

Ex 4 $\int_0^2 \frac{1}{(2x-1)^{2/3}} dx = \int_0^{1/2} \frac{1}{(2x-1)^{2/3}} dx + \int_{1/2}^2 \frac{1}{(2x-1)^{2/3}} dx$

$f(x)$ undefined at $x = 1/2 \Rightarrow f$ not cont on $[0, 2] \Rightarrow$ not integrable via FTC

$$\int_0^{1/2} \frac{1}{(2x-1)^{2/3}} dx = \lim_{t \rightarrow 1/2^-} \int_0^t \frac{1}{(2x-1)^{2/3}} dx = \lim_{t \rightarrow 1/2^-} \int_0^t \frac{1}{2} \cdot u^{-2/3} du = \lim_{t \rightarrow 1/2^-} \left[\frac{1}{2} \cdot u^{1/3} \cdot 3 \right]_{-1}^{2t-1}$$

$$u = 2x-1 \quad du = 2dx$$

$$= \lim_{t \rightarrow 1/2^-} \left[\frac{3}{2} (2t-1)^{1/3} - \frac{3}{2} \cdot (-1) \right] = 0 + \frac{3}{2} = \frac{3}{2}$$

$$\begin{aligned} \int_{1/2}^2 \frac{1}{(2x-1)^{2/3}} dx &= \lim_{t \rightarrow 1/2^+} \int_t^2 \frac{1}{(2x-1)^{2/3}} dx = \lim_{t \rightarrow 1/2^+} \left[\frac{3}{2} (2x-1)^{1/3} \right]_t^2 = \lim_{t \rightarrow 1/2^+} \left[\frac{3}{2} (3^{1/3} - (2t-1)) \right] \\ &= \frac{3^{4/3}}{2} - 0 = \frac{3}{2} \cdot \sqrt[3]{3} \end{aligned}$$

$$\Rightarrow \int_0^2 \frac{1}{(2x-1)^{2/3}} dx = \frac{3}{2} (1 + \sqrt[3]{3})$$