exponential

$f(x) = a^x$

fixed base $\in \mathbb{R}^+$

power

fixed exponent

$f(x) = x^n$

logarithm

$y = \log_a x$ if $a^y = x$

domain of $\log_a x$ is a^y $y \in \mathbb{R}$, which is > 0.

$y = f(x) = \log_a x$

$x = f^{-1}(y) = a^y$

$y = a^x$ → given x obtain y through $f(x)$

$x = a^y \Rightarrow y = \log_a x$

↳ how do we get from y to x?

given x obtain y through $\log_a x$

$\log_a x + \log_a y = m$

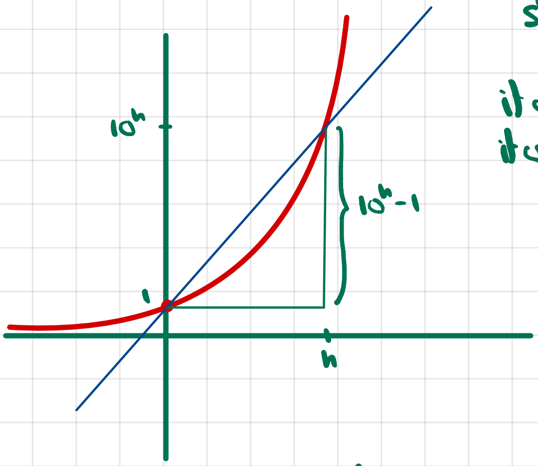
$a^{\log_a x + \log_a y} = a^m$

$a^{\log_a x} \cdot a^{\log_a y} = a^m$

$x \cdot y = a^m$

$\log_a xy = \log_a x + \log_a y$

$y=10^x$ $(0,1)$



$$\text{slope} = \lim_{h\to 0}\frac{10^h-1}{h} = \lim\left(\frac{10^h}{h}-\frac{1}{h}\right)$$

if we tabulate values as $h\to 0$, we reach a constant times 10^0.
if we find the slope on any x:

$$\lim_{h\to 0}\frac{10^{(x+h)}-1}{h} = \lim_{h\to 0}\left(10^x\cdot\frac{10^h}{h}-\frac{1}{h}\right) = 10^x\lim\frac{10^h}{h} - \lim\frac{1}{h}$$

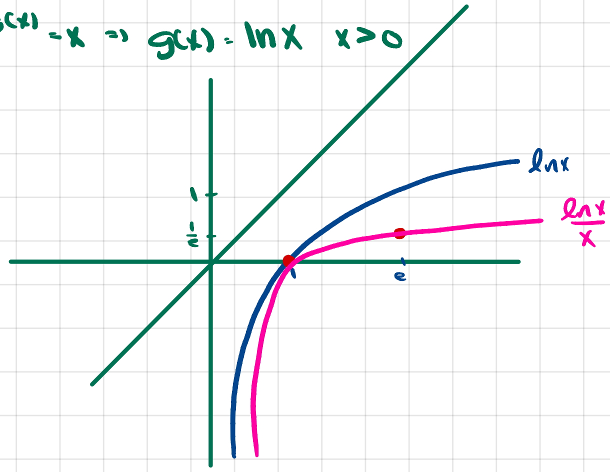
there is a particular choice of base, other than 10, for which the constant $\lim_{h\to 0}\frac{b^h}{h}$ is 1, which makes the slope b^x, the same as the original function. That choice is e.

Inverse of $f(x)=e^x$ $x\in\mathbb{R}$

f is differentiable on open interval $I=\mathbb{R}$, $f'(x)>0$ $\forall x\in I$ $\Rightarrow \exists f^{-1}(x)=g(x)$ differentiable and $g'(x) = 1/f'(g(x))$ $\forall x$ in domain of g

$g(x)=f^{-1}(x) \Rightarrow f(g(x))=x \Rightarrow e^{g(x)}=x \Rightarrow g(x)=\ln x$ $x>0$

$$g'(x) = \frac{1}{e^{\ln x}} = \frac{1}{x}$$



$$f(x) = \frac{\ln x}{x},\ x>0$$

$h(x)=\ln x$ $h'(x)=\frac{1}{x}$
$g(x)=x$ $g'(x)=1$

numerator $=\ln x < x =$ denom.
$f(x)<0$ $x<1$
≥ 0 $x\geq 1$

$$f'(x) = \frac{1}{x}\cdot\frac{1}{x} + \frac{\ln x(-1)}{x^2} = \frac{1-\ln x}{x^2}$$

$f'(x)=0 \Rightarrow \ln x = 1 \Rightarrow x=e$

$f(e)=\frac{1}{e}$

	$\frac{h'}{h} = \frac{1/x}{\ln x}$	$1/x$
1	$1/0 = \infty$	1
e	$\frac{1/e}{1}$	$1/e$
2e	$\frac{1/2e}{\ln 2}$	$1/2e$

Note

$g'(x)=1$
$h'(x)=1/x$ $\Rightarrow h'(x)>g'(x)$ when $\frac{1}{x}>1$, $x<1$ and $h'(x)<g'(x)$ when $x>1$

Intuition might be that since $f=\frac{h}{g}$, after $x=1$, g increases faster than h, therefore f would decrease. That is not how it works. Example:

$\frac{0}{10}\to\frac{1}{20}\to\frac{2}{30}\to\frac{3}{40}$
0 $1/20$ $1/15$ $\sim 1/13$
0.05

what matters is the proportional increase: how much the numer. and denom. are incremented in proportion to their current values

x	$h'(x)/h(x)$ $\frac{1}{x}/\ln x$		$g'(x)/g(x)$ $1/x$
1	$1/1/0=\infty$	$>$	1
e	$1/e$	$=$	$1/e$
2e	$1/2e(\ln 2+1)$	$<$	$1/2e$