41 $y = \frac{x-9}{x^2-3x} \quad x \in [1,2]$

$$\frac{x-9}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$$

$x-9 = A(x-3) + Bx$

$x=3 \Rightarrow B = \frac{-6}{3} = -2$

$x=0 \Rightarrow A = \frac{-9}{-3} = 3$

$$A = \int_1^2 \left[3x^{-1} - 2(x-3)^{-1}\right]dx = \left[3\ln|x| - 2\ln|x-3|\right]_1^2 = (3\ln 2 - 2\ln 1) - (3\ln 1 - 2\ln 2)$$

$= 3\ln 2 + 2\ln 2 = 5\ln 2$

42 $y = \frac{x^2+10x+16}{x^3+8x^2+16x} = \frac{x^2+10x+16}{x(x^2+8x+16)} = \frac{A}{x} + \frac{Bx+C}{x^2+8x+16} \qquad x \in [2,5]$

$\Delta = 64 - 4\cdot 16 = 0$

$x^2+10x+16 = A(x^2+8x+16) + (Bx+C)x = x^2(A+B) + x(8A+C) + 16A$

$A+B=1 \quad \Rightarrow B = 1-A = 0$

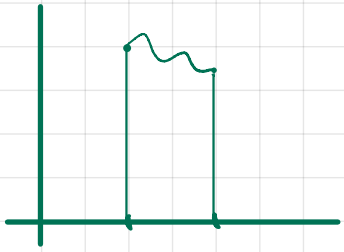
$8A+C=10 \quad \Rightarrow C = 10-8A = 10-8 = 2$

$16A = 16 \Rightarrow A = 1$

$$\int_2^5 \left[x^{-1} + \frac{2}{(x+4)^2}\right]dx = \left[\ln|x| - (x+4)^{-1}\cdot 2\right]_2^5$$

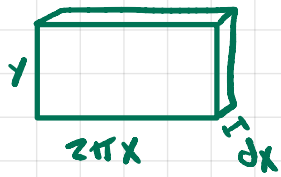
$$= \left(\ln 5 - \frac{2}{9}\right) - \left(\ln 2 - \frac{2}{6}\right) = \ln 5 - \frac{2}{9} - \ln 2 + \frac{1}{3} = \ln\frac{5}{2} + \frac{1}{9}$$

45 $y = \frac{x-9}{x^2-3x}$ $x \in [1,2]$



$f(1) = \frac{-8}{-2} = 4$ $f(2) = \frac{-7}{-2} = 3.5$

method of shells



$$\int_1^2 2\pi \cdot x \frac{x-9}{x^2-3x} = 2\pi \int_1^2 \frac{x-9}{x-3}\,dx = 2\pi \int_1^2 \left(1 - \frac{6}{x-3}\right)dx = 2\pi \left[x - 6\ln|x-3|\right]_1^2$$

$$= 2\pi \left[(2 - 6\ln 1) - (1 - 6\ln 2)\right]$$

$$= 2\pi \left[2 - 1 + 6\ln 2\right]$$

$$= 2\pi (1 + 6\ln 2)$$

$x-3 \overline{)\, x-9}$ (1) $\frac{x-3}{-6}$ $\frac{x-9}{x-3} = 1 - \frac{6}{x-3}$

48 $y = \frac{x^2+10x+16}{x^3+8x^2+16x}$ $x \in [2,5]$

$$\int_2^5 2\pi x \frac{x^2+10x+16}{x^3+8x^2+16x}\,dx = 2\pi \int_2^5 \frac{x^2+10x+16}{x^2+8x+16} = 2\pi \int_2^5 \frac{x^2+8x+16+2x}{x^2+8x+16}\,dx$$

$$= 2\pi \int_2^5 \left(1 + \frac{2x}{x^2+8x+16}\right)dx$$

$\int \frac{x}{(x+4)^2}\,dx = \int \frac{u-4}{u^2}\,du = \int (u^{-1} - 4u^{-2})\,du = \ln|u| - 4(-1)u^{-1}$

$u = x+4$ $du = dx$

$= \ln|x+4| + 4(x+4)^{-1}$

$$= 2\pi \left[x + 2\ln|x+4| + 8(x+4)^{-1}\right]_2^5$$

$$= 2\pi \left[\left(5 + 2\ln 9 + \frac{8}{9}\right) - \left(2 + 2\ln 6 + \frac{8}{6}\right)\right]$$

$$= 2\pi \left[\frac{53}{9} + 2(\ln 9 - \ln 6) - \left(\frac{10}{3}\right)\right] = 2\pi \left[\frac{23}{9} + 2\ln\frac{3}{2}\right]$$