
x	y	x	y	x	y	x	y
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

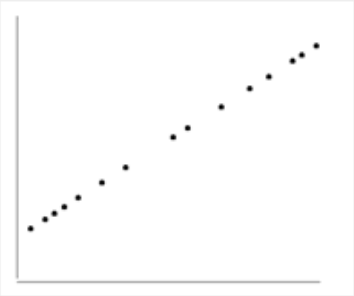
5. What is Pearson's R?

Correlation is a technique for investigating the relationship between two quantitative, continuous variables, for example, age and blood pressure. Pearson's correlation coefficient (r) is a measure of the strength of the association between the two variables.

The first step in studying the relationship between two continuous variables is to draw a scatter plot of the variables to check for linearity. The correlation coefficient should not be calculated if the relationship is not linear. For correlation only purposes, it does not really matter on which axis the variables are plotted. However, conventionally, the independent (or explanatory) variable is plotted on the x-axis (horizontally) and the dependent (or response) variable is plotted on the y-axis (vertically).

The nearer the scatter of points is to a straight line, the higher the strength of association between the variables. Also, it does not matter what measurement units are used.

Values of Pearson's correlation coefficient
Pearson's correlation coefficient (r) for continuous (interval level) data ranges from -1 to +1. Positive correlation indicates that both variables increase or decrease together, whereas negative correlation indicates that as one variable increases, so the other decreases, and vice versa.

r = -1		data lie on a perfect straight line with a negative slope
r = 0		no linear relationship between the variables
r = +1		data lie on a perfect straight line with a positive slope

6. **What is scaling? Why is scaling performed? What is the difference between normalized scaling and standardized scaling?**

Feature scaling is a method used to normalize the range of independent variables or features of data. In data processing, it is also known as data normalization and is generally performed during the data preprocessing step.

Since the range of values of raw data varies widely, in some machine learning algorithms, objective functions will not work properly without normalization. For example, many classifiers calculate the distance between two points by the Euclidean distance. If one of the features has a broad range of values, the distance will be governed by this particular feature. Therefore, the range of all features should be normalized so that each feature contributes approximately proportionately to the final distance.

Another reason why feature scaling is applied is that gradient descent converges much faster with feature scaling than without it.

Normalized Scaling:
Also known as min-max scaling or min-max normalization, is the simplest method and consists in rescaling the range of features to scale the range in [0, 1] or [−1, 1]. Selecting the target range depends on the nature of the data. The general formula for a min-max of [0, 1] is given as:

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

where x is an original value, x' is the normalized value. For example, suppose that we have the students' weight data, and the students' weights span [160 pounds, 200 pounds]. To rescale this data, we first subtract 160 from each student's weight and divide the result by 40 (the difference between the maximum and minimum weights).

To rescale a range between an arbitrary set of values [a, b], the formula becomes:

$$x' = a + \frac{(x - \min(x))(b - a)}{\max(x) - \min(x)}$$

where a, b are the min-max values.

Mean normalization

$$x' = \frac{x - \text{average}(x)}{\max(x) - \min(x)}$$

where x is an original value, x' is the normalized value. There is another form of the mean normalization which is when we divide by the standard deviation which is also called standardization.

Standardized Scaling:
In machine learning, we can handle various types of data, e.g. audio signals and pixel values for image data, and this data can include multiple dimensions. Feature standardization makes the values of each feature in the data have zero-mean (when subtracting the mean in the numerator) and unit-variance. This method is widely used for normalization in many machine learning algorithms (e.g., support vector machines, logistic regression, and artificial neural networks). The general method of calculation is to determine the distribution mean and standard deviation for each feature. Next we subtract the mean from each feature. Then we divide the values (mean is already subtracted) of each feature by its standard deviation.

$$x' = \frac{x - \bar{x}}{\sigma}$$

Where x is the original feature vector, $\bar{x}$ = average (x) is the mean of that feature vector, and σ is its standard deviation.

Scaling to unit length

Another option that is widely used in machine-learning is to scale the components of a feature vector such that the complete vector has length one. This usually means dividing each component by the Euclidean length of the vector:

$$x' = \frac{x}{\|x\|}$$

In some applications (e.g. Histogram features) it can be more practical to use the L1 norm (i.e. Manhattan Distance, City-Block Length or Taxicab Geometry) of the feature vector. This is especially important if in the following learning steps the Scalar Metric is used as a distance measure.

7. You might have observed that sometimes the value of VIF is infinite. Why does this happen?

The extent to which a predictor is correlated with the other predictor variables in a linear regression can be quantified as the R-squared statistic of the regression where the predictor of interest is predicted by all the other predictor variables. The variance inflation for a variable is then computed as:

$$VIF = \frac{1}{1 - R^2}$$

When R-squared reaches 1, VIF reaches infinity. When R-squared reaches 1 then it means multicollinearity exists. Different variables are highly correlated with each other.

