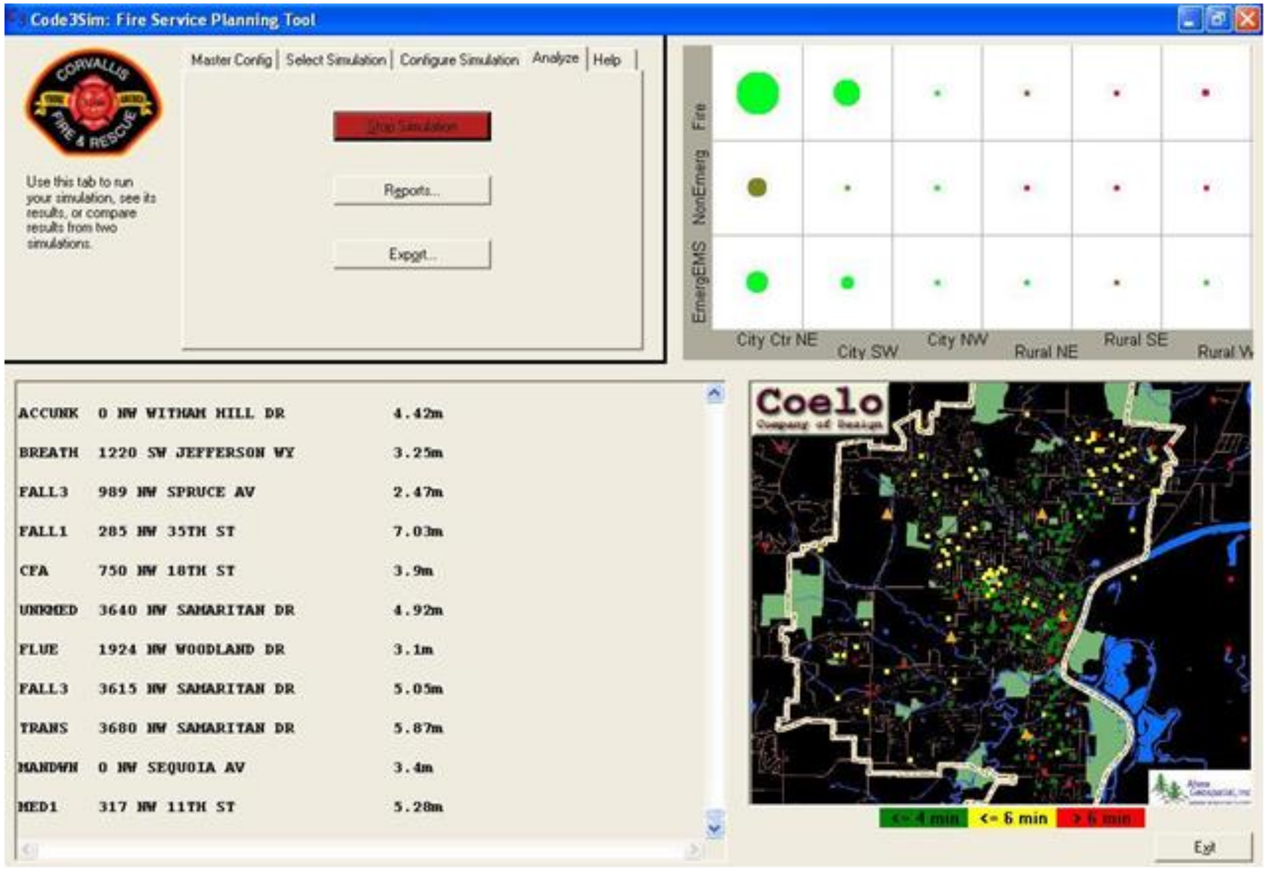
Reinforcement Learning: Introduction and Fundamental Concepts*

* Slides based on Alan Fern

Automated Planning Under Uncertainty

Optimizing Fire & Rescue Response Policies



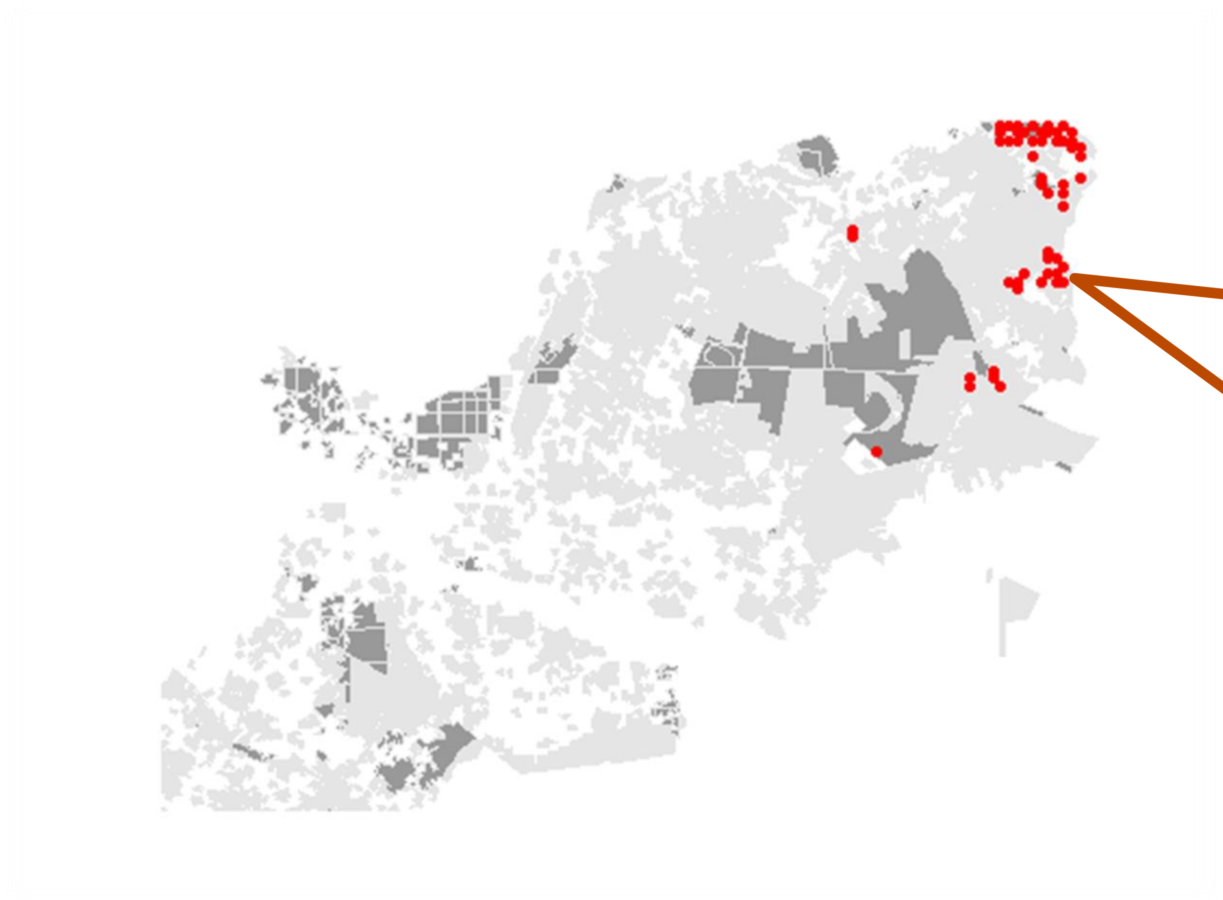
Use this tab to run your simulation, see its results, or compare results from two simulations.

```
ACCUNK  0 NW WITHAM HILL DR        4.42m
BREATH  1220 SW JEFFERSON WY       3.25m
FALL3   989 NW SPRUCE AV           2.47m
FALL1   285 NW 35TH ST             7.03m
CFA     750 NW 18TH ST             3.9m
UNKMED  3640 NW SAMARITAN DR       4.92m
FLUE    1924 NW WOODLAND DR        3.1m
FALL3   3615 NW SAMARITAN DR       5.05m
TRANS   3680 NW SAMARITAN DR       5.87m
MANDWN  0 NW SEQUOIA AV            3.4m
MED1    317 NW 11TH ST             5.28m
```

