7.4

ex 2 $\int \cot^2(3x)\,dx = \int (\csc^2(3x)-1)\,dx = \int (\csc^2(u)-1)\cdot(\tfrac{1}{3}du) = \tfrac{1}{3}(-\cot u - u) + C$

$u = 3x \quad du = 3dx$

$= -\tfrac{1}{3}(\cot(3x) - 3x) + C$

$$\frac{d}{dx}\cot x = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

Ex 3

a) $\int \sin^3 x \cos^2 x = \int \sin^2 x \cos^2 x \sin x\,dx = \int (1-\cos^2 x)\cos^2 x \sin x\,dx = -\int (1-u^2)u^2\,du = -\int (u^2 - u^4)\,du$

$= -\frac{u^3}{3} + \frac{u^5}{5} + C = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

b) $\int \cos^5 x = \int (\cos^2 x)^2 \cdot \cos x\,dx = \int (1-\sin^2 x)^2 \cos x\,dx = \int (1-u^2)^2\,du = \int (1 - 2u^2 + u^4)\,du$

$u = \sin x,\ du = \cos x\,dx$

$= u - 2\frac{u^3}{3} + \frac{u^5}{5} + C$

$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$

Ex 4: $\int \sin^2 x \cos^2 x\,dx = \int \frac{1}{2}(1-\cos 2x)\cdot\frac{1}{2}(1+\cos 2x)\,dx = \frac{1}{4}\int (1-\cos^2 2x)\,dx = \frac{1}{4}\int (1 - \frac{1}{2}(1+\cos(4x)))\,dx$

$= \frac{1}{4}\int \frac{1}{2}(1-\cos 4x)\,dx = \frac{1}{8}\cdot\left(x - \frac{\sin 4x}{4}\right) + C$

ex 5 $\int \cos^4 3x\,dx = \int (\cos^2 3x)^2 = \int \left[\frac{1}{2}(1+\cos(6x))\right]^2 dx = \frac{1}{4}\int (1 + 2\cos 6x + \cos^2 6x)\,dx$

$= \frac{1}{4}\int (1 + 2\cos 6x + \frac{1}{2}(1+\cos 12x))\,dx$

$= \frac{1}{4}\left[x + \frac{2\sin 6x}{6} + \frac{1}{2}x + \frac{1}{2}\cdot\frac{\sin(12x)}{12}\right] + C$

$= \frac{1}{4}\left[\frac{3x}{2} + \frac{\sin 6x}{3} + \frac{\sin 12x}{24}\right] + C$

$= \frac{3}{8}x + \frac{\sin 6x}{12} + \frac{\sin 12x}{96} + C$

1 $\int \sin^2 x\,dx = \int \frac{1}{2}(1-\cos(2x))\,dx = \frac{1}{2}x - \frac{1}{2}\sin(2x)\cdot\frac{1}{2} + C = \frac{x}{2} - \frac{\sin 2x}{4} + C$

2 $\int \cos^2 5x\,dx = \int \frac{1}{2}(1+\cos(10x))\,dx = \frac{1}{2}x + \frac{1}{2}\sin(10x)\cdot\frac{1}{10} + C = \frac{x}{2} + \frac{\sin(10x)}{20} + C$

3 $\int \sec^2(x/2)\,dx = \int \sec^2(u)\cdot 2\,du = 2\cdot\tan(x/2) + C$

$u = \frac{x}{2} \quad du = \frac{1}{2}dx$

4 $\int \tan^2(x/2)\,dx = \int (\sec^2(x/2) - 1)\,dx = 2\tan(x/2) - x + C$

$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| + C$

5 $\int \tan 3x\,dx = -\frac{\ln|\cos(3x)|}{3} + C$

6 $\int \cot 4x\,dx = \frac{\ln|\sin 4x|}{4} + C$

$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx = \ln|\sin x| + C$

7 $\int \sec 3x\,dx = \frac{\ln|\sec 3x + \tan 3x|}{3} + C$

* $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

8 $\int \csc 2x\,dx = -\frac{\ln|\csc 2x + \cot 2x|}{2} + C$

9 $\int \csc^{-2} x\,dx = \int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$

10 $\int \sin^2 x \cot^2 x\,dx = \int \sin^2 x(1-\csc^2 x)\,dx = \int (\sin^2 x - 1)\,dx = \frac{x}{2} - \frac{\sin 2x}{4} - x + C$

$= -\left[\frac{x}{2} + \frac{\sin 4x}{4}\right]$

11 $\int \sin^3 x\,dx = \int \sin x(1-\cos^2 x)\,dx = \int \sin x\,dx - \int \sin x\cos^2 x\,dx = -\cos x + \frac{\cos^3 x}{3} + C$

12 $\int \sin^4 x\,dx = \int (\sin^2 x)^2\,dx = \int \left[\frac{1}{2}(1-\cos 2x)\right]^2 = \int \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x)\,dx$

$= \frac{1}{4}\left(x - \sin 2x + \frac{\cos^3 2x}{3\cdot 2}\right) + C = \frac{1}{4}\left(x - 2\sin x\cos x + \frac{[\cos^2 x - \sin^2 x]^3}{6}\right)$

Alternatively, use reduction formula $\int \sin^n x\,dx = \frac{-\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$

$\int \sin^4 x\,dx = \frac{-\sin^3 x\cos x}{4} + \frac{3}{4}\int \sin^2 x\,dx$

$= \frac{-\sin^3 x\cos x}{4} + \frac{3}{4}\left[\frac{-\sin x\cos x}{2} + \frac{1}{2}x\right] = -\frac{1}{4}\sin^3 x\cos x - \frac{3}{8}\sin x\cos x + \frac{3}{8}x + C$