# 6.5 Force and Work

$W = F \cdot d$

work done by a constant force

Generalization: variable force given by force function $F(x)$

Context: $F(x)$ defined at each point on $[a,b]$

Force acts on particle moving it from a to b.

What is the work done by this force?

Partition $[a,b]$ into n subintervals of length $\frac{b-a}{n}$, $x_i^*$ is arbitrary point in $[x_{i-1}, x_i]$

$\Delta W_i \approx F(x_i^*) \cdot \Delta x$ = work to move particle from $x_{i-1}$ to $x_i$

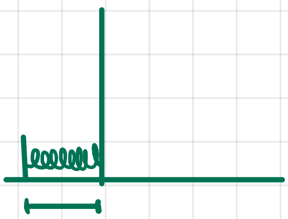
$W = \sum \Delta W_i \approx \sum_{i=1}^{n} F(x_i^*) \Delta x$

Riemann Sum

$W = \int_a^b F(x)\,dx$

Force to hold spring at position x

**Hooke's Law:** $F(x) = kx$, $k>0$, the spring constant



natural length

**Newton's Law of Gravitation**

force required to hold a body at a distance r from the center of the earth

$F(r)$ = holding force

$= \frac{k}{r^2}$, $k>0$

For $r = R \approx 4000$ mi ($\sim 6370$ km), F is called the weight of the body.

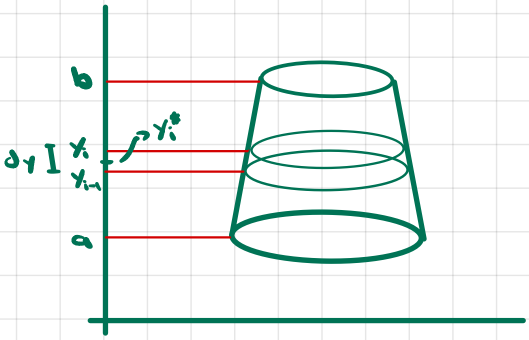
**Power**: rate at which work is done

1 hp defined as 33000 ft·lb/min

# Work Done in Filling a Tank

## constant force acting through different distances

$*$ 1 lb force accelerates a 1 lb mass at $9.80665\ m/s^2 = 32.174\ ft/s^2$




Tank filled in thin horizontal layers of fluid
Different layers lifted different distances

Average value theorem applied to $A(y)$
$A(y)$ contin. on $[y_{i-1}, y_i]$

$$\Rightarrow A(\bar{y}) = \frac{1}{y_i - y_{i-1}} \cdot \int_{y_{i-1}}^{y_i} A(y)\,dy$$

for some $\bar{y}$ in $[y_{i-1}, y_i]$

→ Partition $[a,b]$ into $n$ subintervals $\Delta y$ length.

→ Volume horizontal slice $= \Delta V_i = \int_{y_{i-1}}^{y_i} A(y)\,dy = A(y_i^*)\,dy$

→ $\rho$ = fluid density

→ force required to lift the slice from $y=0$ to final height, somewhere in $[y_{i-1}, y_i]$:

$$F_i = \rho \Delta V_i = \rho A(y_i^*)\Delta y$$

Technically this is actually:

Force in lbf $= (\rho \Delta V_i)$ lbm $\cdot\ 32.\cancel{174}\ ft/s^2 \cdot \dfrac{1\ lbf}{32.17\cancel{4}\ ft/s^2}$

$= \rho \Delta V_i$ lbf

→ work to lift slice $i$:

$$F_i y_{i-1} \le \Delta W_i \le F_i y_i$$

$$\rho\, y_{i-1} A(y_i^*)\Delta y \le \Delta W_i \le \rho\, y_i A(y_i^*)\Delta y$$

Add the work for each slice $i$:

$$\sum_{i=1}^{n} \rho\, y_{i-1} A(y_i^*)\Delta y \le W \le \sum_{i=1}^{n} \rho\, y_i A(y_i^*)\Delta y$$

these are Riemann sums.

Though $y_{i-1}$, $y_i^*$, and $y_i$ are not the same, both sums approach $\int_a^b \rho\, y A(y)\,dy$ as $\Delta y \to 0$

$W = \int_a^b \rho\, y A(y)\,dy$ by the squeeze law of limits