Automated Planning Under Uncertainty

Conservation Planning:

Recovery of Red-cockaded Woodpecker



From http://www.fws.gov/
rcwrecovery/rcw.html

Automated Planning Under Uncertainty

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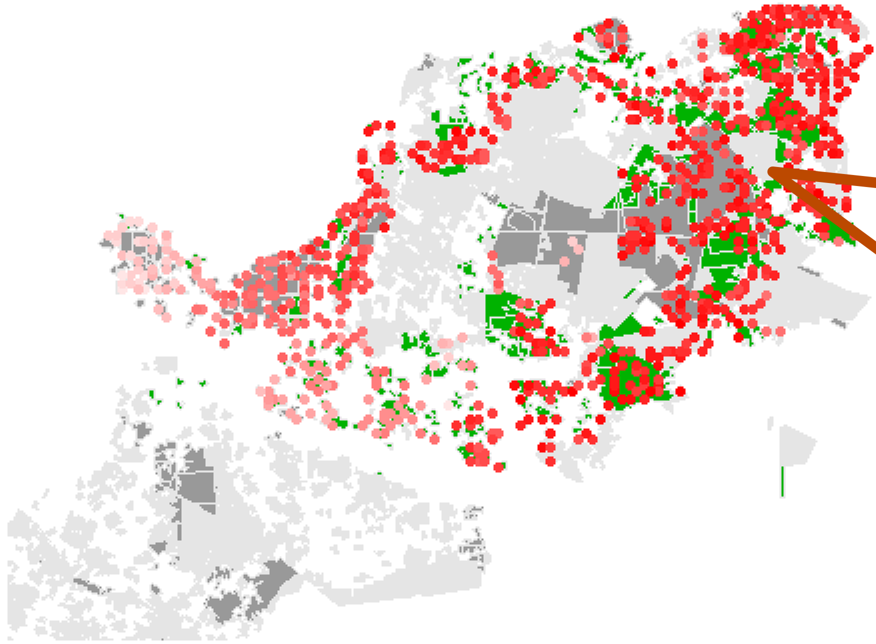
Recovery of Red-cockaded Woodpecker



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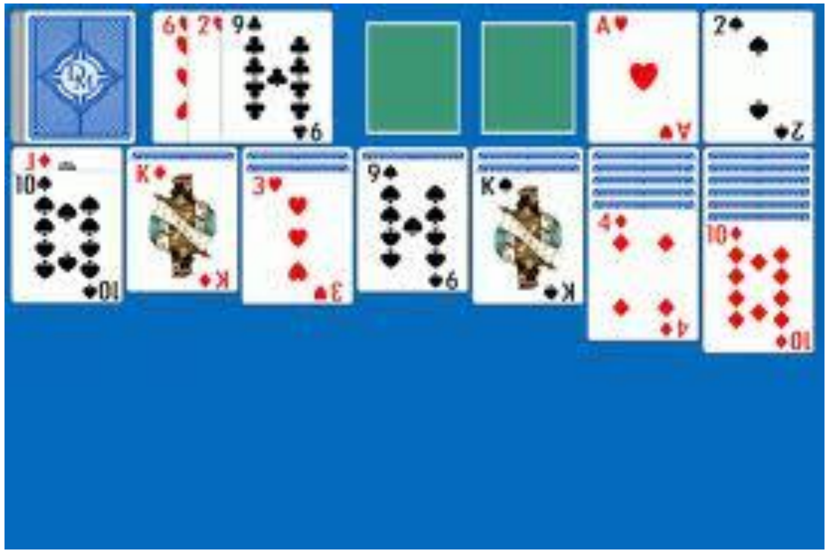
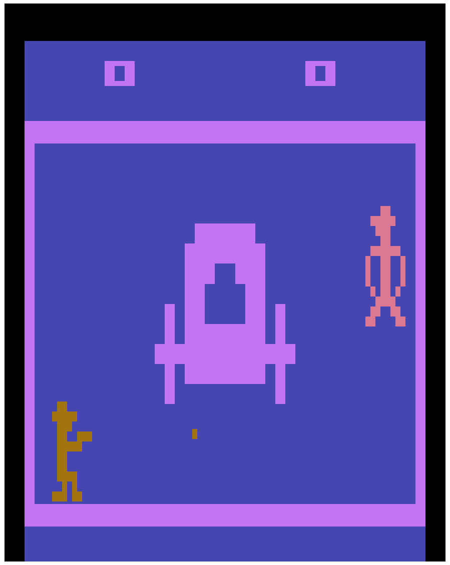
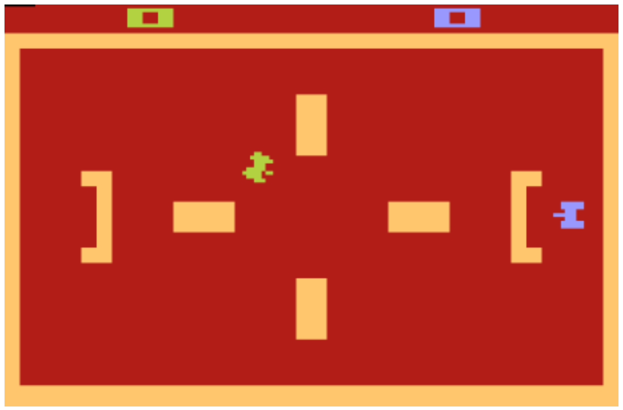
Automated Planning Under Uncertainty


