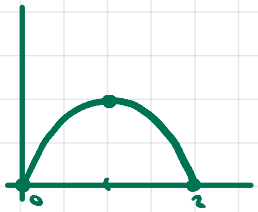
Pset 7

4B-2

e) $y = 2x - x^2$, $y = 0$, y-axis rotation



$2x - x^2 = 0 \Rightarrow x(2-x) = 0 \Rightarrow x = 0$ or $x = 2$

$y' = 2 - 2x = 0 \Rightarrow x = 1$

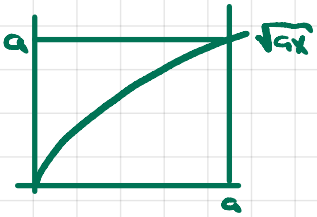
$y(1) = 2 \cdot 1 - 1 = 1$

Volume by shells

$$\int_0^2 2\pi \cdot x \cdot (2x - x^2)\,dx = 2\pi \int_0^2 (2x^2 - x^3)\,dx = 2\pi \left(\frac{2}{3}x^3 - \frac{x^4}{4}\right)\Big|_0^2 = 2\pi\left[\left(\frac{2 \cdot 8}{3} - \frac{16}{4}\right) - (0-0)\right]$$

$$= 2\pi \cdot \left(\frac{4 \cdot 16}{12} - \frac{3 \cdot 16}{12}\right) = 2\pi \cdot \frac{16}{12} = \frac{8\pi}{3}$$

g) $y = \sqrt{ax}$, $y = 0$, $x = a$



Shells

$$\int_0^a 2\pi x \sqrt{ax}\,dx$$

Slices, washers

$\sqrt{ax} = a \Rightarrow ax = a^2 \Rightarrow x = a \Rightarrow y = a$

$$\int_0^a \pi\left(a^2 - (y^2/a)^2\right)dy$$

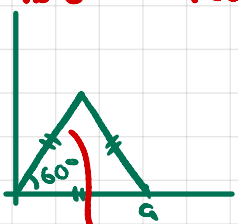
$y = \sqrt{ax} \Rightarrow y^2 = ax \Rightarrow x = y^2/a$

$$= \pi\left[a^2 y - \frac{1}{a^2} \cdot \frac{y^5}{5}\right]\Big|_0^a$$

$$= \pi\left[a^3 - \frac{a^5}{a^2 \cdot 5}\right] = \pi \cdot \frac{5a^3 - a^3}{5} = \frac{4\pi a^3}{5}$$

4B-5

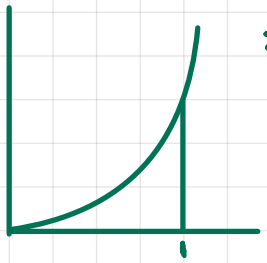
Revolution around x-axis, disks



$y = \tan(\pi/3) \cdot x = \sqrt{3}x$

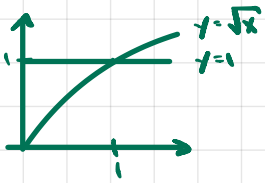
$$2 \cdot \int_0^{a/2} \pi \cdot (\sqrt{3}x)^2\,dx = 2 \cdot \int_0^{a/2} 3\pi x^2\,dx = 6\pi \cdot \frac{x^3}{3}\Big|_0^{a/2} = 2\pi \cdot \left(\frac{a}{2}\right)^3 = \frac{2\pi a^3}{8} = \frac{\pi a^3}{4}$$

4C-2 $0 \le y \le x^2$, $x \le 1$, revolution around y-axis

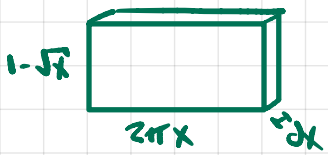


Shells: $\int_0^1 2\pi x \cdot x^2 \, dx = 2\pi \frac{x^4}{4} \Big|_0^1 = \frac{\pi}{2}$

4C-3 $\sqrt{x} \le y \le 1$, $x \ge 0$, revolution around y-axis



Shells



$$V = \int_0^1 2\pi x(1-\sqrt{x})\,dx = \int_0^1 2\pi(x - x^{3/2})\,dx = 2\pi\left(\frac{x^2}{2} - \frac{x^{5/2}}{5/2}\right)\Big|_0^1$$

$$= 2\pi\left(\frac{1}{2} - \frac{2}{5}\right) = 2\pi\left(\frac{5-4}{10}\right) = \frac{2\pi}{10} = \frac{\pi}{5}$$

Disks

$$V = \int_0^1 \pi y^4 \, dy = \pi \frac{y^5}{5} \Big|_0^1 = \frac{\pi}{5}$$

4J-3 The pool has a volume. Each "shell" of circles, centered in the middle of the pool with radius r_i has a volume approx. equal to $2\pi r_i D \, dr = dV_i$

$$V \approx \sum_{i=1}^{n} 2\pi r_i D \, dr$$

The amount of chemical in each dV_i is $dC_i = dV_i \cdot \frac{k}{1+r_i^2}$

The amount in the pool is $C \approx \sum_{i=1}^{n} 2\pi D k \frac{r_i}{1+r_i^2} dr$, a Riemann sum for the integral $\int_0^R 2\pi D k \frac{r}{1+r^2} dr$

$= \cancel{2}\pi k D \ln(1+r^2) \cdot \frac{1}{\cancel{2}} \Big|_0^R$

$= \pi k D \ln(1+R^2)$