8. What is the Gauss-Markov theorem?

In statistics, the Gauss–Markov theorem states that in a linear regression model in which the errors are uncorrelated, have equal variances and expectation value of zero, the best linear unbiased estimator (BLUE) of the coefficients is given by the ordinary least squares (OLS) estimator, provided it exists. Here "best" means giving the lowest variance of the estimate, as compared to other unbiased, linear estimators. The errors do not need to be normal, nor do they need to be independent and identically distributed (only uncorrelated with mean zero and homoscedastic with finite variance). The requirement that the estimator be unbiased cannot be dropped, since biased estimators exist with lower variance. See, for example, the James–Stein estimator (which also drops linearity) or ridge regression. The theorem was named after Carl Friedrich Gauss and Andrey Markov.

9. Explain the gradient descent algorithm in detail.

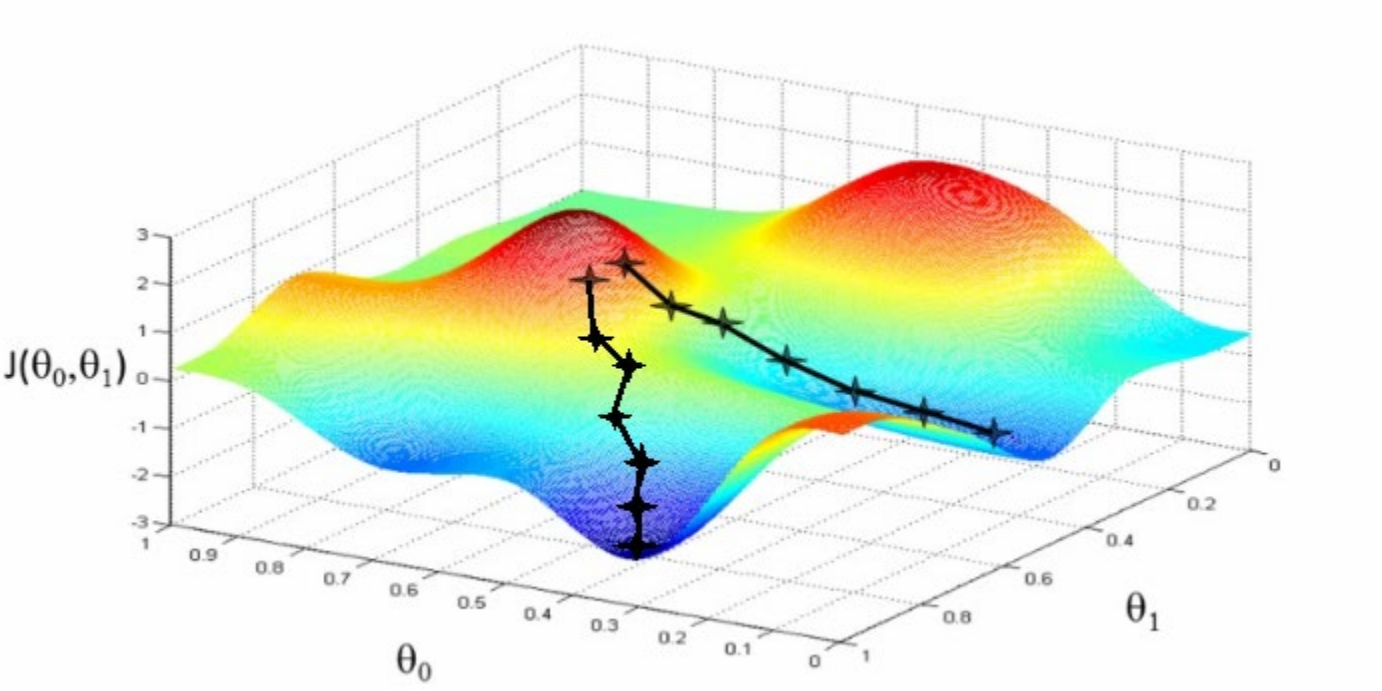
Gradient descent is a first-order iterative optimization algorithm for finding the minimum of a function. To find a local minimum of a function using gradient descent, one takes steps proportional to the negative of the gradient (or approximate gradient) of the function at the current point. If, instead, one takes steps proportional to the positive of the gradient, one approaches a local maximum of that function; the procedure is then known as gradient ascent.

Gradient descent is also known as steepest descent. However, gradient descent should not be confused with the method of steepest descent for approximating integrals.

Suppose you are at the top of a mountain, and you have to reach a lake which is at the lowest point of the mountain (a.k.a valley). A twist is that you are blindfolded and you have zero visibility to see where you are headed. So, what approach will you take to reach the lake?

The best way is to check the ground near you and observe where the land tends to descend. This will give an idea in what direction you should take your first step. If you follow the descending path, it is very likely you would reach the lake.

To represent this graphically, notice the below graph.



Let us now map this scenario in mathematical terms.

Suppose we want to find out the best parameters (θ1) and (θ2) for our learning algorithm. Similar to the analogy above, we see we find similar mountains and valleys when we plot our “cost space”. Cost space is nothing but how our algorithm would perform when we choose a particular value for a parameter.