Klondike Solitaire

Real-Time Strategy Games

AI for General Atari 2600 Games

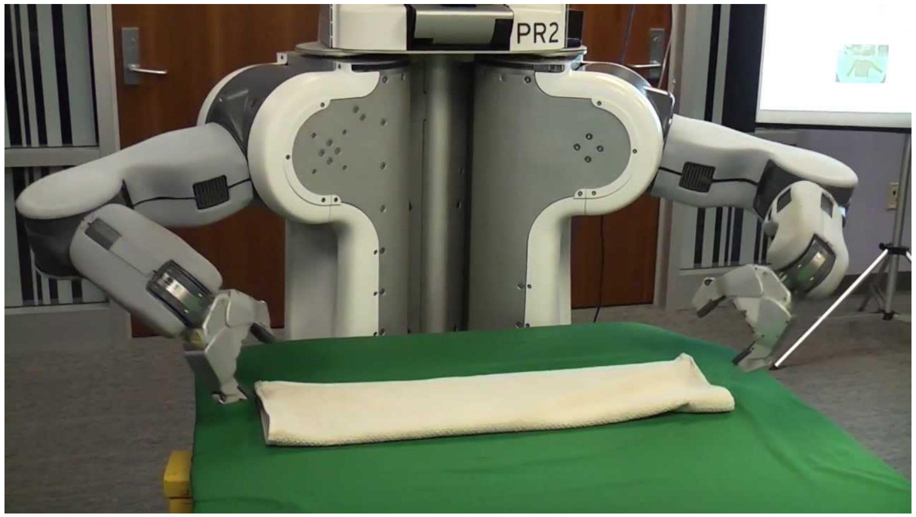




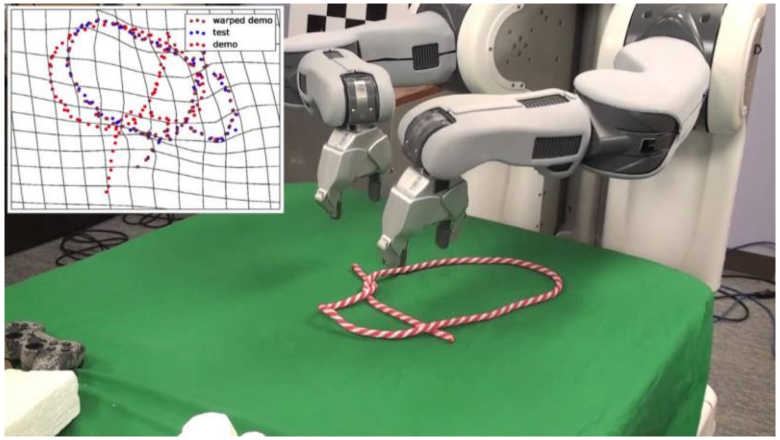
Robotics Control

Helicopter Control

Legged Robot Control



Laundry



Knot Tying

Smart Grids

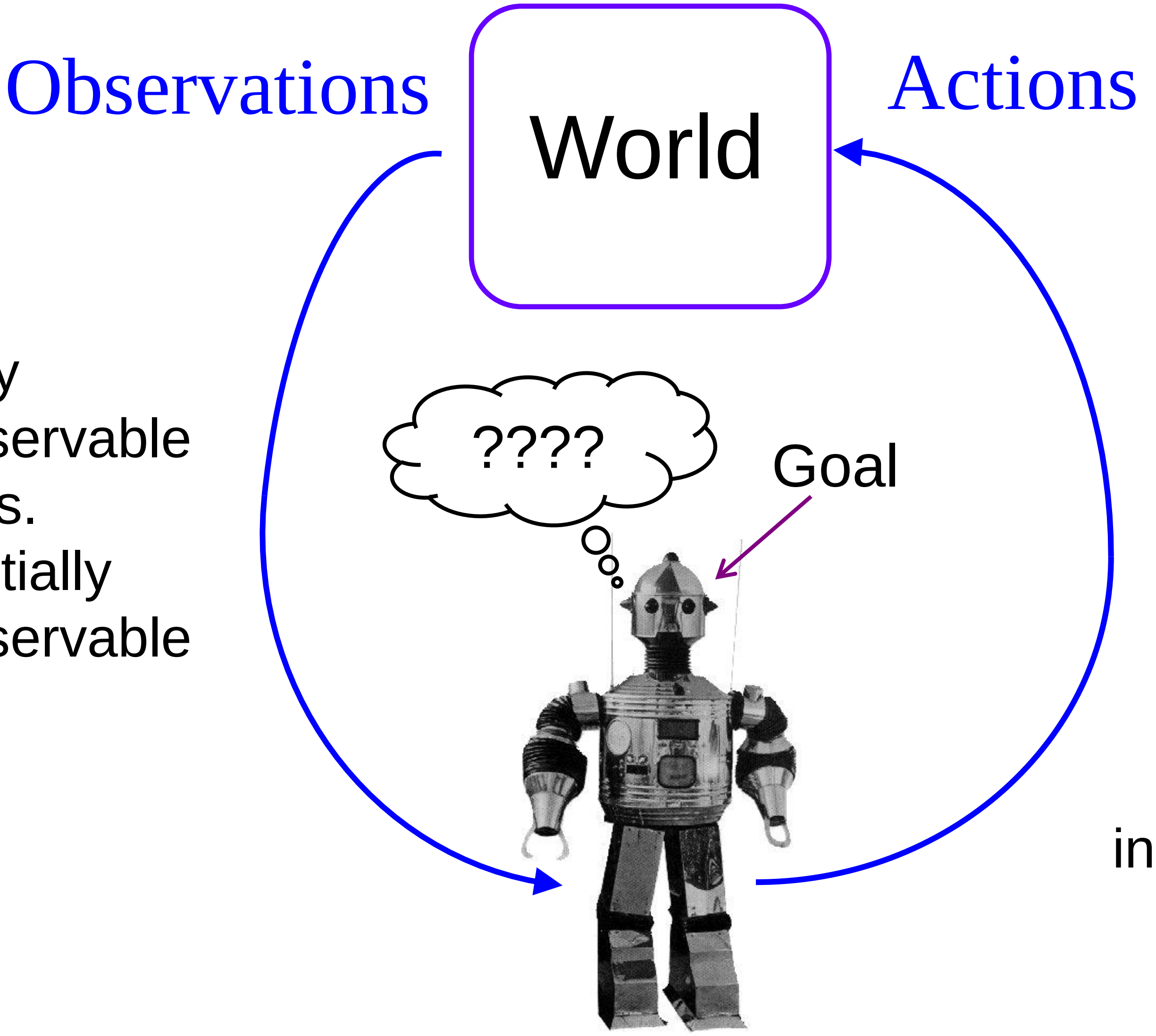
Some AI Planning Problems

- Health Care
  - Personalized treatment planning
  - Hospital Logistics/Scheduling
- Transportation
  - Autonomous Vehicles
  - Supply Chain Logistics
  - Air traffic control
- Assistive Technologies
  - Dialog Management
  - Automated assistants for elderly/disabled
  - Household robots
  - Personal planner

Common Elements

- We have a controllable system that can change state over time (in some predictable way)
  - The state describes essential information about system (the visible card information in Solitaire)
- We have an objective that specifies which states, or state sequences, are more/less preferred
- Can (partially) control the system state transitions by taking actions
- **Problem:** At each moment must select an action to optimize the overall objective
  - Produce most preferred state sequences

Some Dimensions of AI Planning



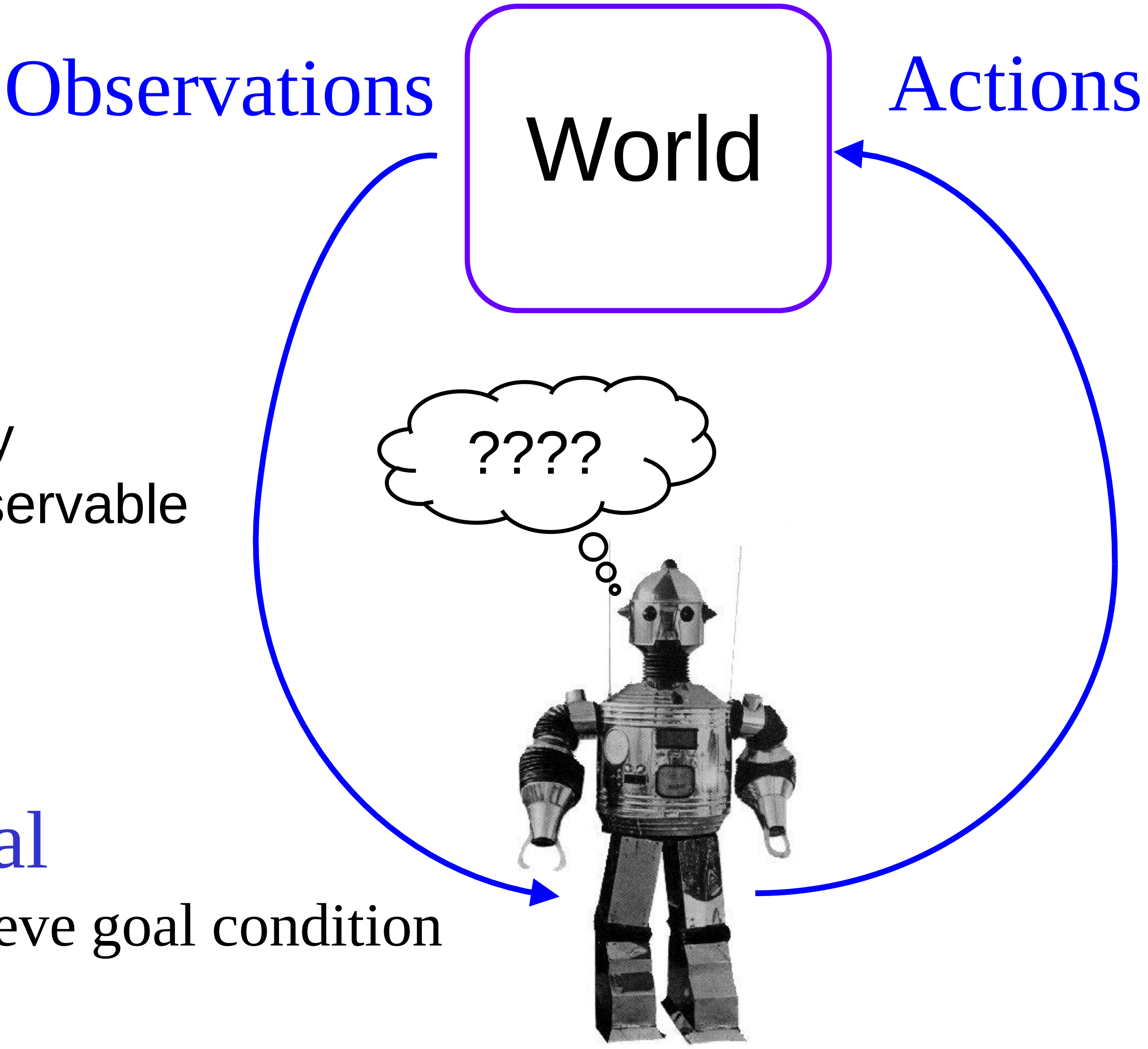
fully observable
vs.
partially observable

