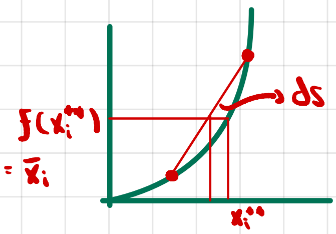
**Ex 4** $y = x^3 \quad x \in [0,2]$, revolution x-axis, surface area of rev.

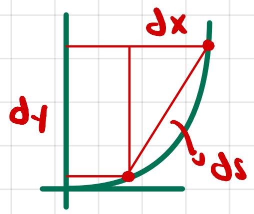



$$A = \int_0^2 2\pi y \, ds = \int_0^2 2\pi x^3 \sqrt{1+(3x^2)^2}\, dx = 2\pi \int_0^2 x^3 (1+9x^4)^{1/2} dx \qquad u = 9x^4 \quad du = 36x^3 dx$$

$$2\pi \int_0^{144} (1+u)^{1/2} \left(\frac{du}{36}\right) = \frac{\pi}{18} \int_0^{144} (1+u)^{1/2} du = \frac{\pi}{18} \cdot \frac{2}{3} (1+u)^{3/2} \Big|_0^{144} = \frac{\pi}{27} \left[ (145)^{3/2} - 1 \right]$$

**Ex 5**

$y = x^2$, $0 \le x \le \sqrt{2}$, revolution y-axis, surface area




$x = \sqrt{y}$, $dx/dy = 1/2\sqrt{y}$, $(dx/dy)^2 = 1/4y$

$ds = \sqrt{1+(dx/dy)^2} \cdot dy$

$$\int 2\pi x \, ds = \int_0^2 2\pi \sqrt{y} \sqrt{1+[1/2\sqrt{y}]^2}\, dy = 2\pi \int_0^2 \sqrt{y\left(1+\frac{1}{4y}\right)}\, dy = 2\pi \int_0^2 \sqrt{y + 1/4}\, dy$$

$$= \frac{2\pi (y+1/4)^{3/2}}{3/2} \Big|_0^2 = \frac{4\pi}{3} (y+1/4)^{3/2} \Big|_0^2 = \frac{4\pi}{3} \left[ \left(\frac{8}{4}+\frac{1}{4}\right)^{3/2} - \left(\frac{1}{4}\right)^{3/2} \right] = \frac{4\pi}{3} \left[ \left(\frac{3}{2}\right)^{3} - \left(\frac{1}{2}\right)^{3} \right]$$

$$= \frac{4\pi}{3} \left[ \frac{27}{8} - \frac{1}{8} \right] = \frac{\cancel{4}\pi}{3} \left[ \frac{\overset{13}{\cancel{26}}}{\cancel{8}_2} \right] = \frac{13\pi}{3}$$

**Alternatively,** $y = f(x) = x^2$

$$\int_0^{\sqrt{2}} 2\pi x \sqrt{1+(2x)^2}\, dx = 2\pi \int x \sqrt{1+4x^2}\, dx$$