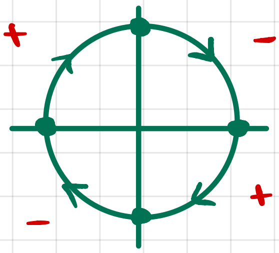
Derivatives of Parametric curves

dy/dx

Ex: $x = \sin\theta \qquad y = \cos\theta$



$$x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$$

$$y'(\theta) = -\sin\theta$$

$$x'(\theta) = \cos\theta$$

θ	x	y
0	0	1
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/2$	1	0
$3\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$
π	0	-1
$5\pi/4$	$-\sqrt{2}/2$	$-\sqrt{2}/2$
$3\pi/2$	-1	0

$$\frac{dy(\theta)}{d\theta} = \frac{dy(x(\theta))}{dx} \cdot \frac{dx}{d\theta}$$

$$\Rightarrow \frac{dy(x(\theta))}{dx} = \frac{dy(\theta)/d\theta}{dx/d\theta} = \frac{-\sin\theta}{\cos\theta} = -\frac{x}{y}$$

$$\frac{d^2y(\theta)}{d\theta^2} = \frac{d}{dx}\left[\frac{dy(\theta)}{d\theta}\right] \cdot \frac{dx}{d\theta}$$

$$\Rightarrow \frac{d}{dx}\left[\frac{dy(\theta)}{d\theta}\right] = \frac{\frac{d^2y(\theta)}{d\theta^2}}{dx/d\theta}$$

in the example

$$\frac{d^2y(\theta)}{d\theta^2} = \frac{d(-\sin\theta)}{d\theta} = -\cos\theta$$

$$\frac{d}{dx}\left[\frac{dy(\theta)}{d\theta}\right] = \frac{-\cos\theta}{\cos\theta} = -1$$

ex:

$x(t) = a(t - \sin t)$

$y(t) = a(1 - \cos t)$

$x'(t) = a(1 - \cos t)$

$y'(t) = a \sin t$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{\cancel{a}\sin t}{\cancel{a}(1-\cos t)} = \frac{\sin t}{1-\cos t}$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{\cos t(1-\cos t) - \sin t(-(-\sin t))}{(1-\cos t)^2} = \frac{\cos t(1-\cos t) - \sin^2 t}{(1-\cos t)^2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}(dy/dx) \Big/ dx/dt = \frac{\cos t(1-\cos t) - \sin^2 t}{(1-\cos t)^2} \cdot \frac{1}{a \sin t}$$