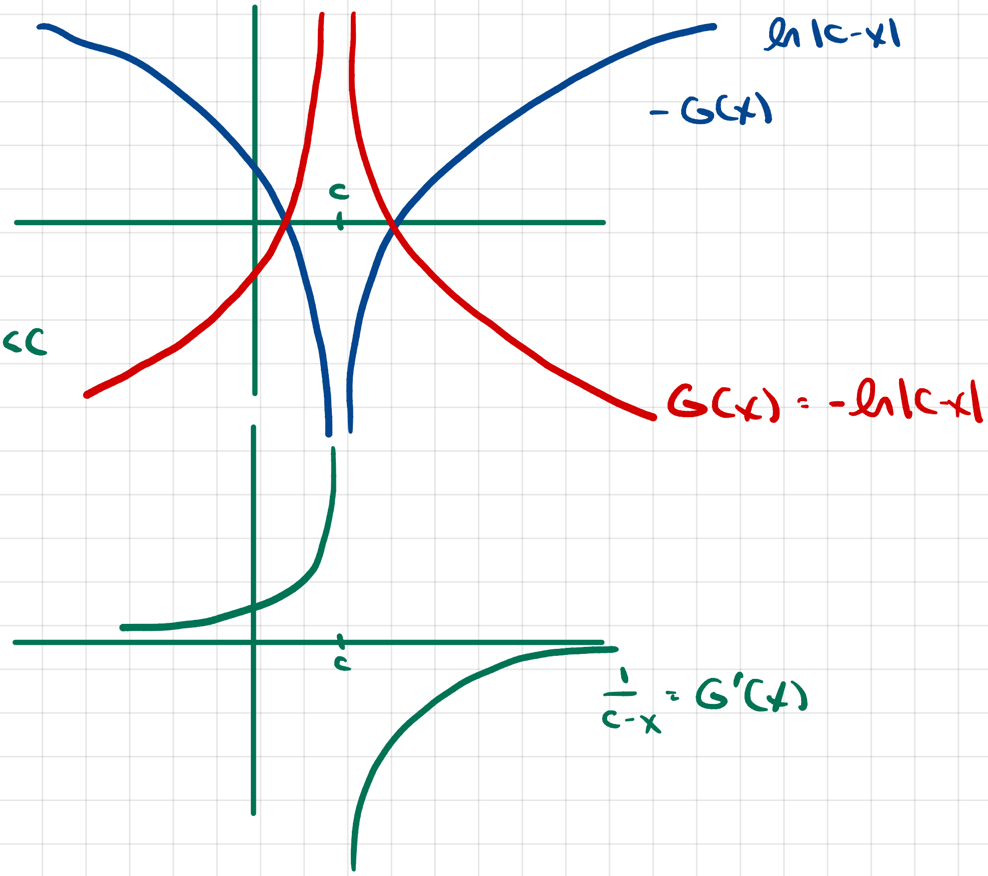
$f(x) = \dfrac{1}{c-x}$

$F(x) = \int f(x)\,dx = -\ln|c-x| + K, \; x \in \mathbb{R}$

$G(x) = -\ln|c-x|$

$= \begin{cases} -\ln(c-x) & x < c \\ -\ln(x-c) & x > c \end{cases}$

$G'(x) = \begin{cases} \dfrac{-1}{c-x}(-1) = \dfrac{1}{c-x} & x < c \\ \dfrac{-1}{x-c} = \dfrac{1}{c-x} & x > c \end{cases}$



what if we say: $F(x) = \int f(x) = -\ln(c-x) + K, \; x < c$

The domain of F is now restricted to $(-\infty, c)$, but f is defined on $\mathbb{R} - \{c\}$, so F in this case does not represent an antiderivative of f since it is not the case that $F' = f$ everywhere f is defined.