So on the y-axis, we have the cost J(θ) against our parameters θ1 and θ2 on x-axis and z-axis respectively. Here, hills are represented by red region, which have high cost, and valleys are represented by blue region, which have low cost.

Now there are many types of gradient descent algorithms. They can be classified by two methods mainly:

- On the basis of data ingestion
  - ✓ Full Batch Gradient Descent Algorithm

- ✓ Stochastic Gradient Descent Algorithm

In full batch gradient descent algorithms, you use whole data at once to compute the gradient, whereas in stochastic you take a sample while computing the gradient.

- On the basis of differentiation techniques
  - ✓ First order Differentiation
  - ✓ Second order Differentiation

Gradient descent requires calculation of gradient by differentiation of cost function. We can either use first order differentiation or second order differentiation.

Challenges in executing Gradient Descent

Gradient Descent is a sound technique which works in most of the cases. But there are many cases where gradient descent does not work properly or fails to work altogether. There are three main reasons when this would happen:

- **Data Challenges**

a. If the data is arranged in a way that it poses a non-convex optimization problem. It is very difficult to perform optimization using gradient descent. Gradient descent only works for problems which have a well-defined convex optimization problem.
b. Even when optimizing a convex optimization problem, there may be numerous minimal points. The lowest point is called global minimum, whereas rest of the points are called local minima. Our aim is to go to global minimum while avoiding local minima.
c. There is also a saddle point problem. This is a point in the data where the gradient is zero but is not an optimal point. We don't have a specific way to avoid this point and is still an active area of research.

- **Gradient Challenges**

a. If the execution is not done properly while using gradient descent, it may lead to problems like vanishing gradient or exploding gradient problems. These problems occur when the gradient is too small or too large. And because of this problem the algorithms do not converge.

- **Implementation Challenges**

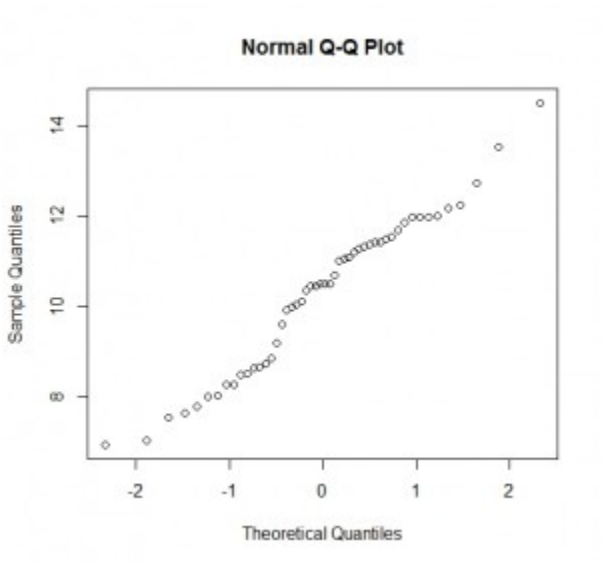
a. Most of the neural network practitioners don't generally pay attention to implementation, but it's very important to look at the resource utilization by networks. For example: When implementing gradient descent, it is very important to note how many resources you would require. If the memory is too small for your application, then the network would fail.
b. Also, it is important to keep track of things like floating point considerations and hardware / software prerequisites.

10. What is a Q-Q plot? Explain the use and importance of a Q-Q plot in linear regression.

The Q-Q plot, or quantile-quantile plot, is a graphical tool to help us assess if a set of data plausibly came from some theoretical distribution such as a Normal or exponential. For example, if we run a statistical analysis that assumes our dependent variable is Normally distributed, we can use a Normal Q-Q plot to check that assumption. It's just a visual check, not an air-tight proof, so it is somewhat subjective. But it allows us to see at-a-glance if our assumption is plausible, and if not, how the assumption is violated and what data points contribute to the violation.

A Q-Q plot is a scatterplot created by plotting two sets of quantiles against one another. If both sets of quantiles came from the same distribution, we should see the points forming a line that's roughly straight. Here's an example of a Normal Q-Q plot when both sets of quantiles truly come from Normal distributions.

This plot shows if residuals are normally distributed. Do residuals follow a straight line well or do they deviate severely? It's good if residuals are lined well on the straight dashed line.



Q-Q plots take your sample data, sort it in ascending order, and then plot them versus quantiles calculated from a theoretical distribution. The number of quantiles is selected to match the size of your sample data. While Normal Q-Q Plots are the ones most often used in practice due to so many statistical methods assuming normality, Q-Q Plots can actually be created for any distribution.

QQ-plots are ubiquitous in statistics. Most people use them in a single, simple way: fit a linear regression model, check if the points lie approximately on the line, and if they don't, your residuals aren't Gaussian and thus your errors aren't either. This implies that for small sample sizes, you can't assume your estimator is Gaussian either, so the standard confidence intervals and significance tests are invalid.