Ex 6 $\sum_{n=1}^{\infty} \frac{2^{2n-1}}{3^n}$

$a_n = \frac{2^{2n}}{2 \cdot 3^n} = \frac{4^n}{2 \cdot 3^n} = \underset{a}{\frac{1}{2}} \cdot \left[\underset{r}{\frac{4}{3}}\right]^n$

geometric series $\sum_{n=1}^{\infty} ar^n$ w/ $r > 1$ $\Rightarrow$ divergent

$\frac{1}{2}\left[\frac{4}{3} + \frac{16}{9} + \frac{64}{27} + \cdots\right]$

Ex 8 $\frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6}$

$= \sum_{n=1}^{\infty} \left(\frac{5}{6}\right)^{2n} \cdot \frac{1}{6}$ geom series w/ $a = 1/6$, $r = \left(\frac{5}{6}\right)^2$

$= \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{\frac{1}{6}}{\frac{11}{36}} = \frac{36}{11} \cdot \frac{1}{6} = \frac{6}{11}$

Ex 9 $\sum_{n=1}^{\infty} (-1)^{n-1} n^2 = 1 - 4 + 9 - 16 + \cdots$

$a_n = (-1)^{n-1} n^2$ $\lim_{n \to \infty} a_n$ does not exist; a_n goes to $+\infty$ and $-\infty$, alternating.

1 $\sum_{n=1}^{\infty} \frac{1}{3^n} = \sum 1\left(\frac{1}{3}\right)^n = \frac{1}{(1-1/3)} = 3/2$

$a=1 \quad r=1/3$ of $\sum ar^n \Rightarrow$ converges

13 $\sum_{n=0}^{\infty} (-1)^n\left(\frac{3}{e}\right)^n = \sum\left(\frac{-3}{e}\right)^n = 1 - \frac{3}{e} + \frac{3^2}{e^2} - \frac{3^3}{e^3} + \frac{3^4}{e^4} - \frac{3^5}{e^5} + \frac{3^6}{e^6} + \cdots$

$= 1 + \left(\frac{3}{e}\right)\left(\frac{3}{e}-1\right) + \left(\frac{3}{e}\right)^3\left(\frac{3}{e}-1\right) + \left(\frac{3}{e}\right)^5\left(\frac{3}{e}-1\right) + \cdots$

$= 1 + \sum\left(\frac{3}{e}-1\right)\left(\frac{3}{e}\right)^{2n+1} = \sum_{n=1}^{\infty} \underbrace{\left(\frac{3-e}{e}\right)\left(\frac{3}{e}\right)}_{a} \underbrace{\left(\frac{9}{e^2}\right)^n}_{r>1} \Rightarrow$ divergent

24 $\sum_{n=0}^{\infty}\left(\frac{99}{100}\right)^n = 1 + \frac{99}{100} + \left(\frac{99}{100}\right)^2 + \cdots = \frac{1}{1-\frac{99}{100}} = \frac{1}{\frac{1}{100}} = 100$

32 $\sum_{n=1}^{\infty} \sin^n 1 = \sin 1 + \sin^2 1 + \sin^3 1 + \cdots$

$= \sin 1\,(1 + \sin 1 + \sin^2 1 + \cdots$

$a = \sin 1$

$r = \sin 1 < 1 \Rightarrow$ converges to $\frac{\sin 1}{1-\sin 1}$

38

a) $0.66666\ldots = \frac{6}{10} + \frac{6}{100} + \frac{6}{1000} + \cdots = \frac{6}{10}\left[1 + \frac{1}{10} + \frac{1}{100} + \cdots\right] = \sum_{n=0}^{\infty}\left(\frac{6}{10}\right)\left(\frac{1}{10}\right)^n = \frac{\frac{6}{10}}{1-1/10}$

$= \frac{6}{10}\cdot\frac{10}{9} = \frac{2}{3}$

41 $0.123123123 = \frac{1}{10} + \frac{2}{100} + \frac{3}{1000} + \frac{1}{10^4} + \frac{2}{10^5} + \frac{3}{10^6} + \cdots$

$= \left(\frac{1}{10}\right)\left(1 + \frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \cdots\right)$

$+ \left(\frac{2}{100}\right)\left(1 + \frac{1}{10^3} + \frac{1}{10^6} + \cdots\right)$

$+ \left(\frac{3}{1000}\right)\left(1 + 1/10^3 + 1/10^6 + \cdots\right)$

$= \sum_{n=0}^{\infty}\left(\frac{1}{10}\right)\left(\frac{1}{10^3}\right)^n + \sum_{n=0}^{\infty}\left(\frac{2}{100}\right)\left(\frac{1}{10^3}\right)^n + \sum_{n=0}^{\infty}\left(\frac{3}{1000}\right)\left(\frac{1}{10^3}\right)^n$

$= \frac{1/10 + 2/100 + 3/1000}{1 - 1/10^3} = \frac{(100+20+3)/\cancel{1000}}{\frac{999}{\cancel{1000}}} = \frac{123}{999}$

Alternatively,

$= \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \cdots$

$= \frac{123}{10^3}\left(1 + \frac{1}{10^3} + \cdots\right) = \sum_{n=0}^{\infty}\left(\frac{123}{10^3}\right)\left(\frac{1}{10^3}\right)^n = \frac{123/10^3}{1-1/10^3} = \frac{123}{999}$

47 $\sum_{n=1}^{\infty}\left(\frac{x-2}{3}\right)^n$

$\left|\frac{x-2}{3}\right|<1 \Rightarrow \frac{x-2}{3}<1 \Rightarrow x-2<3 \Rightarrow x<5$

$x \in (-1,5)$

$\frac{x-2}{3}>-1 \Rightarrow x-2>-3 \Rightarrow x>-1$

$= \sum_{n=0}^{\infty}\left(\frac{x-2}{3}\right)\left(\frac{x-2}{3}\right)^n = \frac{\frac{x-2}{3}}{1-\frac{x-2}{3}} = \frac{\frac{x-2}{3}}{\frac{3-x+2}{3}} = \frac{x-2}{5-x}$