3.8 Problems

1 $f(x) = e^{2x}$
$f'(x) = 2e^{2x}$

2 $f = e^{3x-1}$
$f' = 3e^{3x-1}$

3 $f = e^{x^2}$
$f' = e^{x^2} \cdot 2x$

4 $f = e^{4-x^3}$
$f' = e^{4-x^3}(-3x^2)$

5 $f = e^{1/x^2}$
$f' = e^{1/x^2} \cdot (-2)x^{-3}$

6 $f = x^2 e^{x^3}$
$f' = 2xe^{x^3} + x^2 e^{x^3} \cdot 3x^2 = e^{x^3}(2x + 3x^4)$

7 $g = te^{\sqrt{t}}$
$g' = e^{\sqrt{t}} + te^{\sqrt{t}} \cdot (1/2) \cdot t^{-1/2}$

8 $g = (e^{2t} + e^{3t})^7$
$g' = 7(e^{2t} + e^{3t})^6 \cdot (2e^{2t} + 3e^{3t})$

9 $g = (t^2 - 1)e^{-t}$
$g' = 2te^{-t} + (t^2-1)e^{-t}(-1)$

10 $g = \sqrt{e^t - e^{-t}}$
$g' = (1/2)(e^t - e^{-t})^{-1/2}(e^t - e^{-t}(-1))$
$= \dfrac{e^t + e^{-t}}{2\sqrt{e^t - e^{-t}}}$

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$$f(x) = \ln\left[\frac{4-x^2}{9+x^2}\right]^{\frac{1}{2}} = \frac{1}{2}\left[\ln(4-x^2) - \ln(9+x^2)\right]$$

$$f'(x) = \frac{1}{\cancel{2}}\left[\frac{-\cancel{2}x^2}{4-x^2} - \frac{\cancel{2}x}{9+x^2}\right] = \frac{x^2}{4-x^2} - \frac{x}{9+x^2}$$

44 $f(x) = x^2 \ln\frac{1}{2x+1} = x^2\cdot(\ln 1 - \ln(2x+1)) = -x^2\cdot\ln(2x+1)$

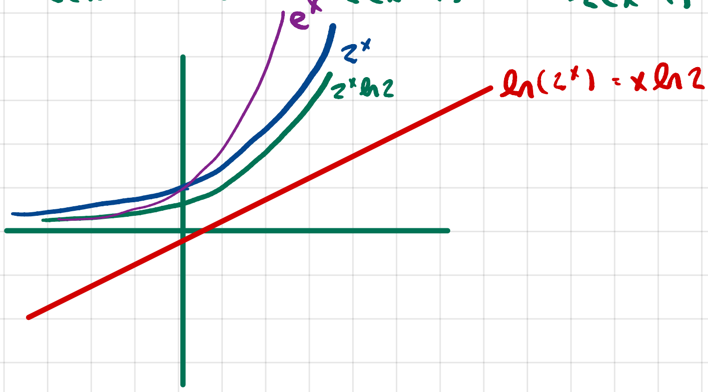
$$f'(x) = -\left[2x\ln(2x+1) + x^2\cdot\frac{2}{2x+1}\right] = -2\left(x\ln(2x+1) + \frac{x^2}{2x+1}\right)$$

46 $f(x) = \ln\frac{\sqrt{x+1}}{(x-1)^3} = \ln\sqrt{x+1} - \ln(x-1)^3$

$$f'(x) = \frac{1}{\sqrt{x+1}}\cdot\frac{1}{2}\frac{1}{\sqrt{x+1}} - \frac{1}{(x-1)^3}\cdot 3(x-1)^2$$

$$= \frac{1}{2(x+1)} - \frac{3}{x-1} = \frac{x-1-3\cdot 2(x+1)}{2(x+1)(x-1)} = \frac{x-1-6x-6}{2(x^2-1)} = \frac{-5x-7}{2(x^2-1)}$$

47 $y = 2^x$



$\ln y = \ln 2^x = x\ln 2$

$\frac{1}{y}y' = \ln 2$

$y' = y\ln 2 = 2^x\cdot\ln 2$

48 $y = x^x$

$\ln y = x\ln x$

$\frac{1}{x^x}\cdot y' = \ln x + \frac{x}{x}$

$y' = x^x(\ln x + 1)$

49 $y = x^{\ln x} \quad x > 0$

$\ln y = \ln x \ln x = \ln^2 x$

$\frac{1}{y}y' = 2\ln x\frac{1}{x} \Rightarrow y' = 2x^{\ln x - 1}\ln x$

$y' = 0 \Rightarrow x^{\ln x}\cdot\ln x = 0 \qquad x^{\ln x} = 0 \ \nexists$

$\qquad \ln x = 0 \Rightarrow x = 1 \Rightarrow y = 1$

$$y'' = 2D_x[x^{\ln x - 1}]\ln x + 2x^{\ln x - 2} = 2x^{\ln x-2}\cdot\ln^2 x + 2x^{\ln x - 2} = \underbrace{2x^{\ln x - 2}}_{>0}\underbrace{(\ln^2 x + 1)}_{>0} > 0$$

$z = x^{\ln x - 1}$

$\ln z = (\ln x - 1)\ln x$

$\frac{1}{z}z' = \frac{1}{\cancel{x}}\ln x + (\ln x - \cancel{1})\frac{1}{x} = \frac{\ln x}{x}$

$z' = x^{\ln x - 2}\ln x$

50 $y = (1+x)^{1/x}$

$\ln y = \frac{1}{x}\ln(1+x)$

$\frac{1}{y}y' = \frac{-1}{x^2}\ln(1+x) + \frac{1}{x}\,\frac{1}{1+x}$

$y' = \frac{\left[-(\ln(1+x))(1+x) + x\right]}{x^2(1+x)}(1+x)^{1/x}$

$= \frac{x - [\ln(1+x)](1+x)}{x^2}\cdot(1+x)^{\frac{1}{x}-1}$

51 $y = (\ln x)^{\sqrt{x}}$

$\ln y = \sqrt{x}\ln(\ln x)$

$\frac{1}{y}y' = \frac{1}{2\sqrt{x}}\ln(\ln x) + \sqrt{x}\,\frac{1}{\ln x}\cdot\frac{1}{x}$

$y' = (\ln x)^{\sqrt{x}}\cdot\left[\frac{\ln(\ln x)}{2\sqrt{x}} + \frac{1}{\ln x\sqrt{x}}\right]$