sole source of change
vs.
other sources

deterministic
vs.
stochastic

instantaneous
vs.
durative

Classical Planning Assumptions
(primary focus of AI planning until early 90's)



fully
observable

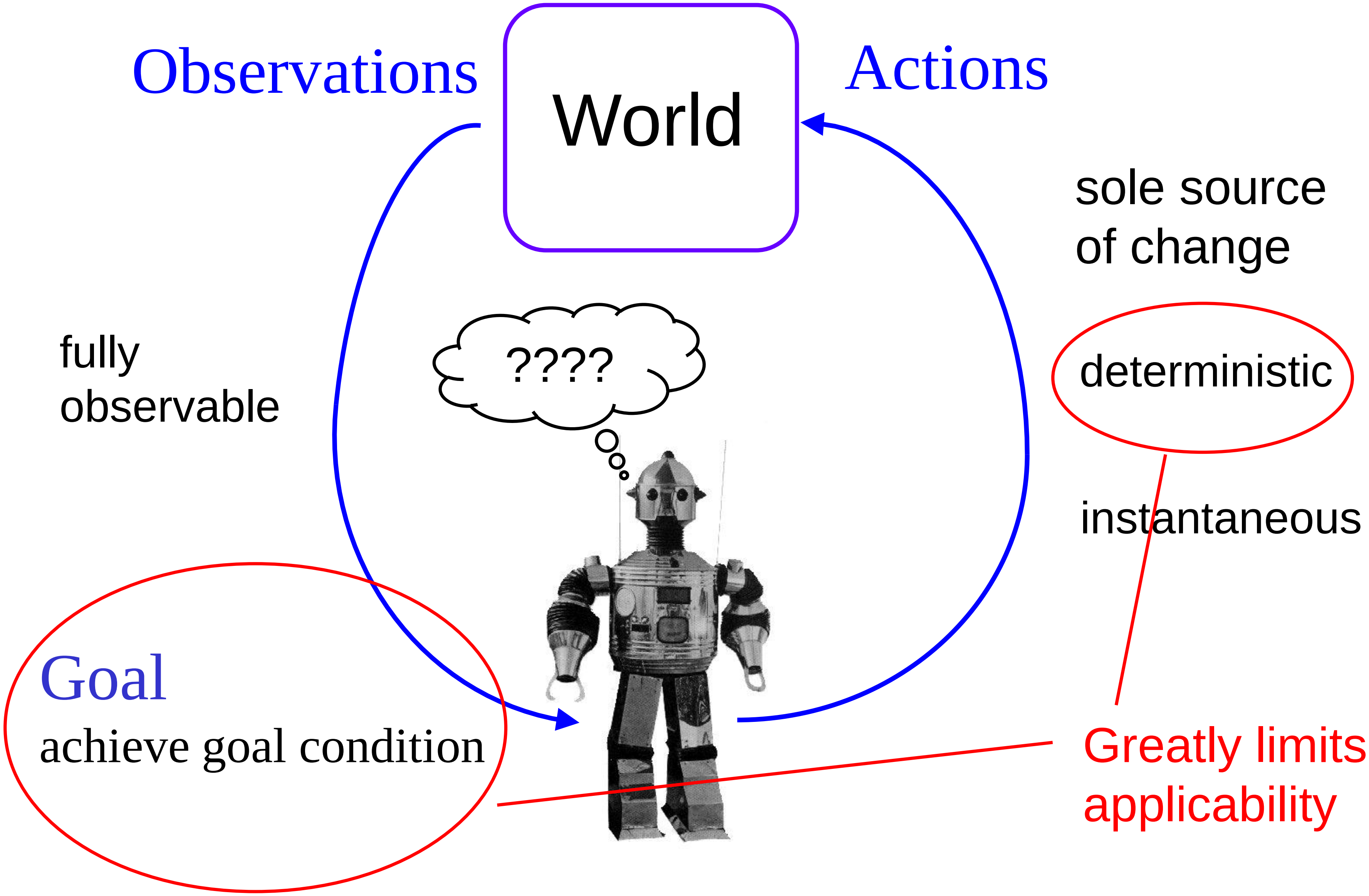
sole source
of change

deterministic

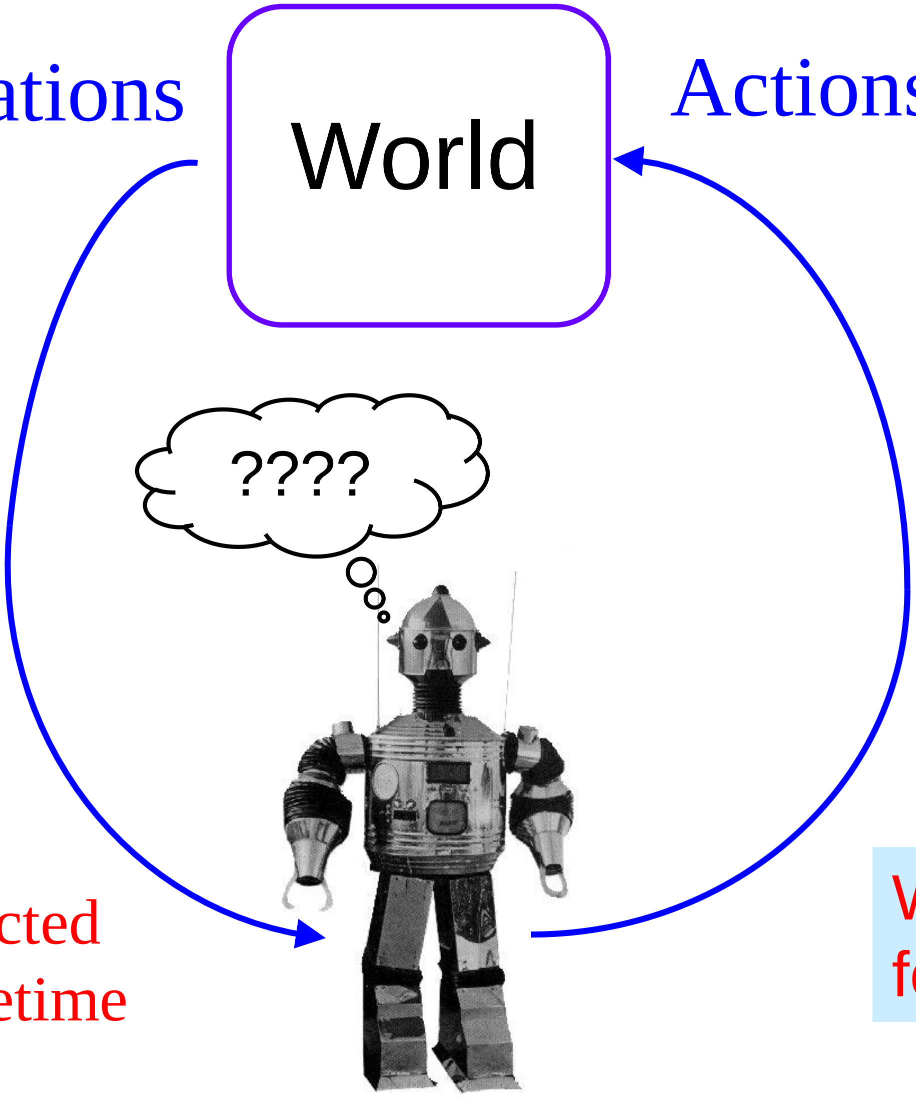
instantaneous

Goal
achieve goal condition

Classical Planning Assumptions
(primary focus of AI planning until early 90's)



