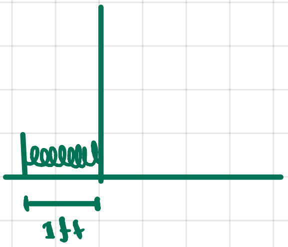
6.5 Force and Work

example 2

Force to hold spring at position x

Hooke's Law: $F(x) = kx$, $k>0$, the spring constant



$F(0.5) = 10 = k \cdot 0.5 \Rightarrow k = 20$

$F(x) = 20x$

$$\int_0^1 20x\,dx = 20\frac{x^2}{2}\Big|_0^1 = 10x^2\Big|_0^1 = 10 \text{ lb}\cdot\text{ft}$$

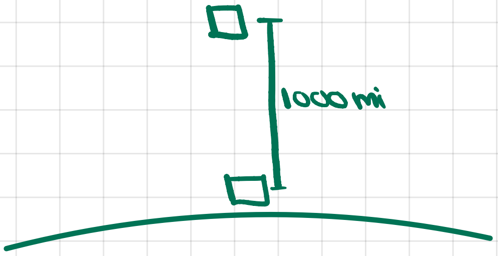
Example 3

force required to hold a body at a distance r from the center of the earth

$F(r)$ = holding force

$= \dfrac{k}{r^2}$, $k>0$

For $r = R \approx 4000$ mi (~ 6370 km), F is called the weight of the body.



□ 1000 lb

$F(r) = \dfrac{k}{r^2} \Rightarrow k = F(r)\cdot r^2 \Rightarrow k = F(R)\cdot R^2 = 1000 \cdot 4000^2 = 16\cdot 10^9 \text{ mi}^2\cdot\text{lb}$

$$W = \int_{4000}^{5000} \frac{16\cdot 10^9}{r^2}\,dr = \frac{16\cdot 10^9 r^{-1}}{-1}\Big|_{4000}^{5000} = -16\cdot 10^9\left(\frac{1}{5000} - \frac{1}{4000}\right) = \frac{16\cdot 10^9}{2\cdot 10^4} = 8\cdot 10^5 \text{ mi}\cdot\text{lb}$$

$8\cdot 10^5$ lb·mi · 5280 ft/mi = $4.224\cdot 10^9$ lb·ft

Rocket generates force. But at what rate? Say the journey to orbit takes 15 minutes. The average rate of work (ie the average power) is:

$\dfrac{4.224\cdot 10^9 \text{ lb}\cdot\text{ft}}{15 \text{ min}} = 0.2816\cdot 10^9$ lb·ft/min. Given hp is 33000 ft lb/min, this is

$8.53\cdot 10^3$ hp. However, the satellite is a small piece of the launched apparatus. Only 2% of the power lifts the satellite, the rest lifts the rocket and fuel.

$8.53\cdot 10^3$ 2%

X   100%

$X = 50 \cdot 8.53\cdot 10^3 \approx 427000$ hp

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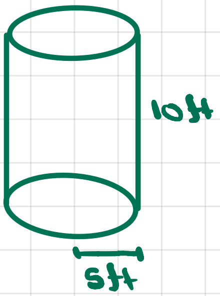

Water pumped in from ground level

$\rho = 62.4$ lb/ft^3, weight density of water

Work to fill tank?

$$\int_0^{10} \overset{\text{height}}{y}\,\underbrace{(\rho\,\underbrace{\pi r^2 dy}_{\text{volume}})}_{\text{mass in lbm = force in lbf}} = 62.4 \cdot \pi \cdot 25 \int_0^{10} y\,dy = 1560\pi \cdot \frac{y^2}{2}\Big|_0^{10} = 780\pi \cdot 100 = 78000\pi \text{ ft} \cdot \text{lbf}$$

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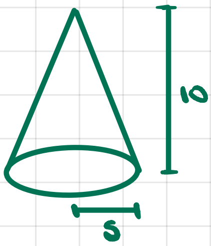
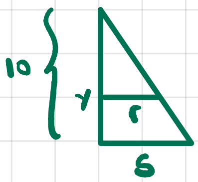


$$\int_0^{10} y\rho\pi\left(\frac{10-y}{2}\right)^2 dy = \frac{62.4\pi}{4}\int_0^{10}(100-20y+y^2)\,y\,dy$$

$$= \frac{62.4\pi}{4}\left[100y^2/2 - 20y^3/3 + y^4/4\right]_0^{10}$$



$$\frac{10-y}{10} = \frac{r}{5}$$

$$\Rightarrow r = \frac{(10-y)}{2}$$