4.9 More Indeterminate Forms

Ex 4 $\lim_{x\to 0} (\cos x)^{1/x^2} = 1^\infty$

$y = (\cos x)^{1/x^2}$

$\lim y = \lim \exp(\ln y) = \exp(\lim \ln y)$

$\lim \ln y = \lim_{x\to 0} \left[\frac{1}{x^2}\cdot \ln(\cos x)\right]$ ($\infty \cdot 0$) $= \lim_{x\to 0} \frac{\ln(\cos x)}{x^2} = \frac{0}{0} = \lim_{x\to 0} \frac{\frac{1}{\cos x}(-\sin x)}{2x} = \lim_{x\to 0} \frac{-\sin x}{2x\cos x} = \frac{0}{0}$

differentiable functions, denominator deriv. $\neq 0$ near $x = 0$

$= \lim_{x\to 0} \frac{-\cos x}{2\cos x - 2x\sin x} = \frac{-1}{2}$

$\Rightarrow \lim \ln y = -\frac{1}{2}$

$= -\frac{1}{2}$

$\lim y = \exp(-1/2) = \frac{1}{\sqrt{e}}$

Ex 5 $\lim_{x\to 0^+} x^{\tan x} = 0^0$

$y = x^{\tan x}$ $\ln y = \ln x^{\tan x} = \tan x \ln x$

$\lim_{x\to 0^+} \ln y = 0\cdot(-\infty) = \lim \frac{\tan x}{1/\ln x}$ $\frac{0}{0}$ $= \lim \frac{\sec^2 x}{\frac{-1}{x(\ln x)^2}} = \lim \frac{-1}{\cos^2 x}\cdot x(\ln x)^2 = \frac{0\cdot(-\infty)^2}{1}$

$= \lim \frac{\ln x}{1/\tan x} = \frac{-\infty}{\infty} = \lim \frac{1/x}{-\sec^2 x/(\tan x)^2}$

$= \lim \frac{1}{x}\cdot \frac{1}{\frac{-1}{\tan^2 x}\cdot\frac{\sec^2 x}{\tan^2 x}} = \lim \frac{1}{x}\cdot(-\sin^2 x) = \frac{0}{0} = \lim \frac{-2\sin x\cos x}{1}$

$= 2\cdot 0\cdot 1 = 0$

$\lim y = e^{\lim \ln y} = e^0 = 1$