Pset 8

4D-2 Avg value of $\frac{1}{x}$ over interval $[a, 2a]$ is of form $\frac{c}{a}$, $a>0$, C constant.

$f(x) = \frac{1}{x}$

$\bar{f} = \frac{1}{2a-a}\int_a^{2a} \frac{1}{x}\,dx = \frac{1}{a}\cdot \ln x \Big|_a^{2a} = \frac{1}{a}(\ln 2a - \ln a) = \frac{1}{a}\cdot \ln 2 = \frac{c}{a}, c = \ln 2$

4D-3

$x = s(t)$, time interval $[a,b]$

$v(t) = s'(t)$

average velocity on $[a,b] = \frac{s(b)-s(a)}{b-a}$

average value of velocity on $[a,b] = \bar{v} = \frac{1}{b-a}\int_a^b v(t)\,dt = \frac{s(b)-s(a)}{b-a}$

4D-5 Average value of $f(t)$ between 0 and x is $g(x)$

Avg value $F(t) = \frac{1}{x}\int_0^x f(t)\,dt = g(x)$

$g'(x) = \frac{-1}{x^2}\int_0^x f(t)\,dt + \frac{1}{x}f(x) = -\frac{1}{x}\cdot g(x) + \frac{1}{x}f(x)$

$\Rightarrow f(x) = g(x) + xg'(x)$