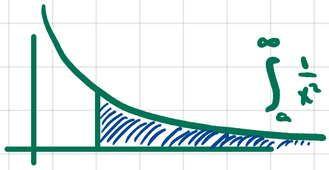
Existence of the integral theorem: f continuous on closed and bounded $[a,b] \longrightarrow f$ integrable on $[a,b]$

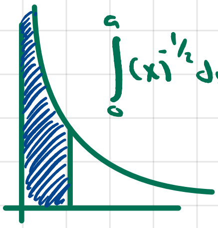
It can happen that



$\int_a^{\infty} \frac{1}{x^2}\,dx$

1 The interval has form $[a,+\infty)$, $(-\infty,c]$, $(-\infty,\infty)$

2 there is infinite discontinuity at some $c \in [a,b]$: $\lim_{x\to c} f(x) = \pm\infty$



$\int_0^a (x)^{-1/2}\,dx$

In which case we have improper integrals

Terms used

Area under curve f

f contin., nonneg on unbounded $[a,+\infty)$ $\Rightarrow$ $A(t) = \int_a^t f(x)\,dx$
$t > a$

we define $\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$ provided the limit exists (as finite number). If either exists or is infinite.

If it exists, $\int_a^{\infty} f(x)\,dx$ converges, if not it diverges.

f cont, positive or neg. on $[a,+\infty)$ $\Rightarrow$ $\int_a^{\infty} f(x)\,dx$ can diverge by oscillation, ie w/o diverging to infinity, e.g. $\int_a^{\infty} \sin x\,dx$.

other cases

$$\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty} \int_t^a f(x)\,dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx$$

$$\int_a^b f(x)\,dx = \lim_{t\to b^-} \int_a^t f(x)\,dx$$ provided existence of latter

infinite integrand, e.g. $f(x)\to\infty$ as $x\to b^-$

$$\int_a^b f(x)\,dx = \lim_{t\to a^-} \int_t^b f(x)\,dx$$

f cont on $[a,b]$