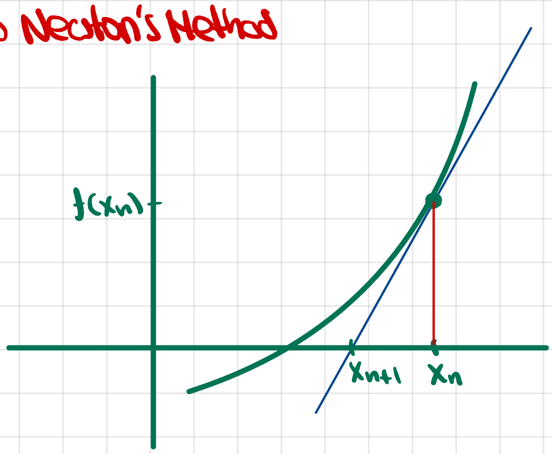
# 3.10 Newton's Method




$$f'(x_n) = \frac{f(x_n) - 0}{x_n - x_{n+1}} \Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ , iterative formula of Newton's method

Example Find $\sqrt{2}$.

$$f(x) = x^2 - 2 = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$$

numerical approx. to the roots using Newton's method means iteratively moving from "guess" to "guess" of what actual number $x = +\sqrt{2}$ or $x = -\sqrt{2}$ actually is.

$$f'(x) = 2x$$

$$x_{n+1} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{2x_n^2 - x_n^2 + 2}{2x_n} = \frac{x_n^2 + 2}{2x_n} = \frac{1}{2}x_n + \frac{1}{2}\frac{2}{x_n} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$$

$$x_0 = 1 \Rightarrow x_1 = 1 - \frac{1-2}{2} = \frac{3}{2} \Rightarrow x_2 = \frac{3}{2} - \frac{\frac{9}{4} - \frac{8}{4}}{3} = \frac{3}{2} - \frac{1}{4 \cdot 3} = \frac{18-1}{12} = \frac{17}{12} \Rightarrow x_3 = \frac{1}{2}\left(\frac{17}{12} + \frac{2 \cdot 12}{17}\right)$$

$$= 577/408$$