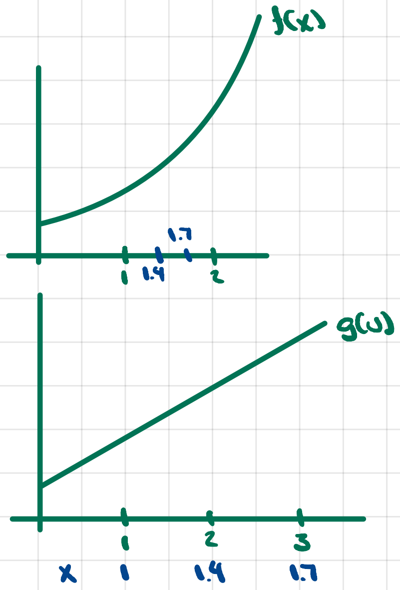
Notes on change of variable

$f(x) = x^2 + 2$

$u(x) = x^2$

$f(u(x)) = g(u) = u + 2$

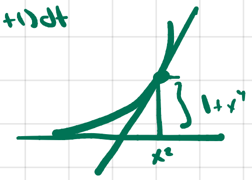
u	x
0	0
1	1
2	$\sqrt{2} \approx 1.4$
3	$\sqrt{3} \approx 1.7$
4	2

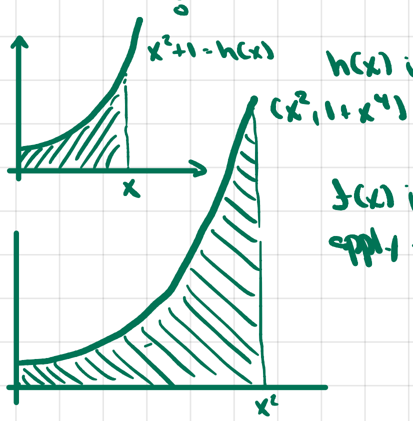


$u(x) = x^2$ is a transformation of x.

To calculate $\frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx}$, we have the constant rate of change $\frac{dg}{du}$ times how fast u changes with x.

Now, take $f(x) = \int_0^{x^2} (t^2+1)\,dt$ and $h(x) = \int_0^{x} (t^2+1)\,dt$





$h(x)$ is the area.

$f(x)$ is a different area. The rate of change is different than before; we can't apply the FTC directly.

If $u(x) = x^2$ then $f(u(x)) = g(u) = \int_0^u (t^2+1)\,dt$, so we're back to the FTC case if we want $\frac{dg}{du} = u^2+1$

$g(u)$ measures the area under u^2+1.

If we want $\frac{dg}{dx}$ we need the rate of change of g when u changes, times how u changes when x changes.

When $x \to x + \Delta x$, $u = x^2 \to (x+\Delta x)^2 = x^2 + 2\Delta x \cdot x + \Delta x^2$, which depends on the level of x.

So at any given u, the increment of u is dependent on x, in a quadratic way. We can expect $\frac{du}{dx}$ to have a fourth power, meaning as x changes, the new area is not based on x^2+1.

$g(u) = \int_0^u (1+t^2)\,dt \Rightarrow g'(u) = 1 + u^2$

$\frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} = g'(u) \cdot u'(x) = (1+u^2) \cdot 2x = (1+x^4) \cdot 2x$