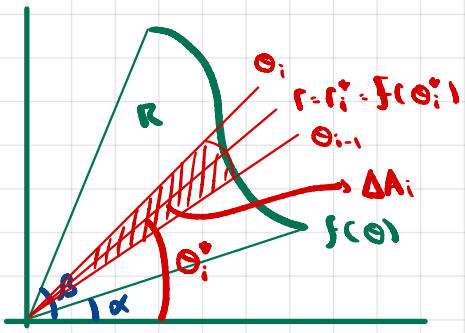
9.3 Area Computations in Polar Coordinates



We want the area of region R

→ bounded by two radial lines $\theta = \alpha, \theta = \beta$, and by curve $r = f(\theta)$ $\theta \in [\alpha, \beta]$

→ we use an approximation.

Partition: $\alpha = \theta_0 < \theta_1 < \cdots < \theta_n = \beta$ of $[\alpha, \beta]$ into n subintervals of length $\Delta\theta = \frac{\beta - \alpha}{n}$

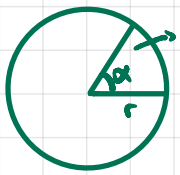
Select θ_i^* in i^{th} subinterval $[\theta_{i-1}, \theta_i]$ $i = 1, \ldots, n$

$\Delta A_i \approx$ area of circular sector with radius $r_i^* = f(\theta_i^*)$ bounded by $\theta = \theta_{i-1}, \theta = \theta_i, f(\theta)$.

$$\approx \frac{1}{2} r_i^{*2} \Delta\theta = \frac{1}{2} f(\theta_i^*)^2 \Delta\theta$$

$$A = \sum_{i=1}^{n} \Delta A_i \approx \sum_{i=1}^{n} \frac{1}{2} f(\theta_i^*)^2 \Delta\theta$$

↙ Riemann Sum for $\int_{\alpha}^{\beta} \frac{1}{2} f(\theta)^2 d\theta$

* 

$A = \pi r^2 \cdot \frac{\alpha}{2\pi} = \frac{\alpha r^2}{2}$