Stochastic/Probabilistic Planning: Markov Decision Process (MDP) Model



Observations

Actions

fully observable

sole source of change

stochastic

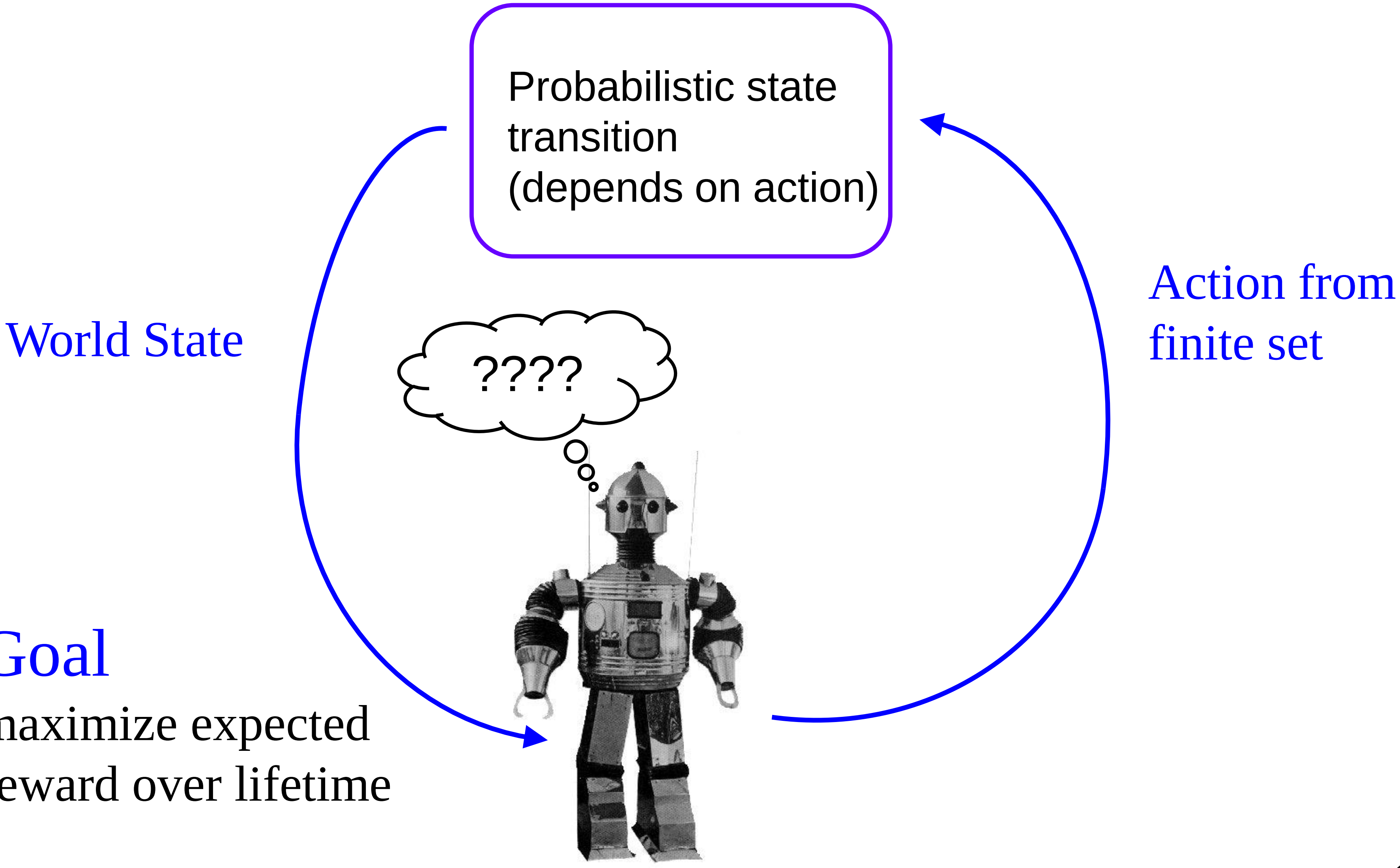
instantaneous

Goal

maximize expected reward over lifetime

We will primarily focus on MDPs

Stochastic/Probabilistic Planning: Markov Decision Process (MDP) Model

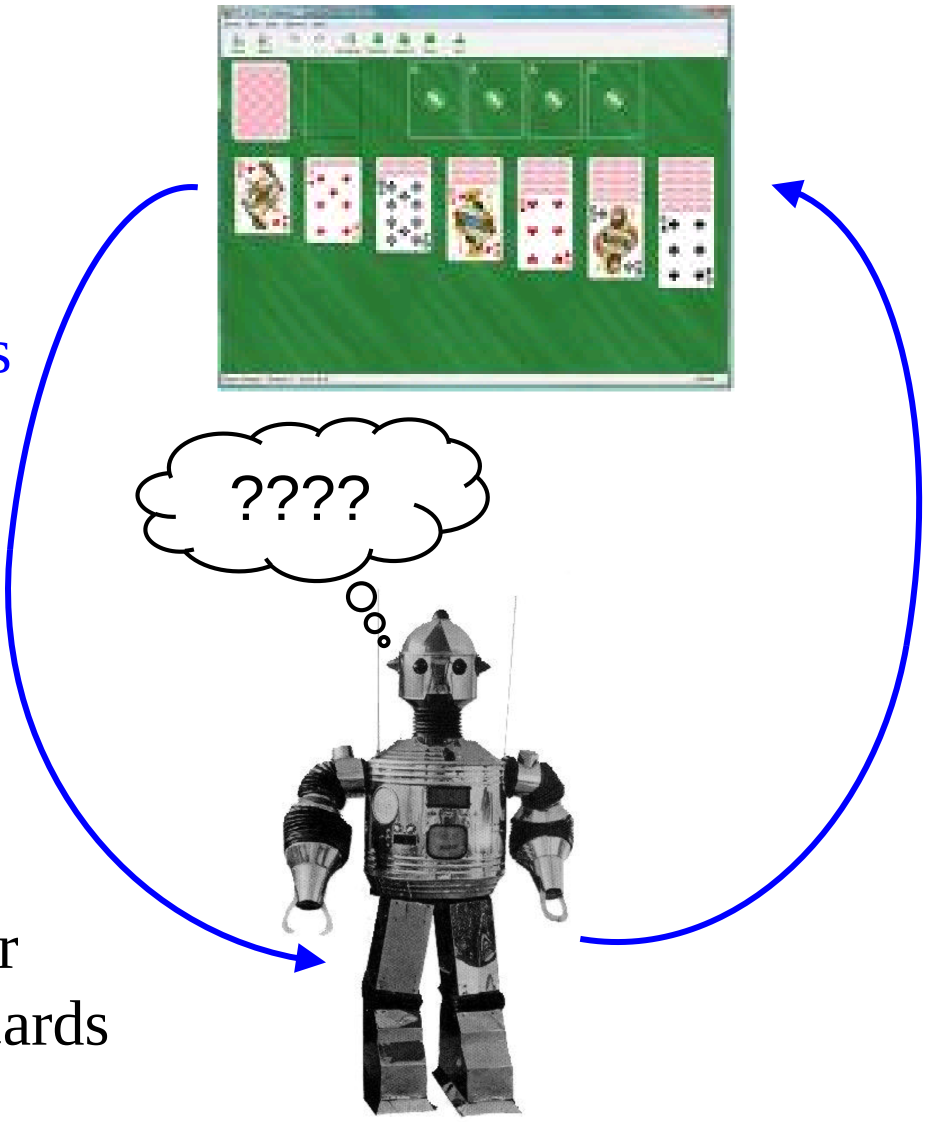


Goal

maximize expected reward over lifetime

Example MDP

State describes all visible info about cards



Action are the different legal card movements

Goal

win the game or play max # of cards

Markov Decision Processes

- An MDP has four components: S, A, R, T:
  - finite **state set** S   (|S| = n)
  - finite **action set** A   (|A| = m)

