9.4 Parametric Curves

- curves so far: graphs of equations $f(x), g(y), F(x,y)=0$
- another type of curve: trajectory of point moving in coordinate plane

motion of point described by $(x(t), y(t))$ ↓ parameter t (independent var.)

parametric curve pair of functions $x=f(t), y=g(t)$ giving x and y continuous functions of $t \in I$ in $\mathbb{R}$

→ **note**

- a given figure in the plane may be the graph of different curves, ie a curve may have different parameterizations.
- any $y=f(x)$ can be parameterized as: $x=t$, $y=f(t)$, t taking values in original domain of f

Tangents

$x=f(t), y=g(t)$ is called smooth if $f'(t)$ and $g'(t)$ are continuous and never simultaneously zero.

- in some neighborhood of each point the graph of a smooth parametric curve → the curve can be described in one or both of the forms $y=F(x)$, $x=G(y)$.
- suppose $f'(t)>0$ on I ⇒ f has inverse ϕ, $t=\phi(x)$ ⇒ $y=g(t)=g(\phi(x))=F(x)$
- $\dfrac{dy}{dt}=\dfrac{dy}{dx}\cdot\dfrac{dx}{dt} \Rightarrow \dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{g'(t)}{f'(t)}=\dfrac{y'}{x'}$ anywhere where $x'=f'(t)\neq 0$

given parametric equations, we can calculate dy/dx without knowing $y=F(x)$; $\frac{dy}{dx}$ as function of t

note $f'(t)=0$ means a vertical tangent in xy space

differentiate again

- $\dfrac{d^2y}{dt^2}=\dfrac{dy'}{dt}=\dfrac{dy'}{dx}\cdot\dfrac{dx}{dt} \Rightarrow \dfrac{d^2y}{dx^2}=\dfrac{dy'}{dx}=\dfrac{dy'/dt}{dx/dt}$

Polar curves as Parametric curves

$r = f(\theta)$ → curve given in polar coordinates
can be regarded as parametric w/ parameter θ

$x = r\cos\theta = f(\theta)\cos\theta$

$y = r\sin\theta = f(\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$