Ex7 $a>0$ $\lim_{n\to\infty} \sqrt[n]{a}$

sequence: $\{\frac{1}{n}\}$ $\lim_{n\to\infty} \frac{1}{n} = 0$

$\Rightarrow \lim f(a_n) = \lim a^{1/n} = f(\lim a_n) = f(0) = a^0 = 1$

$f(x) = a^x$, continuous at $x=0$

Ex 5 $\lim_{n\to\infty} \frac{7n^2}{5n^2-3} = \lim \frac{7}{5-\frac{3}{n^2}} = \frac{\lim 7}{\lim(5-\frac{3}{n^2})} = \frac{\lim 7}{\lim 5 - \lim \frac{3}{n^2}} = \frac{7}{5-0} = \frac{7}{5}$

same series but now the num. and den. are each different: they converge so we can apply:

$\lim \frac{a_n}{b_n} = \frac{\lim a_n}{\lim b_n}$

$a_n = 5$ $\lim a_n = 5$
$b_n = -\frac{3}{n^2}$ $\lim b_n = 0$

$\left.\begin{matrix} \\ \end{matrix}\right\} \lim a_n + b_n = A+B$

Ex 8 $a_n = \frac{4n-1}{n+1}$, $\lim \sqrt{a_n} = ?$

$\lim a_n = \lim \frac{4-\frac{1}{n}}{1+\frac{1}{n}} = \frac{4}{1} = 4$ converges

quotient rule

$f(x) = \sqrt{x}$ continuous at $x=4 \Rightarrow \lim \sqrt{\frac{4n-1}{n+1}} = \sqrt{\lim a_n} = \sqrt{4} = 2$

Ex9 $|r|<1 \Rightarrow \lim_{n\to\infty} r^n = 0$

$\{r^n\} = r, r^2, r^3, r^4, \ldots$

$1, 1, 1, 1, \ldots$
$-1, 1, -1, 1, -1, \ldots$
$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$
$3, 9, 27, 81, \ldots$

$|r^n| = |(-r)^n|$

Assume $0<r<1$ $\frac{1}{r} = 1+a$ for some $a>0$

$\frac{1}{r^n} = (1+a)^n = 1+na+\{\text{positive terms}\} > 1+na$

$\binom{n}{0}1^n n^0 + \binom{n}{1}1^{n-1}n^1 + \ldots$

$\frac{n!}{(n-1)!\,1!} = n$

$0 < r^n < \frac{1}{1+na}$

$0^+ \quad \to 0$

by squeeze law,

$a_n = 0 < r^n < b_n = \frac{1}{1+na}$

$\lim a_n = \lim b_n = 0 \Rightarrow \lim r^n = 0$