Chain Rule - Preliminary Proof

given

$y = f(u)$

$u = g(x)$

we want to compute

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(g(x+\Delta x)) - f(g(x))}{\Delta x}$$

can we write $\Delta y / \Delta x$ as $\frac{\Delta y}{\Delta u} \frac{\Delta u}{\Delta x}$ with

$\Delta u = g(x+\Delta x) - g(x)$

$\Delta y = f(u+\Delta u) - f(u)$

and take the limit as $\Delta x \to 0$?

we must show that $\Delta u \neq 0$ as $\Delta x \to 0$ in $\frac{\Delta y}{\Delta u}$.

If $u' = g'(x) = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} \neq 0$ then we know that $\Delta u \neq 0$ if $\Delta x \to 0$

But we assumed g is differentiable (therefore continuous) at x, and therefore

(... did not understand ; p 133 Penney)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta u} \cdot \frac{\Delta u}{\Delta x} \right) = \lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{dy}{du} \cdot \frac{du}{dx}$$