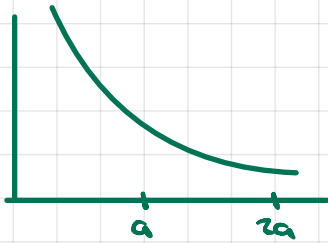


Pset 8

4D-2 Avg value of $\frac{1}{x}$ over interval $[a, 2a]$ is of form $\frac{C}{a}$, $a > 0$, C constant.



$$f(x) = \frac{1}{x}$$

$$\bar{f} = \frac{1}{2a-a} \int_a^{2a} \frac{1}{x} dx = \frac{1}{a} \cdot \ln x \Big|_a^{2a} = \frac{1}{a} (\ln 2a - \ln a) = \frac{1}{a} \cdot \ln 2 = \frac{C}{a}, C = \ln 2$$

4D-3

$x = s(t)$, time interval $[a, b]$

$$v(t) = s'(t)$$

$$\text{average velocity on } [a, b] = \frac{s(b) - s(a)}{b - a}$$

$$\text{average value of velocity on } [a, b] = \bar{v} = \frac{1}{b-a} \int_a^b v(t) dt = \frac{s(b) - s(a)}{b-a}$$

4D-5 Average value of $f(t)$ between 0 and x is $g(x)$

$$\text{Avg value } F(x) = \frac{1}{x} \int_0^x f(t) dt = g(x)$$

$$g'(x) = \frac{-1}{x^2} \int_0^x f(t) dt + \frac{1}{x} f(x) = -\frac{1}{x} \cdot g(x) + \frac{1}{x} f(x)$$

$$\Rightarrow f(x) = g(x) + xg'(x)$$