Section 2.2

Ex 10

$$\lim_{x\to 4} \sqrt[3]{3\sqrt{x^3}+20\sqrt{x}}$$

$f(x)=\sqrt[3]{x}$ $\quad g(x)=3\sqrt{x^3}+20\sqrt{x}$

$\lim_{x\to 4} f(g(x))$

$\lim_{x\to 4} g(4) = 3\cdot\lim\sqrt{x^3} + 20\cdot\lim\sqrt{x}$

$\swarrow$ $\qquad\qquad \searrow = \sqrt{\lim_{x\to 4} x} = \sqrt{4} = 2$

$h_1(x)=\sqrt{x}$

$h_2(x)=x^3$

$$\left.\begin{array}{l}\lim_{x\to 4} h_2(x) = 4^3 \\ \lim_{x\to 4^3} h_1(x) = \sqrt{4^3}\end{array}\right\} \Rightarrow \lim_{x\to 4} h_1(h_2(x)) = h_1(\lim_{x\to 4} h_2(x)) = \sqrt{4^3}$$

$= 3\sqrt{4^3} + 20\cdot 2 = 64$

$\lim_{x\to 64} f(x) = \sqrt[3]{64} = 4$

$\Rightarrow \lim_{x\to 4} f(g(x)) = f(\lim_{x\to 4} g(x)) = 4$

Ex 13 $f(x)=\sqrt{x}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

S 2.3

Ex 2 $\lim_{x\to 0} \frac{\tan 3x}{x} = 3\lim \frac{\tan 3x}{3x} = 3\lim \frac{\tan\theta}{\theta} = 3\lim \frac{\sin\theta}{\theta}\cdot\frac{1}{\cos\theta} = 3\cdot 1\cdot 1 = 3$

$\theta = 3x$

S.2.3

Ex 15 $\lim_{x\to 0} \sqrt[3]{x} = 0$

$|\sqrt[3]{x} - 0| < \varepsilon \Rightarrow |x| < \varepsilon^3$

$|x-0| < \delta = \varepsilon^3 \Rightarrow |f(x)-0| < \varepsilon \Rightarrow \lim_{x\to 0} \sqrt[3]{x} = 0$