3.7 Trigonometric Functions

42 $x = \sec(t^7) = 1/\cos(t^7) = \cos^{-1}(t^7)$

$x' = -1\cdot\cos^{-2}(t^7)\cdot(-\sin(t^7))\cdot 7t^6 = \dfrac{7t^6\sin(t^7)}{\cos^2(t^7)} = 7t^6\cdot\dfrac{\sin(t^7)}{\cos(t^7)}\cdot\dfrac{1}{\cos(t^7)} = 7t^6\cdot\tan(t^7)\cdot\sec(t^7)$

48 $x = \sec(\sqrt{t})\tan(\sqrt{t})$

$\dfrac{d}{dx}\tan x = \sec^2 x = 1/\cos^2 x$

$\dfrac{d}{dx}\sec x = -1\cdot\cos^{-2}(x)\cdot(-\sin x) = \dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\cos x} = \tan x\cdot\sec x$

$\dfrac{dx}{dt} = \tan(\sqrt{t})\cdot\sec(\sqrt{t})\cdot\frac{1}{2}(t)^{-\frac{1}{2}}\cdot\tan(\sqrt{t}) + \sec(\sqrt{t})\cdot\sec^2(\sqrt{t})\cdot\frac{1}{2}t^{-\frac{1}{2}}$

$= \dfrac{\tan^2\sqrt{t}\cdot\sec\sqrt{t}}{2\sqrt{t}} + \dfrac{\sec^3(\sqrt{t})}{2\sqrt{t}}$

49 $x = \csc(1/t^2)$

$\dfrac{d}{dx}\csc x = \dfrac{d}{dx}\left(\dfrac{1}{\sin x}\right) = -\sin^{-2}x\cdot\cos x = -\dfrac{\cos x}{\sin x}\cdot\dfrac{1}{\sin x} = -\cot x\cdot\csc x$

$\dfrac{dx}{dt} = -\cot(t^{-2})\csc(t^{-2})(-2)t^{-3}$
$= \dfrac{2}{t^3}\operatorname{cotg}(t^{-2})\csc(t^{-2})$

61 $y = x\cos x$ tangent at $x = \pi$

$y' = \cos x - x\sin x$
$y'(\pi) = -1 - \pi\cdot 0 = -1$

$y(\pi) = \pi\cdot(-1) = -\pi$

$y - (-\pi) = -1(x - \pi)$
$y + \pi = \pi - x \rightarrow y + x = 0$

64 $y = \frac{3}{\pi}\sin^2\left(\frac{\pi x}{3}\right)$ $x = 5$

sin with smaller amplitude, phase angle slightly less than 2π.

$y(5) = \frac{3}{\pi}\sin^2\left(\frac{5\pi}{3}\right) = \frac{3}{\pi}\cdot\left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{\pi}\cdot\frac{3}{4} = \frac{9}{4\pi}$

$y' = \frac{3}{\pi}\cdot 2\sin\left(\frac{\pi x}{3}\right)\cdot\cos\left(\frac{\pi x}{3}\right)\cdot\frac{\pi}{3}$

$= 2\sin(\pi x/3)\cos(\pi x/3)$
$= \sin\left(\frac{2\pi}{3}x\right)$

$y'(5) = \sin\left(\frac{10\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

Tangent at $(5, 9/4\pi)$

$y - 9/4\pi = -\frac{\sqrt{3}}{2}(x - 5)$

$y = -\frac{\sqrt{3}}{2}x + \frac{9}{4\pi} + \frac{\sqrt{3}}{2}\cdot 5$

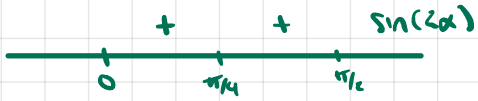
73 $R(\alpha) = \frac{1}{16} V_0^2 \sin\alpha \cos\alpha$

$R'(\alpha) = \frac{V_0^2}{16}\left[\cos^2\alpha - \sin^2\alpha\right] = 0 \Rightarrow \sin^2\alpha = \cos^2\alpha \Rightarrow \alpha = \pi/4$

$\sin\alpha = \pm\cos\alpha$

$R''(\alpha) = \frac{V_0^2}{16}\cdot\left[2\cos\alpha(-\sin\alpha) - 2\sin\alpha\cos\alpha\right]$

$= k\left[-2\cdot 2\cos\alpha\sin\alpha\right] = k\cdot(-2)\cdot\sin(2\alpha)\ ,\ k>0$



$\Rightarrow R'' < 0$ in $[0, \pi/2]$

$\Rightarrow \alpha = \pi/4$ is a maximum (global)

Alternatively,

$R(\alpha) = \frac{V_0^2}{32}\cdot 2\sin\alpha\cos\alpha = k\cdot\sin(2\alpha),\ k>0$

$R'(\alpha) = k\cos(2\alpha)\cdot 2 = 0 \Rightarrow \cos(2\alpha) = 0 \Rightarrow \alpha = \pi/4$

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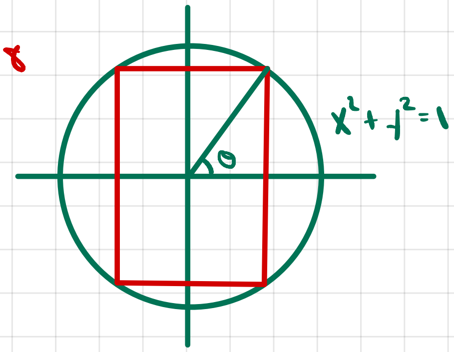

$A = 4\cdot\sin\theta\cdot\cos\theta\ ,\ \theta\in[0,\pi/2]$

Analogous to prob. 73

$\theta = \pi/4$ maximizes A.

Note also that $A(0) = A(\pi/2) = 0$, and $A(\pi/4) = 4$