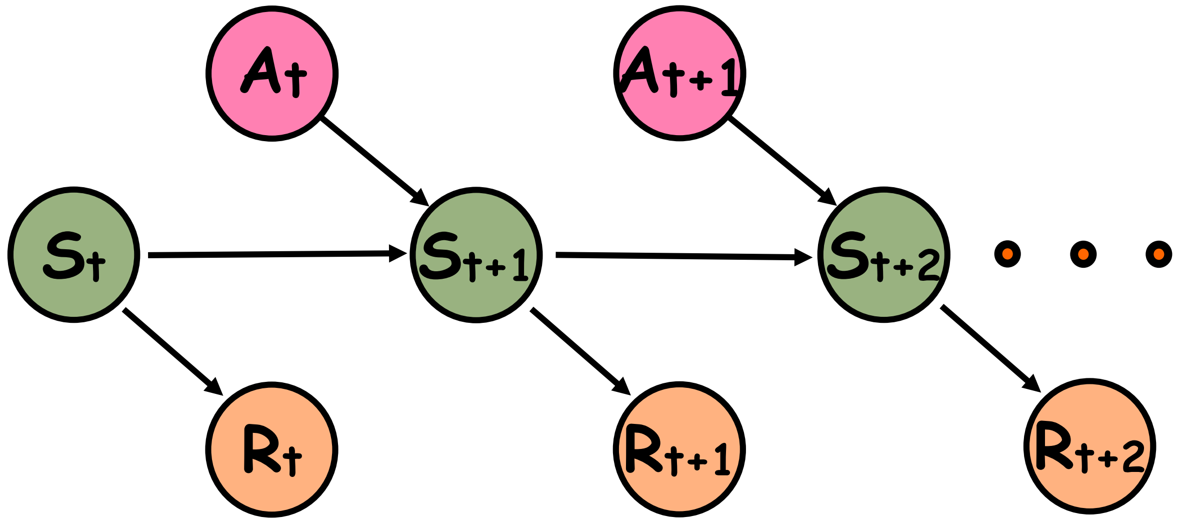
  - **transition function** T(s,a,s') = Pr(s' | s,a)
    - Probability of going to state **s'** after taking action **a** in state **s**
    - How many parameters does it take to represent?

$$m \cdot n \cdot (n-1) = O(mn^2)$$

  - bounded, real-valued **reward function** R(s)
    - Immediate reward we get for being in state s
    - Roughly speaking the objective is to select actions in order to maximize total reward
    - For example in a goal-based domain R(s) may equal 1 for reaching goal states and 0 otherwise (or -1 reward for non-goal states)

Graphical View of MDP

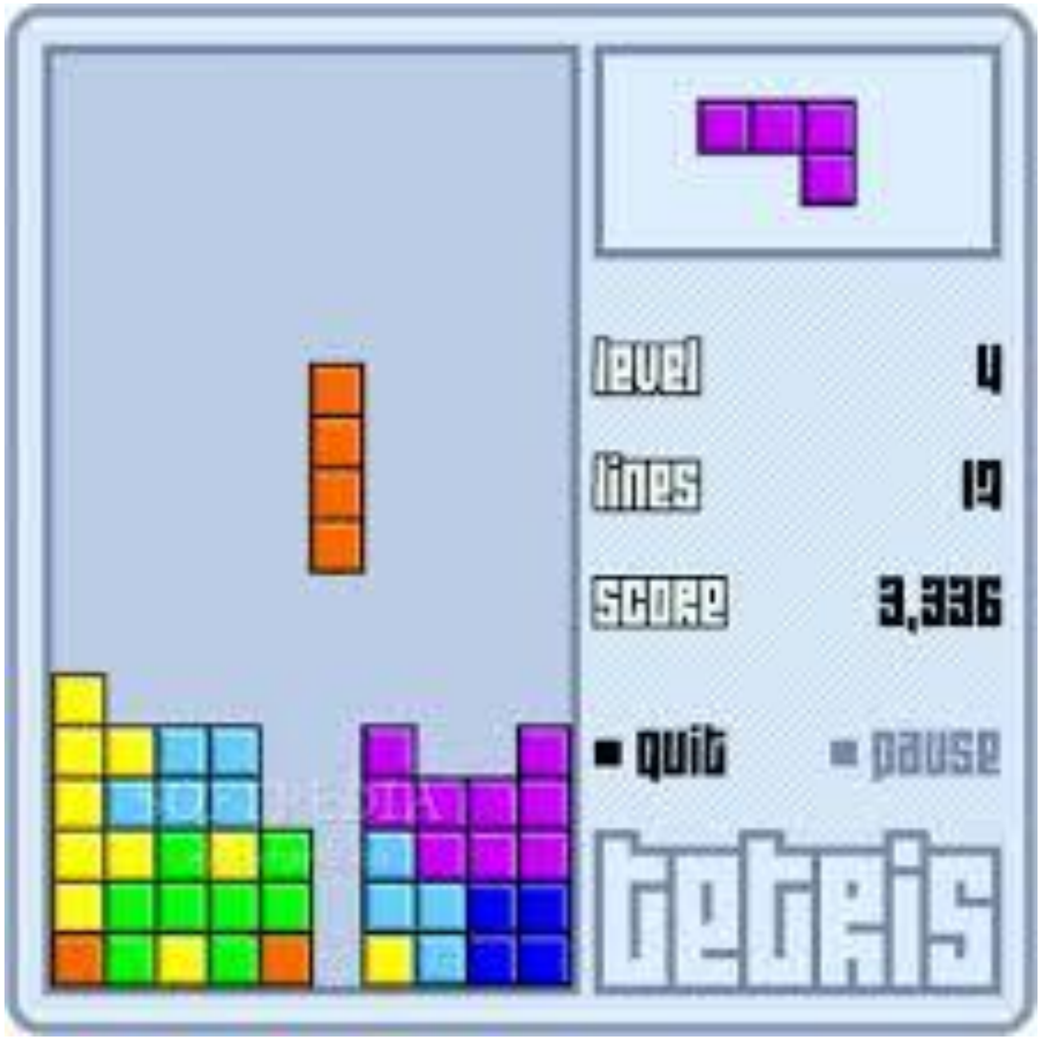
Dynamic Bayesian Network



Assumptions

- **First-Order Markovian dynamics** (history independence)
  - $Pr(S^{t+1}|A^t,S^t,A^{t-1},S^{t-1},..., S^0) = Pr(S^{t+1}|A^t,S^t)$
  - Next state only depends on current state and current action
- **State-Dependent Reward**
  - $R^t = R(S^t)$
  - Reward is a deterministic function of current state and action
- **Stationary dynamics**
  - $Pr(S^{t+1}|A^t,S^t) = Pr(S^{k+1}|A^k,S^k)$ for all t, k
  - The world dynamics and reward function do not depend on absolute time
- **Full observability**
  - Though we can’t predict exactly which state we will reach when we execute an action, after the action is executed, we know the new state

Define an MDP that represents the game of Tetris.



A = ?

S = ?

T(s,a,s') = ?

R(s) = ?

What is a solution to an MDP?

MDP Planning Problem:

Input: an MDP (S,A,R,T)
Output: ????

What is a solution to an MDP?

MDP Planning Problem:

Input: an MDP (S,A,R,T)
Output: ????

- Should the solution to an MDP from an initial state be just a sequence of actions such as (a1,a2,a3, ….) ?
  - Consider a single player card game like Blackjack/Solitaire.
- No! In general an action sequence is not sufficient
  - Actions have stochastic effects, so the state we end up in is uncertain
  - This means that we might end up in states where the remainder of the action sequence doesn’t apply or is a bad choice
  - A solution should tell us what the best action is for any possible situation/state that might arise

Policies ("plans" for MDPs)

- A solution to an MDP is a policy
  - Two types of policies: nonstationary and stationary

- Nonstationary policies are used when we are given a finite planning horizon H
  - I.e. we are told how many actions we will be allowed to take

- Nonstationary policies are functions from states and times to actions
  - π :S x T $\rightarrow$ A, where T is the non-negative integers
  - π (s,t) tells us what action to take at state s when there are t stages-to-go (note that we are using the convention that t represents stages/decisions to go, rather than the time step)

Policies ("plans" for MDPs)

- What if we want to continue taking actions indefinately?
  - Use stationary policies

- A Stationary policy is a mapping from states to actions
  - π :S $\rightarrow$ A
  - π (s) is action to do at state s (regardless of time)
  - specifies a continuously reactive controller

- Note that both nonstationary and stationary policies assume or have these properties:
  - full observability of the state
  - history-independence
  - deterministic action choice

What is a solution to an MDP?

