1 $\int_1^4 \sqrt{t}\,\ln t\,dt = \frac{2}{3}t^{3/2}\cdot\ln t\Big|_1^4 - \underbrace{\int_1^4 \frac{2}{3}t^{1/2}dt}_{\frac{2}{3}\cdot\frac{2}{3}t^{3/2}\Big|_1^4} =$

$u = \ln t \quad du = \frac{dt}{t}$

$dv = \sqrt{t}\,dt \qquad = \left[\frac{2}{3}\sqrt{4^3}\cdot\ln 4\right] - 0 - \frac{4}{9}\left[\sqrt{4^3}-1\right] = \frac{2\cdot 2^3}{3}\cdot\ln 4 - \frac{4\cdot 2^3}{9} + \frac{4}{9}$

$v = \frac{2}{3}t^{3/2}$

$= \frac{4}{9}\left[4\ln 4 - 8 + 1\right] = \frac{4}{9}\left[4\ln 4 - 7\right]$

2 $\int_0^{\pi/4} \tan^4\theta\sec^6\theta\,d\theta$

$= \int_0^{\pi/4} \tan^4\theta\sec^4\theta\sec^2\theta\,d\theta = \int_0^{\pi/4}\tan^4\theta(1+\tan^2\theta)^2\sec^2\theta\,d\theta$

$u = \tan\theta \quad du = \sec^2\theta\,d\theta$

$= \int u^4(1+u^2)^2du = \int u^4(1+2u^2+u^4)du = \int(u^4+2u^6+u^8)du$

$= \frac{u^5}{5} + \frac{2u^7}{7} + \frac{u^9}{9} = \frac{\tan^5\theta}{5} + \frac{2\tan^7\theta}{7} + \frac{\tan^9\theta}{9}\Big|_0^{\pi/4}$

$= \left[\frac{1}{5}+\frac{2}{7}+\frac{1}{9}\right] = \frac{188}{315}$

3 $\int \frac{10}{(x-1)(x^2+9)}\,dx = \int \left[\frac{A}{x-1} + \frac{Bx+C}{x^2+9} \right] dx$

$10 = A(x^2+9) + (Bx+C)(x-1)$

$x=1 \Rightarrow 10A = 10 \Rightarrow A = 1$

$10 = x^2+9 + Bx^2 - Bx + Cx - C$

$= x^2(B+1) + x(C-B) + 9 - C$

$B+1 = 0 \Rightarrow B = -1$

$C - B = 0 \Rightarrow C = B = -1$

$9 - C = 10 \Rightarrow C = -1$

$\Rightarrow \int \left[\frac{1}{x-1} + \frac{-x-1}{x^2+9} \right] dx = \ln|x-1| - \frac{1}{2}\ln|x^2+9| + \frac{1}{3}\ln\left|\frac{3+x}{\sqrt{9-x^2}}\right|$

$\int \frac{1}{x-1}\,dx = \ln|x-1| + C$

$-\int \frac{x+1}{x^2+9}\,dx = -\left[\int \frac{x}{x^2+9}\,dx + \int \frac{1}{x^2+9}\,dx \right] = -\left[\frac{1}{2}\ln|x^2+9| + \frac{1}{3}\ln\left|\frac{3+x}{\sqrt{9-x^2}}\right| \right]$

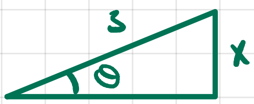
$\int \frac{x}{x^2+9}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|x^2+9|$

$u = x^2+9$
$du = 2x\,dx$

$\int \frac{1}{x^2+9}\,dx = \int \frac{3\cos\theta\,d\theta}{9\sin^2\theta+9} = \int \frac{\cancel{3}\cancel{\cos\theta}\,d\theta}{\cancel{9}_{3}\cos^{\cancel{2}}\theta} = \int \frac{1}{3}\cos^{-1}\theta\,d\theta = \frac{1}{3}\int \sec\theta\,d\theta = \frac{1}{3}\ln|\sec\theta + \tan\theta|$

$x = 3\sin\theta$
$dx = 3\cos\theta\,d\theta$

$= \frac{1}{3}\ln\left|\frac{3+x}{\sqrt{9-x^2}}\right|$



$\cos\theta = \frac{\sqrt{9-x^2}}{3} \Rightarrow \sec\theta = \frac{3}{\sqrt{9-x^2}}$

$\tan\theta = \frac{x}{\sqrt{9-x^2}}$

4 $\int \frac{1}{(5-4x-x^2)^{5/2}} dx$

$$= \int \frac{1}{[-(x^2+4x+4)+9]^{5/2}} dx = \int \frac{1}{[9-(x+2)^2]^{5/2}} dx$$

$x+2 = 3\sin\theta$

$dx = 3\cos\theta$

$$= \int \frac{3\cos\theta \, d\theta}{(9-9\sin^2\theta)^{5/2}} = \int \frac{3\cos\theta \, d\theta}{(9\cos^2\theta)^{5/2}} = \int \frac{3\cos\theta \, d\theta}{9^{5/2} \cdot \cos^5\theta}$$

$$= \int \frac{3 \, d\theta}{3^5 \cos^4\theta} = \frac{1}{3^4} \int \sec^4\theta \, d\theta$$

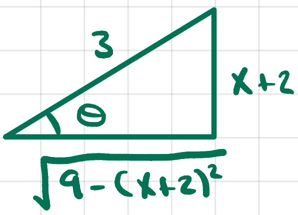
$$= \frac{1}{3^4} \int (1+\tan^2\theta)\sec^2\theta \, d\theta$$

$u = \tan\theta \quad du = \sec^2\theta d\theta$

$= \int u^2 \cdot du = \frac{u^3}{3} = \frac{\tan^3\theta}{3}$

$$= \frac{1}{3^4}\left[\int \sec^2\theta d\theta + \int \tan^2\theta \sec^2\theta d\theta\right]$$

$$= \frac{1}{3^4}\left[\tan\theta + \frac{\tan^3\theta}{3}\right] + C = \frac{1}{3^4}\left[\frac{x+2}{\sqrt{5-4x-x^2}} + \frac{(x+2)^3}{3\sqrt{5-4x-x^2}}\right] + C$$



$$\tan\theta = \frac{x+2}{\sqrt{5-4x-x^2}}$$

5

a) $x = y + y^3 \ , \ y \in [1,4]$

$x' = 1 + 3y^2$

Arc Length $= \int_1^4 \sqrt{1 + (1+3y^2)^2}\, dy$

b) $x = a\cos^3 t$

$y = a\sin^3 t$

$t \in [0, \pi/2]$

$ds^2 = dx^2 + dy^2$

$ds = \sqrt{dx^2 + dy^2} \Rightarrow ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$

$x'(t) = 3a\cos^2(t)(-\sin t)$

$y'(t) = 3a\sin^2(t)\cos t$

$\int_0^{\pi/2} 2\pi \cdot a\sin^3(t) \cdot \left[9a^2\left(\cos^4(t)\sin^2(t) + \sin^4(t)\cos^2(t)\right)\right]^{1/2} dt$

$= \int_0^{\pi/2} 2\pi a\sin^3(t) \cdot 3a\left[\sin^2 t\cos^2 t\left(\cos^2 t + \sin^2 t\right)\right]^{1/2} dt$

$= 6\pi a^2 \int_0^{\pi/2} \sin^3(t) \cdot \sin(t)\cos(t)\, dt = 6a^2\pi \int_0^{\pi/2} \sin^4(t)\cos(t)\, dt$