6.3 Volumes by Method of Cylindrical Shells

second way of computing volumes of solids of revolution, using thin cylindrical shells

cylindrical shell: region bounded by two concentric circular cylinders of the same height h

$V = \pi r_2^2 h - \pi r_1^2 h$

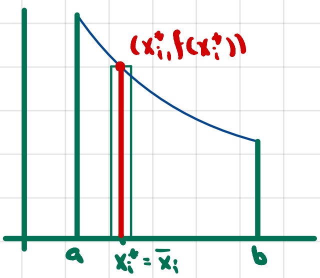
we can also write this as:

average radius $= \dfrac{r_1 + r_2}{2}$

$$\Rightarrow V = \left[2\pi \left(\frac{r_1 + r_2}{2}\right)\right] \cdot (r_2 - r_1) \cdot h = 2\pi \bar{r} \cdot t h$$

average thickness $= t = r_2 - r_1$

General Formula



ΔV_i: solid obtained revolving around y-axis the region under $f(x)$ between x_{i-1} and x_i.
approx equal to approximation with rectangles are revolved around y-axis

$\Delta V_i \approx 2\pi \bar{x}_i f(\bar{x}_i) \Delta x$

$V = \sum_{i=1}^{n} \Delta V_i \approx \sum_{i=1}^{n} 2\pi \bar{x}_i f(\bar{x}_i) \Delta x$

Riemann Sum, as $n \to \infty$ we obtain $\int_a^b 2\pi x f(x)\, dx$