MDP Planning Problem:

Input: an MDP (S,A,R,T)
Output: a policy such that ????

- We don’t want to output just any policy
- We want to output a “good” policy
- One that accumulates a lot of reward

Value of a Policy

- How good is a policy π?
  - How do we measure reward "accumulated" by π?
- **Value function** V: $S \rightarrow \mathbb{R}$ associates value with each state (or each state and time for non-stationary π)
- $V_\pi(s)$ denotes value of policy π at state s
  - Depends on immediate reward, but also what you achieve subsequently by following π
  - An **optimal policy** is one that is no worse than any other policy at any state
- The goal of MDP planning is to compute an optimal

What is a solution to an MDP?

MDP Planning Problem:

Input: an MDP (S,A,R,T)
Output: a policy that achieves an “optimal value”

- This depends on how we define the value of a policy
- There are several choices and the solution algorithms depend on the choice
- We will consider two common choices
  - Finite-Horizon Value
  - Infinite Horizon Discounted Value

Finite-Horizon Value Functions

- We first consider maximizing expected total reward over a finite horizon
- Assumes the agent has H time steps to live

Finite-Horizon Value Functions

- We first consider maximizing expected total reward over a finite horizon
- Assumes the agent has H time steps to live
- To act optimally, should the agent use a stationary or non-stationary policy?
  - I.e. Should the action it takes depend on absolute time?
- Put another way:
  - If you had only one week to live would you act the same way as if you had fifty years to live?

Finite Horizon Problems

- Value (utility) depends on stage-to-go
  - hence use a nonstationary policy

- $V_{\pi}^{k}(s)$ is k-stage-to-go value function for π
  - expected total reward for executing π starting in s for k time steps

$$V_{\pi}^{k}(s) = E\,[\sum_{t=0}^{k} R^{t} \mid \pi, s\,]$$

$$= E\,[\sum_{t=0}^{k} R(s^{t}) \mid a^{t} = \pi(s^{t}, k-t), s^{0} = s\,]$$

- Here R^t and s^t are random variables denoting the reward received and state at time step t when starting in s

Computational Problems

- There are two problems that we will be interested in solving

- **Policy evaluation**:
  - Given an MDP and a nonstationary policy π
  - Compute finite-horizon value function $V_\pi^k(s)$ for any k

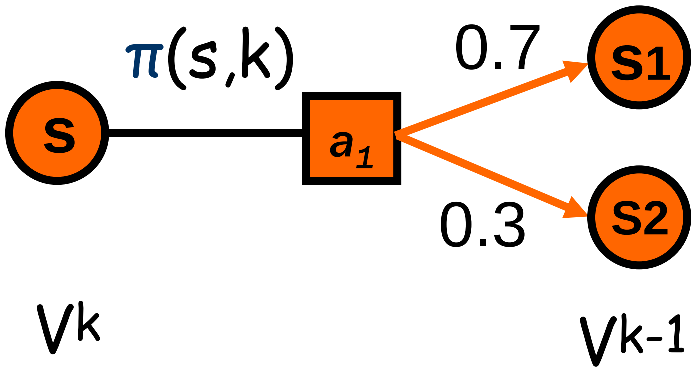
- **Policy optimization**:
  - Given an MDP and a horizon H
  - Compute the optimal finite–horizon policy
  - We will see this is equivalent to computing optimal value function

Finite-Horizon Policy Evaluation

- Can use dynamic programming to compute $V_\pi^k(s)$
  - Markov property is critical for this

(k=0) $$V_\pi^0(s) = R(s), \quad \forall s$$

(k>0) $$V_\pi^k(s) = \underbrace{R(s)}_{\text{immediate reward}} + \underbrace{\sum_{s'} T(s,\pi(s,k),s') \cdot V_\pi^{k-1}(s')}_{\text{expected future payoff with } k\text{-1 stages to go}}, \quad \forall s$$



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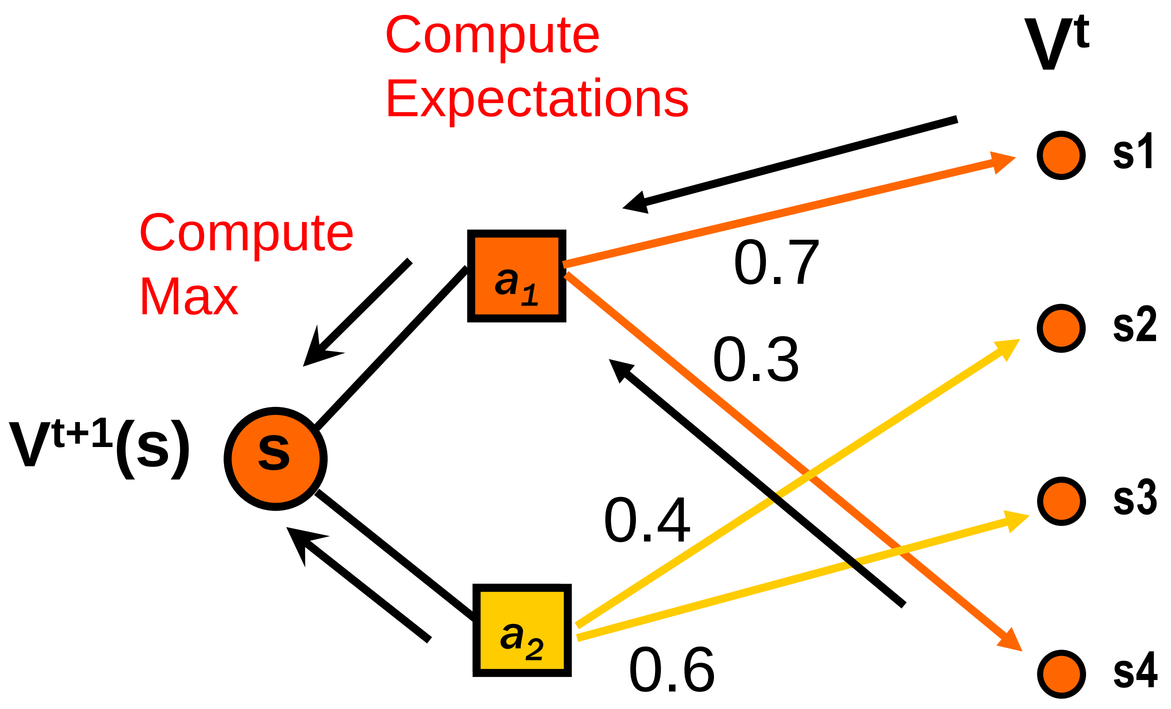
- How many finite horizon policies are there?
  - $|A|^{Hn}$
  - So can't just enumerate policies for efficient optimization

