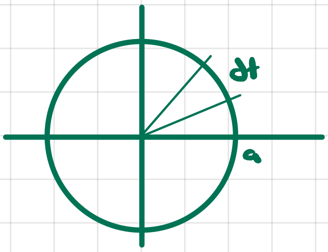
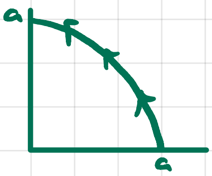
Ex1: $x = a\cos t$, $y = a\sin t$ $\quad t \in [0, 2\pi]$

a) Area



First quadrant area



$x(0) = a \quad y(0) = 0$
$x(\pi/2) = 0 \quad y(\pi/2) = a$

$\int_a^b y\,dx$, traditional area under curve. $= \int_{\pi/2}^{0} a\sin t(-a\sin t)\,dt$

$y = a\sin t \qquad x = a\cos t$
$\qquad\qquad\qquad dx = -a\sin t\,dt$

$$\int a\sin t(-a\sin t)\,dt = -a^2\int \sin^2 t\,dt = -a^2\int \tfrac{1}{2}(1-\cos 2t)\,dt = -\tfrac{1}{2}a^2\int dt + \tfrac{1}{2}a^2\int \cos 2t\,dt$$
$$= -\tfrac{1}{2}a^2 t + \tfrac{1}{2}a^2\cdot\tfrac{1}{2}\sin 2t = \tfrac{1}{2}a^2\left[\tfrac{1}{2}\sin 2t - t\right]$$

now consider the limits of integration.

we have $t \in [0, 2\pi]$, but $\int_0^{\pi/2} y\,dx = \tfrac{1}{2}a^2\left[\left(\tfrac{1}{2}\cdot 0 - \tfrac{\pi}{2}\right) - (0 - 0)\right]$
$$= \frac{a^2}{2}\cdot\left(-\frac{\pi}{2}\right) = -\frac{\pi a^2}{4}$$

negative because we must use limits of int. on t that correspond to the increase val in yx coord, ie from $\pi/2$ to 0.

b) Volume rev. x-axis, cross-sections

Traditional Formula: $\int_{\pi/2}^{0} \pi y^2\,dx = \int_{\frac{\pi}{2}}^{0} \pi a^2\sin^2 t(-a\sin t\,dt) = -\pi a^3\int_{\pi/2}^{0}\sin^2 t\,dt$

$$= -\frac{\pi a^3}{2}\int_{\pi/2}^{0}\sin^3 t\,dt = -\frac{\pi a^3}{2}\int_{\frac{\pi}{2}}^{0}(1-\cos^2 t)\sin t\,dt = -\frac{\pi a^3}{2}\left[-\cos t - \frac{\cos^3 t}{3}\right]\Bigg|_{\frac{\pi}{2}}^{0}$$
$$= -\frac{\pi a^3}{2}\left[\left(-1 - \frac{1}{3}\right) - (0 - 0)\right] = \frac{4}{3}\pi a^3\cdot\frac{1}{2}$$

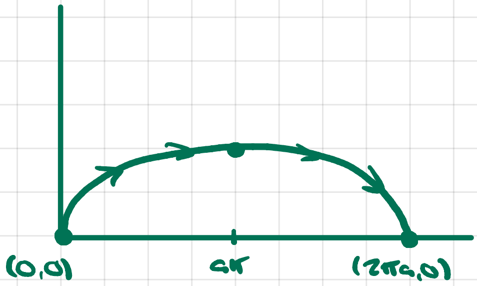
half a sphere

Ex 2:

$x = a(t-\sin t)$ $\quad y = a(1-\cos t)$ $\quad t \in [0, 2\pi]$

Area under curve

$dx = a(1-\cos t)dt$



t	x	y
0	0	0
$\frac{\pi}{2}$	$a(\frac{\pi}{2}-1) = \frac{a(\pi-2)}{2}$	$a(1) = a$
π	$a(\pi - 0) = a\pi$	$a(1-(-1)) = 2a$
2π	$a(2\pi - 0) = a \cdot 2\pi$	$a(1-1) = 0$

$\int \cos^2 t\, dt = \frac{1}{2}\int (1+\cos 2t)dt = \frac{1}{2}\left(t + \frac{\sin 2t}{2}\right)$

$$A = \int_0^{2\pi} a(1-\cos t)\cdot a(1-\cos t)dt = a^2\int_0^{2\pi}(1-2\cos t+\cos^2 t)dt = a^2\left(t - 2\sin t + \frac{1}{2}t + \frac{1}{2}\sin t\cos t\right)\Big|_0^{2\pi}$$

$$= a^2\left[\left(\frac{3}{2}\cdot 2\pi - 2\cdot 0 + 0\right) - \left(\frac{3}{2}\cdot 0 - 2\cdot 0 + 0\right)\right] = 3\pi a^2$$

Arc length

$x'(t) = a(1-\cos t)$

$y'(t) = a\sin t$

$ds = \sqrt{a^2(1-\cos t)^2 + a^2\sin^2 t}\; dt$

$$\int_0^{2\pi}\sqrt{a^2[(1-\cos t)^2+\sin^2 t]}\,dt = \int_0^{2\pi} a\sqrt{1-2\cos t + 1}\; dt = \int_0^{2\pi} a\sqrt{2(1-\cos t)}\; dt = \int_0^{2\pi} a\cdot\sqrt{4\cdot\frac{1}{2}(1-\cos(2\cdot\frac{t}{2}))}\; dt$$

$$= \int_0^{2\pi} a\cdot 2\cdot\sqrt{\sin^2(t/2)}\; dt = \int_0^{2\pi} 2a\sin(t/2)dt = 2a\cdot(-\cos(t/2)\cdot 2)$$

$$= -4a\cos(t/2)\Big|_0^{2\pi} = -4a[\cos\pi - \cos 0] = -4a(-1-1) = 8a$$