6.2 Volumes by Method of Cross Sections

bounded solid region R, volume $V(R)$

R_x : (plane) cross section of R at x

Def : Volume by Cross Sections

R lies along interval $[a,b]$ on x-axis, has continuous cross-sectional area function $A(x)$ $\Rightarrow$ $\underbrace{V(R) = \int_a^b A(x)\,dx}_{\text{Cavalieri's Principle}}$

Solids of Revolution

Revolve around x-axis the region under the graph of $y=f(x)$ over the interval $[a,b]$, where $f(x) \geq 0$.
Each cross section of R at x is a circular disk of radius $f(x)$.

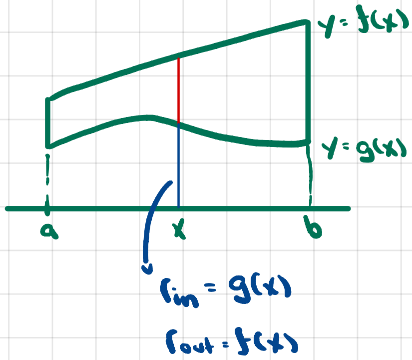
cross-sectional area function $= A(x) = \pi y^2 = \pi[f(x)]^2$

From Cavalieri, $V = \int_a^b \pi y^2\,dx = \int_a^b \pi f(x)^2\,dx$ = volume of solid of revolution around x axis

Revolution around the y-axis

$V = \int_c^d \pi \cdot x^2\,dy = \int_c^d \pi \cdot g(y)^2\,dy$ = volume of solid of revolution around y axis

Revolving Region Between Two Curves



cross-section at x : annular ring bounded by two circles

cross-sectional area

$A(x) = \pi[r_{out}^2 - r_{in}^2] = \pi[y_{top}^2 - y_{bottom}^2] = \pi[f(x)^2 - g(x)^2]$

$V = \int_a^b \pi[f(x)^2 - g(x)^2]\,dx$

Analogously, for $f(y) \geq g(y) \geq 0$ and $c \leq y \leq d$, we can get the volume of the solid in the revolved region between the two curves.

$V = \int_c^d \pi[f(y)^2 - g(y)^2]\,dy$