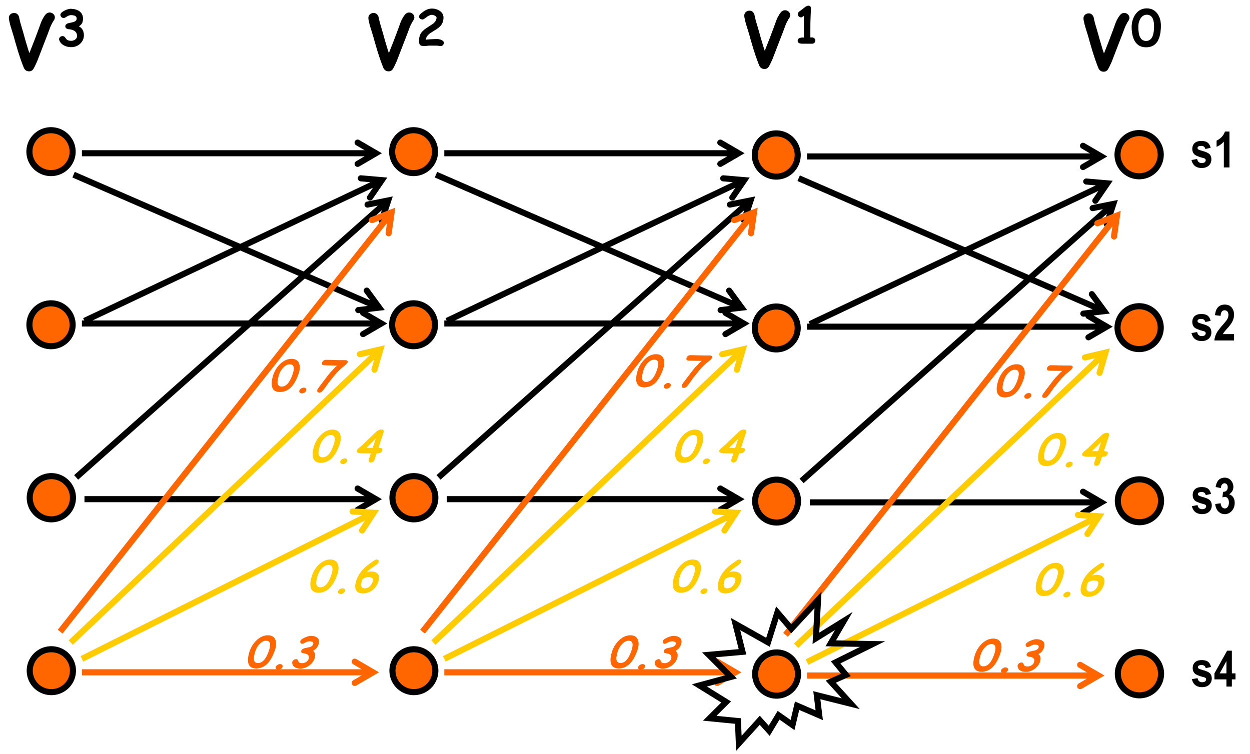
Policy Optimization: Bellman Backups

How can we compute the optimal $V^{t+1}(s)$ given optimal V^t ?



$$V^{t+1}(s) = R(s) + \max \left\{ \begin{array}{ll} 0.7\, V^t(s1) + 0.3\, V^t(s4) & 🟧 \\ 0.4\, V^t(s2) + 0.6\, V^t(s3) & 🟨 \end{array} \right\}$$

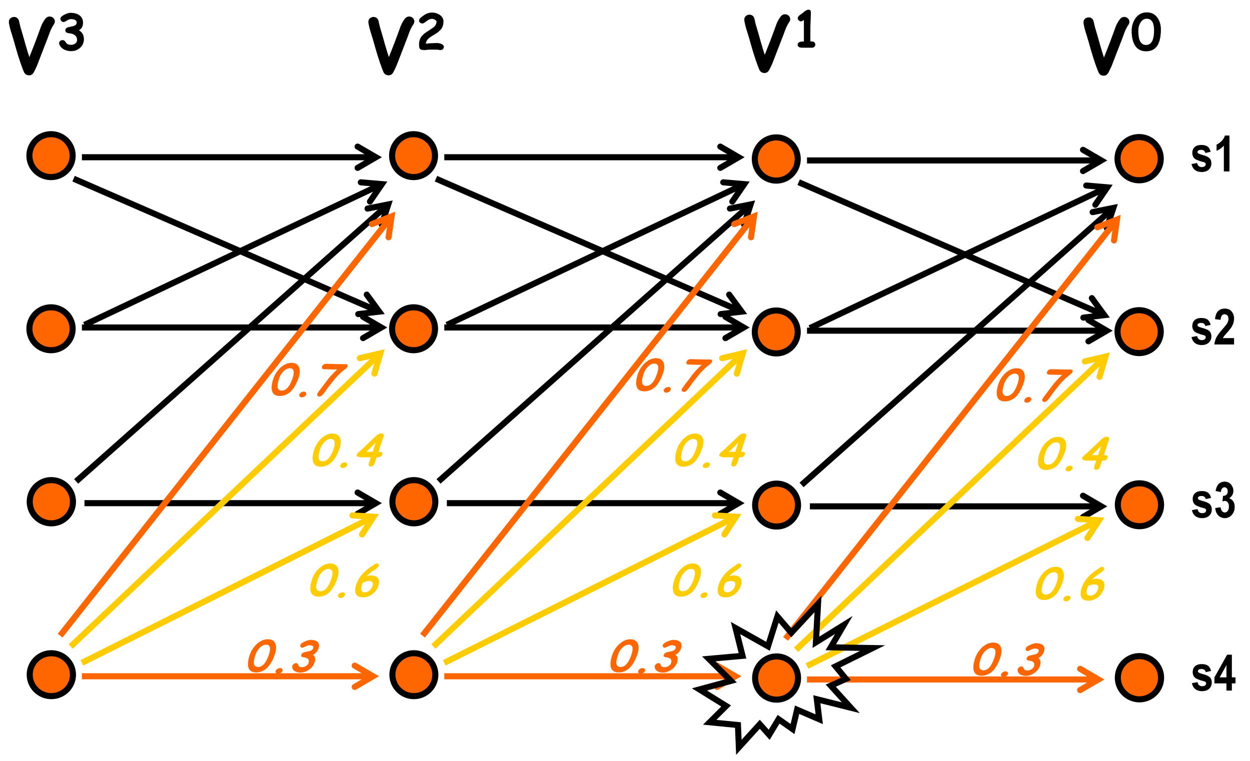
Value Iteration



$V^1(s4) = R(s4) + \max\{0.7\, V^0(s1) + 0.3\, V^0(s4)$ 🟧

$0.4\, V^0(s2) + 0.6\, V^0(s3) \}$ 🟨

Value Iteration



$\Pi^*(s4,t)$ = max {🟧 🟨}

Value Iteration: Finite Horizon Case

- Markov property allows exploitation of DP principle for optimal policy construction
  - no need to enumerate $|A|^{Hn}$ possible policies
- Value Iteration

Bellman backup

$$V^0(s) = R(s), \quad \forall s$$

$$\boxed{V^k(s) = R(s) + \max_a \sum_{s'} T(s,a,s') \cdot V^{k-1}(s')}$$

$$\pi^*(s,k) = \arg\max_a \sum_{s'} T(s,a,s') \cdot V^{k-1}(s')$$

V^k is optimal k-stage-to-go value function

Π*(s,k) is optimal k-stage-to-go policy

Value Iteration: Complexity

- Note how DP is used
  - optimal soln to k-1 stage problem can be used without modification as part of optimal soln to k-stage problem

- What is the computational complexity?
  - H iterations
  - At each iteration, each of n states, computes expectation for m actions
  - Each expectation takes O(n) time
- Total time complexity: $O(Hmn^2)$
  - Polynomial in number of states. Is this good?

Summary: Finite Horizon

- Resulting policy is optimal

$$V_{\pi^*}^k(s) \geq V_\pi^k(s), \quad \forall \pi, s, k$$

  - convince yourself of this (use induction on k)

- Note: optimal value function is unique.
- Is the optimal policy unique?
  - No. Many policies can have same value (there can be ties among actions during Bellman backups).

Discounted Infinite Horizon MDPs

- Defining value as total reward is problematic with infinite horizons (r1 + r2 + r3 + r4 + …..)
  - many or all policies have infinite expected reward
  - some MDPs are ok (e.g., zero-cost absorbing states)
- "Trick": introduce discount factor $0 \leq \beta < 1$
  - future rewards discounted by β per time step

$$V_\pi(s) = E\left[\sum_{t=0}^{\infty} \beta^t R^t \mid \pi, s\right]$$

Bounded Value

- Note: $V_\pi(s) \leq E\left[\sum_{t=0}^{\infty} \beta^t R^{\max}\right] = \frac{1}{1-\beta} R^{\max}$
- Motivation: economic? prob of death? convenience?

Recap: things you should know

- What is an MDP?
- What is a policy?
  - Stationary and non-stationary
- What is a value function?
  - Finite-horizon and infinite horizon
- How to evaluate policies?
  - Finite-horizon
  - Time/space complexity?
- How to optimize policies?
  - Finite-horizon
  - Time/space complexity?
  - Why they are correct?