Example 6

Trapezoidal Approx: $T_n = 1 \cdot \left(\frac{12}{2} + 14 + 17 + 21 + 22 + 21 + 15 + 11 + 11 + 14 + \frac{17}{2} \right)$

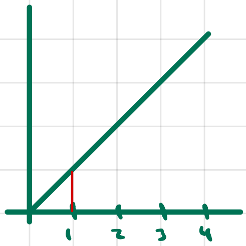
$= 160.5$

Simpson's Approx. $= \frac{1}{3}\left[12 + 4\cdot 14 + 2\cdot 17 + 4\cdot 21 + 2\cdot 22 + 4\cdot 21 + 2\cdot 15 + 4\cdot 11 + 2\cdot 11 + 4\cdot 14 + 17\right]$

$= 161$

Trapezoidal Approxim.

1 $\int_0^4 x\,dx \quad n=4$



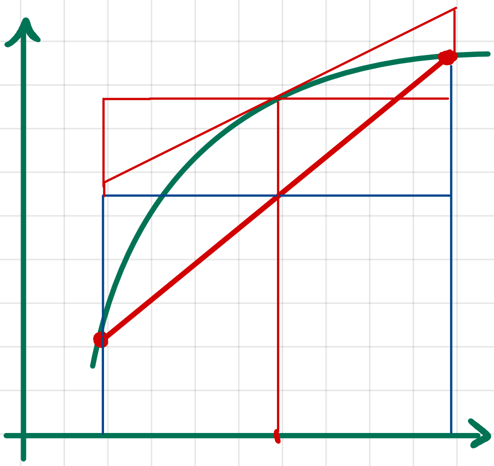
$\approx \frac{1}{2} \cdot (0 + 2\cdot 1 + 2\cdot 2 + 2\cdot 3 + 4)$

$= 8$

$\int_0^4 x\,dx = \frac{1}{2}x^2\Big|_0^4 = \frac{1}{2}(16) = 8$

approx is exact because the trapezoidal does coincide with the actual function, which is linear like the top part of the trapezoid.

1.5 $\int_0^2 e^{-x}\,dx \quad n=4$



$T_n = \sum_{i=1}^{n} \frac{0.5}{2}(y_i + y_{i-1}) = \frac{1}{4}\left[e^{-0} + 2e^{-1} + 2e^{-2} + 2e^{-3} + e^{-4}\right] = 0.88260$

$S_n = \frac{0.5}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 4y_{n-1} + y_n)$

$= \frac{1}{6}(e^{-0} + 4e^{-1} + 2e^{-2} + 4e^{-3} + e^{-4}) = 0.8646$

True value

$-e^{-x}\Big|_0^2 = -\frac{1}{e^2} - (-1) = 1 - \frac{1}{e^2} = 0.864664$